

February 9, 2004
date last saved: 2/9/2004 11:00 AM
date last printed: 3/16/2004 4:23 PM

Quantifying Our Ignorance: Stochastic Forecasts of Population and Public Budgets

Ronald Lee
Demography and Economics
University of California
2232 Piedmont Ave.
Berkeley, CA 94720
rlee@demog.berkeley.edu

This paper was prepared for presentation at the Rand Summer Institute Gala Celebration for the NIA Centers for the Demography and Economics of Aging. Research for this paper was funded by a grant from NIA, R37-AG11761. Tim Miller made valuable contributions to the analysis. I am particularly grateful to my collaborators over the years in this work: first, Lawrence Carter and Shripad Tuljapurkar; and later, Tim Miller, Ryan Edwards, Mike Anderson, and Nan Li.

Introduction

A good deal of demographic research is justified on the grounds that it may lead to improved population forecasts. However, most researchers have never made a forecast, and actual projections are usually done by some form of trend extrapolation with little mention of theory. I believe that most demographers view forecasting as a mechanical and boring exercise, with little intellectual content. And indeed much forecasting, including good forecasts, and my own forecasts, could be fairly characterized in this way. But nonetheless forecasting is one of the most important tasks demographers perform, and it is important that it be done well and to high professional standards. We need forecasts to anticipate population aging, for example, and as inputs for economic, fiscal, environmental, and social service planning. And we need forecasts simply to be able to visualize our collective future. Some kinds of planning depend on demographic patterns many decades in the future, and because of the long term demographic consequences of current population age distribution, demographers can sometimes make useful predictions of these. However, like most kinds of forecasts, population forecasts often turn out to be quite mistaken. It is also a task of demographic forecasters to provide indications of the kinds of errors they may make, and the probabilities of these.

Demographers use accounting identities to translate assumptions about the time path of age specific fertility, mortality and migration into the future population sizes and age distributions they imply. However, demographic rates are only probabilities at the individual level. If a fertility rate for 27 year old women is .5 per year, that means there is a 50% probability that a particular woman will give birth within a year, and a 50% chance that she will not, and similarly for probabilities of death and survival, and migration. This intrinsic uncertainty at the individual level is diminished when we talk about larger groups of women, because it tends to average out, but it never disappears completely. Even if the true rates were known with certainty *ex ante* the outcomes could not be predicted with certainty. Early researchers looked here for the source of uncertainty in population forecasts. However, simple calculations showed that this source of uncertainty became vanishingly small in the larger populations of nations, yet forecasts for large populations were little more successful than for small ones. Then it was recognized that the main source of uncertainty was that the vital rates (that is, the probabilities) themselves change over time, and that we make errors in forecasting these changes (Sykes 1969).

Unfortunately, the analytic tools of pure demography are of little use for the task of forecasting changes in vital rates. The principle analytic technique available is to place current rates in a longer run context through skilled disaggregation – for example, by parity and length of open interval for fertility, or by cause of death for mortality. But disaggregations of this kind are useful mainly when the underlying, disaggregated probabilities are in fact unchanging, and the changes in outcomes are due to changing structures, that is to changing distributions of the population across the different relevant disaggregated categories of risk. However, there is an underlying weakness with this approach: If the distributions are not changing, then the disaggregation is not helpful. On the other hand, if the distributions are irregular and changing, this must reflect past variations in the disaggregated rates. But if the rates have been changing in the past, then

they are likely to change in the future, and we are back to the problem of trying to forecast their changes. Faced with the need to forecast the disaggregated rates, demographers tend to extrapolate their most recently observed levels. Thus for the most part, classic demographic methods are of limited use in a changing world.

For the past 35 years, I have been working on the problem of demographic forecasting under uncertainty. My goal has been partly intellectual, to figure out how to think about the processes at work. But it has also been to develop new forecasting methods that would be more than illustrative, and that might be good enough to inform real world planning.

In this paper, I will explain how this work developed over the years, what directions I tried, and why some failed and some succeeded. In short, I will describe my voyage of discovery. I will not try to provide a comprehensive overview of this large topic, which I attempted to do in Lee (1999) and Lee and Tuljapurkar (2000). However, I will give some indication of the contributions of others, and the directions they have taken.

Fundamental Issues

As a graduate student, my interest in forecasting stemmed from an attempt to understand how and why populations grew and declined, and how the process was related to economic change, age distribution, and accident. In thinking about these questions, I was troubled by a number of questions:

- 1) Theories of Malthus, Easterlin, environmentalists and others tell us that population size and age distribution should themselves have an influence on current vital rates, which implies that population processes should be subject to negative feedback. Yet in practice this is generally ignored in projections. What should we make of this?
- 2) Some analysts have viewed population change as an independent force, itself explaining economic and social changes. Others have viewed population change as endogenous, responding passively to economic change. How can we reconcile the interplay of random and systematic influences on vital rates and population growth?
- 3) How can variations over age and time in vital rates such as fertility or mortality be represented parsimoniously yet realistically?

However forecasting is actually carried out, it explicitly or implicitly involves deep theoretical and empirical assumptions and judgments. Can we ignore feedback? Do more people in the reproductive ages mean more births or fewer births? Will environmental pressures be increased by population growth, and in turn retard that growth? Will mortality rise as population density increases? Does a larger population lead to more rapid technological progress, or to poverty? If, in the end, most population forecasts say nothing about these questions, that reflects an assumption on the part of the forecasters that demographic trends can continue without encountering such feedback, not proof that such economic and environmental interactions with population don't matter. The potential role of such factors is discussed in Lee (1990) and Cohen (1995).

Modeling Variation in Demographic Rates Over Age and Time

There is a tradition in mathematical demography of modeling variation over age and time, closely related to the demographic tradition of constructing families of model age schedules for fertility, nuptiality and mortality. One approach involves fitting nonlinear parametric functions to the observed age schedules, and then letting some or all of the parameters vary over time. For example, Gompertz or gamma functions have been used for fertility, and the Heligman-Pollard nine parameter function has been used to extrapolate mortality rates (McNown and Rogers 1989). Another approach generates new age schedules by transforming an existing one, or a standard; this is sometimes called a relational approach. For example, one simple model assumes an equal additive change to mortality at every age, which is the simplest and most tractable model of all, but which fits actual change only poorly. Another example, this time more realistic but less tractable, assumes that mortality at every age changes by the same proportion. Still more realistic is Brass's logit transform.

The approach I developed was still very simple, but like the logit, both flexible and realistic. It lets each age specific rate have its own additive or multiplicative pattern of change, with the relative sizes of these changes fixed across age:

$$m(x,k) = a(x) + k*b(x)$$

Here $m(x,k)$ can represent either a vital rate at age x for parameter value k , or the logarithm of the rate for the multiplicative version. As k varies, the model generates a family of age schedules of the vital rates m or $\exp(m)$. The model can be fit to a matrix of historical rates varying over age and time, resulting in estimates of the $a(x)$, $b(x)$ schedules, and a time series of the parameter k , call it $k(t)$. We can hold $a(x)$ and $b(x)$ fixed, and then focus attention on analyzing, modeling, and forecasting $k(t)$ without having to worry about all the age specific details. The model has been used mainly for mortality, but also for fertility and migration.

This model also makes it easy to incorporate stochastic disturbances in a natural way, by treating k as a random variable. Then k , for fertility or mortality, can be projected using standard statistical methods, together with its probability distributions. From these, probability distributions for age specific fertility and mortality can be calculated. Other sources of uncertainty, resulting from imperfect fit of the model give above, or from uncertainty about the parameters of the time series models used to forecast k , can also be incorporated.

The Framework

Figure 1 gives a very large scale map of this voyage of discovery, showing fertility and mortality as processes to be modeled, then showing them as inputs to a stochastic population forecast, and finally the use of the stochastic population forecast as an input to stochastic fiscal projections of various kinds. The remainder of the paper takes a close

look at each of these legs of the journey, showing the blind alleys as well as the final route.

Forecasting Mortality: Development of the Lee-Carter Model

Since US life expectancy was widely believed to have reached its natural upper limit in the mid-1960s while I was a graduate student in Demography, I didn't pay much attention to it at the time. Subsequently, mortality began to decline rapidly, and mortality caused larger errors in Census Bureau projections in the 1970s than fertility, so the picture changed. In the late 1980s, Larry Carter spent a semester visiting at Berkeley. We had collaborated earlier on a model of population renewal based on birth-marriage and marriage-birth transitions. This time, we decided to try to use the simple age-time model I had developed for historical work to model and forecast mortality. The single parameter $k(t)$ indexed the intensity of mortality, and we estimated it for the US 1900 to 1989, as shown in Figure 2. Remarkably, the trajectory of $k(t)$ was quite linear, unlike life expectancy which rose at a slowing pace throughout the century. This linearity was striking, because it persisted through important changes like the development of antibiotics and the emergence of the AIDS epidemic. We modeled $k(t)$ as a random walk with drift, and forecasted it along with its probability distribution. From this, we derived the forecasts and probability distributions of age specific death rates and life expectancy. While we were developing this method, Rogers and McKnown (1989, 1992) were developing an approach based on fitting the Heligman-Pollard model mentioned above to the historical mortality data, and then using time series methods to forecast a subset of the estimated parameters. There were lively debates about the relative merits of the two methods. Figure 3 charts this and subsequent efforts to model and forecast mortality. The path ultimately followed is shown by the continuing main line, while the spurs branching off to the sides represent paths tried but abandoned.

Even before the article was published, Statistics Canada invited us to apply it to Canadian data, at Nathan Keyfitz's suggestion (Lee and Nault 1993), and they began to use it partially in their projections. Since then, the model has gradually gained acceptance, and is now also used in some respects by the US Census Bureau, Japan, and the United Nations. Tuljapurkar, Boe and Li (2001) applied it to the G7 countries, and found that as for the US, it predicted life expectancy gains by 2050 that were 2 to 4 years greater than the official projections, and for Japan, 8 or 9 years greater. Lee and Miller (2001) investigated hypothetically how the method would have worked, had it been applied in earlier years. We found that the forecasting errors would have been quite well described by the probability distributions generated by the method. We also found that longer term forecasts tended to understate the future gains in life expectancy, here and in a number of other countries. Recently, Li and Lee (in press) have extended the method for use in countries with mortality data available for only a few irregularly spaced periods.

Forecasting Fertility

For fertility forecasts, I reasoned that once the fertility transition was over, we really had no clue about which way fertility would move, and the best we could do was to model its

level as a trendless (covariance stationary) stochastic process (Lee 1974). I then modeled births as an autoregressive time series, with net maternity rates as the autoregressive parameters, and derived the variance of the best linear forecast of the number of births in relation to the uncertainty of the fertility process by analyzing the renewal equation. From this I saw that the uncertainty in the forecast of births grew linearly with time to a good approximation, and that it was a moving average of past fertility shocks with weights equal to the progeny of a birth cohort at each lag. This work was my first presentation at the Michigan brown bag seminar series as a starting Assistant Professor there, and it was not well received, to say the least. The sociologists thought it was blasphemous to model fertility as a random process. Twenty years later, I got a very similar reaction when I submitted a related paper to *Demography*, which was rejected. The work appeared elsewhere (Lee 1993) and is a key component of the full scale stochastic population forecasts.

In between my first paper, and this elaborated model twenty years later, I had tried many other approaches to forecasting fertility. One line of work modeled an Easterlin-style effect of population age distribution on fertility (Lee 1974b and 1976). In this approach, the same kind of autoregressive birth equation was used, but now fertility was a function of the numbers of earlier births at each lag – that is, it depended on the contemporary population age distribution. Unfortunately, the future did not oblige by conforming to the predictions of the model; no new baby boom occurred, although a brief upswing between 1988 and 1992 looked promising. I also wrote a series of papers on the use of birth expectations data from surveys for forecasting (Lee 1981), which led to a paper on *Aiming at a Moving Target* (Lee 1980), but in the end proved of little use for forecasting. In other work, I considered using New Home Economics type fertility models for forecasting, but did not see how that approach could lead to useful predictions, since it seemed to imply decline without limit as income and female wages rose. Finally, with Larry Carter (1986), I developed a time series model of the joint evolution of births and marriages, each feeding into the other. It was an elegant paper, but marriage waned as a fundamental force driving fertility, and that approach was abandoned. It was a defeat to come back to treating fertility as a stochastic process, with model forecasts heavily conditioned by imposed assumptions for central tendency and, perhaps, for upper and lower bounds (e.g. TFR between 0 and 4). But sometimes it is best to acknowledge defeat, and make peace on the best available terms; that was what I did in Lee (1993). This long journey of discovery for fertility modeling is portrayed in Figure 4.

Figure 5 shows a fertility forecast for the US using this approach. The probability fan seems too wide, with the 95% probability bounds ranging from a TFR of .8 to 3.0. However, those bounds are intended to cover annual ups and downs. If we instead ask for central tendencies, by computing the probability interval for the average TFR along stochastic trajectories up to each horizon, we get a narrower 95% bound ranging from 1.4 to 2.6 for long run forecasts.

The Traditional Treatment of Uncertainty in Population Forecasts

The traditional approach to estimating and communicating the uncertainty of population forecasts is through the construction of scenarios. First, the analyst constructs high, medium and low projections for each of the rates, typically fertility, mortality and net immigration. The high and low trajectories do not have any probabilistic interpretation, but are chosen to span a range which the analyst believes to be plausible. The next step is to bundle these trajectories together to form high, medium and low scenarios. This is done in different ways, depending on the purpose of the projections. For example, in the US, the Census Bureau bundles together high fertility and low mortality (and high immigration) to form the high scenario, yielding the highest rate of population growth, and similarly for the low scenario. But the Social Security Administration bundles together high fertility with high mortality (and immigration), because these generate the lowest old age dependency ratio (OADR), which is the key demographic variable for their financial projections. The Census choice of bundles minimizes the high-low range of the OADR, because the high fertility tends to make the population young, while the low mortality tends to make the population old, and these effects cancel. The Social Security bundles minimize variations in the population growth rate, because high fertility makes the growth rate high, while high mortality makes it low, and these effects cancel.

This problem is illustrated by the 1992 Social Security population projections (Office of the Actuary 1992). For 2970, the High-Low range for the population age 0-19 is ± 34 percent; for the population 20-64, the range is ± 20 percent; and for 65+ it is ± 9 percent. Yet for the total dependency ratio, which is the sum of the first and the third divided by the second, the range is only ± 5 percent, whereas we would expect it to be many times this large. The scenario method inevitably gives probabilistically inconsistent indications of uncertainty for different population variables in the same forecast.

The arbitrariness of the choice of bundles is one of the problems with the scenario approach. Another is that no probabilities can be attached to the high-low range of the projections. Still another is along the scenario trajectories, fertility is always high, medium or low, and mortality likewise. It is therefore implicitly assumed that forecasting errors for fertility and mortality are perfectly correlated over time, so that fluctuations like the baby boom are ruled out by assumption. In addition, errors in fertility and mortality are also implicitly assumed to be perfectly correlated with one another, in the sense that high fertility is always associated with low mortality (or always with high mortality). A final problem is that the high-low bounds cannot have any consistent probabilistic interpretation across different measures, since true uncertainties tend to cancel in larger aggregates like total population, relative to their constituent parts like numbers of births in certain years, or the sizes of particular age groups. In the scenario approach, no such cancellation can take place.

From this point, I will focus on the main direction taken in the research, with less attention to the side routes that turned out to be deadends.

Stochastic Population Forecasts

As a graduate student, I wanted to develop more genuinely probabilistic population forecasts. I realized that most population forecasting errors derived from errors in forecasting the vital rates rather than from individual level uncertainty, as discussed earlier. At first I thought that the answer lay in formulating stochastic models of fertility and mortality, and then using the probability intervals from their fertility and mortality forecasts to set the upper and lower 95% probability bounds for the projection scenarios. However, it soon became clear that this would not do, for all the reasons given above: scenarios do not and cannot work probabilistically. Probabilistic projections required a population projection matrix with stochastic rates which could vary every after every projection step, and from it to derive the probability distribution of the projected population.

I could write down the equations for the stochastic vital rates, perhaps, but I did not know how to derive their implications. A few years later, I began seeing a series of papers by Shripad Tuljapurkar (Tulja), on population renewal in random environments. He was interested in the population dynamics of all species, and he explicitly developed the probability distributions of population variables when reproduction and survival were disturbed by climate, predation, and other partially natural influences (Tuljapurkar 1990). Later, Tulja spent some time at Berkeley and we joined forces to tackle the problem of stochastic population forecasts, combining my work on modeling the vital rates as stochastic processes, and his work on population renewal in random environments.

After several years of work, and improvements due to referees, the results were published in Lee and Tuljapurkar (1994). The paper contained analytic approximations for the actual probability distributions, derived at great cost. In order to check on these analytic results, Tulja also carried out stochastic simulations, which confirmed their accuracy. This exercise taught us that while the explicit mathematical solutions were intellectually satisfying and yielded some insights, the stochastic simulations were far simpler and could be used to estimate probability distributions for any desired functions of the age distributions. We reluctantly abandoned the analytic solutions in our subsequent work. This work, finally brought to a successful resolution, drew on earlier deep theoretical research by Tulja in mathematical population biology, and by me in historical demography.

Figure 6 shows forecasts from 1999 to 2080 of the old age dependency ratio, here defined as (population 65+)/ (population 20-64), with 95% probability intervals and comparisons to the Census and Social Security projections and ranges. The central forecasts of all three are quite similar, although ours are slightly higher due to our forecast of more rapid mortality decline. Our 95% interval is much broader than the High-Low interval of Social Security, which is in turn substantially broader than that of Census, due to the bundling choices for defining their trajectories, as discussed earlier. We find that the High Social Security trajectory is just above the 75th probability bound, and far below the 97.5% probability bound. According to our forecasts, there is a considerable possibility that population aging may be much more severe than that considered by Social Security and Census.

I have emphasized the contributions of Tuljapurkar and myself, but at the same time Juha Alho and his collaborators (1985, 1990, 2001) were working along similar lines and making important contributions. Keyfitz, in unpublished work, suggested a different approach: randomly sampling rates of mortality change from the past, and independently the levels of fertility, as a basis for developing a stochastic forecast. Stoto (1983) and Keyfitz (1981) developed methods for attaching probability intervals to forecasts of population growth rates and population size by analyzing ex post forecasting errors, an approach extensively developed and discussed in National Research Council (2000). Pflaumer (1988) made stochastic population forecasts by randomly sampling vital rates from the high-low range in official forecasts. Lutz, Sanderson and Scherbov (1996) have elaborated on this approach in a series of papers, an approach criticized by Lee (1999) and Tuljapurkar, Lee and Li (in press). Vigorous debates at several international meetings explored the relative merits of these approaches.

But What Are These Good For? Stochastic Social Security Forecasts

We had achieved an important goal, one I had started working on 27 years earlier. But we soon realized that nobody really understood these new projections, or what they were good for. Yes, we could now provide a probability distribution for the forecast of any demographic quantity, simply by examining our stochastic simulation results. But how did, say, the 95% probability bounds differ from the standard High, Medium and Low scenarios? In fact they differed profoundly, but this was difficult to convey.

After trying to explain all this to smart people who were accustomed to the traditional scenarios, and failing to get the point across, we decided that we would need to work out an application ourselves, to illustrate the power of the new stochastic population forecasts. Social Security finances seemed the best place to start, because the Trustees projected over a 75 year horizon every year, and because demography and population aging played key roles in these projections. Furthermore, the Social Security trust fund was the cumulation of net surpluses, and so it should depend on the sum of functions of the demography. Along any stochastic trajectory, there should tend to be some degree of cancellation of errors in our projections, but not in the traditional high-low scenarios used by the Actuaries.

Tulja and I initially developed simulations of the trust fund, with only the demography stochastic. Michael Anderson, who at the time was a graduate student in demography and statistics at Berkeley, programmed the stochastic simulations. Soon, however, we moved on to model productivity growth rates and real interest rates as stochastic processes, and for some purposes, we similarly modeled stock market returns. Our stochastic forecasts for the Trust Fund typically reflected four stochastic inputs out of the eight or ten that were usually viewed as uncertain. The inputs we did *not* treat as stochastic included inflation, disability, and immigration. We believed that the four we included were the most important sources of uncertainty, and would capture most of the overall uncertainty.

Figure 7 shows histograms for the date of exhaustion of the Social Security Trust Fund assuming no change in the payroll tax rate, no investment of the fund in equities, and no change in the currently legislated increase in the normal age of retirement to 67, based on the central assumptions contained in the Trustees Report of 1998 (somewhat more pessimistic than the more recent Trustees assumptions). The median date of exhaustion was then 2032, with 2.5% probability of exhaustion by 2022 and 97.5% by 2072. We consistently find that even the most favorable 2.5% bound shows exhaustion in less than 75 years, in contrast to the Trustees' "Low Cost" forecasts, which suggested that if we were lucky the system would be able to continue robustly in the future.

At the same time we were developing our stochastic forecasts for Social Security, the Congressional Budget Office (CBO) had noted our 1994 paper on stochastic population forecasts, and asked if they could use our stochastic population trajectories as the basis for stochastic Social Security projections of their own. We sent them a set of a thousand stochastic population simulations, and they did indeed develop stochastic forecasts which they published annually for a number of years. In 2001 they published a more elaborate stochastic forecast for Social Security, this time developing their own stochastic population model from scratch. I believe that this effort benefited from the short class we taught in Washington DC for three summers, which I will describe later.

In 1999-2000, I served on the Technical Advisory Panel for Social Security, and I presented our results there. The Trustees and Actuaries had already been advised by earlier committees that they should do stochastic forecasting, so our efforts fit well with those recommendations. Although nothing happened at the time, in 2003 Social Security did develop and publish its own stochastic forecasts for the first time, using methods closely related to ours. They also published comparisons of their stochastic projections to ours and also to a set developed by CBO. The three were remarkably consistent.

Stochastic Fiscal Projections

When President Bush was arguing for tax cuts, the Congressional Budget Office was projecting large surpluses over the next decade, and Greenspan was worrying about what we would do once we had paid back all the government debt. A probabilistic forecast would have shown that not much confidence should have been placed in those projections, and indeed CBO had included probability intervals based on its own analysis of the past performance of its projections, and these showed that it was quite possible that the surpluses would turn to deficits within a few years. I was invited to testify to the Senate Budget Committee about the uncertainties in the fiscal outlook, and the impact of population aging on the Federal budget. My testimony was received with interest, but I doubt that it had much impact. Here is the story of how the probabilistic fiscal forecasts were developed.

Back in 1995, I served on a panel organized by the Committee on Population of the National Academy of Sciences, and Chaired by Jim Smith, to assess the economic and demographic consequences of immigration to the US, for a special bipartisan Congressional Commission. My task was to estimate the fiscal impact of immigrants using a longitudinal design which required that I prepare long run projections of US

government budgets at the federal, state and local levels. I had developed relevant methods during earlier work on estimating the externalities to childbearing, involving the estimation of age profiles of government benefits and taxation, and shifting these over time with productivity growth. Figure 8 illustrates cross-sectional age profiles of this sort for the year 2000, showing the dominating importance of public education, Social Security benefits, Medicare, and institutional Medicaid. Similar age profiles were estimated for various kinds of taxes.

I got advice on modeling government budgets from Alan Auerbach and Robert Inman, two leading Public Finance economists also on the Immigration Panel. I hired Ryan Edwards, then a graduate student in Economics, to work on the budgetary side of these projections. Tim Miller, a demographic researcher at Berkeley, did most of the necessary estimation. With NIA support, this research team worked flat-out for a year, and brought the project to an interesting and successful conclusion. As a byproduct, we had developed the expertise to make long-run deterministic budget projections for all levels of government. If we could do it for one set of demographic and economic assumptions, then we could do it for others, so we had the machinery in place to construct stochastic budget forecasts for all government programs and total taxes and expenditures, exploiting detailed population age distributions and schedules of benefits and costs by age, as driven by productivity growth rates. We were able to draw on both the stochastic population projection methods and the methods for stochastic projections of Social Security. It seemed only natural to take advantage of the situation to construct probabilistic forecasts of federal, state and local taxes and expenditures. We did this, based on seven kinds of taxes and twenty five different age-specific government programs. The results were published in Lee and Edwards, 2002a and b.

Figure 9 shows probabilistic forecasts of government spending as a share of GDP at all levels (Federal, State and Local) combined, but disaggregated by age group in the first three panels and in total in the last panel. These forecasts are conditional on the assumption that current program structures remain constant or vary only according to currently legislated plans such as the increase in the normal retirement age. Panel C shows that no change is projected for the share of age neutral programs, consisting mainly of such items as defense expenditures, police, fire, research: expenditures that cannot be assigned to recipients of any particular age. The flat line with no probability dispersion reflects the assumption made, following CBO assumptions, that such expenditures will be a constant share of GDP in the future. Panel B shows that expenditures on the Young are also expected to be flat over the coming decades, although in this case there is substantial uncertainty, reflecting uncertainty about fertility and therefore the share of children in the population. Panel A shows that expenditures on the elderly are expected to rise strongly, nearly tripling in eighty years, due primarily to the effect of population aging on Social Security, Medicare, and institutional Medicaid (that is, for long term care). The probability distribution is narrow for the first twenty five years or so, reflecting mainly uncertainty about mortality and survival. After this point, however, the much greater uncertainty about fertility begins to affect the projected size of the working age population, which drives projections of GDP, and therefore strongly influences expenditures on the elderly as a share of GDP. Panel D shows the forecast for

expenditures for all age groups combined. Note that we would expect a negative correlation between expenditures on children and the elderly as a share of GDP, since variations in fertility would affect these in opposite directions. This correlation is implicitly taken into account in the probability intervals in Panel D. The central forecast shows that total governmental expenditures as a share of GDP would rise by more than 50% over the next 75 years, which we have seen is due entirely to expenditures on the elderly. The 95% probability interval extends from about 28% of GDP to about 48% of GDP, with uncertainty in the upward direction being greater than downward.

Health Care Costs: The Joker in the Deck

Medicare expenditures were one piece of the federal projections reported above about which we had reservations. However, based on earlier research by HCFA actuaries, Tim Miller (2001) published a paper showing how data on Medicare costs by time until death could be used in projections of Medicare costs. In these projections, as mortality falls, there are two effects on costs. First, there are more old people at every age, so costs tend to rise. Second, at any given age a smaller proportion of people is near death, so costs tend to fall. These two effects largely cancel, as it happens, so it makes little difference to costs whether mortality declines rapidly or slowly. Miller's work paved the way for a subsequent paper on stochastic forecasts of Medicare expenditures (Lee and Miller 2002). In these forecasts, we estimated a stochastic time series model for the growth in Medicare expenditures per enrollee in excess of per capita income growth. This was used as a multiplicative shifter for a schedule relating Medicare costs to years until death. Along each stochastic trajectory, we knew the distribution of deaths by age, so we could apply this schedule to the distribution of deaths to find the Medicare costs implied.

Figure 10 contrasts our probabilistic time until death based forecasts of Medicare costs to the official government projections. Here, probability deciles are indicated by the darkness of the fan. It turns out that probabilistic projections are not always more gloomy than deterministic ones! First, taking time until death into account leads to projections of Medicare costs that are substantially lower than the official projections by the Medicare Trustees, because our forecasts implicitly forecast improving health at every age as mortality falls. Second, we see that while our lower probability decile corresponds closely to their low projection, their high projection is far more pessimistic than our upper 97.5% bound: we find it very unlikely that their high scenario will come to pass.

Population and Fiscal Projections for California

In 2001, I was asked to prepare projections of population aging in California for the state legislature, and I took advantage of the opportunity to enlist Tim Miller and Ryan Edwards in the effort, and to prepare stochastic projections for the population of California and for its budget, through 2050. Tim Miller developed stochastic immigration and internal migration which we incorporated in the demographic the forecasts. Figure 11 shows our projections of the state budget, again assuming current program structure. In contrast to the Federal and general government projections, we find almost no systematic tendency for expenditures to rise relative to state GDP, because for the most part the state does not provide benefits targeted to the elderly, and therefore is not affected by the projected population aging. Revenues are almost flat with almost no uncertainty, since

they are expressed as a share of GDP, and are not much affected by population age distribution. These projections attracted considerable interest in Sacramento, and point towards the application of these methods at the state and local level.

Proselytizing and Training

In the summers of 1998 to 2001, Tulja, Edwards, Anderson and I taught a short intensive class in Washington D.C. on our new methods for stochastic forecasting, with funding from the Social Security Administration. We pitched it to professional government forecasters, and people attended from many government agencies, including Social Security, Census, CBO, OMB, GAO, and Veterans Affairs. Some graduate students and academics, and government forecasters from other countries, attended as well. The lectures seemed to generate a great deal of interest. It is difficult for us to assess the impact of these classes, but in 2001, CBO published its own stochastic forecast for Social Security, and in 2003, Social Security published its own version, as discussed earlier.

Where Next for Stochastic Forecasting?

These general methods for making stochastic forecasts of mortality, fertility, population, Pay-As-You-Go pension systems, health care costs, and full government budgets appear to be reasonably well established, although there is plenty of room for improving each, and for trying completely different strategies. Many questions remain about the best way to handle various details, and work on these questions is ongoing. Doubtless problems will be discovered and the methods will evolve. For example, a book by Tabeau, Jeths and Heathcoate (2001) discusses many approaches to forecasting mortality including Lee-Carter, and a book manuscript by Girosi and King (2003) contains a searching critical analysis of the Lee-Carter method and develops an alternative approach based on covariates and smoothness priors. Denton and Spencer in Canada, and CBO and the Social Security Actuaries are all exploring alternative approaches. Li Nan and I are working on modifying and extending Lee-Carter for use by the United Nations in the mortality component of their global population projections.

I would like to see government agencies develop and use stochastic long run budget projections. This seems particularly important, given the great stresses that population aging will put on the budgets of the industrial world through public pension programs, health care, and long term care. Policy changes today should be taken in light of this sobering long term outlook, but also with a full appreciation of the degree of uncertainty about these pressures. In recent years, both the tax cuts and the Iraq war illustrate the importance for good decision making of taking into account not only the best guess forecast, but also a careful assessment of the degree of uncertainty about that guess, and the expected cost of errors.

One of the most challenging questions in this area is just how policy makers should take uncertainty into account: by acting quickly to build up buffer funds or to contain a situation that might deteriorate if no action is taken, or by postponing action until we have a clearer idea of which direction the cat is going to jump. A useful start has been made on this question by Auerbach and Hassett (2000). But posing the problem as I just did suggests that once the cat jumps, we will know how the future will unfold—rapid

versus slow gains in life expectancy, deficits versus surpluses, and so on. In reality it is much more likely that the future will be no more certain in ten years or twenty or fifty than it is now. The world is not going to choose a direction and then adhere to it thereafter. There is uncertainty about big changes and small every step of the way, without end. We must live and act in the face of this uncertainty as we have always done, but understanding it better should lead to better decisions.

References

- Alho, Juha (in press) "Scenarios, Uncertainty, and Conditional Forecasts of the World Population" *Journal of the Royal Statistical Society-Series A*.
- Alho, Juha M. (1990) "Stochastic Methods in Population Forecasting," *International Journal of Forecasting* v.6, pp.521-530.
- Alho, Juha M., and Bruce D. Spencer (1985) "Uncertain Population Forecasting," *Journal of the American Statistical Association* v.80 n.390 (June) pp.306-314.
- Board of Trustees, Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds. 1998. *The 1998 Annual Report*. Washington DC: U.S. Government Printing Office.
- Carter, Lawrence and Ronald D. Lee (1992) "Modeling and Forecasting U.S. Mortality: Differentials in Life Expectancy by Sex," in Dennis Ahlburg and Kenneth Land, eds, *Population Forecasting*, a Special Issue of the *International Journal of Forecasting*, v.8, n.3 (November) pp.393-412.
- Carter, Lawrence and Ronald Lee "Joint Forecasts of U.S. Marital Fertility, Nuptiality, Births and Marriages Using Time Series Models," *Journal of the American Statistical Association*, 1986.
- Cohen, Joel E. (1995) *How Many People Can the Earth Support?* W. W. Norton & Co., New York.
- Congressional Budget Office of the United States (1996) *The Economic and Budget Outlook: Fiscal Years 1997-2006* (U.S. Government Printing Office, ISBN 0-16-048654-8).
- Girosi, Federico and Gary King (2003) [Demographic Forecasting](http://gking.harvard.edu/files/smooth.pdf), unpublished book manuscript (downloadable at <http://gking.harvard.edu/files/smooth.pdf>).
- Tabeau, Ewa, Anneke Van Den Berg Jeths, and Christopher Heathcote (2001) *Forecasting Mortality in Developed Countries* (Kluwer Academic Publishers, Dordrecht, the Netherlands)
- Keyfitz, Nathan (1981) "The Limits of Population Forecasting," *Population and Development Review* v.7 n.4 (December), pp.579-593.
- Lee, Ronald (1974a) "Forecasting Births in Post-Transitional Populations: Stochastic Renewal with Serially Correlated Fertility," *Journal of the American Statistical Association* 69, no.247 (September) pp.607-617.
- Lee, Ronald (1974b) "The Formal Dynamics of Controlled Populations and the Echo, the Boom and the Bust," *Demography* 11, no. 4 (November, 1974), pp. 563-585.
- Lee, Ronald (1976) "Demographic Forecasting and the Easterlin Hypothesis," *Population and Development Review* (September/December, 1976), pp. 459-468.
- Lee, Ronald (1980) "Aiming at a Moving Target: Period Fertility and Changing Reproductive Goals," *Population Studies*, vol. 34, no. 2 (July), pp. 205-226.

- Lee, Ronald (1981) "Model for Forecasting Fertility From Birth Expectations Data," in Gerry Hendershot and Paul Placek, eds., *Predicting Fertility: Demographic Studies of Birth Expectations* (Lexington Books).
- Lee, Ronald (1990) "Longrun Global Population Forecasts: A Critical Appraisal," in Kinglsey Davis and Michail Bernstam, eds., *Resources, Environment, and Population: Present Knowledge, Future Options*, a Supplement to vol.16, *Population and Development Review*, pp.44-71.
- Lee, Ronald D. (1992) "Stochastic Demographic Forecasting," in Dennis Ahlburg and Kenneth Land, eds, *Population Forecasting*, a Special Issue of the *International Journal of Forecasting*, v.8, n.3 (November) pp.315-328.
- Lee, Ronald D (1993) "Modeling and Forecasting the Time Series of US Fertility: Age Patterns, Range, and Ultimate Level," *International Journal of Forecasting* v.9 (1993), pp.187-202.
- Lee, Ronald (1999) "Probabilistic Approaches to Population Forecasting", in Wolfgang Lutz, James Vaupel and Dennis Ahlburg, eds, Supplement to vol. 24 of *Population and Development Review*, 1999 *Rethinking Population Projections*, pp.156-190.
- Lee, Ronald D. and Lawrence Carter (1992) "Modeling and Forecasting the Time Series of U.S. Mortality," *Journal of the American Statistical Association* v.87 n.419 (September) pp.659-671.
- Lee, Ronald and Ryan Edwards (2001) "The Fiscal Impact of Population Change," in Jane Sneddon Little and Robert K. Triest, eds., *Seismic Shifts: The Economic Impact of Demographic Change*. Federal Reserve Bank of Boston Conference Series No. 46, pp. 220-237.
- Lee, Ronald and Ryan Edwards (2002) "The Fiscal Effects of Population Aging in the US: Assessing the Uncertainties," James M. Poterba, ed., *Tax Policy and Economy* v.16 (NBER: MIT Press, 2002) pp. 141-181.
- Lee, Ronald and Timothy Miller (2002) "An Approach to Forecasting Health Expenditures, with Application to the US Medicare System," *Health Services Research*, v. 37, n. 5, pp. 1365-1386 (October 2002).
- Lee, Ronald, Timothy Miller, and Ryan Edwards (2003) *A Special Report: The Growth and Aging of California's Population: Demographic and Fiscal Projections, Characteristics and Service Needs*, California Policy Research Center, Technical Assistance Program, University of California.
- Lee, Ronald D. And Francois Nault (1993) "Modeling and Forecasting Canadian Mortality: An Application of the Lee-Carter Approach, with Extensions," presented at World Congress of the IUSSP, Montreal, August 1993.
- Lee, Ronald and Shripad Tuljapurkar (1994), "Stochastic Population Projections for the United States: Beyond High, Medium and Low," *Journal of the American Statistical Association*, v.89 n.428 (December) 1175-1189.

- Li, Nan, Ronald Lee and Shripad Tuljapurkar (in press) "Using the Lee-Carter Method to Forecast Mortality for Populations with Limited Data," forthcoming in *International Statistical Review*.
- National Research Council (2000) *Beyond Six Billion: Forecasting the World's Population. Panel on Population Projections*. John Bongaarts and Rodolfo Bulatao, eds. Committee on Population, Commission on Behavioral and Social Sciences and Education, Washington, D.C.: National Academy Press.
- Lutz, Wolfgang, Warren Sanderson, and Sergei Scherbov (1996) "Probabilistic Population Projections Based on Expert Opinion" Chapter 16 in Wolfgang Lutz, ed. (1996) *The Future Population of the World: What Can We Assume Today?*, revised 1996 edition (Earthscan Publications Ltd, London).
- McNown, Robert and Andrei Rogers (1989) "Forecasting Mortality: A Parameterized Time Series Approach," *Demography* 26(4) pp.645-660.
- Pflaumer, Peter (1988) "Confidence Intervals for Population Projections Based on Monte Carlo Methods," *International Journal of Forecasting* v.4 pp.135-142.
- Stoto, Michael (1983) "The Accuracy of Population Projections," *Journal of the American Statistical Association* v.78 n.381 (July) pp.13-20.
- Sykes, Z. M. (1969) "Some Stochastic Versions of the Matrix Model for Population Dynamics," *Journal of the American Statistical Association* v.44 pp.111-130.
- Tuljapurkar, Shripad (1990) *Population Dynamics in Variable Environments* (Springer-Verlag, Lecture Notes in Biomathematics n.85).
- Tuljapurkar, Shripad, Ronald Lee, and Qi Li (in press) "Random Scenario Forecasts versus Stochastic Forecasts," paper presented at the seminar, "How to Deal with Uncertainty in Population Forecasting," forthcoming in *International Statistical Review*.
- Tuljapurkar, S., N. Li and C. Boe, 2000. A Universal Pattern of Mortality change in the G7 Countries. *Nature* 405:789—92.
- U.S. Bureau of the Census (1989), Current Population Reports, Series P-25, No. 1018, *Projections for the Population of the United States, by Age, Sex, and Race: 1988 to 2080*, by Gregory Spencer (U.S. Government Printing Office, Washington, D.C.).
- U.S. Bureau of the Census (1992), Current population Reports, P25-1092, *Population Projections of the United States, by Age, Sex, Race, and Hispanic Origin: 1992 to 2050*. by Jennifer Cheeseman Day. U.S.G.P.O., Washington, D.C.
- Wade, Virginia (1989) *Social Security Area Population Projections: 1989*, Actuarial Study No. 105. Washington, D.C.: Office of the Actuary, Social Security Administration.

Figure 1.

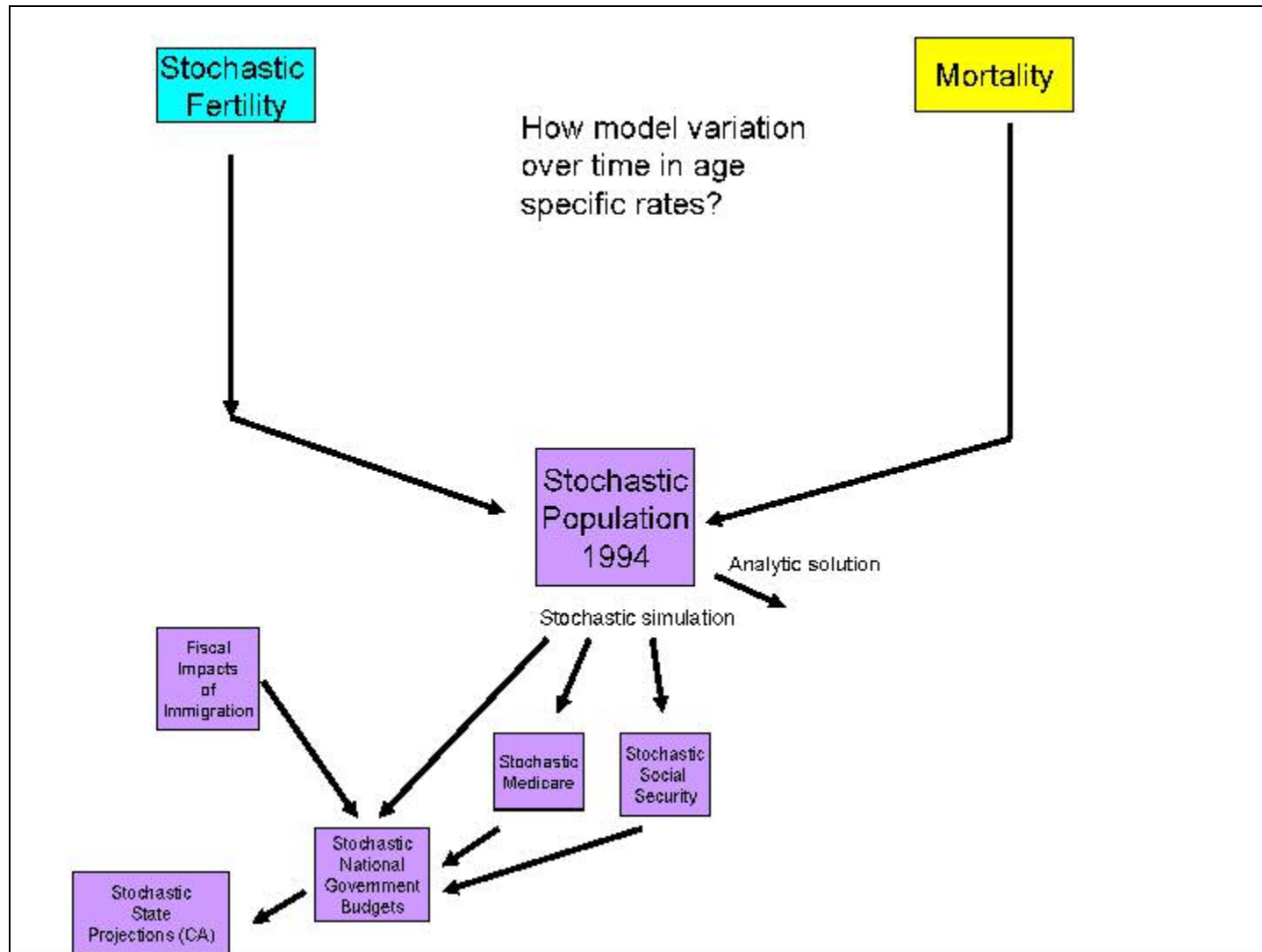
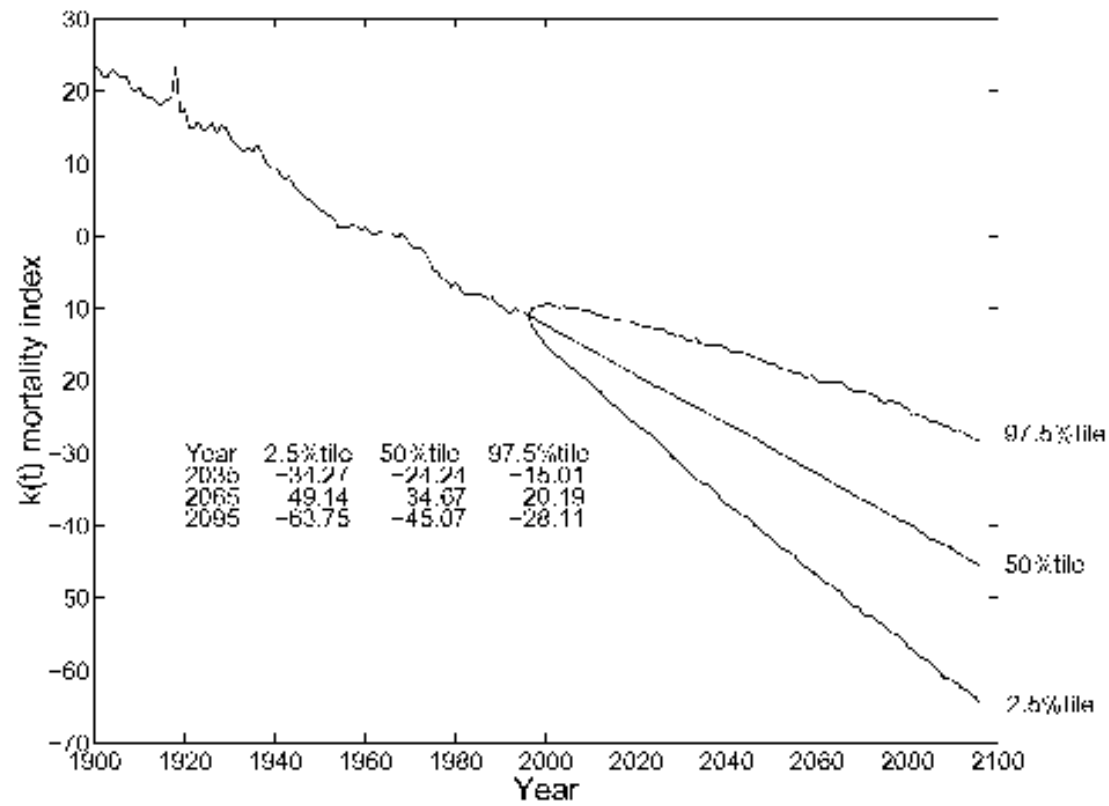


Figure 2. Lee-Carter mortality index $k(t)$, fitted (1990-96) and forecasted (1997-2096)



Source: Lee and Tuljapurkar (2000).

Figure 3.

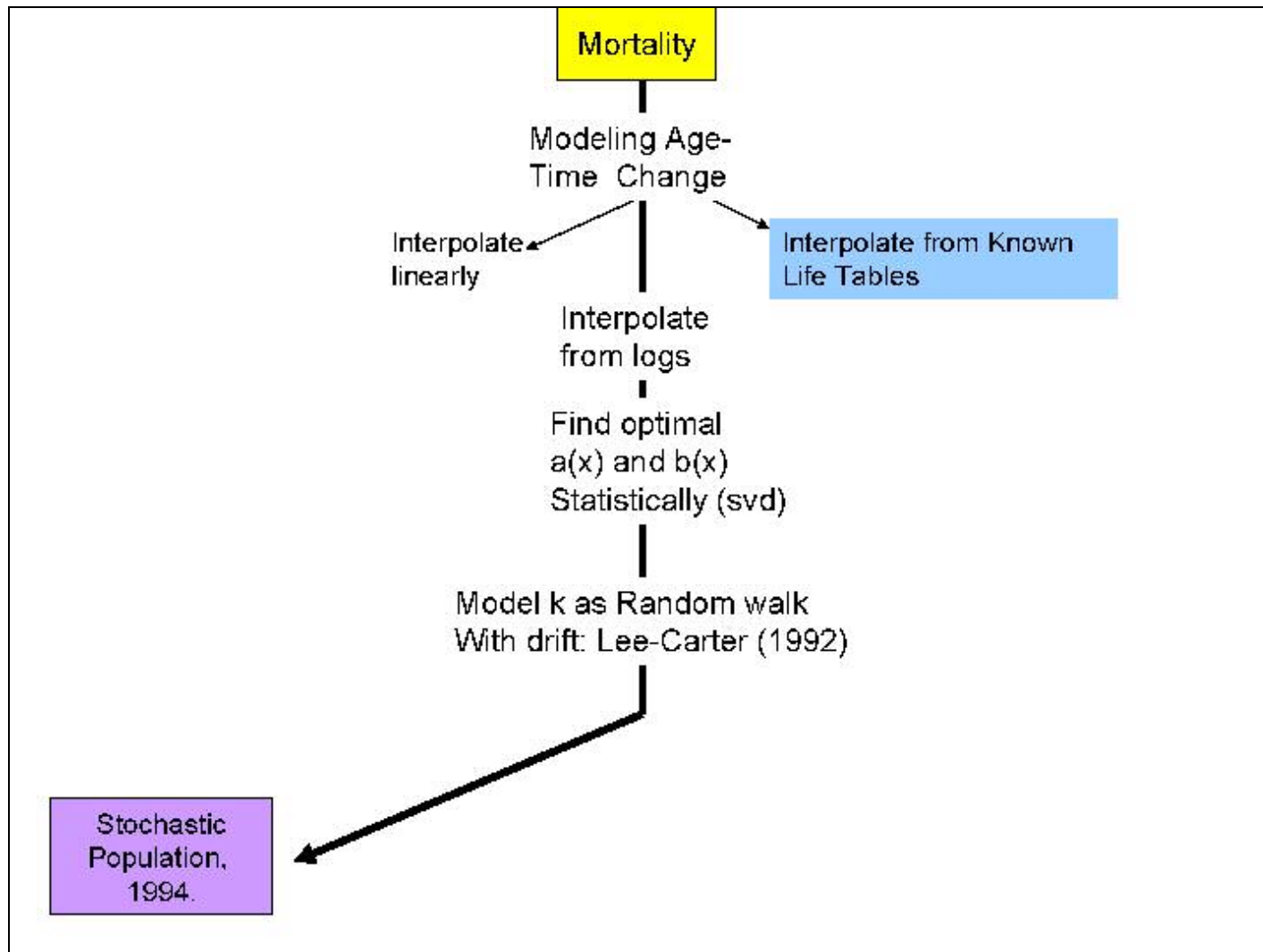


Figure 4.

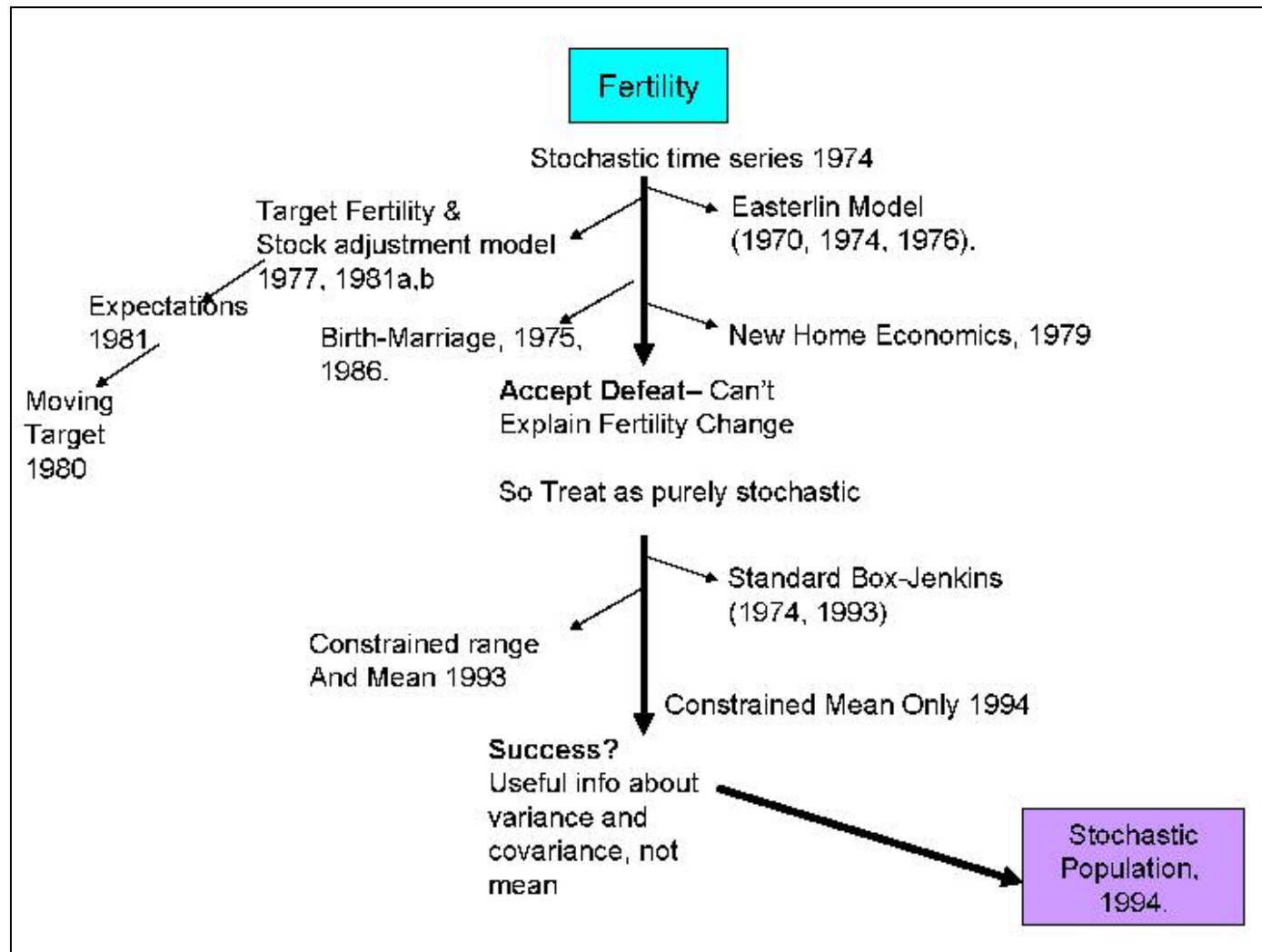
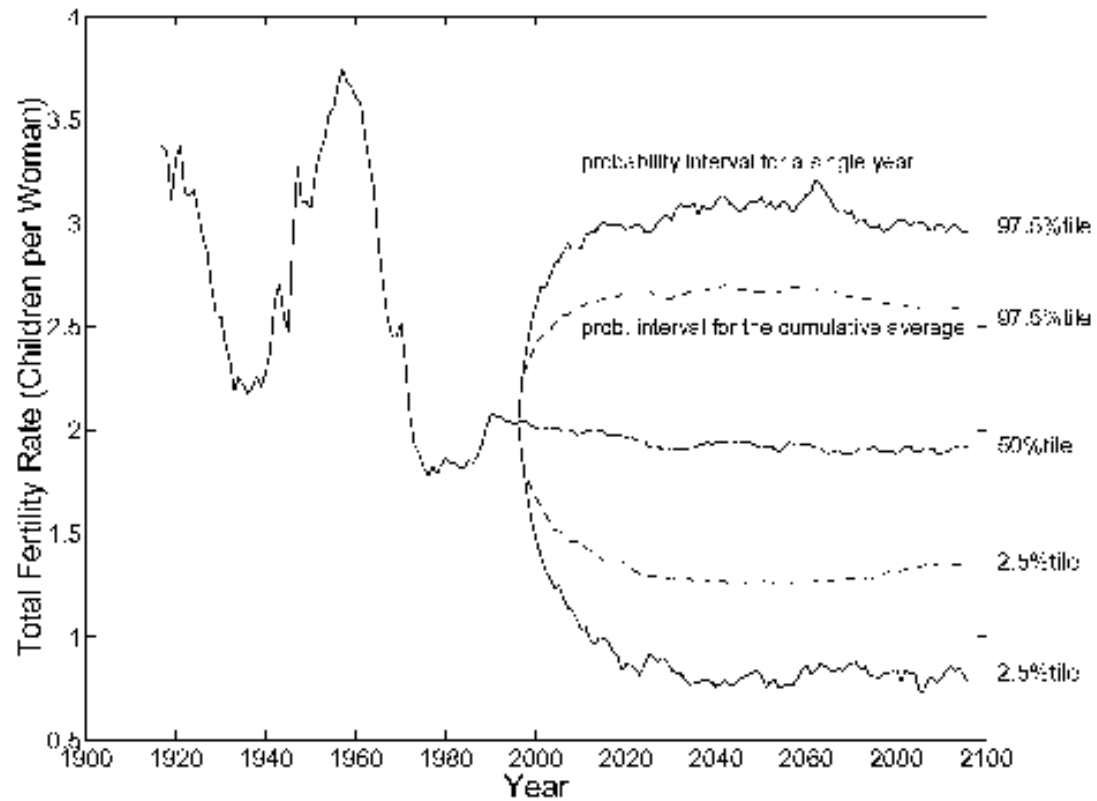
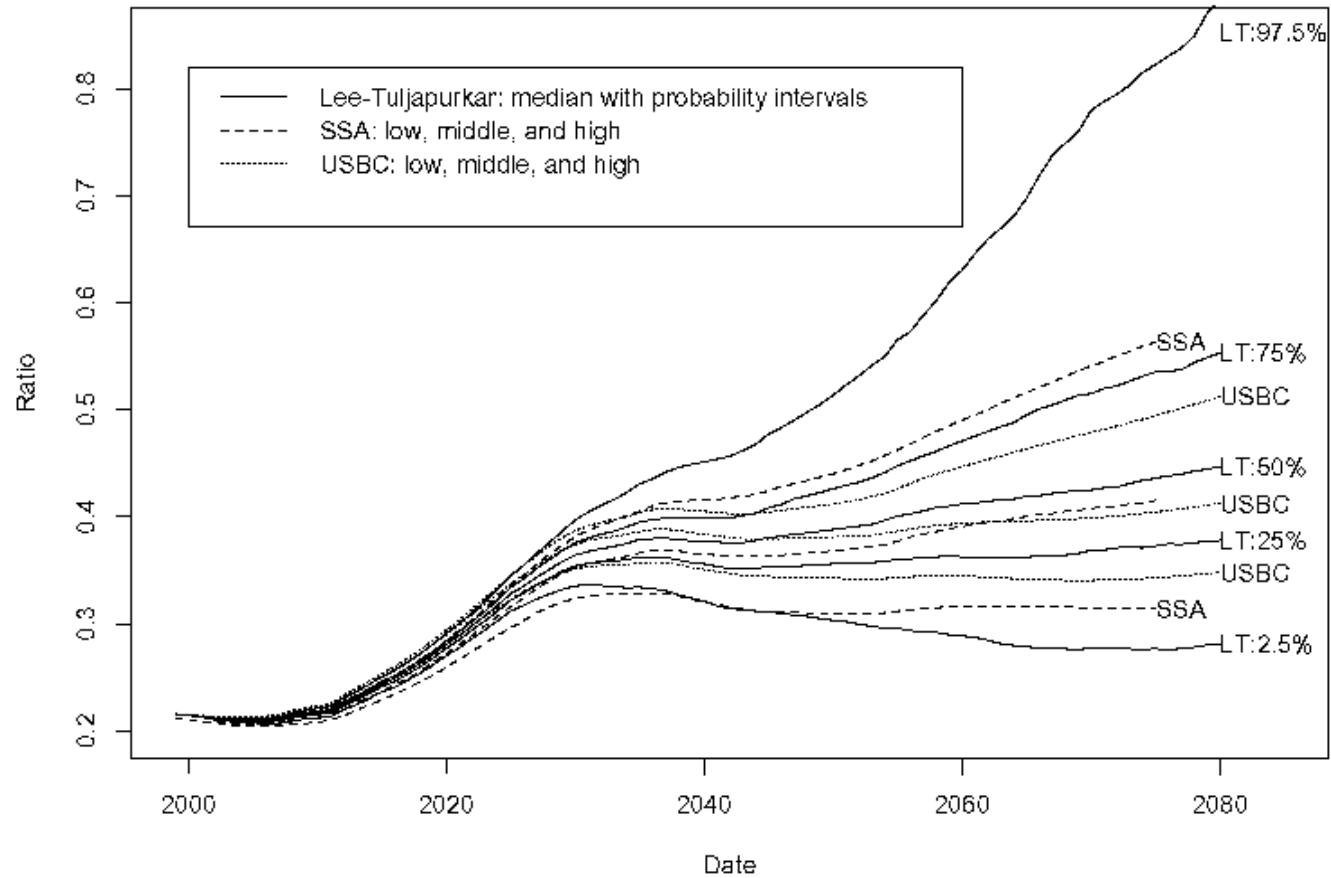


Figure 5. Total Fertility Rate, historical values (1917-1996) and forecasted (1997-2096), with 95% probability intervals for annual values and for the cumulative average up to each horizon



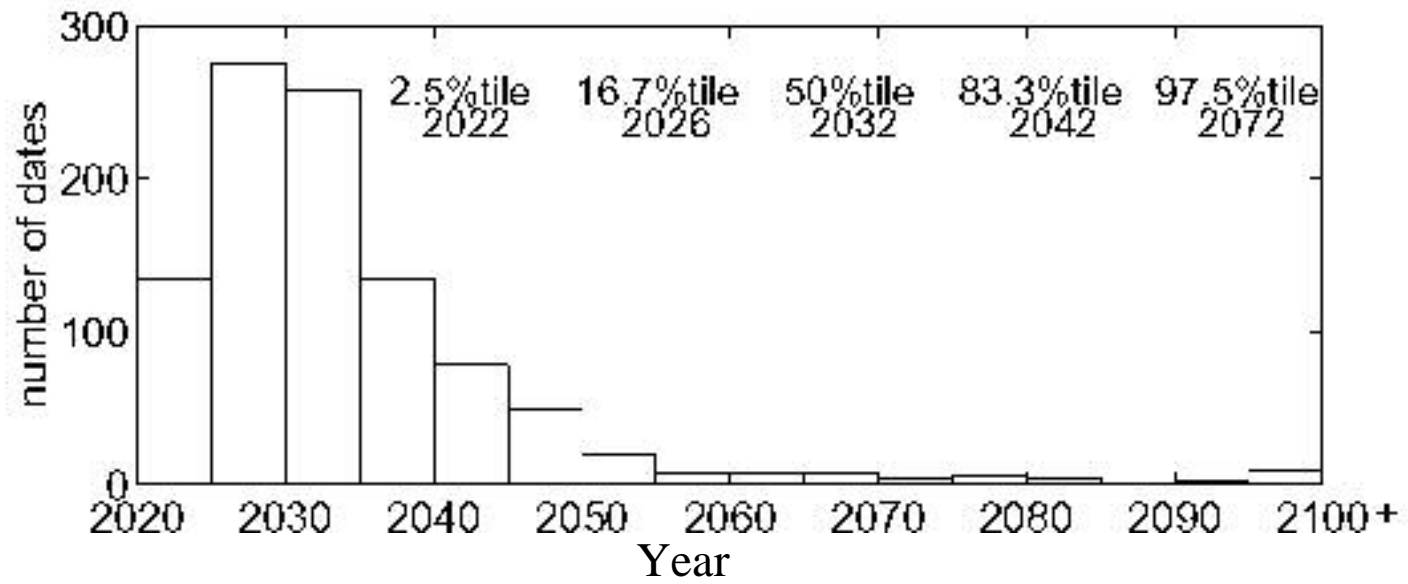
Source: Lee and Tuljapurkar (2000).

Figure 6. Old-age dependency ratio forecasts: 1999 to 2080

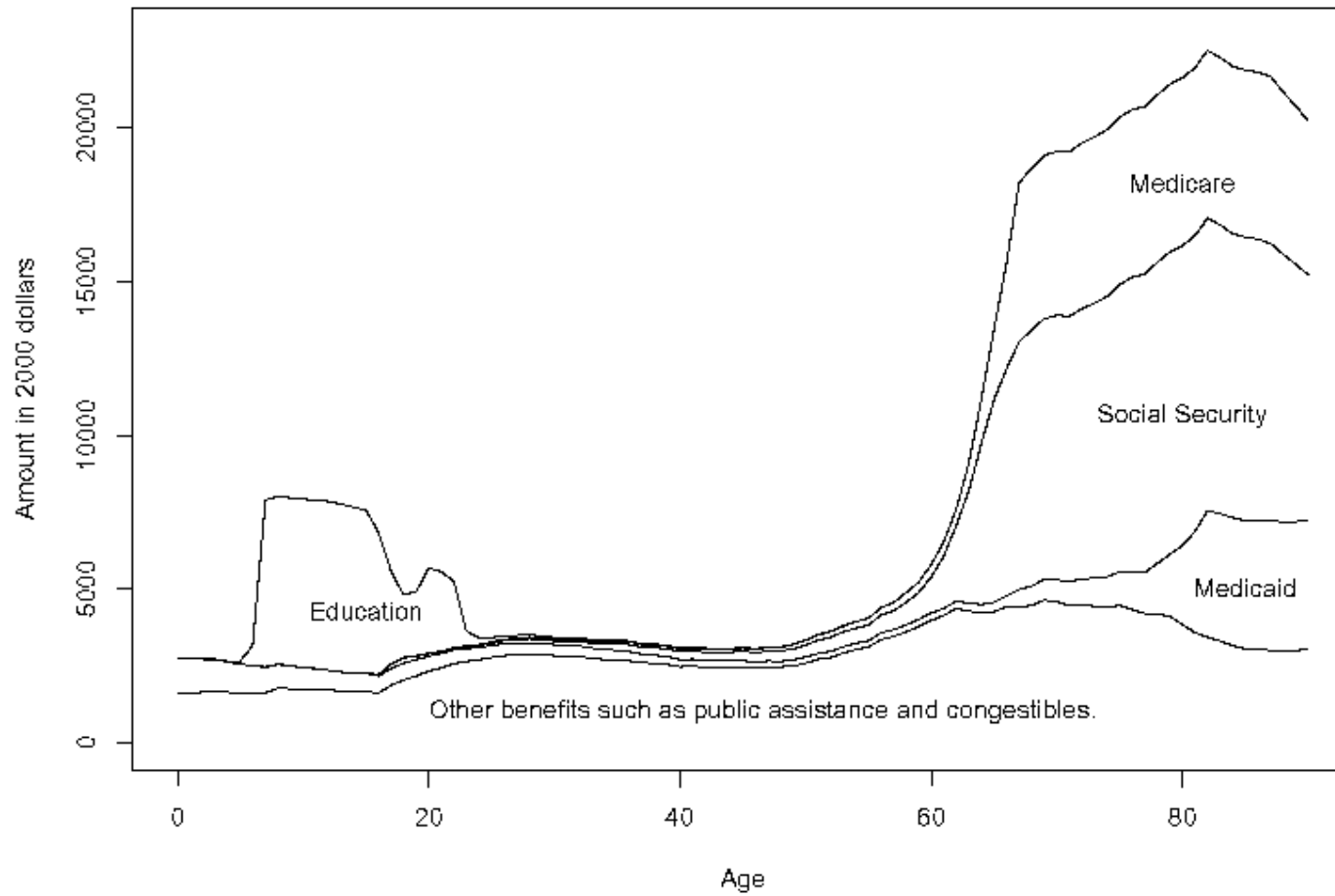


Source: Lee and Edwards (2001).

Figure 7. Histograms of 1,000 dates of exhaustion for social security Trust Funds

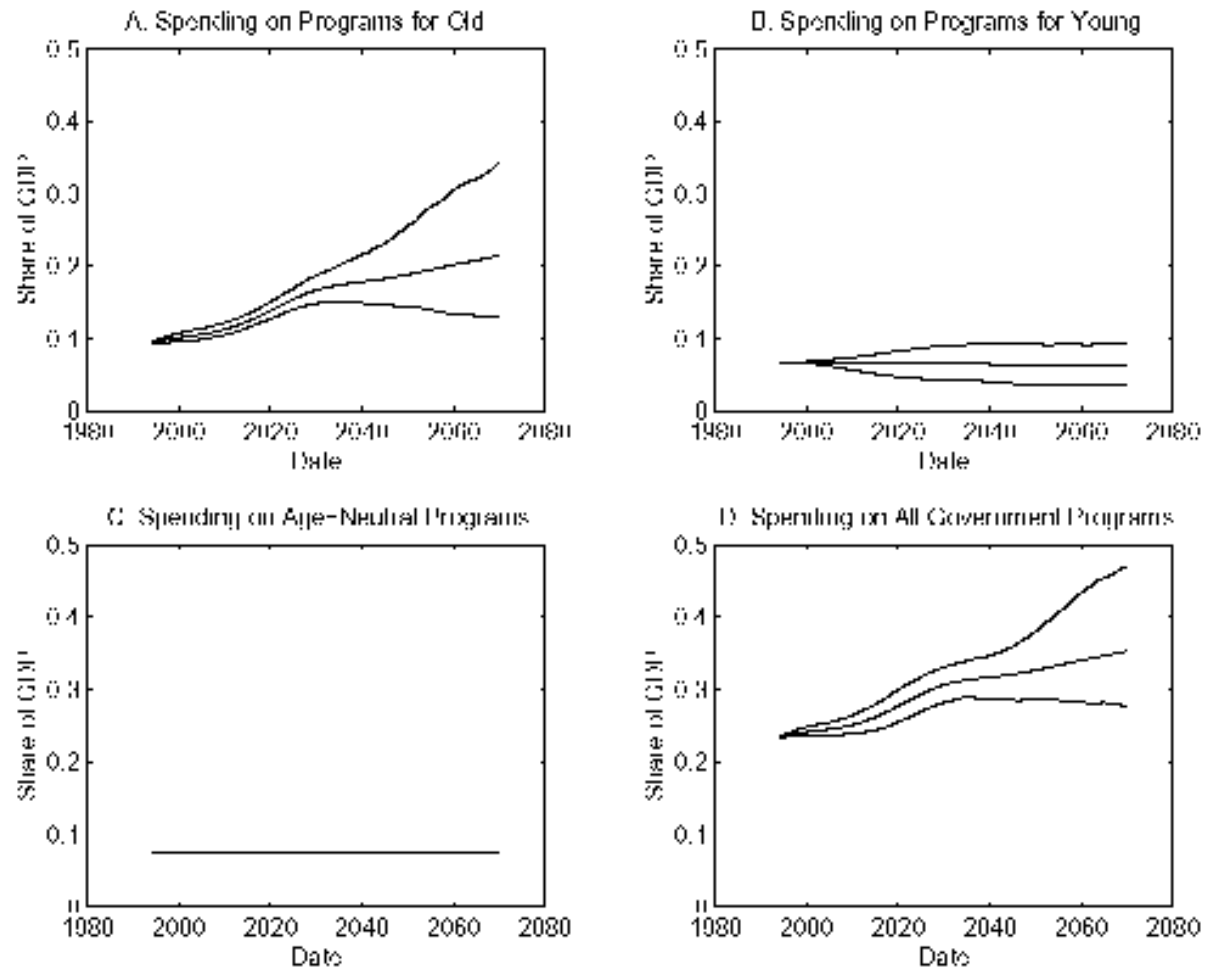


Source: Lee and Tuljapurkar (2000).

Figure 8. Benefits by program and age

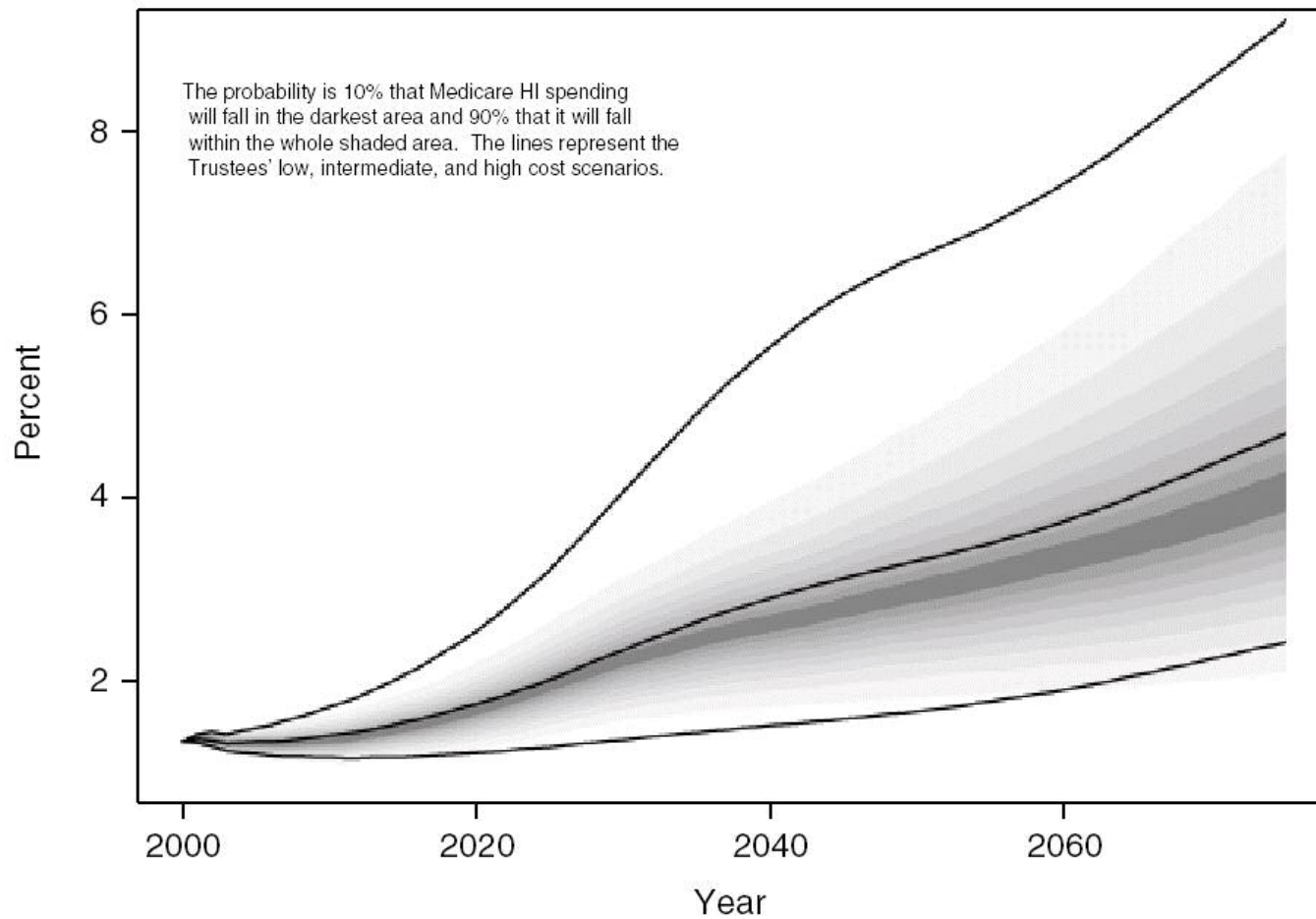
Source: Lee and Edwards (2001).

Figure 9. Fiscal Projections for Spending by Age



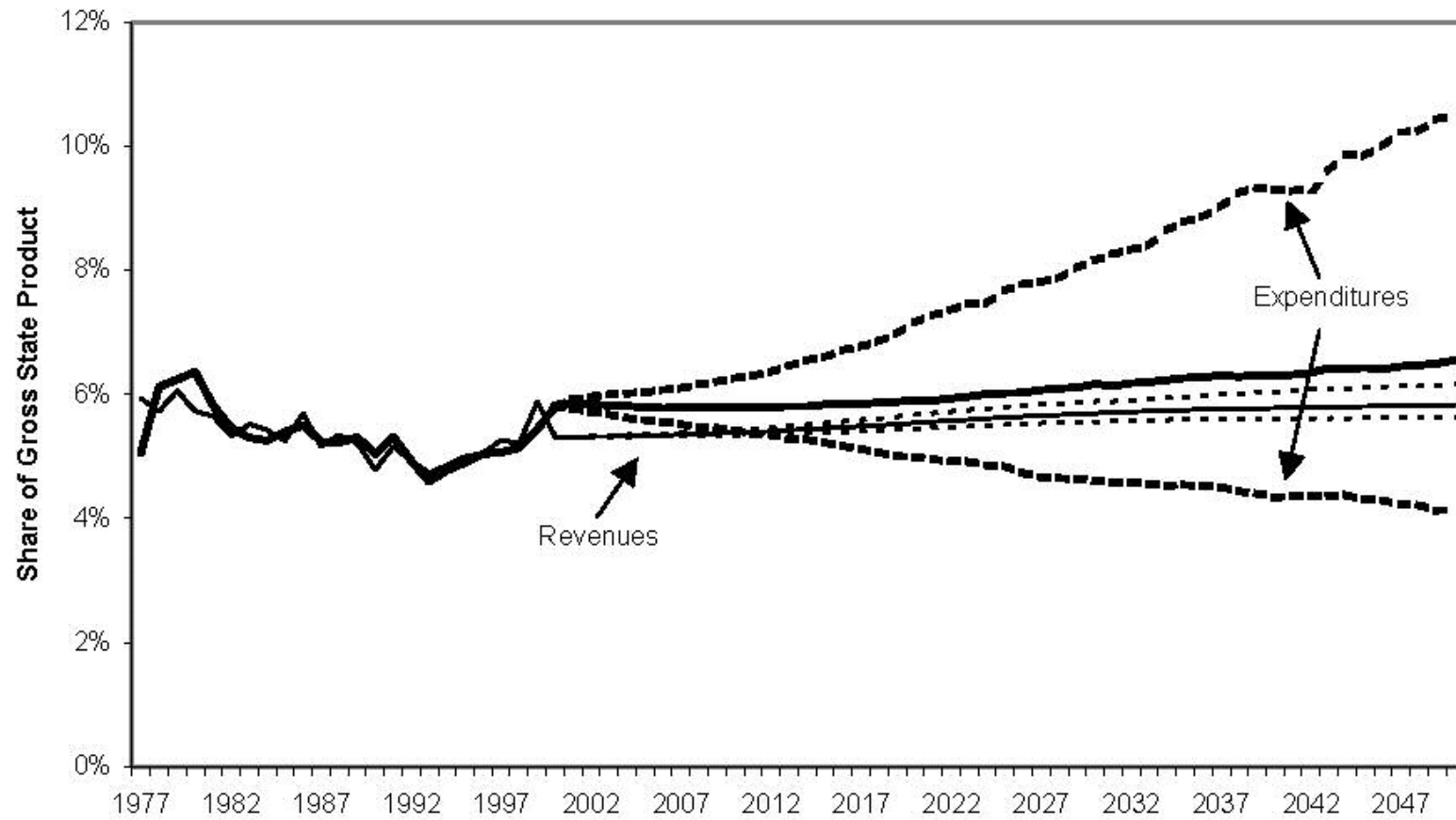
Source: Lee and Tuljapurkar (2000).

Figure 10. Medicare Hospital Insurance Program as Percent of GDP: Lee-Miller Probability Deciles and Trustees Scenarios (Excludes Medicare's Supplementary Medical Insurance Program)



Source: Lee and Miller (2002).

Figure 11. Projections of California's General Fund Revenues and Expenditures as Shares of GSP



Source: Lee, Miller, and Edwards (2003).