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Isospin mixing in charmonium states*

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The "molecular" charmonium models predict both $I = 0$ and $I = 1$ states. The closeness of one of these (4.028 GeV) to one of its main thresholds and the large electromagnetic mass splitting of its daughters may induce a large mixing between states of different isospin. This would manifest itself in deviations from unity of the ratio of charged to neutral decay modes.

The rich structure observed in e^+e^- annihilation in the energy region between the ψ' and 4.4 GeV (Ref. 1) has led to the hypothesis that at least some of these states are four-quark composites,[2-5] or more specifically bound states of $D\bar{D}$, $D\bar{D}^*$, and and $D^*\bar{D}^*$.[1,3-5] The appellation "molecular" charmonium has been given to these states. The isospin of these states can be zero or one. The determination of the $I=1$ character of any of these states would be the cleanest way of distinguishing them from $c\bar{c}$ or "atomic" charmonium states, whose isospin is zero.

Two tests for the existence of $I=1$ states were proposed in Ref. 2: (1) a search for predominantly even π decay modes of any of these resonances and (2) peaks in inclusive charged-π distributions resulting from the decay of a higher-lying resonance into a charged one with lower mass. The first method is, to a large extent, moot as the dominant decay of these resonances is to charmed mesons and not directly to π's. The second method is valid. In this paper we wish to propose another technique, which, even after the hypothetical discovery of such states, will have an interest of its own.

One of the most prominent resonances is the one at 4.028 GeV.[1] The method proposed depends on the existence of a resonance of opposite isospin with a mass near the above one. We shall make this more precise below. The aforementioned rich structure and other estimates[4] make this hypothesis plausible. If the above is true then the closeness of 4.028 GeV to the $D^*\bar{D}^*$ threshold and the D^{*+}-D^{*0} mass difference will induce a sizable mixing of the $I=0$ and $I=1$ states. *This would manifest itself in the deviation of the ratio*

$$R=\frac{\Gamma(4.028\to D^0\bar{D}^0)}{\Gamma(4.028\to D^+D^-)} \tag{1}$$

from unity (in addition to the P-wave phase-space ratio of 1.1). Such mixings have been discussed in connection with the K^0-$\bar{K}^0$ and ω-ρ^0 systems.[6]

Before discussing the mixing formalism, we will review some pertinent experimental facts.[7] The masses of the charmed mesons are $M_{D^0}=1865\pm3$ MeV, $M_{D^+}=1875\pm5$ MeV, $M_{D^{*0}}=2005.7\pm1.5$ MeV, and $M_{D^{*+}}=2010\pm3$ MeV. In spite of the closeness of the (4.028) to the $D^*\bar{D}^*$ thresholds, this resonance decays predominantly into this channel; $\Gamma(4.028\to D^{*0}\bar{D}^{*0}):\Gamma(4.028\to D^0\bar{D}^{*0}+\bar{D}^0D^{*0}):\Gamma(4.028\to D^0\bar{D}^0)=(0.48\pm0.8):(0.47\pm0.08):(0.05\pm0.03)$. Though not precisely established, the widths of the resonances above the ψ appear to be between 30 and 50 MeV.[1] For subsequent estimates we shall use 40 MeV as a typical width; larger values would enhance the deviation of R.

For the sake of discussion let us assume that the resonance at 4.028 GeV has isospin zero and there is an isospin-one state nearby. None of our results would change if the isospin situation were reversed. The Wigner-Weisskopf techniques, as discussed in Ref. 6, applied to this two-level system mix isospins in the (4.028) state:

$$|4.028\rangle=|I=0\rangle+\frac{p-iq}{\Delta}|I=1\rangle\,. \tag{2}$$

The states $|I\rangle$ are eigenstates of isospin. With H_1 the interaction responsible for the decay of this resonance we obtain

$$p-iq=\sum_{\alpha}\int d\vec{k}\,\frac{\langle 0|H_1|\vec{k},\alpha\rangle\langle\vec{k},\alpha|H_1|1\rangle}{M_0-E_k+i\epsilon}\,. \tag{3}$$

Δ is the difference of the diagonal elements of the mass matrix,

$$\Delta=M_1-M_0-\tfrac{1}{2}i(\Gamma_1-\Gamma_0)\,, \tag{4}$$

and

$$\Gamma_i=2\pi\sum_{\alpha}\int d\vec{k}\,\delta(M_0-E_k)|\langle i|H_1|\vec{k},\alpha\rangle|^2\,, \tag{5}$$

where $M_0\approx4.028$ GeV. Restricting the sum over α in (3) to $D^*\bar{D}^*$ intermediate states, we can easily find q, the imaginary part of the off-diagonal element of the mass matrix:

$$q=g_0g_1\left\{\left[1-\left(\frac{2M_{D^{*0}}}{M_0}\right)^2\right]^{3/2}-\left[1-\left(\frac{2M_{D^{*+}}}{M_0}\right)^2\right]^{3/2}\right\}\,. \tag{6}$$

g_0, g_1 (Ref. 8) entail all parts of the matrix elements of H_1 to the $I=0$ and $I=1$ states except for the P-wave phase space. In the same notation the partial width of the $I=0$ state (aside from small corrections due to mixing) to $D^*\bar{D}^*$ is

$$\Gamma_0(D^*\bar{D}^*)=2g_0^2\left\{\left[1-\left(\frac{2M_{D^{*0}}}{M_0}\right)^2\right]^{3/2}+\left[1-\left(\frac{2M_{D^{*+}}}{M_0}\right)^2\right]^{3/2}\right\}. \qquad (7)$$

Assuming[9] $|g_1|\approx|g_0|$ and using the masses and widths discussed previously, we obtain

$$|q|\approx\Gamma_0(D^*\bar{D}^*)/4\approx 5 \text{ MeV}. \qquad (8)$$

More relevant for experimental consequences of this interference is the real part, p, of the mixing. This value is difficult to obtain without a detailed model for the off-shell effects in the decay. We may estimate it by evaluating a crude dispersion integral in M_0 of the imaginary part. For this absorptive part we take the values implied by Eq. (6) from the $D^{*0}\bar{D}^{*0}$ threshold to $M_0+\Gamma_0$. At this value we set the absorptive part to zero (the artificial oscillations due to this sharp cutoff do no affect the real part at M_0). With this procedure

$$|p|\approx 5 \text{ MeV}. \qquad (9)$$

[One often expects real parts to vanish at a resonance peak. However, in this case the imaginary part, Eq. (6), has not reached a maximum at M_0, but is still rising, and we obtain a finite real part.]

Returning to Eq. (2) and Eq. (1) we obtain

$$R=\left|\frac{1-(p-iq)/\Delta}{1+(p-iq)/\Delta}\right|^2 \approx 1-4p/(M_1-M_0). \qquad (10)$$

In the above we have assumed that $M_1-M_0 > \frac{1}{2}(\Gamma_1-\Gamma_0)$. *If the splitting of the $I=1$ and $I=0$ resonances turns out to be of the order of the width this ratio would deviate from unity by as much as 50%.*

This effect would likewise show up in the analogous decays into the $D\bar{D}^*+\bar{D}D^*$ modes. Though the branching ratio into this channel is much higher than into $D\bar{D}$ the latter one is subject to fewer experimental ambiguities. Should a resonance exist just above the DD^* threshold, a similar argument could be repeated for it. Admixtures from nonresonant $D\bar{D}$ backgrounds are not expected to be significant, as recent results from the direct electron counter (DELCO) group[10] indicate that such nonresonant production is very small.

This analysis provides a method for testing the hypothesis that states of opposite isospin are located in the 4 GeV to 4.1 GeV region. If these states are found by other means, the mixing discussed here will still have other interests, especially in studying the properties of the decay Hamiltonian. To base the parameters of this mixing on firmer theoretical arguments we need, in addition to the existence of both isospin states and a knowledge of their masses and width, a better determination of the real part.[11]

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[6]There is an extensive literature on this subject. Three recent sources containing references to earlier articles are P. K. Kabir, *The CP Puzzle* (Academic, New York, 1968); K. Gottfried, in *Proceedings 1971 International Symposium on Electron and Photon Interactions at High Energies*, edited by N. B. Mistry (Cornell Univ. Press, Ithaca, 1972); F. M. Renard, in *Springer Tracts in Modern Physics*, edited by G. Höhler (Springer, New York, 1972), Vol. 63, p. 98.

[7]J. Wiss, presented at the Fifth International Conference on Meson Spectroscopy, 1977 (unpublished); G. J. Feldman *et al.*, Phys. Rev. Lett. 38, 1313 (1977).

[8]We have assumed isospin invariance in the decay matrix elements. A violation of isospin in these matrix elements would yield another source of mixing.

[9]The fact that all the widths above 4 GeV are of comparable magnitude justifies this assumption.

[10]Private communication from R. Burns.

[11]Although the Q value of these decays is very small, Coulomb effects are not expected to be important due to the P-wave character of this decay.