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Quark-Antiquark Binding Force in the Skyrme Model

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Abstract

In the Skyrme model, we can extract both a linear potential and a Coulomb potential between a quark and antiquark; a quark (antiquark) is identified with a part of a certain pion field configuration, which has baryon number $= 1/3 (-1/3)$. We find good agreements with the phenomenological values used in computations of the charmonium spectrum.

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Recently, the Skyrme model¹ has received much attention as a model of hadrons without quarks. The model is described simply by meson fields, but it can yield baryons^{1,2,3} as topological solitons. It has been recognized⁴ that the model gives reasonable values for quantities in hadron physics at low energies. Hence we may ask what the connection is between this model and QCD, and if we can find any relics of quarks in the model. Such a connection has been discussed in Ref. 5.

We shall show here that both a linear potential and a Coulomb potential between a quark and an antiquark can be identified in the Skyrme model. The coefficients in the linear potential (ar) and the Coulomb potential ($-b/r$) are compared with those⁶ used phenomenologically in a potential for charmonium. It turns out that in the Skyrme model we can get reasonable values for the potential between the quarks.

We consider the Skyrme model with $SU(2) \times SU(2)$ symmetry,

$$L = L_1 + L_2 \tag{1}$$

$$L_1 \equiv \frac{f_\pi^2}{16} \text{Tr}(\partial_\mu U \partial_\mu U^\dagger) \quad \text{and} \quad L_2 \equiv \frac{1}{32 e^2} \text{Tr}([\partial_\mu U U^\dagger, \partial_\nu U U^\dagger]^2)$$

with the pion decay constant $f_\pi = 186 \text{ MeV}$.

U is a $SU(2)$ matrix and the second term (L_2) is added to stabilize the skyrmion.^{1,2} For simplicity, we use the Skyrme's ansatz, $U = e^{iS(r)\vec{\sigma} \cdot \vec{n}}$ with $\vec{n} \equiv \vec{r}/r$ and $\vec{\sigma}$ the Pauli matrices. Then the Hamiltonian of the model becomes

$$\begin{aligned}
H = & \int_0^\infty \left(\frac{\pi}{2} f_\pi^2 r^2 + \frac{4\pi}{e^2} \sin^2 S \right) \left(\frac{dS}{dr} \right)^2 dr \\
& + \int_0^\infty \left(\pi f_\pi^2 \sin^2 S + \frac{2\pi \sin^4 S}{e^2 r^2} \right) dr
\end{aligned} \tag{2}$$

and the baryon number density is given by

$$\rho_B = \frac{-1}{\pi} \frac{d}{dr} \left(S - \frac{1}{2} \sin 2S \right) \tag{3}$$

In order to have finite energy, the field $S(r)$ must satisfy the constraints $S(0) = n\pi$ and $S(\infty) = m\pi$, where n and m are integers. These constraints lead to quantization of the baryon number. As is well known, the skyrmion is a soliton-like solution in this model and has baryon number 1. The physical space consists of states with integer baryon number.

Now, let us consider the following field configuration,

$$S(r) = S_0 [\theta(r - r_1) - \theta(r - r_2)] \tag{4}$$

with $r_2 > r_1$,

where $\theta(r)$ is the step function ($\theta(r) = 1$ for $r > 0$ and $\theta(r) = 0$ for $r < 0$) and S_0 is chosen to satisfy the condition,

$$\frac{1}{N} = \frac{1}{\pi} \left(S_0 - \frac{1}{2} \sin 2S_0 \right) \tag{5}$$

where N is identified with N of color gauge group of $SU(N)$. Evidently, the baryon number of this field configuration is localized at r_1 and r_2 , and

equals $\pm 1/N$, respectively. Furthermore, the first term of its energy in eq. (2), which is proportional to $\left(\frac{dS}{dr}\right)^2$, has dominant contributions from the field configuration at r_1 and r_2 (the step function $\theta(r)$ may be smoothed out so as not to produce singular terms in the energy). Hence, we are tempted to think that the field configuration (4) represents a quark at r_2 and an antiquark at r_1 , which interact with each other by the potential given in the second term of the energy (2). Indeed, this potential,

$$V = \pi f_{\pi}^2 \sin^2 S_0 (r_2 - r_1) + 2\pi \sin^4 S_0 \left(\frac{1}{r_1} - \frac{1}{r_2} \right) / e^2 \quad (6)$$

represents a linear potential and a Coulomb potential between the quarks. Note that the distribution of the quarks is localized on the spherical shells, and as $r_2 \gg r_1$, the part of the potential depending only on r_2 is just the potential expected for a quark at r_2 around the antiquark. The confining potential arises from L_1 and the Coulomb potential from L_2 , which was to be expected since L_1 and L_2 are relevant for phenomena with long and short wave length, respectively. Therefore, it is reasonable to interpret the field configuration (4) as a quark and an antiquark interacting with each other by the potential (6).

Obviously, we may consider other more complicated configurations which are not necessarily spherical. Then we will obtain corresponding potential energies of quarks whose wave functions have more complicated distributions. However, in those cases, it is more difficult to extract a potential energy between the quarks as point-like sources. Therefore, we have chosen the above configuration so as to determine the potential, although the configuration is unstable.

Although the quarks can be identified in the Skyrme model, they are not physical states. This circumstance resembles the situation in the bosonized form of the massive Schwinger⁷ model, which doesn't involve elementary fermions as physical states, and where we can identify confining forces between the fermions in a similar manner as stated above.

Numerically, the coefficients in the potential are

$$a \equiv \pi f_{\pi}^2 \sin^2 S_0 = 0.1 \text{ GeV}^2 \quad \text{and} \quad b \equiv 2\pi \sin^4 S_0 / e^2 = 0.31 \quad (7)$$

with $N = 3$ in eq. (5),

where we have used an upper bound on e^2 which has been derived by Balachandran, et al. using P wave $\pi - \pi$ scattering length.

Phenomenologically, a potential as in eq. (6) has been used in the calculation of the charmonium spectrum with the following values⁶ of

$$0.16 < a < 0.2 \text{ (GeV}^2\text{)} \quad \text{and} \quad 0.4 < b < 0.53 \quad (8)$$

We thus find that the Skyrme model yields reasonable values for the quark-antiquark potential (the phenomenological values (8) include effects of the S quark, but they are negligible for our accuracy).

Finally, we show that the behavior of the confining potential (ar) in the large-N limit. In this limit $f_{\pi}^2 \propto N$ and $S_0 \propto N^{-1/3}$, so that

$$a = \pi f_{\pi}^2 \sin^2 S_0 \propto N^{1/3} \quad .$$

This shows that a quark and an antiquark are strongly bound in the large N limit. Namely, the bound state of the quark and the antiquark (meson) is surrounded by an infinite barrier of the potential.

For lack of sufficient knowledge of the parameter e^2 and of terms with higher derivatives than L_2 in eq. (1), we can not infer the behavior of the potential inside the meson as $N \rightarrow \infty$.

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