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Supply Chain Relationships and Contracts: The Impact of Repeated Interaction on Capacity Investment and Procurement

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Consider a firm developing an innovative product. Due to market pressures, production must begin soon after the product development effort is complete, which requires that an upstream supplier invests in capacity while the design of the product and production process are in flux. Because the product is ill-defined at this point in time, the firms are unable to write court-enforceable contracts that specify the terms of trade or the supplier's capacity investment. However, the firms can adopt an informal agreement (*relational contract*) regarding the terms of trade and capacity investment. The potential for future business provides incentive for the firms to adhere to the relational contract. We show that the optimal relational contract may be complex, requiring the buyer to order more than her demand to indirectly monitor the supplier's capacity investment. We propose a simpler relational contract and show that it performs very well for a broad range of parameters. Finally, we identify characteristics of the business environment that make relational contracting particularly valuable.

1. Introduction

Consider a firm developing an innovative product. Due to market pressures, production must begin soon after the product development effort is complete, which requires that an upstream supplier invests in capacity while the design of the product and production process are in flux. At this point in time, the firms cannot write a precise, court-enforceable description of the product and production process, and therefore cannot contract on the price, production capacity, or production quantity. Without a contract, the supplier faces a classic hold-up problem and will underinvest in capacity. Fortunately, to the extent that the firms anticipate repeated business, they can adopt an informal agreement (i.e., a *relational contract*) to reward the supplier for building capacity. The relational contract is sustained, not by the court system, but by the future value of a trusting, cooperative relationship. This paper characterizes the structure of an optimal relational contract that maximizes total expected profit for the innovating firm and its supplier.

This study of relational contracting is motivated by examples from the electronics, automobile, and semiconductor equipment industries. In the electronics industry, Toshiba designs and sells innovative

consumer products. Over the course of the product development process, the design of the product changes, sometimes substantially. If Toshiba's supplier were to delay making production capacity investments until the product's characteristics were fully specified so that the firms could write a court-enforceable procurement contract, the resulting delays in production would be unacceptable. Consequently, Toshiba and its supplier adopt a long-term relational contract to create incentives for the supplier to invest in production capacity. After the supplier's capacity investment and just before large-scale production, Toshiba and its supplier write a short-term, court-enforceable procurement contract. Because the supplier's capacity investment is sunk, Toshiba could negotiate a lower price, but this would damage the long-term trading relationship and prospects for future innovative products. Instead, Toshiba pays a generous price as stipulated by the relational contract (Sako 1992). Similarly, rapid innovation in hybrid vehicles creates difficulties in writing court-enforceable contracts for capacity (Hoyt and Plambeck 2006). The degree to which suppliers are willing to set aside capacity for buyers depends on the depth of the buyer-supplier relationship. Because

Ford has a relatively weak relationship with its hybrid-transmission supplier, it has had more difficulty obtaining capacity than Japanese automakers who have stronger relationships with the supplier (Tierney 2005). As a final example, in the semiconductor equipment industry, a common practice is that the buyer, in advance of placing a binding order, shares a demand forecast with its supplier. The forecast serves as an informal or *soft order* intended to guide the supplier's production decisions. Insofar as the supplier trusts the buyer to purchase in line with the forecast or compensate the supplier on canceling soft orders, the supplier dedicates production resources to meet the buyer's demand forecast (Cohen et al. 2003, Johnson 2003).

The managerial contribution of this paper is to provide insight into how and when supply chain partners should employ relational contracts to provide incentives for capacity investment. To do so, we employ a repeated newsvendor model. We provide a rich characterization of the optimal relational contract. It is complex because the buyer should, under particular circumstances, order more than the realized demand in order to monitor the supplier's capacity investment. We recommend a simpler relational contract without such monitoring and identify broad, plausible conditions under which the simpler relational contract is effective. We also characterize the more narrow circumstances under which the more complex relational contract with monitoring is warranted. Because employing relational contracts—even simple ones—requires significant managerial effort in forethought and coordination, we characterize when relational contracts create the most value and so justify such effort: When the cost of capacity is moderate, the bargaining power is evenly distributed, and the firms do business together frequently.

1.1. Literature Review

Many papers in the supply chain contracting literature examine the impact of *court-enforceable* contracts on capacity investment; Cachon (2003) and Chen (2003) review this literature. If both price and capacity are contractible, then by properly specifying the terms of the contract, the buyer can maximize the total supply chain profit and appropriate it entirely (Cachon and Lariviere 2001). However, Cachon and Lariviere (2001) observe that capacity may not be contractible, and van Mieghem (1999) observes that even the per-unit price may not be contractible, prior to the supplier's capacity investment; it is this setting where price and capacity are noncontractible that we study.

Macaulay (1963) documented that instead of relying on formal, court-enforced contracts, firms rely on informal agreements in procurement. Economists' primary model for the study of such cooperation is the repeated game, in which players face the same "stage game" in every time period (Fudenberg and Tirole 1991). A repeated game typically has many possible Nash equilibria, but the players can agree to adopt one that is mutually advantageous. For example, in Taylor and Wiggins (1997) a buyer inspects every shipment from his manufacturer and rejects faulty items. Taylor and Wiggins show how the buyer can avoid costly inspection by paying a premium for every shipment and threatening to terminate this practice if she later discovers faulty items. Baker et al. (2001, 2002) consider repeated procurement and derive insights regarding ownership structure. Levin (2003) considers a generic, repeated agency problem with moral hazard or hidden information. The principal promises to pay the agent based on the outcome of his action, but cannot write a formal contract. If the principal reneges, the agent will refuse to cooperate in future periods. The maximum credible payment, and hence the strength of incentives for the agent, depends on the value of future cooperation from the agent.

Recently, researchers have explored relational contracts in settings of interest to operations management. Taylor and Plambeck (2007) consider a modeling setting similar to the one here, but focus on comparing the performance of relational contracts that commit the buyer to purchase a fixed quantity versus relational contracts that only specify a per-unit price. Debo and Sun (2004) consider a repeated game in which the supplier sets the wholesale price and the buyer must order before realizing demand. They show that the firms can increase their per-period expected profit by adopting an informal agreement in which the supplier offers a price that is lower than his optimal stage-game price, and the buyer responds by ordering a quantity that is larger than her myopically-optimal order quantity. Tunca and Zenios (2006) model the interplay between relational contracts and supply auctions, with multiple suppliers that differ in quality. For a review of the sociology literature on relational contracts, we refer the reader to Plambeck and Taylor (2006).

This paper is organized as follows. Section 2 introduces a single-period game, and §3 considers the model with repeated interaction. Section 4 characterizes the structure and performance of an optimal relational contract. Section 5 examines the performance of a simpler relational contract and shows how the optimal relational contract changes when the buyer observes the supplier's capacity. Section 6 explains the effects of introducing a random production cost, nonlinear capacity costs, outside options, private information about the cost of capacity, and private information about the demand forecast. Section 7 provides concluding remarks.

2. The Single-Period Game

Consider a simple two-firm supply chain for an innovative product. The downstream firm, denoted the buyer, sells the product to a market in which demand is uncertain. The buyer (she) purchases the product from an upstream supplier (he). Because of long lead times, the supplier must invest in capacity before the firms can contract for production and before demand is known. The firms are risk neutral. This section describes the physical model of capacity investment and examines the case where the firms interact only once. Subsequent sections consider the implications of repeated interaction. Demand ξ is a random variable with distribution function $\Phi(\xi)$ and support Γ. The retail price is r, and the per-unit cost of capacity is c. The assumption that the retail price is fixed is reasonable when the buyer targets a price point. The salvage value of excess product or capacity is assumed to be negligible. We also assume that the production cost is negligible, but relax this assumption in §6. The demand distribution and retail price are known to both firms. However, only the buyer observes the realized demand ξ; this assumption is motivated by the observation that a supplier often lacks visibility into the specifics of the buyer's end market (e.g., individual customers, orders). Further, the buyer does not observe the supplier's capacity; this assumption is reasonable if the buyer is unable to determine what human and physical assets (and their productivity) the supplier has dedicated to the buyer's product.

The sequence of events is as follows:

1. The supplier invests in production capacity K and incurs cost cK (unobserved by the buyer).

2. The buyer observes demand ξ (privately).

3. The firms contract on the wholesale price per unit w. The buyer orders, and the supplier produces and delivers.

Because the product is ill-defined when the supplier initiates his capacity investment (Step 1), at that time the firms are unable to write a court-enforceable procurement contract. However, close to the selling season (Step 3), the product is well-defined and the firms can contractually specify the price.

We assume that in this single-period game, the firms split the ex post gain from trade $r\min(K, \xi)$ according to the generalized Nash bargaining solution with share $\sigma \in (0, 1)$ for the supplier. Specifically, the buyer contracts to pay $w = \sigma r$ per unit and purchases the efficient quantity $q = \min(K, \xi)$.[1] The

profit for the supplier (excluding the sunk cost of capacity) is $\sigma r \min(K, \xi)$, and the profit for the buyer is $(1 - \sigma)r\min(K, \xi)$. The Nash bargaining solution with $\sigma = 1/2$ is the unique outcome that satisfies a set of axiomatic properties including Pareto optimality and independence of irrelevant alternatives (Nash 1950). The economics literature on incomplete contracts and on relational contracts adopts the generalized Nash bargaining solution (e.g., Grossman and Hart 1986, Baker et al. 2002), and interprets $\sigma \in (0, 1)$ as the supplier's bargaining strength. The supplier's bargaining strength is influenced by many factors such as patience for negotiation, whether the buyer or supplier makes the first offer, personal relationships, previous experience in negotiation, relative size, and market forces.

Anticipating the per-unit price σr, the supplier's expected profit when he builds K units of capacity is

$$\sigma r E[\min(K, \xi)] - cK.$$

The supplier faces a newsvendor problem, and his optimal capacity is

$$\underline{K} = \Phi^{-1}([1 - c/\sigma r]^+). \tag{1}$$

The supplier's and buyer's expected profit are

$$\underline{\Pi}_S = \sigma r E[\min(\underline{K}, \xi)] - c\underline{K}$$

$$\underline{\Pi}_B = (1 - \sigma)r E[\min(\underline{K}, \xi)].$$

The total expected profit $r E[\min(K, \xi)] - cK$ is maximized at $\bar{K} = \Phi^{-1}([1 - c/r]^+)$. If the supplier captures all the gain from trade so that all system revenue accrues to him ($\sigma = 1$), then he will build the first-best level of capacity $\bar{K}$. If the buyer captures a portion of the gain from trade ($\sigma < 1$), then the supplier will build a level of capacity that is smaller than the first best $\bar{K}$. This is a classic hold up problem: The supplier invests too little because he will capture only a fraction of the return on investment. If the capacity cost is sufficiently high ($c \geq \sigma r$), then the supplier's incentive to invest is eliminated ($\underline{K} = 0$).

3. A Model of Repeated Interaction

Now suppose that the firms produce and sell a succession of distinct products, repeating the game described in §2 in periods $t = 1, 2, \ldots$. Because the

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[1] Suppose that the supplier and buyer bargain noncooperatively by making alternating offers of the per-unit price as in Rubinstein (1982). Then in the unique subgame perfect equilibrium, by the theorem in Rubinstein (1982, p. 106), the price σr is immediately offered and accepted. The parameter $\sigma \in (0, 1)$ depends on which

firm makes the first offer, the time between offers, the discount factor, and each firm's cost for delay in bargaining. The key to extending Rubinstein's theorem to this setting with private information is to recognize that the expected profit after contracting on per-unit price w is $(r - w)E[\min(K, \xi)]$ for the buyer and $wE[\min(K, \xi)]$ for the supplier. The private information is embedded in the constant multiplier $E[\min(K, \xi)]$ and does not affect the subgame perfect equilibrium.

product produced in each period is distinct, the supplier must make a new capacity "investment" in each period. (This does not necessarily mean that the supplier builds a new production facility every period; instead, the capacity could be thought of as reserved for the buyer's specific product.)[2] The firms are infinitely lived. At the end of each period their game terminates with probability ψ, and the common discount factor is δ'. The termination probability may be a measure of the stability of the firms, economic conditions, or the riskiness of the product market. The discount factor reflects the firms' cost of capital and the length of time between successive products. The effective discount factor is $\delta = (1 - \psi)\delta'$.

With repeated interaction, the firms can adopt an informal agreement (relational contract) that motivates the supplier to build more capacity than in the single-period game §2. For each period $t = 1, 2, \ldots, \infty$, the relational contract specifies that the buyer makes an initial transfer payment to the supplier D^t, orders quantity $q = q^t(\xi) \geq 0$ contingent on her realized demand ξ, and pays the supplier $d^t(q)$ to produce and deliver the q units. The relational contract also specifies that the supplier builds capacity $K^t = \max_{\xi \in \Gamma}\{q^t(\xi)\}$ so that he can produce the requested quantity. We do not place restrictions on the functions $q^t(\cdot)$ and $d^t(\cdot)$ beyond the obvious requirement that the requested quantity $q^t(\cdot)$ be nonnegative; in particular, the functions need not be strictly increasing on $[0, K]$. The sequence of events is as follows:

0. The buyer pays the supplier D^t.

1. The supplier invests in production capacity K^t and incurs cost cK^t (unobserved by the buyer).

2. The buyer observes demand ξ^t (privately) and orders quantity $q^t(\xi^t)$ from the supplier.

3. The supplier produces and delivers quantity q^t, and the buyer pays him $d^t(q^t)$.

Although, for simplicity, we describe the payments as being from the buyer to the supplier, the payments may be negative, indicating that the supplier pays the buyer. The terms $\{D^t, d^t, q^t, K^t\}$ for each period t may depend on all that the firms have commonly observed up to the beginning of period t, for $t = 1, 2, \ldots, \infty$, and are not court-enforceable. Each firm decides dynamically whether or not to adhere to the terms, and seeks to maximize its own expected discounted profit. (The letter "d" in the initial transfer payment D^t and in the quantity-contingent payment $d^t(q)$ is mnemonic

for "discretionary.") A *trigger strategy* is to adhere to the terms until a player publicly fails to do so,[3] and thereafter to play the equilibrium strategy for the single-period game §2. In particular, if either firm fails to adhere to the quantity-contingent payment, the firms revert to Nash bargaining (i.e., the buyer pays σr per unit and purchases the efficient quantity $q = \min(K, \xi)$). The terms $\{D^t, d^t, q^t, K^t\}_{t=1,2,\ldots,\infty}$ and trigger strategies must constitute a perfect public equilibrium, and are then called a *self-enforcing relational contract* (Baker et al. 2001, 2002; Levin 2003).[4] Once the firms coordinate on a self-enforcing relational contract, neither party will subsequently wish to deviate from it unilaterally.

Our problem is to construct an *optimal relational contract*, that is, a self-enforcing relational contract that maximizes the firms' total expected discounted profit. We subsequently explain (see footnote 5) how to set D^1 to implement any allocation of the surplus expected discounted profit generated by an optimal relational contract.

Trigger strategies provide maximal incentives for the firms to adhere to the specified terms of trade, because the noncooperative outcome is the most severe punishment for cheating that is credible. Therefore one may, without loss of generality, restrict attention to trigger strategies in deriving optimal terms of trade (Levin 2003). A concern with trigger strategies is that, if one firm violated the relational contract and punishment ensued, the firms could increase their ongoing profits by renegotiating to resume cooperation. Levin (2003, p. 840) explains how to make the optimal terms of trade immune to renegotiation: If one firm cheats, the firms resume cooperation but the cheater pays a penalty to reduce his ongoing expected discounted profit to the noncooperative outcome. In settings where firms cannot make such make transfer payments to settle up, both firms must suffer in order to punish the cheater. In the psychology literature, Fehr et al. (1997) and references therein provide experimental evidence that people will forgo large amounts of money to punish unfair behavior. Punishment of unfair behavior is associated with neural activation in reward centers of the brain (Quervain et al. 2004). In a laboratory experiment using a repeated trust game, Schweitzer et al. (2006) observed that when a subject is deceived by its partner (the partner promises to make a payment in return for cooperative action and breaks that promise), in subsequent periods the subject tends to distrust his partner and

to behave noncooperatively. This behavioral observation that deception causes significant and enduring harm to trust provides support for focusing on trigger strategies (refusal to cooperate after a partner breaks a promise to pay or a promise to produce) as a first approximation. Atkins et al. (2006) propose a refinement of trigger strategies based on the plausible behavioral assumption that the magnitude of punishment will be proportional to the magnitude of deviation from the relational contract.

4. Optimal Relational Contract

In §4.1 we derive an optimal relational contract and characterize its structural properties. In §4.2 we compare expected profit per period under the optimal relational contract to expected profit in the single-period game. Essentially, we describe how to manage a relationship, and the benefits of doing so.

4.1. Structure of an Optimal Relational Contract

Our model has two-sided asymmetric information and imperfect monitoring. Repeated games with these features typically have optimal equilibria and optimal relational contracts with complex, history-dependent strategies (Abreu et al. 1990, Levin 2003). In contrast, our Proposition 1 characterizes a stationary optimal relational contract, meaning that the firms follow trigger strategies and that the payment terms and actions are identical in every period: For every t, $(D^t, d^t, q^t, K^t) = (D, d, q, K)$, where $D \in R$, $d: [0, K] \to R$, $K \in R^+$ and $q: \Gamma \to [0, K]$. The feasible set of self-enforcing terms of trade in any period depends on the future value of cooperation, which in turn depends on the terms of trade and corresponding capacity investment. Therefore, the optimal relational contract is characterized by solving a fixed-point problem.

PROPOSITION 1. *There exists a stationary optimal relational contract with expected profit per period*

$$\Pi^* = \max[\pi: f(\pi) = \pi],$$

where

$$f(\pi) = \max_{D, d, q, K} \{E[r \min(q(\xi), \xi)] - cK\} \quad (2)$$

subject to, for $\xi \in \Gamma$

$$D - cK + E[d(q(\xi))] = \pi - \underline{\Pi}_B \quad (3)$$

$$q(\xi) \in \arg\max_{q \in [0, K]} \{r \min(q, \xi) - d(q)\} \quad (4)$$

$$r \min(q(\xi), \xi) - d(q(\xi)) \geq (1 - \sigma) r \min(K, \xi) \quad (5)$$

$$d(q(\xi)) + \delta(1 - \delta)^{-1} (\pi - \underline{\Pi}_B)$$

$$\geq \sigma r E[\min(K, \xi^1) \mid q(\xi^1) = q(\xi)] + \delta(1 - \delta)^{-1} \underline{\Pi}_S \quad (6)$$

$$K \in \arg\max_{K' \geq 0} \{-cK' + E[1_{\{q(\xi) > K'\}}(\sigma r \min(K', \xi)$$

$$+ \delta(1 - \delta)^{-1} \underline{\Pi}_S)]$$

$$+ E[1_{\{q(\xi) \leq K'\}} \max(d(q(\xi)) + \delta(1 - \delta)^{-1}(\pi - \underline{\Pi}_B),$$

$$\sigma r E[\min(K', \xi^1) \mid q(\xi^1) = q(\xi)]$$

$$+ \delta(1 - \delta)^{-1} \underline{\Pi}_S)]\}. \quad (7)$$

The stationary optimal relational contract has payment terms, order quantity, and capacity (D^*, d^*, q^*, K^*), a solution of (2)–(7) with $\pi = \Pi^*$.

The proof of this and the subsequent proposition are in the appendix. Here, we simply explain how the stationary optimal relational contract (D^*, d^*, q^*, K^*) works. Assuming an optimal relational contract generates total system expected profit π per period in all future periods, problem (2)–(7) is to find the self-enforcing terms of payment, order quantity, and capacity investment that maximize expected profit for the current period. If π is a fixed point $\pi = f(\pi)$, then the solution to (2)–(7) achieves expected profit π in the current period and is a self-enforcing relational contract. The converse is also true: If a relational contract is self-enforcing, it corresponds to a solution to (2)–(7) with some fixed point $\pi = f(\pi)$. Because the optimal relational contract is the self-enforcing relational contract that maximizes total system expected profit, it corresponds to the largest fixed point of $f(\cdot)$.

We now turn to explaining constraints (3)–(7). The left-hand side of constraint (3) is the supplier's single-period expected profit under the relational contract; the constraint sets the transfer payment D so that the supplier has expected profit per period of $\Pi^* - \underline{\Pi}_B$ and the buyer has expected profit per period of $\underline{\Pi}_B$. The transfer payment D is positive, indicating that the buyer pays the supplier.[5] Transferring the continuation surplus to the supplier maximizes the incentives for capacity investment.

Constraints (4) and (5) ensure that the buyer orders $q = q^*(\xi)$ when her realized demand is ξ, and makes the quantity-contingent payment $d^*(q)$. Because the buyer receives $\underline{\Pi}_B$ in subsequent periods regardless of whether she adheres to the proposed terms, constraint (5) reflects the buyer's current-period profit; the right-hand side is the buyer's current-period expected profit when she does not adhere to the proposed terms, and the firms instead revert to noncooperative bargaining.

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[5] This is not the unique optimal relational contract. The firms can initially reallocate expected discounted profit of $(1 - \delta)^{-1}[\underline{\Pi}_S + (1 - \alpha)(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S)]$ to the buyer and $(1 - \delta)^{-1}[\underline{\Pi}_B + \alpha(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S)]$ to the supplier, for any $\alpha \in [0, 1]$, simply by decreasing the first-period transfer payment by a constant: $D^1 = D^* - (1 - \delta)^{-1}\alpha(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S)$.

After receiving an order $q = q^*(\xi)$, the supplier could refuse to deliver q units for payment $d^*(q)$. Constraint (6) ensures that the supplier's expected discounted profit is greater when he instead adheres to the proposed terms. If the supplier does so, he receives $d^*(q)$ in the current period and expected profit $\Pi^* - \underline{\Pi}_B$ in each subsequent period, which explains the left-hand side of (6). To understand the right-hand side of (6), note that if the supplier does not adhere, the firms revert to noncooperative bargaining in the current period and, because the firms follow trigger strategies, the supplier receives $\underline{\Pi}_S$ in subsequent periods. In the current period, given that the supplier has capacity K and the buyer has ordered $q(\xi)$, the number of units that the supplier expects to sell at the per-unit price σr is

$$E[\min(K, \xi^1) \mid q(\xi^1) = q(\xi)],$$

where the conditional expectation is taken with respect to the random variable ξ^1, conditional on the event $\xi^1 \in \{\tilde{\xi}: q(\tilde{\xi}) = q(\xi)\}$.[6]

Constraint (7) ensures that the supplier builds capacity K^*. If the supplier's capacity is not sufficient to meet the buyer's order $K < q^*(\xi)$, then, because the supplier cannot adhere to the proposed terms, the firms revert to noncooperative bargaining in the current period and the supplier receives $\underline{\Pi}_S$ in subsequent periods. If the supplier's capacity is sufficient to meet the buyer's order $K \geq q^*(\xi) = q$, the supplier will choose to either deliver the order in return for the payment $d^*(q)$ or refuse, whichever maximizes his expected discounted profit.[7] To maximize the supplier's incentive to build the proposed capacity K^* and deliver the order in return for payment $d^*(q)$, the relational contract allocates the continuation surplus $\Pi^* - \underline{\Pi}_B$ to the supplier, which is, as noted above, achieved by setting the transfer payment D appropriately.

Proposition 2 characterizes the payment terms, order quantity and expected profit under the optimal relational contract. The last part of the proposition notes that for a class of demand distributions,

the complexity of the optimal relational contract is limited.

PROPOSITION 2. *The stationary optimal relational contract has expected profit*

$$\Pi^* = rE\min[(K^*, \xi)] - cK^*, \tag{8}$$

initial transfer payment

$$D^* = \Pi^* - \underline{\Pi}_B + cK^* - E[d^*(q^*(\xi))], \tag{9}$$

order quantity

$$q^*(\xi) = \begin{cases} \xi & \text{for } 0 \leq \xi < K_1 \\ K_m + \Delta_m & \text{for } K_m \leq \xi \leq K_m + \Delta_m \\ & \qquad m = 1, \ldots, M \\ \xi & \text{for } K_m + \Delta_m \leq \xi \leq K_{m+1} \\ & \qquad m = 1, \ldots, M \\ K^* & \text{for } \xi > K^*, \end{cases} \tag{10}$$
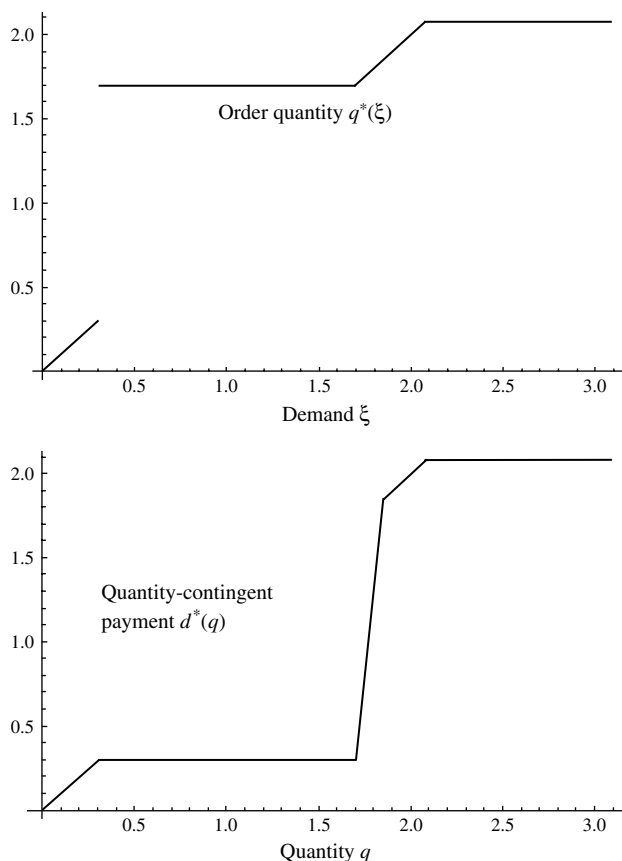
and quantity-contingent payment

$$d^*(q) = \begin{cases} \sigma r q & \text{for } 0 \leq q \leq K_1 \\ d^*(K_m) & \text{for } K_m < q \leq K_m + \Delta_m \quad m = 1, \ldots, M \\ d^*(K_m) + r(q - K_m - \Delta_m) \\ \quad \text{for } K_m + \Delta_m \\ \quad \leq q \leq \min\left[\dfrac{K_m + \Delta_m - d^*(K_m)/r}{1 - \sigma}, K_{m+1}\right] \\ \quad m = 1, \ldots, M \\ \sigma r q \quad \text{for } \dfrac{K_m + \Delta_m - d^*(K_m)/r}{1 - \sigma} \leq q \leq K_{m+1} \\ \quad m = 1, \ldots, M, \end{cases} \tag{11}$$

where M is a nonnegative integer, $0 \leq K_m < K_m + \Delta_m < K_{m+1}$ for $m \in \{1, \ldots, M\}$, and $K_{M+1} = K^*$. If ξ is a normal or truncated normal random variable or if ξ is a continuous random variable whose density is weakly decreasing, then the function $q^*(\xi)$ has at most one point of discontinuity: $M \in \{0, 1\}$.

Figure 1 illustrates Proposition 2 by depicting the optimal quantity-contingent payment and order quantity. In this example, $M = 1$: The order quantity function has one discontinuity. To provide stronger incentives for capacity investment, this optimal relational contract requires the buyer to purchase more than she needs to fulfill demand for moderate levels of demand. Knowing that the buyer will order this larger amount discourages the supplier from cheating by underbuilding capacity, which would otherwise go undetected in periods when demand was

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[6] A point of theoretical interest is that the revelation principle (Meyerson 1979), which allows for restricting attention to mechanisms in which agents report truthfully, breaks in our setting with relational contracting: Because $\max_{\xi \in \Gamma}[\min(K, \xi)] \geq \max_{\xi \in \Gamma} E[\min(K, \xi^1) \mid q(\xi^1) = q(\xi)]$, directly revealing the demand ξ to the supplier would tighten constraints (6) and (7), increasing the supplier's temptation to renege on the relational contract.

[7] The reader might be concerned the supplier could fail to adhere by first delivering q units for payment $d^*(q)$ and then offering to sell additional units at the noncooperative price. However, (5) implies $\sigma r \min(K, \xi) \geq d^*(q(\xi)) + \sigma r[\min(K, \xi) - q^*(\xi)]^+$ for $\xi \in \Gamma$ and $K \geq q^*(\xi)$. Thus, if the supplier has built adequate capacity to meet the buyer's order and chooses not to adhere, he should engage in outright noncooperative bargaining, rather than execute the proposed payment and quantity and subsequently bargain over any residual units.

Figure 1   Optimal Order Quantity and Quantity-Contingent Payment

Notes. Parameters are $r = 10$, $c = 1.25$, $\sigma = 0.1$, $\delta = 0.3$, and ξ is an exponential(1) random variable. The optimal capacity $K^* = 2.08$, and the optimal initial payment $D^* = 8.20$.

low. To maximize the supplier's incentive for capacity investment, the quantity-contingent payment is maximized subject to the constraints that the buyer is willing to order the specified quantity (4) and make the corresponding payment (5). The optimal quantity-contingent payment is continuous and piecewise linear. Initially, (5) binds and the slope is $\sigma r = 1$. For moderate quantities that correspond to the buyer ordering strictly more than realized demand, (4) binds and the slope is zero (excess units must be free, or the buyer will not buy them). For higher quantities that correspond to the buyer ordering exactly her demand, initially (4) binds so the slope is $r = 10$ and then (5) binds so the slope is $\sigma r = 1$.

In summary, having the buyer purchase more than she needs so as to monitor the supplier's capacity investment discourages the supplier from underbuilding capacity, but at the same time limits the magnitude of the discretionary payment, which limits the capacity investment that will be attractive to the supplier. The extent of monitoring in the optimal relational contract reflects this trade-off.

4.2. Performance of an Optimal Relational Contract

Because undertaking the coordination activities to establish a relational contract for procurement is a nontrivial effort, managers should assess the magnitude of the gain that can be reaped from relational contracting before undertaking the coordination effort. This section characterizes the conditions under which relational contracts create the most value and demonstrates that relational contracts can substantially increase the firms' expected profits.

As the firms interact more frequently, so that the discount factor δ increases, cooperation is easier to sustain (constraints (6) are (7) are relaxed), and so the per-period expected profit under an optimal relational contract Π^* increases. When the discount factor is sufficiently large, the first best is achieved $\Pi^* = \overline{\Pi}$. These results, which are straightforward to verify analytically, follow the familiar "folk theorem" for repeated games (Fudenberg and Tirole 1991).

To characterize the impact of the capacity cost and the allocation of bargaining power on the value of relational contracting, we conduct a numerical study. Figure 2 depicts the gain from using an optimal relational contract, with the shaded regions indicating this gain as a percentage of the first best: $(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S)/\overline{\Pi}$. In the gray transparent region, the optimal relational contract achieves the first best; in this region, the gain from using an optimal relational contract varies because the profit without a relational contract $\underline{\Pi}_B + \underline{\Pi}_S$ varies. In all panels, $r = 10$ and ξ is a normal random variable with mean 5 and standard deviation 3, truncated such that its probability mass is distributed over $\xi \geq 0$. In Figure 2(a) $\sigma = 0.5$; in Figure 2(b) $c = 7.5$; in Figure 2(c) $c = 2.5$. The figure is, as described below, representative of a larger numerical study.

The panels depict the intuitive result, noted above, that the gain from an optimal relational contract increases in the discount factor. What is more surprising is that the optimal relational contract may achieve the first best even when the discount factor is relatively small, so that the first best is achieved for a wide range of parameters. Similarly, for a wide range of parameters, the gain from using an optimal relational contract is substantial. Figure 2(a) shows that the gain from using an optimal relational contract is largest when the capacity cost is moderate. The intuition is that when the capacity cost is small, the supplier builds substantial capacity in the single-period game, so system performance without the relational contract is close to the first best. This makes cooperation difficult to sustain because each firm knows it will continue to do quite well if it reneges and cooperation breaks down. When the capacity cost is large and the discount factor is small, it is difficult to provide credible incentives for the supplier to increase

Figure 2   Gain from Optimal Relational Contract

(a)

(b)

(c)
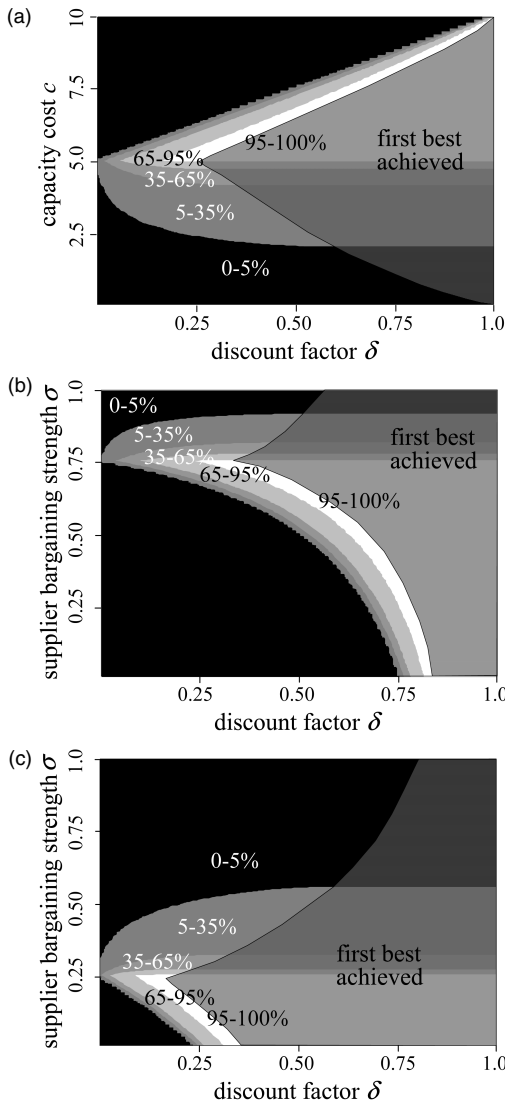
Note. The regions indicate where the optimal relational contract achieves the first best and the gain from using an optimal relational contract as a percentage of the first best.

his capacity investment significantly above the noncooperative level $\underline{K}$. The supplier will not comply with a suggested capacity investment that is much higher than $\underline{K}$, because the high capacity cost makes doing so painful and because the cheating supplier knows it will be able to escape detection when the order quantity is sufficiently small. In contrast, when the capacity cost is moderate, the system performs poorly without a relational contract ($\underline{\Pi}_B + \underline{\Pi}_S \ll \bar{\Pi}$) and building additional capacity is not excessively painful to the supplier, so the gain from an optimal relational contract is relatively large.

Figure 2(b) and (c) show that the gain from using an optimal relational contract is largest when the supplier's bargaining strength in the single-period game σ is moderate. More precisely, the gain from an opti-

mal relational contract is increasing in σ on $(0, c/r)$ and decreasing on $(c/r, 1)$; in Figure 2(b) $c/r = 0.75$ and in Figure 2(c) $c/r = 0.25$. When the supplier's bargaining strength is large ($\sigma > c/r$), in the single-period game the supplier will build nonzero capacity $\underline{K} > 0$ (see (1)), and as σ increases, supplier and total system profit increase. When σ is very large, system performance is close to the first best, and the supplier appropriates most of the system profit. Because the scope for gains from cooperation is limited and the supplier knows he will continue to do quite well if he reneges and cooperation breaks down, the gain from an optimal relational contract is small. In contrast when the supplier's bargaining strength is small ($\sigma < c/r$), in the single-period game the supplier will not build capacity $\underline{K} = 0$ and so outside of a cooperative relationship the firms' profits are zero $\underline{\Pi}_B = \underline{\Pi}_S = 0$. However, as σ increases on $(0, c/r)$, as described above, the buyer is able to credibly commit to a larger quantity-contingent payment, which induces the supplier to invest more in capacity, increasing system profit. Although not depicted in Figure 2, the impact of the standard deviation of demand also depends on the supplier's bargaining strength, and the intuition follows what we described above: When the supplier's bargaining strength is large ($\sigma > c/r$), as the standard deviation decreases, the profits outside the cooperative relationship $\underline{\Pi}_B + \underline{\Pi}_S$ increase, so the gain from an optimal relational contract decreases. In contrast, when the supplier's bargaining strength is small ($\sigma < c/r$), as the standard deviation decreases, the profits outside the cooperative relationship $\underline{\Pi}_B + \underline{\Pi}_S = 0$ are unaffected, so the gain from an optimal relational contract increases.

The qualitative insights in Figure 2 are robust to the remaining parameter specifications. In Figure 2(a), when σ is larger (smaller) the region with gains shrinks and shifts up (expands and shifts down). Further, the qualitative results continue to hold when demand is instead a uniform$(0, 1)$ or exponential(1) random variable. The details of the numerical study, which substantiates these assertions, are in the electronic companion to this paper.[8] To summarize, the gains from the optimal relational contract are largest when the capacity cost c and the supplier's bargaining strength σ are moderate, and the discount factor δ is large. The gain from relational contracting may increase or decrease with the standard deviation of demand, depending on the supplier's bargaining strength and the cost of capacity.

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[8] An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

5. Simple Relational Contracts

Although we have demonstrated that an optimal relational contract has simplifying structural properties (Propositions 1 and 2), an optimal relational contract can still be rather complex and hence difficult to implement. It would be difficult for a manager to explain the scheme in Figure 1 in words, much less a more complex scheme (10)–(11) with multiple discontinuities ($M > 1$). Consequently, in this section we examine two simpler relational contracts. Section 5.1 describes such a relational contract and compares its performance to the optimal relational contract. Section 5.2 shows how the optimal relational contract simplifies when the buyer observes the supplier's capacity.

5.1. No-Monitoring Relational Contract

The driver of complexity in the optimal relational contract is monitoring

$$q(\xi) > \xi, \tag{12}$$

so a natural way to construct a simpler relational contract is to restrict attention to relational contracts that do not involve monitoring

$$q(\xi) \leq \xi. \tag{13}$$

Then an optimal relational contract (the solution to the fixed-point problem (2)–(7) with added constraint (13)) has order quantity

$$q^n(\xi) = \min(K^n, \xi), \tag{14}$$

discretionary payments

$$d^n(q) = \sigma r q \quad \text{for } q \in [0, K^n], \tag{15}$$

$$D^n = \Pi^n - \underline{\Pi}_B + cK^n - \sigma r E[\min(K^n, \xi)], \tag{16}$$

and capacity K^n the solution to

$$f^n(\pi) = \max_K \{E[r\min(K, \xi)] - cK\} \tag{17}$$

subject to

$$K \in \arg\max_{K'} \{-cK' + E[\sigma r \min(K', \xi) + 1_{\{\min(\xi, K) > K'\}}$$
$$\cdot \delta(1-\delta)^{-1}\underline{\Pi}_S + 1_{\{\min(\xi, K) \leq K'\}}$$
$$\cdot \delta(1-\delta)^{-1}(\pi - \underline{\Pi}_B)]\} \tag{18}$$

at the maximal fixed point $\Pi^n = \max[\pi: f^n(\pi) = \pi]$.

We refer to (14)–(18) as the no-monitoring relational contract. It is self-enforcing and achieves total expected discounted profit Π^n in each period. To provide maximal incentives for capacity investment, (15)–(16) are the largest discretionary payments that are self-enforcing, given the no-monitoring order

quantity (14). The initial transfer (16) allocates the gain from the relationship to the supplier, and with this initial transfer, the largest payment the buyer is willing to make (i.e., that satisfies (4) and (5)) is the Nash bargaining payment $\sigma r \min(K, \xi)$. With order quantity (14) and discretionary payment (15), constraint (7), which ensures that the supplier builds the proposed capacity, simplifies to (18). Regardless of the supplier's capacity choice, the supplier receives the Nash bargaining payment. If the supplier's capacity is not sufficient to meet the buyer's order, then cooperation breaks down and the supplier receives $\underline{\Pi}_S$ in subsequent periods; if the supplier's capacity is sufficient, then the supplier receives the gain from the relationship $\Pi^n - \underline{\Pi}_B$ in subsequent periods.

The no-monitoring relational contract performs extremely well under plausible demand distributions. However, when the demand distribution is of a *boom-bust* nature (i.e., probability mass is concentrated at widely separated levels), monitoring can greatly increase expected profit. We establish the former with a numerical study, and the latter with an analytic result. We conclude this subsection by explaining the intuition behind these diverging results.

In our numerical study, we examined the wide set of parameters in Figure 2 (capacity cost $c \in \{0.1r, 0.2r, \ldots, 0.9r\}$, supplier bargaining strength $\sigma \in \{0.1, 0.2, \ldots, 0.9\}$, and discount factor $\delta \in \{0.1, 0.2, \ldots, 0.9\}$). As in the figure, we assumed demand is a truncated normal random variable with mean 5 and standard deviation 3. We also considered the same distribution, but with standard deviation 1, as well as allowing demand to be an exponential(1) or uniform(0, 1) random variable. Subsequently, we refer to this as Parameter Set A. *In all instances, the no-monitoring relational contract performs extremely well relative to the optimal relational contract: The deviation from optimal profit is less than 1%.*

In contrast, the next proposition shows that under boom-bust demand distributions, monitoring can be valuable. Boom-bust demand occurs when the potential customer population exhibits trend-following behavior, so that the product is either a "star" or "dog" (van Ryzin and Mahajan 1999). As an extreme case of boom-bust demand, suppose the probability mass of demand is concentrated at two levels

$$\xi = \begin{cases} H & \text{with probability } \lambda \\ L & \text{with probability } 1-\lambda, \end{cases} \tag{19}$$

where $H > L \geq 0$ and $\lambda \in (0, 1)$. Because our focus is on stochastic demand, we assume that the capacity cost is sufficiently small

$$c < \lambda r \tag{20}$$

so that the demand uncertainty is relevant. If (20) were violated, the high-demand state would be irrelevant (because it would never be optimal to build more than the low-demand level $\bar{K} \leq L$) and so the problem would be equivalent to one with deterministic demand ($\xi = L$ with probability 1). The thresholds c_1, c_2, c_3, c_4, $\bar{L}$, and $\bar{\delta}$ in the next proposition are provided in closed-form, along with the proof of the result, in the electronic companion.

PROPOSITION 3. *Suppose that demand has distribution* (19). *The optimal relational contract yields strictly greater expected profit than the no-monitoring relational contract if and only if the capacity cost satisfies*

$$c \in (c_1, c_2] \cup (c_3, c_4], \tag{21}$$

where $c_1 \leq c_2 \leq c_3 \leq c_4$. There exist $\bar{L} \in (0, H]$ and $\bar{\delta} \in (0, 1]$ such that if either $L < \bar{L}$ or $\delta < \bar{\delta}$, then $c_1 < c_2$. Further, if $\delta < \bar{\delta}$, then for $c \in (c_1, c_2]$, the optimal relational contract achieves the first best ($\Pi^* = \overline{\overline{\Pi}}$) but the no-monitoring relational contract is worthless ($\Pi^n = \underline{\Pi}_B + \underline{\Pi}_S$).

We established in §4 that the gain from an optimal relational contract is largest when the capacity cost is moderate. Although the no-monitoring relational contract performs very well when the capacity cost is moderate, it is less effective on the periphery of moderate capacity costs (21). Here, employing the more complex optimal relational contract (which requires monitoring), increases the range over which relational contracts are effective and increases system profit. The proposition establishes that the loss in system profit can be dramatic: For some intermediate capacity costs, no-monitoring relational contracts are useless, but the optimal relational contract (which requires monitoring) is perfectly effective.

To see why boom-bust demand lends itself to monitoring being useful, consider the case where the buyer would like the supplier to build capacity $K > L$. Without monitoring, the supplier is tempted to cheat by building capacity L and saving the cost of capacity $c(K - L)$. The concentration of demand at L and H means that this cheating will only be detected with probability λ. With probability $1 - \lambda$ the cheating supplier will continue to receive payment from the buyer. The temptation to cheat is especially strong when the low-demand state and the discount factor are small ($L < \bar{L}$, $\delta < \bar{\delta}$), because the supplier's immediate gain from cheating is large and the supplier attaches less value to future profits he could reap from cooperation. Relational contracts with monitoring discourage such cheating, by increasing the likelihood that cheating is detected.

When the density of demand is not concentrated at specific levels, cheating by underbuilding capacity becomes much less attractive, because there are no natural candidate cheating levels that promise large savings in capacity costs and low probabilities of detection. This explains why, in our numerical study (with truncated normal, exponential, and uniform demand), monitoring is of little value. We conclude that the complexity of monitoring is warranted only when the density of demand is concentrated at widely separated levels and the capacity cost falls in a limited range.

5.2. Capacity-Inspection Relational Contract

The optimal relational contract is complex because the buyer should, under particular circumstances, order more than the realized demand in order to monitor the supplier's capacity investment (12). The buyer orders in this way because she cannot directly observe the supplier's capacity investment. However, Dyer (1997) describes how a Japanese automobile manufacturer monitors suppliers' investments in plant, tooling and employees, productivity, and commitments to other buyers, to ensure that each supplier can meet a target level of production. Such capacity inspection is costly and requires deep involvement of the buyer in the supplier's operations. This subsection considers how the structure of the optimal relational contract changes when the buyer can observe the supplier's capacity K and the gain in performance from using this information.

The first main insight is that when the buyer can observe the supplier's capacity K, the optimal relational contract has a simple structure. The structure closely follows that of the no-monitoring relational contract (14)–(18). When the buyer can observe the supplier's capacity, under the optimal relational contract, the buyer verifies that the supplier has built the requested capacity, and then proceeds to order the minimum of capacity and demand (14), paying the Nash bargaining price per unit (15). The only change from (14)–(18) is that the optimal relational contract provides stronger incentives for capacity investment. In the supplier's capacity choice constraint (18), $1_{\{K = K'\}}$ replaces $1_{\{\min(\xi, K) \leq K'\}}$: The supplier receives the gain from cooperation in subsequent periods if and only if she builds the requested capacity. In contrast, when the buyer is unable to observe the supplier's capacity, the supplier has weaker incentives for capacity investment because the supplier can cheat by underbuilding capacity and go undetected when the buyer's requested quantity is sufficiently small (the event $q(\xi) \leq K'$ in constraint (7)). Consequently, the buyer's being able to observe the supplier's capacity increases the profit under the optimal relational contract.

The second main insight is that for a broad set of parameters, capacity inspection does not significantly increase expected profit over either the optimal relational contract of §4 or the simple no-monitoring

relational contract of §5.1. For the range of parameters in Parameter Set A, the loss in system profit from employing either the optimal relational contract or the simple no-monitoring relational contract averages 0.5%, is less than 0.5% in 81.7% of the problem instance, is less than 5% in 97.5% of the problem instances, and is less than 10% in all of the problem instances. Analogous to Proposition 3, one can establish that capacity inspection may be valuable under special circumstances (boom-bust demand and a limited range of capacity costs). Nonetheless, our results suggest that in most settings the firms should adopt the simple no-monitoring relational contract: It performs nearly as well as the capacity-inspection relational contract while avoiding the costs associated with inspection.

6. Extensions

This section describes the effects on the optimal relational contract of (1) a random production cost and a nonlinear capacity cost, (2) outside options, and (3) asymmetric information about the cost of capacity and demand distribution.

6.1. Random Production Cost and Nonlinear Capacity Cost

Because the supplier initiates his capacity investment when the product is ill-defined, it is natural that at this point the supplier's subsequent per-unit production cost, denoted by p, would be uncertain. Suppose that buyer and supplier have common information about the distribution of p, and they both observe the realization of p before the buyer orders.[9] (This is reasonable if the bulk of the production cost is due to commodity material inputs or hourly labor, or if the buyer is otherwise able to become familiar with the production technology and associated costs.) The Nash bargaining price generalizes to $\sigma r + (1 - \sigma)p$. Proposition 1, which characterizes a stationary optimal relational contract, extends by incorporating the production cost and generalized Nash bargaining price into the fixed-point problem (2)–(7). The qualitative insight in Figure 2—that relational contracting substantially increases profit over a wide range of parameter values—remains valid. However, as the production cost increases, continuation of the relationship becomes less valuable, which eventually prevents first-best capacity investment

under the optimal relational contract. Incorporating a positive production cost destroys the simplifying threshold structure of the optimal relational contract (Proposition 2). Nonetheless, our numerical results for Parameter Set A (which assumed $p = 0$) continue to hold with $p \in \{1, 3, 5\}$: First, the loss in system profit from employing the simple no-monitoring relational contract rather than the complex optimal relational contract continues to be less than 1%. Second, the loss in system profit from employing either the optimal relational contract or the simple no-monitoring relational contract rather than the optimal relational contract with capacity inspection continues to be small (the loss averages 0.6%, is less than 0.5% in 80.7% of the problem instance, is less than 5% in 97.1% of the problem instances, and is less than 10% in 99.6% of the problem instances). The main insight from Proposition 3—that with boom-bust demand the performance of the simple no-monitoring relational contract can be poor—continues to hold. However, the last sentence of Proposition 3 does not hold with $p > 0$ because a relational contract with monitoring cannot achieve the first best because excess production is costly.

Finally, Proposition 1 and all but the last line of Proposition 2 continue to hold when the capacity cost is any strictly increasing function of the capacity.

6.2. Outside Options

Each firm may have an alternative supply chain partner, a so-called outside option. For example, at the beginning of each period, a supplier may instead contract to supply a different product to a different buyer. As either firm's outside option improves (expected profit from working with an alternative partner increases), the capacity investment K^* and per-period expected profit Π^* under an optimal relational contract decrease. The intuition is that when a firm decides whether or not to adhere to the relational contract, the firm weighs the expected gain from reneging against its share of the future gain from continued cooperation. As a firm's outside option improves, the temptation to renege becomes more acute.

6.3. Asymmetric Information about Capacity Cost and Demand Distribution

Commonly in practice, the buyer has private information about the demand distribution and the supplier has private information about the cost of capacity. Suppose that the supplier's capacity cost c is a random variable, independent and identically distributed for periods $t = 1, 2, \ldots$. At the beginning of each period, the firms have common information about the distribution of the capacity cost. They may contract to make the initial transfer payment contingent on a cost

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[9] The latter assumption is critical. Without it, the outcome of noncooperative bargaining would not necessarily be the Nash bargaining solution. In particular, if the supplier had private information about p, the theorem on p. 106 of Rubinstein (1982) would no longer apply. One cannot construct a self-enforcing relational contract without understanding the noncooperative bargaining outcome, and hence the temptation to renege on the terms of a relational contract.

report by the supplier. Then the supplier privately realizes the capacity cost. Proposition 1 extends: There exists a stationary optimal relational contract that corresponds to a generalized version of the fixed-point problem (2)–(7). This has the supplier reporting the capacity cost truthfully in each period.

Alternately, suppose that in each period, the buyer privately obtains a forecast that allows her to update the demand distribution, prior to the supplier investing in capacity. The firms may contract to make the initial transfer payment contingent on a forecast-report by the buyer. Again, Proposition 1 extends. However, the buyer shares her forecast truthfully with the supplier if and only if the discount factor is sufficiently large. With a large enough discount factor, the supplier makes the first-best capacity investment. Ren et al. (2005) propose a multi-period-review strategy to induce truthful forecast sharing; the resulting expected discounted profit converges to the first best as the discount factor approaches 1.

7. Discussion

The purpose of this paper is to show how managers should structure informal agreements that motivate a supplier to build capacity. We show that the gain from relational contracting is substantial over a broad range of plausible parameter values, and is greatest when the capacity cost is moderate and bargaining power is evenly distributed. In the special case where the buyer is able to directly observe the capacity investment the supplier has dedicated to her, the optimal relational contract has a simple form. Otherwise, an optimal relational contract may require the buyer to indirectly monitor the supplier's capacity investment by ordering more than the realized demand. We propose a simpler relational contract in which the buyer orders the minimum of her realized demand and requested capacity. We show that this simple relational contract performs extremely well. Indeed, except for rare circumstances, the simple relational contract allows the firms to do nearly as well as they would were the buyer able to directly observe the supplier's capacity investment. We conclude that by properly structuring informal procurement agreements, the firms can avoid having the buyer monitor the supplier's capacity either directly (by the costly, deep involvement in the supplier's operations required to directly audit the supplier's capacity investment) or indirectly (via inflated orders). However, an important feature that is absent from our model in obtaining this conclusion is asymmetry of information regarding the supplier's cost structure and the buyer's demand forecast. Further research is needed to identify simple-yet-effective relational contracts in the presence of such asymmetries.

8. Electronic Companion

An electronic companion to this paper is available as part of the online version that can be found at http://mansci.journal.informs.org/.

Appendix

Definition: Self-Enforcing Relational Contract

Denote the buyer's decision in period t to adhere (or not) to the initial transfer payment D^t by $A_B^t \in \{1, 0\}$ and the buyer's decision to adhere (or not) to the quantity-contingent payment $d^t(q)$ by $a_B^t \in \{1, 0\}$; 1 means that the buyer adheres and 0 that she refuses to do so. Let $A_S^t \in \{0, 1\}$ and $a_S^t \in \{0, 1\}$ denote the supplier's analogous decisions to adhere (or not). If either firm fails to adhere $(A_B^t \cdot A_S^t \cdot a_B^t \cdot a_S^t = 0)$, then noncooperative bargaining occurs as in §2: The supplier produces and delivers $\min(K^t, \xi^t)$ and the buyer pays σr per unit.

A *relational contract* is a complete plan for the relationship. Let $\mathbf{h}^t = \{A_B^m, A_S^m, q^m, a_B^m, a_S^m\}_{m=1,\ldots,t-1}$ denote the public history up to the beginning of period t, and let $\mathcal{H}^t$ denote the set of possible public histories. A relational contract specifies a profile of public payment terms and strategies for the buyer and the supplier. That is, for each period t and contingent on public history at the beginning of period t: $\mathbf{h}^t \in \mathcal{H}^t$, a relational contract describes: (i) The initial transfer payment D^t and the quantity-contingent payment $d^t(q)$; (ii) a strategy for the buyer $\{A_B^t, q^t, a_B^t\}$, where the order quantity q^t and adherence to the quantity-contingent payment a_B^t may depend on the demand ξ^t as well as $\mathbf{h}^t$; (iii) a strategy for the supplier $\{A_S^t, K^t, a_S^t\}$, where the capacity K^t may be contingent on the buyer's adherence to the transfer payment A_B^t as well as $\mathbf{h}^t$, and the adherence to the quantity-contingent payment a_S^t may be contingent on the buyer's A_B^t and order quantity q^t as well as $\mathbf{h}^t$. Furthermore, if $q^t > K^t$, then $a_S^t = 0$. A relational contract is *self-enforcing* if the firms' strategies constitute a perfect public equilibrium (PPE). As defined in Fudenberg et al. (1994), a PPE is a profile of public strategies that, for each period t and public history $\mathbf{h}^t \in \mathcal{H}^t$, constitute a Nash equilibrium from that time onward. In constructing an *optimal relational contract*—a self-enforcing relational contract that maximizes the firms' total expected discounted profit—we will without loss of generality restrict attention to trigger strategies in which each firm adheres to the discretionary payment

if and only if the both firms have adhered in all previous decision epochs

$$A_B^t = \begin{cases} 1 & \text{if } A_B^\tau = A_S^\tau = a_B^\tau = a_S^\tau = 1 \quad \text{for } \tau = 1, \ldots, t-1 \\ 0 & \text{otherwise} \end{cases}$$

$$A_S^t = \begin{cases} 1 & \text{if } A_B^\tau = A_S^\tau = a_B^\tau = a_S^\tau = 1 \quad \text{for } \tau = 1, \ldots, t-1 \\ 0 & \text{otherwise} \end{cases}$$

$$a_B^t = \begin{cases} 1 & \text{if } A_B^\tau = A_S^\tau = a_B^\tau = a_S^\tau = 1 \\ & \text{for } \tau = 1, \ldots, t-1 \text{ and } A_B^t = A_S^t = 1 \\ 0 & \text{otherwise} \end{cases}$$

$$a_S^t = \begin{cases} 1 & \text{if } A_B^\tau = A_S^\tau = a_B^\tau = a_S^\tau = 1 \\ & \text{for } \tau = 1, \ldots, t-1 \text{ and } A_B^t = A_S^t = 1 \\ 0 & \text{otherwise,} \end{cases}$$

and after either firm fails to adhere, the supplier builds his equilibrium capacity for single-period game §2:

if $\quad A_B^t \cdot A_S^t = 0 \quad$ or $\quad A_B^\tau \cdot A_S^\tau \cdot a_B^\tau \cdot a_S^\tau = 0$

for $\tau < t$ then $K^t = \underline{K}$.

Proof of Proposition 1. The proof is achieved in three steps. Initially, we assume that an optimal (but not necessarily stationary) relational contract exists. The first step is to transform it into a *stationary* optimal relational contract. The key to this first step is to show that an optimal relational contract must generate the same ongoing total expected discounted profit in every period (i.e., *sequentially optimal*). The proof of sequential optimality is by contradiction, taking an optimal relational contract that is not sequentially optimal and constructing from it a self-enforcing relational contract that achieves strictly greater expected discounted profit. Then, having established existence of a sequentially optimal relational contract, we convert it to a stationary optimal relational contract. Specifically, we take the first-period strategies and payment terms from the sequentially optimal relational contract, to be used in every period; to make this self-enforcing, we adjust the quantity-contingent payment so that the firms "settle up" at the end of each period. The implication of the first step is that in searching for an optimal relational contract, one may restrict attention to stationary relational contracts. The second step formulates the problem: Find a stationary relational contract that maximizes expected profit, subject to the constraint that the stationary relational contract must be self-enforcing. The function $f(\pi)$ emerges in this second step. Finally, the third step establishes that a stationary optimal relational contract exists, corresponding to the largest fixed point $\pi = f(\pi)$.

Step 1. *Existence of a stationary optimal contract*: Assume that an optimal relational contract exists. Let P denote the total expected discounted profit for the buyer and supplier under the optimal relational contract, and let Π^* denote the deterministic equivalent profit per period:

$$\Pi^* = (1-\delta)P.$$

Also let Π_B^1 and Π_S^1 denote the deterministic equivalent profit per period for the buyer and supplier, respectively

$(\Pi_B^1 + \Pi_S^1 = \Pi^*)$. Consider the following terms of the relational contract for the first period; for brevity in the following analysis, we drop superscripts "1" denoting the first period. The agreement specifies that both firms adhere to the initial transfer payment D, the supplier builds capacity $K = \max_{\xi \in \Gamma}\{q(\xi)\}$, the buyer orders quantity $q = q(\xi)$ on observing demand ξ, and finally, the firms contract for the buyer to pay $d(q)$ and the supplier to produce and deliver q units. Let $\Pi_B^2(q)$ and $\Pi_S^2(q)$ denote the deterministic equivalent profit per period starting from Period 2 with public history $h^2 = \{1, 1, q, 1, 1\}$ for the buyer and supplier, respectively.

The optimal relational contract is self-enforcing in trigger strategies. Therefore the terms for the first period must satisfy the following equilibrium conditions.

(i) By refusing to adhere to the initial transfer payment, the buyer (supplier) could obtain her noncooperative expected profit $\underline{\Pi}_B$ ($\underline{\Pi}_S$) in all periods. Therefore, both buyer and supplier must have greater expected discounted profit under the optimal relational contract:

$$\Pi_B^1 \geq \underline{\Pi}_B \tag{22}$$

$$\Pi_S^1 \geq \underline{\Pi}_S. \tag{23}$$

(ii) The supplier has an incentive to adhere to the quantity-contingent payment $d(q)$ for quantity $q = q(\xi) \leq K$ for $\xi \in \Gamma$,

$$d(q) + \delta(1-\delta)^{-1}\Pi_S^2(q)$$
$$\geq \sigma r E[\min(K, \xi^1) \mid q(\xi^1) = q] + \delta(1-\delta)^{-1}\underline{\Pi}_S, \tag{24}$$

and has incentive to make the target capacity investment: for $K' \geq 0$,

$$-cK + E[d(q(\xi)) + \delta(1-\delta)^{-1}\Pi_S^2(q(\xi))]$$
$$\geq -cK' + E[1_{\{q(\xi)>K'\}}(\sigma r \min(K', \xi) + \delta(1-\delta)^{-1}\underline{\Pi}_S)]$$
$$+ E[1_{\{q(\xi)\leq K'\}}\max(d(q(\xi)) + \delta(1-\delta)^{-1}\Pi_S^2(q(\xi)),$$
$$\sigma r E[\min(K', \xi^1) \mid q(\xi^1) = q(\xi)] + \delta(1-\delta)^{-1}\underline{\Pi}_S)]. \tag{25}$$

(iii) The buyer has an incentive to order a quantity according to $q = q(\xi)$ and adhere to the quantity-contingent payment $d(q)$: for $\xi \in \Gamma$

$$r\min(q(\xi), \xi) - d(q(\xi)) + \delta(1-\delta)^{-1}\Pi_B^2(q(\xi))$$
$$\geq r\min(q(\xi'), \xi) - d(q(\xi')) + \delta(1-\delta)^{-1}\Pi_B^2(q(\xi'))$$
$$\text{for all } \xi' \in \Gamma \tag{26}$$

$$r\min(q(\xi), \xi) - d(q(\xi)) + \delta(1-\delta)^{-1}\Pi_B^2(q(\xi))$$
$$\geq (1-\sigma)r\min(K, \xi) + \delta(1-\delta)^{-1}\underline{\Pi}_B. \tag{27}$$

Furthermore, under the optimal relational contract, given that both firms adhered in the first period, they have an incentive in the second period to adhere to the initial transfer payment. That is, ongoing expected discounted profit from Period 2 is greater than the noncooperative expected discounted profit: for $\xi \in \Gamma$ and $q = q(\xi)$,

$$\Pi_B^2(q) \geq \underline{\Pi}_B \tag{28}$$

$$\Pi_S^2(q) \geq \underline{\Pi}_S. \tag{29}$$

We will now prove that an optimal relational contract is *sequentially optimal* in Period 2: for $\xi \in \Gamma$ and $q = q(\xi)$,

$$\Pi_B^2(q) + \Pi_S^2(q) = \Pi^*. \tag{30}$$

To do so, we employ the technical term *continuation*. By the continuation of a relational contract in period t contingent on history $\mathbf{h}^t$, we mean the terms and strategies specified in the relational contract starting from period t that remain relevant given the history $\mathbf{h}^t$. To establish (30), first observe that if $\Pi_B^2(q) + \Pi_S^2(q) > \Pi^*$, then the continuation of the relational contract in Period 2, contingent on order quantity q and adherence by both firms in Period 1, achieves ongoing total expected discounted profit strictly greater than the total expected discounted profit under the optimal contract. Because the continuation of the relational contract in Period 2 is self-enforcing, this contradicts the optimality of the proposed optimal relational contract. That is, the firms can achieve strictly greater total expected discounted profit by starting in Period 1 with the continuation relational contract specified for Period 2 after observing order quantity q and adherence by both firms in Period 1. Therefore, we must have $\Pi_B^2(q) + \Pi_S^2(q) \leq \Pi^*$. Second, define

$$\mathcal{D} = \{q: \Pi_B^2(q) + \Pi_S^2(q) < \Pi^*\}.$$

If $\mathcal{D}$ is empty, we have established (30). Otherwise, we can construct a self-enforcing relational contract that achieves greater expected profit than the optimal relational contract. Modify the optimal relational contract as follows. For every public history $h^2 = \{1, 1, q, 1, 1\}$ with $q \in \mathcal{D}$, substitute the optimal relational contract for the Period 2 continuation but increase the transfer payment to the supplier in the second period: $D^2(q) = D^1 + (1-\delta)^{-1}(\Pi_B^1 - \Pi_B^2(q))$. Then reduce the transfer payment to the supplier in the first period $D^1 \leftarrow D^1 - \delta(1-\delta)^{-1}E[1_{\{q \in \mathcal{D}\}}(\Pi_S^1 + \Pi_B^1 - \Pi_B^2(q) - \Pi_S^2(q))]$ accordingly. (The notation "$\leftarrow$" in $D \leftarrow D - x$ means that the new value of D is set equal to the previous value of D minus x.) The resulting relational contract provides the same deterministic equivalent profit per period starting from Period 1 for the supplier of Π_S^1, but increases the corresponding profit for the buyer to

$$\Pi_B^1 + \delta E[1_{\{q \in \mathcal{D}\}}(\Pi^* - \Pi_B^2(q) - \Pi_S^2(q))] \geq \Pi_B^1.$$

In contrast, the deterministic equivalent profit per period starting from Period 2 for the buyer is unchanged at $\Pi_B^2(q)$, but the corresponding profit for the supplier increases to

$$\Pi_S^2(q) + 1_{\{q \in \mathcal{D}\}}(\Pi^* - \Pi_B^2(q) - \Pi_S^2(q)) \geq \Pi_S^2(q). \tag{31}$$

These modifications relax constraints (22), (24), (25), and (29), and do not affect (23), (26), (27), and (28), so the resulting relational contract is self-enforcing. In particular, the most complex constraint (25) is relaxed because (25) and (31) imply that for $K' \geq 0$,

$$-cK + E[d(q(\xi)) + \delta(1-\delta)^{-1}(\Pi_S^2(q(\xi))$$
$$+ 1_{\{q \in \mathcal{D}\}}(\Pi^* - \Pi_B^2(q(\xi)) - \Pi_S^2(q(\xi))))]$$
$$\geq -cK' + E[1_{\{q(\xi)>K'\}}(\sigma r \min(K', \xi) + \delta(1-\delta)^{-1}\underline{\Pi}_S)]$$
$$+ E[1_{\{q(\xi) \leq K'\}} \max(d(q(\xi)) + \delta(1-\delta)^{-1}$$
$$\cdot (\Pi_S^2(q(\xi)) + 1_{\{q \in \mathcal{D}\}}(\Pi^* - \Pi_B^2(q(\xi)) - \Pi_S^2(q(\xi)))),$$
$$\sigma r E[\min(K', \xi^1) \mid q(\xi^1) = q(\xi)] + \delta(1-\delta)^{-1}\underline{\Pi}_S)].$$

Therefore, we conclude that an optimal relational contract satisfies $\Pi_B^2(q) + \Pi_S^2(q) \geq \Pi^*$, which completes the proof of (30). It follows immediately from (30) that in the first period,

$$E[r \min(q(\xi), \xi)] - cK = \Pi^*. \tag{32}$$

We now construct a stationary contract that achieves total expected profit per period of Π^*, is self-enforcing, and is therefore optimal. This is accomplished by shifting variation in the firms' continuation expected profits $(\Pi_B^2(q), \Pi_S^2(q))$ into the quantity-contingent payment $d(q)$. The initial transfer payment is

$$D^* = D, \tag{33}$$

and the supplier's capacity is

$$K^* = K. \tag{34}$$

For $\xi \in \Gamma$, the order quantity and quantity-contingent payment are given by

$$q^*(\xi) = q(\xi) \leq K^* \tag{35}$$
$$d^*(q) = d(q) + \delta(1-\delta)^{-1}[\Pi_S^2(q) - \Pi_S^1]. \tag{36}$$

By construction, assuming that the buyer will adhere to the initial transfer payment, order according to $q^*(\xi)$, and adhere to quantity-contingent payment (36), it is optimal for the supplier to adhere to the initial transfer payment, build capacity K^*, and adhere to quantity-contingent payment (36). Under the proposed relational contract, expected discounted profit at the beginning of each period is the same as the initial expected profit under the optimal relational contract: Π_S^1 for the supplier and Π_B^1 for the buyer. Contingent on order quantity q, the ongoing expected discounted profit for the buyer is

$$r \min(q^*(\xi), \xi) - d^*(q) + \delta(1-\delta)^{-1}\Pi_B^1$$
$$= r \min(q(\xi), \xi) - d(q) + \delta(1-\delta)^{-1}\Pi_B^2(q)$$

by substitution of (30) and the definitions (35)–(36). Then from (22), (26), and (27), it is optimal for the buyer to adhere to the initial transfer payment (33), order according to (35), and adhere to the quantity-contingent payment (36). We conclude that the stationary relational contract with payment terms and strategies (D^*, d^*, q^*, K^*) is self-enforcing. Together (32), (34), and (35) imply that the stationary relational contract achieves the optimal expected discounted profit Π^*.

Step 2. *The optimal stationary relational contract*: We have reduced the problem of designing an optimal relational contract to finding the largest scalar π that satisfies

$$\pi = \max_{D, d, q, K, \Pi_B, \Pi_S} \{E[r \min(q(\xi), \xi)] - cK\} \tag{37}$$

subject to, for $\xi \in \Gamma$,

$$q(\xi) \in \arg\max_{q \in [0, K]} \{r \min(q, \xi) - d(q)\} \tag{38}$$
$$r \min(q(\xi), \xi) - d(q(\xi)) + \delta(1-\delta)^{-1}\underline{\Pi}_B$$
$$\geq (1-\sigma)r \min(K, \xi) + \delta(1-\delta)^{-1}\underline{\Pi}_B \tag{39}$$
$$d(q(\xi)) + \delta(1-\delta)^{-1}\Pi_S$$
$$\geq \sigma r E[\min(K, \xi^1) \mid q(\xi^1) = q(\xi)] + \delta(1-\delta)^{-1}\underline{\Pi}_S \tag{40}$$

$$K \in \underset{K' \geq 0}{\arg\max}\{-cK' + E[1_{\{q(\xi) > K'\}}(\sigma r \min(K', \xi) + \delta(1-\delta)^{-1}\underline{\Pi}_S)]$$

$$+ E[1_{\{q(\xi) \leq K'\}} \max(d(q(\xi)) + \delta(1-\delta)^{-1}\Pi_S,$$

$$\sigma r E[\min(K', \xi^1) \mid q(\xi^1) = q(\xi)]$$

$$+ \delta(1-\delta)^{-1}\underline{\Pi}_S)]\} \quad (41)$$

$$D - cK + E[d(q(\xi))] = \Pi_S \quad (42)$$

$$\Pi_B \geq \underline{\Pi}_B, \quad \Pi_S \geq \underline{\Pi}_S, \quad \Pi_B + \Pi_S = \pi. \quad (43)$$

In seeking a stationary, self-enforcing relational contract that maximizes total expected profit, we can allocate the surplus generated by the relationship to the supplier. Specifically, for any feasible solution $\{\hat{D}, \hat{d}, \hat{q}, \hat{K}, \hat{\Pi}_B, \hat{\Pi}_S\}$, we can construct another feasible solution with the same objective value as follows: The transfer payment is $D = \hat{D} + (1-\delta)^{-1}(\hat{\Pi}_B - \underline{\Pi}_B)$ and the capacity is $K = \hat{K}$. For all $\xi \in \Gamma$, the order quantity is $q(\xi) = \hat{q}(\xi)$ and quantity-contingent payment is $d(q) = \hat{d}(q) - \delta(1-\delta)^{-1}(\hat{\Pi}_B - \underline{\Pi}_B)$. The profit allocation is $\Pi_B = \underline{\Pi}_B$, $\Pi_S = \pi - \underline{\Pi}_B$. This simplifies problem (37)–(43) to

$$\pi = \max_{D, d, q, K} \{E[r \min(q(\xi), \xi)] - cK\}$$

$$\text{subject to } (4)–(7).$$

Step 3. *Existence of an optimal relational contract*: Let $\mathscr{P} \subseteq [\underline{\Pi}_B + \underline{\Pi}_S, \overline{\Pi}]$ denote the set of equivalent deterministic profit per period that can be achieved with a self-enforcing relational contract. (Recall that $\overline{\Pi}$ is the first-best total expected profit per period.) To prove that an optimal relational contract exists, we need to show that $\sup\{\pi : \pi \in \mathscr{P}\}$ is an element of $\mathscr{P}$. Observe that the function $f(\pi)$ has the properties $f(\underline{\Pi}_B + \underline{\Pi}_S) = \underline{\Pi}_B + \underline{\Pi}_S$, $f(\pi) \leq \overline{\Pi}$, and $f(\pi)$ is weakly increasing in π for $\pi \in [\underline{\Pi}_B + \underline{\Pi}_S, \overline{\Pi}]$. Therefore Tarski's fixed point theorem implies that $f(\pi)$ has a largest fixed point Π^* in $[\underline{\Pi}_B + \underline{\Pi}_S, \overline{\Pi}]$. Mimicking Lemma 4 in Appendix D of Levin (2003) establishes that this largest fixed point Π^* is an upper bound on the expected profit per period that can be achieved with a self-enforcing relational contract. From Step 2, we know that there exists a stationary, self-enforcing relational contract that achieves expected profit per period of Π^*, corresponding to the solution of problem (2) with $\pi = \Pi^*$. We conclude that this is an optimal relational contract. $\square$

PROOF OF PROPOSITION 2. The proof proceeds in two steps. The first step establishes (8)–(11). The second establishes sufficient conditions under which the optimal order quantity has at most one point of discontinuity: $M \in \{0, 1\}$. The main insights behind the first step are (i) increasing the payment $D + d(q(\xi))$ increases the supplier's incentive to build the capacity specified in the relational contract, (ii) increasing the order quantity $q(\xi)$ above the level of capacity the supplier would choose if he were to cheat on the relational contract increases the supplier's incentive to build the capacity specified in the relational contract, and (iii) the buyer will order $q(\xi) > \xi$ only if excess units are free $d(q(\xi)) \leq d(\xi)$. A relational contract with the structure (8)–(11) maximizes the supplier's incentive to build the specified capacity, by optimizing the trade-off between increasing the order quantity and increasing the payment. This optimal structure becomes apparent as we simplify the fixed-point problem.

The fixed-point problem (2)–(7) with $\pi = \Pi^*$ reduces to

$$\max_{d, q, K}\{E[r \min(q(\xi), \xi)] - cK\} \quad (44)$$

subject to, for $\xi \in \Gamma$

$$q(\xi) \in \underset{q \in [0, K]}{\arg\max}\{r \min(q, \xi) - d(q)\} \quad (45)$$

$$r \min(q(\xi), \xi) - d(q(\xi)) \geq (1-\sigma)r \min(K, \xi) \quad (46)$$

$$d(q(\xi)) + \delta(1-\delta)^{-1}(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S)$$

$$\geq \sigma r E[\min(K, \xi^1) \mid q(\xi^1) = q(\xi)] \quad (47)$$

$$K \in \underset{K' \geq 0}{\arg\max}\{-cK' + E[1_{\{q(\xi) > K'\}}\sigma r \min(K', \xi)]$$

$$+ E[1_{\{q(\xi) \leq K'\}} \max(d(q(\xi)) + \delta(1-\delta)^{-1}(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S),$$

$$\sigma r E[\min(K', \xi^1) \mid q(\xi^1) = q(\xi)])]\}, \quad (48)$$

with optimal initial transfer payment

$$D^* = \Pi^* - \underline{\Pi}_B + cK^* - E[d^*(q^*(\xi))].$$

This establishes (9). Because $r > 0$, any optimal solution to (44)–(48) must satisfy

$$q^*(\xi) \geq \xi \quad \text{for } \xi \leq K^*, \quad (49)$$

so $\min(q^*(\xi), \xi) = \min(K^*, \xi)$ in expected profit (44). This establishes (8). Furthermore, there exists an optimal solution with nondecreasing order quantity $q^*(\cdot)$. To see this, suppose that there exist ξ_1 and ξ_2 such that $\xi_1 < \xi_2$ and $\bar{q} \equiv q(\xi_1) > q(\xi_2)$. Constraint (45) and (49) imply that $d(q(\xi_1)) = d(q(\xi_2))$. Because constraint (47) is satisfied at $\xi = \xi_2$, it will continue to be satisfied if $q(\xi_2)$ is increased to $\bar{q}$. Further, increasing $q(\xi_2)$ to $\bar{q}$ relaxes constraint (48) (strengthens the supplier's incentive for capacity investment), without affecting the other constraints (45)–(46) and objective (44). Because an optimal $q(\cdot)$ is nondecreasing and satisfies (49), we can restrict attention to production quantities of the form

$$q(\xi) \begin{cases} = \xi & \text{for } 0 \leq \xi < K_1 \\ \in (\xi, K_m + \Delta_m] & \text{for } K_m \leq \xi < K_m + \Delta_m \ m = 1, \ldots, M \\ = \xi & \text{for } K_m + \Delta_m \leq \xi \leq K_{m+1} \ m = 1, \ldots, M \\ = K^* & \text{for } \xi > K^*, \end{cases}$$

$$(50)$$

where M is a nonnegative integer (possibly 0), $0 \leq K_m < K_m + \Delta_m < K_{m+1}$ for $m \in \{1, \ldots, M\}$ and $K_{M+1} = K^*$. Increasing the payment $d(q)$ relaxes constraints (47) and (48) without affecting the objective, and (11) is the pointwise maximum payment scheme that satisfies (45) and (46) with order quantity (50). Therefore (11) is optimal. Suppose that (47) is satisfied at $\xi = K_m + \Delta_m$. Then with payment scheme $d(q)$ given by (11) and order quantity satisfying (50), (47) is satisfied for all $\xi \in (K_m, K_m + \Delta_m)$. Suppose that for some $\xi \in [K_m, K_m + \Delta_m)$, $q(\xi) < K_m + \Delta_m$; increasing the order quantity $q(\xi)$ to $K_m + \Delta_m$ relaxes constraint (48) (strengthens the supplier's incentive for capacity investment), without affecting the other constraints (45)–(46) and objective (44). Therefore we conclude that a solution to (44)–(48) has order quantity and quantity-contingent payment of the form (10)–(11).

The second step establishes sufficient conditions under which the optimal order quantity has at most one point of discontinuity: $M \in \{0, 1\}$. Consider a relational contract with the maximal discretionary payments ($q(\xi) = \min(K, \xi)$, $d(q) = \sigma r q$, $D = \Pi^* - \underline{\Pi}_B + cK - \sigma r \min(K, \xi)$). Under this contract, if the supplier intends to cheat and build less capacity than specified in the relational contract, he will choose capacity

$$K_{\text{cheat}} = \arg\max_{K \geq 0} \Pi_{\text{cheat}}(K),$$

where

$$\Pi_{\text{cheat}}(K) = -cK + E\big[\sigma r \min(K, \xi) + \delta(1-\delta)^{-1} \\ \cdot (\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S)1_{\{\xi \leq K\}}\big].$$

For the remainder of the proof, we assume that ξ is a normal or truncated normal random variable or that ξ is a continuous random variable whose density is weakly decreasing. We will show that this implies $\Pi_{\text{cheat}}(K)$ is strictly quasi-concave in K for $K \in [\underline{K}, \bar{K}]$. When ξ is a continuous random variable whose density is weakly decreasing,

$$(\partial^2/\partial K^2)\Pi_{\text{cheat}}(K) \\ = -\sigma r \phi(K) + \delta(1-\delta)^{-1}(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S)(\partial/\partial K)\phi(K) < 0,$$

and $\Pi_{\text{cheat}}(\cdot)$ is strictly concave. Suppose instead that ξ is a normal random variable with mean μ and standard deviation s. Then

$$(\partial^2/\partial K^2)\Pi_{\text{cheat}}(K) \\ = \frac{e^{-(K-\mu)^2/2s^2}}{\sqrt{2\pi}s^3}[\delta(1-\delta)^{-1}(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S)(\mu - K) - \sigma r s^2],$$

which implies that $\Pi_{\text{cheat}}(K)$ is strictly convex on $K \in [0, \mu - \sigma r s^2/\delta(1-\delta)^{-1}(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S))$ and strictly concave on $K \in (\mu - \sigma r s^2/\delta(1-\delta)^{-1}(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S), \infty)$. This, combined with the observation that

$$(\partial/\partial K)\Pi_{\text{cheat}}(K)|_{K=\underline{K}} = \delta(1-\delta)^{-1}(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S)\phi(\underline{K}) > 0,$$

implies that $\Pi_{\text{cheat}}(K)$ is strictly quasi-concave in K for $K \in [\underline{K}, \bar{K}]$. The proof when ξ is a truncated normal random variable is similar.

Next, we will use the quasi-concavity of $\Pi_{\text{cheat}}(K)$ to show that the optimal order quantity function has at most a single jump ($M = 1$) to deter the supplier from building K_{cheat} and adjacent levels of capacity. Clearly, $K_{\text{cheat}} \geq \underline{K}$. If $K_{\text{cheat}} > \bar{K}$, then an optimal solution to (44)–(48) has order quantity (10) and quantity-contingent payment (11) with $M = 0$ and $K^* = \bar{K}$. ($K_{\text{cheat}} > \bar{K}$ implies that (10)–(11) with $M = 0$ and $K^* = \bar{K}$ is feasible and achieves the upper bound on the objective value of $\overline{\Pi}$, the first-best expected profit per period.) For the remainder, we suppose instead that $K_{\text{cheat}} \leq \bar{K}$. We relax problem (44)–(48) by substituting (52)–(56) for the constraint (48) while including $\{K_1, K_2\}$ as decision variables

$$\max_{d, q, K, K_1, K_2} \{E[r \min(q(\xi), \xi)] - cK\} \tag{51}$$

subject to (45)–(47) and

$$-cK + E\big[1_{\{q(\xi)>K\}}\sigma r \min(K, \xi) + 1_{\{q(\xi) \leq K\}} \\ \cdot \max(d(q(\xi)) + \delta(1-\delta)^{-1}(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S), \\ \sigma r E[\min(K, \xi^1) \mid q(\xi^1) = q(\xi)])\big] \\ \geq -cK_1 + E\big[1_{\{q(\xi)>K_1\}}\sigma r \min(K_1, \xi) + 1_{\{q(\xi) \leq K_1\}} \\ \cdot \max(d(q(\xi)) + \delta(1-\delta)^{-1}(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S), \\ \sigma r E[\min(K_1, \xi^1) \mid q(\xi^1) = q(\xi)])\big] \tag{52}$$

$$-cK + E\big[1_{\{q(\xi)>K\}}\sigma r \min(K, \xi) + 1_{\{q(\xi) \leq K\}} \\ \cdot \max(d(q(\xi)) + \delta(1-\delta)^{-1}(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S), \\ \sigma r E[\min(K, \xi^1) \mid q(\xi^1) = q(\xi)])\big] \\ \geq -cK + E[1_{\{q(\xi)>K_2\}}\sigma r \min(K_2, \xi) + 1_{\{q(\xi) \leq K_2\}} \\ \cdot \max(d(q(\xi)) + \delta(1-\delta)^{-1}(\Pi^* - \underline{\Pi}_B - \underline{\Pi}_S), \\ \sigma r E[\min(K_2, \xi^1) \mid q(\xi^1) = q(\xi)])\big] \tag{53}$$

$$K_1 \leq K_{\text{cheat}} \leq K_2 \tag{54}$$

$$q(\xi) = K_2 \quad \text{for } \xi \in [K_1, K_2] \tag{55}$$

$$q(\xi) < K_1 \quad \text{for } \xi \in (0, K_1). \tag{56}$$

To see that this is a relaxation of (44)–(48), note that for any solution of the form (10)–(11), there exist scalars K_1 and K_2 satisfying (54)–(56); also (48) implies (52) and (53). (If the solution has $K_{\text{cheat}} \in [K_m, K_m + \Delta_m]$, then $K_1 = K_m$ and $K_2 = K_m + \Delta_m$ satisfy (54)–(56); otherwise, $K_1 = K_2 = K_{\text{cheat}}$ satisfies (54)–(56).) Now let us characterize the solution to the relaxed problem (51)–(56). Again, an optimal order quantity is nondecreasing and satisfies (49). Constraints (47), (52), and (53) are relaxed by maximizing $d(q)$ subject to (45) and (46). Constraint (45) implies that setting $q(\xi) > \xi$ for any $\xi \in [0, K_1) \cup (K_2, K]$ strictly reduces the maximum feasible $d(q)$. With $q(\xi) = \xi$ for $\xi \in [0, K_1) \cup (K_2, K]$, a necessary and sufficient condition for (47) to hold for $\xi \geq 0$ is that (47) holds for $\xi = K_2^+$. To see sufficiency, note that for $\xi \in [0, K_1)$, an optimal payment is $d(q) = \sigma r q$; for $\xi \in [K_1, K_2]$, the constraint holds because $K_2^+ > E[\min(K, \xi) \mid \xi \in [K_1, K_2]]$; for $\xi > K_2$ an optimal quantity-contingent payment has $d'(q) \geq \sigma r$. Hence there exists an optimal solution to the relaxed problem (51)–(56) of the form

$$d^*(q) = \begin{cases} \sigma r q & \text{for } 0 \leq q \leq K_1 \\ \sigma r K_1 & \text{for } K_1 < q \leq K_2 \\ \sigma r K_1 + r(q - K_2) \\ \quad \text{for } K_2 \leq q \leq \min[(K_2 - \sigma K_1)/(1-\sigma), K^*] \\ \sigma r q & \text{for } (K_2 - \sigma K_1)/(1-\sigma) \leq q \leq K^* \\ d^*(K^*) & \text{for } q > K^* \end{cases} \tag{57}$$

$$q^*(\xi) = \begin{cases} \xi & \text{for } 0 \leq \xi < K_1 \\ K_2 & \text{for } K_1 \leq \xi \leq K_2 \\ \xi & \text{for } K_2 \leq \xi \leq K^* \\ K^* & \text{for } \xi > K^*, \end{cases} \tag{58}$$

where either $K_1 = K_2 = K_{cheat}$ and (57)–(58) is of the form (10)–(11) with $M = 0$, or $K_1 < K_2$ and (57)–(58) is of the form (10)–(11) with $M = 1$. To show that (57)–(58) is an optimal solution for (44)–(48), it is sufficient to show that (57)–(58) satisfies (48). Let $\Pi_{\mathscr{S}}(K')$ denote the maximand in (48). Because for $K \in [0, K_1]$, $\Pi_{\mathscr{S}}(K) = \Pi_{cheat}(K)$ and $\Pi_{cheat}(K)$ is increasing, (52) implies $\Pi_{\mathscr{S}}(K^*) \geq \Pi_{\mathscr{S}}(K)$ for $K \in [0, K_1]$. Because $\Pi_{\mathscr{S}}(K)$ is decreasing on $K \in [K_1, K_2]$ and $K > K^*$, $\Pi_{\mathscr{S}}(K^*) \geq \Pi_{\mathscr{S}}(K)$ for $K \in [K_1, K_2]$ and $K > K^*$. Because for $K \in [K_2, K^*]$, $\Pi_{cheat}(K)$ is decreasing and $\Pi_{cheat}(K) - \Pi_{\mathscr{S}}(K)$ is increasing, $\Pi_{\mathscr{S}}(K)$ is decreasing; therefore, (53) implies $\Pi_{\mathscr{S}}(K^*) \geq \Pi_{\mathscr{S}}(K)$ for $K \in [K_2, K^*]$. Thus, (57)–(58) satisfies (48) and hence is an optimal solution for (44)–(48). We conclude that the optimal order quantity $q^*(\xi)$ has at most one point of discontinuity. $\square$

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