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Authors
Llacer, J.
Spieler, H.
Goulding, F.S.

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UNIVERSITY OF CALIFORNIA

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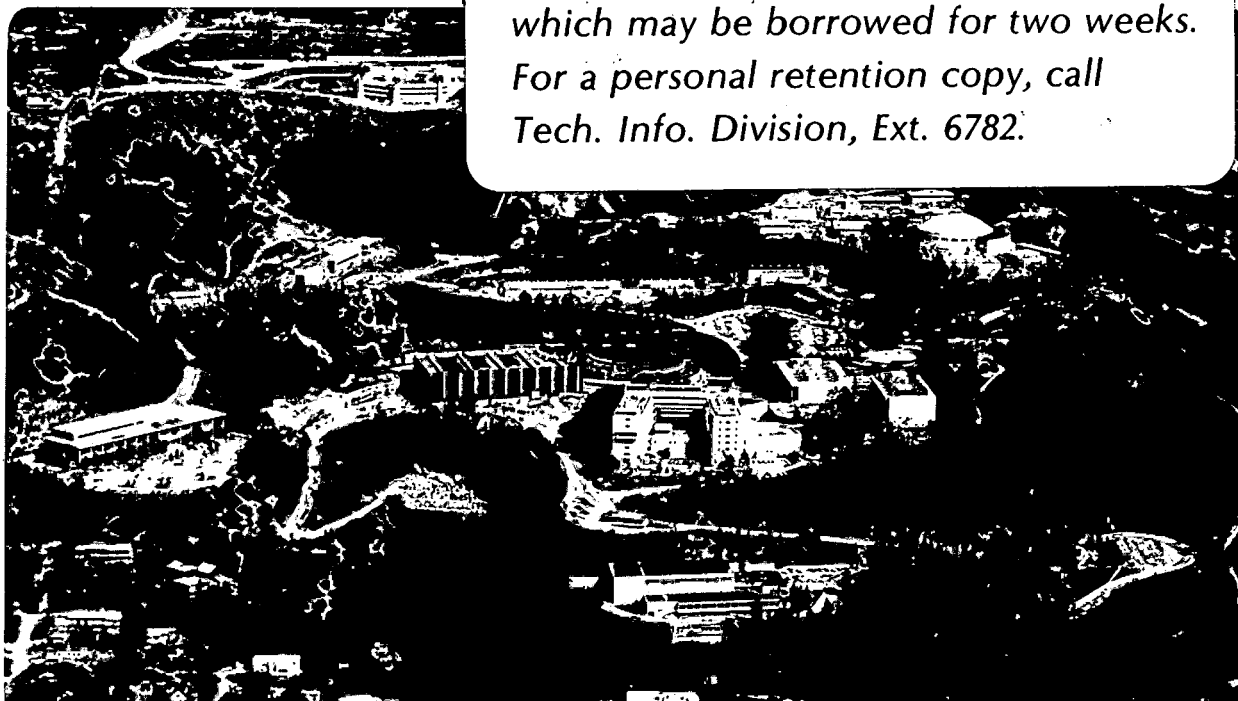
THEORETICAL ANALYSIS OF THE USE OF GERMANIUM DETECTORS FOR TIME-OF-FLIGHT EMISSION TOMOGRAPHY

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THEORETICAL ANALYSIS OF THE USE OF GERMANIUM DETECTORS
FOR TIME-OF-FLIGHT EMISSION TOMOGRAPHY

Jorge Llacer, Helmuth Spieler and Frederick S. Goulding

Lawrence Berkeley Laboratory, University of California
Berkeley, California 94720 U.S.A.

ABSTRACT

A theoretical analysis of the timing capabil-
ities of Ge semiconductor detectors in time-of-
flight positron emission tomography is present-
ed. The effect of detector size on efficiency
and time resolution is discussed. The relevant
noise sources are determined and the optimum fil-
ter is derived to optimize the slope-to-noise
ratio, while minimizing the effect of collection
time variations on time resolution. The perform-
ance of the ideal filter is compared with a sin-
gle RC integrator. For a lower energy threshold
of 200 keV, time resolution of better than 250 ps
seems to be a realistic goal for a detector of
0.5 x 0.5 cm cross section and 3 cm length. This
detector would yield an overall efficiency of 36%
for 511 keV gamma-rays and 80% of the detected
photons would fall in the Gaussian part of the
timing spectrum.

INTRODUCTION

The use of germanium detectors for time-of-
flight positron emission imaging was first pro-
posed by Llacer and Cho in 1973[1]. At that time
the role of time-of-flight techniques in improving
the signal to noise ratio of image reconstructions
was not understood, and the idea was not pursued
experimentally. In more recent papers Kaufman
et al[2,3] have discussed the potential of Ge
detectors for time-of-flight assisted tomography
and presented some preliminary data. Recent dis-
cussions with M. Ter-Pogossian and N. Mullani
have stimulated our interest in reassessing the
timing properties of Ge-detectors in the light of
modern technology. In this paper we present a
theoretical analysis which points out the relevant
parameters in the detector and its associated
electronics and shows their effect on timing
accuracy. The purpose of this analysis is to
define realistic goals and outline ways of
achieving them. Practical aspects of timing with
semiconductor detectors have been treated in a
recent tutorial paper by Spieler[4]. Measurements
to ascertain the limits of available components
in this specific application will begin shortly.

Time resolution in a semiconductor detector
system is determined by the detected energy, the
size and shape of the detector, electronic noise

and the filtering of signal and noise components.
These are treated individually in the following
sections.

DETECTOR GEOMETRY

Size and shape of the detector affect both
efficiency and collection time. The detection
efficiency for 511 keV gamma-rays has been calcu-
lated for three crystal sizes which seem appro-
priate for this application. The length of the
detectors is 3 cm in the direction of the incoming
gamma-rays. The square cross sections have sides
of 0.5, 0.75 and 1 cm. The calculation has been
carried out by a Monte Carlo code written by
S. E. Derenzo[5], which has been validated in
independent measurements. Table 1 shows the
results of the calculation for five different
energy thresholds--assuming perfect energy reso-
lution. Uniform perpendicular illumination of
the entrance face is assumed.

Table 1

Detection Efficiency of a Single Ge Detector
for 511 keV Photons. The detector's length is
3 cm.

Energy Threshold (keV)	Side Dimension (cm)		
	0.5	0.75	1.0
100	52 %	52 %	52 %
200	36 %	38 %	39 %
300	16 %	19 %	22 %
400	6.4%	8.3%	10.3%
510	6.0%	7.8%	9.8%

Efficiency at lower energies is practically iden-
tical for the three sizes, whereas the efficiency
at 510 keV increases by about 60% when the side
dimension is increased from 0.5 to 1.0 cm.

SIGNAL GENERATION

The process of signal generation in a semi-conductor detector will be reviewed briefly in order to demonstrate the origin of the waveforms from which the timing information must be derived. Consider a charge in a semiconductor depletion layer of thickness d, Fig. 1. We shall assume that the electric field in the depletion region is uniform and high enough ($> 10^3$ V/cm) that the electrons and holes generated by ionizing radiation move at a constant saturation velocity v_s. As a single carrier moves, the change in induced charge at each contact is $dq/dt = qv_s/d$. This current is constant during the transit time.
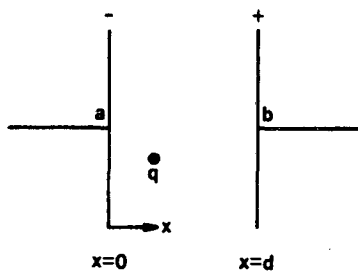
x=0   x=d

XBL 825-10009

Fig. 1. A unit charge q at a distance x from contact a of a semiconductor detector of thickness d.

Depending on the position along x where a gamma-ray interaction occurs, the current pulses will have different shapes. At any time the current flow is due to the simultaneous movement of electrons and holes, or due to one of these if the other has been collected. Consider first a number of electron-hole of pairs generated very near contact a in Fig. 1. The current i will be almost entirely due to the motion of electrons. The total collected charge Q will be given by

$$Q = \int_0^{\tau_t} i \; dt = \frac{q \; E_d}{\epsilon} \qquad (1)$$

where $\tau_t = d/v_s$ is the transit time, E_d is the detected energy in eV, ϵ is the average energy needed to generate one electron hole pair (2.98 eV in Ge) and q is the electronic charge. This current pulse is shown in Fig. 2a. The transit time for a 1 cm thick detector is approximately 100 ns and decreases linearly with detector thickness.

Next, consider a gamma-ray interaction at x = d/2. Both electrons and holes will contribute to the current flow, but current will only flow during $\tau_t/2$ (Fig. 2b). Since the currents due to electrons and holes add, the instantaneous current is twice that of the first case. Examples

for interactions at x = d/4 and d/16 are shown in Figs. 2c and 2d, respectively.

There will be a large number of events where 511 keV gamma-rays will lead to more than one interaction in a Ge detector (Compton scattering followed by photoelectric absorption, for example). Figure 2e shows the current pulse for a hypothetical event in which two interactions occurred at x = d/4 and d/2, each depositing an energy $E_d/2$. This general class of event will be rather frequent: Derenzo's Monte Carlo code[5] shows that for a 200 keV energy threshold and a detector with a side dimension of 1 cm, 48% of the detected particles have one single inter-action, 29% undergo two interactions and the remaining 33% suffer three or more interactions.
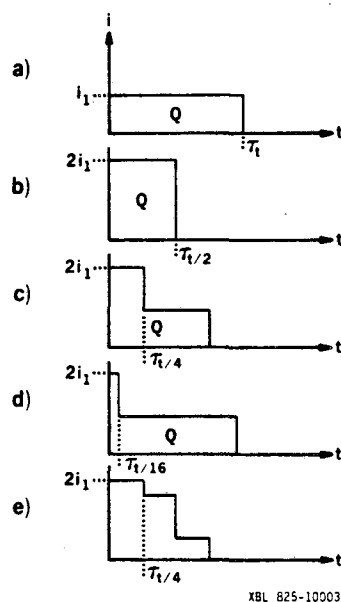
Fig. 2. Current pulse shapes in a semiconductor detector: a) For a detected event infinitesimally near one of the contacts, b) Single event occurring at x = d/2, c) Single event at d/4, d) Single event at d/16, e) Two events, each depositing Q/2, occurring at d/4 and d/2 respectively.

It is important to note that, except in the case of a single interaction arbitrarily close to one of the contacts, all current signals initially attain a current $2i_1$, where $i_1 = Q/\tau_t$; that is, nearly all signals begin with a dual carrier component of equal magnitude.

EQUIVALENT INPUT CIRCUIT

Signal

A simplified model of a detector input circuit is shown in Fig. 3. The detector is represented by a current source and a parallel capacitance,

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which is lumped together with the input capacitance of the amplifier to form the total input capacitance, C_i. The amplifier is assumed to exhibit infinite input impedance. The parallel resistance R_i should be made very high, so that its noise contribution in the passband of interest is negligible. This will be discussed in the following section on noise. Assuming that $R_i \gg \infty$, the signal current will be integrated by the input capacitance C_i, forming a voltage pulse V_i, and after passing through a suitable amplifier of very high bandwidth is then filtered by an optimum linear filter designed to optimize timing.
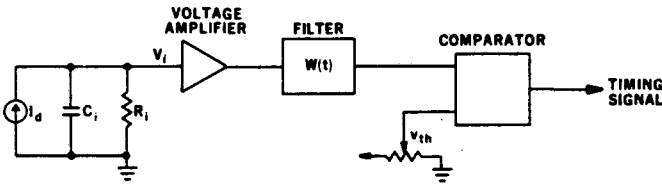

Fig. 3. Simplified model of the timing circuit.

For the analysis a simple leading edge discriminator with a fixed threshold voltage is used to determine the timing of the pulse. In practice, the signal amplitudes vary and some form of amplitude compensation should be used, (e.g. a constant fraction discriminator). The essential limitations of the system can, however, be studied assuming a fixed signal amplitude at the input of a leading edge discriminator.

In principle, either the current or the voltage signal of the detector could be used[4]. As mentioned above, the voltage signal is formed by integrating the detector current on the input capacitance C_i. Compared to current mode operation into a 50 Ohm input impedance this may provide more than an order of magnitude improvement in signal-to-noise ratio for the detector sizes of interest in this application, even if a 'noiseless' input termination is used[4]. The reasons for this will be explained later. The following analysis will, therefore, be restricted to the voltage mode.

The voltage signals V_i for the five cases in Fig. 2 are shown in Fig. 4. Clearly, the shapes of these signals show that it is necessary to set the trigger threshold so low, that it remains in the two carrier part of the pulse transition for a sufficiently large proportion of events.
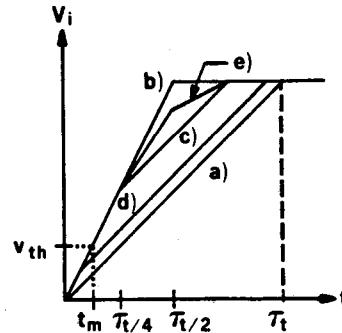
Fig. 4. Voltage waveforms v_i corresponding to the current waveforms in Fig. 2.

Noise

The equivalent input noise sources are shown in Fig. 5. We assume that shot noise associated with the detector leakage current and the equivalent input noise current of the amplifier are negligible, a condition which is readily fulfilled. The noise source v_s accounts for the equivalent series noise resistance of the amplifier and any series resistance in the input signal path. It generates a mean square noise voltage

$$v_s^2 = 4kTR_s \cdot \Delta f \qquad (2)$$

where Δf is the noise bandwidth of the system.

The parallel noise source v_p is primarily due to biasing networks shunting the signal input. It would also include a shunt resistor used for impedance matching. Its spectrum is attenuated at high frequencies by the input capacitance, yielding a mean square noise voltage

$$v_p^2 = \int_0^\infty \frac{4kTR_i}{1 + (\omega R_i C_i)^2} \, df \qquad (3)$$

This contribution can only be neglected, if R_i is sufficiently large ($> 10^5$ Ohms in this application). A 50 Ohm matching resistor, for example, incurs a significant noise penalty.
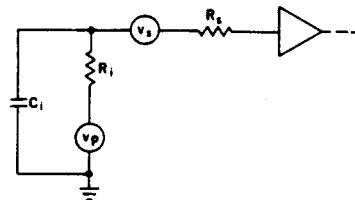
Fig. 5. Equivalent input noise sources.

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This 'skirt' could be eliminated by using the detector current pulse for timing (i.e. choosing the input time constant $\tau_i = R_i C_i$ to be small compared to the collection time τ_t. However, resolution is severely degraded, since the additional noise incurred by the large bandwidth required for this mode outweighs the increase in slope of the signal transition. An optimum filter designed for a 1 cm thick detector operated with a 50 Ohm input resistance and $t_m = 400$ ps results in time resolution nearly seven times worse than for the voltage mode as described above. In practice the degradation will be even greater, since the average spectral noise density will be higher at the gigahertz bandwidths required. Many investigators who believe they are timing on the current pulse actually have bandwidth limitations in their systems - e.g., in the connections between the detector and the preamplifier, in subsequent amplifier stages or in the timing discriminator (Ref. 4) - which effectively integrate the signal, resulting in an inefficient form of voltage mode operation.

The curves of Fig. 8 correspond to the time resolution for a single detected energy E_d. In practice, however, the gamma-ray spectra in the Ge detector will have a range of energies and the timing spectra will therefore be a composite of individual Gaussians of different widths. In order to estimate the effective timing Gaussian width for a realistic situation, the detected energy spectrum of annihilation gamma-rays after passing through a lucite absorber of 12 cm thickness was calculated by Monte Carlo methods, providing a probability distribution P(E). A Ge detector of dimensions 0.5 x 0.5 x 3 cm was assumed in the simulation. This yields the composite time response function

$$P(t) = \int_{E=200}^{511} \frac{P(E)}{\sigma\sqrt{2\pi}} \exp\left(-t^2/2\sigma^2\right) dE \qquad (15)$$

where $\sigma = 3.4 \times 10^{-8}/E(keV)$ for the parameters indicated above. Figure 9 shows the resulting line shape. It corresponds very closely to a Gaussian function with 240 ps FWHM, which is what would be expected for a single detected energy $E_d = 330$ keV.
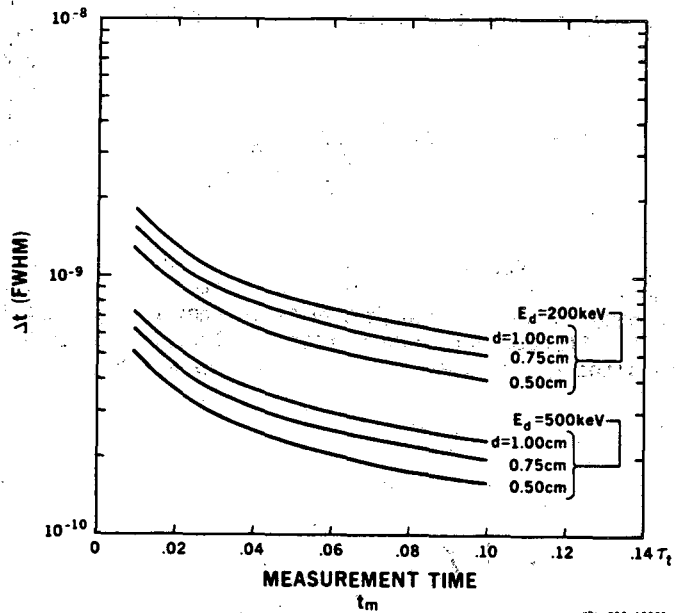

Fig. 8. Calculated time resolution FWHM as a function of the chosen measurement time t_m, for detected energies of 200 and 500 keV. The detectors are 3 cm in length, with cross sections of 0.5 x 0.5, 0.75 x 0.75 and 1.0 x 1.0 cm.
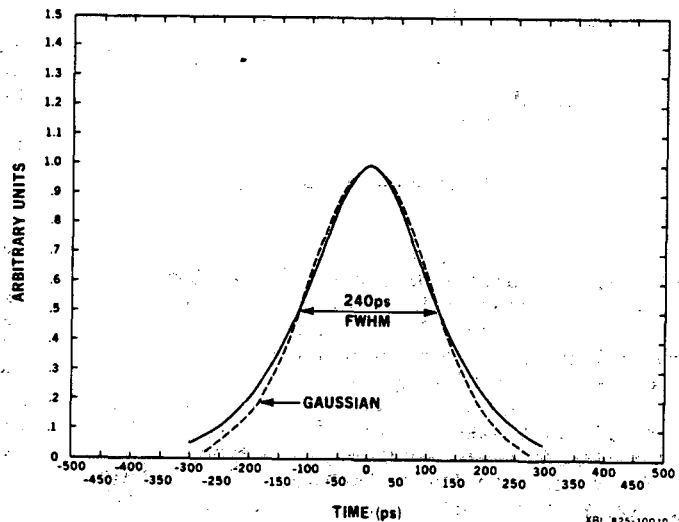

Fig. 9. Calculated timing spectrum for an energy distribution due to 511 keV gamma-rays passing through a 12 cm lucite absorber and weighted by the efficiency characteristics of the detector.

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CONCLUSION

The results of this analysis show that Ge detectors offer excellent time resolution and reasonable efficiency, with overall performance quite competitive with CsF scintillation detectors. They also have additional useful features for time-of-flight positron emission tomography: small dimensions, providing reduced pixel size over scintillator arrays and a physically compact assembly, despite the requirements of vacuum and cooling.

Clearly, this analysis should be verified experimentally. In particular, considerable work is required to determine the best input amplifying device. However, the technology for fabricating large arrays of Ge detectors is well developed and we feel that this may be the right time to seriously investigate the use of germanium detectors in this promising application.

ACKNOWLEDGMENTS

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