Real-time water allocation policies calculated with bankruptcy games and genetic programing

Omid Bozorg-Haddad, Elman Athari, Elahe Fallah-Mehdipour and Hugo A. Loáiciga

ABSTRACT

Population growth coupled with increased urban and agricultural water use have exacerbated water shortages worldwide. Conflicts among water users frequently arise over scarce water. The application of conflict resolution methods has the potential to resolve such conflicts. Bankruptcy games is a branch of game theory applicable to problems dealing with conflict resolution. This study addresses water allocation to urban-industrial, agricultural, and environmental water uses downstream of the Zarrineh-roud dam, Iran, which diverts water from the Zarrineh-roud River, an important tributary to Lake Urmia. Lake Urmia has been severely stressed by reduction of its water inputs. Water allocation is posed in this study as a bankruptcy game in which the allocation to stakeholders is optimized with proportional (P), adjusted proportional, constrained equal award (CEA), and constrained equal losses methods. The CEA was chosen as the best allocation method based on performance criteria and the Bankruptcy Allocation Sustainability Index. Monthly, real-time, water allocation rule curves were calculated with genetic programming.

Key words | bankruptcy games, conflict resolution, genetic programming, Lake Urmia basin

Omid Bozorg-Haddad (corresponding author)
Elman Athari
Elahe Fallah-Mehdipour
Department of Irrigation & Reclamation Engineering, Faculty of Agricultural Engineering & Technology,
College of Agriculture & Natural Resources, University of Tehran,
Karaj,
Tehran,
Iran
E-mail: *obhaddad@ut.ac.ir*

Hugo A. Loáiciga
Department of Geography,
University of California,
Santa Barbara, CA 93016-4060,
USA

INTRODUCTION

Lake Urmia, Iran, has been impacted by diversion of some of its tributary water to meet agricultural, urban, and municipal water demands. The allocation of water to Lake Urmia and other water stakeholders in its drainage areas is posed in this work as a bankruptcy game and tackled with game theory. Game theory is a mathematical technique applied to the analysis of problems involving dynamic and conflicting interactions among agents in a strategic environment. The problems solved, which are basically about decision pitting rival parties, are framed as a game involving players trying to outdo each other. Game theory has been applied to fundamental problems in mathematics and economics (Von Neumann & Morgenstern 1944; Nash 1950; Shapley 1953; Bondareva 1963; Packel 1981; O'Neill 1982; Rubinstein 1982; Aumann & Maschler 1985; Curiel *et al.* 1988; Mas-Colell 1989; Dagan & Volij 1993).

doi: 10.2166/ws.2017.102

Sheikhmohammady & Madani (2008) applied bankruptcy methods to the allocation of water, petroleum, and gas resources of the Caspian Sea to Azerbaijan, Iran, Kazakhstan, Turkmenistan, and Russia. Ansink & Weikard (2009) applied bankruptcy methods to the sequential allocation of resources. Madani (2010) applied two-by-two games to water resources problems. Zarezadeh *et al.* (2012) employed bankruptcy rules to allocate Qiziluzan-Sefidroud River's water to eight provinces in Iran. Madani & Zarezadeh (2012) demonstrated the effectiveness of bankruptcy procedures in solving water disputes with a hypothetical groundwater bankruptcy problem. Madani *et al.* (2014) introduced a bankruptcy procedure to resolve transboundary river water allocation conflict in which the stakeholders' demands exceeded the total available water. Mianabadi *et al.* (2014) developed a bankruptcy method for water resources allocation. Their method considered the contribution of each beneficiary of water resources in accordance with the United Nation's Watercourses Convention (1997). Mianabadi *et al.* (2015) presented a weighted bankruptcy solution to the problem of scarce resources allocation with an application to the allocation of the Tigris River's water between Turkey, Syria, and Iraq. Sechi & Zucca (2015) proposed a method based on bankruptcy games to allocation the resources in complex supply systems under water-shortage conditions.

Performance criteria introduced by Hashimoto *et al.* (1982) are herein applied to evaluate the performance of model solutions to the water allocation problem under scarcity being solved. Several applications of classical optimization methods to water resources systems can be found in Simonovic & Mariño (1980), Rosenthal (1981), Yeh (1985), Hiew (1987), Barros *et al.* (2001), Labadie (2004) and also metaheuristic optimization tools (Wardlaw & Sharif 1999; Momtahen & Borhani Darian 2005; Jalali *et al.* 2007; Bozorg-Haddad *et al.* 2008; Afshar *et al.* 2009). Application of meta-heuristic methods to water resources systems, of which genetic programming (GP) is an example, can be found in Afshar *et al.* (2011), Fallah-Mehdipour *et al.* (2012), Fallah-Mehdipour *et al.* (2013a, 2013b, 2013c), Bozorg-Haddad *et al.* (2014), Asgari *et al.* (2015), Akbari-Alashti *et al.* (2014), Fallah-Mehdipour *et al.* (2014), Orouji *et al.* (2014), and Ashofteh *et al.* (2015), among others. The Hashimoto *et al.* (1982) reliability, resiliency and vulnerability indices have been applied in several studies, among which are those by Moy *et al.* (1986), Vogel & Bolognese (1995), Zongxue *et al.* (1998), Minville *et al.* (2009), and Sandoval-Solis (2011).

This paper employs nonlinear programming (NLP) and GP as optimization algorithms to solve bankruptcy games dealing with water allocation in the Urmia lake basin, Iran. This appears to be the first study implementation of GP for water allocation under uncertainty among competing stakeholders.

Methodology

This section presents three subsections: (1) reservoir operation modeling with NLP; (2) game theory and conflict resolution with the bankruptcy approach; and (3) real-time water allocation using GP.

Reservoir operation modeling with NLP

Reservoir water releases are commonly considered as the decision variables in reservoir operation problems. Alternatively, the reservoir operation rule curve may be the decision variable, in which case various coefficients and rule curve formulas of the type written in Equation (1) become the unknowns to be solved for (Fallah-Mehdipour *et al.* 2012):

$$R_t = F(Q_t, S_t) \tag{1}$$

in which R_t = release volume during the tth time period (10^6 m^3); S_t = storage volume at the beginning of the tth time period (10^6 m^3); Q_t = inflow volume to the reservoir during the tth time period (10^6 m^3); and F is a linear or nonlinear function obtained with GP.

There are several formulas used as a rule curve such as linear equations in Equation (2) that have been previously applied in Bozorg-Haddad *et al.* (2008):

$$R_t = a \times S_t + b \times Q_t + c \tag{2}$$

in which a, b, and c are equation coefficients obtained by regression analysis of time series data.

Bolouri-Yazdeli *et al.* (2014) proposed quadratic and cubic equations as shown in Equations (3) and (4) as rule curves:

$$R_t = a \times S_t^2 + b \times Q_t^2 + c \times S_t + d \times Q_t + e \tag{3}$$

$$R_t = a \times S_t^3 + b \times Q_t^3 + c \times S_t^2 + d \times Q_t^2 + e \times S_t + f \times Q_t + g \tag{4}$$

in which a', b', c', d', e', f', g' are coefficients and decision variables which are calculated by optimization.

Equations (2)–(4) have a predetermined structure. Better equations may exist based on other kind of mathematical relations without predetermined linear or nonlinear structure (Fallah-Mehdipour *et al.* 2012). In this study, the reservoir operation rule curve equation is determined by GP and the results will be compared with standard operating policy (SOP) of a reservoir. The SOP is the historical, non-optimized, rule curve of a reservoir.

The minimization of the sum of relative deviations between release and demand volumes is herein considered as the objective function shown in Equation (5):

$$Min.\ Z = \sum_{t=1}^{T} \left(\frac{|D_t - R_t|}{D_t} \right) \quad (5)$$

in which Z = objective function to meet downstream water demand; T = number of operation periods; and D_t = downstream water demand during the tth period (10^6 m^3).

The continuity Equation (6) governs the evolution of storage in a reservoir:

$$S_{t+1} = S_t + Q_t - R_t - SP_t - Loss_t \quad (6)$$

in which S_{t+1} = reservoir storage volume at the beginning of the $(t+1)$st time period (10^6 m^3); SP_t = spill from reservoir during the tth time period (10^6 m^3); and $Loss_t$ = evaporative reservoir losses during the tth time period (10^6 m^3). The $Loss_t$ is calculated during each period based on Equation (7):

$$Loss_t = Ev_t.\overline{A_t} \quad (7)$$

in which $\overline{A_t}$ = average of reservoir surface during period t (km^2) which is calculated by Equation (8); and EV_t = evaporation depth during the tth time period (m).

$$\overline{A_t} = \left(\frac{A_t + A_{t+1}}{2} \right) \quad (8)$$

in which $A_t$is a function of storage S_t obtained by fitting a cubic equation to the area-volume data.

The constraints on releases and storage are as follows:

$$0 \leq R_t \leq D_t \quad (9)$$

$$S_{Min.} \leq S_t \leq S_{Max.} \quad (10)$$

where $S_{Min.}$ and $S_{Max.}$ are the minimum and maximum values of reservoir storage (10^6 m^3), respectively.

Hashimoto *et al.* (1982) evaluated reservoir performance using a reliability index that reflects the number of times a system incurs failure; a resiliency index for the time it takes a system to return to normal following a failure; and a vulnerability index that measures the intensity of failures. The former three indexes are employed in this study to describe the performance of a reservoir system.

Reliability

The reliability index takes two forms: (1) time-based reliability and (2) volumetric reliability. According to the definition by Hashimoto *et al.* (1982), the time-based reliability is the probability of non-failure periods. While, the volumetric reliability equals to ratio of releases summation to sum of demands during the operation period. The time-based and volumetric reliabilities are defined by Equations (11) and (12):

$$\alpha_t = 1 - \frac{f}{T} \quad (11)$$

$$\alpha_v = \frac{\sum_{t=1}^{T} R_t}{\sum_{t=1}^{T} D_t} \quad (12)$$

in which α_t and α_v = time-based and volumetric reliability indexes; respectively. f = number of failure periods and T = number of operation periods.

Resiliency

This index measures a system's capacity to recover after failure. Hashimoto *et al.* (1982) defined resiliency by Equation (13):

$$\gamma = \frac{1}{f/f_s} = \frac{f_s}{f} \quad (13)$$

in which γ = resiliency rate; and f_s = number of failure series, where a failure series is a sequence of consecutive failure periods that is preceded and followed by non-failure periods.

Vulnerability

This index measures the severity of system failure. Hashimoto *et al.* (1982) defined the vulnerability index as the average of the maximum shortages that occur in each sequence of failure periods:

$$\delta = \frac{\sum_{k=1}^{f_s} Max(Sh_k)}{f_s} \tag{14}$$

in which $\delta =$ vulnerability index; and $Sh_k =$ maximum shortage during the kth failure series.

System operators attempt to maximize reliability and resiliency and minimize the vulnerability.

Game theory and conflict resolution with bankruptcy allocation rules

The terms of the bankruptcy change with changes in the conflict between stakeholders who are engaged in a zero-sum competition whereby each stakeholder attempts to maximize his utility. When a system fails to meet the demands of each stakeholder, the basic question is how to allocate the available resources between the stakeholders. Answering this question is difficult when the utility functions and payoffs of the stakeholders are not well defined.

The following are the best-known bankruptcy methods applied to water allocation of a finite amount of water available:

Proportional (P) rule for water allocation

The share of each stakeholder is calculated on the basis of Equation (15). In accordance with the P method, the amount of allocated water is proportional to the demands of the stakeholders:

$$(\text{sup}_i)_P = \lambda c_i \quad i = 1, 2, \ldots, m \tag{15}$$

in which $(\text{sup}_i)_P =$ water share of the ith stakeholder by applying the P method; $c_i =$ the water demand of the ith stakeholder, and $\lambda =$ satisfaction coefficient that is always positive and is calculated with Equation (16) as follows:

$$\lambda = \frac{E}{C} \tag{16}$$

in which $E =$ the total available water, and $C =$ sum of all the stakeholders' water demands $= \sum_{j \neq i} c_i$.

Adjusted proportional rule for water allocation

This method first calculates the value of υ_i according to the values of E and $C = \sum_{j \neq i} c_i$ as written in Equation (17):

$$\upsilon_i = \text{larger of} \left\{ \{0, E - \sum_{j \neq i} c_i\} \right\} \quad i = 1, 2, \ldots, m \tag{17}$$

in which $\upsilon_i =$ the initial allocation, which is the minimum amount of water that can be allocated to the ith stakeholder, and $j =$ counter for all stakeholders except the ith stakeholder. Next, the value of the water allocated to each stakeholder is improved by Equation (18):

$$c_i^* = \text{smaller of} \left\{ (c_i - \upsilon_i), \left(E - \sum_{i=1}^{N} \upsilon_i \right) \right\} \quad i = 1, 2, \ldots, m \tag{18}$$

in which $c_i^* =$ improved water allocation to the ith stakeholder, and $N =$ the set of all of the stakeholders.

In the third stage of this method, the water allocation to stakeholders (sup_i^*) is calculated according to the improved allocations and the remaining water available as follows:

$$\text{sup}_i^* = \lambda^* c_i^* \tag{19}$$

$$\lambda^* = \frac{E - \sum_{i=1}^{N} \upsilon_i}{\sum_{i=1}^{N} c_i^*} \tag{20}$$

in which $\text{sup}_i^* =$ secondary allocation to the ith stakeholder, and $\lambda^* =$ satisfaction coefficient corresponding to the state of improved water allocations. Lastly, the water allocation to the ith stakeholder obtained with the adjusted proportional (AP) method ($(\text{sup}_i)_{AP}$) equals the sum of the initial (υ_i) and secondary (sup_i^*) allocations as expressed by Equation (21):

$$(\text{sup}_i)_{AP} = \upsilon_i + \text{sup}_i^* \quad i = 1, 2, \ldots, m \tag{21}$$

Constrained equal awards rule for water allocation

Based on this rule, the water share of each stakeholder (or player) equals the smaller of the player's water demand (or claim) or the value β according to Equations (22) and (23):

$$E = \sum_{i=1}^{n} \text{smaller of}\,(\beta,\ c_i) \tag{22}$$

$$(sup_i)_{CEA} = \text{smaller of}\,(\beta, c_i) \quad i = 1,\ 2,\ \ldots,\ m \tag{23}$$

in which $(sup_i)_{CEA}$ = the water share of each stakeholder (or player) by using the constrained equal award (CEA) method and Equation (22) is calculated by trial and error or with optimization algorithms; $\beta$is calculated by Equation (24), which has one solution provided that $C \geq E$.

$$E = \sum_{i=1}^{n} \text{larger of}\,(0, c_i - \beta) \tag{24}$$

Constrained equal losses rule for water allocation:

This method allocates water to each stakeholder using Equation (25) as follows:

$$(sup_i)_{CEL} = \text{larger of}\,(0, c_i - \beta) \quad i = 1,\ 2,\ \ldots,\ m \tag{25}$$

in which *Max.* (x, y) = the larger of the two arguments x or y, and β is defined by Equation (24). Water allocations to the stakeholders are made at specific time steps, i.e. the system is a time-dependent system.

The P rule for water allocation

The P rule prescribes water allocation by solving the following minimization problem (Equations (28)–(31)) in which the minimization is with respect to the allocations to the ith stakeholder at the tth time step ($sup_{i,t}$) defined as follows:

$$\lambda_{P_{i,t}} = \frac{sup_{i,t}}{C_{i,t}} \quad i = 1,\ 2,\ \ldots.,\ m;\ t = 1,\ 2\ldots,\ T \tag{26}$$

where $\lambda_{P_{i,t}}$ = the relative allocation coefficient (or the satisfaction coefficient) to the ith stakeholder at the tth time step (state variable); $C_{i,t}$ = claim of the ith stakeholder at the tth time step;

$$\lambda_{P_t} = Max.\,(\lambda_{P_{i,t}}) \tag{27}$$

λ_{P_t} = the maximum relative allocation coefficient at the tth time step (state variable).

$$Min. \sum_{t=1}^{T} \left(\lambda_{P_t} - \prod_{i=1}^{m} \lambda_{P_{i,t}} \right) \tag{28}$$

Subject to:

$$\lambda_{P_{i,t}} \leq \lambda_{P_t} \quad i = 1,\ 2,\ \ldots.,\ m;\ t = 1,\ 2\ldots,\ T \tag{29}$$

$$0 \leq sup_{i,t} \leq C_{i,t} \quad i = 1,\ 2,\ \ldots.,\ m;\ t = 1,\ 2\ldots,\ T \tag{30}$$

$$\sum_{i=1}^{m} sup_{i,t} \leq E_t \quad t = 1,\ 2\ldots,\ T \tag{31}$$

in which E_t = total resources available at the tth time step. The multiplication term in the objective function ensures the existence of a single answer for the optimization model and causes the allocation coefficients ($\lambda_{P_{i,t}}$) to be close to each other (Madani *et al.* 2014). The lowest value of the objective function is achieved when the relative allocation coefficients are equal to each other, $\lambda_{P_{i,t}} = \lambda_{P_t}$ for $i = 1, 2, \ldots., m, t = 1, 2\ldots, T$.

The AP rule for water allocation

The AP method addresses the water allocation to the ith stakeholder when all the other stakeholders have been awarded their water claims. The initial allocation to the ith stakeholder in this case equals the remaining water once the water allocations to all other stakeholders have been made. If there are no remaining resources, then the initial allocation to the ith stakeholder equals zero. The initial water allocations to all other stakeholders (stakeholder $j = 1, 2, \ldots, m, j \neq i$) are modified in the next step of the AP method. The modified claim of each stakeholder equals his initial allocation minus the amount of remaining water. The P rule is subsequently applied to the remaining resources and the modified or improved claims. Optimization of water allocation with the AP rule in dynamic, time-dependent, systems such as reservoir systems is as follows:

$$Min. \sum_{t=1}^{T}\left(\lambda^*_{AP_t} - \prod_{i=1}^{m}\lambda^*_{AP_{i,t}}\right) \quad (32)$$

Subject to:

$$B_{i,t} = \sum_{j\neq i} C_{j,t} \quad \forall i, i \neq \text{j} \quad (33)$$

$$\upsilon_{i,t} = \text{larger of}\left\{0, E_t - B_{i,t}\right\} \quad \forall i \quad (34)$$

$$C^*_{i,t} = \text{smaller of}\left\{(C_{i,t} - \upsilon_{i,t}),\ \left(E_t - \sum_{i=1}^{m}\upsilon_{i,t}\right)\right\} \quad \forall i \quad (35)$$

$$\lambda^*_{AP_{i,t}} = \frac{\text{sup}^*_{AP_{i,t}}}{C^*_{i,t}} \quad i = 1, 2, \ldots, m \quad (36)$$

$$\lambda^*_{AP_{i,t}} \leq \lambda^*_{AP_t} \quad i = 1, 2, \ldots, m \quad (37)$$

$$0 \leq sup^*_{i,t} \leq C^*_{i,t} \quad i = 1, 2, \ldots, m \quad (38)$$

$$\text{sup}_{i,t} = \upsilon_{i,t} + \text{sup}^*_{i,t} \quad i = 1, 2, \ldots, m \quad (39)$$

in which $B_{i,t}$ = total claims of the stakeholders except that of the ith stakeholder; $\upsilon_{i,t}$ = the initial allocation to the ith stakeholder at the tth time step; $\lambda^*_{AP_{i,t}}$ = the relative allocation coefficient applied to the adjustment of the claims; $\lambda^*_{AP_t}$ = the maximum value of the relative allocation coefficient applied to the claims; $C^*_{i,t}$ = adjusted claim of the ith stakeholder at the tth time step, and $\text{sup}^*_{i,t}$ = allocation to the ith stakeholder at the tth time step when adjusting the claims (these are the decision variable). Based on the aforementioned equations, the $B_{i,t}$ parameter represents the sum of other stakeholders' claims against the player i in period t, which is employed to calculate the initial allocation to each stakeholder. Moreover, $\text{sup}^*_{i,t}$ is less than $C^*_{i,t}$, the adjusted claim of stakeholder i in time step t.

The CEA rule for water allocation

The CEA method first allocates the water claimed by the stakeholder with the smallest water claim. If there is water available after supplying the first stakeholder (i.e., if $E_t \geq 0$), then this first stakeholder is removed from the water allocation process and the remaining water is distributed among the other stakeholders in a similar fashion, that is, satisfying with the extant smallest claim first, and proceeding sequentially (Madani & Dinar 2013). Optimal water allocation with the CEA rule in dynamic, time-dependent systems such as reservoirs is given by:

$$Min. \sum_{t=1}^{T}\left(\lambda_{CEA_t} - \frac{\prod_{i=1}^{m}\lambda_{CEA_{i,t}}}{\lambda^{m-1}_{CEA_t}}\right) \quad (40)$$

Subject to:

$$\lambda_{CEA_{i,t}} = \text{sup}_{i,t} \quad i = 1, 2, \ldots, m \quad (41)$$

$$\lambda_{CEA_{i,t}} \leq \lambda_{CEA_t} \quad i = 1, 2, \ldots, m \quad (42)$$

$$0 \leq \lambda_{CEA_{i,t}} \leq C_{i,t} \quad i = 1, 2, \ldots, m \quad (43)$$

in which t = time period; $\lambda_{CEA_{i,t}}$ = allocation to the ith stakeholder at the tth time step (decision variable), and λ_{CEA_t} = the maximum amount of allocation at the tth time period. The value of $\lambda_{CEA_{i,t}}$ may be larger than one. Therefore, to achieve converge in the objective function (40) its second (multiplicative) term was divided by $\lambda^{m-1}_{CEA_t}$.

The constrained equal losses rule for water allocation

The constrained equal losses (CEL) rule allocates water by prioritizing those stakeholders with the largest demands, starting with the stakeholder with the largest demand, followed by the stakeholder with the second largest demand, and proceeding likewise and sequentially. The solution process is: CEL specifies a loss for each of player and substracts this loss from the claims of each stakeholder ($C_i - \lambda_{CEL_i}$). If C_i is less than λ_{CEL_i}, the value of allocation to the player is zero. The final allocation to each stakeholder equals $Max.\{0, (C_i - \lambda_{CEL_i})\}$. Optimization modeling of resources allocation by CEL rule in time-dependent systems is as follows:

$$Min. \sum_{t=1}^{T} \left(\lambda_{CEL_t} - \frac{\prod_{i=1}^{m} \lambda_{CEL_{i,t}}}{\lambda_{CEL_t}^{m-1}} \right) \tag{44}$$

Subject to:

$$\lambda_{CEL_{i,t}} = C_{i,t} - \sup_{i,t} \quad i = 1, 2, \ldots, m \tag{45}$$

$$\lambda_{CEL_{i,t}} \leq \lambda_{CEL_t} \quad i = 1, 2, \ldots, m \tag{46}$$

$$0 \leq sup_{i,t} \leq C_{i,t} \quad i = 1, 2, \ldots, m \tag{47}$$

in which $\lambda_{CEL_{i,t}}$ = the water shortage applied in supplying the water claim of the ith stakeholder at the tth time step, and λ_{CEL_t} = the maximum shortage among all the stakeholders at the tth time period.

Stability evaluation

The water allocation rules presented above are based on diverse concepts of fairness and introduce a variety of strategies for water allocation among stakeholders. The rules are unequally acceptable to water stakeholders, depending on perceived differential benefits resulting from them (Madani & Lund 2011). A common method to choose among the various rules is balloting by stakeholders to select one among them (Sheikhmohammady & Madani 2008; Madani et al. 2014). The plurality index measures the acceptability of allocation rules by determining the number of stakeholders who prefer each possible allocation rule, so that the rule with most supporters has the highest acceptability (Dinar & Howitt 1997).

It is possible that the plurality index selects a rule that does not earn the support of the majority of the stakeholders, rendering it of questionable acceptability. Other methods to measure the acceptability of allocation rules (Read et al. 2014); Loehman et al. (1979) employed the power index (α_i), which was developed by Shapley & Shubik (1954). This index is described by Equation (48) and it evaluates the power of players in game theory problems in which the players look for the best way to accrue incremental benefits resulting from cooperative allocation, that is, allocation that is acceptable to the majority of stakeholders:

$$\alpha_i = \frac{x_i - x'_i}{\sum_{i=1,2,..,m} (x_i - x'_i)} \quad i = 1, 2, \ldots, m \quad \sum_{i=1,2,\ldots,m} \alpha_i = 1 \tag{48}$$

in which x_i = incremental water allocated to the ith player; x'_i = ith player's allocation in the current situation (non-cooperative state), and m = the number of stakeholders.

High power-index values are associated with stakeholders with currently low water allocations who would benefit most with the allocation governed by the power index. For this reason, an alternative, more sustainable solution may be achieved when the power index is distributed among players (Dinar & Howitt 1997). One proposal to accomplish this is by employing the coefficient of variation of the players' power index to evaluate the sustainability of water allocation solutions as a sustainability index (Sus_α):

$$Sus_\alpha = \frac{\sigma_\alpha}{\bar{\alpha}} \tag{49}$$

in which σ_α = standard deviation of the players' power index, and $\bar{\alpha}$ = the average of the players' power index. Low values of the sustainability index defined by Equation (49) indicates greater sustainability of water allocation.

Madani et al. (2014) proposed the Bankruptcy Power Index (BPI) for bankruptcy allocation of resources as follows:

$$BPI_i = \frac{sup_i\, \upsilon_i}{\sum_i (sup_i\, \upsilon_i)} \quad i = 1, 2, \ldots, \sum_i BPI_i = 1, \tag{50}$$

$$sup_i = \sum_{t=1}^{T} sup_{i,t} \quad i = 1, 2, \ldots, m \tag{51}$$

$$\upsilon_i = \sum_{i=1}^{T} \upsilon_{i,t} \quad i = 1, 2, \ldots, m \tag{52}$$

in which BPI_i = BPI for the ith stakeholder; and m = of the number of stakeholders.

According to the aforementioned equations, the *BPI* for *i*th stakeholder is calculated for planning horizon using allocation to the *i*th stakeholder (sup_i) and the initial allocation to the *i*th stakeholder (υ_i).

The *bankruptcy allocation sustainability index (BASI)* employs the coefficient of variation of the players' *BPI* to evaluate the various bankruptcy rules:

$$BASI = \frac{\sigma_{BPI}}{\overline{BPI}} \tag{53}$$

in which σ_{BPI} = standard deviation of the players' *BPI* and $\overline{BPI}$ = the average of the players' *BPI*. High values of the BASI indicates that the allocation method is unstable and likely unsustainable.

Real-time water allocation with GP

GP is a metaheuristic algorithm which explores the decision space of an optimization problem randomly and calculates the decision variables' values near to optimal solutions based on evolutionary- and naturally-inspired phenomena. At the first step, a few random possible solutions are generated that are metaphoric of chromosomes in genetics. Chromosomes (i.e. possible solutions) are evaluated with an objective function and are ranked according to the value of the objective function that they attain. During the optimization process, the populations that are generated in each step are modified randomly by crossover and mutation operators. GP is an artificial intelligence algorithm that expresses complex problems with mathematical and logical relations. Its search process and convergence to an optimal solution is similar to those of the genetic algorithm (GA), with the exception that the decision variables are not always expressed in numerical form, but, rather, include operators, functions, and coefficients. This property gives enhanced capabilities to GP for calculating appropriate relations in an optimization problem.

Table 1 lists resemblances and differences between the GA and the GP operators. GP treats the operators, functions, and coefficients of relations as decision variables, which constitutes a key advantage. As a result, no preset relation is imposed on the GP algorithm, and the best relation is obtained with operators, functions, and coefficients. The GP has ample capability to generate optimal reservoir operating rules as a function of reservoir inflow and storage.

A code in GP consists of mathematical operators (for example $\{\pm, \times, \div\}$) and various functions (for example $\{\sin, \cos, power(x^y), \sqrt{}\}$) that create random relations between independent variables (consisting of reservoir inflow and storage) and water releases (R_t), which serve as the dependent variable in reservoir operating rules. The release value obtained by GP is evaluated with the objective function (Equation (5)) in each algorithmic iteration. The GP generates a rule curve relation in each iteration and the R_t values and their associated values of the objective function are calculated and ranked. The releases are improved from one iteration to the next with mutation and crossover operators until reaching a stopping criterion. The GP outputs consist of the operating rule curve, the optimal reservoir releases, the values of the objective function at the optimal solution, reservoir storage, and spill volumes.

Table 1 | Comparison of the GA and GP

	Algorithm features					
Algorithm name	Basis of the algorithm	Algorithmic process	Decision variables	Decision variable sets	Generation of new solutions with the crossover operator	Generation of new solutions with the mutation operator
GA	Based on evolution theory	Random search and iteration	Gene (represents numbers)	Chromosome	Exchanging several genes between two chromosomes randomly	Generating new random values for numerical decision variables with random genes
GP	Based on evolution theory	Random search and iteration	Node (represents operators, functions, and numbers)	Tree	Exchanging several branches between some trees	Randomly generating new operators, functions and coefficients in nodes

Case study

The Zarrineh-roud River, Iran, supplies a large part of Lake Urmia's water. Lake Urmia has experienced declining water levels and ecosystems degradation in the past years due to diversions of its native inflow by various regional water users or stakeholders. The Zarrineh-roud reservoir is the largest dam in the Urmia lake basin and a key regulator of inflow to Lake Zurmia. The Zarrineh-roud Sub-basin was herein selected as the case study with the purpose of determining optimal operating rules for the Zarrineh-roud reservoir employing bankruptcy rules to optimally allocate water to local stakeholders.

The Urmia lake basin is located between northern latitudes 37°4′ and 38°17′ and eastern longitude 45° to 46°. Figure 1 shows the Urmia lake basin with locations of reservoirs which have more than 10 (10^6 m^3) capacity. Lake Urmia's average annual evaporation and lake precipitation equal $4{,}467.9 \times 10^6$ m^3 and $1{,}381.2 \times 10^6$ m^3, respectively (Abbaspour & Nazaridoust 2007). The average annual volume of groundwater feeding Lake Urmia at normal level equals 210.7×10^6 m^3 (Abbaspour & Nazaridoust 2007). The total average annual volume of water (except for lake precipitation) that enters the lake equals $2{,}675.7 \times 10^6$ m^3. This implies that the average annual water shortage to meet Lake Urmia's ecological requirements equals 411×10^6 m^3 $(= (4{,}467.9 - 1{,}381.2 - 2{,}675.7) \times 10^6$ $m^3)$. Lake inflows support its ecosystem, including the brine shrimp (*Artemia* species). A summary of data applied in the calculation of the ecological needs of Lake Urmia is listed in Table 2. There are 17 main rivers in the Lake Urmia basin that constitute its tributary water sources. The volumes of water to be provided by the 17 rivers to correct the Lake Urmia water deficit are listed in Table 3. This work investigates several options to reduce river water-use and to eliminate or reduce Lake Urmia's water deficit.

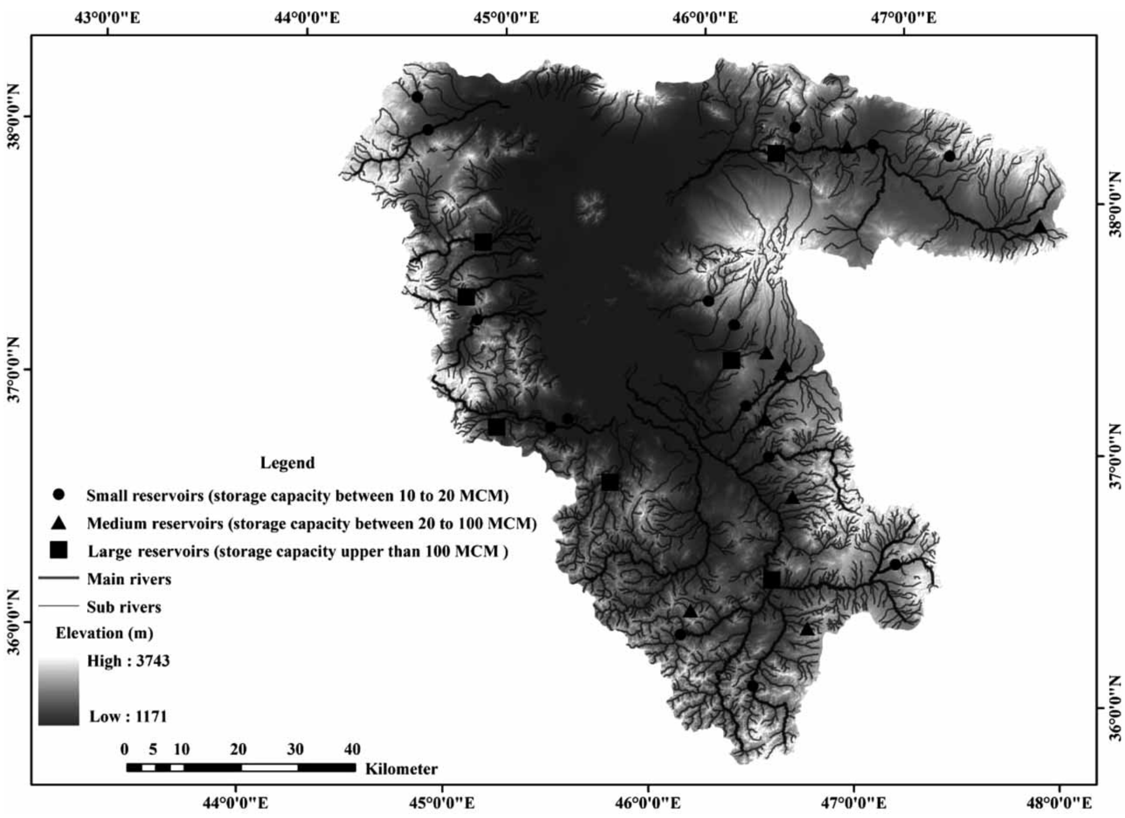

Figure 1 | Locations of small, medium, and large reservoirs in Urmia basin.

Table 2 | Lake Urmia's ecological claims

Quality-quantity indicators for Lake Urmia associated with ecological requirements	Value (unit)
Tolerance threshold of salt of Lake Urmia	240 (mg/L)
Lake water level at salt threshold	1,274.1 (m)
Ecological water surface	4,652.2 (km^2)
The volume of annual evaporation from the ecological surface	4,467.9 ($10^6\,m^3$)
The volume of annual rainfall on the ecological surface	1,381.2 ($10^6\,m^3$)
The Lake Urmia ecological water storage	3,086 ($10^6\,m^3$)

The geometric characteristics of the Zarrineh-roud reservoir

The maximum and minimum storages of the Zarrineh-roud reservoir are equal to 762 ($10^6\,m^3$) and 107.6 ($10^6\,m^3$), respectively. The Zarrineh-roud reservoir was built to supply a variety of water uses including municipal, industrial, and agricultural uses. Table 4 lists the values of the monthly requirements for each of these stakeholders. Table 5 lists the values of average amounts of precipitation, evaporation and net precipitation at Zarrineh-roud reservoir.

Table 3 | Share of water supply to Lake Urmia by main rivers needed to correct the lake's water deficit

River name	Long-term average discharge ($10^6\,m^3$)	Share of the water for Lake Urmia ($10^6\,m^3$)
Aji-chay	428.7	35.14
Azarshahr-chay	31.41	2.57
Qala-chay	82.75	6.78
Javan-chay	11.28	0.92
Sufi-chay	125.41	10.28
Marduq-chay	86.44	7.08
Leylan-chay	62.01	5.08
Zarrineh-roud	1,838.87	150.71
Simiineh-roud	560.38	45.93
Mahabad-chay	274.85	22.53
Godar-chay	398.67	32.67
Baranduz-chay	281.31	23.06
Shahr-chay	169.57	13.9
Roze-chay	44.37	3.64
Nazlu-chay	417.89	34.35
Zola-chay	164.38	13.47
Sinikh-chay	27.82	2.28
Sum	5,006.13	411

RESULTS

The results of the long-term operation of Zarrineh-roud reservoir

The optimized long-term operation of Zarrineh-roud reservoir was calculated with NLP for a 57-year period. The most recent 5 years were selected to test the calculated rule curves. The values of monthly water demand equaled the sum of the four downstream stakeholders' claims. The lowest value of total water demand occurs in October and equals $18.87 \times 10^6\,m^3$. The maximum value of water demand occurs in July and equals $74.263 \times 10^6\,m^3$. The objective function of the optimization model (see Equation (5)) was calculated to be 40.497.

Figure 2 shows the optimal monthly reservoir releases calculated with NLP and water demand for a 684-month long period. It is seen in Figure 2 that there are large deficits of water supply from period 564 through 600. The values of the performance criteria are listed in Table 6. Table 6 shows that NLP achieved low resiliency and time-based reliability and high value of volumetric reliability.

Water allocation to downstream stakeholders by means of P, AP, CEA, and CEL

Water allocations are released to downstream users during each operational period following the calculation of optimal releases. Each water receiver or beneficiary is considered as a player (stakeholder) in the four bankruptcy methods herein considered (P, AP, CEA, and CEL). Figure 3 depicts the water allocations assigned to each of the stakeholders with the four implemented methods. It is observed in Figure 3(a) that the CEL and CEA methods assigned the lowest and highest water allocations to the urban sectors, respectively. The CEL method assigned zero water to the urban sector in some periods. The P method also assigned

Table 4 | The amounts of different water claims downstream from the reservoir

Water demand (10^6 m^3)	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sum
Agricultural	88	0.0	0.0	0.0	0.0	0.0	22	102	229	245	22	179	1,091
Environmental	1.56	4.91	8.32	9.52	12.2	80.48	159.84	125.02	11.94	3.47	1.81	1.46	420.55
Urban and industrial	14.2	12.2	12.7	13.27	13.01	13.41	11.1	13.01	13.4	14.3	14.5	13.2	158.03
Lake Urmia	0.56	1.76	2.98	3.41	4.37	28.84	57.28	44.8	4.29	1.24	0.65	0.52	150.71
Sum	104.32	18.87	24.00	26.20	29.59	122.73	250.22	284.83	258.62	263.74	242.97	194.19	1,820.29

Table 5 | Monthly average values of precipitation, evaporation, and net evaporation at the Zarrineh-roud reservoir

	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sum
Average precipitation (mm)	60.78	127.64	115.4	86.46	80.75	136.99	202.52	219.62	79.00	26.09	7.69	16.69	1,125.62
Evaporation (mm)	175.60	84.20	59.0	42.90	39.90	55.10	77.00	132.60	172.60	207.80	217.80	195.10	1,431.8
Net evaporation (mm)	86.82	−43.44	−56.40	−43.56	−40.85	−81.89	−125.52	−87.02	93.80	181.71	210.11	178.41	272.18

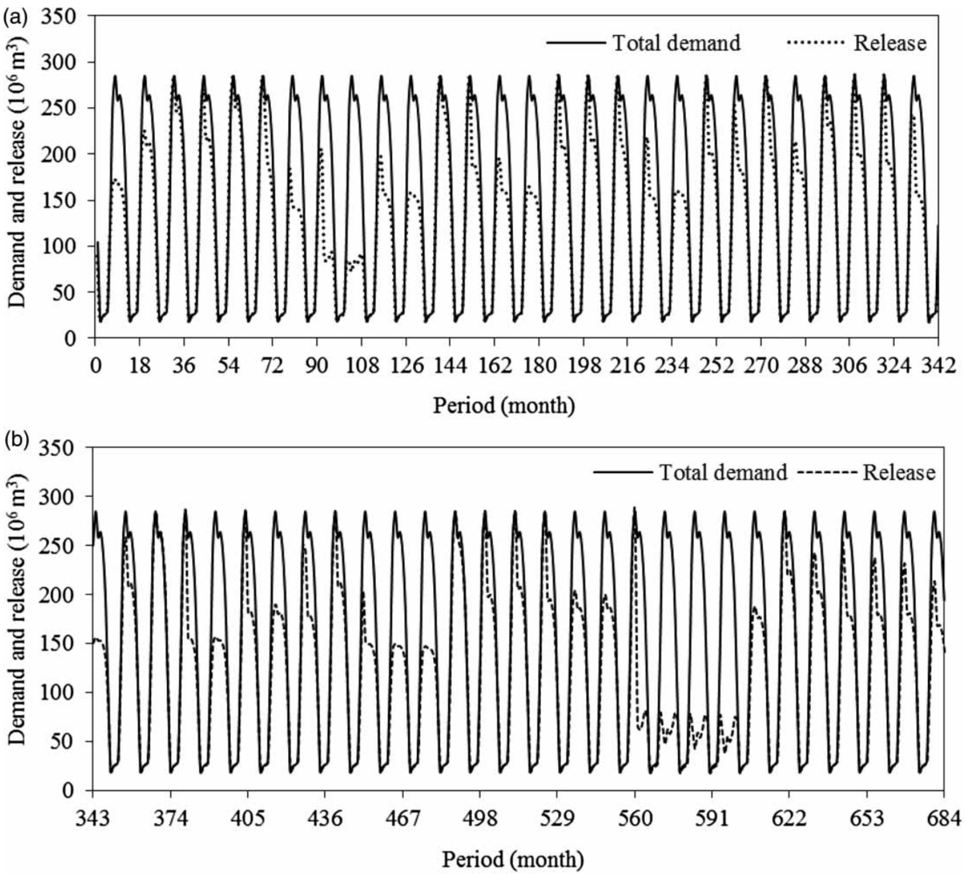

Figure 2 | Values of total water demand and release during the operational period, (a) period 1 to 342 and (b) period 343 to 684. Change period to 343–684 in the horizontal axis.

Table 6 | Values of the performance criteria for reservoir operation optimized with NLP

	Performance criteria		
Sum of relative deficits (Equation (5) the objective function)	Resiliency	Volumetric reliability	Time-based reliability
40.497	0.134	0.842	0.467

very small amounts of water in dry periods. This is so because in dry periods the water deficit is high and water allocation to the urban sector is drastically reduced given that the P method divides the water deficit equally among stakeholders. The AP method produced low water allocations in dry periods similar to those calculated with the CEA method. The provision of urban water is a high priority, and applying high water deficits to this sector has detrimental economic and social consequences. Therefore, the CEL, AP, and P methods are not well suited to apportion water to the urban sector, which exhibits relatively low water claims, when compared with the agricultural sector.

It is seen in Figure 3(b) that the CEA and AP approaches assigned relatively large allocations to the environmental sector. The CEL method allocated the lowest water volume to the environmental sector. The CEA, AP, P and CEL methods calculated the largest, second largest, and third largest water allocations to the environmental sector, respectively (see Figure 3(c)). The CEL assigned the largest water allocation to the agricultural sector as shown in Figure 3(d), followed by the CEA, P, and AP methods which provided the second, third, and fourth largest water allocations to the agricultural sector, respectively. The CEL method, therefore, performed well in supplying stakeholders with large water claims. But the water allocations by the CEL method could have severely detrimental effects on other stakeholders.

Table 7 lists the values of the calculated four performance criteria herein applied to water allocations to

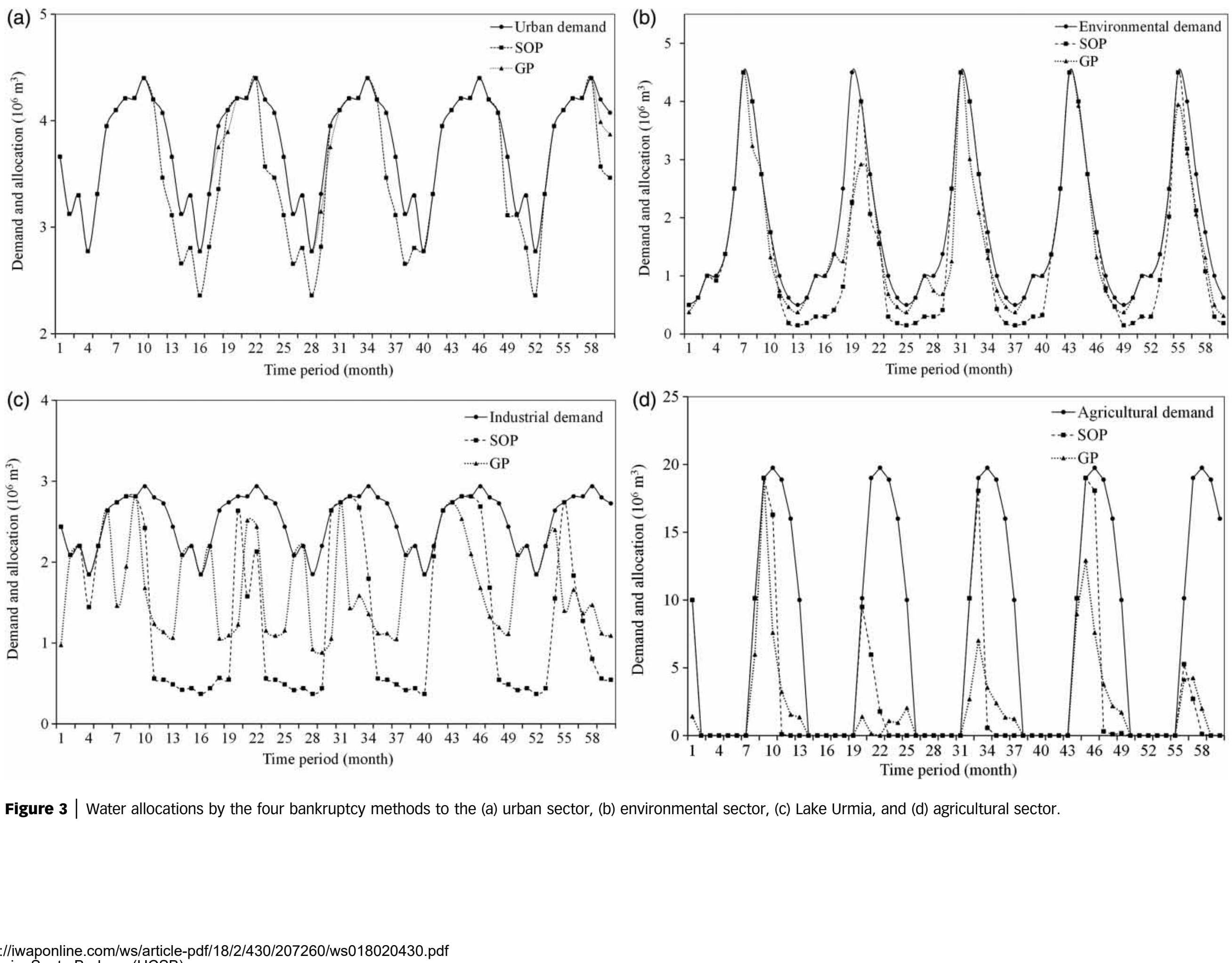

Figure 3 | Water allocations by the four bankruptcy methods to the (a) urban sector, (b) environmental sector, (c) Lake Urmia, and (d) agricultural sector.

Table 7 | Comparison of the performance criteria of the bankruptcy methods with respect to the water allocations to various stakeholders

Sector	Performance criteria															
	Volumetric reliability				Time-based reliability				Resiliency				Vulnerability			
Stakeholder	P	AP	CEA	CEL	P	AP	CEA	CEL	P	AP	CEA	CEL	P	AP	CEA	CEL
Urban-industrial	0.82	0.94	0.96	0.45	0.47	0.36	0.67	0.19	0.19	0.25	0.38	0.052	14.16	2.26	1.49	63.02
Environmental	0.81	0.92	0.9	0.52	0.53	0.53	0.76	0.19	0.16	0.25	0.28	0.052	14.57	4.63	8.13	53.68
Lake Urmia	0.82	0.87	0.96	0.45	0.56	0.43	0.94	0.19	0.21	0.25	0.57	0.052	14.07	8.6	3.61	59.87
Agricultural	0.77	0.73	0.77	0.77	0.48	0.46	0.49	0.46	0.16	0.15	0.16	0.15	16.4	21.93	16.59	18.35

stakeholders with bankruptcy methods. Recall that the performance criteria are the volumetric reliability, time-based reliability, resiliency, and vulnerability. It is evident in Table 7 that, concerning the water allocation to the urban sector, the CEA had the best performance because its values of the volumetric reliability, time-based reliability, and resiliency are higher (better) than those of the other methods, and the amount of vulnerability is the lowest (better). The AP method had the second best performance in the urban sector after that of the CEA method. Also, the CEL method was not successful in supplying this sector's water claim. The AP method performed best in providing environmental water needs having the best values of volumetric reliability and vulnerability. The CEA method had the best values of time-based reliability and resiliency in supplying the environmental needs. The best values of the performance criteria in supplying the lake's claim belong to the CEA, AP and P methods, respectively, in decreasing order of performance quality. The CEL method has acceptable performance in supplying the agriculture sector. Yet, in some cases it incurred significant failures, showing a relatively high vulnerability index. Overall, the CEA method performed best with water supply to the agricultural sector. The aforementioned index named the 'Bankruptcy Allocation Stability Index' (BASI) is commonly employed in bankruptcy games for the purposes of comparing and choosing the best method of conflict resolution. A low value of BASI signifies that an implemented allocation method has acceptable performance in the allocation problem in question. Table 8 lists the values of BASI for the four bankruptcy methods implemented in this work. The results shown in Table 8 indicate that the P method has the best acceptability having the lowest (best) BASI among the bankruptcy methods. It is followed in decreasing order of acceptability by the CEA, AP and CEL methods. The P method allocates available resources by considering the merits of the water claims by stakeholders relative to the available water, and assigns water shortage (or deficit)

Table 8 | The values of BASI for all water allocation method

	P	AP	CEA	CEL
BASI	0.075	0.57	0.368	0.67

ratios that are consequent with those merits. The resulting water allocation is likely to be acceptable to stakeholders for the same reason that a low value of BASI has a likely high acceptability among stakeholders. Applying equal ratios of water deficit to the allocations to stakeholders is sound in situations when the various claims have very similar magnitudes. However, an allocation method chosen based on the BASI would be questionable when the water claims of stakeholders are significantly different to each other, such as is the case with the claims of the stakeholders located downstream of the case-study reservoir. These considerations make evident that it is necessary to evaluate an allocation method's BASI and the four performance criteria to make a more suitable selection method for water allocation. The results listed in Tables 7 and 8 establish that the CEA method achieved the best water allocations judged by the performance criteria and the BASI.

Figures 4–7 depict the water allocations to the urban, environmental, lake, and agricultural stakeholders, respectively, calculated with the CEA method. These figures confirm the CEA method's suitable performance in supplying the stakeholders with low water claims. This does not mean that that 100% of their water claims are supplied at all times. Rather, relatively low water deficits were applied to these stakeholders in dry periods. The CEA might seem unfair in comparison with the P method at first sight. Yet, further analysis reveals that the high water use by the agricultural sector has a destabilizing effect on water claims. For this reason, the agricultural water claim is managed such that, in dry periods, its water allocation is allocated rationally. This is accomplished by applying low water deficits to stakeholders with low claims (such as the urban sector) during times of heightened scarcity.

Calculation of monthly allocation rule-curves with GP

Real-time rule curves describing the monthly water allocations to the four stakeholders (urban, environmental,

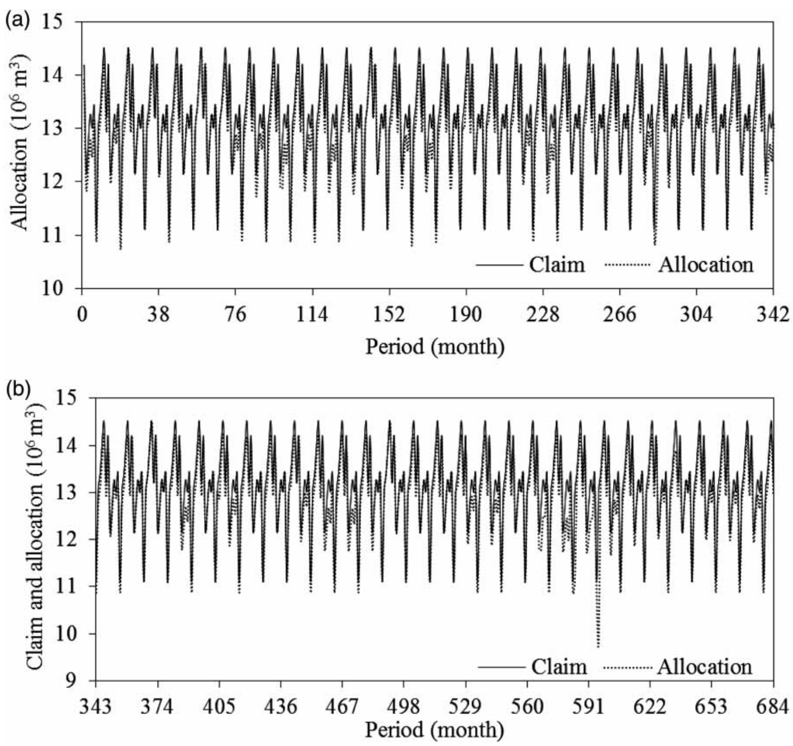

Figure 4 | Water allocations to the urban sector by the CEA method, (a) period 1 to 342 and (b) period 343 to 684.

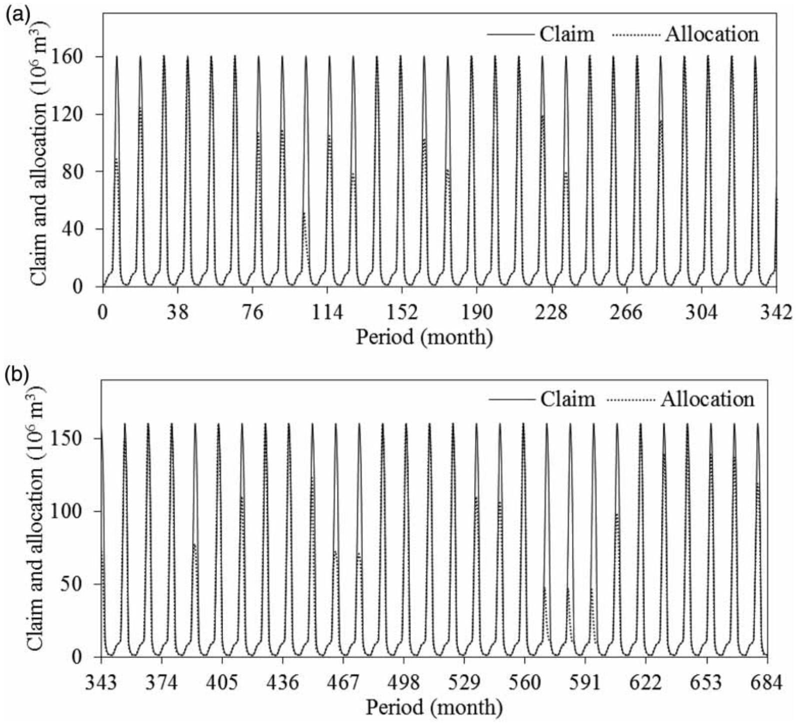

Figure 5 | Water allocations to the environmental sector by the CEA method, (a) period 1 to 342 and (b) period 343 to 684.

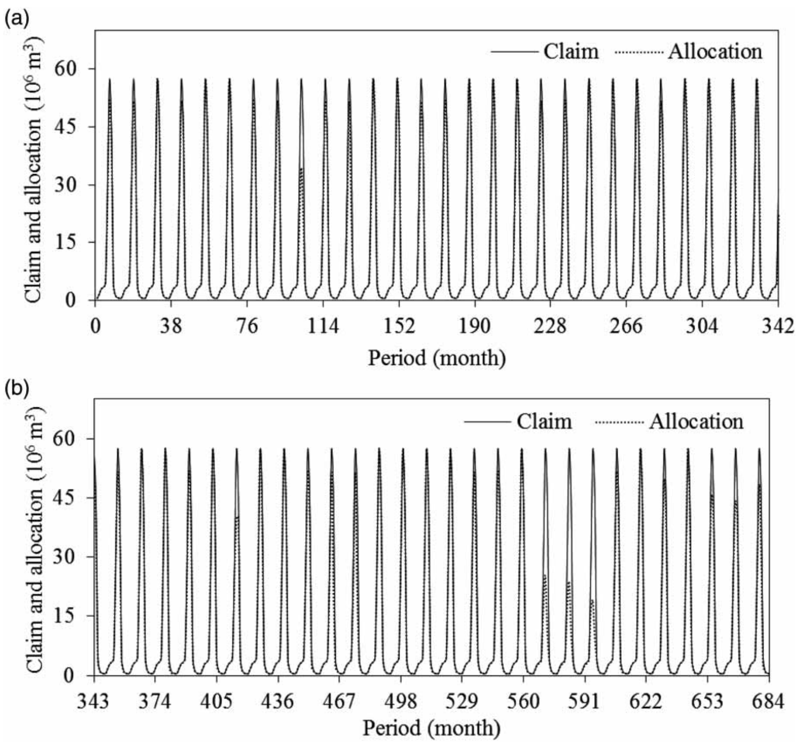

Figure 6 | Water allocations to Lake Urmia by the CEA method, (a) period 1 to 342 and (b) period 343 to 684.

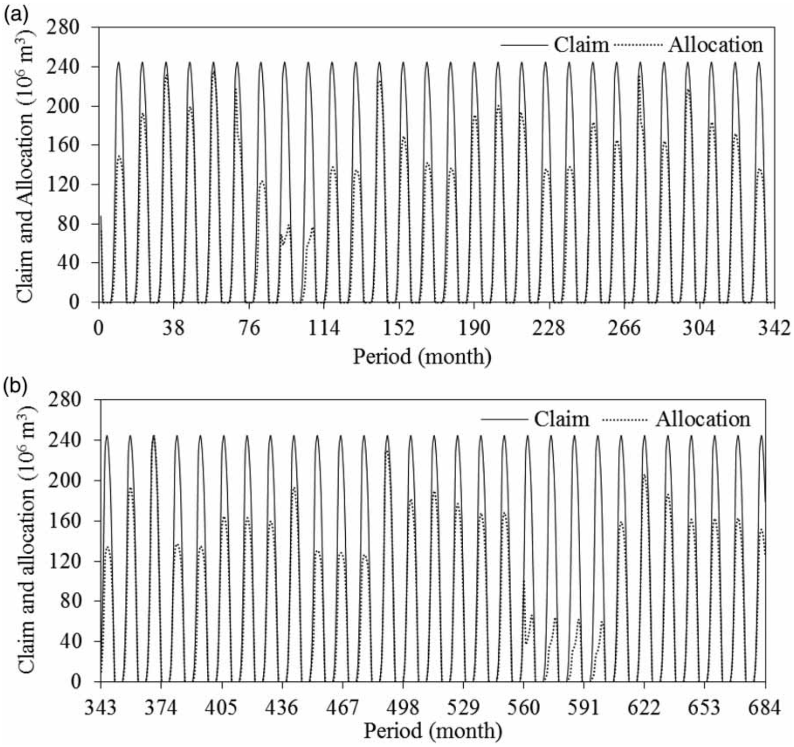

Figure 7 | Water allocations to the agricultural sector by the CEA method, (a) period 1 to 342 and (b) period 343 to 684.

Lake Urmia, and agricultural) were calculated with GP, for a total of 48 (=4 stakeholders times 12 months annually) real-time operating rules of the form given by Equation (54):

$$R_{i,k} = f_{i,k}(Q_t, S_t) \quad (54)$$

in which i = counter for stakeholders (i = 1, 2, 3, 4); k = counter of the periods (months) (k = 1, 2, 3, 4)); $R_{i,k}$ = allocation volume to the ith stakeholder in the kth period and $f_{i,k}$ = the function of the real-time water allocation equation for the ith stakeholder in the kth period (i.e. the rule curve i, k).

The GP was calibrated with the first 40 years of the data for the Zarrineh-roud reservoir (70% of the data) and tested with the last 17 years of data (30% of the data). The allocations to the stakeholders were written as functions of reservoir inflow during any period and of the reservoir storage at the beginning of the same period. These functions constitute the reservoir operation rules. Functions and operators were applied in GP to calculate the monthly rule curves. The objective function was the minimization of the root mean square error (RMSE) between calculated real-time and the long-term allocation volumes. The number of trees and chromosomes implemented in GP were 3 and 30, respectively, and the number of iterations were 200,000 in each of the five runs of GP. Three-dimensional (3D) charts displaying the monthly rule curves and the objective function (RMSE) in March are portrayed in Figure 8 as an example of the results concerning the calculated rule curves.

Evaluation of real-time water allocations

The real-time rule curves calculated with GP were tested with 5 years of monthly time series of reservoir inflow by evaluating its allocation of the Zarrineh-roud reservoir's water resources to downstream stakeholders. The GP-calculated rule curves were compared with the optimal releases and water allocations calculated with NL and CEA (or

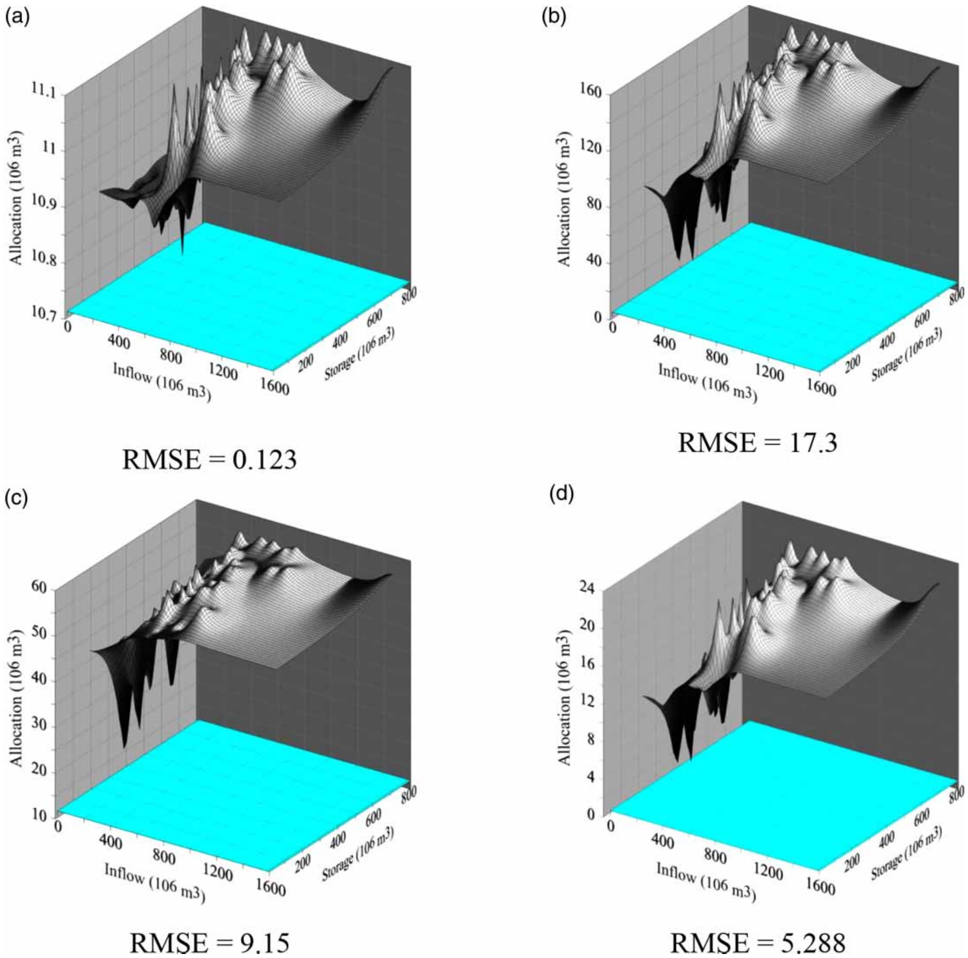

Figure 8 | 3D charts depicting the calculated monthly rule-curves and the value of the objective function (RMSE) in March, (a) urban sector, (b) environmental sector, (c) the Lake Urmia sector, and (d) the agricultural sector.

NLP-CEA). The NLP-CEA method gives the global optimal solution. The purpose of the comparison to test how well the real-time operation rules calculated with GP approximate the long-term water allocations obtained with NLP-CEA. Figure 9 shows the allocation to stakeholders by GP and NLP-CEA. Figure 9 demonstrates that GP approximates the NLP-CEA water allocations to the stakeholders (urban, Lake Urmia, environmental, and agricultural) with negligible errors. Of special significance in these figures is the difference between the water allocation values by the two methods in the periods when the claims are largest. Claims are largest during dry periods when the water system faces water shortages. Evidently, GP calculates near optimal water allocation during dry periods, thus making it a useful optimization method for real-time reservoir operation. Table 9 lists several correlation indexes between NLP-CEA's and GP's results. Table 9's results prove that the performance of GP in estimating the water allocations is acceptable. Table 10 lists the performance criteria of water allocations to stakeholders by GP and NLP-CEA. It is evident from Table 10 that the values of time-based reliability and resiliency are lower than those volumetric reliabilities. This means that both methods minimized water shortages in each period to prevent large shortages overall. The GP's performance criteria are quite close to those of the NLP-CEA method, proving its capacity to determine sound real-time water allocation to stakeholders.

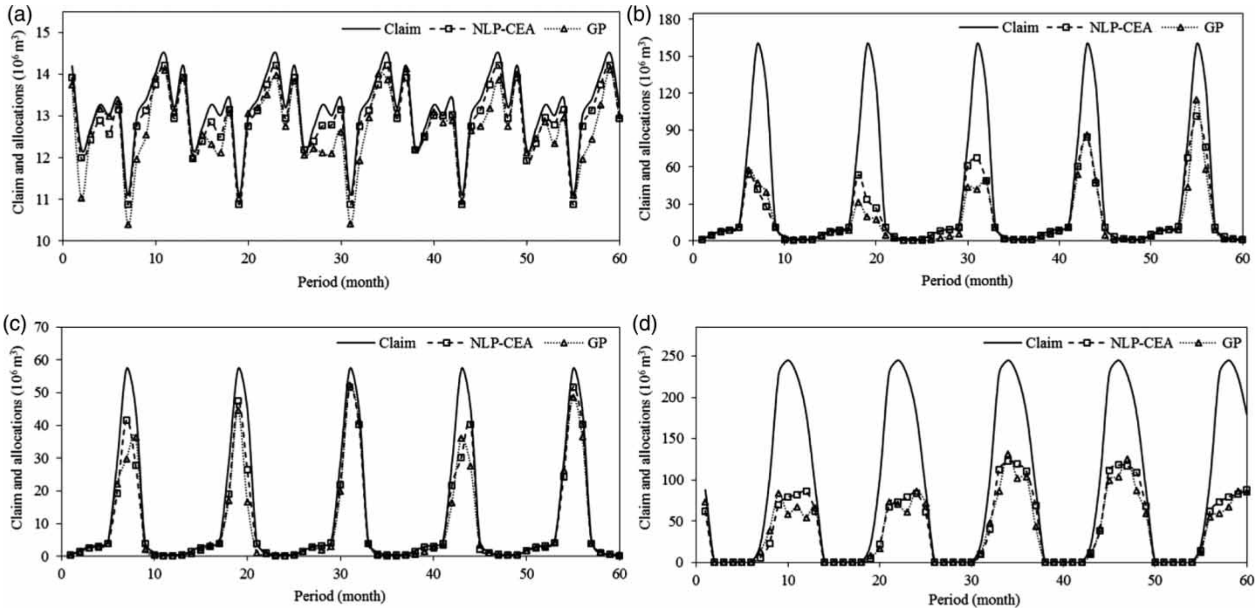

Figure 9 | Water allocations obtained by NLP-CEA and the calculated real-time allocation rules to the (a) urban sector, (b) environmental sector, (c) Lake Urmia, and (d) agricultural sector.

CONCLUDING REMARKS

Long-term optimal Zarrineh-roud reservoir releases were calculated with NLP. Thereafter, the four bankruptcy methods, P, AP, CEA and CEL, were applied to optimize water allocations among the four stakeholders receiving water from the reservoir. The CEA method exhibited the best water allocation performance based on time-based reliability, volumetric reliability, resiliency, and vulnerability criteria. Moreover, the acceptability of these methods was analyzed with the BASI stability index. The P method has the best acceptability among the four bankruptcy methods, followed by the CEA method. Overall, the CEA method was chosen as the best method of water allocations to downstream-reservoir stakeholders (urban, environmental, Lake Urmia, and agricultural).

The CEA method was applied in real-time with GP. GP calculated optimal monthly water allocations to the four

Table 9 | Various correlation indexes between GP and NLP-CEA

	Stakeholder			
Criteria	Urban-industrial	Environmental	Lake Urmia	Agricultural
RMSE	0.3685	7.36	3.18	15.96
MAE	0.287	3.88	1.54	9.63
R	0.93	0.95	0.96	0.89
R^2	0.865	0.9	0.92	0.79

MAE, mean absolute error; RMSE, root mean square error.

Table 10 | Comparison of GP and NLP-CEA with the four performance criteria

	Performance criteria							
	Time-based reliability		Volumetric reliability		Resiliency		Vulnerability	
Stakeholder	GP	NLP-CEA	GP	NLP-CEA	GP	NLP-CEA	GP	NLP-CEA
Urban-industrial	0.0	0.0	0.93	0.97	0.0	0.0	1.13	0.027
Environmental	0.0	0.0	0.603	0.72	0.0	0.0	13.17	7.88
Lake Urmia	0.0	0.0	0.704	0.78	0.0	0.0	8.41	4.82
Agricultural	0.0	0.262	0.48	0.685	0.0	0.143	36.47	11.09

stakeholders, thus yielding 48 optimized real-time rule curves. The GP-calculated rule curves were compared with the optimal water allocations from the NLP-CEA method over a 5-year period. The comparison results demonstrated that the GP water allocations approximate very closely the optimal allocations.

REFERENCES

Abbaspour, M. & Nazaridoust, A. 2007 Determination of environmental water requirements of Lake Urmia, Iran: an ecological approach. *International Journal of Environmental Studies* **64** (2), 161–169.

Afshar, A., Zahraei, A. & Mariño, M. A. 2009 Large-scale nonlinear conjunctive use optimization problem: decomposition algorithm. *Journal of Water Resources Planning and Management* **136** (1), 59–71.

Afshar, A., Shafii, M. & Bozorg-Haddad, O. 2011 Optimizing multi-reservoir operation rules: an improved HBMO approach. *Journal of Hydroinformatics* **13** (1), 121–139.

Akbari-Alashti, H., Bozorg-Haddad, O., Fallah-Mehdipour, E. & Mariño, M. A. 2014 Multi-reservoir real-time operation rule using fixed length gene genetic programming (FLGGP). *Proceedings of the Institution of Civil Engineers, Water Management*, 167 (10), 561–576.

Ansink, E. & Weikard, H. P. 2009 Sequential sharing rules for river sharing problems. In: *Paper presented at the 17th Annual Conference of the European Association of Environmental and Resource Economists*, Amsterdam, The Netherlands.

Asgari, H. R., Bozorg-Haddad, O., Pazoki, M. & Loáiciga, H. A. 2015 Weed optimization algorithm for optimal reservoir operation. *Journal of Irrigation and Drainage Engineering* **142** (2), 04015055.

Ashofteh, P. S., Bozorg-Haddad, O., Akbari-Alashti, H. & Mariño, M. A. 2015 Determination of irrigation allocation policy under climate change by genetic programming. *Journal of Irrigation and Drainage Engineering* **141** (4), 04014059.

Aumann, R. J. & Maschler, M. 1985 Game theoretic analysis of a bankruptcy problem from the Talmud. *Journal of Economic Theory* **36** (2), 195–213.

Barros, M. T., Lopes, J. E., Yang, S. L. & Yeh, W. W. G. 2001 *Large-scale Hydropower System Optimization*. IAHS Publication, Wallingford, Oxon., UK, pp. 263–268.

Bolouri-Yazdeli, Y., Bozorg-Haddad, O., Fallah-Mehdipour, E. & Mariño, M. A. 2014 Evaluation of real-time operation rules in reservoir systems operation. *Water Resources Management* **28** (3), 715–729.

Bondareva, O. N. 1963 Some applications of linear programing methods to the theory of cooperative games. *Problemy Kybernetiki* **10**, 119–139.

Bozorg-Haddad, O., Afshar, A. & Mariño, M. A. 2008 Honey-bee mating optimization (HBMO) algorithm in deriving optimal operation rules for reservoirs. *Journal of Hydroinformatics* **10** (3), 257–264.

Bozorg-Haddad, O., Karimirad, I., Seifollahi-Aghmiuni, S. & Loáiciga, H. A. 2014 Development and application of the bat algorithm for optimizing the operation of reservoir systems. *Journal of Water Resources Planning and Management* **141** (8), 04014097.

Curiel, I. J., Pederzoli, G. & Tijs, S. H. 1988 *Reward Allocations in Production Systems*. Springer, Berlin, Heidelberg, Germany, pp. 186–199.

Dagan, N. & Volij, O. 1993 The bankruptcy problem: a cooperative bargaining approach. *Mathematical Social Sciences* **26** (3), 287–297.

Dinar, A. & Howitt, R. E. 1997 Mechanisms for allocation of environmental control cost: empirical tests of acceptability and stability. *Journal of Environmental Management* **49** (2), 183–203.

Fallah-Mehdipour, E., Bozorg-Haddad, O. & Mariño, M. A. 2012 Real-time operation of reservoir system by genetic programing. *Water Resources Management* **26** (14), 4091–4103.

Fallah-Mehdipour, E., Bozorg-Haddad, O. & Mariño, M. A. 2013a Extraction of optimal operation rules in aquifer-dam system: a genetic programing approach. *Journal of Irrigation and Drainage Engineering* **139** (10), 872–879.

Fallah-Mehdipour, E., Bozorg-Haddad, O. & Mariño, M. A. 2013b Developing reservoir operational decision rule by genetic programing. *Journal of Hydroinformatics* **15** (1), 103–119.

Fallah-Mehdipour, E., Bozorg-Haddad, O. & Mariño, M. A. 2013c Prediction and simulation of monthly groundwater levels by genetic programing. *Journal of Hydro-Environment Research* **7** (4), 253–260.

Fallah-Mehdipour, E., Bozorg-Haddad, O. & Mariño, M. A. 2014 Genetic programing in groundwater modeling. *Journal of Hydrologic Engineering* **19** (12), 04014031.

Hashimoto, T., Stedinger, J. R. & Loucks, D. P. 1982 Reliability, resilience, and vulnerability criteria for water resource system performance evaluation. *Water Resources Research* **18** (1), 14–20.

Hiew, K. L. 1987 *Optimization Algorithms for Large-Scale Multireservoir Hydropower Systems*. Colorado State University, Fort Collins, CO, USA.

Jalali, M. R., Afshar, A. & Mariño, M. A. 2007 Multi-colony ant algorithm for continuous multi-reservoir operation optimization problem. *Water Resources Management* **21** (9), 1429–1447.

Labadie, J. W. 2004 Optimal operation of multireservoir systems. *Journal of Water Resources Planning and Management* **130** (2), 93–111.

Loehman, E., Orlando, J., Tschirhart, J. & Winstion, A. 1979 Cost allocation for a regional wastewater treatment system. *Water Resources Research* **15**, 193–202.

Madani, K. 2010 Game theory and water resources. *Journal of Hydrology* **381** (3–4), 225–238.

Madani, K. & Dinar, A. 2013 Exogenous regulatory institutions for sustainable common pool resource management: application to groundwater. *Water Resources and Economics* **2**, 57–76.

Madani, K. & Lund, J. R. 2011 A Monte-Carlo game theoretic approach for multi-criteria decision making under uncertainty. *Advances in Water Resources* **34** (5), 607–616.

Madani, K. & Zarezadeh, M. 2012 Bankruptcy methods for resolving water resources conflicts. In: *2012 World Environmental and Water Resources Congress*, ASCE, Reston, VA, USA, pp. 2247–2252.

Madani, K., Zarezadeh, M. & Morid, S. 2014 A new framework for resolving conflicts over transboundary rivers using bankruptcy methods. *Hydrology and Earth System Sciences* **18** (8), 3055–3068.

Mas-Colell, A. 1989 An equivalence theorem for a bargaining set. *Journal of Mathematical Economics* **18**, 129–139.

Mianabadi, H., Mostert, E., Zarghami, M. & van de Giesen, N. 2014 A new bankruptcy method for conflict resolution in water resources allocation. *Journal of Environmental Management* **144**, 152–159.

Mianabadi, H., Mostert, E., Pande, S. & van de Giesen, N. 2015 Weighted bankruptcy rules and transboundary water resources allocation. *Water Resources Management* **29** (7), 2303–2321.

Minville, M., Brissette, F., Krau, S. & Leconte, R. 2009 Adaptation to climate change in the management of a Canadian water-resources system exploited for hydropower. *Water Resources Management* **23** (14), 2965–2986.

Momtahen, S. & Borhani Darian, A. 2005 Genetic algorithm (GA) method for optimization of multireservoir system operation. *Water and Wastewater* **56** (7), 11–20.

Moy, W. S., Cohon, J. L. & ReVelle, C. S. 1986 A programing model for analysis of the reliability, resilience, and vulnerability of a water supply reservoir. *Water Resources Research* **22** (4), 489–498.

Nash Jr, J. F. 1950 The bargaining problem. *Econometrica: Journal of the Econometric Society* **18** (2), 155–162.

O'Neill, B. 1982 A problem of rights arbitration from the Talmud. *Mathematical Social Sciences* **2** (4), 345–371.

Orouji, H., Bozorg-Haddad, O., Fallah-Mehdipour, E. & Mariño, M. A. 2014 Flood routing in branched river by genetic programing. *Proceedings of the Institution of Civil Engineers, Water Management* **167** (2), 115–123.

Packel, E. W. 1981 A stochastic solution concept for n-person games. *Mathematics of Operations Research* **6** (3), 349–362.

Read, L., Madani, K. & Inanloo, B. 2014 Optimality versus stability in water resource allocation. *Journal of Environmental Management* **133**, 343–354.

Rosenthal, R. E. 1981 A nonlinear network flow algorithm for maximization of benefits in a hydroelectric power system. *Operations Research* **29** (4), 763–786.

Rubinstein, A. 1982 Perfect equilibrium in a bargaining model. *Econometrica: Journal of the Econometric Society* **50** (1), 97–109.

Sandoval-Solis, S. 2011 Water Planning and Management for Large Scale River Basins Case of Study: the Rio Grande/Rio Bravo Transboundary Basin, PhD Dissertation, The University of Texas at Austin, Austin, TX, USA.

Sechi, G. M. & Zucca, R. 2015 Water resource allocation in critical scarcity conditions: a bankruptcy game approach. *Water Resources Management* **29** (2), 541–555.

Shapley, L. S. 1953 Stochastic games. *Proceedings of the National Academy of Sciences* 39 (10), 1095–1100.

Shapley, L. S. & Shubik, M. 1954 A method for evaluating the distribution of power in a committee. *American Political Science Review* **48**, 787–792.

Sheikhmohammady, M. & Madani, K. 2008 Sharing a multi-national resource through bankruptcy procedures. In: *Proceeding of the 2008 World Environmental and Water Resources Congress* (R. W. Babcock & R. Walton, eds). American Society of Civil Engineers, Honolulu, HI, USA, pp. 1–9.

Simonovic, S. P. & Mariño, M. A. 1980 Reliability programing in reservoir management: 1. Single multipurpose reservoir. *Water Resources Research* **16** (5), 844–848.

Vogel, R. M. & Bolognese, R. A. 1995 Storage-reliability-resilience-yield relations for over-year water supply systems. *Water Resources Research* **31** (3), 645–654.

Von Neumann, J. & Morgenstern, O. 1944 *Games and Economic Behavior*. Princeton University Press, Princeton, NJ, USA.

Wardlaw, R. & Sharif, M. 1999 Evaluation of genetic algorithms for optimal reservoir system operation. *Journal of Water Resources Planning and Management* **125** (1), 25–33.

Yeh, W. W. G. 1985 Reservoir management and operations models. *Water Resources Research* **21** (12), 1797–1818.

Zarezadeh, M., Madani, K. & Morid, S. 2012 Resolving transboundary water conflicts: lessons learned from the Qezelozan-Sefidrood River bankruptcy problem. In: *World Environmental and Water Resources Congress 2012*, Albuquerque, NM, USA, May 20–24.

Zongxue, X., Jinno, K., Kawamura, A., Takesaki, S. & Ito, K. 1998 Performance risk analysis for Fukuoka water supply system. *Water Resources Management* **12** (1), 13–30.

First received 10 February 2017; accepted in revised form 16 May 2017. Available online 24 June 2017