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Contract No. W-7405-eng-48

ELASTIC SCATTERING OF 340-MEV PROTONS BY DEUTERONS

D. D. Clark
(Thesis)

June 25, 1953

Berkeley, California

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ELASTIC SCATTERING OF 340-MEV PROTONS BY DEUTERONS

D. D. Clark

Radiation Laboratory, Department of Physics,
University of California, Berkeley, California

June 25, 1953

ABSTRACT

The differential cross section for the elastic scattering of 340-Mev protons by deuterons has been measured at seven angles from 40° to 150° in the center of mass system. A coincidence counting system using scintillation counters detected both product particles, the elastic effect being separated from the inelastic through identification of the scattered deuterons by photographically recorded pulse heights produced in a counter telescope. The resulting cross section is peaked strongly at small angles and exhibits a rise in the large angle (pick-up) region. Comparison is made with an approximate theoretical curve calculated by a method of Chew.

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I INTRODUCTION

One of the most fundamental problems of nuclear physics is that of a complete understanding of nucleon-nucleon interactions. With this knowledge and a suitable theoretical apparatus, a very large body of nuclear phenomena could be organized into a coherent "predictable" whole. Numerous approaches to the study of this problem exist, but one of the most direct and necessary is through experiments on scattering.

A great deal of effort, both experimental and theoretical, has been expended on neutron-proton and proton-proton scattering over a wide range of energies.¹ In the low energy region our understanding is quite good, but at higher energies, where probing of the details of the presumed potentials is possible, theoretical interpretation even on a phenomenological basis has been unable to give complete accord with experiment, and meson theory has been too difficult to apply. The Lévy potential² is a start in the application of meson theory, but it again is limited to the lower energies; moreover, questions have been raised regarding its theoretical foundations.³

The place of nucleon-deuteron scattering in the attempt to learn more about nuclear forces is twofold. First, in a manner analogous to the ortho- and para-hydrogen experiments with very slow neutrons,^{4, 5, 6} we might expect interference effects between the scattered waves from the two components of the deuteron, which would give us information as to the relative signs of the nucleon-nucleon phase shifts. Second, as targets of neutrons do not exist, neutron-deuteron scattering is our closest approach to n-n scattering, and a comparison of n-d and p-d scattering can answer the question of whether or not nuclear forces are

charge-symmetric. Moreover, though nucleon-deuteron scattering is a three-body problem, theoretical treatment -- at least for small angles of deflection -- is possible at the higher energies by use of the impulse approximation.^{7, 8}

Experimental work to date in the higher energy region has been that of Stern⁹ (192-Mev d-p; 63 Mev in the center of mass); Cassels, Stafford, and Pickavance¹⁰ (145-Mev p-d; 95 Mev c.m.); and Schamberger¹¹ (240-Mev p-d; 156 Mev c.m.). Recently, Ashby completed a 32-Mev p-d experiment¹² (21 Mev c.m.). Powell has made a cloud chamber study of 90-Mev n-d (59 Mev c.m.),¹³ and a counter technique in which the deuteron is identified is currently being used by Youtz and Kerth¹⁴ on the same problem. Bratenahl¹⁵ has studied the pick-up region (near 180°) at energies from 95 to 138 Mev.

The present experiment was undertaken because of the usefulness of any additional high-energy scattering data, and in particular because the impulse approximation becomes increasingly valid with higher energy. The proton beam energy used was 340 Mev, and the available energy in the center of mass is 218 Mev.

II THE EXPERIMENTAL PROBLEM

The process of measuring the cross section consisted of the measurement of the four quantities designated below:

- D = number of elastic p-d events observed per unit integrated beam
- N = number of target deuterons per cm^2 measured in the direction of the beam
- n = number of beam protons per unit beam
- $\Delta\Omega$ = solid angle subtended by the defining counter

(Unit beam is usually designated as "integrator volt", or I.V., because of the manner in which the beam intensity is measured.)

The cross section is then:

$$\left(\frac{d\sigma}{d\Omega} \right)_{\text{lab.}} = \frac{D}{n N \Delta\Omega}$$

The heart of the experimental problem is the measurement of D, and the nature of the detecting system is in large part dictated by the difficulties or lack thereof in making this measurement. It is therefore of importance to consider the character of the events being studied in the present experiment, and of the principal sources of background.

One useful characteristic of the elastic process is that it is a two-body one in which both emergent particles are charged; thus a coincidence counting system in which both are detected is quite feasible and helpful. Such an arrangement was employed; because of the short duty cycle of the cyclotron, scintillation counters and relatively fast (about 2×10^{-8} sec.) coincidence circuits were used. When one counter angle is fixed and the other made variable, the coincidence counting rate should be peaked at one particular angle -- that for elastic scattering.

Two characteristics of the desired events make it difficult to obtain accurate values of D. One is that for a large range of angles the cross section is between 40 and 100 microbarns per steradian; this affects the statistical accuracy which can be obtained in a reasonable

amount of running time. The second and much more important is the small ratio of the elastic cross section to that for the principal background -- the inelastic process, in which the deuteron is broken apart, and two protons and one neutron are the emergent particles. This ratio is of course small because of the high energy of the beam particles, and the low binding energy of the deuteron. In the words of Chew's analysis¹⁶ of nucleon-deuteron scattering, the sticking factor is very small.

The results of Bloom¹⁷ (190-Mev inelastic d-p scattering) and of Stern⁹ furnish information on the relative intensities of elastic and inelastic events for the energy employed by them. For observation at 45° in the laboratory system, Stern obtains $(d\Omega/d\Omega)_{\text{elastic}} = 2.07 \text{ mb/ster}$; Bloom obtains for those inelastic collisions in which both protons suffer significant momentum changes ("p-p type inelastic collisions") a value of 5.7 mb/ster. The ratio of the number of elastic events to the sum is therefore about 0.27. From Chew's paper,¹⁶ the sticking factor in this case is nearly the same as for 40° center-of-mass scattering in the present experiment. As the sticking factor decreases rapidly with increasing angle of scattering, we may expect in the present experiment that at no angle will the above ratio be more favorable than 0.27, and that it will decrease by a factor of 10 or more at the larger angles.

The smallness of this ratio takes on added importance when one realizes that if in the laboratory system the target particle is the deuteron, and the beam particle the proton, the kinematics of the inelastic process is such that there is a wide range of possible included angles between the emergent protons. This can easily be seen from an impulse treatment of the kinematics, which is carried out in some detail in Appendix B.

Consider the case in which the beam proton and the proton in the deuteron collide; in our approximation, the neutron of the deuteron will be relatively unaffected. As the beam energy is high, and the binding energy of the deuteron and the kinetic energy carried off by the neutron are small, the collision is very much like one between two free protons. Because of the internal momentum of the deuteron, however, the "target" proton in most cases is moving in the laboratory system at the moment of collision. For the most probable value of internal momentum -- zero -- the included angle between the emergent protons is that of collisions

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Fig. 1. Deuterium effect for 90° center-of-mass elastic scattering, without identification of deuteron. Proton counter fixed at 60° from beam, and deuteron counter angle variable, with 3° resolution. To enhance elastic effect compared to inelastic, the conjugate-counter solid angle has been made smaller than necessary to count all partners of elastically scattered particles striking defining counter.

A. Without absorber preceding proton counter.

B. With 22.5 gm/cm^2 copper preceding proton counter.

Elastically scattered proton has 35.2 gm/cm^2 total range.

between equal-mass particles, one of which is stationary -- 90° , or somewhat less when relativity and binding energy are taken into account. But when the internal momentum is not zero at the time of collision, the included angle can be quite different from 90° . Moreover, low-energy protons result from collisions of the beam proton with the target neutron, and again the included angle between the two protons can vary widely. Detection efficiency for the neutron is small.

This situation is in contrast to that existing when the beam particle is the deuteron, and the target a proton. As shown in the work of Bloom,¹⁷ when the beam energy is high, internal momentum in the deuteron contributes to the inelastic process simply a small uncertainty in the angle and energy of the beam proton. The target proton is stationary, so the included angle is about 90° , with only a small spread in the peak.

In our case the wide range of included angles means that coincidences due to inelastic processes will be observed at the elastic peak angle; this would not put us into serious difficulty except for the predominance of the inelastic events, which actually swamp out the elastic peak. Figure 1A shows this effect as observed in an early exploratory run.

A closer look at the inelastic process (see Appendix B) shows that the inelastically scattered protons striking either coincidence counter have a spectrum of energies; this makes impossible the complete separation of inelastic and elastic events by use of discrimination on the basis of a single parameter such as range or specific ionization. However, the bulk of the inelastic events may be eliminated by use of the fact that the proton partners in the elastic events have just slightly more range than any inelastic proton striking the proton counter. Figure 1B shows the variation of counting rate with included angle in the same case as for Fig. 1A, but with 22.5 gm/cm^2 of copper preceding the elastic proton counter. This curve demonstrates the existence of the p-d elastic process (which was hardly doubted) but the method is not suitable for cross-section measurements. An unknown amount of inelastic events is included, the method is slow, and at some angles a prohibitive amount of absorber would be necessary.

Consequently, a method has been utilized which identifies the scattered deuteron as a deuteron, while retaining the coincidence sys-

tem to suppress general background. Identification can be accomplished by a number of means, in each of which two or more parameters of the particle are measured. These parameters can be specific ionization, range, energy, momentum, velocity, etc. Each combination of parameters has advantages and disadvantages; for the present application, specific ionization and energy were chosen. This system affords adequate separation of protons and deuterons, and the equipment necessary is lightweight and physically compact. Its principal disadvantage results from the requirement of a thick counter to measure the total energy of the impinging particle; at high energies the range is sufficiently great that corrections for nuclear absorption and deuteron stripping occurring during the stopping process can become excessive.

In the system employed, the specific ionization is approximately measured by the pulse height produced in a relatively thin counter, and the total energy by the pulse height from a counter thick enough to stop the particle. These pulse heights are displayed on an oscilloscope screen and photographed. The counting apparatus is more completely described in the chapter on experimental arrangement, and the details of analysis of the films are discussed in the chapter on reduction of data.

III EXPERIMENTAL ARRANGEMENT

General

The experiment was conducted in the scattered external proton beam of the 184-inch cyclotron. Figure 2 shows schematically the layout of the cyclotron. In the bombardment area (the "cave") were located the targets, counters, and beam monitor, all mounted on a scattering table, which in turn rested on a movable platform, allowing easy alignment of the apparatus with the proton beam. The scattering table was designed by Wiegand and Kelly and has been used in numerous previous experiments, including those of Chamberlain, Segrè, and Wiegand,¹⁸ and Stern.⁹ It permitted remote-control selection of three different targets. The counters were mounted on two movable arms pivoted about a point under the target in use. The angular positions of one arm with respect to a fixed "zero degree" line on the table, and of the other arm with respect to the first, were indicated by calibration marks every 0.5° . Cables between the cave and the counting area supplied power to the cave apparatus and brought out the signals to the counting area, where the remainder of the equipment was located.

Figure 3 combines a schematic diagram of the physical arrangement of the apparatus in the cave and a block diagram of the electronics. During the progress of the experiment, many of the components shown on the block diagram underwent considerable development; consequently only their general nature is described below. Portions of the electronic equipment were designed, or designed and constructed, by Prof. Owen Chamberlain and Dr. Clyde Wiegand.

Collimation and Beam Monitor

The snout collimator (see Fig. 2) used in all runs was a 4-foot-long cylindrical brass tube, of $1/2$ inch internal diameter through the first 33 inches, and of $3/4$ inch for the last 15 inches. At the position of the counting apparatus, 99 percent of the beam was contained in a circle of less than 2 cm diameter.¹⁹

Fig. 2 Schematic layout of cyclotron showing external beam path.

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Fig. 3. Schematic diagram of apparatus in cave, and block diagram of electronics. Power supplies not shown, for simplicity.

The beam intensity was measured by a 2-inch deep argon-filled ionization chamber operated at 1000 v. The collection electrode was connected to a 1 μ f integrating capacitor, accurately calibrated; the voltage on the capacity was measured by a standard UCRL electrometer employing feedback, and recorded on a Leeds and Northrup recording millivoltmeter. The ion chamber was calibrated against a Faraday cup. The chamber was built for the p-p scattering experiments of Chamberlain, Segrè, and Wiegand, and it and the method of calibration are more fully described by them.¹⁸

Targets

The deuterio-polyethylene and carbon targets were uniform disks of 1-3/4 inches diameter suspended by nylon threads. The CD₂ targets, which contained 94 percent D and 6 percent H, were either 0.1312 or 0.3264 gm/cm² thick, and the carbon targets were of thicknesses giving in most cases nearly the same number of carbon atoms per cm², and in the others giving the same stopping power as the CD₂ targets. The polyethylene targets were formed under heat and pressure in a polished steel mold from powder purchased from Los Alamos.

Counters

The deuteron telescope had two different forms. When the range of the deuterons to be detected was greater than 7 gm/cm² of copper (70° to 150° center-of-mass angles), the telescope consisted of four counters, which in the order seen by an incident particle were: a defining counter furnishing a signal for the coincidence circuit, a thin counter to measure the specific ionization, a thick counter to measure the total energy, and a "pass-through" counter to give a signal when an energetic background particle did not stop in the energy counter. This last was included to aid in the reading of the film, as a major portion of the background protons were of large range compared to the elastic deuterons. When the deuteron range was 2.6 grams/cm² or less (40° to 50° center-of-mass angles), the telescope consisted of three counters:

one serving the double function of furnishing defining-coincidence and dE/dx signals, one to measure the energy, and one as a pass-through counter.

For some angles aluminum absorber was added to the telescope to decrease the range of the incident deuteron. An aluminum absorber wedge was also used to bring the deuterons, whose energy varied with angle across the face of the defining counter, to approximately the same energy before entering the pulse height counters. The wedge and absorber were placed between counters 1 and 2 in the long-range array and in front of the entire telescope in the short-range array.

The areas of the counter faces were chosen so that no deuterons entering the defining counter would be scattered out of the energy counter by multiple Coulomb scattering before stopping, even when the maximum amount of absorber used was present. The magnitude of the scattering was calculated, and a liberal safety margin allowed. The calculation method was also checked against measurements made by Kirschbaum on the spreading of the full-energy proton beam in stopping in lead,²⁰ and found to be adequate.

The dE/dx counter thickness was chosen so that (1) the range bite was not so large as to make the energy loss unrepresentative of the specific ionization (and thus spoil the separation), and (2) the energy loss was not so small that the Landau effect²¹ and statistical fluctuations in the number of photoelectrons released from the photocathode would decrease the pulse-height resolution of the counter to a point that would mask the separation.

Ideally the energy counter should have sufficient depth to stop the desired elastically scattered deuterons. However, the maximum deuteron range encountered in the experiment was 44.7 gm/cm^2 of copper, and it was not thought feasible to construct a counter with good pulse-height resolution with that large a stopping power. Consequently, the energy counter in the long-range telescope was only 15 gm/cm^2 copper equivalent in depth.

Figure 4 is a schematic drawing of the two telescopes; the scintillator dimensions and counter spacings are drawn approximately to scale. The phosphor, method of optical coupling to the photomultiplier tube, and type of phototube are also shown. The liquid phosphor em-

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Fig. 4. Schematic diagrams of the two deuteron telescopes. Scintillator dimensions and counter spacings approximately to scale. All counters used phenylcyclohexane mixture except the one marked stilbene. All counters viewed end-on were square, except pass-through counter which was circular. Thicknesses in gm/cm² copper equivalent.

ployed was one developed by Kallmann²² 0.3 percent p-terphenyl and 0.001 percent diphenylhexatriene (by weight) dissolved in phenylcyclohexane. The lucite light pipes were of the adiabatic taper type discussed by Garwin,²³ except for the large energy counter, whose area in the direction of viewing was much larger than the 5819 photocathode area. In this case the pipe tapered in toward the phototube in both dimensions. The light pipes were typically 7 inches to 10 inches long, and were coupled to the tube either by gasketed lucite cups filled with mineral oil, or by a nonhardening viscous cement described by R. L. Shipp.²⁴

Accurate pulse-height distribution curves for monochromatic particles incident on the counters were not taken for all counters. However, energy calibration data taken during runs permit an estimate for the 100-Mev protons incident on the long-range telescope; for both the dE/dx and the energy counters, the distribution had about 15 percent full-width at half maximum. The width for the former is believed to be about half due to Landau effect, and about half due to photoelectron statistics.

Theoretically, for protons and deuterons giving the same energy pulse height, the dE/dx pulse height of the deuterons should be about 1.4 to 1.7 times that of the protons, depending upon the relative energy losses in the two counters. The lower figure is for larger proportions of loss in the dE/dx counter. The factor actually observed with the long-range telescope was usually about 1.4. For the short-range telescope the separation was only about 1.4 where it should have been 1.7, because of the partial saturation of the stilbene dE/dx counter at the relatively high values of dE/dx encountered.²⁵ Poor pulse-height resolution of the stilbene counter (about 25 percent) made the protons and deuterons overlap to some extent. Figures 11 and 12 in the chapter on reduction of data demonstrate these separation factors.

For the other coincidence counter, designated as the conjugate counter or proton counter, two different counters were employed at different times. One was a stilbene crystal in an aluminum reflecting housing viewed by a 1P21, and the other was a liquid scintillator very similar to the long-range telescope dE/dx counter. The heights and widths were respectively 8.8 and 4.2 cm for the stilbene counter, and 5.95 and 11.89 cm for the liquid scintillator. All photomultiplier tubes were magnetically shielded.

Electronic and Photographic Apparatus (see Fig. 3)

The coincidence circuit employed was at different times a germanium diode circuit designed by Prof. Chamberlain or a circuit that was essentially a Garwin circuit.²⁶ The chassis contained two double-coincidence circuits; of the three inputs, one was fed to both circuits. There were five outputs: one from each coincidence circuit, and three singles rate outputs, one for each input. The output signals were of sufficiently high level and long enough duration to drive a scaler directly. Pulse-height discrimination was accomplished at the inputs by biases on the input elements.

Signals from the coincidence counters were brought to the counting area by coaxial cable. The signal from one counter was amplified and fed to the input common to both coincidence circuits; that from the other was split, one leg being delayed (by a length of coaxial cable), and the other not, before being amplified and fed to the remaining inputs. All amplifiers were of the distributed type. The output of the coincidence circuit fed by the delayed signal gave a continuous monitor of the accidental coincidence rate.* All five outputs were connected to scalars; also the sweep of the Tektronix model 517 oscilloscope was triggered by either the direct or the delayed coincidence output as desired. The "gate output" of the oscilloscope was fed to a scaler to count the number of sweeps. It was found impossible to adjust the trigger amplifier control so that the number of sweeps exactly equaled the number appearing on the coincidence scaler. However, the deuteron counting rate as seen on the film was found to be insensitive within statistics even to rather large discrepancies in the two readings. This merely implies that the deuterons were well on the counting plateau.

The signals from the pulse-height counters were taken directly from the anode or last dynode of the phototubes (depending upon the sign of pulse desired) and transmitted over coaxial cable to the counting

* The fine structure of the proton beam determines the delay necessary. The particles emerge in bursts of about 0.5×10^{-8} sec. in duration separated by 6×10^{-8} sec. intervals. (About 350 bursts comprise each "beam pulse"; these beam pulses occur 60 times per second.) The coincidences observed when the signals in one channel are from one burst, and those in the other channel from the following burst are related to the accidental rate. The exact relation depends upon the processes being observed and the counter geometry employed.

area, where they were suitably delayed with respect to one another in order to separate them on the oscilloscope trace. They then passed to the signal input of the oscilloscope through a common distributed amplifier and a common delay which controlled their position on the sweep. Regulated supplies were used throughout the pulse-height circuits, and the stability as checked by a pulse generator was excellent. Since the over-all sensitivity remained quite constant during runs, but sometimes appeared different on successive days, it is thought likely that the magnetic shields for the 5819 photomultiplier tubes were affected in the process of turning off the cyclotron magnetic field in the evening or turning it on again in the morning.

The oscilloscope face was coupled to a General Radio 35-mm oscilloscope camera by a brass hood furnished with a peephole. The film was moved continuously at rates of about 1 foot per minute by a variable-speed motor. The aperture used was $f/1.5$, and the 24-kv acceleration potential of the oscilloscope was found quite ample to produce easily developable traces on Eastman Linagraph Pan film even at writing speeds of 200 cm per microsecond. The film was developed either in Eastman D-19 or concentrated Atkinson A-72.

IV EXPERIMENTAL PROCEDURE

The experimental procedure followed can best be described by giving the steps of a typical run in chronological order.

Preparation

Calculations were carried out to decide the most suitable "geometries" for the angles to be covered during the day. To be chosen for each angle were target thickness, target angle with respect to the beam, counter distances, absorber sizes, and conjugate-counter height and width. The last had to be large enough to accept with a sizable safety margin all proton partners of elastic deuterons passing through the defining counter. The formulas used for this calculation are given in appendix C. The maximizing of the rate of taking data was also a consideration.

Routine checks were made on the operating condition of the apparatus.

Lining up

After the cyclotron crew had tuned up the proton beam entering the cave so that the collimation was adjudged satisfactory on the basis of x-ray films exposed to the beam, the movable platform was adjusted to bring the scattering table into proper alignment with the beam. This was accomplished by a system of fiducial markers and the use of film exposures.

Concurrently the beam monitor circuit operation was checked.

Adjustment of Electronic Apparatus

Previously pulse generator signals had been used to set discriminator levels and amplifier gains to suitable values. With a CH_2 target in place, and the counters set to observe p-p scattering, a series of routine adjustments were then made on the electronic gear.

(1) The coincidence rates were observed visually while the photomultiplier voltages were varied to check roughly for the existence of a counting plateau and to verify that no apparent coincidence counts appeared at the outputs when high-level input pulses were fed to only one input at a time.

(2) With voltages set at reasonable points on the "visual" plateau, various cable lengths were inserted in the inputs of both coincidence circuits to find the maximum coincidence rate per unit beam. This balancing was necessary because of possible unequal delays introduced by amplifiers, cables, phototubes, etc. The delay curves were about 4×10^{-8} sec. wide at half maximum. Figure 5 shows a typical curve.

(3) Detailed counting plateaus were then taken by plotting direct coincidences per unit beam versus photomultiplier voltage; in this operation the voltage was varied on one counter at a time. As the carbon portion of the target could not be expected to give a plateau, the coincidences due to carbon had to be subtracted. This was done by subtracting the delayed coincidences as experience showed that these were very nearly equal to the carbon effect. The photomultiplier voltages were set for the remainder of the day at points well on the plateau; Fig. 6 shows a typical plateau and the operating point chosen.

(4) A check was then performed with a CH_2 target on the accuracy of the assumption that the delayed coincidences were equal to the accidentals. The beam level was varied, and the coincidences per unit beam plotted against beam level for both coincidence outputs. Figure 7 is an example. On this plot the direct coincidences curve should vary linearly with beam level, the intercept at zero beam giving the systematic coincidences per unit beam. The delayed coincidences curve should also be a straight line, but should pass through the origin. If the delayed and accidental coincidence rates were equal, the two lines would be parallel. Otherwise, the delayed values would have to be multiplied by a factor (equaling the ratio of the slopes) to get the actual accidental rate. This check always showed the slopes to be equal.

(5) A measurement of the beam energy was then carried out by plotting a Bragg curve: the ratio between the charges received by two ionization chambers placed in the beam with a variable amount of copper absorber inserted between them, plotted as a function of absorber thickness. Figure 8 shows a typical curve; the 0.8 maximum point was taken as indicating the beam energy.²⁷ The value found for the beam energy was usually 340 Mev; on one occasion it was 346, but there may have been an error in the measurement.

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Fig. 5. Typical delay curves.

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Fig. 6. Typical plateau.

Fig. 7. Typical comparison of direct and delayed coincidence rates to find factor relating delayed and accidental coincidence rates.

Fig. 8. Typical Bragg curve.

(6) The gains of the pulse height counters were then adjusted. Sufficient absorber was used so that the p-p scattered protons would stop in the energy counter. The energy losses in the pulse-height counters of these protons had previously been calculated from range-energy curves and compared with the energy losses expected from the elastic deuterons at the angles to be covered on that run. The photo-multiplier voltages of the pulse-height counters were adjusted so that the pulses seen on the oscilloscope were of such height that at no elastic p-d angle would the desired deuteron pulses saturate the amplifying circuits. The trigger amplifier control of the oscilloscope was adjusted so that the sweep rate exceeded slightly the coincidence rate. Camera focus and oscilloscope focus and intensity were checked.

(7) Some of these p-p scattered pulses were then photographed to give energy calibration points for interpretation of the deuteron data.

Elastic p-d data

(1) The counters were set at calculated positions for observing a given case of elastic p-d scattering. Using a CD_2 target, the beam level was adjusted to a suitable value; we did not want the real/accidental ratio to be either too high or too low, nor to have too many spurious pulses on the oscilloscope traces.

(2) Data were then taken on the film with six arrangements: in turn CD_2 , C, and blank ($\equiv$ no) targets were put in position; with each the direct and the delayed coincidence outputs were used to trigger the oscilloscope. Deuterons from the blank target were found to be virtually nonexistent, and as they would represent only a very small correction to the calculated cross section, in some runs no blank data were taken.

(3) Periodic checks on stability of the pulse-height circuits were made by photographing the pulses observed when a pulse generator was connected to points in the circuits preceding the amplifier input. The stability was found to be quite satisfactory, so the frequency of the checks was decreased in later runs.

(4) Further checks performed to verify that the desired effect, and only that effect, was being measured are discussed in the chapter on presentation of data.

(5) For measurements at 130° and 150° center-of-mass scattering, the deuterons were too energetic to be conveniently stopped in the energy counter. The method used to separate the inelastic and elastic effects at these angles is discussed in the next chapter following an explanation of the reading of the films.

Following the run

The film was developed and read. Reading of the film is described in the next chapter.

V REDUCTION OF DATA

Reading of film -- general

Reading of the developed film was done on a microfilm viewer, on the screen of which was superimposed a lucite grid with rulings every 2 mm. Pulse heights could be read to 1 mm, and typical heights encountered were from 10 to 60 mm. The sweep speed normally used was 10^{-7} second/cm on the oscilloscope face, and the pulses were about 4×10^{-8} second wide at their bases. The camera reduced the pulse size by a factor of 4 on the 35-mm film, while the viewer gave an image 10 times as large as that on the film.

A typical section of film is shown in Fig. 9. A single high-energy proton or deuteron would produce a pattern of two pulses (or three, if it were energetic enough to pass through the energy counter) at a position on the sweep that could vary slightly owing to time jitter in the sweep triggering. The relative spacings of the two or three pulses, which were determined by the delays introduced for that purpose, were not always exactly the same but varied somewhat with the position of the pattern on the sweep. This was because of a slight nonlinearity in the sweep. However, the position and internal spacings of the pattern were sufficiently uniform so that, with extremely few exceptions, it was always possible to know which pulses should be read even when spurious pulses due to additional particles were found on the sweep.

The heights of the pulses from the dE/dx and energy counters were tabulated for each pattern; only the presence or lack thereof was recorded for the pulses from the pass-through counter. The pair of numbers from each pattern was plotted as a point on graph paper with dE/dx pulse height as ordinate, and energy pulse height as abscissa. Patterns without a pass-through pulse were plotted as dots, and those with one as crosses. Figure 10 shows the typical results (for the long-range telescope) when several hundred points were plotted; a CD_2 target was in the beam in this case. The general features of the plot are as follows: (1) a band identifiable as the locus of protons, (2) an "island" due to a group of nearly monoenergetic deuterons (presumed to be from elastic p-d events),

Fig. 9. Typical section of film, showing shape and spacing of pulses. d indicates a deuteron, p a proton, b a blank sweep (no particle) and x a pass-through (a particle not stopped in the energy counter).

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Fig. 10. Typical plot of data from film for the long-range telescope (70° center-of-mass scattering). See text for explanation of the features of the plot.

(3) a tail consisting almost entirely of points from patterns including a pass-through signal (crosses), and (4) a few widely scattered points.

The collection of deuterons from p-d events into an island, despite the variation of the deuteron energy across the finite width of the defining counter, was accomplished by a wedge of aluminum absorber, as mentioned previously.

The tail is due to protons which are too energetic to be stopped in the energy counter; they produce a dE/dx pulse height which is proper for their energy, but a pulse height from the energy counter which is much below the value observed for those which end their range just barely inside the energy counter. In a sense, both counters are dE/dx counters in this region, and, for both, the pulse heights decrease as the particle energy increases. A very energetic deuteron of course falls into a similar tail belonging to the deuteron locus. Furthermore, the deuteron and proton tails coincide in part.

Some of the widely scattered points are due to particles which scatter out of the counters and thus do not lose in the counters the proper amounts of energy, and some to the simultaneous passage through the telescope of two scattered particles. The probability of the latter type of event was very small at the counting rates employed; the duration of the pulses was short enough so that only when two or more particles were scattered into the telescope during the same burst (of 0.5×10^{-8} sec. duration*) would their pulses be superimposed so as to appear as a single pulse. Some of the widely scattered points may be tritons, though none have been positively identified as such.

Of all the sweeps photographed in a typical case with long-range telescope and CD_2 target and triggering by the direct-coincidence output, approximately 3 percent of the sweeps showed deuterons from elastic p-d events. The remaining sweeps could be grouped approximately as follows: sweeps with no pulses of readable size, 45 percent; sweeps with only one pulse, 8 percent; sweeps exhibiting pass-through pulses, 21 percent; and all others (mainly protons), 23 percent. About one percent of the sweeps gave evidence either of two scattered particles

* See footnote on page 18 on the structure of the beam.

in the same burst, or of the presence of tritons. This one percent was of course distributed over a wide area on the graph, and the probability that any apparent deuterons in the island were due to superposition was very small. Probably no more than two percent of the deuterons in the island could be due to this cause. The large number of sweeps with no pulses could be attributed to two causes: (1) low-energy products of the bombardment and (2) particles scattered from the beam collimator at such angles that only the defining counter of the telescope was traversed. In later runs, more adequate shielding from the collimator and use of thin absorbers in front of the coincidence counters reduced this percentage. With the short-range telescope the number was considerably less, because the defining counter was also the dE/dx counter.

Procedure for reading deuteron data

The analysis to separate deuterons from protons was carried out by three different methods: one for the $70^\circ - 110^\circ$ range of center-of-mass scattering, one for the $130^\circ - 150^\circ$ range, and one for the $40^\circ - 50^\circ$ range. We will discuss each method in the above order.

In the following we will designate data taken with a CD_2 target and with triggering of the oscilloscope from the direct-coincidence output as " CD_2 -direct" data and use analogous terms for the other arrangements; for example, "C-delayed".

(1) $70^\circ - 110^\circ$ range

For this group of angles the long-range telescope was employed, and aluminum absorbers (up to two inches in thickness) were used, if necessary, to cause the deuterons to stop in the energy counter.

The first step in reading the film was to read that taken with p-p scattering. A plot of these data gave a small island on the proton locus; from its position and the computed energy losses in the dE/dx and E counters for these protons, a calibration of the pulse heights on the viewer in terms of energy loss was obtained. The expected position of the deuteron island for elastic p-d deuterons for the angle of scattering to be read was then calculated. Owing to slight saturation effects in the energy counter, the actual island was usually observed at a slightly different position from that calculated.

Next, several hundred sweeps of CD_2 -direct data were read and plotted to establish the approximate locations of the proton locus and the deuteron island, and to check that the general behavior of the apparatus was as expected. After this check, only those sweeps were read which had a dE/dx pulse height greater than one-half that for particles in the approximate island; the other sweeps were merely counted. This procedure reduced considerably the time for reading the film, and by including neighboring portions of both the proton and deuteron loci, not only increased the number of points on the basis of which to determine the island boundaries, but also gave a running check on the stability of the pulse-height system. CD_2 -delayed, C-direct, C-delayed, blank-direct, and blank-delayed data were then also read on the basis of the same criterion for the dE/dx pulse height.

It was necessary to choose, on the basis of all sweeps read, the outlines of the island on the graph which would be taken to include all elastic p-d deuterons. While the separation of protons and deuterons was reasonably unambiguous, the presence on the deuteron locus of a few deuterons of varying energies due to the carbon portion of the target made it unreasonable to count all observed deuterons on the locus as coming from elastic p-d scattering. Despite the smallness of the number of deuterons, the grouping was close enough that the outlines of the island could be drawn without much difficulty. (Islands of larger sizes were tried, and it was found that the number of deuterons in the added area, after subtraction of carbon counts and accidentals, was zero within statistics.) The outline on Fig. 10 shows the island chosen in one case. (The outline was drawn on the basis of many more points than are shown in the figure.)

Using the island thus chosen, the number of deuterons for each of the six arrangements of target and triggering was counted. The subtraction of carbon counts and accidental counts is discussed in a later section.

(2) 130° - 150° range

For these angles the deuterons to be observed had ranges from 36 to 45 gm/cm^2 of copper. The amount of absorber (placed in front of the telescope) that would be necessary to cause these deuterons to stop in the energy counter was considered excessive. Consequently, only about one

inch of aluminum absorber was used; the desired deuterons were thus allowed to fall on the "tail" of the deuteron locus. Because of the partial overlap of the proton and deuteron "tails", there was the possibility that some of the inelastic protons could be confused with the deuterons. Consequently, absorber was placed in front of the proton counter to reduce the effect of the inelastic scattering, as discussed in Chapter II. Film data were then taken not only with the proton counter at the proper angle to observe the elastic events, but also at angles to either side of the elastic angle, to evaluate the amount of inelastic scattering included.

The reading of the film containing the data taken in the above manner was carried out in the same steps as for the 70° - 110° range: energy calibration, reading of data for all six arrangements of target and triggering, and choice of the deuteron island (now on the "tail"). Only sweeps with pass-through pulses were read, however, after the first several hundred.

The number of points found in the island when the proton counter was set at other angles than the elastic angle was found to be zero within statistics after the subtraction of contributions due to accidentals and carbon. We therefore felt justified in considering that all the apparent deuterons at the elastic angle were indeed deuterons, and the statistical counting errors were computed on that basis. However, the systematic error in the separation of protons and deuterons has been taken somewhat larger for these angles than for the 70° - 110° range.

(3) 40° - 50° range

For this group of angles the short-range telescope was used, and absorbers were employed, if necessary, to reduce the range of the elastic deuterons.

As previously mentioned, the separation of protons and deuterons was not so good in this telescope as with the other. Consequently, the data from the film had to be treated in a different manner.

Let us consider again the method of plotting points on a graph of dE/dx pulse height versus energy pulse height. The proton and deuteron loci are approximately parallel curves (as can be seen from Fig. 10, for example). If a group of equally spaced curves parallel to the proton curve is drawn in, and the distribution of points in the "channels" thus created is tabulated, the result is an approximate mass spectrum

of the particles seen by the telescope. In our work the data from p-p scattering can be used to determine the proton locus as the basis for drawing the channels. Fig. 11 shows, for example, the mass spectrum obtained with a portion of long-range telescope data.

The data for 40° and 50° were therefore treated as follows: first the film with the p-p scattering data (CH_2 target) was read and plotted. These data were taken without a wedge preceding the telescope in order that a band of proton energies would be accepted. Channels (different of course from those in Fig. 11) were then drawn in and the mass spectrum of these protons tabulated. Figure 12C shows the spectrum obtained. Then CD_2 -direct data were read and plotted as a mass spectrum using the same channels; Fig. 12A shows the result for a portion of the 40° runs. The proton portion of this curve was then subtracted, channel by channel, utilizing the proton spectrum from p-p scattering as a standard. As the number of protons in the proton portion of the CD_2 data was not the same as in the p-p scattering, the proper ratio to be used in the subtraction was determined by a weighted average of the observed ratios for each of the channels 0 through minus 3 separately, the weights being assigned according to the error in the ratio for each channel due to counting statistics. Figure 12B shows the deuteron spectrum obtained after the subtraction. The errors shown for this curve are the standard deviations resulting from combining the errors from the proton spectrum, from the ratio calculation, and from the number of counts in each channel for the CD_2 data. The counts in channels 0 through 8 were then totaled, and the standard deviation of the total determined by combining those from each channel.

This same treatment was also applied to data from the other arrangements of target and triggering.

Because of the uncertainties involved, the systematic error due to the separation method has been considered larger for these angles than for the 70° - 110° range.

Subtraction of effects due to carbon and accidental coincidences

It has been mentioned several times that deuterons from the carbon target were observed. This was rather surprising since the cross

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Fig. 11. "Mass spectrum" for a portion of data taken with long-range telescope.

Fig. 12. "Mass spectra" for a portion of the 40° center-of-mass scattering data.
 A. With CD_2 target, showing both protons and deuterons.
 B. Same but with protons subtracted by method explained in text.
 C. Proton spectrum used in subtraction (from p-p scattering).

section expected for pick-up²⁸ at 340-Mev bombarding energy is very low, and especially since the detecting apparatus used required a coincidence of the deuteron with a charged particle at an angle with respect to the deuteron which was characteristic of p-d scattering (94° to 137° in the present experiment). Two possible mechanisms were developed to explain the observations. One was that the beam proton is able to knock a complete deuteron out of the carbon nucleus, thus approximating the kinematics of a p-d elastic process. The angle between the emergent particles, though not a unique quantity because of internal momentum, would probably be peaked at the same values as for elastic p-d scattering.

The second mechanism, thought to be more likely, was that a nucleon-nucleon collision takes place inside the carbon nucleus, and that the recoil (or even the deflected) particle in emerging from the nucleus picks up a partner at the surface to form a deuteron. The pick-up probability for the beam proton would be enhanced by the reduction in momentum following the collision. The angle between the proton and deuteron for this process would vary, but perhaps be peaked near 90° . It might therefore be possible to distinguish between these two mechanisms by measuring the deuteron rate as a function of the angle between the two counters.

In the present experiment, however, the deuterons from carbon were treated only as background, and no attempt was made to study them further. It was subsequently learned that the second process is the topic of a theoretical paper by Bransden.²⁹ Hess³⁰ is at present investigating experimentally high-energy deuteron production from various elements under bombardment by 340-Mev protons.

In the present experiment, CD_2 -direct data contained deuterons not only from the desired events but also from carbon and from accidental coincidences. Those from the last two causes had to be subtracted. We will develop now the formulas that were used. We define the following symbols:

$$CD_2\text{-dir} = \text{observed deuterons/unit time from } CD_2 \text{ target with triggering from direct-coincidence output}$$

analogously for CD_2 -del, etc.

$$D_d = \text{number of deuterons per unit time incident on deuteron telescope and due to deuterium in target}$$

- D_c = number of deuterons per unit time incident on deuteron telescope and due to carbon in target
- P_d = number of charged particles per unit time incident on conjugate counter and due to deuterium in target
- P_c = number of charged particles per unit time incident on conjugate counter and due to carbon in target
- f = fraction of deuterons from carbon which (1) strike the telescope and (2) have partners striking the conjugate counter
- τ = effective resolving time of the coincidence circuits

In the quantities D_d and D_c , only those deuterons which are in the island chosen are to be included.

We can then write down the following formulas for the coincidence rates (we have simplified some expressions by using the fact that $D_c \ll P_c$):

$$CD_2\text{-dir} = D_d + fD_c + 2\tau (1-f) [D_c P_d + D_c P_c]$$

$$CD_2\text{-del} = 2\tau [(D_d + D_c) (P_d + P_c)]$$

$$C\text{-dir} = fD_c + 2\tau (1-f) D_c P_c$$

$$C\text{-del} = 2\tau D_c P_c$$

In deriving these expressions we have assumed that the geometry is such that the partners of all deuterons from deuterium strike the conjugate counter and that therefore in the case of triggering from the direct-coincidence output, these deuterons cannot create any surplus counts due to accidental coincidences with an uncorrelated particle entering the conjugate counter. (Similarly for the fraction f of deuterons from carbon.) However, in the case of triggering from the delayed-coincidences output all deuterons contribute to the counting rate by making accidental coincidences.

The quantity that we must subtract from $CD_2\text{-dir}$ to obtain the counting rate for elastic p-d scattering (which is D_d) can be seen to be $fD_c + 2\tau (1-f) [D_c P_d + D_c P_c]$. There is no way of subtracting exact-

ly this quantity unless the singles rates and τ are known. However, we note that C-dir differs from this quantity by only a small amount, which is $2\tau(1-f)D_cP_d$. Furthermore, we can estimate the error made in calculating D_d by the formula $D_d = (CD_2\text{-dir}) - (C\text{-dir})$ by using the expression

$$\frac{2\tau(1-f)D_cP_d}{CD_2\text{-del}} = \frac{(1-f)}{\left(1 + \frac{D_d}{D_c}\right)\left(1 + \frac{P_c}{P_d}\right)}$$

For one of our cases (90° center-of-mass scattering) we have data from Hess³⁰ for the cross section for the deuteron production from carbon, which enables us to evaluate f and D_d/D_c . The ratio P_c/P_d may be calculated from the observed singles rates. We obtain upon substitution of these numbers $2\tau(1-f)D_cP_d = 0.09(CD_2\text{-del})$. $CD_2\text{-del}$ was observed in one run at this angle to be $0.37 D_d$, so in this case the error made in the subtraction by the above formula was to give a value for D_d only 3 percent too high. In most of the experiment, moreover, the counting rates were low enough that $CD_2\text{-del}$ was a good deal smaller than in the case above. Consequently, the formula adopted for subtraction of deuterons from carbon and accidentals was $D_d = (CD_2\text{-dir}) - (C\text{-dir})$, and a systematic error of 3 percent was assigned for the subtraction.

Note was also taken of the ratio of the number of carbon atoms/cm² in the CD_2 target to that in the C target; in most cases this ratio was about 1.05. As previously stated, the deuteron rates from the blank target, when measured, were always zero, so no account of blank had to be taken in making the subtraction.

Corrections for attenuation in absorbers and counters

Corrections to the observed data had to be made because of attenuation by inelastic scattering and by stripping of the deuterons, both of which could take place in the absorbers and in the counters themselves.

For attenuation of the elastic protons due to the absorber that precedes the conjugate counter employed in some cases, the cross sections measured by Kirschbaum³¹ were used.

For attenuation of the deuterons, a total cross section for all processes except diffraction scattering for deuterons of an average energy of about 140 Mev was available.³² As data for other deuteron energies was not available, and as it is believed that the attenuation cross section is not strongly dependent on energy, the value for 140 Mev was used throughout. An error of 20 percent was assigned to the cross section employed.

In the calculation for deuteron attenuation, all counters preceding the energy counter were treated as absorbers, because any processes deflecting or breaking up the deuteron in these counters would cause the deuteron either to be lost completely or to give a pulse-height pattern not characteristic of a deuteron. In the energy counter, however, a few stripping events would not cause a reduction of the energy pulse height large enough to make the deuteron have a pulse-height pattern not in the deuteron island. The number of such events was estimated, and the error in the estimate was assumed to be 50 percent.

Table I in the next chapter shows (along with the experimental results) the correction factor for attenuation which was applied at each angle.

Target

The target material was analyzed chemically and found to have a ratio of total hydrogen to carbon content of 1.97 ± 0.02 (numbers of atoms). Further analysis by mass spectrometry showed the deuterium fraction (by number) of the hydrogen content to be (94 ± 1) percent. Both analyses were carried out by Dr. Amos Newton of this laboratory.

A very thin foil of the target material was also analyzed by infrared absorption techniques. This analysis, which was done by Professor George Pimentel of the University of California Chemistry Department, gave values from 5 percent to 7 percent for the H^1 fraction.

A value of (94 ± 1) percent was therefore used in the calculations, and the number of carbon atoms was taken as one-half the number of H^1 plus H^2 atoms.

Sample calculation

We give as an example the calculation of the first case of 110° scattering listed in Table I.

Targets. The CD_2 target was 0.1312 gm/cm^2 thick, and therefore contained 0.941×10^{22} deuterons per cm^2 and 0.497×10^{22} carbon atoms per cm^2 . (The purity of the target was of course taken into account.) The CD_2 target was oriented with its normal 11.2° from the beam, so the surface density of deuterons in the beam direction was $N = 0.941 \times 10^{22} / \cos 11.2^\circ = 0.959 \times 10^{22}$, and of carbon atoms was 0.507×10^{22} . The carbon target was at 12° to the beam, giving 0.530×10^{22} carbon atoms per cm^2 in the beam direction.

Beam current. One volt on the recording millivoltmeter (one "integrator volt") indicated $C/q\mu$ protons in the beam, where C was the capacity employed (0.985×10^{-6} farad), q the elementary electronic charge (1.602×10^{-19} coulomb), and μ the "multiplication factor" of the ion chamber. This multiplication factor (not related to gas multiplication as in proportional counters) is due to the formation of a large number of ion pairs by each proton passing through the chamber, and may be defined as the ratio of the charge received by the ionization chamber caused by the passage of a single proton to the charge carried by the proton.

The calibration with the Faraday cup had shown that $\mu = 1040$ for 340-Mev protons in an ionization chamber of effective depth of 2 inches filled with argon to 100 cm Hg absolute pressure at 21°C . The chamber employed in taking the data under discussion had a depth of 2.028 inches, and was filled to 82.5 cm Hg at 21.0°C . Therefore

$$\mu = 1040 \times \frac{82.5}{100} \times \frac{2.028}{2} = 867.4$$

Hence one integrator volt was equivalent to $n = 0.985 \times 10^{-6} / (1.602 \times 10^{-19}) (867.4) = 0.709 \times 10^{10}$ protons.

Solid angle. The face of the defining counter was $2.00 \times 2.05 \text{ cm}$, or 4.10 cm^2 in area, and the midpoint of the counter was located at 80.2 cm from the target position. The solid angle subtended was therefore $\Delta\Omega = 4.10 / (80.2)^2 = 6.38 \times 10^{-4}$ steradian.

Number of events observed. From analysis of the films, we had the following numbers of events and their associated statistical errors:

$$\text{CD}_2\text{-dir} = 49 \pm 7 \text{ in } 10.224 \text{ I.V.}, \text{ or } 4.79 \pm 0.69 \text{ per I.V.}$$

$$\text{C-dir} = 1 \pm 1 \text{ in } 3.074 \text{ I.V.}, \text{ or } 0.325 \pm 0.325 \text{ per I.V.}$$

$$\text{blank-dir} = 0 \pm 1 \text{ in } 0.509 \text{ I.V.}, \text{ or } 0 \pm 1.96 \text{ per I.V.}$$

Therefore the number of elastic p-d events per integrator volt was

$$D = 4.79 \pm 0.69 - 0.957 (0.325 \pm 0.325) - (1 - 0.957) (0 \pm 1.96) =$$

4.48 ± 0.762 , where 0.957 is the ratio of the number of carbon atoms per cm^2 (perpendicular to the beam) in the carbon target to that in the CD_2 target.

Cross section. Therefore the laboratory cross section, uncorrected for attenuation, was

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{lab.}} = \frac{D}{nN\Delta\Omega} = \frac{4.48 \pm 0.0762}{0.709 \times 10^{10} \times 0.959 \times 10^{22} \times 6.38 \times 10^{-4}} =$$

$$0.103 \pm 0.018 \text{ mb/ster.}$$

To convert to the center-of-mass system, $d \cos \theta_{\text{lab.}} / d \cos \theta_{\text{c.m.}} = 0.293$, giving $(d\sigma/d\Omega)_{\text{c.m.}} = 0.0303 \pm 0.0053 \text{ mb/ster.}$

Attenuation correction. No absorber was present in front of the proton counter. For that in front of the deuteron telescope, and for the telescope counters themselves, we used $I/I_0 = \exp(-\rho\sigma x)$ where I_0 = the number of elastically scattered deuterons actually incident on the telescope, I = the observed number of deuterons, ρ = numerical density of nuclei, σ = attenuation cross section per nucleus, and x = thickness of absorbing material in cm. The resulting correction factor was 1.59 ± 0.12 , and the final value for the cross section was therefore 0.0482 mb/ster with 17 percent standard deviation counting error, as listed in Table I.

VI PRESENTATION OF DATA AND DISCUSSION OF ERRORS

Table I shows the experimental results obtained. For each center-of-mass scattering angle studied, the following quantities are tabulated: (1) corrected center-of-mass differential cross section in millibarns per steradian; (2) absorber correction factor employed; (3) statistical error (standard deviation); (4) error in absorber correction; (5) error in counting deuterons in the film analysis (separation of protons from deuterons and choice of deuteron island); (6) combined systematic error from other sources; and (7) total error, combined from all sources. Measurements taken under different experimental conditions are shown on separate lines.

In Fig. 13, the averaged differential cross section in the center-of-mass system is plotted on a logarithmic scale versus center-of-mass scattering angle. In this figure the error indicated is the total error.

During the course of the experiment several checks were performed to make certain that all the elastic p-d scattering events were being observed, and that no other processes were contributing to the apparent counting rates. Some of these checks were on the operation of the electronic apparatus, as discussed in the chapter on experimental procedure. Two others, the results of which could be known only after reading the film, will be discussed now.

It was necessary to make sure that the conjugate-counter solid angle as calculated was sufficiently large to catch all proton partners to elastic deuterons striking the defining counter. An example of this check is found in the comparison of the two lines for 90° scattering in Table I. Data on the second line were taken with a solid angle twice as large as for the data on the first line. Another example is in the comparison of the two lines of data for 70° scattering, which were taken on different runs; for the second line the solid angle was 66 percent larger than for the first line. However, in neither example do the counting statistics allow any definite conclusion. Nevertheless, because the formulas used in calculating the solid angle were based on similar formulas used in previous work by Stern and by Chamberlain, Segrè, and Wiegand, and because a liberal safety margin was allowed, we do

TABLE I --- EXPERIMENTAL RESULTS AND ERRORS

angle (c.m.)	$\frac{d\sigma}{d\Omega}$ (c.m.)	attenuation correction factor	statistical error (s.d.)	error in att'n corr.	error in separation	other syst. errors	total error
40°	1.04 (mb/ster)	1.05	8%	2.0%	15%	10%	20%
50°	0.392	1.07	11	2.0	15	10	21
70°	0.104	1.11	32	2.2	5	9	16
	0.109	1.11	13	2.2	5	9	
	0.108 (avg)	1.11	12	2.2	5	9	
90°	0.0402	1.28	18	5.8	5	8	16
	0.0581	1.28	23	5.8	5	8	
	0.0468 (avg)	1.28	12	5.8	5	8	
110°	0.0482	1.59	17	9.8	5	8	17
	0.0399	1.57	13	10.0	5	8	
	0.0424 (avg)		10		5	8	
130°	0.0440	1.50	20	8.6	10	9	26
150°	0.101	1.46	11	8.6	10	11	21

The cross sections given have been corrected for attenuation. Angles are known to 2° in the center-of-mass system, and resolution in the center-of-mass system is 5.7° for 40° and 50°, and 2.8° for the others.

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Fig. 13. Differential cross section for elastic p-d scattering. Both coordinates in center-of-mass units. Total (statistical and systematic) errors shown. For comparison, the theoretical cross section discussed in Chapter VII is included.

not believe that more than a few percent of the proton partners were not counted.

A second check carried out was to verify that the deuteron counting rate was peaked at the calculated angle for the elastic events, and zero to either side. Figure 14 shows the deuteron counts per integrator volt as a function of the angular setting of the proton counter. The peak is seen to exist and to be at the expected angle. This check also shows that the conjugate-counter solid angle is not seriously in error.

Because the very low counting rates per unit time for the elastic events required that most of the running time be spent in measuring the cross section, these two checks were not performed at every angle. The reliability of the formulas used in calculating the angular settings of the counters was considered well established by those checks that were performed. The adequacy of the solid angle of the conjugate counter was less well established.

A word should be said about the reasons why the measurements were not extended to center-of-mass scattering angles less than 40° or greater than 150° . The fundamental reasons are that the range of the proton for angles larger than 150° is too small to allow the coincidence method to be used reliably, and that the range of the deuteron for angles smaller than 40° is too small to permit the identification method to be employed successfully. In addition, for angles greater than 150° , the deuteron telescope angle would be well into the region of diffraction scattering from carbon, and the accidental coincidences would therefore be excessive.

Discussion of errors

The following is a list of the known sources of systematic error; the size of the error in the cross sections which was assigned to each of these sources is also given.

1. Geometry

a. The calibration of the scattering table angles is accurate to about $1/2$ degree.

b. The alignment of the table with respect to the beam was accurate within 1 degree. This results in a 5 percent uncertainty in

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Fig. 14. Elastic peak as shown by the variation of the number of deuterons from CD₂ per integrator volt versus angle of proton counter with respect to beam. Deuteron counter was fixed at 54°.

the cross section at 40° , 50° , 70° , and 150° , and negligible uncertainty at the other angles, where the cross section is slowly varying.

- c. The distance of the defining counter was known to 5 mm. in 50 or more cm. (2 percent error in solid angle)
- d. Measurement of the defining counter area (1 percent).
- e. Uncertainty in effective defining counter area (long-range telescope) due to the lucite box surrounding the liquid scintillator (3 percent).
- f. Possibility of nonuniform sensitivity over the area of the stilbene defining counter (short-range telescope) (3 percent).
- g. Loss due to insufficient conjugate-counter solid angle (3 percent).

2. Beam current

- a. Calibration of ionization-chamber multiplication factor (2 percent).
- b. Saturation of ionization chamber (i.e., collection of all ions) (1 percent).
- c. Calibration of condenser (0.3 percent).
- d. Recorder accuracy and reading of recorder (1 percent).

3. Targets

- a. Polyethylene target composition (carbon/hydrogen ratio) (2 percent).
- b. Polyethylene target purity (deuterium/total hydrogen ratio) (1 percent).
- c. Grams/cm² of targets (2 percent).
- d. Target angle with respect to beam was known to 2 degrees. Part of the error was contributed by uncertainty in alignment of scattering table with respect to beam. This results in an error of about 3 percent at 40° and 50° , 2 percent at 70° , 1 percent at 90° and 110° , 5 percent at 130° , and 6 percent at 150° .

4. Subtraction of counts from carbon and accidentals (3 percent)

5. Telescope efficiency

- a. Losses due to multiple Coulomb scattering and diffraction scattering (3 percent).
- b. Increase in number of apparent deuterons due to simultaneous passage of two particles through telescope (2 percent).
- c. Increase in number of deuterons due to pick-up by protons incident on telescope is negligible. (Hadley and York²⁸ report that the total cross section for 90-Mev incident neutrons for production of deuterons of energy greater than 27 Mev and between 0° and 25° is only 12 millibarns for carbon.)

6. Counting of deuterons (separation of protons from deuterons and choice of deuteron island) has been assumed to contribute 15 percent at 40° and 50° , 5 percent at 70° , 90° , and 110° , and 10 percent at 130° and 150° .

7. Attenuation corrections contribute errors which range from 2 percent to 10 percent; these are listed separately in Table I.

8. Variation in beam energy could contribute as much as 4 percent error, as the cross section changes fairly rapidly with energy.

VII CONCLUSIONS

Comparison with theory

Theoretical analysis of high energy nucleon-deuteron scattering has been carried out by Chew^{7,16,33,34}, Wu and Ashkin³⁵, Gluckstern and Bethe³⁶, Bransden³⁷, Horie, Tamura, and Yoshida³⁸, and Chamberlain and Stern³⁹. Some of these authors have treated the problem with central forces only, and some have also considered tensor forces. The approximations that have been used include the Born approximation and the impulse approximation. From a survey of these papers it may be concluded (1) that tensor forces must be included to obtain better agreement with experiment (as is true for nucleon-nucleon scattering also) and (2) that the best approximation available -- the impulse approximation -- is reliable only for fairly small-angle scattering (less than about 100° center of mass), which involves only small momentum transfers to the deuteron.

As no published theoretical predictions giving numerical values for the present experiment exist, we will make some approximate calculations with which to compare our experimental results. We adopt the impulse approximation; further simplifying assumptions will be discussed later.

The impulse approximation, originated by Chew, is based on the following conditions in the problem of nucleon-deuteron scattering: (1) the de Broglie wavelength of the incident nucleon must be small compared to the size of the deuteron; (2) the collision time must be short compared to the period of the deuteron. These assumptions are equivalent to considering that the scattering of the incident wave is done by two scattering centers acting independently and suddenly. By "independently" it is meant that the scattering at one center is affected only negligibly by the scattering at the other; both centers of course perturb the incident wave, and the interference is coherent. It is believed that it is not necessary that the perturbation of the incident wave be small, in contrast to the condition required for the validity of the Born approximation.

We now summarize some results of Chew. The following general formula relates nucleon-nucleon scattering parameters to the elastic proton-deuteron cross section:

$$\frac{d\sigma_{p-d}}{dK^2} = \left\{ \left| r_{np}^o + r_{pp}^o \right|^2 + \frac{2}{3} \left| \underline{r}_{np}' - \underline{r}_{pp}' \right|^2 \right\} S(\underline{K})$$

where $\underline{K}$ is the momentum transfer

$S(\underline{K})$ is the sticking factor (i. e., probability that the deuteron will remain bound after suffering a momentum change of $\underline{K}$)

r_{np}^o is the amplitude for n-p scattering without spin-flip

$\underline{r}_{np}'$ is the amplitude for n-p scattering with spin-flip

similarly for r_{pp}^o and $\underline{r}_{pp}'$.

We note that

$$\frac{d\sigma_{np}}{dK^2} = \left| r_{np}^o \right|^2 + \left| \underline{r}_{np}' \right|^2$$

and similarly for

$$\frac{d\sigma_{pp}}{dK^2}$$

We see that if the division between spin-flip and non-spin-flip scattering were known, we would have an immediate relation between the n-p and p-p experimental results and those for p-d scattering. However, this division is not known experimentally.

(In the case of no coupling between spin and orbital angular momentum,

$$r^o = \frac{1}{4} (3r^t + r^s), \quad \underline{r}' = \frac{1}{4} (r^t - r^s) \underline{\sigma}_p$$

where r^t and r^s are the triplet and singlet amplitudes respectively, and $\underline{\sigma}_p$ is the proton spin. However, again we have no experimental knowledge separating singlet and triplet scattering amplitudes.)

Therefore to evaluate $d\sigma_{pd}/d\Omega$ in terms of experimental nucleon-nucleon results in order to compare the present results with a theoretical curve, we must make some further assumptions. (The

alternative is to make extensive calculations, based on assumed n-p and p-p interactions.) We follow Chew (reference 33) in assuming that (1) S scattering only takes place and (2) there is no spin dependence except that forced by the Pauli principle. Chew then obtains

$$\frac{d\sigma_{pd}}{d\Omega} = \frac{16}{9} \left\{ \frac{d\sigma_{np}}{d\Omega} + \frac{3}{4} \frac{d\sigma_{pp}}{d\Omega} + \left[\frac{d\sigma_{np}}{d\Omega} \frac{d\sigma_{pp}}{d\Omega} \right]^{\frac{1}{2}} \cos \Delta \right\} S(\theta)$$

where θ is the center-of-mass scattering angle,

$d\sigma_{np}/d\Omega$ and $d\sigma_{pp}/d\Omega$ are center-of-mass differential cross sections for the same laboratory bombarding energy as in the p-d case,

Δ is the difference in phase shift between n-p and p-p scattering. The factor 16/9 arises from evaluating the momentum transfer $\underline{K}$, which for the two-body problem is $\underline{K} = k \sin(\theta/2)$ and for the three-body problem is $\underline{K} = (4/3) k \sin(\theta/2)$, where in each expression k is the incident nucleon laboratory momentum.

The assumption of S scattering requires that $d\sigma_{np}/d\Omega$ and $d\sigma_{pp}/d\Omega$ be constants. Of course $d\sigma_{np}/d\Omega$ is angular-dependent, but a suitable average value is not far from the truth when only the middle range of scattering angles is being considered.

We further assume (with Chew) that $\Delta = 0$. This would require $d\sigma_{np}/d\Omega = d\sigma_{pp}/d\Omega$ but we assign, respectively, values of 2 mb/steradian (extrapolated to 340 Mev from 90- and 260-Mev n-p results; references 40 and 41) and 4 mb/steradian (from 340-Mev p-p scattering; reference 18). The effect of $\Delta = 0$ is taken as strictly true only in evaluating $\cos \Delta$.

We obtain $d\sigma_{pd}/d\Omega = 13.95 S(\theta)$ mb/steradian.

The sticking factor $S(\theta)$ is defined in reference 16 (and 33)

by

$$S^{\frac{1}{2}}(\underline{K}) = \int d\underline{r} \psi_0^2(\underline{r}) e^{\frac{i}{2}(\underline{K} \cdot \underline{r})}, \quad \underline{K} = \frac{4}{3} k \sin \frac{\theta}{2}$$

where $\psi_0(\underline{r})$ is the ground state wave function of the deuteron. We

have assumed $\psi_0(r)$ to be the Hulthen wave function

$$\left[\frac{a\beta(a+\beta)}{2\pi(a-\beta)^2} \right] \frac{e^{-ar} - e^{-\beta r}}{r}$$

and have evaluated a and β from recent values for the binding energy of the deuteron and the triplet effective range.⁴¹

The resultant theoretical curve is shown in Fig. 13, along with the present experimental results. The agreement we obtain is not very good, but is no worse than that obtained by Chew using the same assumptions in treating elastic p-d scattering at lower energies.³⁴ We note in particular that up to about 110° the angular dependences of the theoretical and experimental cross sections agree; this is because of the dominance of the sticking factor in determining the angular variation. For the smaller angles of scattering, the rise in the experimental n-p cross section would in a more exact theoretical treatment cause the theoretical curve to rise more rapidly. Beyond 110° the present theory is in very poor agreement, as it does not take into consideration the pick-up process which causes the rise evident at 150° . The impulse approximation itself is not applicable in this region, in which occur large momentum transfers, which imply for elastic scattering high deuteron internal momentum components and therefore close spacing of the two nucleons in the deuteron.

The magnitude of the theoretical curve is at all applicable points about twice the experimental values. As our theoretical assumptions are crude, definite conclusions about the origins for this factor are not possible. However, the indication is that to produce agreement with experiment, the interference term in the p-d cross section should be taken very small or even negative (destructive).

That the interference between the n-p and p-p scattered amplitudes is indeed destructive may be seen by making the same comparison of the p-d cross section with n-p and p-p data as was made by Chamberlain and Stern³⁹ for Stern's experiment. They define the

quantity δ as

$$\delta = \frac{\frac{9}{16} \frac{1}{S(\theta)} \frac{d\sigma_{pd}(\theta)}{d\Omega}}{\frac{d\sigma_{np}(\theta')}{d\Omega} + \frac{d\sigma_{pp}(\theta')}{d\Omega}}$$

where the nucleon-nucleon cross sections are the experimental values for the same laboratory bombarding energy as in the p-d case and for angles θ' (center-of-mass) for which the magnitude of the momentum transfer $\underline{K}$ is the same as in the p-d case for angle θ ; i.e.,

$$\underline{K} = \frac{4}{3} k \sin \frac{\theta}{2} = k \sin \frac{\theta'}{2}$$

(Strictly speaking, the nucleon-nucleon cross sections should be those for a laboratory energy that deviates slightly from the above criterion by an amount that depends upon θ , but as the deviation is small and the nucleon-nucleon cross sections are not known to an accuracy that would allow the correction to be significant, we are justified in assuming that the laboratory energy is to be taken as the same for all three experimental cross sections.)

The quantity δ will be greater than one for constructive interference, equal to one for zero interference contribution, and less than one for destructive interference.

Table II gives the values of δ deduced from the present experiment for the p-d angles most suitably analyzed in this manner; values of θ' , $S(\theta)$, and the cross sections used are also tabulated.

TABLE II

θ	θ'	$\frac{d\sigma_{np}(\theta')}{d\Omega}$	$\frac{d\sigma_{pp}(\theta')}{d\Omega}$	$S(\theta)$	$\frac{d\sigma_{pd}(\theta)}{d\Omega}$	δ
40°	54.2°	1.5 mb/ster	4 mb/ster	0.117	1.04 mb/ster	0.91
50°	68.6°	1.1	4	0.0628	0.392	0.69
70°	99.8°	0.96	4	0.0203	0.108	0.60

From the values of δ we see that the interference is destructive and becomes increasingly so as θ increases. Very similar results were obtained by Chamberlain and Stern for 192-Mev d-p scattering, which is equivalent to 95-Mev p-d scattering; they also found that theoretical calculations based on central forces only could not give agreeing values of δ but that the inclusion of tensor forces produced reasonable agreement.

Comparison with experiments at lower energies

Figure 15 compares the present results with data obtained by other workers at lower energies. It is seen that our curve exhibits the expected magnitude and angular dependence.

Fig. 15. Comparison of present experimental results with experimental data at lower energies.

VIII ACKNOWLEDGMENTS

The author wishes to express his appreciation for the continuous advice and guidance of Professor Owen Chamberlain throughout the experiment, and for the helpful discussions held with Professor Emilio Segrè. Dr. Clyde Wiegand gave much valuable advice and portions of the apparatus were designed and constructed by him.

The author is also indebted to various graduate students, especially Mr. John Garrison and Mr. Gordon Pettengill, for assistance in construction and testing of some of the equipment and in executing the runs at the cyclotron.

He is grateful to Dr. Amos Newton and Professor George Pimentel for the analyses of the target material, to Miss Greta Kemp for reading portions of the films, and to the cyclotron crews directed by Mr. James Vale for providing at all times the proton beam that was desired.

This work was performed under the auspices of the United States Atomic Energy Commission; the author is doubly indebted to the AEC, since during a major portion of the work he was the holder of an AEC Pre-Doctoral Fellowship.

APPENDIX A. KINEMATICS, ELASTIC PROCESS

Given below are the general formulas for the relativistic kinematics of an elastic two-body collision, and a numerical table of the kinematic parameters of the present experiment.

Let a particle of rest mass m_1 be incident with kinetic energy T_1 on a particle of mass m_2 at rest (we refer to laboratory coordinates). We denote by Φ the laboratory angle (with respect to the beam) to which the incident particle is deflected, and by (H) the angle at which the target particle is scattered. The center-of-mass scattering angle is denoted by θ .

Employing the Lorentz transformation, we obtain for β ($= v/c$) and γ ($= 1/\sqrt{1-\beta^2}$) of the center-of-mass system as viewed from the laboratory system

$$\beta = \frac{\sqrt{(\epsilon + 1)^2 - 1}}{\epsilon + \rho + 1}$$

$$\gamma = \frac{\epsilon + \rho + 1}{\sqrt{\rho^2 + 2\rho(\epsilon + 1) + 1}}$$

where $\epsilon = \frac{T_1}{m_1 c^2}$ and $\rho = \frac{m_2}{m_1}$. Further calculation gives

$$\cot (H) = \gamma \tan \frac{\theta}{2}$$

$$\tan \Phi = \frac{\sin \theta}{\gamma \cos \theta + \gamma \left(\frac{\epsilon + 1 + \frac{1}{\rho}}{\epsilon + 1 + \rho} \right)}$$

The kinetic energies of m_1 and m_2 in the laboratory system following the collision are given by

$$T_2' = 2 m_2 c^2 (\gamma^2 - 1) \sin^2 \frac{\theta}{2}$$

$$T_1' = T_1 - T_2'$$

and the available energy in the center-of-mass system by

$$\frac{T_1}{\gamma} = (m_1 + m_2) c^2 \left(1 - \frac{1}{\gamma}\right).$$

Finally, the factor for transforming the cross section between laboratory and center-of-mass frames is

$$\frac{d \cos \theta}{d \cos \Theta} = \frac{\gamma^2}{4 \cos \Theta} \left[1 + \left(\frac{1}{\gamma^2} - 1 \right) \cos^2 \Theta \right]^2$$

For the present experiment we identify the proton with m_1 and the deuteron with m_2 ; then $p = 2$, $\epsilon = \frac{340}{938} = 0.3625$, $\beta = 0.2752$, and $\gamma = 1.0401$.

Table III lists for each value of θ the quantities Θ , T_d' , R_d , Φ , T_p' , R_p , and $\frac{d \cos \Theta}{d \cos \theta}$ where R_d and R_p are in gm/cm² of copper.

TABLE III

θ degrees	(H) degrees	T_d' Mev	R_d gm/cm ²	Φ degrees	T_p' Mev	R_p gm/cm ²	$\frac{d \cos(H)}{d \cos \theta}$
0	90	0	0	0	340	93	
10	84.8	2.4		6.2	337.6	92	2.98
20	79.6	9.2		12.4	330.8	88	1.49
30	74.4	20.5	0.45	18.7	319.5	86	1.00
40	69.2	35.9	1.20	25.1	304.1	79	0.748
50	64.1	54.8	2.52	31.6	285.2	71	0.602
60	59.0	76.7	4.55	38.3	263.3	62	0.515
70	53.9	101.0	7.3	45.2	239.0	54	0.435
80	48.9	126.9	11.2	52.4	213.1	44	0.385
90	43.9	153.5	15.4	60.0	186.5	35.2	0.346
100	38.9	180.1	20.3	68.1	159.9	27.3	0.316
110	33.9	206.0	25.7	76.7	134.0	20.0	0.293
120	29.0	230.2	31.2	86.2	109.8	14.1	0.274
130	24.1	252.1	36.2	96.8	87.9	9.5	0.260
140	19.3	271.1	41.3	108.9	68.9	6.3	0.249
150	14.4	286.4	44.7	123.0	53.6	3.95	0.241
160	9.6	297.7	46.2	139.5	42.3	2.60	0.235
170	4.8	304.6	49.0	158.8	35.4	1.87	0.232
180	0	307.0	50.0	180.0	33.0	1.65	0.231

APPENDIX B. KINEMATICS, INELASTIC PROCESS

For purposes of comparing the kinematics of the elastic and inelastic events to determine in part the nature of the background produced by inelastic scattering, it is sufficient to use nonrelativistic equations for both processes; the deviations from the relativistic results are small for either process, and in making the comparison, the major portions of the deviations cancel.

The inelastic kinematics may be handled by an impulse treatment mentioned in chapter II: the incident proton is assumed to strike only one nucleon of the deuteron, and the other nucleon is assumed to be unaffected by the collision, maintaining after the collision the same vector momentum it had due to internal motion in the deuteron before the collision. This vector momentum is equal but opposite to the vector momentum of the struck particle at the time of collision, as the deuteron as a whole is at rest. We may therefore write the energy-conservation law as

$$T_1' + T_2' + T_2 = T_1 + B$$

where T denotes kinetic energy, the subscripts 1 and 2 refer to the incident and struck particles respectively, and B is the deuteron binding energy. Primed quantities are postcollision values, and unprimed quantities are precollision ones. The term T_2 on the left side is the kinetic energy carried off by the unstruck particle.

The momentum-conservation law need be applied only to the two nucleons involved in the collision, as the unstruck particle does not suffer any momentum change.

These assumptions have received experimental support (for collisions in which the deuteron internal momentum is not assumed unreasonably large) in the observations of Bloom¹⁷ in studying 192-Mev d-p inelastic scattering.

We have solved these equations graphically by constructing a laboratory-system momentum diagram, an example of which is shown in Fig. 16.

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Fig. 16. Momentum diagram (laboratory system) for inelastic scattering observable at elastic-scattering angles. See Appendix B for further explanation.

(The example is for a p-d center-of-mass scattering angle of 90° .) As we were most interested in the inelastic events yielding protons emergent in the directions for elastic scattering, we set up a non-rectangular coordinate system whose axes lie along the elastic scattering directions. As the example drawn in dotted lines shows, any point on the page will satisfy the momentum-conservation requirements. The curved line, found by trial and error, shows the locus of those points which also satisfy the energy-conservation law. This curve then shows what momentum range will be covered by the inelastically scattered protons proceeding in the directions of elastically scattered particles. (With some thought, one can also read the qualitative features of the distribution.)

We note in particular that inelastic protons emerging at the deuteron angle have a spectrum of ranges surrounding the deuteron range, and that a few inelastic protons emerging in the elastic proton direction have a range almost as great as the elastic proton itself. These facts lead to the requirement of a detection method that identifies the deuterons.

APPENDIX C. CALCULATION OF CONJUGATE-COUNTER HEIGHT AND WIDTH REQUIREMENTS

The calculation of the conjugate-counter solid angle necessary to insure that all proton partners to elastic deuterons striking the defining counter strike the conjugate counter may be divided into two parts, one giving the width requirement, and one the height requirement.

We define the following symbols:

x = conjugate counter distance from target in cm

y = defining counter distance from target in cm

ϕ = angular setting of conjugate counter

(H) = angular setting of defining counter

α = angle of target normal with respect to beam (positive toward conjugate counter, negative toward deuteron counter)

t = target thickness in gm/cm^2

R_d, R_p = deuteron and proton ranges in gm/cm^2

R_d', R_p' = deuteron and proton ranges in cm of air

w_c, h_c = width and height of conjugate counter in cm

d = diameter of beam at the target in cm

To the width of the conjugate counter there are four contributions:

(1) effect of finite width of proton beam, (2) effect of multiple scattering of both proton and deuteron in target, (3) effect of multiple scattering of both in the air paths between target and counters, and (4) the "projected" width; i. e., the width required by the finite width of the defining counter. Equations for these quantities can be derived from simple geometrical considerations, and will only be stated here.

It should be remarked first, however, that the multiple scattering effects were calculated by the formula

$$\theta_{\text{r. m. s.}}^2 = \frac{Z}{M} \frac{\Delta E}{E}$$

where Z is the atomic number of the scattering nuclei

M the mass of the scattered particle

m the electron mass

ΔE the energy loss in the scattering material

E the average energy of the scattered particle while in the scattering material.

This formula is approximate but sufficient for our purposes. It is obtained by dividing the usual multiple coulomb scattering expression for $\theta_{r.m.s.}^2$ by the usual expression for dE/dx , considering the ratio of the logarithmic factors to be one. $\Delta E/E$ may also be approximated by $\frac{1}{n} \frac{\Delta R}{R}$ where n is the range exponent in $R = kE^n$, ΔR the range bite of the scattering material, and R the range of the scattered particle. The angular width required for the multiple scattering contribution is taken as $2 \times 3.5/\sqrt{2}$ times the r.m.s. angle; the 2 since θ is the semivertical angle, the 3.5 as the factor to include virtually all the Gaussian distribution, and the $\sqrt{2}$ since we wish the deflection as projected onto a line rather than in two dimensions. We can then write down for the four contributions to w_c enumerated above:

$$(1) \quad x \left\{ \frac{d}{y \cos \alpha} \left[\cos (\Theta + \alpha) - \frac{y}{x} \cos (\Phi - \alpha) \right] \right\} \text{ cm.}$$

$$(2) \quad x \left\{ 0.150 \sqrt{\frac{t}{n R_d \cos (\Theta + \alpha)}} \sqrt{1 + \frac{1.96 R_d \cos (\Theta + \alpha)}{R_p \cos (\Phi - \alpha)}} \right\} \text{ cm.}$$

$$(3) \quad x \left\{ 0.0933 \sqrt{\frac{y}{R_d}} \sqrt{1 + \frac{1.99 \times R_d}{y R_p}} \right\} \text{ cm.}$$

$$(4) \quad x \left\{ \frac{d\Phi}{d\Theta} \frac{w_d}{y} \right\} \text{ cm.}$$

The second term in the second radical in each multiple-scattering term is due to the contribution to multiple scattering by the proton, and the factors 1.96 and 1.99 arise from the ratio of the deuteron and proton masses multiplied by the ratio of the range exponents for deuteron and proton respectively.

For the height of the conjugate counter there were three contributions: (1) the combined effect of finite defining counter height and beam diameter, (2) effect of multiple scattering in the target, and (3) effect of multiple scattering in the air paths. These con-

tributions to h_c are given by

$$(1) \quad x \left\{ \frac{(d + h_d) \sin \bar{\phi}}{y \sin (H)} + \frac{d}{x} \right\} \text{ cm.}$$

$$(2) \quad x \left\{ 0.150 \frac{y}{x} \sqrt{\frac{t}{R_d \cos (H + \alpha)}} \sqrt{1 + \frac{1.96 R_d x}{R_p y}} \right\} \text{ cm.}$$

$$(3) \quad x \left\{ 0.0933 \sqrt{\frac{y}{R_d}} \sqrt{1 + \frac{1.99 x^3}{y^3} \frac{R_d'}{R_p'}} \right\} \text{ cm.}$$

The calculated heights and widths, which typically were about 7.5 cm and 3.5 cm respectively, were increased by at least 2 cm as a safety margin to determine the solid angle actually to be used in runs.

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