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Essays on Mortgage Risk

by

Alan J. Neale

A dissertation submitted in partial satisfaction of the
requirements for the degree of
Doctor of Philosophy

in

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in the

GRADUATE DIVISION
of the
UNIVERSITY OF CALIFORNIA, BERKELEY

Committee in charge:
Professor Roger Craine, Chair
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Fall 2012

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Alan J. Neale

Abstract

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Doctor of Philosophy in Economics

University of California, Berkeley

Professor Roger Craine, Chair

This dissertation contributes three essays in areas of mortgage risk that are rapidly growing in importance. The first essay develops a fully dynamic optimization model for a borrower's redefault decision on a modified mortgage incorporating real-world frictions relevant for default decisions. Solutions to the model reveal large differences across modification structures and a basic pecking order for redefault performance controlling for resulting mortgage present value. Further, empirical tests utilizing unique and extensive data on modified loans offer broad agreement with the predictions of the model.

The second essay provides one of the most complete studies for termination behavior of non-U.S. mortgages to date, jointly estimating the competing risks of prepayment and default in a grouped duration mixed proportional hazard framework applied to Singapore mortgages. The study tests option-theoretic motivations for prepayments and defaults as well as "trigger event" explanations, explores comparative results to U.S. mortgage studies, examines unique institutional characteristics of this market impacting option-theoretic motivations for loan termination, and documents that variation in sources of borrower equity matter for the exercise of default options.

The final essay argues that the estimation of tail credit risk in residential mortgage portfolios remains relatively poorly understood, and that many common approaches to the problem have been incomplete or inadequate. In addition to laying out the fundamental components of sound portfolio credit risk assessment, the essay develops competing models for realistic dynamics of underlying risk factors, such as home prices. Particular attention is paid to identifying the properties of these models most consequential for the estimated distribution of losses, and to measures of implied sensitivity to geographic diversification.

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Chapter 1

Introduction

Real estate is among the largest asset classes in the U.S. economy, and indeed in many economies, and the mortgage debt that finances it similarly constitutes one of the largest and most important debt markets. Efficient mortgage markets and broad-based investor participation in those markets is of critical importance. Achieving those goals requires obtaining as complete an understanding of mortgage risk as possible. That fact has not been lost on the many participants in, and observers of, the mortgage market, and an immense amount of research on mortgage risk has been developed over the last couple of decades. Much of this research has focused on modeling prepayment risk, owing largely to the intermediation of credit risk in the U.S. market resulting from the activities of the Government Sponsored Enterprises, Fannie Mae and Freddie Mac, as well as other various agencies and entities.

Additionally, mortgage risk modeling has advanced considerably from the development of theoretical foundations that reveal a mortgage to be a debt contract with embedded call and put options. Cash flows on a mortgage can thus be understood as deriving from borrower decisions about exercising or preserving these embedded prepayment and default options, and, since the exercise of one option extinguishes the other, that these options are best viewed in a simultaneous competing risk framework.

Yet, the mortgage market has recently undergone tremendous change that suggests our understanding of mortgage risk is at best incomplete and still evolving. The mortgage crisis in the U.S. beginning in 2007 is a stark example of this change and the corresponding implications for our understanding of mortgage risk. This dissertation contributes three essays that further extend our understanding of mortgage risk. Each essay deals with a specific area of mortgage risk that has gained considerable importance in recent years, and will continue to do so, but that heretofore have not received much research attention.

First, the tremendous levels of delinquencies and defaults resulting from the crisis have created an unprecedented level of loan modification activity. Already defaulted mortgages represent a very different set of risk characteristics and considerations relative to a traditional mortgage loan, and collective experience with these loans

has not been extensive prior to the crisis. Policy makers and loan services alike have consequently struggled with how best to structure modification programs, and differing views about how best to restructure troubled loans have emerged. Chapter 2 examines the specific question of how different loan modification structures should be expected to work. The chapter begins with an examination of option-theoretic principals for mortgage default and develops a model for the default decisions of a dynamically optimizing borrower in relation to the structure of his modified loan contract. The chapter further tests the findings of these theories with a unique and extensive data set on modified loan performance.

Second, significant growth in mortgage markets of more newly emerging economies has been occurring in recent years, particularly in Asian economies such as China, Singapore, Hong Kong, and Korea, to name a few. Growth in these markets have tended to be far higher than in the U.S. and other developed economies in recent periods, and given the potential size of these markets it may be inevitable that new markets and mechanisms for funding mortgages—analogue to securitization in the U.S.—will emerge in these economies. However, these markets can differ in significant ways from the U.S., where most research on mortgage risk has been centered, and much less is known about how features of mortgage risk in these markets compare to our understanding from the U.S. context. Consequently, Chapter 3 looks at the performance of Singapore mortgages, providing one of the most extensive analysis of that market to date. Specifically, the chapter estimates joint, competing risk models for prepayment and default utilizing an extensive data set on Singapore mortgages tracked through periods of significant variation in variables that determine the value of embedded options. We compare results of these models with those from research on U.S. mortgages, and examine how closely mortgages in this market relate to option-theoretic predictions. Furthermore, we also examine the implications of several unique market and institutional features that should impact mortgage risk in that market.

Finally, Chapter 4 concludes with an examination of portfolio credit risk models. In particular, estimating the tail risk of credit mortgage portfolios remains a tremendously difficult yet incredibly important task. The mortgage crisis of recent years have witnessed the insolvency of a wide range of mortgage risk taking institutions, and have divulged the naive nature of risk assessment methods employed by rating agencies and others prior to the crisis. Given the widespread fallout of the crisis, it would not be an exaggeration to claim that refinements to portfolio credit risk assessments are critical to the future functioning of the mortgage market. Chapter 4 argues that, aside from the proliferation of new products and underwriting features whose risk characteristics were perhaps not well appreciated prior to the crisis, the foundation for better estimating credit risk lies with better understanding the dynamic processes for the state variable risk drivers such as home prices and interest rates, and their effect on tail risk. In addition to discussing foundational elements of credit risk assessments, this essay estimates several models for state variable processes and

explores the relative strengths and weaknesses of each, and, crucially, the implications for estimation of credit tail risk in mortgage portfolios.

Chapter 2

Default Risk on Modified Mortgages

2.1 Introduction

Mortgage lenders often seek to renegotiate the terms of a mortgage contract upon default on the mortgage by the borrower. Renegotiation from the lender's perspective is aimed foremost at mitigating the costs of default to the lender, and so will seek to balance the costs of concessions given in modification to the cost of default in the absence of modification. A critical component to assessing the costs of a modification to the lender is the expected rate of redefault. The form of the modification determines a new schedule of cash flows on the mortgage, but redefault probabilities will primarily determine the expected cash flows on the mortgage. Lenders often face trade-offs in designing modifications in the sense that reducing scheduled cash flows may also reduce redefaults, and they will generally seek to optimize over this trade-off. Additionally, lenders must decide *how* to reduce scheduled cash flows. Different structures may in fact involve the same or similar schedule of cash flows, yet may have drastically different implications for redefault probabilities.

While most lenders have long had established programs, modifications have historically played a very small role in mortgage servicing activities since defaults themselves have generally occurred at very low rates. This picture, however, has dramatically changed with the high default rates seen since the beginning of the mortgage crisis in 2007. According to data compiled by the Office of the Controller of the Currency (OCC), in the third quarter of 2011 nearly 7% of mortgage loans serviced by the nation's largest mortgage servicers were modified or in a trial period of pre-modification (Office of the Controller of the Currency, 2011). Modifications have therefore grown to become an increasingly important part of mortgage servicing activities.

Similarly, loan modification structures have grown much more complex in recent periods. Whereas modifications in the past have tended to follow fairly simple

structures geared toward helping borrower's cope with temporary financial set-backs—often highly idiosyncratic to specific circumstances of a borrower—modifications today are increasingly geared toward addressing the impacts of more systemic and severe macroeconomic dislocations to national labor and housing markets. A broad array of structures has now been implemented by lenders that offer substantial and long-lived reductions to mortgage payments. Nevertheless, many observers have noted that the performance track record of these modification efforts have to date been less than encouraging, as fewer than anticipated households have participated and a substantial fraction of loans redefault within a short period after modification. For example, data from the OCC indicates that roughly 20%-25% of loans modified since the second quarter of 2010 were 60 or more days delinquent just nine months after modification (Office of the Controller of the Currency, 2011).

This experience has prompted considerable debate over what forms of loan modification work best in terms of avoiding redefault by the borrower. Particularly noteworthy are the many calls to force lenders to do more widespread principal reduction modifications, reducing the face value of debt owed by the borrower to levels closer to the value of their home. Proponents of principal reduction argue that modifications are prone to fail unless they address the negative equity of the borrower through debt forgiveness. They further argue that lenders and servicers are reluctant to offer them for a variety of reasons, including conflicts of interest and other contract frictions (see, for example, *National Mortgage Servicing Standards* (2011) for an overview of many of these arguments). In this view, (a) debt forgiveness dominates other forms of modification, at least for the substantial portion of loans owing more than their house value, and (b) good public policy therefore requires promoting more of it than may naturally occur in the market. These arguments have been highly persuasive in policy circles, as evidenced by changes made to the government's Home Affordable Modification Program (HAMP) to emphasize principal reduction, and the efforts by State's Attorney Generals to require principal reductions as part of a settlement with major servicers over foreclosure processing errors¹.

A substantial amount of theory and empirical evidence has now well established a relationship between higher rates of mortgage default and low or negative borrower equity, a literature that we review briefly below. This connection between borrower equity and default is an often cited rationale of proponents for expanding the use of principal reduction. In option-theoretic models of mortgage default, for example, the borrower compares the value of his home to the value of his mortgage liability, and defaults when the former is sufficiently below the latter (that is, when his equity is sufficiently low). However, while it is true that the value of the mortgage liability will generally reduce with the face value of debt, other forms of modification that do not

¹HAMP, to be discussed further below, was changed in 2011 to emphasize principal reduction in the evaluation process for determining what form of modification a borrower is to receive, and provides compensation incentives for lenders to provide them. However, to date HAMP stops short of making them mandatory, although calls continue to do so.

reduce the face value of debt can also render a mortgage's value substantially below the face value. "Book" equity, in other words, is not the measurement that directly matters, and arguments about the efficacy of principal reduction relative to other modification structures have tended to overlook or neglect this point. Consequently, it is not clear from standard option-theoretic models of default just how various modification structures should stack up. Moreover, empirical studies relating default to low borrower equity do not resolve the issue either. These studies most often examine non-modified loan samples and periods where mortgage value and face value are not too dissimilar, and in fact often use face value of debt as a proxy for mortgage value. More specifically, these studies do not examine whether differences in the sources of variation in borrower equity make any difference for default behavior. Likewise, there are very few empirical studies that directly examine redefaults on modified mortgages, as there has generally been a lack of sufficiently detailed loan information and a very short history of observations upon which to base such studies.

This study contributes to our understanding of how redefault behavior varies across loan modification structures, both from a theoretical and empirical perspective. We begin by showing that standard option models for default generally have little to say about how modification structure matters, and suggest extensions to the standard model that may be important to an understanding of modified loan behavior. We construct a dynamic structural model of optimal default decisions that extends the basic option model to allow for important frictions relevant for predicting behavior across various modification structures. Particular attention is paid in our model to costs of sub-optimal default stemming from exogenous events outside the borrower's control, and to tax effects that interact with the structure of the modification cash flows. For a range of plausible parameter values of our model we find principal reduction modifications do indeed outperform other forms of loan modification, all other things being equal. For a typical negative equity loan receiving a modification, for example, we find principal reductions can result in up to $2/3$ fewer redefaults than some of the worst performing structures, controlling for the impact modification has on the net present value of the remaining mortgage payments.

Having analyzed loan modification structures through the lens of a theoretical model, we then turn to examine empirical evidence on modification performance using a unique loan-level data set on modified loans. This data allows us to test the predictions of the extended option model, and our empirical results match quite closely with the predictions of our model. Specifically, we find a similar advantage to principal reduction over other modification forms in the data, and document a consistent pecking order as well as similar quantitative impacts as predicted by the option model. To our knowledge this is the first study that confirms borrower equity remains an important determinant for default in the context of modified loans and that sources of variation in equity are additionally important determinants.

The rest of the chapter is organized as follows. Section 2.2 reviews additional background on loan modifications and the related literature. Section 2.3 reviews briefly

a standard option model of default, and then presents a dynamic structural model of redefault extending the basic model to include important frictions. Using the extended model, we parameterize solutions of the model to simulate and compare the performance of major modification structures, including principal reduction modifications. Section 2.4 introduces the empirical modeling and data, and presents empirical results used to examine the questions of relative performance of major modification types. Section 2.5 concludes the chapter.

2.2 Background and Literature

Traditional modification programs have tended to focus on remediating temporary hardship of the borrower, often related to idiosyncratic life-events such as loss of employment or income, death, illness, or divorce. For the most part traditional programs have not involved large adjustments to contractual terms of the mortgage, and have tended to involve changes or adjustments limited in duration. Some of the more common approaches, for instance, have included capitalizing past due interest and extending the term of the mortgage (resetting the loan current in the process), or offering temporary forbearance of payments (similarly extending the term and resetting the loan to current). The most common forms of modification in traditional programs, therefore, did not involve changes that would make a large difference to the value of the mortgage liability of the borrower. Rather, they sought to intervene and reverse a trajectory of default caused by temporary obstacles to a borrower's ability to catch up on past due payments and resume normal repayment of the loan. For a good overview of these more traditional programs, see Cutts (2008).

Modifications in more recent periods often go much further by seeking to make more substantial and permanent changes to contractual terms that can significantly impact the liability value of the mortgage. A few widespread programs have been developed and promulgated by the U.S. government. A well-known example of a government program is the HAMP program discussed above. Another prominent example of a government program is that of the Federal Deposit Insurance Corporation (FDIC) implemented on portfolios of failed institutions managed by them. In addition, many lenders and servicers have developed their own proprietary modification programs. The design of these newer programs generally involve several elements, such as eligibility criteria for who will be offered a modification, a menu of specific changes to the debt contract that will determine a pattern of future scheduled cash flows, and a specification for what will occur in the event the borrower defaults on his loan as modified. The specific contractual changes in newer programs invariably seek to lower the borrower's payment, the ratio of loan amount to property value (LTV), or both. Some of the most common contractual changes include:

- **Rate Reduction/Term Extension.** The borrower's loan interest rate is reduced, term extended, or both in order to lower his payment to a target level. Rate reductions can be a single new permanent lower rate, or involve a lower rate that steps up over time according to some schedule. In most cases, however, the rate is fixed according to a schedule, and not pegged to a fluctuating market rate of interest.
- **Principal Forbearance.** A portion of the borrower's principal balance is set aside for purposes of determining the amount of interest due and amortization in the loan payment (i.e., a non-interest bearing and non-amortizing principal portion), lowering his payments. This can result in a balloon payment for the deferred amount due at the expiry of the mortgage.
- **Principal Reduction.** Forgives a portion of the borrower's outstanding balance, often to achieve a target loan-to-value (LTV) ratio. This also reduces the borrower's payment by recasting the amount upon which interest and amortization payments are based.

Combinations of the above changes can be implemented in any individual loan modification, and many programs have determined an order of prioritization or “waterfall” for which terms to consider adjusting to meet defined criteria of the program. These criteria could include lowering the borrower's payment to a target level, lowering LTV, and other considerations. As an example, the HAMP program has employed a waterfall of lowering rate, extending term, and forbearing principal (all within defined limits) until the borrower's payment can be made 31% or less of gross income. Lenders will typically compare an estimate of the net present value (NPV) of expected cash flows on the new mortgage to anticipated recoveries from proceeding with foreclosure to decide if offering the modification is economic in the first instance. As previously mentioned, lenders participating in HAMP are required to evaluate the NPV of principal reduction modifications, but are not currently required to choose a principal reduction over the /rate/term/forbearance structure above². Modified loans are also often fully re-underwritten, whereby the borrower's employment status and income are confirmed, the value of the collateral property is updated, and the borrower may have to undergo a trial period of making payments under the modified terms for a short period (often 3 months) before the modification is considered permanent.³

²HAMP will provide payments to lenders that choose principal reductions, however, partially offsetting the costs to the lender. Committing tax payer funds to partially pay for principal reductions clearly demonstrates the policy preference for principal reductions currently in favor. HAMP also provides other payments to borrowers and servicers in connection with successful modifications.

³Additional and more detailed descriptions of newer government program structures such as HAMP can be found on program administration websites. For HAMP, for example, see <http://www.allregs.com/tpl/Main.aspx>.

A large literature has examined the structural dynamic modeling of the choices of mortgage borrowers, primarily related to option-theoretic approaches to modeling default and prepayment behavior of mortgagors, starting, for example, with Dunn and McConnell (1981a,b), Dunn and Spratt (1986), Kau, Keenan, Muller, and Epperson (1992), Kau and Keenan (1995). Stanton (1995) and Downing, Stanton, and Wallace (2005) are examples of studies that model the impacts of exogenous early termination on prepayment option exercise. Kau, Keenan, and Kim (1993) provide an example of modeling the impacts of exogenous early termination on default. Very few papers, however, model redefault behavior on modified mortgages. Foote, Girardi, Goette, and Willen (2009) provides an example of a simple 2-period model of a borrower's redefault decision that shares some elements with our model below. However, we are aware of no studies that treat the decision to redefault on a modified mortgage in a fully dynamic optimization framework.

The empirical literature testing option theories of mortgage terminations is likewise extensive, as for instance in Green and Shoven (1986), Schwartz and Torous (1989), Deng, Quigley, and Van Order (2000), and Deng and Quigley (2003), among others. Empirical studies of terminations for modified mortgages are much fewer. Quercia, Ding, and Ratcliffe (2009) provide estimates of redefault rates using logistic regression on a sample of subprime loans from the ABS securitization market. They document payment relief as an important determining factor in redefault probabilities, but are not able to provide more direct and conclusive evidence on the efficacy of different modification structures in their study. More recently, Goodman, Ashworth, Landy, and Yang (2012), also using subprime ABS data, document principal reductions perform better than alternative structures even controlling for payment relief. Our empirical evidence below looks beyond the subprime securitization segment, and studies loan behavior from a broad sector of the mortgage market, including prime loans. Additionally, our data is sufficiently rich in detail on the underlying pre-modification and post-modification contracts that we are able to go beyond controlling for the immediate impact on borrower payment and control for impact on the present value of all remaining loan payments.

2.3 Modeling Redefault

2.3.1 Redefault in a Basic Option Model

Consider a standard option theoretic model of mortgage default.⁴ To fix ideas and focus our attention on redefault of a modified mortgage, assume the mortgage is infinitely-lived and non-prepayable. Since mortgages typically have maturities of 30

⁴The basic option model framework below draws heavily from Kau and Keenan (1999).

years, and modified mortgages often extend terms to 40 years, the assumption of a perpetual term is likely not distorting in the majority of cases. Most mortgages are of course prepayable, and indeed option models of mortgage termination normally focus on the prepayment option, as this has been the predominant risk faced by many investors in mortgages historically. Moreover, the prepayment option can in turn significantly impact the value of the option to default and therefore effect optimal default decisions. In the context of a modified mortgage, however, the borrower is effectively locked out of normal refinancing channels owing to his prior default. In other words, the prepayment option on a modified mortgage has little or no value, and consequently will not have a significant effect on the borrower's decision to redefault. Since we do not consider prepayments, we also consider interest rates as fixed.

Let home prices H_t be governed by a geometric Brownian motion process

$$dH = \alpha H dt + \sigma H dz, \quad (2.1)$$

where dz is an increment of a Wiener process. Since α is the expected capital gain from holding housing an increment of time, it can be written as the total required return on housing less the service flow “dividend” accruing to the owner of housing via implicit rents. Given our assumptions, the value of the mortgage with face value F paying a fixed coupon c will depend only on the single state variable H and can be written as $M(H)$. Applying Ito's Lemma to $M(H)$ to calculate the incremental appreciation of the mortgage, adding the coupon payment c , and equating to a required rate of return ρ produces a standard ordinary differential equation for M :

$$\frac{\sigma^2}{2} H^2 M'' + \alpha H M' - \rho M + cF = 0. \quad (2.2)$$

A solution for M and the default boundary of house prices H^* can be determined by solving equation (2.2) subject to the boundary conditions

$$\lim_{H \rightarrow 0} M(H) = 0 \quad (2.3)$$

$$M(H^*) = H^* \quad (2.4)$$

$$M'(H^*) = 1. \quad (2.5)$$

The default boundary H^* in this case turns out to be

$$H^* = \frac{\gamma}{\gamma - 1} \frac{cF}{\rho}, \quad (2.6)$$

where γ is a root to the quadratic equation

$$\gamma(\gamma - 1)\sigma^2/2 + \gamma\alpha - \rho = 0. \quad (2.7)$$

Given the process (2.1), default boundary (2.6) and a current value $H_t > H^*$ the probability of default can be determined by the probability that H falls below H^* . Similarly, if default has already occurred so that $H_t \leq H^*$ the mortgage can be modified to lower H^* further and the probability of redefault can likewise be determined by now referencing the new threshold $H^{**} < H^*$.

Since the default boundary in (2.6) is simply the scaled present value of the mortgage, it is obvious that lowering F —that is, a principal reduction—will lower H^* and impact the redefault probability accordingly. Alternatively, reducing c —that is, a rate reduction—by the same proportion would have an equal impact on the loan present value, default boundary, and associated probability of redefault. Thus, there is no benefit to principal reduction over rate reduction in a standard model such as this. Said another way, rate reductions can have an equal impact on the value of the mortgage and default decisions as principal reductions. If principal reductions are to have some benefit over modifying other contractual terms, such must come from extending the basic option theory model.

We consider extensions to the basic model that introduce frictions with potentially important influences on borrower decisions. First, we allow for random stopping events that force the borrower to terminate the mortgage early, possibly through default. Such termination events can represent exogenous shocks stemming from a variety of underlying causes, such as loss of employment or income, or a need to relocate or move. Following Kau, Keenan, and Kim (1993), we allow for the possibility of exogenous stopping events forcing early sub-optimal terminations in the future and potentially feeding back to decisions about optimal default in the present. We examine these effects in the context of modified mortgages to understand the implications for various changes to contractual terms of the mortgage. The form of the modified contract may have a strong influence on what a borrower decides to do when a termination event occurs. For example, by reducing the balance owed a principal reduction may make it less likely the borrower defaults in the event of exogenous termination and more likely he sells his home and pays off the mortgage⁵. Thus, the possibility of forced termination may introduce differential costs and considerations to the borrower's decisions regarding optimal default. Second, we model the impact of tax effects. Since mortgage interest is deductible, the form of modification can impact the tax liability of the borrower, which may in turn impact their decisions regarding optimal default.

To explore these extensions to the basic model we develop below a finite horizon discrete time model closely related to the option model above. Some of the details of extending the model will be more convenient to handle in a discrete time framework, and using a finite horizon also allows us to explore principal deferments and term extensions that are ignored in the infinite horizon example above. To this end, we

⁵While this would represent an exogenous prepayment of the mortgage, it would not be associated with exercise of a financial prepayment option.

model a borrower's decision to default on his modified mortgage contract as a decision for whether to make the contractual payment under the new terms of the contract at each specified due date, or to cease making payments and forfeit the property to the lender through foreclosure. This formulates the default decision as an optimal stopping problem, fully analogous to modeling the exercise of an American option in continuous time as above⁶. Lastly, because payments are actually due in discrete intervals of time (normally monthly), it is natural to assume the borrower's decisions are likewise made in the same discrete points of time taken over each period of the remaining term of the modified mortgage.

2.3.2 Dynamic Programming Extension of the Model

In each period $t = 1, \dots, T$ following modification of the mortgage, up to and including the maturity period T of the new contract, the borrower must decide whether to redefault or continue paying on the contract as agreed. More precisely, the borrower seeks to choose a sequence of optimal controls, $\{u_t\}_{t=1}^T$, over time so as to maximize the expected discounted payoffs resulting from these decisions, to be defined further below. The control is a binary variable in this context, and we let $u_t = 0$ correspond to a decision to continue paying on the mortgage, and $u_t = 1$ correspond to a decision to default. Redefault terminates the mortgage and results from a decision not to make the contractual payment for the period. We further assume this results in immediate forfeiture of the property to the lender through foreclosure and no further cash flows on the mortgage.⁷

If the borrower continues to pay on his mortgage he will receive a period payoff equal to the value of rental services provided by the housing less the after-tax value of payments he makes on his mortgage inclusive of property taxes and insurance.⁸ Specifically, let r_t represent the value at time t of the rental services provided by the borrower's home. We assume r_t evolves stochastically over time according to the Markov process

$$r_{t+1} = (1 + \alpha)r_t + \sigma r_t \epsilon_{t+1}. \quad (2.8)$$

⁶The continuous time model above will be a limiting case of a more basic version of the discrete model below that doesn't consider exogenous terminations, tax effects, and other contract features as below.

⁷The assumption of immediate foreclosure is one of convenience for exposition. In reality, foreclosure (and eviction) may take time to complete which could benefit the borrower by having cost-free housing for a period of time. However, this can be subsumed into default costs to be discussed below.

⁸Most mortgage contracts require monthly payments that will include principal and interest on the mortgage loan, as well monthly amounts of property tax and insurance to be escrowed to meet those obligations as they come due. The mortgage interest and tax components are commonly deductible from borrower income in determining their state and federal income tax liability.

Equation (2.8) is simply a discrete time formulation of the standard geometric Brownian motion process (2.1), which is why we preserve the use of the drift parameter α and volatility parameter σ for convenience. We measure time in months, and assume $\epsilon \sim \text{i.i.d. standard normal}$ for all t .

Home prices are related to rents through a standard assumption of market efficiency for housing, which implies prices are the expected discounted present value of rents

$$H_t = \sum_{j=1}^{\infty} \frac{E_t(r_{t+j})}{(1+\rho)^j}. \quad (2.9)$$

Here $E_t(\cdot)$ is the expectation operator conditional on information at time t , and ρ , as before, is a periodic discount factor applicable to housing. In the housing price literature ρ often represents a market-based user cost of capital that depends on mortgage rates, depreciation, and other components, adjusted for income and property taxes (see, for instance, Dougherty and Van Order (1982)). For simplicity we assume ρ is constant through time, and we also assume $\rho > \alpha$ to ensure that house prices are bounded. The relationship in (2.9) expresses price as a “fundamental value” and says there are no bubbles in housing, and that prices reflect the best available current information on the value of the implicit rental service stream housing will provide in the future. An active area of research has examined the efficiency of the housing market, including testing the present value relation in (2.9) directly (see for example, Case and Shiller (1989), Meese and Wallace (1994)). Much of this literature has documented that home prices can deviate from market efficiency relationships as above, especially in the short run, but that these deviations have tended to be consistent with the high transactions costs associated with trading in housing assets (Meese and Wallace, 1994). Regardless of whether short-run deviations from efficiency exist, we focus exclusively on measures of fundamental value to examine the behavioral incentives faced by a rational borrower under a modified loan contract.

Along with (2.8) these assumptions allow us to re-express the relation in (2.9) as

$$H_t = \frac{1+\alpha}{\rho-\alpha} r_t = \Lambda r_t. \quad (2.10)$$

This expresses house prices as proportional to rents and implies that home prices themselves will follow a geometric Brownian motion.

The modified loan contract specifies a sequence of required principal and interest payments to service the loan $\{p_t\}_{t=1}^T$. We assume this sequence is fully specified and deterministic in the new loan agreement. In most cases, including the modification forms considered here, p_t can be expressed formulaically in terms of underlying contractual parameters. For example, we may write for $t = 1, \dots, T-1$

$$p_t = c_t \eta(c_t, T-t) \widehat{B}_t \quad (2.11)$$

and

$$\widehat{B}_t = (1 + c_t - c_t \eta(c_t, T - t)) \widehat{B}_{t-1}. \quad (2.12)$$

Here, c_t denotes the loan interest rate in effect for the period t , $\widehat{B}_t$ a reference balance upon which interest and amortization will apply, and $\eta(c_t, T - t)$ a function from the standard annuity formula given as

$$\eta(c_t, T - t) = \frac{(1 + c_t)^{T-t}}{(1 + c_t)^{T-t} - 1}. \quad (2.13)$$

For some of the structures we consider the final period payment, p_T could also reflect a balloon amount from the deferred principal in a forbearance structure. Note also that the reference balance $\widehat{B}_t$ need not equal the actual contractual balance B_t , such as under a principal forbearance. The assumption that p_t is deterministic stems from the assumption that the loan rate c_t is likewise deterministic, which is highly realistic in most modification situations. While interest rates on traditional ARM or Hybrid mortgages are indexed to a market rate of interest, and thus stochastic, most loan modifications today seek to remove the related interest rate risk from the loan by fixing a predetermined schedule of rates.⁹ Though equations (2.11) and (2.12) describe principal and interest dynamics for most structures we consider, the key point here is that principal and interest payments over time will be fully determined sequences of specified interest rates, amortization rates, reference balances, and reference maturities.

We adjust the payment p_t to reflect the tax benefits of deducting interest payments from taxable income. Letting τ represent the borrower's marginal effective tax rate, we calculate the tax-adjusted principal and interest payment as

$$\dot{p}_t = p_t - \tau c_t \widehat{B}_t = (1 - \tau/\eta) p_t. \quad (2.14)$$

For simplicity, we assume τ is constant over the planning horizon. However, equation (2.14) makes clear that the after-tax payment will vary over time and according to the new interest terms of the contract through the function η . Specifically, rate reduction modification will experience some dilution of benefit from lower tax deductions, and tax benefits will tend to decay over time as principal amortization accelerates.

As mentioned above, in addition to principal and interest components borrower payments often must also include property taxes and insurance amounts that are escrowed by the servicer¹⁰. We specify the property tax and insurance portion of the borrower's payment to be proportional to the assessed value of the home:

⁹The rate schedule, however, may adjust deterministically, as is commonly done with rate reduction modifications where the rate steps-up over time.

¹⁰For cases where escrow accounts were not established on the original loan, many modification programs require that one be established under the new loan agreement.

$$J_t = \theta H_t, \quad 0 < \theta < 1. \quad (2.15)$$

We assume the parameter θ , representing the sum of the tax and insurance premium rates, is constant over time¹¹. The after-tax portion of this payment is thus

$$J'_t = (1 - \tau)\theta\Lambda r_t. \quad (2.16)$$

Termination can also occur for exogenous reasons, which we take to be governed by a Poisson arrival process with intensity parameter λ . Roughly speaking, this implies the probability of exogenous termination over the next interval of time dt , conditional on termination having not yet occurred, is approximately λdt . Exogenous early termination can be thought of as a shock forcing termination that may itself stem from a variety of underlying causes beyond the control of the borrower. We define the state variable z_t to take the value of one if a termination event has arrived in period t or earlier and the value zero otherwise. If $z_t = 1$ and $z_{t-1} = 0$, then a terminal cash flow will accrue and no further cash flows need to be considered in our problem.

The arrival of an early termination event forces either sale of the property if the borrower has sufficient equity or otherwise forces default on the loan. Specifically, if the sales price, assumed to be the value of the property as given by (2.9) above, net of costs to sell are greater than the outstanding loan amount, then the loan is terminated without default and the borrower keeps the net sales proceeds after repayment of the loan. However, if net sales proceeds would not be sufficient to repay the loan the borrower must evaluate whether to default by comparing the cost of default to the shortfall in proceeds to repay the loan. We will specify default costs explicitly below, but note here that for reasonable values of costs to default we assume the borrower is not constrained to default in this situation—that is, we assume he has sufficient other financial means or ability to borrow unsecured credit to make up the shortfall if doing so is less costly than defaulting. In practice, credit constraints may be prominent, and they could be included in our modeling framework by introducing a new state variable representing the credit constraint condition of the borrower. However, doing so would not qualitatively add much to the analysis, and we make the simplifying assumption of no constraints for greater tractability.

Finally, costs to sell include real estate commissions, taxes, and other fees, which we take to be proportional to the sales price, so that net sales proceeds can be expressed as $(1 - \kappa)H_t$, $1 > \kappa > 0$. This completes the specification of the relevant state variables and transition dynamics of the model. We can collect the state variables

¹¹Though tax and insurance rates are assumed constant, tax and insurance payments will vary monthly with home prices in this specification. In reality there may be limitations on how these payments will adjust over time in different jurisdictions, for example, when tax assessments are made only periodically or otherwise subject to restrictions in reassessment amounts.

into a state vector $x_t = (r_t, p_t, H_t, z_t)$, with transition dynamics for the components of x described as above. We next turn to describing the payoff functions of the dynamic program.

Loan termination can incur costs for the borrower. In particular, we assume default is costly to the borrower and model the termination payoff from default to summarize these costs, which may include costs of moving, loss of reputation, availability and cost of future credit, and costs associated with renting housing rather than owning¹². These costs can be substantial, and may vary by household characteristics such as household income, family size, number of school-aged children, among other factors. We assume default costs are a simple multiple of rents:

$$\psi_t = -\phi r_t, \phi \geq 0. \quad (2.17)$$

Assuming the mortgage has not terminated prior to period $T + 1$ (the period following the final decision period), the borrower will receive the termination payoff $\Omega_{T+1} = H_{T+1}$. In the final decision period T , if a random stopping event has not arrived, the borrower chooses between continuing the mortgage or defaulting. If he continues he receives the continuation value

$$C_T(x_T) = (1 - (1 - \tau)\theta\Lambda)r_T - (1 - \tau/\eta)p_T + (1 + \delta)^{-1}E_T(H_{T+1}), \quad (2.18)$$

where δ is the periodic discount rate applied to future payoffs in the borrower's optimization. The first term on the right hand side of (2.18) is the rent benefit net of income tax-adjusted property tax expense, the second term is the tax-adjusted loan payment, and the last term is the expected discounted value of next period's home price. If he chooses to default he receives the default penalty $\psi_T = -\phi r_T$.

In the case a random stopping event has arrived in period T , the borrower receives the termination payoff

$$\Omega_T = \max(\psi_T, (1 - \kappa)H_T - B_T). \quad (2.19)$$

The value function for period T is thus fully determined as

$$V_T = \begin{cases} \max_{u_T}(C_T, \Omega_T) & \text{if } z_T = 0 \\ \Omega_T & \text{if } z_T = 1, z_{T-1} = 0. \end{cases} \quad (2.20)$$

For periods t prior to T , the continuation value, termination value, and value functions are similarly given as

¹²Moving costs should be thought of more generally than direct expenditures to move, and should be measured as the value the borrower would place on having the option to not move. Costs associated with renting include the risks of rising rents and non-renewal of lease, risks to which homeownership is a hedge.

Description	Parameter	Value/Range
Drift in rent & home prices	α	0.015
Volatility in rents & home prices	σ	0.05
Property tax/insurance	θ	0.0124
Default Penalty	ϕ	0-12
Marginal income tax rate	τ	0.2
Discount rate	δ	0.05
Housing required return/user cost	ρ	0.06
Exogenous stopping intensity	λ	0-0.2
Cost to sell	κ	0.06

Table 2.1: Summary of Selected Parameter Values for Model Simulation

$$C_t(x_t) = (1 - (1 - \tau)\theta\Lambda)r_t - (1 - \tau/\eta)p_t + (1 + \delta)^{-1}E_t(V_{t+1}), \quad (2.21)$$

$$\Omega_t = \max(\psi_t, (1 - \kappa)H_t - B_t), \quad (2.22)$$

and

$$V_t = \begin{cases} \max_{u_t}(C_t, \Omega_t) & \text{if } z_t = 0 \\ \Omega_t & \text{if } z_t = 1, z_{t-1} = 0. \end{cases} \quad (2.23)$$

Equations (2.21)-(2.23) define the standard recursive Bellman equations for the borrower's optimal stopping problem.

2.3.3 Parameterization and Numerical Solutions

In order to explore solutions to the model we first determine values for key model parameters. Table 2.1 provides a summary description of the main parameters of the model and the values or value ranges selected. Most parameters are calibrated to represent reasonable/typical values of the items they represent. For example, parameters for the drift in rents and home prices are roughly consistent with data on implicit rents from the Bureau of Economic Analysis, and the property tax rate is consistent with the average property tax rate in the U.S. as calculated by the Tax Foundation. For other parameters we explore solutions to the model under alternative calibrations, in particular, for parameters determining the default penalty and the stopping intensity.

Given a parameter selection as above, and given a sequence $\{p_t\}_{t=1}^T$ determined by a loan modification structure, the recursions in (2.18)-(2.23) are solved backward from period T to period 1. Our solution framework maps the state space onto a fine-grained discrete lattice and evaluates at each node of the lattice the expectation in the

continuation value (2.21) through one-step-ahead simulation of the state process. The solution of the model determines a boundary condition for rents in each period, r_t^* , $t = 1, \dots, T$, such that $r_t \leq r_t^*$ implies the borrower will choose to default. In general, r_t^* will lie below $(1 - \tau/\eta)/(1 - (1 - \tau)\theta\Lambda)p_t$, such that the borrower is willing to endure “negative carry” on the loan in order to avoid a default penalty and loss of potential excess rents over borrowing costs in the future. For sufficiently low r_t , however, paying the negative carry on the loan is not outweighed by the costs of defaulting. Since the boundary solution will depend on the state process, which includes both the process for r_t and the (deterministic) process for p_t , different modification types will imply different default boundaries for rents. Once the boundary r_t^* for a given modification structure has been determined, and given an initial value for rents, r_0 , we can determine the probability of redefault on the modified loan structure by reference to the stochastic rent process in (2.8), together with the process for λ .

Exogenous terminations turn out to have a significant impact on endogenous decisions to default within the model. Figure 2.1 illustrates the simulated frequencies of redefault for solutions of the model under a range of values for the parameter λ for a benchmark loan. Specifically, the Figure shows the likelihood of a borrower choosing to default in the absence of an exogenous stopping event. Higher values of λ essentially raise the default boundary r^* , and the probability that a borrower exercises his default option grows significantly with higher values of λ . This is in addition to the impact a higher background rate of exogenous terminations has on forced defaults. Thus, overall redefault rates will be highly sensitive to the value of λ .

Figure 2.2 shows the sensitivity of solutions to the model, again for a benchmark loan, assuming different values for the cost to default ϕ . Overall, redefault rates vary by roughly 10 percentage points over a five year horizon as default costs vary from 0 to 12 months of the current value of rent. While this is measurable sensitivity, it is not as important a factor as λ in impacting redefault probabilities. More importantly, neither the value of ϕ nor the value of λ fundamentally alters the relative performance of the different modification structures we explore below. Therefore, without loss of generality, we fix the values of these parameters at the intermediate levels $\phi = -6$ and $\lambda = 0.05$ and turn our focus to evaluating the relative performance of different structures.

We analyze solutions to the model under four prominent forms of loan modification: principal reduction, rate reduction, rate reduction with a term extension, and principal deferment. In each case we alter the corresponding terms of the contract so as to render the present value of remaining promised payments to a common level. This allows us to explore the effects of different modification forms separate from the standard net present value effects they entail. Figure 2.3 shows the results of simulating the model for a 125 LTV loan with a 5% coupon and 330 months of remaining term. Consider first a comparison of the extreme outcomes in Figure 2.3. In the principal reduction scenario the loan balance is reduced so as to make the

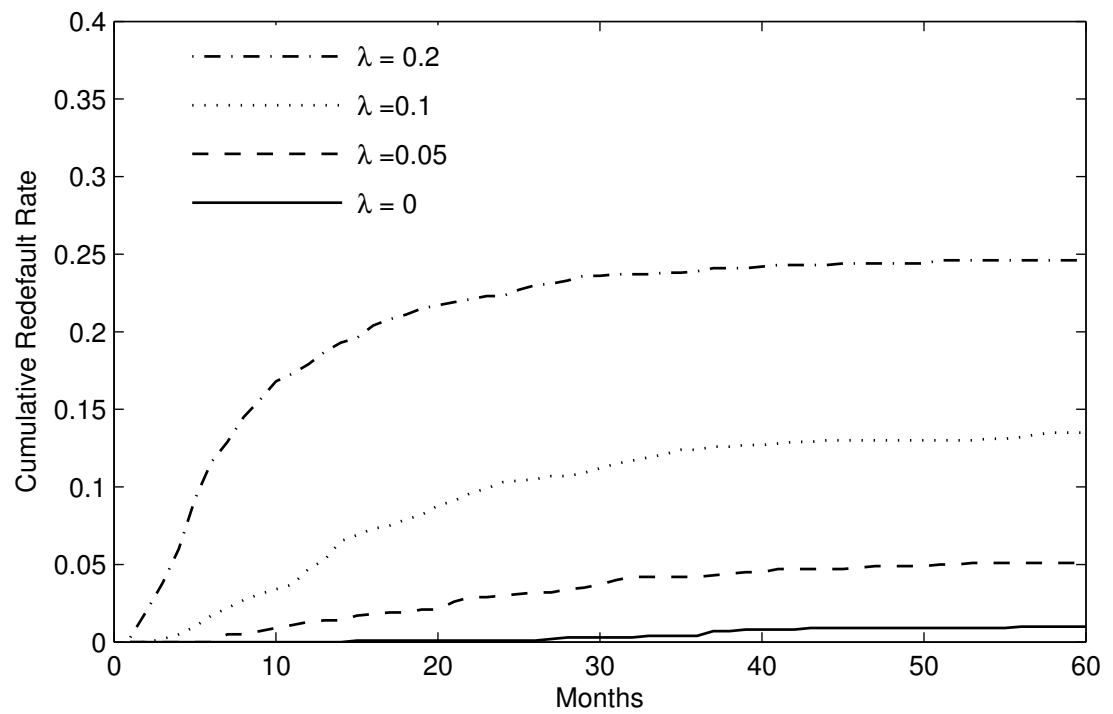


Figure 2.1: Simulated Endogenous Redefault Rates by Stopping Intensity: 100 LTV, 330 month remaining term, 5% coupon loan; $\phi = -6$.

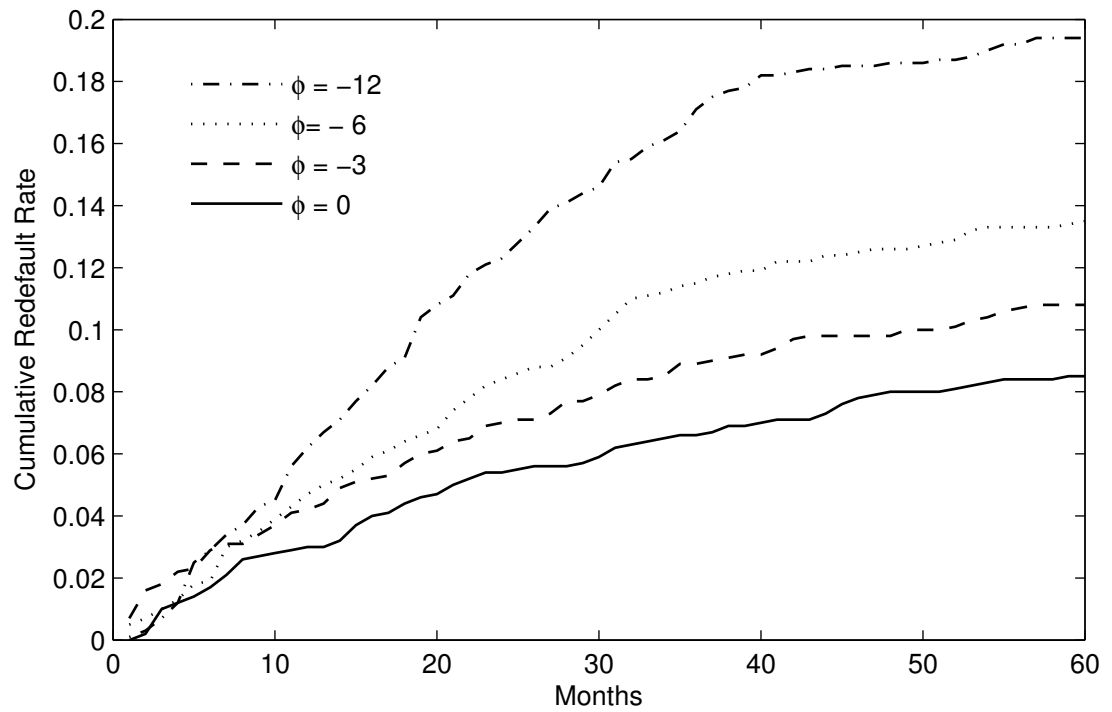


Figure 2.2: Simulated Total Redefault Rates by Default Penalty: 100 LTV, 330 month remaining term, 5% coupon: $\lambda=0.05$.

loan's LTV 100 percent. At just under a 15% redefault rate over the first 5 years following modification this scenario provides the best redefault performance among the structures analyzed. By contrast, the rate reduction scenario provides the highest redefault rate. This scenario reduces the coupon to roughly 3% to match the payment, and therefore the present value, of the principal reduction scenario. Despite their matched payments and present value, rate reduction results in an additional 40% of loans redefaulting.

There are two important reasons for the relatively poor performance of rate reduction over principal reduction in the model. First, although the payment is the same in both modification forms the composition of the payment is very different. In particular, the rate reduction modification shifts more dollars toward principal repayment and fewer dollars toward interest payments. This implies that the after-tax value of the rate reduction payment is higher relative to the principal reduction payment, which makes the borrower less willing to endure a negative carry on the loan¹³. Second, by leaving the loan principal balance unchanged a borrower under the rate reduction modification remains vulnerable to defaulting for exogenous reasons. Should an exogenous termination event occur, a borrower will more frequently find defaulting to be his best option in a rate reduction modification than under a principal reduction modification. Given the background rate of exogenous terminations, therefore, rate reduction structures will ex-post default at higher rates. Borrowers of course understand this ex-ante, and so will consider the expected costs of sub-optimal default embedded in their modification structure, which will influence their willingness to continue with the mortgage today. This is one reason, for example, why the higher rate of principal paydown in a rate reduction modification has little value. There is no benefit to a faster up-front schedule of principal amortization if a strong likelihood exists that this money will be lost and default penalties incurred regardless.

How strong are the relative influences of these two factors in explaining the performance differences between rate reduction and principal reduction structures? The principal deferment scenario can offer clues. The principal deferment scenario reduces the borrower's payment to a similar level as the principal reduction and rate reduction scenarios by deferring roughly 26% of the loan balance to the final period of the loan. This structure therefore retains much of the tax-saving features of principal reduction since the relative amount of interest is similar¹⁴. At the same time, this structure shares the higher ex-post default and higher ex-ante sub-optimal default costs of exogenous early termination as the rate reduction structure, since 125% of the property value remains to be repaid at the point of modification. From Figure 2.3 it is evident that the effect of sub-optimal termination costs dominate, though

¹³This result assumes principal reduction is itself not taxed, as is currently the law under the Mortgage Forgiveness Debt Relief Act of 2007. If allowed to expire in 2012, taxation of principal forgiveness amounts would strongly alter the results here.

¹⁴Interest in this example is 92% of the interest under a principal reduction scenario.

the tax effect remains important.

The last scenario depicted in Figure 2.3 is rate reduction combined with term extension. Extending the term out to 480 months (normally the maximum a lender will consider) will lower the borrower's payment but will not impact the mortgage present value. Additionally lowering the rate, however, will further lower the borrower's payment and will lower the mortgage present value. This scenario turns out to be very similar to the principal deferment scenario under the assumed parameters. In this example, the rate must be reduced to around 3.5% to achieve a comparable present value. This results in a slightly lower payment than under principal deferment, but also a slightly smaller amount of interest than paid in the principal deferment scenario. The after-tax payments are therefore nearly the same, and because both have a similar schedule of loan balances the payouts from early termination are nearly the same. Figure 2.3 shows that redefault behavior are consequently very similar for these modifications.

We do not depict a pure term extension modification in Figure 2.3. As already noted, merely extending the term does not effect the present value of a mortgage, so a pure term extension would not be comparable to the other scenarios depicted in Figure 2.3 on that basis. Moreover, while extending the term, say out to 480 months, does lower the payment, it is insufficient in our model to avoid redefault—that is, it is not enough to change a rational borrower's mind on his default decision. In this sense, a pure term extension is the worst performing structure among those considered here.

Results of our model for loans with higher pre-modified LTVs draw a similar conclusion, if not a starker contrast between the structures considered here. Figure 2.4, for example, shows results for a loan with a 150 pre-modified LTV. Principal is reduced in the debt forgiveness scenario to again bring the LTV down to 100, so results for this scenario are unchanged from the prior example, except of course the change in mortgage present value in this example is more significant than before. Rate reduction now performs very poorly, however. Here the interest rate must be reduced to very low levels to match the payment and present value of the principal reduction modification, increasing the borrower's tax liability and offsetting the benefit of a lower payment. Moreover, a shortfall in the value of the home relative to the face amount of debt is much more likely in the event of an exogenous stopping, so the expected costs associated with a forced default are much higher if the borrower chooses to continue (for example, he will likely not recover principal paydowns in the event of exogenous stopping later). Principal deferment and rate reduction with term extension are again the intermediate scenarios in this example, although principal deferment now does relatively better. This results from the fact that for a higher balloon amount to be paid at maturity, a deferment modification must have a lower payment relative to the rate reduction/term extension modification to match in present value, and the deferment modification will maintain a higher interest tax benefit. As before a pure term extension does not prevent the borrower from redefaulting

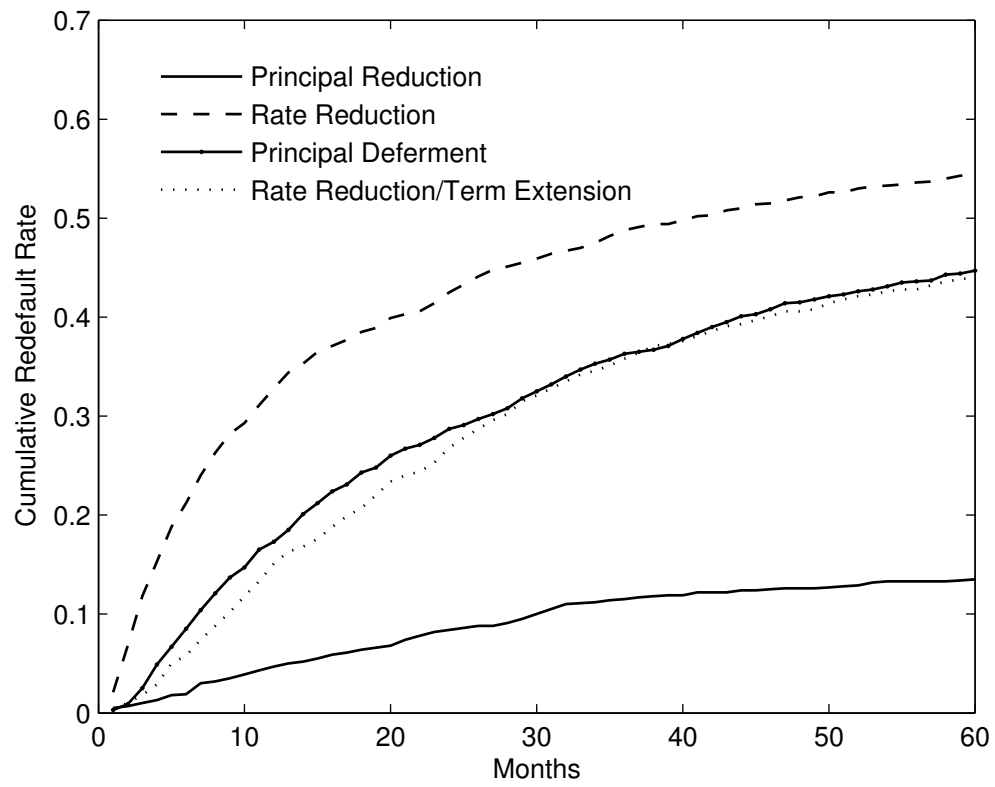


Figure 2.3: Simulated Total Redefault Rates for a 125 LTV Loan: 5% coupon, 330 month remaining term loan (before modification), $\lambda = .05$, $\phi = -6$.

at all, and so is not considered in Figure 2.4.

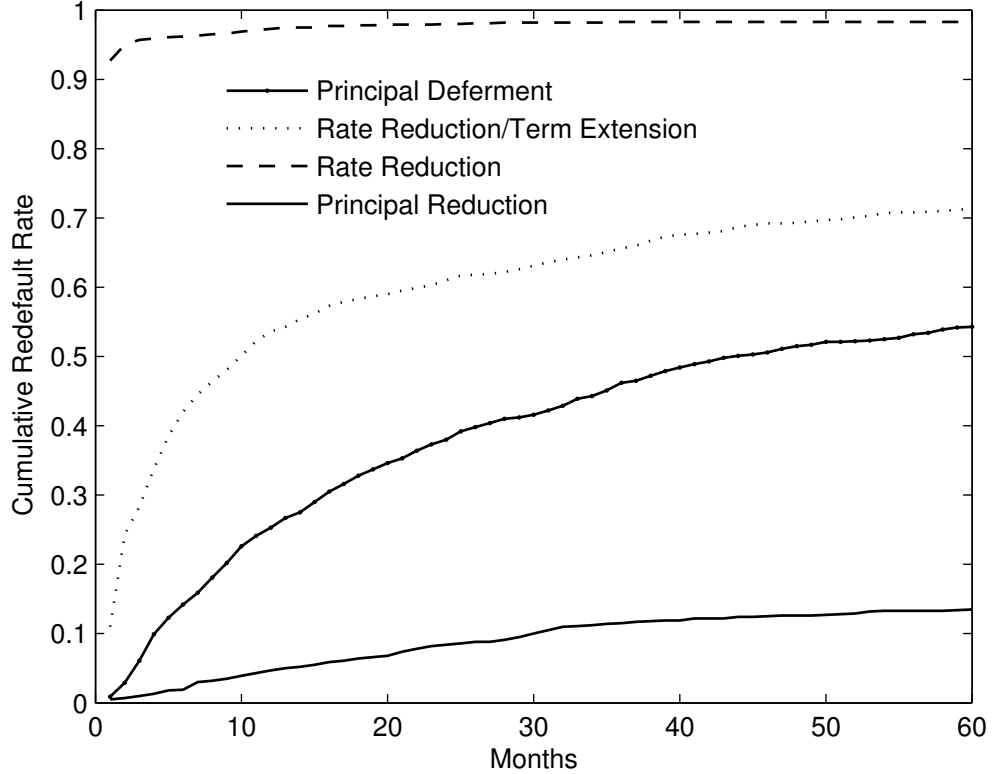


Figure 2.4: Simulated Total Redefault Rates for a 150 LTV Loan: 5% coupon, 330 month remaining term loan (before modification). $\lambda = .05$, $\phi = -6$.

We next turn our attention to an empirical study of the behavioral relationships examined in the option model developed above.

2.4 Empirical Analysis

2.4.1 Model

Structural models of default as above base default probabilities and timing on the first-passage or first-arrival times T of the underlying state process¹⁵. For instance,

¹⁵Note that in this section we maintain a standard use of notation for T representing a random termination time. Although this differs from the prior section where T represented the final decision

letting T_s represent the random time until rents or home prices fall to the default threshold, T_λ the random first-arrival time of the exogenous stopping process, and $T = \min(T_s, T_\lambda)$, the probability of default in the interval $(0, t)$ is given by $\Pr(T < t)$.¹⁶ For empirically modeling default this has naturally led to the choice of a wide class of reduced-form survival or duration models where the direct object of study are the termination times T of various agents, possibly censored, conditional on relevant observable characteristics of the agents. This choice of reduced-form modeling has been applied to study default and termination behavior of a wide class of loans, including corporate loans and mortgages¹⁷. In the case of mortgages, proportional hazard duration models such as those developed by Cox (1972) and Cox and Oates (1984) have been adopted to study the effects of option-incentive measures on mortgage termination behavior, and the framework has been further extended to include additional measures of "trigger event" effects (see, for instance, Deng, Quigley, and Van Order (2000) and Deng and Quigley (2003)). We adopt the same approach to study loan modification performance across different modification structures. Specifically, we estimate the relative hazard rates of redefault across different modification structures, controlling for the impact the modified contract has on the present value of the remaining loan payments, as well as controlling for other economic variables and borrower characteristics. These estimates provide a means to test the main implications of our option model above.

Again, letting T represent the duration of a modified mortgage until redefault, the hazard function h for the Cox model is specified as the product of a baseline hazard function and a set of proportional factors:

$$\lim_{dt \rightarrow 0} \Pr(t < T < t + dt | t < T; z) = h(t|z) = h_0(t) \exp(z'\beta), \quad (2.24)$$

where $h_0(t)$ is a baseline hazard depending exclusively on time and common to agents, z a vector of agent-specific covariates with elements that may be time-invariant or may depend on time, and β a vector of parameters common to all agents. The hazard thus represents the instantaneous rate of redefault on a mortgage at time t conditional on the borrower having not redefaulted prior to t , a concept also commonly referred to as the conditional default rate (CDR) in the mortgage analysis literature (e.g. see Hayre (2001)). The survivor function giving the probability of surviving to time t is thus easily derived as

period, there should be no risk of confusion as it should be clear from the context which notation applies.

¹⁶This assumes for simplicity of exposition that exogenous stopping results in default. However, it is straightforward to extend the argument to include probabilistic assessments for default occurring given a stopping event. The main point remains that default probabilities are directly related to first arrival times.

¹⁷For applications to corporate loans, see Duffie and Singleton (2003), especially chapter 3.

$$S(t|z) = \exp\left(-\int_0^{\infty} h(u|z)du\right). \quad (2.25)$$

Observations on T are naturally subject to censoring when a loan is not observed to have defaulted by the end of the observation period. Thus, the econometrician observes in practice duration realizations t that are the smaller of the default variate T or a censoring time C . The data therefore consists of pairs (t, d) , with t a realization of $\min(T, C)$ and d an indicator for whether duration has been censored. Censoring times for our data are fixed and therefore assumed independent of duration (non-informative censoring), so that each mortgage i contributes to a likelihood for the observed data with a term

$$L_i = f(t_i|z_i)^{d_i} S(t_i|z_i)^{1-d_i}, \quad (2.26)$$

where $f(\cdot)$ is the density function for duration and $d_i = 0$ indicates censoring of loan i . Substituting the fact that $f = h/S$ and taking logs yields the log-likelihood equation for the data:

$$\mathcal{L} = \sum_{i=1}^n \left(d_i \log h(t_i|z_i) - \int_0^{t_i} h(u|z_i)du \right). \quad (2.27)$$

Inference using the full likelihood equation (2.27) can proceed if one specifies a parametric form for the baseline hazard h_0 , or under discretization methods such as those applied to analysis of grouped duration data (we explore such grouped duration models in the subsequent chapter).

A now very common alternative is to leave the baseline hazard unspecified and perform limited information or semi-parametric inference by factoring the likelihood into partial likelihoods for the rank and order of the duration data. By basing inference on the partial likelihood for the rank of duration data, the baseline hazard cancels from the partial likelihood allowing one to leave it unspecified, while retaining consistent estimates of the parameter vector β with all the standard asymptotic properties and consistent estimates of its covariance matrix (see Lancaster (1990), especially chapter 9, and Kalbfleisch and Prentice (2002) for further details). In this case, the log partial likelihood simplifies to

$$\mathcal{L}_p = \sum_{i=1}^n d_i \left[z_i' \beta - \log \left(\sum_{j=1}^n I(t_j \geq t_i) \exp(z_j' \beta) \right) \right], \quad (2.28)$$

where $I(\cdot)$ is an indicator function taking the value 1 when the expression in its argument is true. In our application to modification redefault, we follow this approach by focusing our attention on the parameters of interest β describing the relative

importance of modification structure attributes, treating the baseline hazard rates as nuisance parameters.

A well-known concern with inference in proportional hazard models such as the Cox partial likelihood model is the effect on parameter estimates of unobserved heterogeneity stemming from unobserved or missing covariates, even when such are uncorrelated with observed covariates. As shown in a number of studies, such as those of Ridder and Verbakel (1983) and Gail, Wieand, and Piantadosi (1984), the general effect of unobserved heterogeneity is to bias the coefficients toward zero, though standard errors and statistical tests remain approximately valid. A proposed solution to the problem is to specify a mixed proportional hazard model with a random variable from a mixing distribution multiplicatively entering the hazard function

$$h(t_i) = v_i h_0(t_i) \exp(z_i' \beta). \quad (2.29)$$

This is essentially a random effects version of the Cox model, where the random effect typically represents unmeasured covariates. Lancaster (1990) discusses this model extensively assuming a Gamma mixing distribution with unit mean for identifiability and variance $\sigma^2 = 1/\omega$, and suggests estimation via the EM algorithm treating the v_i as missing data. More recently, Therneau, Grambsch, and Pankratz (2000) show that solutions equivalent to EM estimation of the Gamma model can be achieved by maximizing over β, v , and ω a penalized partial log-likelihood

$$\mathcal{L}_{pp} = \mathcal{L}_p - g(v; \omega), \quad (2.30)$$

where $\mathcal{L}_p$ is the standard Cox partial likelihood as in (2.28) now based on the hazard (2.29) augmented to include the v_i , and $g(\cdot)$ is a penalty function given as

$$g(v; \omega) = -\omega \sum_{i=1}^n (\log v_i - v_i). \quad (2.31)$$

This treats the v_i as additional regression parameters in the partial likelihood that are constrained by the penalty function (2.31), which in this case is the portion of the logarithm of the log-gamma density depending only on v . See Therneau, Grambsch, and Pankratz (2000) and Therneau and Grambsch (2010) for further details on estimation and inference for penalized Cox models, which we make use of in our results below.

2.4.2 Data

The empirical analysis is based on data from a large national mortgage servicer and consists of loans modified between January 2009 and February 2012. Each loan record contains detailed information on the pre-modification contract, allowing us to observe both the history and scheduled future values of key contractual parameters

such as balance, required payment, interest rate, and remaining term, from the point of origination until modification. In addition, the data includes a detailed history of payment status, whether the loan had been previously modified, as well as detailed underwriting information on the original loan, including borrower credit score, income, total monthly debt service, original loan-to-value (LTV), origination channel, property location, occupancy status, and loan purpose, among other variables. The data similarly contains details of the new loan contract, such as the new term, schedule of payments, schedule of interest rates, and scheduled principal, inclusive of any forgiven or deferred principal amounts.

The analysis is confined to fully reunderwritten loans that were underwater (that is, had a mortgage value exceeding the property value) prior to modification and received adjusted terms that lowered the present value of remaining mortgage payments. Confining analysis to this segment of loans allows us to more directly examine the implications from the option model above for how different methods of loan present value reduction matter for subsequent redefault performance. Since each loan is re-underwritten, we also know the updated property value, updated borrower income, and borrower credit score at modification. In total the sample consists of 19,837 loans observed monthly from modification until termination or censoring in August 2012, for a total of 364,891 loan-month observations. Payment performance of each loan is tracked by month from modification until redefault, which is defined as relapsing to 60 days or more past due, or until censoring. Nearly all non-defaulting loans are censored at August 2012, but a very small number of loans (250) prepay prior to the final observation date. This confirms that we need not model the competing risk of voluntary prepayment for these loans, and we treat the prepaid loans as censored at their termination date in this analysis.

Roughly 1/3 of all loans redefault within the observation time-frame, and Figure 2.5 shows redefault rates by year of modification and modification type. Several features in the Figure stand out. First, Panel A of Figure 2.5 shows redefault rates decline for each successive year of modifications. This is consistent with broader industry trends and is almost surely explained by a transition away from older traditional structures that did not significantly reduce mortgage present value to structures that more aggressively reduce present value. Panels B, C, and D show redefault performance for loans receiving a principal reduction, principal deferment, and rate reduction/term extension, respectively, compared to loans that do not receive each respective treatment. It is evident that loans receiving a principal reduction perform better than other loans, followed closely by loans receiving a principal deferment. Loans receiving a rate reduction and term extension perform only modestly better than loans that do not. Note that the "treatments" in Panels B-D are not mutually exclusive, and a given loan may receive more than one (or all) of the treatments (thereby contributing to the performance depicted in more than one panel). Interestingly, these basic performance differences appear to be broadly in line with the basic "pecking order" developed in our theoretical analysis above, and our modeling

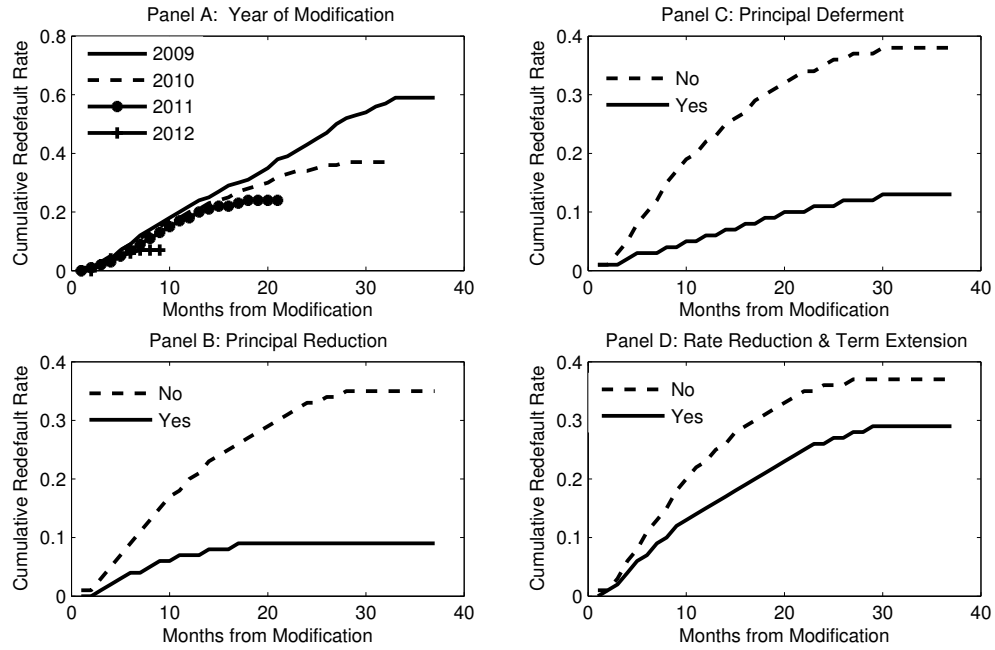


Figure 2.5: Redefault Rates for Loans in the Sample: Panel A shows cumulative redefaults by year of modification. Panel B shows cumulative redefaults for loans receiving a principal reduction versus all other loans. Panel C shows cumulative redefaults for loans receiving a principal deferment versus all other loans. Panel D shows cumulative redefaults for loans receiving a rate reduction and term extension versus all other loans. A given loan can receive more than one of these treatments and therefore appear in more than one panel.

specifications below will seek to confirm if these relationships hold up more generally as predicted by theory after controlling for the resulting impact on payments and loan present value.

The key explanatory variables are those that describe the modification structure and resulting present value of mortgage payments. To that end, we construct a series of indicator variables, "*Prin_Red*", "*Prin_Def*", "*Rate_Red*", and "*Term_Ext*", representing "treatment effects" for whether the loan received a principal reduction, principal deferment, rate reduction, or term extension, respectively. In addition, we compute a series of "dose effect" variables, "*Prin_Red%*", "*Prin_Def%*", "*Rate_Term%*", describing the percentage contribution to the total reduction in mortgage present value from principal reduction, principal deferment, and rate reduction combined with term extension, respectively.¹⁸ Since term extension alone

¹⁸These dose effect variables correspond to a total differentiation of the present value formula.

generally contributes little or nothing to present value reduction, only the combined effect with rate reduction is calculated in "*Rate_Term%*".

These modification structure variables need to be evaluated controlling for the resulting mortgage present value. For this we follow Deng, Quigley, and Van Order (2000) and compute for each loan a time-varying path for the borrower's current equity position relative to her property value, defined as

$$EQ(t) = \frac{H(t) - M(t)}{H(t)}. \quad (2.32)$$

$H(t)$ is simply the appraised value at the time of modification updated to the current period using changes in a repeat sales home price index for the MSA of the loan. $M(t)$ is the present value of remaining mortgage payments discounted by the current monthly mortgage interest rate relative to the remaining term of the mortgage¹⁹. The equity ratio compares the present value of the borrower's mortgage payments, M , to his current home value, H , in each period, measuring the (normalized) payoff to the default option should it be exercised. Modification structures can have a powerful impact on "*EQ*", and represents the main theoretical channel by which default behavior can be influenced.

In addition to the main theoretical variables above, we also include the time-varying explanatory variables for the degree of payment reduction and the current debt-to-income (DTI) level of the borrower. Specifically, "*Pmt_Red%*" represents the percentage change in the current month's payment relative to the pre-modification payment, and "*DTI*" represent the current payment as a fraction of borrower income. Both of these variables may be time-varying as the current month's payment can adjust, such with step-up rate reduction modifications. These variables are often cited as important for modification performance, or can be explicit targets for loan modification structures (such as with HAMP and DTI). Finally, we include time-varying variables for other "trigger events" using the monthly MSA-level unemployment rate, "*UR*", and the borrower's quarterly updated credit score, "*Score*". The local unemployment rate may proxy for labor market conditions and the likelihood that a borrower experiences a reduction in income post-modification. Updated credit scores can further proxy for borrower financial difficulty, but can also proxy for potential reputation costs to defaulting.

Table 2.2 provides summary information for the key explanatory variables in the data, showing mean values at the time of termination or censoring of the loan. Note that principal reduction has been relatively much less prevalent comprising just under 10% of the treatments in the sample, whereas rate reduction has been a much more extensively used treatment. Most loans remain in negative equity position after

¹⁹Specifically, we use the Freddie Mac Primary Mortgage Market Survey (PMMS) rates matching to the term closest to remaining loan term. However, we find results of the empirical analysis are extremely similar and qualitatively unchanged using alternative discount rates, such as the pre-modified loan coupon.

Variable	Mean	Standard Deviation
<i>Prin_Red</i>	0.09	0.29
<i>Prin_Def</i>	0.20	0.40
<i>Rate_Red</i>	0.95	0.21
<i>Term_Ext</i>	0.58	0.49
<i>Prin_Red%</i>	0.02	0.08
<i>Prin_Def%</i>	0.06	0.14
<i>Rate_Red%</i>	0.30	0.16
<i>EQ</i>	-0.55	1.06
<i>Pmt_Red%</i>	0.35	0.21
<i>DTI</i>	0.24	0.08
<i>UR</i>	9.35	2.40
<i>Score</i>	603	85
Observations	19,837	

Table 2.2: Descriptive Statistics on Modified Loans

modification and the distribution is somewhat skewed, ranging from a low of 100% negative equity to a high of 55% positive equity.

2.4.3 Specifications and Results

Table 2.3 provides partial maximum likelihood estimates for parameters of a basic treatment effects model without considering unobserved heterogeneity. The first four variables in the table are the main treatment indicators for structural changes to a modified loan contract that lower loan payments. All four treatment effect parameters are highly statistically significant with signs and magnitudes very much in accordance with the results from our theoretical model. Controlling for the resulting mortgage present value and loan payment reductions, the results suggest it does indeed matter how those reductions are achieved: principal reductions have the greatest impact, reducing the default hazard by over 60% all else equal; principal deferments have the second greatest impact, reducing the hazard by about 54%, followed by rate reductions at around 47%. Term extensions by themselves actually slightly increase the default hazard by around 6%. At first glance this last result may appear to contradict the pecking order of our option model results where we showed in Figures 2.3 and 2.4, for example, that a term extension combined with a rate reduction improves over a rate reduction alone.

However, recall that the reason for this outcome in the option model simulations was that combining rate reduction with term extension lowered the borrower's pay-

Variable	Coeff Estimate	Std Error	P-Value	Hazard Ratio
	β			$\exp(\beta)$
<i>Prin_Red</i>	-0.9215	0.0894	<.001	0.398
<i>Prin_Def</i>	-0.7791	0.0554	<.001	0.459
<i>Rate_Red</i>	-0.6326	0.0893	<.001	0.531
<i>Term_Ext</i>	0.0586	0.0281	0.037	1.060
<i>EQ</i>	-0.0167	0.0048	<.001	0.983
<i>Pmt_Red%</i>	-1.4980	0.0882	<.001	0.224
<i>DTI</i>	0.0106	0.0164	0.52	1.011
<i>UR</i>	0.0127	0.0049	0.01	1.013
<i>Score</i>	-0.0107	0.0002	<.001	0.989
log likelihood	-58,045			

Table 2.3: Partial Maximum Likelihood Estimates of Basic Treatment Effects without Heterogeneity

ment by a greater amount to achieve a given reduction in present value. Because we have entered payment reduction in the hazard model, one need only consider the combined effects of the treatment variables along with the payment reduction variable to check the overall consistency of the hazard estimates with the simulation results of the option model. For instance, in the simulated example of Figure 2.3 present value in all modification structures was reduced to par with property value, while payments were reduced by about 20% for principal reduction, principal deferment, and rate reduction, and by about 30% for rate reduction and term extension. Table 2.4 shows the resulting hazard ratios for the variable combinations resulting from these structures using the estimated coefficients of the regression model and holding all other factors fixed. The last row of Table 2.4 shows that the hazard model ranking of structures in this stylized example is in fact very similar to the option model results in Figure 2.3: principal reduction does best, rate reduction does worst, whereas principal deferment and rate reduction with term extension are close to each other. Repeating this analysis for the example in Figure 2.4 likewise shows an agreement in ranking, and specifically rate reduction with term extension begins to move closer in performance to pure rate reduction as the option model predicts. As a final point related to the results on treatment effects, we point out that the results here suggest that a pure term extension is counter-productive if accompanied by no reduction in loan payments, such as when term extension is used to offset higher payments from capitalizing past due interest.

Variable	Coefficient	Principal Red.	Principal Def.	Rate Red.	Rate Red. & Term Ext.
<i>Prin_Red</i>	-0.9215	1	0	0	0
<i>Prin_Def</i>	-0.7791	0	1	0	0
<i>Rate_Red</i>	-0.6326	0	0	1	1
<i>Term_Ext</i>	0.0586	0	0	0	1
<i>Pmt_Red%</i>	-1.4980	0.2	0.2	0.2	0.3
$\sum z_i \beta_i$		-1.2211	-1.0787	-0.9322	-1.0084
$\exp(\sum z_i \beta_i)$		0.295	0.34	0.394	0.359

Table 2.4: Relative Hazards Applied to Numerical Example of the Option Model

Turning to the other variables of the model, we see that as with prior studies of mortgage default borrower equity remains a predictive variable in redefault on modified mortgages. Though we discussed the estimated results for payment reduction above, we note that the results here confirm other findings on modified loan performance that reducing payments is a strong contributor to lowering redefaults (for example, see Goodman, Ashworth, Landy, and Yang (2012) and Quercia, Ding, and Ratcliffe (2009)). Increases in DTI raise the default hazard as expected. Interestingly, however, this effect is not significantly different from zero, suggesting once controlling for payment reduction and present value, the level of DTI is not important. This may have important implications for programs that explicitly target a DTI level. Finally, both "trigger event" variables are meaningful and significant in explaining conditional redefault rates. Each percentage point increase in the local unemployment rate increases the hazard by 1.3%, and each additional point in the current credit score (higher scores resulting from better credit history profiles) lowers the hazard 1.1%.

The model results in Table 2.3 are an interesting starting place for analysis, but a possible concern with the model involves the potential endogeneity of the treatment effects. The treatments are not randomly assigned, but rather chosen in some combination by the servicer. Endogeneity of treatments can potentially impart bias into the model estimates. We address this issue by estimating the model in a 2-stage "control-function" approach, along the lines suggested by Blundell and Powell (2003,2004). Specifically, in a first stage we model the probability that a borrower will receive a given treatment, specifying this probability to depend on a list of variables potentially important to the servicer's treatment selection but not directly relevant for post-modification performance. We then estimate the treatment effects model as above in a second stage, adding the predicted probabilities in the first stage as control

variates in the regression²⁰.

For the first stage of modeling we run for each treatment separately a logistic regression for the probability that a borrower will receive that treatment, using the same set of explanatory variables in each regression. These explanatory variables include an extensive, and arguably an exhaustive, set of factors a servicer could use in determining which treatments to include in the modification, including: product and vintage characteristics, such as the original loan type (e.g. whether a fixed or adjustable rate), and the year of origination of the original loan and remaining term; original LTV and LTV trend leading up to modification; whether and how a borrower's income and assets were documented on the original loan, and what any verified or stated income amounts were on the original loan; the borrower's current income; the occupancy status of the property (e.g. whether the property is the principal residence of the borrower); the original and current loan size (e.g. whether the loan was or remains a jumbo loan above conforming limits); the loan purpose (e.g. whether to purchase the home or refinance it); the borrower's original credit score and score trend leading up to modification; the channel by which the loan was originated (e.g. whether "retail" or "wholesale" originated), the loan interest rate and difference relative to current market rates; and whether the loan has been modified before, and number of times if so. Additional factors include the state in which the property is located (which can speak to how easily foreclosure can be pursued as an alternative to modification), and the quarter that modification takes place (which can capture the possibility that different treatments may be emphasized differently at different points in time).

Table 2.5 shows results of the 2-stage maximum likelihood estimates for the treatment effects model with the endogeneity control variates. The results are very similar to the prior model, with the ordering of treatment effects remaining intact. In fact, the separation between treatments as indicated by the relative magnitudes of their respective coefficients and resulting hazard ratios are greater after controlling for potential endogeneity. For principal reductions and deferments, for example, the estimated reduction in hazard is greater in the 2-stage results than in the basic model. Evidently, endogeneity bias works in the direction of muting the estimated impact of principal reduction and deferment, and we can see from the estimated coefficients of their respective control variates, "*Pr_Prin_Red*" and "*Pr_Prin_Def*", that indeed

²⁰This method is also sometimes referred to as a propensity score method in the biostatistics literature. Therneau and Grambsch (2010), for example, discuss how propensity score corrections can also include using scores to weight regression, propensity score matching, and stratified regression. In the econometrics literature, the predicted probabilities only (and not the actual treatment values) are typically entered in the regression, as with classical IV estimation. For non-linear models, however, this classic IV estimation does not retain its linear model properties. The control function approach typically calls for entering in addition to treatment variables, the residuals from a prior stage IV estimation. For the basic treatment effects model here, we have tried both of these other forms of correction—predicted values and residuals—and find the qualitative results unaffected by the choice.

Variable	Coeff Estimate	Std Error	P-Value	Hazard Ratio
	β			$\exp(\beta)$
<i>Prin_Red</i>	-1.0371	0.0964	<.01	0.354
<i>Prin_Def</i>	-0.8088	0.0581	<.01	0.445
<i>Rate_Red</i>	-0.5419	0.0925	<.01	0.582
<i>Term_Ext</i>	0.0497	0.0309	0.11	1.051
<i>EQ</i>	-0.0167	0.0048	<.01	0.983
<i>Pmt_Red%</i>	-1.6092	0.0972	<.01	0.200
<i>DTI</i>	0.0073	0.0183	0.69	1.007
<i>UR</i>	0.0139	0.0051	0.01	1.014
<i>Score</i>	-0.0108	0.0002	<.01	0.989
<i>Pr_Prin_Red</i>	0.4130	0.1867	0.03	1.511
<i>Pr_Prin_Def</i>	0.1470	0.0864	0.09	1.158
<i>Pr_Rate_Red</i>	-0.4003	0.3161	0.21	0.670
<i>Pr_Term_Ext</i>	0.0504	0.0657	0.44	1.052
log likelihood	-57,783			

Table 2.5: 2-Stage Partial Maximum Likelihood Estimates of Treatment Effects without Heterogeneity

loans more likely to receive a principal reduction or deferment have higher expected redefault hazards, all else equal. This is consistent with servicers reserving these more potent treatments for the harder cases.

Table 2.6 reports 2-stage estimates of the treatment effects model also allowing for individual-specific gamma distributed unobserved heterogeneity. The results are qualitatively very similar to those in Table 2.5, although the estimated impacts for the treatments, borrower equity, and payment reduction are now somewhat greater (consistent with the attenuation toward zero of coefficients in the presence of unobserved heterogeneity). A test for the variance of the random effect can be obtained by comparison of the likelihoods of the models with and without ω , producing a chi-square statistic of 172 with 1 degree of freedom and a p-value close to 0.

An alternative view of the importance of different modification structures can be obtained by examining the effect of measured treatment dose, where as described above treatment dose corresponds to a decomposition of change in present value from principal reduction, principal deferment, and rate reduction with term extension. Since treatment dose is a continuous variable, the control-function approach calls for

Variable	Coeff Estimate	Std Error	P-Value	Hazard Ratio
	β			$\exp(\beta)$
<i>Prin_Red</i>	-1.2039	0.1003	<0.01	0.300
<i>Prin_Def</i>	-0.9494	0.0590	<0.01	0.387
<i>Rate_Red</i>	-0.6948	0.1016	<0.01	0.499
<i>Term_Ext</i>	0.0631	0.0327	0.12	1.065
<i>Eq</i>	-0.0503	0.0144	0.01	0.951
<i>Pmt_Red%</i>	-2.1732	0.1030	<0.01	0.114
<i>DTI</i>	-0.0021	0.0179	0.93	0.998
<i>UR</i>	0.0130	0.0054	0.05	1.013
<i>Score</i>	-0.0134	0.0002	<0.01	0.987
<i>Pr_Prin_Red</i>	0.6966	0.1932	<0.01	2.007
<i>Pr_Prin_Def</i>	0.1962	0.0920	0.08	1.217
<i>Pr_Rate_Red</i>	0.0241	0.3323	0.95	1.024
<i>Pr_Term_Ext</i>	0.1266	0.0697	0.14	1.135
ω	1.0493		<0.01	
log likelihood	-57,701			

Table 2.6: 2-Stage Partial Maximum Likelihood Estimates of Treatment Effects with Gamma Heterogeneity

estimating a first-stage regression for the continuous endogenous regressors on a set of exogenous covariates and instruments, and entering the residuals from this regression into the second-stage estimation procedure. Table 2.7 shows results of estimates for treatment dose where we enter first-stage residuals from linear regressions of dose on the same right-hand side variables considered in our logistic modeling of treatment probability. For this specification, however, we remove the payment reduction variable since it represents a quantity very similar to the sum of our dose effect variables, the later effectively representing a decomposition of the payment and present value reduction²¹. The results indicate the same rank ordering for the modification structures as in our prior specifications. To get a sense for the magnitude of estimated effects, consider a loan receiving a 30% reduction in present value through principal reduction. The default hazard for this loan is estimated to be proportional by $\exp(-4.37 * 0.3)$, or roughly 27% of the hazard of the same loan not receiving any principal reduction. This compares to 42% and 69% proportional hazard reductions for deferment and rate reduction/term extension, respectively. The effects of borrower equity, DTI, unemployment, and credit score are little changed from the treatment effects specifications.

Lastly, Table 2.8 gives results for the dose effect model allowing for individual gamma random effects. A likelihood ratio test for ω confirms the significance of the random effects. Moreover, similar to the treatment effect heterogeneity model, the impact of including a random effect in the dose effect specification is to increase the relative importance of principal reduction and borrower equity. However, most other variable impacts are little changed. More importantly, the basic results of all specifications examined here confirm the order of priority predicted by our option model for various loan modification structural changes.

2.5 Conclusion

This chapter has developed a simple and straightforward extension of an option model of default to show that a dynamically optimizing rational borrower facing a modified schedule of mortgage cash flows will behave very differently depending on the contractual changes that determine those cash flows. In particular, a rational borrower may care about the tax effects of how cash flows are restructured, and whether or how the loan can be resolved if future events beyond their control force them to terminate the loan. Principal reduction modifications perform relatively better with respect to redefault rates under these circumstances. Since our analysis

²¹If rates in the modification are constant, for example, the payment reduction and percent change in present value would be identical values. For some loans, however, such as those with a step-up rate modification, the initial payment reduction can be more than the present value reduction. Even in these cases, however, the variable magnitudes are extremely similar loan by loan.

Variable	Coeff Estimate β	Std Error	P-Value	Hazard Ratio $\exp(\beta)$
<i>Prin_Red%</i>	-4.3774	0.468115	<.01	0.0126
<i>Prin_Def%</i>	-2.9030	0.184904	<.01	0.0549
<i>Rate_Red%</i>	-1.2304	0.094044	<.01	0.2922
<i>EQ</i>	-0.0173	0.004805	<.01	0.9828
<i>Pmt_Red%</i>	-	-	-	-
<i>DTI</i>	0.0153	0.013543	0.26	1.0154
<i>UR</i>	0.0133	0.005003	<0.01	1.0134
<i>Score</i>	-0.0110	0.000181	<0.01	0.9891
<i>Resid_Prin_Red%</i>	-0.3054	0.213447	0.15	0.7369
<i>Resid_Prin_Def%</i>	-0.2570	0.072748	<0.01	0.7734
<i>Resid_Rate_Red%</i>	-1.1521	0.329995	<0.01	0.3160
log likelihood	-58,003			

Table 2.7: 2-Stage Partial Maximum Likelihood Estimates of Dose Effects without Heterogeneity

Variable	Coeff Estimate	Std Error	P-Value	Hazard Ratio
	β			$\exp(\beta)$
<i>Prin_Red%</i>	-4.9028	0.512084	<0.01	0.00743
<i>Prin_Def%</i>	-3.2564	0.204876	<0.01	0.03853
<i>Rate_Red%</i>	-1.5953	0.116999	<0.01	0.20284
<i>Eq</i>	-0.0520	0.018005	<0.01	0.94931
<i>Pmt_Red%</i>	-	-	-	-
<i>DTI</i>	0.0151	0.022473	0.5	1.01522
<i>UR</i>	0.0125	0.006312	0.047	1.01259
<i>Score</i>	-0.0133	0.000217	<0.01	0.98682
<i>Resid_Prin_Red%</i>	-0.2185	0.250521	0.38	0.80376
<i>Resid_Prin_Def%</i>	-0.3668	0.089240	<0.01	0.69292
<i>Resid_Rate_Red%</i>	-0.9488	0.401912	0.018	0.38721
ω	0.868		< .01	
log likelihood	-57,940			

Table 2.8: 2-Stage Partial Maximum Likelihood Estimates of Dose Effects with Gamma Heterogeneity

controls for the present value of the scheduled cash flows, it follows that relative expected cash flows should be strongly influenced by the differential redefault rates of various modification structures.

We provide empirical evidence that corroborates the predictions of the option model for how different structures perform relative to one another. The hazard regressions presented here provide evidence that, controlling for other key factors that may impinge on redefaults, principal reduction modifications do perform better than other structures. Likewise, rate reductions, deferments, and extensions perform relative to one another as predicted by the option model. The empirical results also confirm the findings of other studies that the degree of payment reduction is an important determinant of modification success, and our results also show that default option variables like borrower equity found to be significant in the general mortgage default literature remain significant in the context of redefault of modified mortgages.

While the results of this chapter provide compelling and credible evidence for how various modification structures should be expected to perform relative to one another, there are several potential areas for further extending the research. First, the option model and empirical models of this chapter can potentially be integrated. An ongoing area of econometric research involves integrating dynamic programming optimization problems into econometric estimation. The technique involves iterative solutions for the structural dynamic optimization problem for a given value of parameters, and entering the solution into a maximum likelihood estimator, repeating until convergence to a maximum. In this way the parameters of the structural model are estimated empirically and in a way fully consistent with the structural solutions of the dynamic optimization model. Rust (1987) is a prominent example of the approach, and Eckstien and Wolpin (1989) provide an early survey of the methods. However, this approach remains extremely computational difficult to implement, often accompanied by difficult identification issues, making applications to date very limited and often relying on strong simplifying assumptions for tractability. Consequently, research in this area has focused on developing more tractable techniques.

Secondly, and more broadly, we do not address in this study issues of bargaining behavior that may explain a great deal more about how loan modification programs are actually selected and implemented in practice. In particular, game-theoretic considerations to modification activity could go a long way to explaining the relative prevalence of different modification forms, and yield important insights into the policy desirability or optimality of emphasizing different modification structures generally. Since our focus here has been on the narrower questions of expected performance given modification, we do not solve the broader policy debate surrounding loan modifications. We leave these issues for future research.

Chapter 3

Competing Risks in Non-U.S. Residential Mortgages

3.1 Introduction

A large empirical literature examines prepayment and default behavior of mortgage holders in the context of the U.S. mortgage market (see references and discussion of Chapter 2 for example). This literature has both increased our understanding of the determinants of prepayment and default, and our understanding of how closely observed behavior accords to option theoretic models. For instance, in their extensive study of fixed rate mortgages in the U.S., Deng, Quigley, and Van Order (2000) document that:

1. Option-related incentives do a good job of explaining mortgage termination behavior. In particular, variables that proxy for the degree to which a borrower is "in-the-money" relative to exercise of a borrower's prepayment call option appear to explain prepayment behavior, and variables representing a borrower's payoff on a default put option appear to explain default behavior.
2. The simultaneity of option value emphasized in theory, whereby high value in one option may impinge on optimal exercise behavior of the other option, appear born out by the data. In particular, factors important for one option are predictably also important for the other option.
3. Non-option variables, such as those that proxy for adverse life events, are also important in explaining behavior. Therefore, the basic option models by themselves provide an incomplete picture of mortgage terminations.
4. Unobserved borrower heterogeneity appears to be an important factor in understanding mortgage termination behavior. In particular, borrower's may vary

in unobserved ways in their astuteness to option exercise or their vulnerability to adverse life events.

However, much less is understood about mortgage termination behavior of households outside of the U.S., especially of mortgage terminations in markets of emerging or more newly developed economies. This study partly fills that gap by examining mortgage termination behavior in the Singapore market. We utilize a unique set of data spanning over a decade of mortgage performance that encompass periods of dramatic fluctuations in the drivers of mortgage option value, such as home prices and interest rates.

There are a number of reasons why studying mortgage terminations outside of the U.S. context, and for Singapore in particular, may provide valuable insights. First, for relatively emerging mortgage markets understanding the degree to which factors such as (1)-(4) above hold true, and how mortgages perform more generally, is important to potentially developing these markets further. Because real estate is typically among the largest asset classes of any economy, achieving access to a broad-base of efficient financing can be very important, and doing so requires a well-developed understanding of the risks and returns in financing mortgages. Secondly, emerging markets often have unique institutional features that can provide valuable comparisons for better understanding how the U.S. market would behave under alternative institutional structures. This too increases our understanding of mortgage behavior more generally.

In the case of Singapore specifically, several unique institutional features of that market make it a particularly interesting case study for examining mortgage optionality relative to the U.S. On the prepayment side, for example, mortgages in Singapore are nearly universally adjustable rate mortgages (ARMs) with rates resetting whenever the underlying benchmark rate changes. Since a mortgage is in the money for prepayment when the present value of remaining mortgage payments at current market rates exceeds the face value of the debt, synchronous rate adjustments for ARMs should mean they will seldom be in the money. However, in Singapore, and especially in the periods covered in our data (to be further described below), the benchmark rate for ARMs are not market rates derived from public securities markets. Rather, benchmark rates are typically a bank's own prime rate adjusted at the bank's discretion. Such adjustments are, of course, typically in response to changes in other market rates—for example, in response to a change in the bank's cost of funds—and so are not completely disconnected from market rates.¹ However, the possibility of variation in the speed of rate adjustments or rate changes that are non-synchronous with broad market rate trends introduces the potential for significant prepayment optionality, similar to the various rate index products in U.S. ARM mortgages (see, for example, Stanton and Wallace (1999) for discussion of how rate index and other

¹In addition, changes to bank's prime rate may subject to regulatory review and constraint.

ARM features impact ARM prepayment optionality). The degree to which Singapore mortgages retain their prepayment optionality under these market conditions is therefore an empirical question.

Other factors may also contribute to differences in prepayments for Singapore mortgages. For example, fixed rate mortgages (FRMs) have been relatively uncommon in Singapore. Thus, incentives to switch product types—as when a U.S. household refinances out of an ARM to a FRM in response to interest rate risk or yield curve movements—should not be a factor for prepayments in this market. Additionally, many borrowers in the U.S. refinance their mortgages at market rates even above their existing contract rate in order to access equity from their homes (cash-out refinancing), resulting in termination of their existing mortgage. By contrast, in many cases a Singapore borrower will be able to extract funds from their existing mortgage without a need to refinance into a new loan simply by requesting a new disbursement from their lender.² Of course, exogenous prepayments may still occur as a result of borrower's moving or as a result of trading-up or trading-down to match changes in household size, income, or other determinants of housing demand. All of these differences have the potential to impart very different prepayment behaviors for mortgage loans in this market relative to the U.S.

On the default side, there are equally important differences in this market. For example, mortgages in Singapore are full-recourse loans, where the borrower remains fully liable for any deficiencies in the amounts recovered by the lender in the sale of the foreclosed property. U.S. borrowers are technically subject to deficiency judgments in some states and circumstances, but lenders rarely seek judgments in practice (Ghent and Kudlyak, 2009). To the extent legal rules and institutions in Singapore make such lender recourse more easily obtained it may be difficult for a borrower to find himself in the money vis-à-vis a default option. There would be little point to default if doing so merely entailed substituting one form of liability (a mortgage) for another, possibly a much more expensive form of liability (asset levies, garnished wages, etc.). Some economists have advocated making modified mortgages in the U.S. full recourse in exchange for principal reduction modifications, in essence having lenders buy back the default option from the borrower.³ Strong recourse on a mortgage loan should render the option value of default moot, and we should expect to see little correlation between variables measuring option payoff and default decisions of borrowers. Further understanding the effects of recourse, therefore, is one area that could have implications for our understanding of the efficacy of alternative policies in and outside of the U.S.

A second unique aspect of mortgage lending in Singapore relevant to credit risk is that mortgage borrowers often use retirement funds to finance down payments on

²This practice, often called a "top-up" can be executed by adjustment to the current mortgage terms, or by creating a new linked mortgage. By contrast, in the U.S. the prior mortgage will nearly always be terminated.

³See Stiglitz (2010), page 107, for an example of this argument.

their mortgages. U.S. tax code similarly allows households to borrow (within limits) from their 401k or IRA accounts for purchasing a house, or take limited distributions without penalty for first time purchases. In Singapore, however, the use of retirement funds has had a more complicated history that makes it a particularly interesting case study. Mandatory retirement contributions are made by employees and employers to Singapore's Central Provident Fund (CPF), and since 1970 the CPF has allowed designated contributions to be used toward home purchase. CPF funds applied to a down payment or to make mortgage payments has enjoyed at different times a varying degree of priority to the proceeds of property sales, whether a forced sale or a voluntary sale. For a home purchased prior to September 2002, for instance, the CPF balances applied to purchase had absolute priority over any mortgage balances owed in the event of sale (forced or otherwise). In a sense, the borrower's equity portion coming from her use of retirement funds was protected from claims upon termination. Where retirement funds have priority to sales proceeds, the mortgage is effectively collateralized by the residual value of the property, that is, the difference between the property value and the retirement funds. This means that in considering the option value of default the borrower compares the value of his mortgage to this residual value of the property, which may be highly leveraged to property value declines for borrowers whose principal source of down payment is their CPF funds. The degree to which equity protection matters to how borrowers choose to exercise their default options is likewise an empirical question.

We find evidence that mortgage borrowers in Singapore are in fact responsive to incentives to exercise their prepayment and default options, and document that several aspects of items (1)-(4) found important for U.S. mortgages similarly hold true for Singapore mortgages. We further explore in this study the effects of equity protection on mortgage defaults, and do not find strong evidence that this is a particularly important determinant of default behavior. This suggests that default options are not at least "ruthlessly" exercised. This study is the first of which we are aware to examine the joint competing risks of prepayment and default in the Singapore market, and to document that similar factors determine terminations in this market as in other more developed markets such as the U.S.

The rest of the chapter proceeds as follows. Section 3.2 discusses further background of the Singapore market and provides a brief review of studies examining mortgage terminations in Singapore. Section 3.3 introduces the empirical model used in the study, and Section 3.4 describes the data and empirical results. Section 3.5 concludes the chapter.

3.2 The Singapore Market and Related Literature

The home ownership rate in Singapore is quite high, standing at 88.6% at the end of 2011.⁴ The housing market is segmented into public and private housing sectors, and public policies toward these segments have strongly promoted home ownership. The Housing Development Board (HDB) of Singapore is one of the most important players in developing housing policies for the country (Ong, 2008). The HDB develops and manages public housing, as well as finances the purchase of public housing at subsidized rates. Since public housing represents the majority of Singapore housing, HDB financing is a large share of the overall mortgage market and large factor in the high rate of home ownership. However, private housing has been increasing significantly in share in recent years, and Ong (2008) reports that this share is expected to grow to 30% of the overall housing market. The private segment is predominately financed through traditional banking sector mortgages with no government subsidy.

However, as noted above, the purchase of private and public housing can be facilitated by using retirement savings. Contributions to an individual's CPF fund is mandatory for both employees and their employers, and though contribution rates generally vary by age and income they are currently typically about 36% of an individual's income.⁵ Within limits CPF savings can be used as a down payment for a mortgage loan or to service the debt on a mortgage loan. Figure 3.1 shows that the use of these funds for housing has been growing steadily, and represent a major source of funding for Singapore housing. At the end of 2011, for example, about 150 billion Singapore dollars (roughly \$US 117 billion) of CPF funds had been committed to finance housing.

Upon sale of the property prior to reaching retirement age, however, these CPF funds must be repaid from the proceeds with accumulated interest (currently at 2.5% per annum). Prior to a change in the law in late 2002, CPF funds had priority over mortgages in the sale proceeds. This was true for either a voluntary sale or a forced sale resulting from foreclosure.⁶ Transactions using CPF funds for home purchase after September 2002 do not give CPF funds priority in sale, but rather give the purchase mortgage first priority. Furthermore, since September 2002 a mortgage holder who originally purchased prior to September 2002 can request their CPF funds be subordinated to their purchase mortgage, which some borrowers will do to secure a rate reduction from their lender. However, any subsequent or additional mortgages originated beyond the original purchase mortgage—for example to take cash out or

⁴See housing and other statistics at www.singstat.gov.sg

⁵See <http://mycpf.cpf.gov.sg> for further details on the CPF contribution rates and other aspects.

⁶It is important to note that in Singapore a lender does not take title to real property as a result of foreclosure, but rather can force a sale through a legal process with claim to proceeds of the sale. Any excess proceeds after satisfying the mortgage debt or other priority claims would be returned to the borrower.

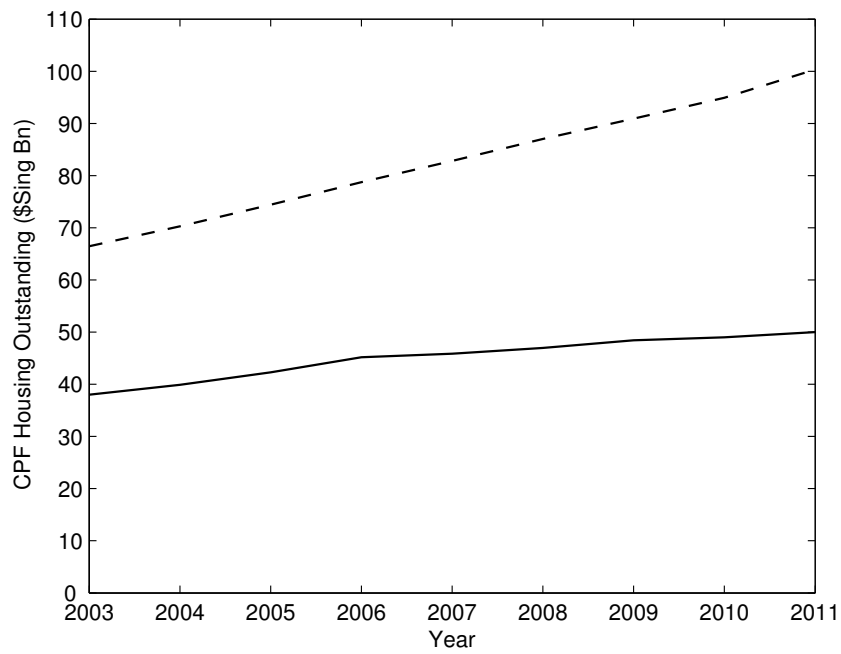


Figure 3.1: CPF Savings Applied to Finance Housing by Year: solid line represents private housing and dashed line public housing. Source: Singapore Central Provident Fund (available at <http://mycpf.cpf.gov.sg/CPF/About-Us/CPF-Stats>)

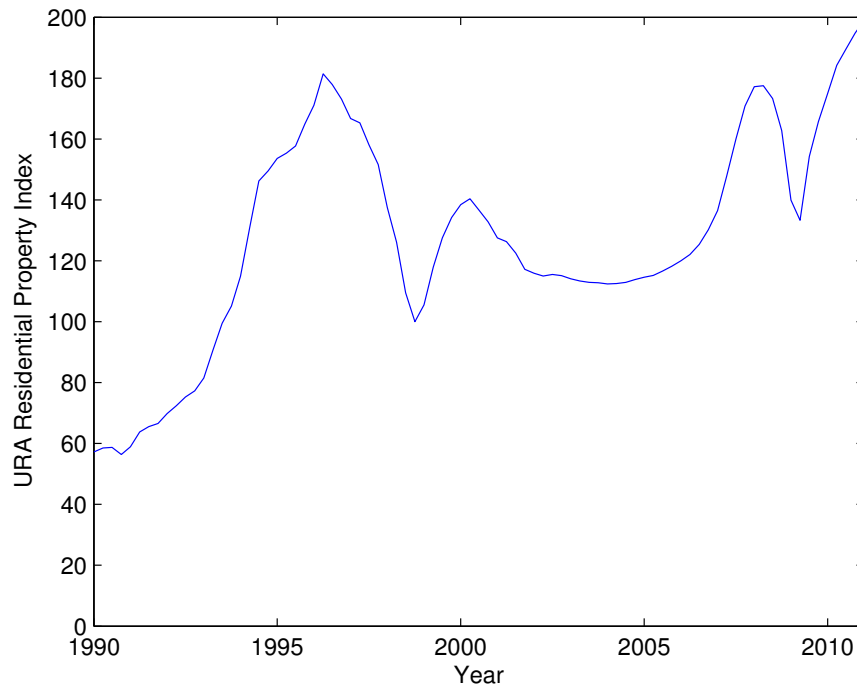


Figure 3.2: Home Price Index for Residential Housing in Singapore. Source: Urban Redevelopment Authority of Singapore.

to refinance a purchase mortgage—will be subordinate to any CPF funds used to buy the property unless the borrower agrees to subordinate the CPF funds. This aspect of mortgage finance, that is protected borrower equity, adds a wrinkle to option-theoretic views of mortgage default that we explore below.

The Singapore property market has undergone periods of rapid price growth and decline in recent decades. Figure 3.2 shows the home price index published by the Urban Redevelopment Authority (URA) for Singapore residential properties covering 1990 to 2011. From 1990 to the peak in mid-1996 home prices more than tripled. Subsequently, until the end of 1998, prices declined by 45%, mainly as a result of anti-speculation measures imposed by the government and the Asian financial crisis (see Edelstien and Laum (2004) and Quigley (2001), for example). Since 2000, prices have fallen further before rising sharply again, with an intermittent large dip coinciding with the onset of the global financial crisis in 2008. These booms and declines rival the experience of the U.S. housing market in recent years, and will provide ample conditions to examine the effects of growth and declines in borrower equity, and even deeply negative equity, in our empirical work below.

Mortgage rates have also fluctuated significantly over these periods. Figure 3.3

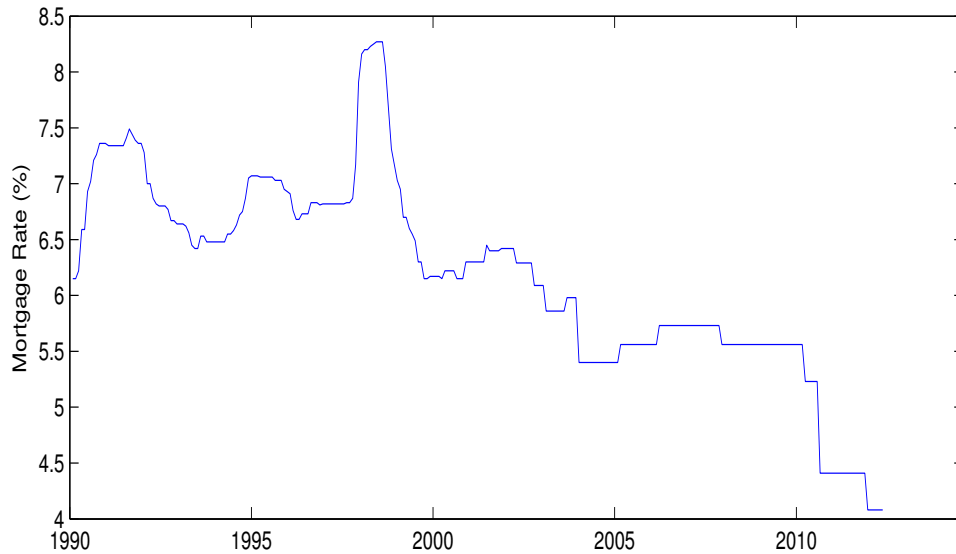


Figure 3.3: Singapore Mortgage Rates 1990-2012. Source: Monetary Authority of Singapore benchmark survey of largest 10 banks and finance companies.

shows benchmark rates underwent periods of high short-term volatility, though they have trended downward over the last couple of decades.

A relatively small literature has examined mortgage termination behavior in Singapore. Ong (2000) studies prepayments by looking at property record transaction data as opposed to direct observation of mortgage behavior. Ong, Thang, and Maxim (2002) and Lee and Ong (2005) study prepayments on mortgages with an emphasis on explanatory factors related to household mobility. Ong, Thang, and Maxim (2002) also relate prepayments to other asset market returns. Teo (2004) and Ong, Sing, and Teo (2007) study delinquency risk as a proxy for default risk. Interestingly, these studies do not find a strong relation between delinquency and classic predictors of credit risk in U.S. mortgages, such as loan-to-value ratio (LTV) and debt-to-income rate (DTI). In addition, these studies do not find evidence that CPF fund protection contribute in any way to delinquency rates on mortgages that utilize them. The current study is the most comprehensive of Singapore mortgage terminations—and non-U.S. mortgages more generally—of which we are aware of to date. It utilizes a deep historical data set on loan-level performance, it is the first study of which we are aware to examine default terminations directly for Singapore mortgages, and the first to consider prepayments and defaults for this market in a joint, competing risk framework.

3.3 The Model

The mixed proportional hazard model for grouped duration data employed by Deng, Quigley, and Van Order (2000) provides the framework for our empirical analysis to follow. This framework is similar to the Cox framework utilized in Chapter 2, but extends it in several important dimensions. First, it considers the simultaneous competing risks of prepayment and default, allowing for these risks to be correlated. Consequently, parameters for prepayment and default hazard functions are likewise estimated jointly and simultaneously rather than independently as in standard Cox frameworks. Second, the model allows for unobserved borrower heterogeneity to effect both prepayment and default behavior in a flexible manner. Random effect parameters, for example, can have mutually reinforcing effects on the competing risks, increasing or lowering both hazards, or opposite and complimentary effects, increasing one while lowering the other. Lastly, the framework does not rely on a partial likelihood estimation but uses a full maximum likelihood estimation procedure. The model is set in discrete time, grouping durations into monthly intervals corresponding to the data. We adapt the model to test the degree to which prepayment and default related option variables explain prepayment and default behavior in Singapore mortgages, as well as examining additional "trigger event" variables.

Let T_p and T_d represent discrete random variables for the duration of the mortgage until prepayment or default, respectively. The (latent) joint survivor function in this case represents the joint probability of T_p exceeding t_p and T_d exceeding t_d , so that $S(t_p, t_d) = 1 - F(t_p, t_d)$, for some joint cumulative distribution function F . In practice, we will of course only observe the sooner of these termination times, $t = \min(t_p, t_d)$, but the distinction between events remains relevant. Conditioning on both a set of observable factors and random effects, the joint survivor function can be written in grouped duration form as:

$$S(t_p, t_d | r, H, Z, X, \zeta_p, \zeta_d) = \exp \left\{ -\zeta_p \sum_{t=1}^{t_p} \exp(\alpha_{pt} + \phi_{pt}(r, H, Z) + \beta'_p X) \right. \\ \left. - \zeta_d \sum_{t=1}^{t_p} \exp(\alpha_{dt} + \phi_{dt}(r, H, Z) + \beta'_d X) \right\}. \quad (3.1)$$

In this formulation the terms $\phi_{wt}(\cdot)$, $w = p, d$, are time-varying functions representing option related variables depending on interest rates, r , home prices, H , and other option-related variables, Z , all of which will generally be time-varying in value; X is a vector of possibly time-varying borrower-specific covariates that may include "trigger events" and other non-option related variables; and α_{wt} represents the period's integrated base-line hazard rate:

$$\alpha_{wt} = \log \int_{t-1}^t h_{ow}(u) du, \quad w = p, d. \quad (3.2)$$

These baseline hazards are estimated non-parametrically in a full maximum likelihood estimation procedure together with the other parameters of the model, although they are not generally parameters of interest in testing option theories with the data.

The ζ terms represent random effects that are assumed to be discretely and jointly distributed along M mass points, such that the support of the random vector $\zeta = (\zeta_p, \zeta_d)$ consists of pairs (ζ_{ip}, ζ_{id}) occurring with probabilities π_i , $i = 1, \dots, M$. These heterogeneity terms can be motivated by assuming there are M discrete groups of people sharing common unobserved characteristics. Clearly, as M becomes large, any number of continuous mixing distributions can be closely approximated. The parameters of the mixing distribution can be estimated along with other parameters of the model in the likelihood function.

McCall (1996) shows how to derive the likelihood function for this model. Let $\theta_w(t|\zeta)$, $w = p, d, c$, represent the probability of loan termination or censoring in period t through prepayment, default, or end of data collection, respectively. These probabilities can be written in terms of the survivor function (suppressing for the moment all of the conditioning arguments in (3.1)) as:

$$\theta_p(t|\zeta) = S(t, t) - S(t + 1, t) - A(t), \quad (3.3)$$

$$\theta_d(t|\zeta) = S(t, t) - S(t, t + 1) - A(t), \quad (3.4)$$

and

$$\theta_c(t|\zeta) = S(t, t), \quad (3.5)$$

where $A(t)$ is an adjustment term to account for the fact that durations are measured in discrete time, and given as

$$A(t) = \frac{1}{2} (S(t, t) + S(t + 1, t + 1) - S(t, t + 1) - S(t + 1, t)). \quad (3.6)$$

Integrating over the mixing distribution yields the unconditional probability of termination by means w in period t :

$$\theta_w(t) = \sum_{i=1}^M \pi_i \theta_w(t|\zeta_i). \quad (3.7)$$

Finally, the log-likelihood is given as

$$\log \mathcal{L} = \sum_{j=1}^n \sum_w d_{wj} \log(\theta_w(t_j)), \quad (3.8)$$

where, for $w = p, d, c$, the d_{wj} are indicators for individual j taking the value 1 if termination is by prepayment, default, or censoring, respectively, and zero otherwise.

3.4 Empirical Analysis

3.4.1 Data

Our data comes from a commercial bank that lends to finance or refinance either leasehold or freehold private properties in Singapore. The data consists of monthly observations on a sample of 3,850 loans originated from 1995 through 2000, and tracked until the date of prepayment or default on the loan or the censoring date of January 2012. Observed defaults over the period number 173 loans, or about 4.5% of the sample. This is a relatively high rate of default, especially given that most loans were originated with relatively high down payments and low loan-to-value (LTV) ratios. However, as Figure 3.2 makes clear, all of the loans experienced significant erosions in property values over the observation period, generating significantly more observations on loans with high current LTVs and low borrower equity than would otherwise be the case. Observed prepayments over the period include 3,113 loans, and the remaining loans are censored. Panels A and B of Figure 3.4 show the prepayment and default rates for loans in the sample by vintage. Interestingly, the vintages 1995 and 1996, originated near the peak of the housing market, experienced the greatest default rates. This is similar to recent experience in the U.S. for the housing correction beginning in 2006.

For each loan, the data includes information at the point of origination, including loan amount, property value, CPF funds utilized toward purchase, borrower's total debt-to-income ratio (inclusive of non-mortgage debt), borrower income, loan purpose, property occupancy, property type, and all key loan contract details, such as rate, payment, and term. For each monthly observation, the data includes current rate and payment, current balance, delinquency status, total CPF funds that have been applied to the loan though additional contributions to service the debt or accrued interest to these funds, as well as prepayment or default status.

The main variables of interest in the model are the option related variables:

$$\phi_{wt}(r, H, Z), \quad w = p, d. \quad (3.9)$$

Following Deng, Quigley, and Van Order (2000), we specify the prepayment call option variables and default put option variables in (3.9) as below.

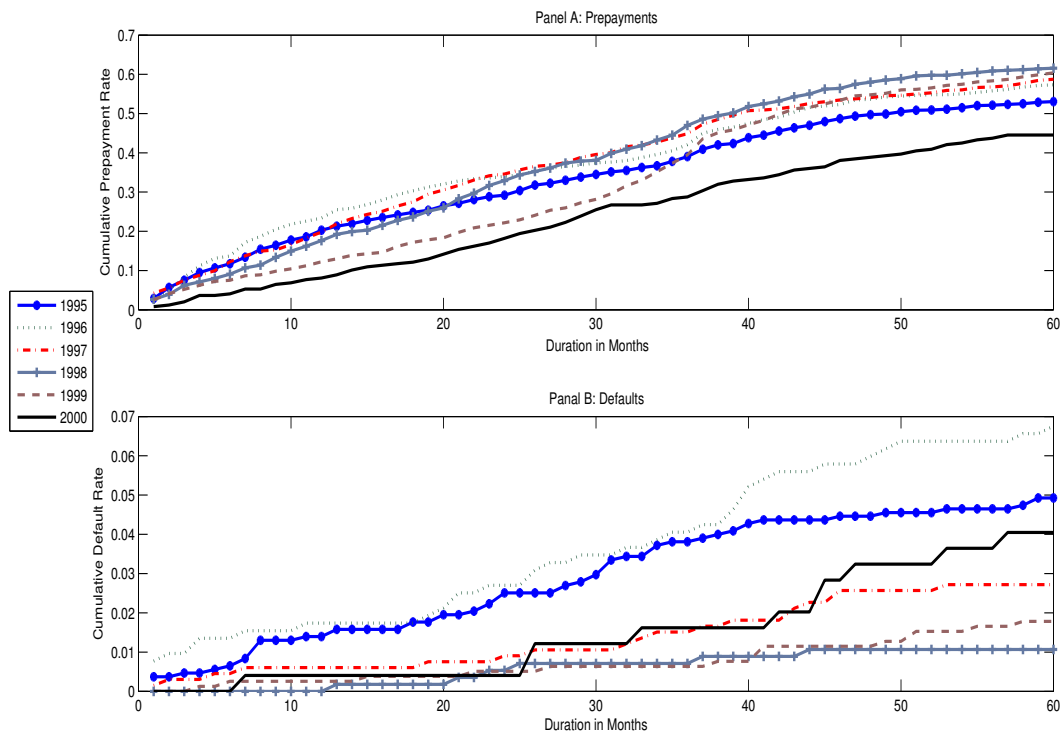


Figure 3.4: Summary Performance on Sample Loans by Vintage : Panel A shows the cumulative rate of prepayment and Panel B the cumulative rate of default.

The prepayment call option variable is represented by the mortgage premium value, giving the difference between the present values of the mortgage payment stream remaining over k periods taken at the the current market rate and the mortgage coupon:

$$\begin{aligned} Call_Option(t) &= \frac{\sum_{\tau=1}^k \frac{P(t)}{(1+r_t)^\tau} - \sum_{\tau=1}^k \frac{P(t)}{(1+c_t)^\tau}}{\sum_{\tau=1}^k \frac{P(t)}{(1+r_t)^\tau}} \\ &= \frac{M(t) - \bar{M}(t)}{M(t)}, \end{aligned} \quad (3.10)$$

where r_t is the market mortgage rate at time t and c_t the mortgage coupon at time t .⁷ This expresses how "in-the-money" the prepayment call option is in each period.

For each loan period we estimate the current home value by applying from loan origination until the observation month the cumulative change in the URA index by property type to the value of the home at origination:

$$H(t) = H(0) \left[\frac{URA(t)}{URA(0)} \right]. \quad (3.11)$$

In deriving the home price index, individual house prices are assumed to follow a non-stationary lognormal diffusion process (see Case and Shiller (1989)). Under those assumptions the term in brackets in (3.11) is a lognormally distributed variable whose variance grows with time. Therefore, there is a dispersion around the point estimate provided in (3.11) that we will account for in estimating the equity position and payoff to the default put option. Specifically, given (3.11) a point estimate of borrower equity is given as

$$E(t) = H(t) - M(t) - S(t), \quad (3.12)$$

where H and M are as above, and S represents the borrower's protected CPF savings financing the property. In the U.S. context S would not appear in the equity calculation, but in Singapore protected equity implies (3.12) is the value the borrower keeps or remains liable for upon liquidation of the property, including the exercise of his default option. It is essentially the (negative) of the payoff to exercising the default put option, where the borrower sells the residual asset $H - S$ at a strike of M . It is common to normalize equity as in (3.12) by dividing by the home price H .

Rather than represent a point estimate for the expectation of the borrower's equity status, we characterize the distribution of his equity position by computing the probability that his equity is negative, providing our main put option variable:

⁷We use data from Monetary Authority of Singapore on monthly mortgage rates representing the average offer rate of the 10 largest institutions as a proxy for r .

$$Put_Option(t) = \Pr(E(t) < 0) = \Phi \left(\frac{\log(M(t) + S(t)) - \log(H(t))}{\omega(t)} \right), \quad (3.13)$$

where Φ is the standard normal CDF and ω a variance parameter for individual home prices that will grow with the length of time t . In our empirical work we parameterize ω using the estimates in Hwang and Quigley (2010) in their study of Singapore home price dynamics.

Protected equity affords us the opportunity to examine a unique situation whereby equity is negative solely due to the protected savings portion. This situation occurs when a borrower's home value remains larger than her loan, yet she can rationally increase her wealth through default. The probability that a borrower has negative equity solely due to protected CPF savings is simply the difference between the probability of negative equity given by (3.13) and the probability that $H < M$, or

$$Put_CPF(t) = Put_Option(t) - \Phi \left(\frac{\log M(t) - \log H(t)}{\omega(t)} \right). \quad (3.14)$$

In some sense a test for whether "*Put_CPF*" is a significant determinant of default is a test of the "ruthlessness" by which borrowers may default. We explore this further in our specifications below.

Finally, in addition to the key option related variables above, we consider several non-option variables in order to explore additional determinants of prepayment and default. These include borrower income, both at origination and current debt-to-income ratios, the current unemployment rate, and an indicator for whether the loan is to an investor or an owner occupant. We also include indicators for whether the loan was for a purchase or a refinance, and indicators for the property type. The most common property type in the data is a condominium, which serves as the baseline property type. Our property indicators then represent whether the property is an apartment, terrace housing, or all other property types (which primarily include detached and semi-detached homes). We also include a time-period indicator to capture the regime change in the treatment of CPF equity after September 2002. In particular, if lenders reduce loan margins subsequent to this date to reflect the advantage of a priority lien position, it could present refinance opportunities and thereby impact prepayments. However, since only new originations after this date are primarily impacted by this change, it is less clear what impacts this could have on defaults. Table 3.1 provides descriptive statistics for the loans in the sample calculated at the point of termination or censoring.

3.4.2 Specifications and Results

Table 3.2 presents results of maximum likelihood estimates for two basic models of prepayment and default based purely on our measures of the financial value of option

	Prepaid		Defaulted		Censored	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
<i>Call_Option</i>	-0.0116	0.128	-0.0427	0.087	-0.044	0.108
<i>Put_Option</i>	0.1002	0.172	0.283	0.246	0.014	0.042
<i>Put_CPF</i>	0.057	0.111	0.114	0.150	0.011	0.036
<i>Current DTI</i>	0.288	0.270	0.398	0.466	0.255	0.261
<i>Apartment</i>	0.137	0.344	0.138	0.346	0.198	0.399
<i>Terrace</i>	0.179	0.383	0.236	0.426	0.168	0.374
<i>Other Property</i>	0.094	0.292	0.086	0.282	0.065	0.247
<i>Investor</i>	0.167	0.373	0.150	0.358	0.131	0.337
<i>Refi</i>	0.116	0.320	0.098	0.298	0.086	0.281
<i>Refi Cash Out</i>	0.215	0.411	0.213	0.411	0.278	0.448
<i>Unemployment Rate</i>	2.943	0.741	3.18	0.647	2.323	0.645
<i>Income</i>	10,339	8963	11,056	8,720	10,036	7,045
<i>LTV at Origination</i>	0.505	0.275	0.559	0.289	0.474	0.299

Table 3.1: Descriptive Statistics for Loans at Termination Date by Termination Type

exercise, without considering borrower heterogeneity. The results support the options theory of mortgage termination. Model 1, for example, shows the prepayment hazard increases with the degree to which the prepayment option is in the money, with a highly significant coefficient value. Similarly, the default hazard increases with the degree to which the default option is expected to be in the money, and also highly statistically significant. Model 2 simply reestimates the model entering the option variables in binned ranges rather than as continuous variables, allowing us to inspect the effects on the hazard in different ranges of the explanatory variables. We see from these results that the greater the prepayment option is in the money the larger the prepayment hazard becomes, suggesting prepayment accelerates when the call option becomes deeply in the money. Similarly, the default hazard continues to increase through all ranges in the probability of negative equity.

Cross-effect terms are likewise significant in the model. Specifically, the prepayment hazard decreases with the probability of negative equity, suggesting borrowers in the money for default are less likely to prepay, as anticipated by the theory. Interestingly, the degree to which borrowers are in the money for prepayment also increases the hazard of default. Both the call variable and the put variable depend inversely on interest rates, and lower rates in particular increase the value of both options. Lower rates increase the underlying asset value for the call option (the mortgage) relative to the strike, but also raises the strike of the put option. This feature may explain the estimated cross-effect in the data. We note that, taken together, these results are

	Model 1		Model 2	
	Prepay	Default	Prepay	Default
<i>Call_Option</i>	1.940 (257.99)	1.9597 (14.44)		
<i>Put_Option</i>	-0.8457 (56.71)	2.1816 (57.84)		
$-0.3 \leq Call_Option < -0.15$			0.55917 (44.37)	1.749 (2.91)
$-0.15 \leq Call_Option < 0$			0.7149 (124.55)	2.605 (6.71)
$0 \leq Call_Option < 0.15$			0.8163 (122.25)	2.800 (7.62)
$0.15 \leq Call_Option$			0.9864 (126.25)	-11.53 (0.0008)
$0.2 \leq Put_Option < 0.3$			-0.2872 (13.50)	0.7289 (7.30)
$0.3 \leq Put_Option < 0.4$			-0.2336 (6.85)	0.9327 (11.27)
$0.4 \leq Put_Option < 0.5$			-0.4786 (20.16)	1.1219 (17.69)
$0.5 \leq Put_Option$			-0.6014 (40.36)	1.2710 (40.14)
<i>Loc1</i>	0.0297 (3.57)	.0019 (1.25)	0.0245 (3.49)	.0030 (1.11)
-LL	20,933		21,753	

Table 3.2: Maximum Likelihood Estimates for Competing Risks of Prepayment and Default without Heterogeneity. Wald ratios are shown below in parentheses. The mass point is set to 1 in the estimation (a fixed effect).

very similar qualitatively to those of Deng, Quigley, and Van Order (2000) for their study on U.S. mortgages.

Table 3.3 presents maximum likelihood estimates extending the basic model to include additional explanatory variables as well as unobserved heterogeneity. The specification in Model 3 adds several additional explanatory variables to Model 1, which appears to improve the model fit significantly. Thus, as in the U.S. case, purely financial option explanations for mortgage terminations are not sufficient to fully predict or explain loan terminations in Singapore. For prepayments, the estimated impact of *Call_Option* increases slightly, while the cross-term effect decreases slightly. Several of the additional variables also appear to help explain prepayments. Current DTI lowers the prepayment hazard, which may be understandable to the degree that this variable proxies for current liquidity and financial resources of the borrower. While property type does not appear to be important overall, loans financing apartments do prepay slower. This could be a result of differences in household mobility for families in this property segment. Investor loans also tend to prepay faster. It's possible this group is more astute in recognizing and acting on prepayment opportunities, or that their holdings periods are shorter than owner occupants strictly as a result of the investment nature of the transaction. Lastly, the results suggest that both loan purpose and the post-2002 time period are significant contributors to prepayment. The later result confirms that borrowers may have had opportunities to refinance at lower rates by switching lien priority after the change in law.

On the default side, the estimated impact of *Call_Option* is also very slightly increased, and a few additional variables appear significant. Interestingly, higher income appears to be predictive of a higher default hazard. Relative to option theory, higher income borrowers may show greater awareness of the default option value of their mortgage; relative to trigger event explanations higher income borrowers may also have on average higher volatility of income. Both DTI and the unemployment rate increases the default hazard as one would expect. The result for unemployment confirms that, just as in the U.S., trigger events are also important for defaults in Singapore.

Models 4 and 5 explore the effects of modeling borrower unobserved heterogeneity. Model 4 shows estimates for a model with two distinct groups of borrowers (two mass points) and Model 5 extends to 3 groups of borrowers. The results do not significantly impact the estimated effects of our variables in Model 3. Moreover, the estimated impacts of heterogeneity given by the point estimates of the location parameters are not very economically meaningful. For example, groups differ in their prepay hazards by at most 5%, and in their default hazards by even less. In addition, many of the location or mass point parameters are not statistically significant. In contrast to Deng, Quigley, and Van Order (2000), therefore, we do not find evidence of significant borrower heterogeneity for Singapore. One reason for this could be the significantly larger loan sample they estimate their results on, in conjunction with the significant

regional variation in their loan sample that they document as an important source of performance variation. In contrast, our data sample is much smaller and confined to the relatively small and homogeneous geography of Singapore, and therefore may not be sufficient to identify heterogeneity effects. Since including random effects in the model does not appear to provide additional information, we drop them from our final specifications.⁸

Our final specifications examine the question of how ruthless exercise of default options are in Singapore. As discussed above, in the absence of penalties or other transactions costs to default, the protection of CPF savings presents an opportunity for borrowers to enhance wealth by defaulting whenever equity is negative. This includes situations where equity is negative *solely* due to the amount of CPF savings funding the property. Since in this scenario the mortgage remains below the value of the property, defaulting in these situations is arguably a more ruthless form of default option exercise. To examine this issue, Table 3.4 provides results for specifications adding *Put_CPF*, representing the probability of negative equity solely due CPF savings. Model 6 extends Model 1 by decomposing *Put_Option* into *Put_CPF* and its complement, $Put_Loan = Put_Option - Put_CPF$; Model 7 similarly extends Model 3.

The results suggest that as a source of negative equity CPF by itself does not strongly influence the hazard of default. Although the estimated coefficients imply defaults rise with *Put_CPF*, the effect on the default hazard is much weaker than for *Put_Loan*. For example, raising the probability of negative equity by 10 percentage points is estimated to increase the default hazard by roughly 9% if purely due to CPF and by 30% if relative to the loan. Moreover, these estimated impacts are highly statistically significant in the case of *Put_Loan*, but at best significant at the 10% level of confidence for *Put_CPF*. It would appear that sources of variation in negative equity is an important factor in determining default. Declines in home prices, for example, are more relevant than increases in protected savings. We also note that these results contrast with prior results by Teo (2004) and Ong, Sing, and Teo (2007) on delinquency studies, which find that neither CPF equity nor total equity matter for predicting delinquency.

Taken together, our results suggest that (a) negative equity and financial incentives for default remain important despite the recourse nature of the loans, and (b) ruthless default on protected CPF portions of negative equity is not especially prevalent. One difficulty in interpreting these seemingly somewhat contradictory results is the possibility that recourse and protected CPF may interact or correlate in specific ways. For example, we do not know the complete asset position of the borrower vis-a-vis her non-housing assets, which would be an important factor for recourse. It is quite possible that when equity is negative solely due to CPF recourse is more binding

⁸As a further robustness check with respect to the number of mass points we estimate the prepayment and default specifications here independently using the Gamma heterogeneity model of Chapter 2. We also find no significant random effects in that framework.

	Model 3		Model 4		Model 5	
	Prepay	Default	Prepay	Default	Prepay	Default
<i>Call_Option</i>	2.5422 (326.9)	2.064 (12.83)	2.534 (317.8)	1.900 (10.43)	2.535 (318.27)	1.896 (10,36)
<i>Put_Option</i>	-0.6818 (32.77)	2.038 (40.94)	-0.6787 (32.36)	2.073 (42.23)	-0.6763 (32.14)	2.061 (41.17)
<i>Log Income</i>	0.0431 (1.42)	0.5344 (13.14)	0.0431 (1.43)	0.5310 (12.98)	0.0424 (1.38)	0.5363 (13.23)
<i>Current DTI</i>	-0.2188 (7.26)	0.8001 (27.51)	-0.2187 (7.27)	0.7893 (26.39)	-0.2465 (7.42)	0.8407 (24.63)
<i>Apartment</i>	-0.1318 (5.32)	-0.3947 (2.62)	-0.1318 (5.32)	-0.4106 (2.81)	-0.1313 (5.27)	-0.408 (2.78)
<i>Terrace</i>	0.0411 (0.63)	0.1658 (0.69)	0.0405 (0.62)	0.1564 (0.62)	0.0387 (0.56)	0.1608 (0.65)
<i>Other Property</i>	0.1010 (1.57)	0.1103 (0.12)	0.1006 (1.55)	0.1038 (0.10)	0.1002 (1.54)	0.0965 (0.09)
<i>Investor</i>	0.1990 (14.06)	0.1486 (0.45)	0.1988 (14.02)	0.1491 (0.45)	0.1977 (13.87)	0.1572 (0.50)
<i>Refi</i>	-0.2579 (16.98)	-0.3938 (2.09)	-0.2593 (17.11)	-0.4024 (2.18)	-0.2597 (17.16)	-0.4004 (2.17)
<i>Refi Cash Out</i>	-0.3886 (54.98)	-0.1407 (0.44)	-0.3926 (54.25)	0.1755 (0.68)	-0.3921 (54.15)	0.1748 (0.67)
<i>Post Sept 2002</i>	0.3589 (50.92)	-0.0203 (0.01)	0.3630 (50.04)	0.0353 (0.03)	0.3626 (49.95)	-0.0285 (0.02)
<i>Unemployment Rate</i>	0.0111 (0.15)	0.2834 (5.4)	0.0083 (0.08)	0.2469 (3.98)	0.0078 (0.07)	0.2503 (4.09)
<i>Loc1</i>	0.0183 (3.33)	.0009 (0.98)	0.0081 (0.58)	0.0002 (0.14)	0.0071 (0.21)	0.0002 (0.14)
<i>Loc2</i>			0.0531 (3.01)	0.0028 (0.71)	0.006 (0.51)	0.0002 (0.12)
<i>Loc3</i>					0.0463 (2.88)	0.027 (0.67)
<i>Mass1</i>			0.658 (0.45)		0.294 (0.19)	
<i>Mass2</i>					0.329 (0.47)	
-LL	19,956		19,956		19,955	

Table 3.3: Maximum Likelihood Estimates for Competing Risks of Prepayment and Default with Unobserved Heterogeneity. Wald ratios are shown below in parentheses. The last mass point is normalized to 1 in estimation.

	Model 6		Model 7	
	Prepay	Default	Prepay	Default
<i>Call_Option</i>	1.954 (262.1)	2.006 (15.25)	2.557 (331.6)	2.167 (14.14)
<i>Put_Loan</i>	-0.289 (3.02)	3.026 (85.72)	-0.1265 (0.52)	2.672 (53.02)
<i>Put_CPF</i>	-1.356 (62.14)	0.718 (1.97)	-1.175 (43.80)	0.8875 (2.72)
<i>Log Income</i>			0.0248 (0.47)	0.4575 (9.28)
<i>Current DTI</i>			-0.251 (9.29)	0.7232 (20.61)
<i>Apartment</i>			-0.1329 (5.41)	-0.4013 (2.68)
<i>Terrace</i>			0.0412 (0.65)	0.1243 (0.38)
<i>Other Property</i>			0.099 (1.50)	0.0652 (0.04)
<i>Investor</i>			0.1923 (13.13)	0.1378 (0.38)
<i>Refi</i>			-0.2379 (14.37)	-0.3247 (1.41)
<i>Refi Cash Out</i>			-0.386 (54.18)	-0.1581 (0.55)
<i>Post Sept 2002</i>			0.363 (51.93)	-0.0126 (0.004)
<i>Unemployment Rate</i>			0.012 (0.17)	0.2972 (5.97)
<i>Loc1</i>	0.0256 (3.49)	.0011 (1.09)	0.0141 (3.21)	.0008 (0.77)
-LL	20,925		19,948	

Table 3.4: Maximum Likelihood Estimates for Competing Risks of Prepayment and Default by CPF Contribution to Negative Equity. Wald ratios are shown below in parentheses.

than when equity is negative in general. This could occur, for instance, if higher CPF values correlate with higher non-housing assets, which may be quite plausible. By contrast, when equity is negative in general, there may be on average very little additional wealth of the borrower which could make recourse much less effective. Without specific knowledge of these non-housing asset positions, it will be difficult to fully ascertain the conditions under which recourse may be more or less effective, and how recourse may interact with CPF equity.

3.5 Conclusion

This study has examined prepayment and default terminations in Singapore mortgages within a standard competing risk framework, and shows that many of the same determinants found to be important in the U.S. context remain important in the case of Singapore. Because of the pricing structure of ARMs in the Singapore market, the option value to prepayment must come predominately from index rate dynamics and changing conditions affecting loan rate margins (such as senior lien finance opportunities). Our results document that these factors are indeed important for determining prepayments in Singapore. Additional factors determining prepayments are those related to household mobility as associated with property types households select, as well as the loan purpose.

On the default side, we find strong evidence that default terminations are highly responsive to default option payoff variables. Our results here do not indicate an important role for protected CPF equity and ruthless default decisions, but rather suggest total equity is most relevant. This suggests that recourse to default may have an impact in some circumstances but is not sufficient to eliminate default option value. This finding could have important policy implications relative to the effectiveness or desirability of recourse in mortgage markets more generally.

Chapter 4

Portfolio Credit Risk in Residential Mortgages

4.1 Introduction

The past couple of decades have seen enormous advances in our understanding of the risks associated with single-family mortgage lending.¹ Modeling of mortgage default risks at the loan level has increased in breadth and sophistication, receiving boosts from the option theoretic models treating the decision to default as the exercise of a default option that competes with the exercise of a prepayment option. Modeling of the expected loss on defaulted mortgages—that is, loss severity—also has advanced, particularly with publication by the Federal Housing Finance Agency (FHFA, formerly the Office of Federal Housing Enterprise Oversight (OFHEO)) of a risk-based capital rule which incorporated modeling of loss severity. The pace of advances in modeling of prepayment risk, while not a topic of this chapter, has been even more torrid. The study of prepayment risk has been boosted not just by methodological improvements but by the tremendous expansion of mortgage securitization, and, by and large, from the researcher’s point of view, timely surges in refinancing activity. These advances have improved our understanding of expected losses: the probability of default and loss given default on individual mortgages, and the expected loss of mortgage pools, including lender portfolios as well as pools underlying mortgage-backed securities. A better understanding of the central tendency of both loan level and portfolio losses has significant practical benefits—for example, it is of first order importance for correctly pricing mortgages and mortgage credit risk products.

The financial crisis of 2008 emanating from trouble in the mortgage market beginning in 2007 has of course added new elements to our understanding of mortgage credit risk. First, with respect to expected loss, the effects of various forms of risk-layering

¹This chapter is adapted from Calem, Case, and Neale (2004). Any remaining errors are my own.

associated with the proliferation of new products and greatly expanded credit criteria have now been evidenced. These expanded forms of risk layering include many now well publicized examples, such as the granting of mortgage credit to very low credit quality borrowers with no or very low down-payments; foregoing the verification of assets, employment, or income; applying loose standards for appraisals; and emphasizing products that entailed substantial mortgage payment adjustments in the future. Arguably, many participants in the mortgage market were caught off guard by the potency of interactions in many of these risk layered product and underwriting features, but mortgage risk modeling has and will continue to further develop by encompassing many of these newer elements. Second, the significant pace of home price declines that ensued across the country has added to the data by which we can examine the sensitivities of default to home price declines. Both of these factors enrich the data by which we can estimate conditional expected losses for loans and loan portfolios.

Beyond the first moment of the loss distribution, however, the recent modeling advances and crisis experience have not significantly improved our understanding of the tail-risk characteristics of mortgage pools. That is, while we have learned a great deal about loan-level credit risk (and its pool analogue, expected loss), we still know little about covariance in loan-level credit risk and little, therefore, about the shape of the distribution of possible losses on a mortgage pool. In particular, we know little about the right-hand tail of that distribution. Grenadier and Hall (1996) provided an initial contribution, developing a framework for evaluating risk-based bank capital standards that recognized the importance of diversification. Calem and LaCour-Little (2004) advanced the topic by quantifying portfolio-level risk in the context of a non-parametric approach to simulating house price and interest rate scenarios. Alternative, parametric approaches to simulating risk factor scenarios have not been explored.

In this study we hope to advance the understanding of portfolio-level mortgage credit risk in several ways. First, we enumerate the elements of a model of portfolio-level credit risk. Second, we simulate portfolio losses using both non-parametric and parametric models and consider the relative advantages of each approach. Third, we vary the structure of the parametric models to identify those features to which the tail-loss estimates appear to be most sensitive. Fourth, we demonstrate the sensitivity of portfolio-level credit risk to the geographic composition of the mortgage portfolio.

The lack of progress in understanding tail losses in mortgage portfolios may owe in part to the regulatory regimes in which mortgage portfolios have been held and in part to a traditional emphasis, particularly in the securitization area, on stress testing approaches. Banks and other depository institutions, for example, ought to be interested in understanding the probabilities associated with very large losses in their mortgage portfolios, but their attention may have been focused on implementing regulatory measurements of credit risk capital that have tended to be either insensitive to portfolio-level risk or provide an incomplete picture of portfolio-level risk.

lio risk. The government-sponsored secondary mortgage market participants (Fannie Mae and Freddie Mac) presumably are similarly interested in understanding tail-loss probabilities, but they have little incentive to publicize whatever they have learned. Their regulator, the FHFA, along with the rating agencies that evaluate the credit risk of securitized pools of mortgages, evaluate credit risk exposure by the predicted impact of an arbitrarily chosen stress scenario.² In evaluating structured products such as Collateralized Debt Obligations (CDOs) with underlying assets comprised of tranches of securitized mortgage pools, the rating agencies did use credit risk correlation assumptions in their assessments. However, those assumptions implicitly assumed extremely low probabilities that individual underlying mortgages would simultaneously experience adverse performance—in part due to a view that housing market outcomes across geographies have very low correlation. In retrospect, those assumptions appear to be based on very little evidence even for data available at the time. As a consequence, severe losses have been experienced on a large class of AAA rated securities.

The negotiation of the Basel II agreement on risk-based regulatory capital standards for banking organizations focused considerable attention on modeling credit loss distributions for the purpose of quantifying economic capital charges to support credit portfolios. Economic capital is generally understood using the “value at risk” (VaR) standard: the capital required to cover portfolio losses with a given level of certainty over a given time horizon (e.g. 99.9% over one year). The computation of economic capital, then, is defined as the computation of a selected quantile of the distribution of portfolio losses over the horizon. Thus, the Basel II process has focused attention on methodological issues and empirical results in portfolio-level credit risk, although most of the published research to date has been in the context of corporate loan portfolios rather than of mortgage portfolios.

In addressing economic capital for residential mortgages, developers of the Basel II proposal relied in part on the nonparametric approach of Callem and LaCour-Little (2004), which we revisit here, as well as on rating agency models and industry surveys.³ The simulation results presented here may provide some useful benchmarks for assessing the Basel II requirements, which are essentially unchanged for the current Basel III proposals.⁴

The chapter proceeds as follows. The next section presents a general framework for understanding portfolio credit risk models. Section 4.3 describes the three different

²Purchasers of mortgage-backed securities, and private mortgage insurers, likewise have a clear interest in understanding tail losses, although in a curiously truncated way. Securities investors are interested specifically in the probability that aggregate losses will exceed the credit support provided by junior tranches of a given security, resulting in a loss to any senior tranche.

³See Callem and Follain (2003) for details.

⁴In particular, the calculation of risk weighted assets for Basel III are unchanged from Basel II. Throughout the remaining parts of the chapter, therefore, Basel II and Basel III can be used as interchangeable terms.

models we use to simulate portfolio credit losses. Section 4.4 presents results and demonstrates sensitivity of the results to varying the geographic composition of the portfolio to which the model is applied and (in the case of the parametric models) to certain structural assumptions. The chapter closes with a discussion of what the results seem to imply regarding the most important routes for further research.

4.2 Elements of a Portfolio-Level Credit Model

The capital charge associated with any individual loan can be understood as the incremental contribution of that loan to the aggregate required economic capital for that portfolio. It is simple to see, then, that the probability of default (PD) and loss given default (LGD) on individual mortgages are essential to determining the incremental economic capital associated with those mortgages. It should be equally clear, though, that the expected loss ($EL = PD * LGD$) on an individual mortgage is not the same as that loan's incremental contribution to economic capital—that is, its incremental contribution to a selected (q) percentile of the portfolio loss distribution.

There are two general approaches to the problem of quantifying economic capital charges.⁵ In the “internal models” approach the problem is expressed as the problem of estimating the joint distribution over credit losses at the level of the individual mortgage. But the incremental contribution of a given loan depends on how that loan affects the diversification of assets in the entire portfolio; as a result, the capital charge associated with any loan is a function of the characteristics not only of that loan but also of the portfolio in which that loan is held.

Although important advances have been made in internal modeling, development of this approach is still at an early stage and the modeling and data complexities would make consistent validation of internal models quite difficult. For these (and other) reasons, financial institution regulatory agencies have elected to express the problem of estimating economic capital charges in a simpler and more restrictive way: as the problem of estimating the incremental contribution of any loan to the aggregate required economic capital for that portfolio, based solely on the characteristics of that individual loan.

The difference between the two approaches is essential, and especially important in the context of mortgage portfolios: it is that, while economic capital charges in the internal models approach depend on the characteristics of the portfolio as well as of the loan, capital charges in the simplified approach are portfolio-invariant. As Gordy (2003) has shown, the two approaches are identical (that is, contributions to VaR are portfolio-invariant) only if two assumptions are met. The first assumption is almost certainly met in practice by any reasonably-sized single-family mortgage portfolio: it is that no exposure in the portfolio accounts for more than an arbitrarily small share of total exposure—that is, the portfolio has near-infinite granularity.

⁵This discussion borrows heavily from Gordy (2000 and 2003).

The second assumption, however, is much more problematic, especially for mortgage portfolios. Estimating the joint distribution of credit losses is accomplished by assuming that correlations in unconditional default probability across borrowers arise because they share dependence on a single systematic risk factor—that is, a macroeconomic factor that affects (to some degree) all borrowers in the portfolio.⁶ Note that the single factor may be an aggregate index representing an invariant combination of multiple risk factors, such as a weighted sum of random variables where the weights are the same for all loans in the portfolio. For single-family mortgages considered in an option-theoretic framework, the unconditional probability of default is most directly a function of the probability that the default option will become “in the money”—that is, that borrower’s equity will become negative by enough to offset the costs of exercising the option—which means that the unconditional probability of default is a function of house price appreciation patterns. To extend this to a two-trigger default framework, unconditional probabilities of default may be functions of combinations of risk factors: house prices (which contribute to the probability that the equity trigger will be pulled) along with unemployment or other income shocks, which contribute to the probability that the ability-to-pay trigger will be pulled.

Crucially, while both house price appreciation patterns and income or unemployment shocks have a common national component, they also differ very clearly across regions of the country. In modeling terms, there are multiple systematic risk factors, implying that the incremental contribution of any individual mortgage to the economic capital required to cover losses in that portfolio with a given level of certainty (q) is not portfolio-invariant, but depends rather on the composition of the portfolio. This, of course, is no surprise: a mortgage that contributes to the diversification of assets in a portfolio should require less economic capital than the same mortgage that, because it is placed into a portfolio of similar mortgages, contributes to concentration rather than to diversification of assets.

Let us summarize the elements of a portfolio-level model of credit risk that are suggested by this discussion. The first element is the set of systematic risk factors that contribute, in varying degrees, to the probability of default of all borrowers in the portfolio. For a mortgage portfolio, the systematic risk factor impinging on a given mortgage can be summarized by the rate of price appreciation for the property underlying that mortgage; the set of systematic risk factors encompasses regional or local variation in these price appreciation rates.

The second element is the sensitivity of a loan’s conditional default probability to variations in the systematic risk factors—factor loadings, in the language of credit risk modeling. If there were a single systematic risk factor (or invariant combination of multiple risk factors represented by a single, aggregate index), then the factor loading

⁶The conditional default probability is the probability of default conditional on a particular realization of the systematic risk factors, and is idiosyncratic to the individual borrower. The unconditional default probability is the probability of default given all observable information prior to the realization of the systematic risk factors.

would indicate the sensitivity of the conditional default probability to changes in overall macroeconomic conditions (summarized by house price appreciation rates) in the country. Since there are multiple systematic risk factors—one for each region of the country—we have instead a set of factor loadings, each one indicating the sensitivity of the conditional default probability of a given loan to changes in overall macroeconomic conditions in a given region. The factor loading will be much larger for the systematic risk factor pertaining to the region in which a property is located than for the risk factors pertaining to other regions, and this difference in factor loadings will show up as a difference in the required economic capital associated with that loan depending on whether it is placed in a portfolio of loans primarily from the same region or primarily from other regions.

The third element is the remaining probability of default on each individual loan that is idiosyncratic to that loan—that is, the default risk that is not correlated across loans in the portfolio based on their common sensitivity to variations in the set of systematic risk factors. Recent research has contributed to a solid understanding of the multiple factors contributing to PD—including idiosyncratic factors such as loan terms, characteristics of the property, or the borrower’s credit score—but it has not generally emphasized the difference between those factors that are idiosyncratic to the individual borrower and those that are common across borrowers.

Finally, the fourth element is the loss given default on each individual loan. Although the discussion here has suggested that only (conditional) default probabilities may be sensitive to the set of systematic risk factors, clearly they may affect (conditional) LGDs as well, presumably with a different set of factor loadings. Because of this, while recent research has contributed to our understanding of the factors affecting LGD—for example, the role of state foreclosure laws—it would be beneficial to have a better understanding of those factors that are sensitive to systematic risk as well as those factors that are idiosyncratic to individual mortgages.

Note that we have not mentioned explicitly the contribution of any given mortgage to the diversification of a mortgage portfolio, which is certainly of first-order importance in estimating the contribution of that mortgage to the distribution of portfolio losses.⁷ But that is simply because we are using different terminology: the effect of diversification is reflected in the set of systematic risk factors and the factor loadings on each factor. For example, if there are four systematic risk factors—reflecting, say, macroeconomic conditions in the Northeast, South, Midwest, and West—then a mortgage on a property located in the Northeast would have a large factor loading on the Northeast macro factor and small factor loadings on the South, Midwest, and West

⁷We have also not mentioned loan size, or exposure to a given borrower, for two reasons. First, for granularity purposes, in practice essentially any single-family mortgage portfolio will be considered sufficiently granular. Second, in contrast to other loan types such as credit cards or home equity lines of credit, for single-family mortgages the exposure is essentially equal to the principal balance and is therefore known to the lender. Of course, loan size at origination may also be a determinant of (idiosyncratic) default probability and/or loss given default.

macro factors. If that mortgage were placed in a portfolio dominated by mortgages from the other regions (and therefore mortgages that had large factor loadings on one of the other factors but small factor loadings on the Northeast factor) then the capital charge associated with that mortgage would be relatively small. That is, the diversification benefit of any given mortgage is reflected in a difference between the (regional or local) factor to which that mortgage is most sensitive and the (regional or local) factors to which the other mortgages in the portfolio tend to be sensitive.

Finally, we also have not mentioned the asset value correlation, which measures the extent to which the values of the assets (properties) underlying all loans in the portfolio move together over time: if the asset value correlation is large, then the default correlation will tend to be large as well: that is, whatever defaults occur will tend to occur together in time. The asset value correlation plays a prominent role in the Basel II/III regulatory capital framework, and is the main parameter whose value has been established by regulators while other parameters will be estimated by financial institutions. The asset value correlation is merely the square of the factor loading in the context of a model with a single systematic risk factor.

As noted, financial institution regulatory agencies have elected to compute regulatory capital charges as the incremental contribution of any loan to the aggregate required economic capital for that portfolio based solely on the characteristics (PD and LGD) of that individual loan, and this portfolio-invariant capital charge equals the internal-models capital charge only under the assumption that there is a single systematic risk factor. For Basel II purposes, then, the set of factor loadings (on multiple systematic risk factors) is reduced to a single factor loading or asset value correlation. The Basel II mapping (of PD and LGD to a capital charge) thus may be viewed as an approximation to the capital charges suggested by the Callem and LaCour-Little (2004) portfolio credit risk model, which has multiple systematic risk factors, and by rating agency models and other sources.⁸

4.3 Three Portfolio Credit Risk Models for Single-Family Mortgages

In this section we develop and compare three alternative approaches to modeling portfolio credit risk for residential mortgages. Each of the models incorporate all of the elements identified in the previous section—systematic risk factors, and factor loading, idiosyncratic PD, and LGD—that determine the shape of the right-hand tail of the distribution of mortgage portfolio losses. The models are distinguished solely

⁸The Basel II formula requires as an input the annualized average PD for the mortgages, although the capital charge it calculates corresponds to a life-of-the loan value-at-risk measure. The formula was constructed in this fashion for consistency with the framework applied to corporate loans, where the input is also a one-year PD and an explicit “maturity adjustment” is added to the capital charge for loans with maturity greater than one year.

by the way they represent and simulate the systematic risk factors underlying house price appreciation and interest rate dynamics. Conditional on the same set of local house price scenarios and interest rate paths, each model would yield identical outcomes. That is, all three models are calibrated with the same “factor loadings” and the same specification of idiosyncratic PD and LGD. More precisely, they have the same calibration of conditional default and prepayment transition probabilities and conditional expected loss severities. The models are implemented by iterating random draws of the systematic risk factors and calculating cumulative discounted expected losses conditional on each draw.

House price appreciation rates, of course, are of first-order importance in determining the probability of default on a mortgage, and there is no question that there is sharp regional (or even local) variation in house price appreciation. Freddie Mac and FHFA, for example, publish separate house price indices across the nine Census divisions, 50 states, and some 384 MSAs, and Fiserv CSW and Corelogic compute separate proprietary price indices even for some ZIP code areas. Price appreciation patterns do not necessarily differ, however, across any pair of MSAs (much less ZIP code areas), and estimating separate price indices for areas that in fact share a common price index reduces the available sample size and therefore the precision of the estimated indices. Nor do differences necessarily correspond to state or Census division boundaries: areas in different states (or divisions) can share a common price index, just as areas within the same state may have different indices.

Case and Calem (2004) used FHFA’s MSA-level price indices to group metropolitan areas into nine districts based on differences in underlying factors that should continue to generate differences in price appreciation patterns. The implicit assumption underlying this exercise is that systematic (non-idiosyncratic) changes in house prices are common to all properties not just in a given MSA but in all of the MSAs within a given district. Indeed, some house price changes are common to all MSAs nationwide, so Case and Calem (2004) decomposed the estimated real house price changes in each MSA into (1) a national component common to all MSAs, (2) a district component common to all MSAs in a given district but not across all districts, and (3) a residual local component common to all loans in a given MSA but not common across the MSAs in a district. Note that district components may be correlated across a given pair of districts (though not across all districts), and local components may be correlated across a given pair of MSAs within the district (though not across all MSAs within the district).

Two of the models described below (the nonparametric simulation procedure and one of the parametric procedures) employ the Calem and Case (2004) decomposition as a basis for constructing the systematic risk factors underlying house price appreciation. These models, then, are ones in which there are nine primary sources of systematic risk in house price appreciation, corresponding to the sum of a common national component and one of the nine common district components of house price

appreciation patterns.⁹ This is not, of course, the only way to represent the systematic risk factors underlying house price appreciation, and the third model described below employs an empirical, factor analysis approach to develop and calibrate such systematic risk factors.

In estimating and calibrating the models below, and applying them to representative loan pools, we emphasize data from pre-crisis periods prior to 2005. Focusing on these periods allows us to fully align the data with that used to determine the district assignments in Case and Callem (2004). It also allows us to abstract from the effects of loose underwriting practices and the high degree of risk layering that occurred with increasing intensity from 2005 onward and focus instead on risk capital for more traditionally underwritten mortgages. Finally, it also allows us to evaluate the estimates of credit risk capital in light of the actual stress losses occurring as a result of the steep declines in home prices beginning in 2006.

4.3.1 Non-Parametric Simulation of Systematic Risk Factor Realizations

Generating systematic risk factor realizations in the mortgage portfolio credit risk context means generating a distribution of prospective house price paths for the properties underlying the loans in a mortgage portfolio, along with contemporaneous mortgage interest rate paths. Although we cannot predict house price or interest rate shocks, we have a large set of observed (estimated) historical house price changes: that is, the quarterly price changes implied by the MSA-level FHFA price indices, along with historical series for interest rates. Following Callem and LaCour-Little (2004), we first associate each district with a district selected at random, and then associate each MSA within that district with an MSA selected at random from the associated district (sampling with replacement in each case). Next we select at random an eight-year period starting as early as 1982Q1 and as late as 1993Q4. Finally, each property supporting a mortgage in our portfolio is assigned the observed house price appreciation pattern for its associated MSA during the randomly selected eight-year period. We also assign the quarterly, average mortgage interest rate pattern published by Freddie Mac for the same eight-year period, with the note rate on each mortgage assumed to be the same as the published interest rate for the initial quarter in that series. In addition, we draw the historical series of interest rates on large, 6-month certificates of deposit for the selected period, for use as a discount factor.

The effect of this resampling approach is threefold. First, it preserves the historically observed bivariate distribution of eight-year house price appreciation patterns and contemporaneous mortgage interest rates. Second, it distributes realizations of this bivariate distribution randomly across MSAs in all districts, implying that no

⁹Unless the portfolio contains concentrations of loans in particular MSAs, the local components do not contribute significantly to systematic risk.

district or MSA is systematically more (or less) subject to macroeconomic stress than any other district or MSA. For example, historically we observed a very stressful price appreciation episode in several Texas MSAs during the early 1980s. The resampling approach preserves the probability that this stressful episode will recur, but not its identity as a “Texas” episode: under the resampling approach it could occur with the same likelihood in any other district. Third, because the approach involves bootstrap-type sampling with replacement, historical price declines observed for a single city or region will replicate, leading to more broad-based declines than have been observed historically.

4.3.2 Parametric Simulation of National, Regional, and Local Components of House Price Appreciation

Among the advantages of the resampling approach is that it obviates the need for parametric assumptions concerning the distribution of the systematic risk factors as well as parametric assumptions concerning spatial and temporal correlation among them: that is, the characteristics of the joint distribution of historically observed quarterly house price appreciation rates and mortgage interest rates (across time as well as across districts and MSAs) is preserved in the set of house price appreciation and mortgage interest rate patterns that are generated through this approach. Conversely, of course, the resampling approach cannot generate realizations of regional price declines other than those that have been observed in at least one region (or local price declines observed in at least one city) during the historic period. The advantage of a parametric approach, then, is that—if the parameters of the model are estimated correctly—it can generate realizations of the systematic risk factors that were not observed historically but that may well be observed prospectively.

Our first parametric model incorporates three elements for each of three components (national, district-level, and local) of house price appreciation defined by Case and Calem (2004): (1) the mean for each component (quarterly trend rate of nominal appreciation at each level) (2) the variance of quarterly house price shocks at each level (3) the lag structure of quarterly house price changes at each level—that is, the temporal correlation of each component of house price change. In addition, the model incorporates the covariance of mortgage interest rates with the national component of house price change. Because we cannot distinguish the degree to which they reflect fundamental economic commonalities, we ignore remaining correlation of quarterly shocks across districts and MSAs in observed historical house prices. That is, we rely solely on the national component and on the district definitions of Case and Calem (2004) to incorporate covariance across MSAs.

In estimating district-specific and MSA-specific mean real house price appreciation rates, the first question that we confront is whether there may be long-run differences across MSAs, regardless of whether differences have been observed during

District	Appreciation Rates
National	4.50%
Northwest/Great Basin	4.56%
West South Central	2.03%
Inland Northeast/Mid-Altantic	5.11%
West North Cental	4.26%
East North Central	4.09%
California	5.56%
Coastal Northeast	6.66%
South	4.14%
Florida/Southeast	3.96%

Table 4.1: National and District Long Run Appreciation Rates

the historic period. Assuming a well-functioning market for housing, equilibration processes would tend to imply that long-run appreciation rates must be equal across MSAs, otherwise supply would expand accordingly. On the other hand, house prices are determined in part by amenities in finite supply, such as Big Sur or Marblehead coastline. To the extent that demand for such limited amenities is income-elastic and substitutes are unavailable, house price appreciation rates may well differ even in the long run. For this reason we generate parametric realizations of the systematic risk factors under both assumptions, first with district-specific long-run trend appreciation rates and then with a common national long-run trend appreciation rate.

These long-run trends, as well as the local, regional, and national components of quarterly deviations from the long-run trend, are estimated from quarterly MSA-level house price indices published by FHFA. The sample consists of observations from 188 metropolitan areas over the period 1982 through 2003. Long-run trend appreciation rates for this sample period are shown in Table 4.1.

The first step after calculating the long-run trends is to separate the national and district-level components from the local component of house price appreciation. This is accomplished by regressing MSA-level quarterly price changes, APP_{it} , expressed as the difference from either the national or district-level long-run trend, on a series of dummy variables D_{jt} for district j (of MSA i) and quarter t :

$$APP_{it} = \beta_{jt}D_{jt} + \epsilon_{it}. \quad (4.1)$$

The estimated coefficients from this regression, which we denote β_{jt} , represent the combined regional and national components, and the residuals represent MSA-level house price shocks.

Next, we separate the national component of house price appreciation from the district-level component and estimate the lag structure and variance of the district-level components. This is accomplished by regressing the combined regional and

national components β_{jt} on a set of lagged values $L_j(\beta_{jt})$ and a series of dummy variables N_t representing each quarter:

$$\beta_{jt} = \gamma_t N_t + L_j(\beta_{jt}) + \eta_t. \quad (4.2)$$

After exploring a number of lag specifications for (4.2), we determined that one with three lags, representing the district-level component in the previous quarter, its average value over the previous four quarters, and its average value four years prior (the average value over the 13th through the 16th lagged quarters) most closely fit the data for each district. Moreover, we found that two districts (California and Northeast) had lagged structures that differed from those in other districts but were similar to each others' and that otherwise, lag structures were quite similar across districts. Hence, we base our final coefficient estimates on a specification that estimates a single set of lag coefficients for California with Northeast and another, common set of lag coefficients for all other districts.

We then determine the lag structure of the national component and its relation to market interest rates. Specifically, we regress the quarterly observations of the national component of house price appreciation, represented by the estimated coefficients γ_t from (4.2), on their lagged values and the contemporaneous five-year treasury bill interest rate, R_t :

$$\gamma_t = \alpha_0 + \alpha_1 R_t + \alpha_2 \gamma_{t-1} + \nu_t. \quad (4.3)$$

Finally, for each individual MSA, we estimated a regression equation to determine the lag structure of its local component:

$$\epsilon_{it} = L_i(\epsilon_{it}) + \mu_{it}. \quad (4.4)$$

All of the information required for the simulation of systematic risk factor realizations (quarterly local, district-level, and national appreciation rates) is thus obtained. This information consists of: a long-run trend appreciation rate (common national or district-specific); the lag structures of the regional and district-level components; the relation of the national component to its lagged values and the contemporaneous 5-year t-bill rate, and the standard deviations of the national, local, and district level shocks ν_t , η_t , and μ_{it} , respectively.

Results for the national and district level components are reported in Tables 4.2 and 4.3 using a common national trend and district specific trends, respectively. A range of different lag structures and variances are obtained for the local components of MSA house price appreciation. The mean value of the standard deviation of local shocks μ_{it} is 3.4%.

Parameter	Results for National Component	
Intercept	0.0151	
Interest Rate	-0.0025	
Lagged National	-0.3703	
	Results for Regional Component	
	California and Northeast	All other districts
Lag 1 Qtr	0.5495	0.1940
Lag 1 Yr	0.3479	0.6991
Lag 4 Yr	-0.1095	-0.0744
District/National	Standard Deviation of Shocks	
National	2.01%	
Northwest/Great Basin	2.06	
West South Central	2.77	
Inland Northeast/Mid-Altantic	1.80	
West North Cental	1.41	
East North Central	1.65	
California	2.24	
Coastal Northeast	2.69	
South	1.78	
Florida/Southeast	1.68	

Table 4.2: Results for National and District Level Components Assuming a Common National Trend

Parameter	Results for National Component	
Intercept	0.0169	
Interest Rate	-0.0029	
Lagged National	-0.3799	
	Results for Regional Component	
	California and Northeast	All other districts
Lag 1 Qtr	0.5495	0.1953
Lag 1 Yr	0.3507	0.6715
Lag 4 Yr	-0.1032	-0.1208
District/National	Standard Deviation of Shocks	
National	2.01%	
Northwest/Great Basin	2.06	
West South Central	2.77	
Inland Northeast/Mid-Altantic	1.80	
West North Cental	1.41	
East North Central	1.65	
California	2.24	
Coastal Northeast	2.69	
South	1.78	
Florida/Southeast	1.68	

Table 4.3: Results for National and District Level Components Assuming District Specific Trends

4.3.3 Latent Factor Model for House Price Appreciation

Latent risk factor models have proven to be useful tools for modeling the dynamics of many financial variables, including asset returns and the term structure of interest rates. A principle advantage of these models is that they can often provide a reasonable representation of the dynamics of a multivariate return process without the need for a detailed understanding of the underlying structural process. In our application to house price returns, a factor model has particular appeal to the extent that it can parsimoniously capture both the persistence in MSA housing returns and the correlation of returns across MSAs in a freely parameterized model. Indeed, it is this explicit reliance on a freely parameterized time series model to capture multivariate return dynamics as closely as possible that perhaps most sharply distinguishes this approach from the alternative approaches considered in this study. At the same time, the absence of an explicit economic interpretation for each of the latent risk factors may be viewed as a limitation of the approach. For instance, since some of the observed correlation among MSAs may reflect coincidence of idiosyncratic shocks as opposed to economically meaningful common factors, it is possible to “overfit” the data by including too many risk factors.

We specify a seemingly unrelated time series model for MSA house price returns where each MSA follows a lagged adjustment process in its own returns. However, innovations to MSA returns are given by a dynamic factor model. Specifically, we model a set of n distinct MSA returns as

$$y_{it} = \phi_i^{-1}(L)\zeta_{it}, \quad i = 1, \dots, n, \quad (4.5)$$

where for the i th MSA, y_{it} , $\phi_i(L)$, and ζ_{it} represent the (de-measured) house price return, lag polynomial, and return innovation, respectively. The return innovation is assumed to be a white noise process that can be further related to a linear combination of a set of common unobserved risk factors $f_t \in \mathbb{R}^k$ and an MSA-specific shock u_{it} :

$$\zeta_{it} = \lambda_i' f_t + u_{it}, \quad \lambda_i \in \mathbb{R}^k, \quad i = 1, \dots, n. \quad (4.6)$$

This formulation implies that MSA-level returns are not directly related to one another in any causal sense, but innovations to returns share a common dependence to a set of generic economic risk factors. Allowing each MSA to have its own lag specification also ensures that we account for observed differences across MSAs in the persistence of returns and that linear dependence will be absent in the innovations.

Equations (4.5) and (4.6) are estimated by maximum likelihood after a lag specification search is performed for each MSA in the sample. For this model we chose to work with the 62 most populated MSAs in the country over the period 1982Q1 to 2004Q2.¹⁰ We identify seven factors ($k = 7$) as contributing to the return innova-

¹⁰These MSAs represent nearly 80% of all U.S. households, and so will comprise a large portion of holdings for large portfolio lenders—and 100% in the case of our sampled portfolios below.

	Lag Structure			Average Variance Explained by Factor						
	Min	Max	Mean	1	2	3	4	5	6	7
Northwest/Great Basin	1	4	2.3	2	5	18	8	5	5	2
West South Central	1	4	2.4	3	1	41	1	2	5	6
Inland N.E./Mid-Atlantic	0	3	2.3	25	9	4	22	6	2	3
West North Central	0	5	2	2	3	2	18	15	6	1
East North Central	1	5	3.4	31	7	3	3	6	16	5
California	1	3	1.3	5	51	5	5	1	4	1
Coastal Northeast	2	2	2	51	8	4	4	2	1	4
South	0	3	2	5	0	6	3	16	6	3
Florida/Southeast	2	4	3.1	13	5	17	20	1	1	4

Table 4.4: Summary Information for the Latent Factor Model Grouped by District

tions, and the fitted model reproduces the empirical mean, variance, and correlations in MSA returns quite closely.

Table 4.4 presents summary information for the fitted models. Not surprisingly, we find strong similarities in the estimated model structures for geographically related MSAs, both in terms of the order of lagged dependence and the loadings on individual factors. The similarities of within-district factor loadings, with respect to both the sign and magnitude of the estimated parameters, are a consequence of the strong empirical correlations for these MSAs. For example, all MSAs in our sample located in the Coastal Northeast load positively and heavily on the first factor, with typically about 51% of the innovation variance explained by variation in that factor. In contrast, factor 1 does little to contribute to innovations in the Northwest (explaining only about 2%), and in fact contributes to weaker correlation with the northeast since all MSAs in the northwest load negatively on it. Similar patterns can be seen across the other regions: factor 2 is a considerable source of return innovations in California, and likewise factor 3 for the Southwest, and so on. It appears, then, perhaps not surprisingly, that many of these latent factors are representing regional shocks.

4.3.4 Calibration of Interest Rate Dynamics for the Parametric Models

As noted above, the non-parametric model resamples rate history coinciding with the resampled home price history. For our parametric models, however, we need to additionally specify an interest rate process. For this purpose, we implement a single-factor Cox, Ingersoll, and Ross (1985) (CIR) model for the term structure of

interest rates. The CIR model specifies a single latent factor, interpreted as the short rate of interest r , that follows a diffusion process given as

$$dr = (\kappa(\theta - r) - \lambda r) dt + \sigma\sqrt{r}dz, \quad (4.7)$$

where κ is a parameter describing the rate of reversion to the long-run mean θ , λ a parameter representing the price of interest rate risk, σ a volatility parameter, and dz an increment of a Wiener process. We discretize and calibrate the parameters of (4.7) following the procedures in Backus, Foresi, and Telmer (1998), using data on U.S. Treasuries from 1982 to 2004.

The term structure of interest rates represent additional systematic risk factors directly and indirectly impacting our loss estimates. Recall the national component of house prices in (4.3) depends on intermediate-term interest rates directly. Higher rates dampen home price appreciation (through affordability and other mechanisms), and vice-a-versa. Fluctuations in rates are therefore inversely correlated with home prices in this specification of a parametric model. The latent factor model, on the other hand, does not include interest rates as a determinant of home price returns, implying home prices and interest rates are uncorrelated in that framework. In either parametric modeling framework—the hierarchical local-regional framework or the latent factor framework—interest rates may have a material influence on defaults through prepayments. We model the performance of loans in a competing risk environment, so interest rates drive fluctuations in prepayments. Because prepayment competes with default, higher rates of prepayment stemming from lower market interest rates can often be reflected in lower defaults.

4.3.5 Calibration of Default Probability and Loss Given Default

The next step after generating realizations of the systematic risk factors is to calibrate the factor loadings (for both PD and LGD) as well as the idiosyncratic components of conditional default probabilities and conditional LGDs. As noted, the factor loadings represent the sensitivity of conditional default probabilities and conditional LGDs to realizations of the systematic risk factors, while the idiosyncratic components of conditional PDs and conditional LGDs represent those risk factors that are not systematic but idiosyncratic to a particular loan. These calibrations are implicit in a specification of conditional expected default and prepayment probabilities and conditional expected loss severities, for which we rely on the empirical survival curves and loss severity estimates employed by Calem and LaCour-Little (2004). These survival curves are approximations to proprietary, conditional probability estimates for transitions prepayment and transitions to 90-day delinquency and from delinquency to foreclosure (or foreclosure alternative) from the LoanPerformance Risk Model. The loss severity calculations are based on part on an empirical equation for net recoveries

on foreclosed loans, estimated using FHFA data. Given these calibrations, PD varies with the original loan-to-value ratio (LTV) and loan size and with the borrower's initial credit rating (FICO) score, and LGD varies with LTV, loan size, and median income of the neighborhood where the property is located.

4.3.6 Calibration of Loan Characteristics, Horizon, and Discount Factor

Probability distributions over cumulative discounted credit losses are generated using each of the three models for various specified risk segments within a number of hypothetical bank portfolios. An eight-year horizon is specified. The interest rate on large, 6-month certificates of deposit is employed as the discount factor, following Calem and LaCour-Little (2004).

Risk segments are defined by an original loan-to-value ratio and borrower credit rating (FICO score). Ten Hypothetical bank portfolios are constructed by randomly sampling 1000 loans from the 2003 Home Mortgage Disclosure Act data of each of the ten largest mortgage lending institutions (with sampling restricted to the 30 cities having the largest concentration of loans for each institution).

4.4 Results

As noted, the three credit models described in the previous section differ only in the ways in which realizations of the systematic risk factors underlying house price appreciation patterns are generated. Conditional on these realizations—the set of local house price appreciation scenarios—each model yields identical outcomes because each uses the same factor loadings (sensitivity of PD and LGD to the systematic risk factors) and the same specification for the idiosyncratic components of PD and LGD.

This means that variation among the models in terms of the estimated capital required to cover unexpected losses with a given level of confidence will indicate the sensitivity of economic capital estimates to variations in the parametric or non-parametric assumptions underlying the exercise. In addition to this, however, we would also like an appreciation of the sensitivity of capital requirements to variation in portfolio weights, reflecting for example the degree of geographic concentration. For this reason we estimate economic capital using the different modeling frameworks applied to a sample of loans from the 30 MSAs with the largest number of loans in the actual portfolio of each of the banks with the ten largest mortgage portfolios. While each of these ten largest banks is likely to have a fairly diversified mortgage portfolio, this will enable us to investigate differences in required economic capital that arise solely because of differences in the portfolio weights assigned to each of that bank's cumulative 30 largest markets.

The right-hand tails of the unexpected loss distribution (where unexpected losses are defined as total losses minus expected loss) estimated using the three modeling frameworks are shown in Figures 4.1 and 4.2. Panel A of Figure 4.1 shows the estimated tail of the unexpected loss distribution using the non-parametric (resampling) approach, with each line showing the results for the portfolio of loans sampled by one of the ten largest banks (not individually identified). Similarly, Panel B of Figure 4.1 shows the estimated tail of the unexpected loss distribution using the latent factor model. Panels C and D of Figure 4.2 show the tails estimated using the hierarchical parametric model with, respectively, (Panel C) a common national mean real house price appreciation rate but district-specific volatilities, and (Panel D) district-specific mean real house price appreciation rates and district-specific volatilities. While we can compute the estimated tails using several different illustrative values for LTV ratio and credit rating (as arguments in the estimation of PD and LGD), the graphs show the tails estimated using an LTV ratio of 80 percent with FICO scores of 620 (solid lines) and 740 (dotted lines).

Table 4.5 summarizes the results by reporting the estimated capital that would be required to cover unexpected losses at a level of confidence that would earn a BBB+ rating (98 percent for an eight-year horizon). Model 1 represents the nonparametric model, Model 3 the latent factor model, while Models 2A, 2B, and 2C are variants of the district-based parametric model that incorporate a common national mean and volatility, common national mean and district-specific volatilities, and district-level means and volatilities, respectively. Not surprisingly, the results under all of the models suggest considerable differences in the estimated tail losses for hypothetical portfolios of mortgages with the same credit score and LTV but held at different banks—that is, with different geographic distributions. For example, the range of lowest to highest estimates across portfolios vary from 140 basis points under the nonparametric model to 440 basis points under the factor model, variation that reflects the differing geographic weights across portfolios. The overall level of capital, however, appears to be less sensitive to the districts in which each bank’s mortgage portfolio tends to concentrate. Estimated tail losses are comparatively large, for example, in the portfolios held by Bank 8 (relatively heavily weighted in the Midwest and North Central districts but light in California and the Northwest) and by Bank 5 (relatively heavily weighted in California but light in the Midwest, North Central, and Southwest). In contrast, estimated tail losses are comparatively small in the portfolios held by Bank 4 (relatively heavy in the Northwest and light in the Coastal Northeast), Bank 7 (relatively heavy in the South and the Northeast, and light in the Southwest and California), and Bank 2 (relatively heavy in the Midwest and light in the Inland Northeast).

The estimates also suggest that the choice of model framework is equally important to the estimated credit capital of mortgage portfolios. Overall, the resampling approach yields smaller estimates of BBB+ tail losses than the other methods: on average about 217 basis points for mortgages with low credit scores, compared to an

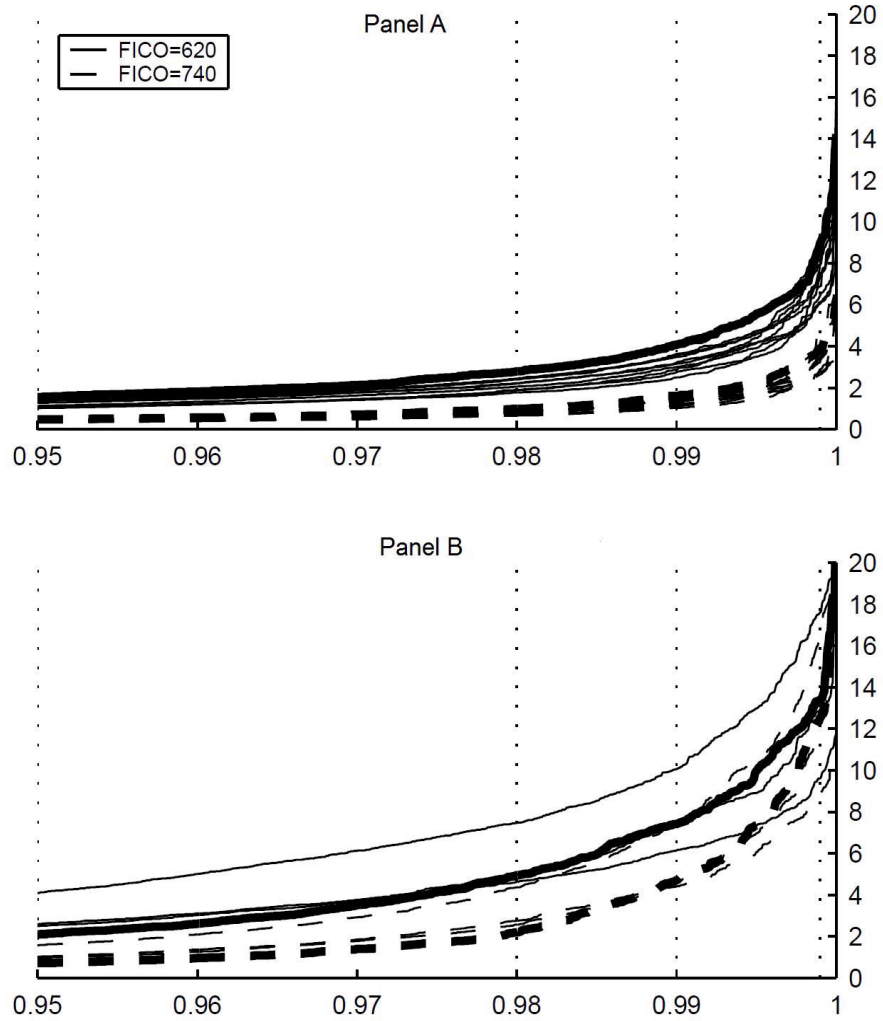


Figure 4.1: Unexpected Loss Distributions for Resampling and Latent Factor Models. Panel A = resampling model, Panel B = latent factor model. Both panels illustrate the tail distribution by hypothetical portfolios of 80 LTV mortgages. Percentiles of the distribution are on the x-axis, unexpected loss (loss minus expected loss) in percent is on the y-axis. Solid lines are for 620 FICO, dashed lines are for 740 FICO.

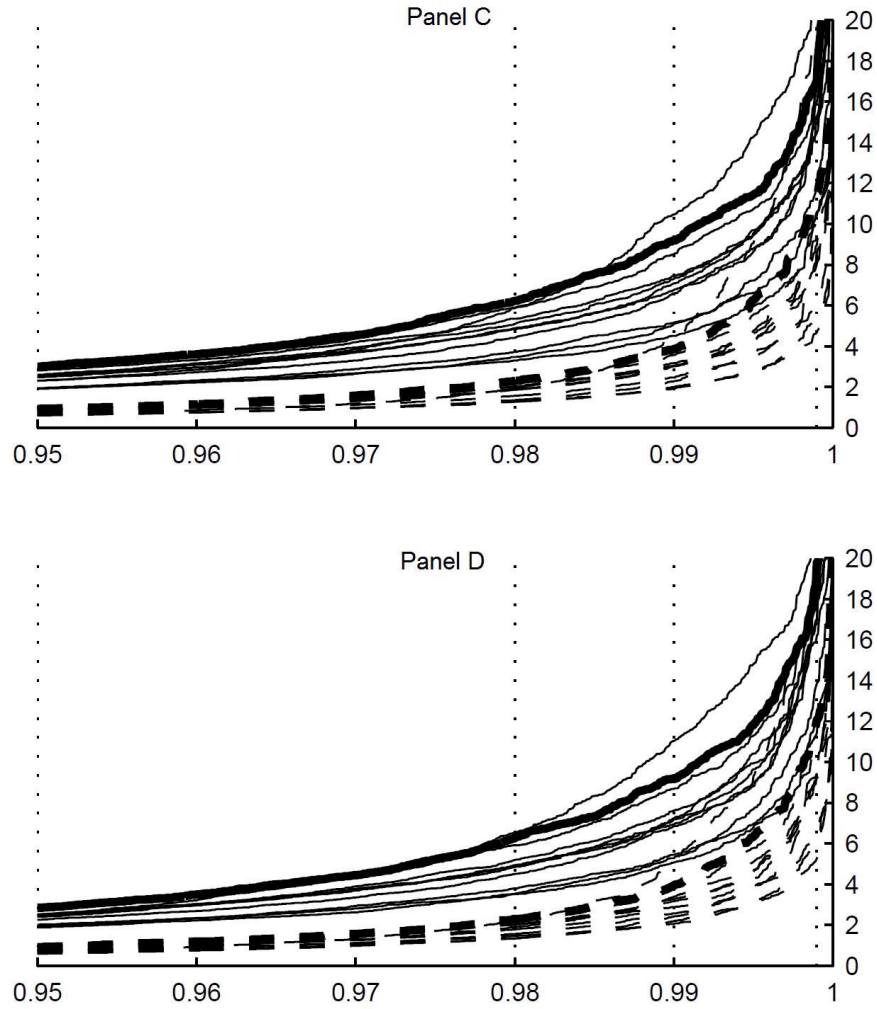


Figure 4.2: Unexpected Loss Distributions for Hierarchical Parametric Models. Panel C assumes a common national mean appreciation and regions-specific volatilities; Panel D assumes district-specific appreciation rates and volatilities. Both panels illustrate the tail distribution by hypothetical portfolios of 80 LTV mortgages. Percentiles of the distribution are on the x-axis, unexpected loss (loss minus expected loss) in percent is on the y-axis. Solid lines are for 620 FICO, dashed lines are for 740 FICO.

	Non-Parametric (Resampling)	Parametric Models			Latent Factor
	Model 1	Model 2A	Model 2B	Model 2C	Model 3
FICO 620					
Bank 1	143	390	441	451	412
Bank 2	189	341	371	380	300
Bank 3	239	451	534	485	574
Bank 4	176	315	346	353	341
Bank 5	283	542	589	591	746
Bank 6	220	446	487	485	460
Bank 7	205	340	328	352	312
Bank 8	187	673	586	653	699
Bank 9	243	463	508	519	497
Bank 10	221	428	483	494	541
FICO 740					
Bank 1	85	136	156	157	145
Bank 2	69	123	134	135	105
Bank 3	84	164	199	178	213
Bank 4	68	115	127	148	139
Bank 5	98	192	212	218	275
Bank 6	82	173	186	190	435
Bank 7	81	135	127	135	274
Bank 8	65	223	193	222	236
Bank 9	85	176	201	201	255
Bank 10	76	165	185	204	221

Table 4.5: BBB+ Risk Capital for Hypothetical Mortgage Portfolios (in Basis Points)

average of around 5 percent under the parametric methods. In one respect this is not surprising, because under the resampling approach individual worst-case realizations of the systematic risk factors are limited to the most severe than were actually observed during the historic period; in contrast, any parametric approach generates a full set of realizations drawn from the distribution given by the parametric assumptions, including worst-case realizations that may be more severe than have been observed in past years.

In particular, in the district-based parametric models that we estimated for this study house price appreciation rates are also driven by mortgage interest rates, so worst-case realizations of the interest rate paths are likely to also generate particularly bad house price scenarios. In addition, as noted above, worst-case interest rate scenarios can also significantly slow the level of estimated prepayments, exposing more loans to the possibility of default. Continued research to develop better specifications of the interest rate process and the relationship between interest rates and house prices is one area where further refinements are clearly needed.

Comparing the different versions of the parametric models, a few features stand out. First, the results for all of the parametric models are fairly similar, with the latent factor model providing slightly higher estimates on average. This may be partially a result of the fact that the district-based model does not fully incorporate cross-MSA correlation while the latent factor model freely parameterizes cross-MSA correlations. The results of the district-based models appear to be only slightly sensitive to the question of whether MSAs have different long-term mean rates of house price appreciation or whether they are constrained to share a common national long-term mean appreciation rate, and only slightly more sensitive to the question of whether MSAs share a common national volatility of house prices. The version with a common national mean and common volatility resulted in smaller estimated tail losses than the version with district-specific appreciation rates and district-specific volatility. Still, the differences among parametric models with different parameter assumptions are not nearly as great as the differences among modeling approaches, nor are they nearly as great as the differences among banks with different mortgage portfolio geographic distributions.

Finally, it is interesting to compare the results of the models above to the actual experience through the housing crisis over the last few years for mortgage pools originated in 2004 and prior (most accurately reflecting our loan period sampling). According to recent surveillance estimates from Standard and Poors for RMBS vintages of 2004 and prior, lower credit quality subprime loans will experience a total lifetime stress loss of about 625 basis points and higher credit quality prime loans about 100 basis points.¹¹ Expected loss for such pools were typically in the range of 300-400 basis points for subprime and 30-40 basis points for prime, implying un-

¹¹See Standard and Poors (2012). Note that significantly higher losses are expected to accrue to more recent vintages with extensive layered risk properties, which, as discussed above, are outside of our focus here.

expected loss of 225-325 basis points for subprime and 60-70 basis points for prime. Most of the estimates in Table 4.5 fall well within these ranges.

4.5 Conclusions

The goal of this exercise was not so much to estimate precisely the tails of the unexpected loss distribution for mortgage portfolios as to compare modeling approaches and to identify the assumptions or parameters that appear to be most important in generating good estimates of tail losses. The three general approaches taken to generate realizations of the set of systematic risk factors—nonparametric resampling, a standard parametric generation, and generation using a parametric latent risk factor model—all appear to be promising avenues for further development, but all are in their early stages of development.

In particular, the versions of district-based parametric models presented here do not yet incorporate cross-MSA correlation in house price movements, a limitation that presumably biases downward the estimated tail losses. On the other hand, the interest rate process and estimated sensitivity of house prices to interest rates clearly merit additional development.

In a parametric modeling framework the results suggest that calibration of house price volatilities and lag structures, while relevant, are not of very substantial importance in estimating the tails of the unexpected loss distribution; we will have to see whether this finding holds in further research. The question of whether long-term house price appreciation rates share a common national mean or can differ across districts (or MSAs) of the country seems to be more important—a very interesting finding, because the question of whether house prices can differ over a long term or whether supply and demand forces will equilibrate regional house prices eventually has been raised in many other contexts but, surprisingly, has not been answered with any degree of empirical support. Continued theoretical and empirical research into the likelihood that observed differences in mean appreciation rates across MSAs and across regions can persist indefinitely will be important in estimating portfolio tail losses parametrically.

The resampling approach is more straightforward than the parametric modeling approach because it substitutes historic observation for parametric assumptions and estimation of the mean, variance, and cross-MSA correlations in the systematic risk factors. The results do not enable us to make a definitive judgment as to whether the resampling approach is superior to any parametric approach. We suspect that the parametric assumptions and estimates incorporated into our parametric models are not yet refined enough to estimate tail losses confidently. On the other hand, the fact that the resampling approach generates much smaller estimated tail losses suggests that the limitation of the resampling approach—the fact that worst-case scenarios are limited to the worst scenarios observed historically—may be a material

shortcoming of this modeling approach. This potential shortcoming of the resampling approach in turn raises the question whether Basel II risk weighted assets calculations for residential mortgage credit risk may be too low.

Finally, there is virtually no limit to the types of extensions to the modeling frameworks considered here for estimating the joint dynamics for these fundamental risk factors in mortgage credit risk. These could include, for example, incorporating time-varying correlation, regime switching frameworks, and more fully integrated home price and interest rate processes, to name only a few. This study demonstrates that all such extensions are likely to have meaningful consequences for our estimates and understanding of tail risk in residential mortgages. This leaves open a very wide spectrum of possibilities for future research.

Bibliography

Backus, D., Foresi, S., & Telmer, C. (1998). *Discrete-time models of bond pricing* (NBER Working Paper 6736). Cambridge, MA: National Bureau of Economic Research.

Blundell, R.W., & Powell, J.L. (2003). Endogeneity in nonparametric and semiparametric regression models. In M. Dewatripont, L.P. Hansen, & S.J. Turnovsky (Eds.), *Advances in economics and econometrics: Theory and applications* (Eighth World Congress, Vol. II). Cambridge, UK: Cambridge University Press.

Blundell, R.W., & Powell, J.L. (2004). Endogeneity in semiparametric binary response models. *Review of Economic Studies* 71(3), 655-679.

Calem, P.S., Case, B., & Neale, A.J. (2004). *What do we know about portfolio-level credit risk of single-family mortgages?* Unpublished manuscript.

Calem, P.S., & Follain, J.R. (2004). *The asset correlation parameter in Basel II for mortgages on single-family residences* (Consultative Paper). Washington D.C.: Board of Governors of the Federal Reserve System.

Calem, P.S., & LaCour-Little, M. (2004). Risk-based capital requirements for mortgage loans. *Journal of Banking and Finance*, 28(3), 647-672.

Case, B., & Calem, P.S. (2004). *Co-movements in U.S. house price appreciation patterns*. Unpublished manuscript, Board of Governors of the Federal Reserve System, Washington, D.C.

Case, K. E., & Shiller, R. (1989). The efficiency of the market for single family homes. *American Economic Review*, 79(1), 125-137.

Cox, D.R. (1972). Regression models and life tables (with discussion). *Journal of the Royal Statistical Society B*, 34(2), 187-220.

Cox, D.R., & Oakes, D. (1982). *Analysis of survival data*. London, UK: Chapman and Hall.

- Cox, J.C., Ingersoll, J.E., & Ross, S.A. (1985). A theory of the term structure of interest rates. *Econometrica*, 53(2), 385-487.
- Cutts, A.C., & William, A. M. (2008). *Interventions in mortgage default: policies and practices to prevent home loss and lower costs* (Working Paper 08-01). Washington. D.C.: Freddie Mac Corporation.
- Deng, Y., & Quigley, J. M. (2003). *Woodhead behavior and the pricing of residential mortgages* (Working Paper Series 2003-1005). Lusk Center, University of Southern California.
- Deng, Y., Quigley, J.M., & Van Order, R. (2000). Mortgage terminations, heterogeneity and the exercise of mortgage options. *Econometrica*, 68(2), 275-307.
- Dougherty, A., & Van Order, R. (1982). Inflation, housing costs, and the consumer price index. *The American Economic Review*, 72(1), 154-164.
- Downing, C., Stanton, R., & Wallace, N. (2005). An empirical test of a two-factor mortgage prepayment valuation model: how much do house prices matter? *Real Estate Economics*, 33(4), 681-710.
- Duffie, D., & Singleton, K.J. (2003). *Credit risk: Pricing, measurement, and management*. Princeton, NJ: Princeton University Press.
- Dunn, K. B., & McConnel, J. J. (1981a). A comparison of alternative models for pricing GNMA mortgage-backed securities. *Journal of Finance*, 36(3), 471-483.
- Dunn, K. B., & McConnel, J. J. (1981b). Valuation of mortgage-backed securities. *Journal of Finance*, 36(3), 599-617.
- Dunn, K. B., & Spratt, C. S. (1986). *The effect of refinancing costs and market imperfections on the optimal call strategy and the pricing of debt contracts* (Working Paper). Pittsburgh, PA: Carnegie-Mellon University.
- Eckstein, Z., & Wolpin, K. I. (1989). The specification and estimation of dynamic stochastic discrete choice models: a survey. *The Journal of Human Resources*, 24(4), 564-598.
- Edelstein, R., & Lum, S.K. (2004). House prices, wealth effects, and the Singapore macroeconomy. *Journal of Housing Economics*, 13(4), 343-367.
- Foote, C. L., Gerardi, K. S., Goette, L., & Willen, P.S. (2009). *Reducing foreclosures* (Public Policy and Discussion Papers No. 09-2.). Boston, MA: Federal Reserve Bank of Boston.

- Gail, M.H., Wieand, S., & Piantadosi, S. (1984). Biased estimates of treatment effect in randomized experiments with nonlinear regressions and omitted covariates. *Biometrika*, 74(3), 431-444.
- Ghent, A. C., & Kudlyak, M. (2009). *Recourse and residential mortgage default: theory and evidence from U.S. states* (Working Paper No 09-10). Richmond, VA: Federal Reserve Bank of Richmond.
- Goodman, L. S., Ashworth, R., Landy, B., & Yang, L. (2012). Mortgage modification activity—recent developments. *The Journal of Fixed Income*, 21(4), 55-68.
- Gordy, M.B. (2000). A comparative anatomy of credit risk models. *Journal of Banking and Finance*, 24(1-2), 119-149.
- Gordy, M.B. (2003). A risk-factor model foundation for ratings-based bank capital rules. *Journal of Financial Intermediation*, 12(3), 199-232.
- Green, J., & Shoven J.B. (1986). The effects of interest rates on mortgage prepayment. *Journal of Money, Credit and Banking*, 18(1), 41-50.
- Grenadier, S., & Hall, B. (1996). Risk-based capital standards and the riskiness of bank portfolios: credit and factor risks. *Regional Science and Urban Economics*, 26(3-4), 433-464.
- Hayre, Lakhbir (Ed.) (2001). *Solomon Smith Barney guide to mortgage-backed and asset-backed securities*. New York, NY: John Wiley and Sons.
- Hwang, M., & Quigley, J.M. (2010). House price dynamics in time and space: predictability, liquidity, and investor returns. *Journal of Real Estate Finance and Economics*, 41(1), 3-23.
- Kalbfleisch, J.D., & Prentice, R.L. (2002). *The statistical analysis of failure time data*. Hoboken, NJ: Wiley.
- Kau, J. B., & Keenan, D. C. (1995). An overview of the option-theoretic pricing of mortgages. *Journal of Housing Research*, 6(2), 217-244.
- Kau, J. B., & Keenan, D. C. (1999). Patterns of rational default. *Regional Science and Urban Economics*, 29(6), 765-785.
- Kau, J. B., Keenan, D. C., & Kim, T. (1993). Transaction costs, suboptimal termination and default probabilities. *Journal of American Real Estate and Urban Economics Association*, 21(3), 247-263.
- Kau, J. B., Keenan, D. C., Muller, W. J., III, & Epperson, J.F. (1992). A generalized valuation model for fixed-rate residential mortgages. *Journal of Money, Credit and Banking*, 24(3), 279-299.

Lancaster, T. (1990). *The econometric analysis of transition data*. Cambridge, UK: Cambridge University Press.

Lee, N.J. & Ong, S.E. (2005). Upward mobility, house price volatility and prepayment risk. *Journal of Housing Economics*, 14(2), 127-146.

McCall, B.P. (1996). Unemployment insurance rules, joblessness, and part-time work. *Econometrica*, 64(3), 647-682.

Meese, R., & Wallace, N. (1994). Testing the present value relation for housing prices: Should I leave my house in San Francisco? *Journal of Urban Economics*, 35(3), 245-266.

National mortgage servicing standards and conflicts of interest: Hearing before the Subcommittee on Housing, Transportation, and Community Development of the Committee on Banking, Housing, and Urban Affairs, *United States Senate*, 112 Cong. (2011) (testimony of Laurie Goodman). Retrieved from [http://banking.senate.gov/public/index.cfm?FuseAction=Files.](http://banking.senate.gov/public/index.cfm?FuseAction=Files.View&FileStore_id=484c5b2b-6924-459f-898e-3ae075feeb15)

[View&FileStore_id=484c5b2b-6924-459f-898e-3ae075feeb15.](http://banking.senate.gov/public/index.cfm?FuseAction=Files.View&FileStore_id=484c5b2b-6924-459f-898e-3ae075feeb15)

Office of the Controller of the Currency (2011). *OCC mortgage metrics report: Disclosure of national bank and federal savings association mortgage loan data*. Washington, D.C.: Department of Treasury. Retrieved from <http://www.occ.treas.gov/publications/publications-by-type/other-publications-reports/mortgage-metrics-2011/mortgage-metrics-q3-2011.pdf>

Ong, S.E. (2000). Prepayment risk and holding period for residential mortgages in Singapore. *Journal of Property Investment and Finance*, 18(6), 586-601.

Ong, S.E. (2008). Mortgage market in Singapore. In D. Ben-Shahar, C.K.Y. Leung, & S.E. Ong, (Eds.), *Mortgage markets worldwide*. Oxford, UK: Blackwell Publishing.

Ong, S.E., Sing, T.F., & Teo, A. (2007). Delinquency and default in ARMs: The effects of loss aversion and protected equity. *Journal of Real Estate Finance and Economics*, 35(3), 253-280.

Ong, S.E., Thang, D., & Maxam, C. (2002). Mortgagor motivations in prepayments for adjustable rate mortgages. *Review of Urban and Regional Development Studies*, 14(2), 97-116.

Quercia, R. G., Ding, L., & Ratcliffe, J. (2009). *Loan modifications and redefault risk: an examination of short-term impact* (Working Paper). Center for Community Capital, University of North Carolina at Chapel Hill. Retrieved from http://www.ccc.unc.edu/documents/LM_March3_%202009_final.pdf.

- Quigley, J. M. (2001). Real estate and the Asian crisis. *Journal of Housing Economics*, 10(2), 129-167.
- Ridder, G., & Verbakel, W. (1983). *On the estimation of the proportional hazard model in the presence of heterogeneity* (actuarial science and econometrics report 22/83). Amsterdam, NL: University of Amsterdam.
- Rust, J. (1987). Optimal replacement of GMC bus engines: an empirical model of Harold Zurcher. *Econometrica*, 55(5), 999-1035.
- Schwartz, E.S., & Torous, W.N. (1989). Prepayments on fixed-rate mortgage-backed securities. *Journal of Finance*, 44(2), 357-392.
- Standard and Poors. (2012). *Methodology and assumptions: U.S. RMBS surveillance credit and cash flow analysis for pre-2009 originations*. Retrieved from http://www.standardandpoors.com/spf/upload/Events_US_US_SF_Event_rmbscriteria8912.pdf.
- Stanton, R. (1995). Rational prepayment and the valuation of mortgage-backed securities. *The Review of Financial Studies*, 8(3), 677-708.
- Stanton, R., & Wallace, N. (1999). Anatomy of an ARM: The interest rate risk of adjustable rate mortgages. *Journal of Real Estate Finance and Economics*, 19(1), 49-67.
- Stiglitz, J. E. (2010). *Freefall: America, free markets, and the sinking of the world economy*. New York, NY: W.W. Norton.
- Teo, A. H. (2004). Delinquency risk in residential ARMS: A hazard function approach. *Journal of Real Estate Portfolio Management*, 10(3), 243-258.
- Therneau, T. M., Grambsch, P.M., & Pankratz, V. S. (2000). Penalized survival models and frailty (Technical Report No 66). Mayo Foundation. Retrieved from <http://mayoresearch.mayo.edu/mayo/research/biostat/upload/66.pdf>.
- Therneau, T. M., & Grambsch, P.M. (2010). *Modeling survival data: Extending the Cox model*. New York, NY: Springer.