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Author

Glendenning, N.K.

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CAN THE HADRONIC MASS SPECTRUM BE DISCOVERED THROUGH
HIGH ENERGY NUCLEAR COLLISIONS?*

Norman K. Glendenning and Yasha Karant

8 August 1977

Nuclear Science Division
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

Abstract

It is suggested that the general form of the hadronic mass spectrum can be determined by studying the decay of hot hadronic matter produced in very high energy nuclear collisions. In a thermodynamic model we find that at presently attainable energies the temperature and the hadron populations in the matter depend strongly on the form of the hadronic spectrum.

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One of the fundamental properties of matter is the mass spectrum of the hadrons. Besides being an object of interest in particle physics, it has profound cosmological significance. The spectrum starts with the pion and rapidly becomes dense up to masses of about 2 GeV. Thereafter, according to our present knowledge, but most likely due to the difficulty of measurement, there are only a few additional known particles and resonances. The experimental spectrum is shown in Fig. 1 with the exception of isolated recent discoveries. According to the bootstrap hypothesis the spectrum continues beyond the known region and in effect, exponentially. The hypothesis can be stated simply as

follows: from among the known particles or resonances select two (or more) and combine their quantum numbers. The multiplet so obtained are also particles or resonances (at something like the sum of the masses). Add these to the pool of particles and continue. The spectrum thereby generated by Hamer and Frautschi¹ is also shown in Fig. 1. The implication is astonishing. The number of particles and resonances (each charge and spin state counted individually) grows so fast that at masses of only 2.5 GeV the number in a mass interval of the pion mass, expected from the bootstrap hypothesis, is $\sim 10^4$. The number of known hadrons is $\sim 10^2$. If new particles were discovered at the rate of one a day it would require of the order of a hundred years to confirm the bootstrap prediction by a direct count, and that in only such a small mass interval and at such a low mass!

We suggest as an alternative that it may be possible to determine the general behavior of the spectrum without the necessity of discovering the individual particles and resonances. The decay of hadronic matter produced in ultra high energy collisions between nuclei will depend upon the type and masses of particles that can energetically be produced; that is, on the mass spectrum of the hadrons. After that obvious statement, there remain two important questions. At what energy does the outcome of the collision depend sensitively on the unknown spectrum and what is the dynamical description of the reaction? We shall here assume that the colliding nucleons, assisted by particle production and the resulting subsequent collisions, attain a temporary state of equilibrium among the large number of hadrons, and shall answer the first question in the context of that dynamics. We conjecture that the energy at which sensitivity is achieved will not depend crucially on the dynamics assumed, although the observable signals may. To the nuclear physicist, the

possibility that an equilibrium state would be temporarily reached in a nuclear collision at high energy may seem remote. Nonetheless thermodynamic models in particle physics are not new, and they enjoy some success.² A collision involving initially many nucleons makes even more viable a statistical thermodynamic theory than one involving initially only two.

It is the aim of this research to determine whether the thermodynamic properties of hot hadronic matter occupying a volume of the order of nuclear dimensions is sufficiently sensitive to the unknown hadronic mass spectrum at attainable energies as to encourage an attempt to discover it experimentally in nuclear collisions. We shall refer hereafter to such hadronic matter as a nuclear fireball, it being suggested by the thermodynamic bootstrap theory of hadrons due to Hagedorn³ and already used in the context of nuclear collisions.^{4,5}

The partition function and momentum distribution function for an ideal relativistic gas of Fermions or Bosons of mass m and statistical weight $g = (2J+1)(2I+1)$ occupying a volume V at temperature T are (Hagedorn³) (units are $\hbar = c = k_{\text{Boltzman}} = 1$)

$$Z(V,T) = \frac{gV}{2\pi^2} m^2 T \sum_{n=1}^{\infty} \frac{(\mp)^{n+1}}{n^2} K_2\left(\frac{nm}{T}\right), \quad \begin{matrix} (F) \\ (B) \end{matrix} \quad (1)$$

$$f(p,T) d^3p = \frac{gV}{2\pi^2} \frac{p^2 dp}{\exp\left(\frac{1}{T} \sqrt{p^2 + m^2}\right) \pm 1}, \quad \begin{matrix} (F) \\ (B) \end{matrix} \quad (2)$$

from which the various thermodynamic quantities can be calculated. We want to describe a gas of Baryons and Mesons distributed in mass according to some unknown functions $\rho_B(m)$ and $\rho_M(m)$ for which Baryon number is conserved. This conservation can be achieved as usual by introducing a chemical potential

μ for the Baryons.⁶ The average number and energy^{*} for Baryons and Mesons are

$$B = \frac{VT}{2\pi^2} \int_{m_N}^{\infty} dm \rho_B(m) m^2 \sum_1^{\infty} \frac{(-)^{n+1}}{n} K_2\left(\frac{nm}{T}\right) \exp\left(\frac{n\mu}{T}\right) \quad (3)$$

$$E_B = \frac{VT}{2\pi^2} \int_{m_N}^{\infty} dm \rho_B(m) m^3 \sum_1^{\infty} \frac{(-)^{n+1}}{n} \left[K_1\left(\frac{nm}{T}\right) + \frac{3T}{nm} K_2\left(\frac{nm}{T}\right) \right] \exp\left(\frac{n\mu}{T}\right) \quad (4)$$

$$M = \frac{VT}{2\pi^2} \int_{m_{\pi}}^{\infty} dm \rho_M(m) m^2 \sum_1^{\infty} \frac{1}{n} K_2\left(\frac{nm}{T}\right) \quad (5)$$

$$E_M = \frac{VT}{2\pi^2} \int_{m_{\pi}}^{\infty} dm \rho_M(m) m^3 \sum_1^{\infty} \frac{1}{n} \left[K_1\left(\frac{nm}{T}\right) + \frac{3T}{nm} K_2\left(\frac{nm}{T}\right) \right] \quad (6)$$

We shall consider collisions between identical nuclei with isospin $I=0$. In that case charge conservation is implied already. For a grid of values of T , we solve the first equation for the value of μ which yields the baryon density desired. The remaining equations then tell us the corresponding energy and meson number.

It may seem strange that a strongly interacting hadronic system is discussed in terms of an ideal gas. However Hagedorn has argued convincingly, on the basis of statistical mechanical techniques introduced by Beth Uhlenbeck and Belenkij⁷ that the hadronic spectrum is the manifestation of the hadronic interactions, and that by introducing the complete hadronic mass spectrum one has accounted for their interactions completely.

*In statistical mechanics, we are of course working in the grand canonical ensemble, making the identification of "microscopic" conservation with the ensemble averages.

We consider two extreme possibilities for the mass spectrum of hadrons. One is a representation of the current experimental situation. We use a discrete spectrum of the known non-strange hadrons up to mass ~ 1680 MeV with their widths (14 mesons and 6 baryons). We continue this with a continuum of constant density equal to the average in the region 1820-2520 MeV, which is 27.5 per pion mass. The other is a Hagedorn mass spectrum normalized to agree with the average experimental density in five pion mass intervals centered at 1400 MeV and with a slope in agreement with the bootstrap iteration of Fig. 1. This continuous spectrum we use for $m > 1680$, and we use the discrete experimental spectrum of 20 hadrons mentioned above for the low mass end. The number of baryons and mesons in the continuous region are assumed to be equal. In summary the extreme assumptions are

$$\rho_{\text{exp}}(m) = \begin{cases} 20 \text{ discrete hadrons} & , \quad m < 12 m_{\pi} \\ 27.5 \text{ per pion mass} & , \quad m > 12 m_{\pi} \end{cases} \quad (7)$$

$$\rho_{\text{Hagedorn}}(m) = \begin{cases} 20 \text{ discrete hadrons} & , \quad m < 12 m_{\pi} \\ \frac{1.12 e^{m/T_0}}{(m/T_0)^3} / \text{pion mass} & , \quad m > 12 m_{\pi} \end{cases} \quad (8)$$

$$T_0 = 0.958 m_{\pi} \quad , \quad m_{\pi} = 140 \text{ MeV}$$

Corresponding to the above extreme assumptions, the temperature of the fireball is plotted in Fig. 2 as a function of the total center of mass energy per nucleon for a symmetric collision. For a perfectly inelastic collision, this is the C.M. collision energy per nucleon including rest mass. If the hadron spectrum increases exponentially the temperature cannot exceed T_0 ,³ but the temperature can increase without bound for the less rapidly increasing

spectrum. We find a large difference in the temperature for the two assumed spectra at energies in range of the CERN storage rings. This difference is dramatically reflected in the populations of the various hadronic states, the lower mass ones being illustrated in Fig. 3. This encourages us to believe that even at presently attainable energies, one might be able to discover the form of the hadronic spectrum in ultra high energy nuclear collisions.

The next steps in our research will involve the study of several different models for the expansion of the fireball to a freezeout density as well as including the other hadronic degrees of freedom, such as the additional flavors. We hope that the large difference between the two assumed spectra will persist to the freezeout density where thereafter it will be preserved in the spectra of observed particles.

We interpret our suggestion and result as follows. The true dynamics is without doubt much more complicated than equilibrium thermodynamics. The other extreme dynamics is a sequence of independent collisions. In this extreme there would be no advantage to studying nuclear collisions over the traditional hadron-hadron collisions. Most likely the truth lies somewhere between the extremes. Extensive theoretical developments in understanding the dynamics of high energy nuclear collisions, assisted by much experimentation are needed before we can even contemplate extracting the hadronic mass spectrum from data on nuclear collisions. What this paper shows is that the properties of hot hadronic matter are very sensitive to the hadronic mass spectrum. It therefore provides the motive for pursuing what is surely a long and difficult task.

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FOOTNOTES AND REFERENCES

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FIGURE CAPTIONS

Fig. 1. The density of different hadrons, ρ , in unit interval is plotted as a function of the mass in units of the pion mass. The experimentally known particles and resonances with their multiplicities are shown in the shaded areas. The dotted histogram is a bootstrap iteration¹ on the known spectrum and the solid curve is a Hagedorn type spectrum, fitted to the bootstrap.

Fig. 2. The temperature of hot hadronic matter assumed to be produced in a symmetric nuclear collision is plotted as a function of the C.M. total energy per nucleon of the colliding nuclei for a volume corresponding to about the initial density of nucleons. The curve labelled "Experimental" corresponds to a mass spectrum that approximates the known spectrum Eq. (7), while that labelled "Hagedorn" corresponds to a Hagedorn spectrum, Eq. (8). ($B/V = 1/4$).

Fig. 3. Corresponding to the two situations described in Fig. 2, the meson and baryon populations are shown. Since baryon number is conserved, all low mass baryon populations eventually decrease as higher mass ones (not shown) increase. Figure 3(a) corresponds to the experimental spectrum; Figure 3(b) corresponds to the Hagedorn spectrum.

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Fig. 1

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Fig. 2

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Fig. 3(a)

FOR HAGEDORN SPECTRUM
BARYON AND MESON POPULATIONS

XBL 779-4876

Fig. 3(b)

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