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February 1981

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Optimal Energy Extraction From a Hot Water Geothermal Reservoir

KAMAL GOLABI,¹ CHARLES R. SCHERER,² CHIN FU TSANG,³ AND SASHI MOZUMDER⁴

An analytical decision model is presented for determining optimal energy extraction rates from hot water geothermal reservoirs when cooled brine is reinjected into the hot water aquifer. This applied economic management model computes the optimal fluid pumping rate and reinjection temperature and the project (reservoir) life consistent with maximum present worth of the net revenues from sales of energy for space heating. The real value of product energy is assumed to increase with time, as is the cost of energy used in pumping the aquifer. The economic model is implemented by using a hydrothermal model that relates hydraulic pumping rate to the quality (temperature) of remaining heat energy in the aquifer. The results of a numerical application to space heating show that profit-maximizing extraction rate increases with interest (discount) rate and decreases as the rate of rise of real energy value increases. The economic life of the reservoir generally varies inversely with extraction rate. Results were shown to be sensitive to permeability, initial equilibrium temperature, well cost, and well life.

INTRODUCTION

This paper is concerned with the extraction of energy from a hot water geothermal field. The central question is what extraction rate maximizes the net discounted value of the energy extracted when the cooled brine is reinjected.

The determination of the rate of geothermal energy extraction is important for both conceptual and practical policy reasons. A great deal of conceptual work has been done on the theory of socially optimal natural resource extraction, beginning with *Hotelling* [1931]. This has been summarized in a working paper by *Peterson and Fisher* [1976]. Other recent work includes the proceedings of the *Symposium on Exhaustible Resources* [1974] and work by *Heal* [1976], *Solow and Wan* [1976], *Pearce and Rose* [1975], *Schulze* [1974], *Gordon* [1967], *Herfindahl* [1967], and *Smith* [1968]. However, as *Peterson and Fisher* [1976, p. 17] point out,

In their current state, these models are excellent vehicles for teaching concepts and techniques of dynamic optimization, especially in the presence of externalities. Unfortunately, they cannot be used to manage actual natural resources, because their functional forms are too simple and their empirical content too low.

Accordingly, one goal of the present study is the introduction of some 'empirical content' into the theoretical optimal extraction literature, providing a bridge between theory and practical application. To this end a particular extractable resource and technology has been selected, and a hydrothermal expression relating the rate of heat energy extraction and the 'quality' of the remaining unextracted energy has been introduced. For the case of a production-reinjection geothermal well doublet with homogeneous aquifer the expression describes the relationship between geothermal brine pumping rate and the temperature of the production well brine over time.

The practical policy implications of this work pertain to the economics of geothermal extraction as well as to decisions on how rapidly and for how long private and public sector geothermal fields should be developed. These are questions of

relevance to both private energy companies and federal government agencies such as the U.S. Geological Survey and the U.S. Bureau of Land Management, the agency that inventories and leases land for mineral exploration.

The following specific questions are addressed in this paper:

1. At what rate and for what duration of time should a geothermal reservoir be exploited, and to what degree should the brine be cooled before reinjection in the aquifer in order to maximize the present worth of profits?
2. To what extent are these decisions dependent on the pertinent geohydrological and economic parameters? In particular, how are these decisions affected by variations or uncertainty in these parameters (especially interest rates and rates of growth in energy value)? How cost beneficial would it be to obtain additional information regarding these parameters before the extraction facilities are designed?
3. What is the economic worth of a geothermal reservoir, and in what manner and to what extent is this value dependent on physical geohydrological parameters and economic parameters?
4. To what extent can regulatory agencies influence the economics and rate of exploitation of geothermal energy by manipulating factors such as royalty and land rents?

BACKGROUND

The hot water geothermal system referred to in this paper is a saturated aquifer bounded top and bottom by impermeable aquicludes (see Figure 1). The water in the aquifer (before extraction) is in thermal equilibrium with both aquicludes and with the aquifer matrix. The heat source is assumed to be from magmatic intrusions into the earth's outer crust. Although a thermal gradient exists everywhere from the center to the surface of the earth, a geothermal field, also referred to as an 'anomaly,' may be detected (actually defined) by an unusually steep local thermal gradient near the earth's surface.

With respect to the next few decades the energy in the intrusive heat source is assumed to be infinite. However, since the rate of heat transfer across the aquicludes to the aquifer matrix and fluid is governed by the rate of thermal conductivity, it is possible to cool the aquifer to a temperature below which the remaining heat has no economic value. The regeneration of the aquifer is relatively slow, so the energy of the geothermal field can be considered to be nonrenewable in the future of economic relevance.

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From a strictly hydraulic point of view the aquifer may be renewable, in the sense that any groundwater system is renewable. However, if the aquifer is overdepleted, the matrix might be permanently compacted, reducing future hydraulic yield and inducing subsidence of the overburden. Since this subsidence may be highly undesirable for unrelated reasons, it will be necessary to reinject the spent brine; hence the introduction above of a production-reinjection doublet. Good examples of potential damage from subsidence are found in the California Imperial Valley, where substantial subsidence could seriously disturb the vertical alignment of the irrigation canals, and in the Wairaki fields in New Zealand [Atherton *et al.*, 1976; Stilwell *et al.*, 1975].

The hydraulic operation of a hot water geothermal system may be described as follows. Water is pumped from the production well (at point 1 in Figure 1) up to the surface, where it enters the heat exchanger (point 2). After it leaves the heat exchanger (point 3), it is piped to the reinjection well (point 4) and pumped back into the aquifer (point 5). It then moves through the aquifer toward point 1. The flow is turbulent from point 1 to point 5 and laminar from point 5 to point 1. Initially, when pumping begins, the entire aquifer is in equilibrium at temperature T_0 . Water is pumped out at that temperature and cooled to T_i as the energy is extracted. After the water is pumped back into the aquifer, it is heated by the aquifer matrix as it moves from point 5 to point 1. For some period the water will be heated back to T_0 by the time it arrives at point 1.

As heat is transferred from the aquifer matrix to the fluid, the temperature of the matrix decreases until the time τ , when the matrix can no longer heat the fluid to T_0 by the time the fluid reaches point 1. When this happens, the production well temperature at point 1 (and hence at point 2) will begin to drop. Denoting the time-variable production well temperature as T_0' , this process of temperature degradation over time can

Fig. 1. The system.

be plotted as shown in Figure 2. The point in time τ when temperature begins to decline below T_0 is called 'breakthrough,' referring to the time when the reduced fluid temperature breaks through to the production well. Breakthrough is inversely proportional to Q , the extraction rate. The post-breakthrough rate of decrease in temperature also depends on Q and reinjection temperature T_i . As $T_0' \rightarrow T_i$, the cost (per Btu) of heat extraction increases, and there is a time when it is no longer 'economical' to extract more heat though there may be a great deal of heat energy left in the aquifer.

HYDROTHERMAL RELATIONSHIP

The relationship between production well temperature and time for a given flow rate is defined in this section. Hot brine is withdrawn at the rate Q and reinjected at the same rate. The temperature of the reinjected fluid at time t is denoted by T_i' . For the first τ years ($0 \leq t \leq \tau$), $T_0' = T_0^0$, where $T_0^0 \equiv T_0$ is the initial equilibrium temperature of the unexploited reservoir, and τ denotes the breakthrough time. The breakthrough time is described by the following relationship [see Tsang *et al.*, 1976]:

$$\tau(Q) = t_u/6 \quad (1)$$

where t_u is a unit for time,

$$t_u = \frac{2\pi h D^2 \rho_a c_a}{8760 Q \rho_f c_f} \quad (2)$$

and h is the aquifer thickness (meters), D the well separation (meters), Q the pumping rate (cubic meters per hour), and ρ_a , ρ_f , c_a , and c_f the densities and specific heats of the aquifer and the fluid, respectively. The relationship between the heat capacities of the aquifer matrix and the fluid is given by

$$\rho_a c_a = \phi c_f \rho_f + (1 - \phi) \rho_R c_R \quad (3)$$

where ϕ is the porosity of the aquifer and ρ_R and c_R are the density and specific heat of the aquifer matrix.

For the purpose of this analysis the temperature drop in surface pipes is neglected so that the temperature of the fluid entering the heat exchanger is T_0' and the injection temperature T_i' equals the temperature of the fluid leaving the heat exchanger. After breakthrough, T_0' is determined by a function $\bar{g}(T_i', t/t_u)$ which gives the ratio of the temperature drop through the heat exchanger experienced by the brine at time t to that at time zero:

$$\frac{T_0' - T_i'}{T_0 - T_i} = \bar{g}(T_i', t/t_u) \quad (4)$$

It can be shown that the variation in T_i' is small and $\bar{g}$ can be approximated by a function g which is valid for invariant T_i [Golabi and Scherer, 1977a]:

$$\frac{T_0' - T_i}{T_0 - T_i} = g(t/t_u) \quad (5)$$

Using the results of the Gringarten and Sauty [1975] model, an expression for g has been developed [Tsang *et al.*, 1976] and is given by

$$\begin{aligned} g(t/t_u) &= 1 & t \leq \tau \\ g(t/t_u) &= \gamma_1 e^{-\phi_1 t/t_u} + \gamma_2 e^{-\phi_2 t/t_u} + \gamma_3 e^{-\phi_3 t/t_u} & t \geq \tau \end{aligned} \quad (6)$$

Fig. 2. Temperature versus time plot for a given flow rate.

where $\phi_1 = 0.0138$, $\phi_2 = 0.656$, $\phi_3 = 8.006$, $\gamma_1 = 0.338$, $\gamma_2 = 0.337$, and $\gamma_3 = 1.368$.

The thermodynamics of heat transfer on the surface are worked out on the assumption that the energy will be used for nonelectrical steam production (e.g., space heating). Hot water from a building complex enters the heat exchanger at the saturation temperature of steam (at the desired pressure) T_s (at point 6 in Figure 1), is heated, leaves the heat exchanger as steam at T_s (at point 7), is circulated throughout the building complex, losing heat to the building in the process of phase change, and returns as condensate to the heat exchanger (at point 6).

Aside from piping heat losses there are only two ways in which heat can leave the doublet system (which includes reservoir as well as surface equipment): by transfer to the steam cycle and by heat loss from the heat exchanger. The heat loss is considered in the model section. For now it is convenient to neglect heat losses in the heat exchanger and assume that all the heat removed from the brine at any given time is used to generate steam.

The effectiveness ϵ of a heat exchanger is defined in terms of the hot and cold brine temperatures T_0' and T_i and the temperature of the cold side of the exchanger, T_s . The maximum transferable heat (given an infinite exchange area) is $QC_f\rho_f(T_0' - T_s)$. However, since heat transfer becomes very expensive as T_i approaches T_s , the heat actually transferred is usually less than the maximum defined above. Accordingly, ϵ is defined as the heat actually transferred divided by the maximum amount of heat that could be transferred given an infinite transfer surface area [Edwards et al., 1973, p. 253]:

$$\epsilon = \frac{QC_f\rho_f(T_0' - T_i)}{QC_f\rho_f(T_0' - T_s)} = \frac{T_0' - T_i}{T_0' - T_s} \quad (7)$$

For a given Q , (7) implies that ϵ increases to unity as $T_i \rightarrow T_s$. In addition, the effectiveness of a heat exchanger is a function of the number of transfer units $NTU(t)$ at time t [Edwards et al., 1973, p. 243],

$$\epsilon = 1 - e^{-NTU(t)} \quad (8)$$

where

$$NTU(t) = k(t)A/Q \quad (9)$$

and

$$k(t) = 0.00488 U(t)/c_f\rho_f$$

$U(t)$ is the overall heat transfer coefficient at time t (Btu per hour per square foot per degree Fahrenheit), $c_f\rho_f$ is the heat capacity (calories per cubic centimeter per degree Celsius), and A is the heat exchanger area (square meters). The units of k are meters per hour, making NTU in (9) dimensionless. (Metric units are generally used in this study. However, for cases where data are commonly available in British units, the latter system of units is used along with the appropriate conversion factors in the equations as coefficients.)

Although T_i may be assumed to be constant with time, its value obviously affects heat removed per unit of time (for a given Q) and hence discounted net revenues. That is, lower values of T_i yield greater heat removals per unit of time but cause the field to cool more rapidly. Furthermore, to achieve lower values of T_i (for a given T_s), larger, and hence more expensive, heat exchangers are required. To see this, note from (7), (8), and (9) that for a given Q ,

$$\frac{T_0 - T_i}{T_0 - T_s} = 1 - e^{-k(0)A/Q} \quad (10)$$

yielding

$$A = \frac{Q}{k(0)} [\ln(T_0 - T_s) - \ln(T_i - T_s)] \quad (11)$$

which implies that $A \rightarrow \infty$ as $T_i \rightarrow T_s$.

ECONOMIC MODEL

Costs and Benefits

The costs considered in this study reflect the 'opportunity cost' to society of resources (steel, concrete, pumps, well-drilling services, etc.) used in extracting geothermal energy, resources that could have been put to some alternative use. There are no primary 'externalities' associated with this energy extraction, since spent brines will be reinjected, preventing subsidence and escape to the atmosphere of noxious gases. Secondary impacts, such as population influx to build and operate the geothermal system, are assumed to be small and are therefore neglected.

The value or benefits of the extracted energy are computed as follows. If there are other sources of energy (including imports) that are serious competition for geothermal energy, then geothermal energy benefits are the cost to the customer of the least expensive alternative energy source other than geothermal energy. For example, a customer using geothermal energy to generate steam for space heating has the option of generating steam with an oil- or coal-fired boiler, each of which also has some social opportunity cost. The lowest of these costs is therefore taken as the 'price' or value of the geothermal energy. In a purely competitive energy market this least expensive price is the only market price, and the geothermal energy company is simply a price taker attempting to maximize profits.

From a societal viewpoint the goal is to maximize the gross value (benefits) of the geothermal resource to consumers, net of the social cost of other opportunities precluded by this energy extraction, that is, benefits minus extraction costs. From

an entrepreneurial viewpoint, maximization of revenues minus costs (profits) is a standard objective.

Treatment of Time

The essential factor in the theory of optimal extraction is time. Indeed, the major question is, 'how much now and how much later?' In order to structure a framework in which to examine this question it is necessary to state explicitly how time affects the costs and benefits of the extraction process and how time affects one's perception of these costs and benefits. The latter is reflected in the standard concept of the social discount rate.

The effect of time on the costs and benefits of the geothermal energy extraction problem is quite straightforward. As Hanke et al. [1975] point out, it is appropriate to disregard inflation in a dynamic analysis if real (as opposed to inflated) costs and benefits are used and, in addition, if the discount rate is not compensated for inflation. This approach to inflation was taken in this study. All real costs are assumed to be constant except for electric pumping energy, which increases at the same rate as the value of produced heat energy. These values are escalated over time as follows:

$$P_t = P_0 e^{rt} \quad (12)$$

where P_t is the price (value) of energy at some time t , P_0 is the price (value) of energy at time $t = 0$, and r is the time rate of increase of real energy price.

P_0 is the cost of the least expensive alternative method of producing 10^6 Btu of steam. At the present time this alternative is producing steam in a boiler heated by fuel oil. The components composing the cost are capital, fuel, and operating costs. On the basis of an interest rate of i and component life of 25 years, P_0 (dollars per 10^6 Btu) is given by

$$P_0 = 0.1876 \text{ CRF}(i, 25) + 1.086 \quad (13)$$

where

$$\text{CRF}(i, n) = \frac{i(1+i)^n}{(1+i)^n - 1} + m \quad (14)$$

where m is the cost of insurance and local taxes expressed as a percentage of original capital costs.

Since $T_0 - T_i$ decreases with time, a constant flow rate model necessarily yields a decreasing Btu output over time. For a given Q the annual heat output curve could be plotted and used to compute the annual supplement required each year. This energy, which would be purchased at price P_n , is not considered in this study.

Extraction Model

The question of when and how much energy to extract from a geothermal aquifer depends on the relative benefits and costs of the energy now and into the future. Since the real value of the energy increases with time, perhaps extraction should be postponed to a time when the net social value (benefits minus costs) is greater. On the other hand, pumping energy costs increase at the same rate, and a positive discount rate discounts these greater future values, so the rate of increase in value of energy and the discount rate work against each other in determining when and how much energy to extract. Moreover, the temperature-time profile for a particular pumping rate Q implies a significant trade-off between energy obtained now and later. If energy is extracted rapidly at first,

the temperature will decrease rapidly, seriously diminishing the quality of the heat in the future. Finally, for a given pumping rate, more heat can be extracted by lowering the reinjection temperature, but for achieving lower reinjection temperatures, larger and more costly heat exchangers are required.

The amount of heat recovered per unit of time is the product of the flow rate, the heat capacity of the fluid, and the temperature drop experienced by the hot brine in the heat exchanger. For the first τ years this temperature drop is $T_0 - T_i$. From that time until the termination of the project at time L the temperature drop is governed by (6). Since a certain amount of heat is lost in the heat exchange, the approach of the brine temperature at the heat exchanger inlet is limited to $\delta^\circ\text{C}$ of the brine outlet temperature ($\delta \geq 0$). This restricts the 'optimal life' of the project to L_δ , where L_δ is such that

$$T_0^{L_\delta} - T_i = \delta \quad (15)$$

The optimization problem is therefore

$$\begin{aligned} \max_{Q, T_i, L} \Pi = & (1 - \eta) \int_0^{\tau(Q)} 34.76 P_0 e^{rt} Q c_f \rho_f (T_0 - T_i) e^{-it} dt \\ & + (1 - \eta) \int_{\tau(Q)}^L 34.76 P_0 e^{rt} Q c_f \rho_f (T_0 - T_i) g(t/t_u) e^{-it} dt \\ & - C(Q, T_i, L) \end{aligned} \quad (16)$$

subject to

$$g(L/t_u) \geq \frac{\delta}{T_0 - T_i} \quad T_i \geq T_s$$

where η is the royalty for the geothermal lease paid as a fraction of the value of produced energy, i is the discount rate, L is the project life (years), $C(Q, T_i, L)$ is the cost function describing the present worth of total capital and operating costs, and 34.76 is a conversion factor to yield revenues in dollars per year. By taking time in days a closer approximation to the discounted profits could be obtained. However, for simplicity, time is computed in years.

Now define

$$\beta = \pi h D^2 \rho_a c_a / 26280 c_f \rho_f$$

$$\tau = \beta / Q$$

$$t_u = 6\beta / Q$$

$$\psi_j = \phi_j / 6$$

$$\alpha = i - r$$

$$a = 34.76 P_0 c_f \rho_f (1 - \eta)$$

where the ϕ_j are the exponential parameters of g in (6). For $t > \tau$, (6) can therefore be written as

$$g(t/t_u) = \gamma_1 e^{-\psi_1 Q t} + \gamma_2 e^{-\psi_2 Q t} + \gamma_3 e^{-\psi_3 Q t} \quad (17)$$

Thus the maximand in (16) reduces to

$$\begin{aligned} \Pi = & a Q (T_0 - T_i) \int_0^{\beta/Q} e^{-\alpha t} dt \\ & + a Q (T_0 - T_i) \int_{\beta/Q}^L \left(e^{-\alpha t} \sum_{j=1}^3 \gamma_j e^{-\psi_j Q t} \right) dt - C(Q, T_i, L) \end{aligned} \quad (18)$$

yielding

$$\Pi = aQ(T_0 - T_i) \left[(1 - e^{-a\beta/Q})/\alpha + \sum_{j=1}^3 \gamma_j \frac{(e^{-(\Psi_j Q + \alpha)\beta/Q} - e^{-(\Psi_j Q + \alpha)L})}{\Psi_j Q + \alpha} \right] - C(Q, T_b, L) \quad (19)$$

COST FUNCTION

This section develops the cost function $C(Q, T_b, L)$. The components of the cost function are (1) costs for well drilling and casing and their maintenance, (2) well assemblies, (3) pumps and their operation, (4) pipes and pipe cleaning, (5) heat exchangers, and (6) rent and salaries. The individual functions for these costs are summarized below. A complete discussion of their derivation is given by Golabi and Scherer [1977a].

Well and Well Assembly Costs

The costs for wells and their casing are determined by the thickness of the impermeable strata and the level of drilling difficulty in each individual field. These phenomena are specific to a particular geothermal aquifer, so the costs of wells (WC), well assembly (WA), and the annual cost for maintenance of each pair of production injection wells (WM) are assumed to be known.

Pump Costs

Because of the very large drawdowns generated in the production well at high production rates, it is not economical and is probably infeasible for a single pump to lift the brine from the aquifer, pump it through the pipeline and heat exchanger to the injection well, and overcome the pressure buildup in the injection well. Therefore two pumps are used for each doublet. The first is a vertical turbine pump installed in the production well. This pump lifts the brine to the surface and sends it through the piping system and heat exchanger to a second pump, the latter being a horizontal pump capable of pumping the brine back to the aquifer through the injection well.

The vertical turbine pump in the production well consists of five component assemblies: (1) the drive, an electric motor, (2) the discharge assembly, on which the motor is mounted, (3) the vertical lineshaft, (4) the column assembly, through which the lineshaft extends, and (5) the bowl unit.

$$\text{drive cost} = 14.97 \left[\frac{(0.003644)(Q)(SG)}{\text{Eff}_v} (\Delta P + z + b) + 0.0234 (\Delta P + z) \right] + 1907 \quad (20)$$

where SG is the specific gravity of the fluid, Eff_v is the efficiency of vertical pump, ΔP is the steady state drawdown (meters), z is the static level of the brine, that is, the vertical distance between the discharge and the free pool when no water is being pumped (meters), and b is the friction losses (meters). (Costs are in dollars.)

$$\text{shaft cost} = \left[0.001339(Q) \frac{SG}{\text{Eff}_v} (\Delta P + z + b) + 0.0768(\Delta P + z) - 10.132 \right] [\Delta P + z] \quad (21)$$

$$\text{column assembly cost} = [z + 4 + (Q) \ln(D/r_w)/2\pi Kh] [0.1313 Q + 1.323 Q^{1/2} - 4.36] \quad (22)$$

where r_w is the well radius (meters), $K = 1.1653 \times 10^{-11}$ kγ/hr (meters per hour), k is the intrinsic permeability of the aquifer (millidarcies), γ is the specific weight of water (pound force per cubic foot), and μ is the absolute viscosity (pound force second per square foot).

$$\text{bowl unit cost} = c_1 + (c_2 - c_1)(Q - 50)^2/40000 \quad (23)$$

where c_1 is the cost of a bowl unit capable of lifting 50 m³/hr (220 g/min) of water a distance of $\Delta P(50) + z$ m and c_2 is the cost of a bowl unit capable of lifting 250 m³/hr (1100 g/min) of water a distance of $\Delta P(250) + z$ m.

The discharge assembly cost constitutes a minor portion of the pump costs, and is taken as the cost of a 10 × 10 F standard fabricated steel head recommended by the pump manufacturer for the range of flow rates considered here.

The cost of the injection pump is

$$24Q + \frac{(0.0546)(Q)(SG)}{\text{Eff}_H} \left[\frac{(Q \ln(D/r_w))}{2\pi Kh} \right] + 1907 \quad (24)$$

where Eff_H is the efficiency of its motor.

The total pump capacity costs PM are the sum of the above cost functions.

Pump operating costs are based on the escalating real costs of energy. Let R_t be the price of electricity (dollars per kilowatt hour) at time t supplied to the motors of the vertical and horizontal pumps. Then, pump operating cost at time t is given by

$$\text{PO}(t) = R_t k_m \left[\frac{23.8(Q)}{\text{Eff}_v} (SG)(\Delta P + z + b) + 152.86 (\Delta P + z) + \frac{23.8(Q)(SG)}{\text{Eff}_H} \cdot \Delta P \right] \quad (25)$$

where k_m is a multiplier indicating annual maintenance costs of pumps and their motors.

Surface Pipe Costs

Assuming a flow velocity of 6 ft/s and pipe specification in accordance with standard industrial practice (G. Lombard, personal communication, 1976), the initial pipe costs are

$$\text{PP} = k_p D [0.1313 Q + 1.323 Q^{1/2} - 4.36] \quad (26)$$

where k_p is a cost multiplier to reflect pipe support and installation costs.

The pipe-cleaning cost PC is assumed to be proportional to the length of the pipeline:

$$\text{PC} = p_c D \quad (27)$$

where p_c is the expected annual cost of cleaning 1 m of pipe.

Heat Exchanger Costs

The heat exchanger cost is

$$\text{HE} = 5000 + 150.7 \frac{Q}{k(0)} [\ln(T_0 - T_i) - \ln(T_i - T_s)] \quad (28)$$

Cost Function for One Doublet

All elements of the cost function that depend on Q have been presented, and an expression for annual costs of one doublet can soon be written. But first it is necessary to review assumptions on equipment life and well life.

The useful life of pumps and heat exchangers is taken as 10 years, and that of pipes and well assemblies as 25 years. Well life is defined as the duration of time over which the well remains free of chemical scale and otherwise clear of local blockages which would impair its ability to transmit fluid. Since the life of a geothermal well may vary from field to field, it is assumed as an input parameter. Payments for the cost of each type of equipment and accrued interest are distributed uniformly over the lifetime of the equipment. Furthermore, each piece of equipment (with the exception of the wells) has a salvage value equal to a percentage of its remaining unpaid costs if it is sold before its lifetime is up.

With these lifetimes, capital recovery factors for annualization of capital costs can be specified. However, since near-term termination dates are likely for this project, the annualization of costs must be carefully treated lest portions of loans on capital investment appear to be 'forgiven' when, in fact, they cannot be. This special treatment takes the form of 'extra costs' which are associated with project termination prior to completion of lifetime cycles of various components.

To better visualize these extra termination costs, suppose that L , the life of the project, is 23 years and the useful life of wells, WL , is 10 years. Also, let L_1 , L_2 , and L_3 be the smallest multiples of 10, 25, and well life, respectively, which contain L , and let s_1 , s_2 , s_3 , and s_4 be the salvage values of pumps, heat exchangers, pipes, and well assemblies, respectively (as a fraction of their remaining payments). Then L_1 and L_3 are 30 years, and L_2 is 25 years. The wells which have been drilled at the beginning of the twenty-first year still have a useful life of 7 years. Since wells do not have any salvage value, we assume that the remaining payments on the well cost become immediately payable. The present value of this cost is

$$WC \cdot CRF(i, 10) \int_{23}^{30} e^{-it} dt \quad (29)$$

The other pieces of equipment, however, have salvage values. As an example, consider pipes which have a life of 25 years. The present value of the unpaid cost is

$$PP \cdot CRF(i, 25) \int_{23}^{25} e^{-it} dt \quad (30)$$

Since a percentage of this cost, namely, s_3 , can be recovered by resale, the termination cost for pipes is

$$(1 - s_3)PP \cdot CRF(i, 25) \int_{23}^{25} e^{-it} dt \quad (31)$$

The total of all costs which vary with Q , $q(Q, T_b, L)$, can now be written for one doublet pumped at flow rate Q for L years with injection temperature T_i :

$$q(Q, T_b, L) = 0 \quad Q = 0$$

$$\begin{aligned} q(Q, T_b, L) = & \int_0^L [(PM + HE)CRF(i, 10) \\ & + (WA + PP)CRF(i, 25) + (WC)CRF(i, WL) \\ & + WM + PC + PO(t)]e^{-it} dt + [(1 - s_1)PM \\ & + (1 - s_2)HE]CRF(i, 10) \int_L^{L_1} e^{-it} dt + [(1 - s_3)PP \end{aligned}$$

$$\begin{aligned} & + (1 - s_4)WA]CRF(i, 25) \int_L^{L_2} e^{-it} dt \\ & + (WC)CRF(i, WL) \int_L^{L_3} e^{-it} dt \quad 0 < Q \leq \bar{Q} \end{aligned} \quad (32)$$

where $CRF(i, \quad)$ is the capital recovery factor and $\bar{Q}$ is the maximum flow rate from each doublet.

Disregarding labor and land costs, $C(Q, T_b, L) = q(Q, T_b, L)$ for $Q \leq \bar{Q}$, where $\bar{Q}$ is determined by the assumption in the Gringarten-Sauty hydrothermal model that the flow into the production is laminar and by the technology of brine pumps.

Total Cost Function for All Values of Q

Denote by S the present value of total labor costs and land rent for the geothermal reservoir paid during the life of the project, that is,

$$S = \int_0^L (\text{annual rents} + \text{annual salaries})e^{-it} dt$$

Note that S is independent of the extraction rate.

All of the above can now be combined to state the cost function for a cluster of doublets arranged close together so that the distance between all production wells and the distance between injection wells are small in comparison to D . Since there are certain fixed costs that must be paid for each doublet, the total cost function $C(Q, T_b, L)$ is a step function of Q (see Figure 3) with jumps equal to the present value of well and overhead assembly costs plus fixed capital costs of pumps and heat exchangers. Suppressing the dependence of C and q on T_i and L ,

$$\begin{aligned} C(Q) &= 0 & Q &= 0 \\ C(Q) &= q(Q) + S & 0 < Q \leq \bar{Q} \\ C(Q) &= q(\bar{Q}) + q(Q - \bar{Q}) + S & \bar{Q} < Q \leq 2\bar{Q} \end{aligned} \quad (33)$$

and in general,

$$C(Q) = nq(\bar{Q}) + q(Q - n\bar{Q}) + S \quad n\bar{Q} < Q \leq (n+1)\bar{Q}$$

for $n = 1, 2, \dots$

Optimization Algorithm

Since T_i may appropriately be expressed as a function of Q and L for purposes of optimization [see Golabi and Scherer, 1977a], the algorithm used to obtain the optimal solution to the model is a grid search over values of L and Q . L is varied from $L_{\min}$ to $L_{\max}$ in increments of L_{inc} . For each L , Q is varied from $Q_{\min}$ to $Q_{\max}$ in increments of Q_{inc} . The values of $L_{\min}$, $L_{\max}$, L_{inc} , $Q_{\min}$, $Q_{\max}$, and Q_{inc} used in this study are $L_{\min} = 0$ years, $L_{\max} = 250$ years, $L_{\text{inc}} = 1$ year, $Q_{\min} = 50$ m³/hr, $Q_{\max} = 5000$ m³/hr, and $Q_{\text{inc}} = 5$ m³/hr. For each Q and L the δ constraint is checked to make sure the difference between the production and injection temperature does not fall below $\delta^\circ\text{C}$.

DATA

The optimization is carried out using data which in the authors' best judgment reflect pertinent costs in 1976. The geohydrological data have generally been chosen in the mid-range of values associated with known hot water geothermal

Fig. 3. Total discounted costs versus pumping rate for given project lives.

resources. The following are subsequently referred to as the 'basic data.'

RESULTS

Results for the Basic Data

This section presents optimization results for a set of real discount rates from 6% to 15%, a set of rates of increases in the real price of energy, r , ranging from 0 to 3%, and the basic data.

Profits. The present worth of maximum profits, Π , is shown in Table 1. The values across the top row represent discount rates, while those at the bottom denote values for P_0 , the 1976 value of 10^6 Btu of 5-psi pipeline steam (which depends on interest rate (equation 13)). The left column contains values for r . For each i and r the optimal profit is given in the table. As is expected, the table shows that optimal profits increase as r increases and decrease as i increases. Profits vary substantially over this range.

Optimal pumping rate. Table 2 presents the optimal pumping rate Q^* , which increases as i increases and decreases as r increases. These trends are consistent with economic intuition, whereby when r is higher, heat is extracted more slowly, thus reserving a larger amount for the future, when value is higher. But for a given r , heat is extracted at a higher rate as i increases, leaving less for the future, when energy value (although increasing) is heavily discounted. The absolute value of Q^* is less than $\bar{Q} = 500$ m³/hr in all cases shown here. This means that only one set of wells is used to make one doublet. With the data used in the computations it does not seem feasible to use clusters of wells for each doublet.

Optimal project life. Table 3 presents optimal project lives. The economic lives are nonincreasing in i and nondecreasing in r . Thus as future profits are discounted more heavily, a greater amount of heat is extracted per unit of time over a shorter period in comparison to when the discount rate is not as high. On the other hand, when the energy is expected to increase rapidly in value with time, extraction of heat over a longer period of time is more profitable.

The daggers on some of the lifetimes in Table 3 indicate the

beginnings of ranges of optimal values for L . That is, the present worth of maximum profits remains constant for lifetimes greater than the numbers indicated by daggers until a time L_δ , after which it decreases. However, since the project lives within the range $L^+ - L_\delta$ are equally desirable from a profit perspective, they are all associated with the same extraction rate and heat exchanger area. Accordingly, which L^* is actually 'best' is of little operational and design importance.

For the basic model and $\delta = 6$, $L_\delta \geq L_{\max}$, which was chosen as 250 years. Therefore Π^* was effectively constant from L^+ to 250 years for these results. In general, the L constraint was rarely active.

Optimal injection temperature. Optimal values are close to 110°C. Although there is some small variation of T_i^* with r and i , the most important observations on T_i^* are that (1) the value of T_i^* is remarkably stable with respect to i and r and (2) this value of T_i^* is within 1°C of T_s (the temperature of 5-psi steam: 109°C). The resulting high costs of heat exchanger equipment are evidently offset by the value of the extra energy extracted by having T_i^* close to T_s . Although injecting the brine at a high temperature prolongs the duration of time that the production temperature remains above a specified level, it has no effect on prolonging the economic lifetime of the geothermal reservoir.

Optimal breakthrough time. Table 4 gives the optimal breakthrough time τ^* for this range of i and r . These times are inversely proportional to Q^* . Note that the optimal breakthrough times occur very early in the project, namely, during the first 2 years—long before L^* .

Sensitivity Analysis

This section discusses the sensitivity of the optimal decision variables to changes in the parameters of the model. The discount rate is 10% for these analyses, and the real annual rate of growth in energy value is 2.4% as given by the *Federal Energy Administration* [1977].

Well life. Expected well life has a great effect on profits and optimal reservoir life. For example, when expected well life is 10 years, L^* is 10 years, and when well life is 25 years,

TABLE 1. Maximum Profits Π^*

	i				
	0.06	0.08	0.10	0.12	0.15
r					
0.000	374.	317.	266.	220.	160.
0.010	554.	456.	375.	308.	226.
0.020	830.	620.	505.	412.	303.
0.024	1065.	728.	561.	457.	337.
0.030	1508.	938.	688.	537.	389.
P_0	1.104	1.107	1.110	1.113	1.118

Π^* is the present worth of maximum profits in thousands of 1976 dollars. Units of P_0 are dollars per 10^6 Btu.

L^* is also 25 years. Furthermore, the predominance of 25-year optimal lives in Table 3 is due to the fact that the well life was chosen as 25 years in the basic model. In fact, it seems that with the exception of the case where $r = 0$ the optimal life is either equal to the well life or to L^* , the point where $\Pi[L, Q^*(L)]$ reaches a plateau.

The optimal pumping rate Q^* is also influenced by this parameter. For example, when well lives are 10 and 25 years, Q^* is 430 and 385 m^3/hr , respectively. Finally, well life affects maximum profits. When expected well life increases from 10 to 25 years, maximum profits increase from \$421,000 to \$561,000. However, when well life increases another 15 years, from 25 to 40 years, maximum profits only increase from \$561,000 to \$607,000, suggesting diminishing returns to increasing well life. It appears that additional information on the expected lifetime of geothermal wells would be valuable in selecting the best pumping rate and in correctly estimating optimal expected reservoir life and the net economic worth of the reservoir (profits).

Aquifer porosity and permeability. Porosity ϕ and intrinsic permeability k are important in the management of this resource, because it involves fluid flow in a porous medium and because extensive information on porosity and permeability of the aquifer is available only at the very high cost of obtaining many exploratory cores. Hence sensitivity of Π^* , Q^* , and L^* to these two aquifer parameters is of special interest. If porosity is 10% instead of 20% (base case), the optimal profit is only \$493,000 instead of \$561,000, a decrease of 12%. If porosity is 30%, the maximum profit is \$628,000, an increase of 12%. However, Q^* is virtually constant, so it would appear that this major design parameter is virtually insensitive to porosity over this range of porosities. This suggests that major expenditures to accurately determine porosity in order to compute the 'correct' pumping rate would probably not be warranted.

On the other hand, the effect of variation in intrinsic permeability is more significant. Reduction in permeability from 200 millidarcies (base case) to 150 millidarcies reduces the optimal profits from \$561,000 to \$242,000, a reduction of 57%, and optimal pumping rate from 385 to 310 m^3/hr , a reduction of about 19%. An increase in permeability to 250 increases the profits to \$859,000, an increase of 53%, while Q^* increases 13% to 430 m^3/hr . From elementary fluid mechanics we recall that permeability governs pumping costs. As permeability increases, the drawdown in the production well decreases (for a given Q), thereby requiring smaller pumps and less energy for pumping the same amount of flow. As a result, profits are increased. Unlike porosity, expenditures to obtain accurate information on permeability may be very important, for profits

and design flow rate are indeed sensitive to values of permeability.

Initial aquifer equilibrium temperature. Among the physical geothermal parameters the one with the greatest impact on profits and the optimal pumping rate is the initial aquifer equilibrium temperature T_0 . This is the temperature at which the geothermal aquifer fluid and the aquifer matrix are in thermal equilibrium. With regard to the general extraction problem it is analogous to the 'grade' of an ore deposit. Optimal profits and flow rates as functions of temperature are shown in Table 5. These are the only examples of Q^* greater than $\bar{Q}$. In general, as the equilibrium temperature increases, it is optimal to extract energy at a higher rate and terminate the project in a shorter period. The only exception to this trend occurs when the temperature is $150^\circ C$. Since the profit is rather low at this temperature, the well cost has a severe impact on the profits, thus making it imperative to terminate the project at $L = 25$. Even in this case the total discounted profits if the project is terminated at $L = 25$ are about the same as if the project life had been, say, 200 years (\$561,000 versus \$560,000).

Electric pumping costs. The economic parameter that most influences profits and optimal flow rate is the present cost of electricity for pumping. The total discounted optimal profits increase by 43% when electricity cost decreases from 3 (base case) to 2.5¢/kWh and by 130% when it decreases to 2¢/kWh. It is interesting that the optimal pumping rate increases by only 10.5% from 385 to 420 m^3/hr when electricity cost decreases from 3 to 2.5¢/kWh, while it increases by 142% to 920 m^3/hr when electricity cost is decreased to 2¢/kWh. Pump operating cost clearly plays a major role in the economics of geothermal energy production. It follows that with regard to the economic viability of geothermal energy these results are somewhat conservative for application to an artesian system. That is, if pumping were not necessary, profits would be much higher (and so might Q^* , unless large Q required pumping).

Royalty. The behavior of profits with respect to changes in royalty are more uniform. Optimal profits decrease by 37% when royalty is increased from 10% (base case) to 15% and increase by 38% when royalty is decreased to 5%. However, although these changes in royalty amounts cause significant changes in profits, they hardly affect Q^* . The amount of royalty paid is of more concern to the investor than to the design engineer and is not a highly effective policy instrument for influencing extraction rate.

Well cost. The effect of changes in well costs for each doublet is indeed significant. An increase of \$200,000 over well cost of \$600,000 (base case) reduces optimal profits by 52%, while the same amount of decrease in well cost increases the

TABLE 2. Optimal Pumping Rate Q^*

	i				
	0.06	0.08	0.10	0.12	0.15
r					
0.000	380.	390.	400.	405.	420.
0.010	365.	375.	385.	395.	410.
0.020	325.	375.	385.	395.	410.
0.024	320.	345.	385.	395.	410.
0.030	310.	340.	360.	380.	405.
P_0	1.104	1.107	1.110	1.113	1.118

Units of Q^* are cubic meters per hour; units of P_0 are dollars per 10^6 Btu.

TABLE 3. Economic Reservoir Life L^*

r	i				
	0.06	0.08	0.10	0.12	0.15
0.000	20.	20.	20.	20.	20.
0.010	25.	25.	25.	25.	25.
0.020	250.†	25.	25.	25.	25.
0.024	250.†	215.†	25.	25.	25.
0.030	250.†	220.†	165.†	125.†	25.
P_0	1.104	1.107	1.110	1.113	1.118

† Start of an optimal range of values.

Units of L^* are years; units of P_0 are dollars per 10^6 Btu.

profits by 57%. Although these changes in well cost do not significantly alter the optimal pumping rate, it is clear that well cost plays a major role in determining the economic viability of this type (nonelectric) of geothermal project.

Well maintenance costs. The effect of the annual well maintenance cost is the least among the major economic parameters. An increase of 50% in these costs reduces optimal profits by only 5%, while doubling these costs reduces profits by less than 10%. Again, the optimal pumping rate and economic lifetime remain unchanged. This is an interesting result when it is considered together with the substantial effect of expected well life on profits. Although no data on the relationship between well maintenance expenditures and well life are available, it is reasonable to speculate that a good well maintenance program could be fairly cost beneficial up to a point. However, recalling that gains in profits were increasing at a decreasing rate as well life increased, there would probably be some 'optimal' maintenance expenditure beyond which expenditure would outweigh costs. The conclusion of engineering-economic import is that potential gains in profits may justify further investigation in this area of 'optimal well maintenance.'

CONCLUSIONS

This study has shown that physical and economic data can be combined in an operational model for optimal extraction of geothermal energy to heat space (for human habitation or product drying). Depending on particular values of the discount rate and the rate of increase of real energy value r , extraction rates and anticipated reservoir lives have been determined. Extraction rate tends to increase with the discount rate and decrease as r increases. As is expected, a greater emphasis on the present (larger i) 'tilts' the design decisions on optimal pumping rate toward more rapid heat energy extraction rates. That is, heat is extracted at a higher rate, leaving less for the future, when profits are heavily discounted.

TABLE 4. Optimal Breakthrough Time τ^*

r	i				
	0.06	0.08	0.10	0.12	0.15
0.000	1.79	1.75	1.70	1.68	1.62
0.010	1.86	1.81	1.77	1.72	1.66
0.020	2.09	1.81	1.77	1.72	1.66
0.024	2.13	1.97	1.77	1.72	1.66
0.030	2.20	2.00	1.89	1.79	1.68
P_0	1.104	1.107	1.110	1.113	1.118

Units of τ^* are years; units of P_0 are dollars per 10^6 Btu.

Regardless of discount rate and r the optimal reinjection temperature is within 1° of the temperature of the low-pressure steam produced by this system. The high costs of heat exchanger equipment resulting from this proximity are evidently offset by the value of the extra energy extracted due to this proximity. In the context of the Gringarten-Sauty hydrothermal model a higher injection temperature has no effect on prolonging the economic lifetime of the project. Although reinjecting the brine at a higher temperature prolongs the duration of the (postbreakthrough) time that the production temperature remains above a specified level, it is not actually profitable to do so.

In the examples studied in this paper (with the set of physical data used) the optimal breakthrough times occur quite early in the project, namely, during the first 2 years. The amount of heat which is extracted during this period is far less than that extracted after the breakthrough time. For example, in the base case the heat extracted before the breakthrough time is less than 12% of the total heat extracted during the life of the project. For geothermal space heating, termination of the extraction process at the breakthrough time may be premature.

One of the purposes of this paper was to estimate the net economic worth of a geothermal reservoir (present worth of revenues minus costs of production). Representing this value with 'profits' as defined above, the value of a reservoir increases with r and decreases as the discount rate increases, as would be expected. This value increases rapidly with the initial temperature of the aquifer and decreases as electric pumping costs increase. Initial temperature proved to be an especially important parameter. Extraction rate moves in the same direction as profits when these two key economic inputs are varied.

Well costs and royalties also affect profits, though they have little influence on pumping rate, which suggests that royalty is not a promising pressure point for public regulation of the extraction rate from a given reservoir. However, this is not to say that rents, royalties, and other incentives are unimportant in influencing the profit-maximizing entrepreneur's decision on the timing of production (when to commence). Indeed, they do offer some potential in this area, as is discussed by Golabi and Scherer [1977b, 1981].

The assumed permeability of the aquifer was also found to significantly affect the value of the reservoir and the optimal pumping rate (higher permeabilities result in higher profits and pumping rates). Aquifer porosity, on the other hand, was relatively unimportant in this regard.

The sensitivity analyses show that well life has a substantial effect on both the economic life of the project and the maximum profits. On the other hand, the reduction in profits from increasing expenditures for well maintenance costs is minimal. Therefore savings from a good well maintenance program might outweigh the costs. However, the gains in profits in-

TABLE 5. Sensitivity to Initial Aquifer Temperature

T_0 , °C	Π^* , dollars	Q^* , m ³ /hr	L^* , years
150	561	385	25
160	1,983	875	150
170	5,008	2400	75
180	10,479	4900	50

Here $i = 0.01$, and $r = 0.024$.

crease at a decreasing rate as well life increases, so there will probably be some optimal maintenance expenditure beyond which marginal expenditures would outweigh marginal costs.

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