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SUPERCONDUCTING WEAK LINKS

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ABSTRACT

Calculations of static $I_0-V_0$ curves, and of the heights of steps
induced by rf radiation, have been carried out for current biased Joseph-
son junctions with both a phase-independent quasiparticle conductance
$G_0$ and a phase-dependent quasiparticle pair "interference" conductance
$G_1 \cos \phi$. Little or no effect of the term in $G_1 \cos \phi$ is found for junc-
tions with zero capacitance. It thus appears that this term can be
neglected in many practical applications of weak links. Substantial
effects are found for the case of finite junction capacitance. The
term in $G_1 \cos \phi$ then enhances the hysteresis of the static $I_0-V_0$ curve
by an amount which roughly corresponds to a factor of 2-5 increase in
capacitance, and causes the static voltage to increase above the value
expected from the shunt conductance alone. The most easily observable
effect of the term in $G_1 \cos \phi$ on the rf induced step heights is a
pronounced shift in the values of rf current for which maxima occur for
low values of normalized rf frequency.

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According to the original work of Josephson\textsuperscript{1} as developed by Josephson\textsuperscript{2,3} and by Nam,\textsuperscript{4} the current in a superconducting tunnel junction is given by

\[ I(t) = I_c \sin \phi(t) + \left[ G_0(V) + G_1(V) \cos \phi(t) \right] V(t). \quad (1) \]

The first term on the right is the usual phase dependent Josephson pair tunneling current. The second term, $G_0 V$, is the dissipative quasiparticle tunneling current neglecting coherence.\textsuperscript{2} The third term, which is both dissipative and phase dependent, has been described\textsuperscript{5,6} as an interference term between the pair and quasiparticle currents when the effects of coherence on the quasiparticle distribution are included.\textsuperscript{7} Because the quasiparticle current in a tunnel junction biased well below the gap is small, the interference term has usually been neglected. Recent experiments on the Josephson plasma resonance in Pb-Pb tunnel junctions by Pedersen \textit{et al.}\textsuperscript{6} have shown evidence for the existence of the term in $G_1 \cos \phi$ and obtained a value for the ratio $\gamma = G_1/G_0$ of $-0.9 \pm 0.2$. A subsequent calculation by Poulsen\textsuperscript{5} from the microscopic theory of tunnel junctions has shown that, for small voltages, the ratio is essentially independent of $V$, and is equal to $-0.93$ for the junctions used by Pedersen \textit{et al.}

It is of interest to explore whether a term in $G_1 \cos \phi$ with $\gamma$ of this order is present in the proper description of weak links other than tunnel junctions. Since the shunt conductance $G_0$ is large in point contacts, Dayem bridges, and proximity effect bridges, the existence of such a term might be of considerable practical importance for device
applications. The theory of such weak links is not developed sufficiently to answer this question reliably. In order to obtain an experimental answer, we have calculated such experimentally observable quantities as the static $I_0 - V_0$ characteristic and the height of rf induced steps in the presence of voltage-independent $G_1$ and $G_0$ with a constant negative ratio. In the absence of a shunt capacitance, little or no effect due to the $\gamma G_0 V \cos \phi$ term is obtained. When nonzero capacitance is introduced, observable effects are predicted. None of the predicted effects, however, are dramatic enough for the existence of the term in $\gamma G_0 \cos \phi$ to be verified from published data.

We introduce the junction shunt capacitance into Eq. (1) in the usual way,\textsuperscript{8,9} letting $I = C \frac{dV}{dt}$ for the capacitor, and replacing $V$ with $\frac{h}{2e} \frac{d\phi}{dt}$. Using convenient dimensionless units\textsuperscript{8} we obtain, for the time evolution of the phase in a junction biased with a constant $I_0$,

$$\beta c \frac{d^2 \phi}{d\tau^2} = i_0 \cdot \sin \phi - \frac{d\phi}{dt} (1 + \gamma \cos \phi),$$

(2)

where $i_0 \equiv I_0 / I_c$, $\tau \equiv 2e I_c t / h G_0$, and $\beta_c = 2eI_c C / h G_0^2$. Taking $G_0$ and $\gamma$ to be constant we have integrated Eq. (2) numerically to find the periodicity of $d\phi/d\tau$ and used this to determine the dependence of the time averaged dc voltage $V_0 \equiv \frac{I_c}{G_0} \left( \frac{d\phi}{dt} \right)$ across the junction on the applied dc current $I_0$ for given values of $\gamma$ and $\beta_c$.

Static $I_0 - V_0$ characteristics corresponding to earlier calculations for $\gamma = 0$\textsuperscript{8,9} and also for $\gamma = -0.95$ are shown in Fig. 1 for several values of the capacitance parameter $\beta_c$. These static characteristics are independent of $\gamma$ for zero junction capacitance, as can be shown
by analytic integration of Eq. (2) with $\beta_c = 0.10$. For a given $\beta_c > 0$, and a given bias current, the dc voltage increases as $\gamma$ goes from 0 to -0.95. Note that for $\beta_c$ and $\gamma$ both nonzero, the $I_o - V_o$ characteristic drops below the asymptotic line $I_o = V_o G_o$. This corresponds to a power dissipation greater than $I_o^2 / G_o$ over a wide range of current. The voltage at which $I_o = V_o G_o$ scales as $\beta_c^{-1/2}$ for $\beta_c \geq 0.1$. Similar calculations for $\gamma = -0.5$ show smaller deviations, with the crossover moving towards higher currents.$^{11}$

The most pronounced effect of including the $\gamma G_o \cos \phi$ conductance is the increase in the hysteresis of the $I_o - V_o$ curve at nonzero $\beta_c$. Defining a hysteresis parameter $\alpha \equiv I_{i min} / I_o$ in the usual way,$^{8,9}$ we plot $\beta_c$ vs $\alpha$ in Fig. 2 for $\gamma = 0$, -0.5 and -0.95. For purposes of comparison, we have also shifted the curve for $\gamma = 0$ down by an amount corresponding to a reduction in $\beta_c$ by a factor 0.45 to show that the shape of the curve is most strongly affected in the range of small hysteresis ($\alpha \sim 1$).

That the $\beta_c^{-1/2}$ scaling law is an inherent feature of the model is most easily shown from the definition of $\beta_c$:

$$\beta_c \equiv \frac{2 e I_c}{\hbar c} \left( \frac{C}{G_o} \right)^2 \omega_p^2 \tau_o^2$$

where $\omega_p = (2 e I_c / \hbar c)^{1/2}$ is the zero-phase plasma frequency and $\tau_o = C / G_o$ is the time constant of the junction. When $\omega_p \tau_o \leq 1$ (large $\beta_c$) the junction is lightly damped in the sense that the phase evolution is governed by the plasma frequency $\omega_p$ and the junction time constant can be neglected; in this regime the modulation of the phase during a cycle (and consequently the deviation of the $I_o - V_o$
characteristic from the asymptote) is governed solely by $\gamma$ and the
dimensionless ratio $(d\phi/dt)/\omega_p = (V G_c/I_c)\beta_c^{-1/2}$. Since a given feature
of the $I_o-V_o$ characteristic, such as crossing the asymptotic conductance,
will occur at the same value of $(d\phi/dt)/\omega_p$ for all junctions having the
same $\gamma$ and $\beta_c > 1$, the voltage $V_x$ at which such a feature occurs will
scale as $V_x = \beta_c^{-1/2} I_c G_o^{-1}[(d\phi/dt)/\omega_p] = \text{constant} \times \beta_c^{-1/2}$. When
the junction is heavily damped, $\beta_c << 1$, the phase response will be
governed by the junction time constant $\tau_o$ and the scaling law breaks
down. Our calculations show that $\beta_c < 0.1$ is necessary before deviations
from the $\beta_c^{-1/2}$ law become significant.

The deviation of the static $I_o-V_o$ curves for nonzero $\gamma$ can be most
easily understood by examining Fig. 3, in which we plot a single period
of the time evolution of the phase for various values of $\gamma$ and $\beta_c$
at a fixed value of $i_o = 1.2$. First consider the case $\gamma = \beta_c = 0$. For
$\phi$ near $\pi/2$, the bias current flows primarily through the superconducting
junction, so there is little dissipation in the shunt resistor. When
$\phi$ is near $3\pi/2$, the junction current is opposite to the bias current,
so the current (and dissipation) in the shunt resistor is large. As
the phase spends more time near $\pi/2$ during a cycle than near $3\pi/2$,
the average dissipation is less than the value $I_o^2/G_o$ for the shunt
alone. This corresponds to a time-averaged forward supercurrent, and
an average voltage $V_o < I_o/G_o$.

As $\beta_c$ is increased for $\gamma = 0$, the phase-time characteristics
change as shown in Fig. 3(a). The inertial effect of the capacitance
smooths out the ac supercurrent, equalizing the time spent near $\pi/2$ and
3π/2. As βc → ∞ at constant i₀, the voltage approaches the value
V₀ = I₀/G₀ and the time-averaged supercurrent vanishes.

When we choose γ = -0.95, we obtain the phase-time characteristics shown in Fig. 3(b). A careful comparison of Fig. 3(a) with Fig. 3(b) shows that the ratio of the time spent near π/2 to that spent near 3π/2 is independent of γ for βc = 0. As we increase βc from zero, the decreased conductance near φ = 0 shortens the junction time constant and decreases the time spent near φ = π/2 while the increased time constant near φ = π increases the time spent near φ = 3π/2. There is a regime where the phase actually spends more time near 3π/2 than near π/2, giving a time-averaged reverse supercurrent, so that the dissipation and the time-averaged voltage are greater than for the conductance alone; V₀ > I₀/G₀. As βc → ∞ the capacitor again smooths out the time evolution of the phase so that the I₀ - V₀ characteristic approaches the asymptotic value, V₀ = I₀/G₀.

Because of the present interest in ac Josephson effect devices, the possible influence of a nonzero γ on the ac response of various types of weak links is of practical importance. In order to explore this question we include an rf current source i_rf cosΩ_rf t in the right-hand side of Eq. (2), where Ω_rf = ℏω_rf G₀/2eIc and i_rf = I_rf/Ic. As a measure of the rf response of the junction we have computed the dependence of the height of the radiation induced steps with n = 0, 1 and 2 on i_rf. For βc = 0, no effects of nonzero γ were observed for values of reduced frequency Ω_rf < 0.16. The effects of nonzero γ increase with Ω_rf, but remain quite small even for Ω_rf as large as 1.7.
They include a small shift in the positions of the minima, a lifting of the minima away from zero step height, and a small decrease in the height of the subsidiary maxima. All of these effects would be difficult to observe. Since junctions for applications generally have $\alpha = 1$, step heights were also computed for $\beta_c = 0.15$, the largest value giving no hysteresis. The largest effects then occur for small values of the product $\Omega_{rf} \beta_c = \omega_{rf} C/G_o$. Neither $\beta_c = 0.15$, nor $\gamma = -0.95$ alone has an appreciable effect on the step heights, but in combination they produce a marked shift in the positions of the subsidiary maxima, as shown in Fig. 4 for $\Omega_{rf} = 0.16$.

Even though a finite $\gamma$ increases the harmonic content of the phase-time curve, the heights of integral order steps are not very sensitive to $\gamma$. For these steps, the rf period averages over $n$ periods of the phase oscillation. Although we have not explored the $\gamma$-dependence of subharmonic steps, these might have a different sensitivity, as several rf cycles occur during one phase period.

We conclude that a phase dependent quasiparticle-pair conductance term $yG_o \cos \phi$ produces potentially observable effects on the static $I_o - V_o$ curves and the radiation induced steps of Josephson junctions shunted by both resistance and capacitance. In no case, however, are the effects large enough to be recognized in published data on weak links, or to require the inclusion of this term in an equivalent circuit used for device design.

It will, in fact, be difficult to establish unambiguously the existence of the $yG_o \cos \phi$ conductance by direct comparison with $I_o - V_o$ curves for any type of junction. As $G_o$ can only strictly be considered
constant near zero bias, unshunted tunnel junctions will have a very small $G_0$ value. Externally shunted junctions are obviously unsuitable, while for the several extant types of weak link the need for simultaneous determination of $\beta$, $G_0$, and $\gamma$ for comparison with Fig. 1 makes such a comparison highly unlikely. Even with optimal values of applied rf fields, the shifts in the amplitudes and positions of the rf steps are presently smaller than the accuracy with which the induced rf currents can be estimated. We do, however, note that the position of the first minima of the $n = 0$ step shown in Fig. 4 does not shift, while successive minima occur at progressively lower rf currents. Thus an investigation of the relative positions of the successive minima of the $n = 0$ step as a function of applied rf power appears to be the most promising and sensitive method for investigating the behavior of the $\gamma G_0 \cos \phi$ term in weak link devices.

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REFERENCES

10. We thank Y. Taur for first pointing out this result.
FIGURE CAPTIONS

Fig. 1. Normalized static $I_o-V_o$ curves for various values of the junction capacitance parameter $\beta_c$ and pair quasiparticle interference parameter, $\gamma$, in the absence of rf fields.

Fig. 2. Plot of the junction capacitance parameter $\beta_c$ against the junction hysteresis parameter $\alpha$ for various values of the pair-quasiparticle interference conductance $\gamma G_0 \cos \phi$. The dashed line is the $\gamma = 0$ curve shifted downward by an amount corresponding to a reduction in $\beta_c$ by a factor of 0.45.

Fig. 3. Influence of a finite pair-quasiparticle interference conductance $\gamma G_0 \cos \phi$ on the time evolution of the junction phase $\phi$. (a) With zero interference current ($\gamma = 0$), at various values of $\beta_c$. (b) With finite interference current ($\gamma = -0.95$), at the same values of $\beta_c$.

Fig. 4. Plot of the height of the rf-induced dc current steps for $n = 0, 1, \text{and } 2$ as a function of the rf current at the reduced frequency $\Omega_{rf} = 0.16$. The locations of the subsidiary maxima are shifted by the combined effects of a small junction capacitance $\beta_c = 0.15$ and a finite quasiparticle-pair interference term $\gamma = -0.95$. 
Fig. 1

Normalized Voltage $V_o G_o / I_C$

Normalized Current $I_o / I_C$

- $\gamma = -0.95$
- $\gamma = 0$

$\beta_c = 0.1$

$0.1$

$2$

$10$

$10$

$I_o$

$V$

$C$

$G_o$

$\gamma G_o$

$\cos \phi$

$\sin \phi$

$I_C$
Fig. 2
Fig. 3
Fig. 4
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