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An Empirical Study of Statistical Financial Models: Portfolio Optimization and Evaluation

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Abstract of the Thesis

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This paper provides a review of statistical models in finance for portfolio optimization and portfolio performance evaluation. Based on the assumptions of modern portfolio theory, we discuss five portfolio optimizing models. We then classify portfolio performance evaluation measures into four generalized categories, including the most common performance/risk ratios, the incremental return, the preference-based measures, and the market timing measures. Under each category we review the typical measures with their advantages and drawbacks, and discuss approaches to refine on the drawbacks.

In the empirical study section that follows we build five portfolios based on the portfolio optimizing models. Eleven performance evaluation measures are applied to the portfolios, and are compared according to their effectiveness.

The thesis of Weikang Fan is approved.

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Mark Stephen Handcock, Committee Chair

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To my dear parents . . .

for their constant support for my education

and endless love through my life

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CHAPTER 1

Introduction

The literature on portfolio management goes back to the 1950s. Economist Harry Markowitz introduced modern portfolio theory in a 1952 essay, which established the mathematical framework for the portfolio management models in later years. Modern portfolio theory assumes that returns follows a Gaussian distribution, measures return by expected values and quantifies risk by variance. The defect is obvious: these assumptions does not match the real world. In reality the market is highly chaotic and unlikely to follow a Gaussian distribution. The returns may not be symmetrically distributed; even if it is, the investors may not have a symmetrical preference over the returns. Investors are usually risk-averse, and concerned more about the losses than the gains. We naturally conceive risk as asymmetric, which brings about the needs for new risk measures without a fixed assumption of distribution.

In this thesis, we start from the classical Markowitz model, and discuss several other widely used portfolio optimizing methods proposed in later studies. We will evaluate the performance of models through both the basic expected value-variance scheme and some improved measures, which work in general non-Gaussian cases.

This chapter briefly introduces the background of the thesis. Chapter 2 will present the idea of diversification and how it helps us to invest portfolios and reduce risk. We discuss five portfolio optimizing models: the classical Markowitz model, the single index model, the constant correlation model, the multigroup model, and the multi-index model. For each model, we will show the algorithms to find the optimal portfolio.

Chapter 3 summarizes the four categories of portfolio performance evaluation measures.By category we first review the basic measures and their strengths and weakness. Then we discuss their later modifications and extensions that improve the basic measures and correct their defects.

Chapter 4 conducts an empirical study on some of the portfolio optimizing models and performance evaluation methods discussed in Chapter 2 and 3. We collect historical return rates for multiple stocks and the market index in two periods, using data in the first period to construct the optimal portfolios and data in the later period to evaluate the portfolio performance. We analyze the performance of different optimizing models and the effects of selected evaluation measures.

Chapter 5 is the conclusion. It provides a brief summary of the empirical study results, reviews the limitations of the study, and discusses the future work that can improve upon the limitations.

CHAPTER 2

Portfolio Optimization

2.1 Basic Setup

2.1.1 Assumptions

Throughout this paper we follow two assumptions:

Assumption 1. Investors are risk-averse and rational; their goal of portfolio optimization is maximizing the expected return of the portfolio while trying to minimize the risk at the same time. This is the basic assumption of Modern Portfolio Theory.

Assumption 2. Short sales are allowed. Short selling a stock is to sell a stock that you don't own. When short sales are allowed, investors are able to achieve a broader range of portfolio combinations. Thus for generalization we assume short sales are allowed in this paper.

2.1.2 Concepts

(1) Return rate and its mean and variance Suppose that P_{it} is stock i's price at time t. Then the return of the stock i at time t is

$$R_{it} = \frac{P_{it} - P_{i,t-1}}{P_{i,t-1}}.$$

The mean and the variance of stock i's return are:

$$\bar{R}_i = \frac{1}{n}\sum_{t=1}^{n} R_{it}, \quad \sigma_i^2 = \frac{1}{n-1}\sum_{t=1}^{n}(R_{it} - \bar{R}_i)^2.$$

The covariance between the returns of stocks i and j is:

$$cov(R_i, R_j) = \sigma_{ij} = \frac{1}{n-1}\sum_{t=1}^{n}(R_{it} - \bar{R}_i)(R_{jt} - \bar{R}_j).$$

The correlation coefficient ρ between stocks i and j is:

$$\rho_{ij} = \frac{cov(R_i, R_j)}{\sigma_i \sigma_j}.$$

(2) Risk Risk is the possibility that the return of an investment will be different than expected. Investors are always glad to have a return higher than expected, thus risk usually just indicates the likelihood of losing some or all of the original investment. There are many kinds of risks; below we introduce some risks associated with investing in the stock markets:

Systematic Risk It may also be called market risk. It is the risk for the entire market to decline.

Unsystematic Risk It refers to the risk that a single stock may decrease in value independent of the entire stock market.

Business Risk It is the risk that a company fails to stay in business.

Regulatory Risk It is the risk that the changes in relevant regulations may influence the stock value and even the entire stock market.

Opportunity Cost Risk It refers to the risk that investors may be able to achieve a higher rate of return by using the money to make another investment instead of the original one.

Liquidity Risk It is the risk associated with the marketability of the stock. A stock would be riskier when it is less marketable than others.

In financial theories we suppose that there is a positive relationship between risk and return. If an investor is willing to take more risk on an investment, he or she will expect a higher potential return in compensation for the higher risk.
The concept of risk is not as explicit as return. The return rate has a clear and unique mathematical definition, but this does not go for the risk. Actually there are various kinds of risk measures, which will be discussed in later sections. A very common and easy-to-compute risk measure is the standard deviation of the stock's historical return. The standard deviation of stock i is denoted by

$$\sigma_i = \sqrt{var(R_i)} = \{\frac{1}{n-1}\sum_{t=1}^{n}(R_{it} - \bar{R}_i)^2\}^{\frac{1}{2}}$$

(3) Investing a portfolio Portfolio is the term for a group of assets. For simplicity here we just consider the case where all the assets are stocks. Suppose that the portfolio is constituted by N stocks, and the fraction of available funds invested in stock i is x_i. Then the expected return of the portfolio is defined as:

$$E(R_p) = E(\sum_{i=1}^{N} x_i R_i) = \sum_{i=1}^{N} x_i \bar{R}_i$$

. The variance of the portfolio is:

$$var(R_p) = var(\sum_{i=1}^{N} x_i R_i) = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \rho_{ij}\sigma_i\sigma_j$$

.

(4) Risk-free rate Risk-free rate of return, denoted by R_f, is the return rate of an investment with zero risk. Theoretically it is the minimum return of an investment; in practice we usually use the interest rate on the U.S. Treasury bill as the risk-free rate for models on U.S. financial market.

Table 2.1: An example of diversification

Investment	Market Condition			Mean	Standard
	Good	Average	Poor	Return	Deviation
Stock 1	0.15	0.10	0.05	0.10	0.041
Stock 2	0.05	0.10	0.15	0.10	0.041
Portfolio	0.10	0.10	0.10	0.10	0

(5) Market return Market return, denoted by R_m, is the return rate of the market portfolio. Theoretically market portfolio is defined as the portfolio consisting of all assets in the market, with weights to be the proportion of asset market value relative to the total market value. In practice we usually use the returns of stock market indexes, like $S\&P500$ and $NASDAQ$, as the market return.

2.2 Diversification and Equal Allocation Portfolio

The well-known proverb "Don't put all your eggs in one basket!" conveys the idea of "diversification". When applied to investment decision making, it suggests us to invest in a portfolio rather than a single asset. "Diversification" is a key concept of risk management and a very useful method to reduce risk.

To see this, suppose that we have two stocks, stock 1 and stock 2; we build a stock portfolio constituted by 50% in stock 1 and 50% in stock 2. Suppose that the stock market have 3 kinds of future states, good, average, and poor. The return rates and the standard deviations of the stocks and portfolios are presented in Table 2.1. The mean return is defined as

$$\bar{R} = \frac{R_{Good} + R_{Average} + R_{Poor}}{3},$$

and the standard deviation is defined as

$$\sigma = \frac{1}{3} \sum_{s} (R_s - \bar{R})^2, \quad s \in \{Good,\ Average,\ Poor\}.$$

From Table 2.1 we can find that while stock 1, stock 2, and the portfolio share the same mean return of 0.10, they have different standard deviations, i.e. risks.

$$\sigma_{portfolio} = 0 < \sigma_1 = \sigma_2 = 0.041$$

The portfolio has lower standard deviation than both stock 1 and stock 2; by investing in the portfolio rather than single stocks we diversify away the risk. Since investors are assumed to be risk-averse, they would always prefer the portfolio rather than stock 1 and stock 2.

Based on the idea of diversification we choose equal allocation as the first way to optimizing the portfolio. Suppose that we have a given set of N stocks, then the allocation vector is

$$x = (x_1, \cdots, x_N) = (\frac{1}{N}, \cdots, \frac{1}{N}).$$

It is a very rough optimizing method, and we mainly use the equal allocation portfolio as a contrast to be compare with other well-designed portfolios.

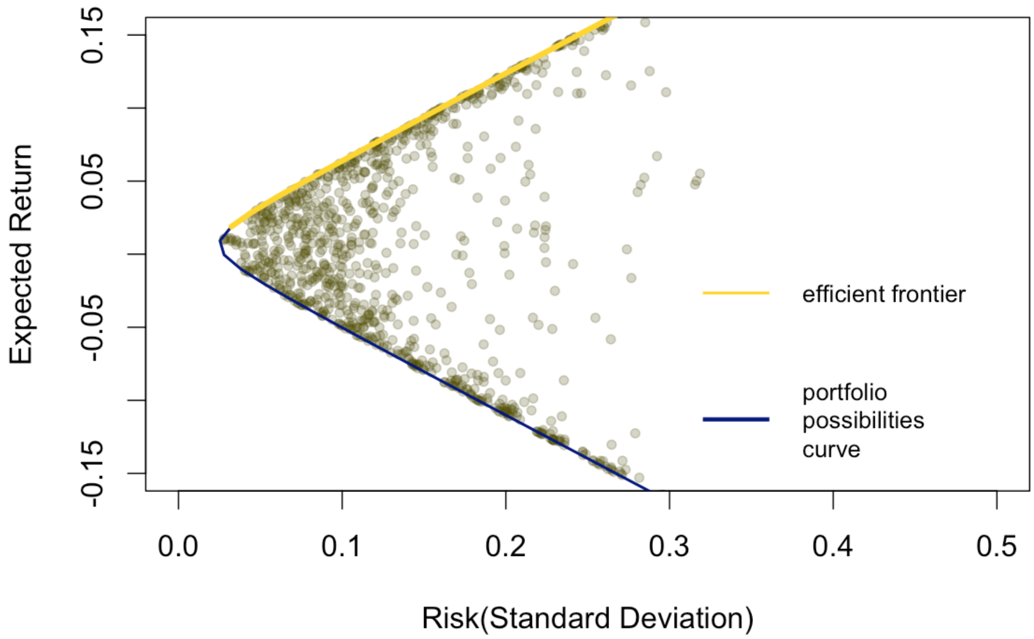
2.3 Classical Markowitz Model

By changing the allocation vector $x = (x_1, \cdots, x_N)$ we can obtain numerous portfolios constituted by a given set of N stocks. We plot the expected return $E(R_p)$ against the standard deviation $\sigma_p = \sqrt{var(R_p)}$ for all the possible portfolios in a graph, then each portfolio is represented by a point in the graph. The boundary of the cloud of points is called portfolio possibilities curve.

The efficient frontier is defined as the set of points representing portfolios with the highest $E(R_p)$ for a give value of σ_p. It is actually the upper half of the portfolio possibilities curve.

Figure 2.1 plots the efficient curve and the portfolio possibilities curve. The green points represents the portfolios constituted by the give set of assets; the red

Figure 2.1: Classical Markowitz model: efficient frontier and portfoliio possibilities curve



curve represents the efficient frontier, and the blue curve represents the possibilities curve (its upper part is overlapped with the red curve).

Mathematically the efficient frontier is the set of portfolios that satisfy the constrained minimization problem:

$$min \ \ \frac{1}{2}\sigma^2$$

such that:

$$\sum_i x_i = 1,$$

$$E = \sum_i x_i \bar{R}_i,$$

$$\sigma^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i x_j \rho_{ij} \sigma_i \sigma_j,$$

where σ^2 is the variance of a portfolio on the frontier with expected return E.

Suppose that the risk-free rate is R_f. Draw a line that passes the point $(0, R_f)$ and is tangent to the efficent frontier, then the tangency point represents the optimal portfolio. The steps for calculating the optimal portfolio are:

Step 1. Compute the excess return vector $\mathbf{R} = (\bar{R}_1 - R_f, \cdots, \bar{R}_m - R_f)^T$;

Step 2. Compute the variance-covariance matrix $\mathbf{\Sigma} = (\sigma_{ij})_{N \times N}$;

Step 3. Compute $\mathbf{Z} = \mathbf{\Sigma}^{-1}\mathbf{R}$;

Step 4. Compute $x_k = \frac{z_k}{\sum_i z_i}$ for $k = 1, \cdots, N$, then $x = (x_1, \cdots, x_N)$ is the allocation vector of the optimal portfolio.

2.4 Single Index Model

The single index model states that

$$R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it},$$

where R_{it} is the return of stock i at time t and R_{mt} is the return of the market at time t. Assume

$$E(\epsilon_i) = 0, \quad var(\epsilon_i) = \sigma^2_{\epsilon_i}, \quad E(\epsilon_i \epsilon_j) = 0,$$

$$cov(R_m, \epsilon_i) = 0, \quad var(R_m) = \sigma^2_m, \quad E(R_m) = \bar{R}_m.$$

The steps for calculating the optimal portfolio are:

Step 1. Run regression $R_{it} = \alpha_i + \beta_i R_{mt} + \epsilon_{it}$ for every stock i;

Step 2. Compute the excess return to beta $= \frac{\bar{R}_i - R_f}{\beta_i}$ for every stock i;

Step 3. Compute the the cut-off point C^*

$$C^* = \frac{\sigma_m^2 \sum_{j=1}^{N} (\bar{R}_j - R_f)\beta_j/\sigma_{\epsilon_j}^2}{1 + \sigma_m^2 \sum_{j=1}^{N} \beta_j^2/\sigma_{\epsilon_j}^2};$$

Step 4. Rank stocks based on the excess return to beta ratio. Stocks with excess return to beta greater than C^* will be held long; stocks with excess return to beta smaller than C^* will be held short.

Step 5. Compute $z_i = \frac{\beta_i}{\sigma_{\epsilon_i}^2}(\frac{\bar{R}_i - R_f}{\beta_i} - C^*)$;

Step 6. Compute $x_k = \frac{z_k}{\sum_i z_i}$ for $k = 1, \cdots, N$, then $x = (x_1, \cdots, x_N)$ is the allocation vector of the optimal portfolio.

2.5 Constant Correlation Model

The constant correlation model is similar to the single index model. It assumes that the pairwise correlation coefficients of different stocks are equal. The average correlation coefficient is defined as

$$\rho = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \rho_{ij},$$

where ρ_{ij} is the correlation coefficient between stock i and stock j.
The steps for calculating the optimal portfolio are:

Step 1. Calculate the variance-covariance matrix $\Sigma = (\sigma_{ij})_{N \times N}$ for all the N stocks;

Step 2. From Σ we can obtain standard deviation of stock i, $\sigma_i = \sqrt{\sigma_{ii}}$, and the correlation coefficient between stock i and stock j, $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$;

Step 3. Compute the excess return to standard deviation $= \frac{\bar{R}_i - R_f}{\sigma_i}$ for every stock i;

Step 4. Compute the average correlation coefficient $\rho = \frac{1}{N(N-1)} \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} \rho_{ij}$;

Step 5. Compute the cut-off point C^*

$$C^* = \frac{\rho}{1 - \rho + N\rho} \sum_{j=1}^{N} \frac{\bar{R}_j - R_f}{\sigma_j};$$

Step 6. Rank stocks based on the excess return to standard deviation ratio. Stocks with excess return to standard deviation greater than C^* will be held long; stocks with excess return to standard deviation smaller than C^* will be held short.

Step 7. Compute $z_i = \frac{1}{(1-\rho)\sigma_i}(\frac{\bar{R}_i - R_f}{\sigma_i} - C^*)$;

Step 8. Compute $x_k = \frac{z_k}{\sum_i z_i}$ for $k = 1, \cdots, N$, then $x = (x_1, \cdots, x_N)$ is the allocation vector of the optimal portfolio.

2.6 Multigroup Model

In the multigroup model we suppose that the stocks are grouped by industry. For simplicity, we discuss the case of two industries, where stocks 1, 2, 3 belong to industry 1 and stocks 4, 5, 6 belong to industry 2. We assume that the correlations within the first group are the same for all pairs in group 1 (call it ρ_{11}), and similarly the correlations within the second group are the same for all pairs in group 2 (call it ρ_{22}). We also assume that the correlations for all pairs of stocks between the first group and the second group are the same (call it ρ_{12}). Namely we have

Group1: $\rho_{11} = \rho_{12} = \rho_{13} = \rho_{23}$

Group 2: $\rho_{22} = \rho_{45} = \rho_{46} = \rho_{56}$

Between Group 1 and Group 2: $\rho_{12} = \rho_{14} = \rho_{15} = \rho_{16} = \rho_{24} = \rho_{25} = \rho_{26} = \rho_{34} = \rho_{35} = \rho_{36}$

Thus the original correlation matrix ρ equals

$$\rho = \begin{bmatrix} 1 & \rho_{11} & \rho_{11} & \rho_{12} & \rho_{12} & \rho_{12} \\ \rho_{11} & 1 & \rho_{11} & \rho_{12} & \rho_{12} & \rho_{12} \\ \rho_{11} & \rho_{11} & 1 & \rho_{12} & \rho_{12} & \rho_{12} \\ 1 & \rho_{11} & \rho_{11} & 1 & \rho_{22} & \rho_{22} \\ 1 & \rho_{11} & \rho_{11} & \rho_{22} & 1 & \rho_{22} \\ 1 & \rho_{11} & \rho_{11} & \rho_{22} & \rho_{22} & 1 \end{bmatrix}$$

We use $\bar{\rho}$ to represent the group correlation matrix:

$$\bar{\rho} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{12} & \rho_{22} \end{bmatrix}$$

Suppose that there are p groups, and we have N_i stocks in group i. The steps for calculating the optimal portfolio are:

Step 1. Compute the excess return $= \bar{R}_i - R_f$ and standard deviation σ_i for every stock i;

Step 2. Compute the group correlation matrix $\bar{\rho} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{12} & \rho_{22} \end{bmatrix}$;

Step 3. Calculate the matrix $A = \begin{bmatrix} 1 + \frac{N_1 \rho_{11}}{1-\rho_{11}} & \frac{N_1 \rho_{12}}{1-\rho_{11}} \\ \frac{N_2 \rho_{12}}{1-\rho_{22}} & 1 + \frac{N_2 \rho_{22}}{1-\rho_{22}} \end{bmatrix}$

Step 4. Calculate the vector $C = \begin{bmatrix} \sum_{i=1}^{N_1} \frac{\bar{R}_i - R_f}{\sigma_i (1-\rho_{11})} \\ \sum_{i=1}^{N_2} \frac{\bar{R}_i - R_f}{\sigma_i (1-\rho_{22})} \end{bmatrix}$

Step 5. Calculate the matrix $\Phi = A^{-1} C = \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix}$

Step 6. Compute $z_i = \frac{1}{\sigma_i(1-\rho_{kk})}[\frac{\bar{R}_i - R_f}{\sigma_i} - \sum_{g=1}^{p} \rho k g \Phi_g$ for each stock i and the group k it belongs to;

Step 9. Compute $x_i = \frac{z_i}{\sum_i z_i}$ for $i = 1, \cdots, 6$, then $x = (x_1, \cdots, x_6)$ is the allocation vector of the optimal portfolio.

2.7 Multi-index Model

In the multi-index model we continue to suppose that the stocks are grouped by industry. For simplicity, we discuss the case of two industries, where stocks 1, 2 belong to industry 1 and stocks 4, 5 belong to industry 2. We assume that there is a group index for each group; the stocks in a group are linearly related to the corresponding group index, and the group index is linearly related to the market index. The model is:

$$R_i = \alpha_i + \beta_i I_j + \epsilon_i$$

$$I_j = \gamma_j + b_j R_m + c_j$$

where

$$E(\epsilon_i \epsilon k) = 0 \text{ for } i = 1, \cdots, n, \ k = 1, \cdots, n, \ i \neq k$$

$$E(c_j c_l) = 0 \text{ for } j = 1, \cdots, p, \ l = 1, \cdots, p, \ j \neq l$$

$$E(\epsilon_i c_j) = 0 \text{ for } i = 1, \cdots, n, \ j = 1, \cdots, p$$

According to the assumptions above we can derive the variance and covariances for the stocks.

The variance for stock i, denoted by σ_i^2, is:

$$\sigma_i^2 = \beta_i^2 \sigma_j^2 + \sigma_{\epsilon_i}^2, \text{ and } \sigma_j^2 = b_j^2 \sigma_m^2 + \sigma_{c_j}^2$$

$$\Rightarrow \sigma_i^2 = \beta_i^2 (b_j^2 \sigma_m^2 + \sigma_{c_j}^2) + \sigma_{\epsilon_i}^2.$$

Suppose that stock i and k belong to the same group j. Then the covariance for stock i and k denoted by σ_{ik} is:

$$\sigma_{ik} = \beta_i \beta_k (b_j^2 \sigma_m^2 + \sigma_{c_j}^2).$$

Suppose that stock i and k belong to different groups: stock i belongs to group j, and stock k belongs to group l. Then the covariance for stock i and k denoted by σ_{ik} is:

$$\sigma_{ik} = \beta_i \beta_k b_j b_l \sigma_m^2.$$

Then we solve the problem below for an optimal portfolio:

$$\begin{cases} \bar{R}_1 - R_f &= z_1 \sigma_1^2 + z_2 \sigma_{12} + z_3 \sigma_{13} + z_4 \sigma_{14} \\ \bar{R}_2 - R_f &= z_1 \sigma_{21} + z_2 \sigma_2^2 + z_3 \sigma_{23} + z_4 \sigma_{24} \\ \bar{R}_3 - R_f &= z_1 \sigma_{31} + z_2 \sigma_{32} + z_3 \sigma_3^2 + z_4 \sigma_{34} \\ \bar{R}_4 - R_f &= z_1 \sigma_{41} + z_2 \sigma_{42} + z_3 \sigma_{43} + z_4 \sigma_4^2 \end{cases}$$

The steps for calculating the optimal portfolio are:

Step 1. Run the regressions below for every stock i in every group j:

$$R_i = \alpha_i + \beta_i I_j + \epsilon_i,$$

$$I_j = \gamma_j + b_j R_m + c_j$$

;

Step 2. Calculate the matrix M:

$$M = \begin{bmatrix} 1 + \frac{\beta_1^2}{\sigma_{\epsilon_1}^2}[\sigma_{c_1}^2 + b_1^2 \sigma_m^2] + \frac{\beta_2^2}{\sigma_{\epsilon_2}^2}[\sigma_{c_1}^2 + b_1^2 \sigma_m^2] & [\frac{\beta_1^2 b_1 b_2}{\sigma_{\epsilon_1}^2} + \frac{\beta_2^2 b_1 b_2}{\sigma_{\epsilon_2}^2}]\sigma_m^2 \\ [\frac{\beta_3^2 b_1 b_2}{\sigma_{\epsilon_3}^2} + \frac{\beta_4^2 b_1 b_2}{\sigma_{\epsilon_4}^2}]\sigma_m^2 & 1 + \frac{\beta_3^2}{\sigma_{\epsilon_3}^2}[\sigma_{c_2}^2 + b_2^2 \sigma_m^2] + \frac{\beta_4^2}{\sigma_{\epsilon_4}^2}[\sigma_{c_2}^2 + b_2^2 \sigma_m^2] \end{bmatrix}.$$

Step 3. Calculate the vector R:

$$R = \begin{bmatrix} \sum_{i=1}^{2} \frac{(\bar{R}_i - R_f)\beta_i}{\sigma_{\epsilon_i}^2} \\ \sum_{i=3}^{4} \frac{(\bar{R}_i - R_f)\beta_i}{\sigma_{\epsilon_i}^2} \end{bmatrix}.$$

Step 4. Calculate the vector Φ:

$$\Phi = \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = M^{-1}R.$$

Step 4. Calculate z_i for $i = 1, 2, 3, 4$:

$$\begin{cases} z_1 &= \frac{\beta_1}{\sigma^2_{\epsilon_1}}[\frac{\bar{R}_1 - R_f}{\beta_1} - [(\sigma^2_{c_1} + b_1^2\sigma^2_m)\Phi_1 + b_1 b_2 \sigma^2_m \Phi_2]] \\ z_2 &= \frac{\beta_2}{\sigma^2_{\epsilon_2}}[\frac{\bar{R}_2 - R_f}{\beta_2} - [(\sigma^2_{c_1} + b_1^2\sigma^2_m)\Phi_1 + b_1 b_2 \sigma^2_m \Phi_2]] \\ z_3 &= \frac{\beta_3}{\sigma^2_{\epsilon_3}}[\frac{\bar{R}_3 - R_f}{\beta_3} - [(\sigma^2_{c_2} + b_2^2\sigma^2_m)\Phi_2 + b_1 b_2 \sigma^2_m \Phi_1]] \\ z_4 &= \frac{\beta_4}{\sigma^2_{\epsilon_4}}[\frac{\bar{R}_4 - R_f}{\beta_4} - [(\sigma^2_{c_2} + b_2^2\sigma^2_m)\Phi_2 + b_1 b_2 \sigma^2_m \Phi_1]] \end{cases}$$

Step 6. Compute $x_i = \frac{z_i}{\sum_i z_i}$ for $i = 1, \cdots, 4$, then $x = (x_1, \cdots, x_4)$ is the allocation vector of the optimal portfolio.

CHAPTER 3

Portfolio Evaluation

3.1 Basic Concepts

In section 2.1.1 we assume that investors' goal of portfolio optimization is maximizing the expected return of the portfolio while trying to minimize the risk at the same time. Thus when we evaluate the performance of a portfolio, we are actually evaluating the return of a portfolio relative to its risk. Compared with return, which has a explicit definition making it easy to calculate, risk is rather difficult to measure. Various risk measures leads to various portfolio evaluation methods, which can be generalized into 4 categories as follows:

Performance/Risk Ratios Evaluate the portfolio by the ratio of dividing the performance by a risk measure. A typical measure is Sharpe's ratio.

Incremental Return The portfolio performance is represented by an absolute return obtained from subtracting a penalty from the measure of wealth. A typical measure is Jensen's alpha.

Preference-based Measures We design individualized portfolio performance measures by introducing utility functions to represent the investors' risk preferences. A typical measure is generalized Sharpe ratio.

Market Timing Market timing is the strategy to make investment decisions by attempting to predict the future market movements. Typical measures are the Merton-Henriksson market timing measure and the Treynor-Mazuy market timing measure.

In the following sections we will discuss these portfolio evaluation methods in detail.

3.2 Performance/Risk Ratios

3.2.1 Sharpe Ratio and Adjusted Sharpe Ratio

The Sharpe ratio is defined as the ratio of the mean return in excess of the risk free rate divided by its standard deviation:

$$Sharpe\ ratio = \frac{\bar{R}_p - R_f}{\sigma_p}.$$

The higher the Sharpe ratio, the better the portfolio performance. Sharpe ratio exhibits obvious advantages. From its definition we can see that Sharpe ratio uses the standard deviation of returns to measure total portfolio risk, which assumes normally distributed returns. And to calculate Sharpe ratio we only need the expected return and the standard deviation of the portfolio. Sharpe ratio is easy to understand and calculate; although it is proposed as early as 1966, it is still widely used nowadays.

However, Sharpe ratio shows many weaknesses as well. First, it is only useful in ranking different portfolios; the value itself it is meaningless. Second, the ratio relies on a constant risk-free rate and does not introduce any benchmark portfolio. Third, when Sharpe ratio is negative, it will increase along with the risk, which is hard to interpret. Fourth, returns may deviate from normal distribution.

To refine on the weaknesses some statistical adaptations are proposed in later years. For example, Israelsen's modified Sharpe ratio exponentiates the denominator with the excess return divided by its absolute value and thus widens the range of values. Some other improved measures, such as adjusted for skewness Sharpe ratio and adjusted for skewness and kurtosis Sharpe ratio, introduce higher moments to solve the non-Gaussian distribution problem.

3.2.2 Sharpe Ratio Based on VaR and CVaR

VaR refers to the risk measure "Value at Risk", and CVaR refers to the measure Conditional Value at Risk.

Given a confidence level $\alpha \in (0,1)$, suppose that the loss of a portfolio is L, then Value at Risk (VaR) at confidence level α is defined as the smallest number l such that the probability of loss exceeding l is at most $(1-\alpha)$, namely

$$VaR_{\alpha}(L) = \inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \alpha\}.$$

For instance, given confidence level $\alpha = 5\%$, VaR is the minimum loss among the worst 5% of the cases.

In original Sharpe Ratio we use the standard deviation to measure risk. As for Sharpe ratio based on Value at Risk, VaR is the risk indicator instead. To calculate Sharpe ratio based on VaR, we divide the VaR by the initial value of the portfolio and use this percent of loss as the denominator in the Sharpe ratio. This measure refines the original Sharpe ratio on its weakness of unable to distinguish between upside and downside risks.

On the other hand, VaR has its own drawbacks too. First, it is sensitive to the confidence level α. Second, VaR has many local extremes resulted in unstable ranking criterion. Third, it does not measure losses exceeding VaR.

Conditional Value at Risk (CVaR), also known as expected shortfall and expected tail loss, refines VaR on its third drawback. It is the probability for a specific loss to exceed VaR. Suppose $\alpha \in (0,1)$ and $L \in L_p(\mathscr{F})$ is the payoff of a portfolio at some time in the future, then CVaR is defined as

$$\begin{aligned} CVaR_\alpha &= \frac{1}{\alpha}\int_0^\alpha VaR_\gamma(L)d\gamma \\ &= -\frac{1}{\alpha}(E[L\mathbb{1}\{L \leq l_\alpha\}] + l_\alpha(\alpha - P[L \leq l_\alpha])) \end{aligned}$$

where $l_\alpha = \inf\{l \in \mathbb{R} : P(L \leq l) \geq \alpha)\}$ and $\mathbb{1}$ is the indicator function.

CVaR can be used as an alternative of VaR that is more sensitive to the tail of the loss distribution. Compared with Sharpe ratio based on VaR, Sharpe ratio based on CVaR used CVaR as the risk measure in the denominator instead of VaR.

3.2.3 Treynor Ratio and Treynor-Black Ratio

Before introducing Treynor ratio, we first explain a new measure of risk: the beta. Beta measures the volatility, or systematic risk, of a portfolio in comparison to the entire market. To calculate beta, we consider the following regression of the portfolio P's excess return on market excess return:

$$R_p - R_f = \alpha_p + \beta_p(R_m - R_f) + \epsilon_p.$$

The beta is the coefficient of market return in the regression, so it is also called "beta coefficient", and can be presented as

$$\beta = \frac{cov(R_p, R_m)}{var(R_m)} = \frac{\sigma_{im}}{\sigma_m^2} = \rho_{im}.$$

The Treynor ratio, also known as the reward-to-volatility ratio, is a risk-adjusted measurement of a return, based on systematic risk. It is the excess returns over the risk-free rate per unit of additional risk compared with the market. Mathematically it is defined as

$$Treynor\ ratio = \frac{\bar{R}_p - R_f}{\beta_p}.$$

The higher the Treynor ratio, the better the portfolio performance.

We see that Treynor ratio is very similar to Sharpe ratio. The only difference is that Sharpe ratio uses standard deviation to measure the total risk, while Treynor ratio uses beta to measure the systematic risk. Therefore Treynor ratio shares some limitations with Sharpe ratio: it is simply a ranking criterion with meaningless value, and it relies on the choice of market return rate. Compared with Sharpe ratio, an additional drawback is that portfolios with different total risk but equal systematic risk will have the same rank. On the other hand, an additional strength is that the risk measure beta it uses allows it to evaluate an aggregation of portfolios, while Sharpe ratio can only be used to evaluate a single portfolio.

A modified version, Treynor-Black ratio (also known as Treynor-Black Appraisal ratio), consider the alpha coefficient in the regression at the numerator instead of excess return, and the standard deviation of the residuals at the denominator instead of beta:

$$Treynor - Black\ ratio = \frac{\alpha_p}{\sigma(\epsilon_p)}.$$

Alpha is an alternative measure of excess return, and we will discuss its advantages later.

3.2.4 Other measures

Omega Ratio Given a threshold return target r, omega ratio is defined as the probability weighted ratio of gains versus losses with respect to r, namely

$$\Omega(r) = \frac{\int_r^\infty (1 - F(x))dx}{\int_{-\infty}^r F(x)dx}$$

where F is the cumulative distribution function of the return. The higher the omega ratio is, the more gains over losses the portfolio produces, and the better the portfolio performance is. When r is zero, the omega ratio is equivalent to the Bernardo-Ledoit gain-loss ratio, which is defined as the expectation of the positive part of the returns divided by the expectation of the negative part of returns. Earlier we mentioned that the Sharpe ratio is hard to interpret when it is negative; an advantage of the omega ratio is that it utilizes the information discarded by Sharpe ratio, i.e. the negative part of returns.

Sortino Ratio Sortino ratio, like the omega ratio, refines Sharpe ratio on its inability to utilize the negative part of returns, but in a different way.

First we introduce semivariance, which is defined as

$$semivariance = E[(R_p - \bar{R}_p)^2 \mathbb{1}\{R_p \leq \bar{R}_p\}],$$

and semi-deviation is the square root of the semivariance. Semivariance, like VaR and CVaR, measures the downside risk (risk of the actual return being below the expected return).

Sortino ratio, as a modification of the Sharpe ratio, is defined as

$$Sortino\ Ratio = \frac{\bar{R}_p - T}{DR},$$

where T is s target return rate and DR is the target semi-deviation, i.e.

$$DR = \sqrt{E[(R_p - T)^2 \mathbb{1}\{R_p \leq T\}]}.$$

Sharpe ratio and Sortino ratio would give similar results under a symmetrical return distribution, but differ greatly when the return distribution is skewed.

Gini Ratio The Gini ratio, as another modification of Sharpe ratio, uses Gini coefficient as the risk measure. Suppose that we have the historical return rate for the portfolio in time $t = 1, \cdots, T$, the Gini coefficient is defined as

$$G = \frac{\sum_{i=1}^{T} \sum_{j=1}^{T} |R_{p,i} - R_{p,j}|}{2T^2 \bar{R}_p}.$$

And Gini ratio is the ratio of excess return from the risk-free rate divided by the Gini coefficient:

$$Gini\ ratio = \frac{\bar{R}_p - R_f}{G}.$$

Gini coefficient measures the spread of return rates among themselves, and does not rely on any fixed central point like the mean or a target return rate. Therefore Gini ratio is more informative than Sharpe ratio when the return has a non-Gaussian distribution.

Minimax Ratio Minimax ratio is the ratio of expected excess return over risk-free rate divided by the maximum loss. As Sharpe ratio measures risk with standard deviation, the square root of the variance, it can be seen as based on a l_2 risk measure. Then the minimax ratio is based on a l_∞ risk measure. Minimax ratio has an obvious and severe weakness that it is easily impacted by outliers.

Information Ratio Information ratio, also known as the appraisal ratio, is the ratio of expected excess return over a benchmark divided by the standard deviation of the excess return. The Treynor-Black ratio we discussed earlier is a special case of information ration using the market return as the benchmark.
The information ratio mainly has two disadvantages. First, it relies on the sensitivity of the portfolio return to the benchmark. If the portfolio return is not sensitive to benchmark, there will be little variation. Second, it treats the upside risk and downside risk from the benchmark equally. There is an information ratio based on semivariance that refines original information value on the second disadvantage.

3.3 Incremental Return

3.3.1 Jensen's Alpha and Its Variations

Jensen's alpha is a measure of the marginal return. To calculate it, we consider the regression below:

$$R_p - R_f = \alpha_p + \beta_p(R_m - R_f) + \epsilon_p.$$

Then the α_p coefficient is Jensen's alpha.
Alpha, as the excess return of a portfolio relative to the benchmark market return, evaluates the performance of an investment against a market index. When alpha is positive, the portfolio outperforms the market, and when alpha is negative the

portfolio underperforms he market.

Like Sharpe ratio Jensen's alpha is a very classical and popular measure, which is easy to calculate and interpret but has many weaknesses as well. First, it relies on the choice of the benchmark, the market portfolio. Second, it does not measure risks.

An adaption of Jensen's alpha, the total risk alpha, brings risk in to account. Given a target risk σ_p, we first build a portfolio BP with risk σ_p by combining the market portfolio and the risk-free asset. Then we build a portfolio P with a risk of σ_p too. Then the total risk alpha of portfolio P is $R_P - R_{BP}$, where portfolio BP acts as the benchmark.

Another adaption is standardized Jensen's alpha, which is defined as the original Jensen's alpha divided by its standard deviation. The advantage of standardized Jensen's alpha is that when two portfolios share the same Jensen's alpha, we can use standardized Jensen's alpha to rank them.

3.3.2 M^2 Index

The M^2 index, also called risk-adjusted performance (RAP), measures the incremental return relative to market risk.

For a portfolio P with expected return $\bar{R}_p$ and standard deviation σ_p, and a market portfolio with expected return $\bar{R}_m$ and standard deviation σ_m, we build a portfolio BP with risk σ_m by combining the portfolio P and the risk-free asset. Suppose that the expected return of portfolio BP is R_{BP}, then the M^2 index of portfolio P is

$$R_{BP} - R_m = \frac{\sigma_m}{\sigma_p}\bar{R}_p + (1 - \frac{\sigma_m}{\sigma_p})R_f - \bar{R}_m = \frac{\sigma_m(\bar{R}_p - R_f) - \sigma_p(\bar{R}_m - R_f)}{\sigma_p}$$

When we use M^2 index to rank portfolios, the higher the M^2 index value, the better the portfolio performance. Although there is a benchmark market portfolio involved, the benchmark is just used to scale the return and does not affect the final ranking of portfolios. Mathematically M^2 index is is a linear function of the Sharpe ratio; thus it has the same drawbacks as Sharpe ratio.

3.4 Preference-based Measures

Before explaining preference-base measures, we first introduce utility functions. Suppose that X is a consumption set. A consumer's utility function $u : X \to \mathbb{R}$ represents a preference relation on X:

$\forall x, y \in X,\ u(x) \leq u(y)$ if and only if the consumer wants y at least as much as x.

Utility functions map the consumption set to real number set, and help us quantify consumers' preference while keeping the preference order. The simplest ways to build preference-based measures is to directly introduce utility functions to represent investors' preference. Suppose that the investor of a portfolio has utility function $u(\cdot)$, then we can define a new measure as

$$\frac{E[u(\bar{R}_p - R_f)]}{\sigma_p}.$$

The new measure keeps the general form of Sharpe ratio and maximizes the utility of the investor instead of the expected portfolio return.

A typical preference-based measure, the generalized Sharpe ratio suggests an exponential utility function to represent investors' preference and relates it to the Sharpe ratio. The preference-based measures that directly introduce utility for preference like this share a common drawback: the complexity in computation. When we calculate the measures, we have to solve a utility maximizing problem first; this can be troublesome for some utility functions.

3.5 Market Timing

3.5.1 Treynor-Mazuy Market Timing measure

The Treynor-Mazuy market timing regression is

$$R_{p,t+1} - R_f = a_p + b_p(R_{m,t+1} - R_f) + \Lambda_p(R_{m,t+1} - R_f)^2 + v_{t+1}$$

where the coefficient Λ_p measures the market timing ability. In this case $\Lambda_p > 0$ indicates market timing ability: when the market moves, the portfolio return goes up or down in the same direction as the market, but by a disproportionate amount. When $\Lambda_p > 0$, although different with Jensen's alpha a_p can be used for portfolio evaluation as well. A positive a_p usually indicates a well-designed portfolio.

3.5.2 Merton-Henriksson Market Timing Measure

The Merton-Henriksson market timing regression is

$$R_{p,t+1} - R_f = a_p + b_p(R_{m,t+1} - R_f) + \Lambda_p max(R_{m,t+1} - R_f, 0) + u_{t+1}$$

where the coefficient Λ_p measures the market timing ability.
When $\Lambda_p = 0$, a_p is Jensen's alpha and b_p is the beta we use to measure risk. Here $\Lambda_p \neq 0$ indicates market timing ability: when the market moves, the portfolio return moves in the same direction as the market by a proportional amount.
The term $max(R_{m,t+1}, 0)$ is a dummy variable that takes different values when the market goes up and down, and it allows the investor to choose between an upside beta and a downside beta. When the market goes up, investors will choose the higher upside beta, as more funds are invested in the risky asset; when the market goes down, investors will choose the lower downside beta, as more funds are invested in the risk-free asset.

CHAPTER 4

Empirical Study

4.1 Data Selection

I collect the historical monthly return rates of 25 stocks and a market index S&P 500 during the period January 2008 to December 2015 from Yahoo Finance (http://finance.yahoo.com). I use the data from January 2008 to December 2012 to find the optimal portfolios and the data from January 2013 to December 2015 to evaluate the performance of the portfolios.
The 25 stocks belongs to 5 sectors: Consumer Goods, Utility, Basic Materials, Healthcare, and Industrial Goods, with 5 stocks in each of the above 5 sectors. The stock names and their notations are listed by sector in the Table 4.1.

4.2 Portfolio Optimization

I build five portfolios as follows:

Equal Allocation Portfolio (denoted by EA) Invest a proportion of $1/25 = 0.04$ in each of the 25 stocks; the expected portfolio return is 0.01093141, and the standard deviation is 0.06553477.

Classical Markowitz Model (denoted by CMM) With the risk-free rate to be $R_f = 0.001$, an optimal portfolio is built based on the classical Markowitz model. The allocation proportions, the expected return, and the standard devia-

Table 4.1: Names and notations of the market index and 25 stocks

Consumer Goods	Utility
Unilever PLC (ULVR.L)	AGL Energy Ltd (AGL.AX)
Apple Inc (AAPL.MX)	Algonquin Power (AQN.TO)
Associated British Foods PLC (ABF.L)	Centrica PLC (CNA.L)
Anglo-Eastern Plantations PLC (AEP.L)	Cesc Ltd (CESC.BO
Archer-Daniels-Midland Company (ADM)	NTPC Ltd (NTPC.NS)
Industrial Goods	**Healthcare**
Bodycote PLC (BOY.L)	Unilever PLC (ULVR.L)
ACC Ltd (ACC.BO)	ACADIA Pharmaceuticals Inc. (ACAD)
Caterpillar Inc. (CAT)	Achillion Pharmaceuticals, Inc (ACHN)
Deere & Company (DE)	Advanced Proteome Therpt (APC.V)
Escorts Ltd (ESCORTS.NS)	Alnylam Pharmaceuticals, Inc. (ALNY)
Basic Materials	**Market Index**
Anglo American PLC (AAL.L)	S&P 500 (^GSPC)
Chambal Fertilisers (CHAMBLFER.NS)	
Deepak Fertilisers (DEEPAKFER.NS)	
Dhanuka Agritech Ltd (DHANUKA.BO)	
E I du Pont de Nemours (DUPP.PA)	

Table 4.2: Classical Markowitz model: optimal portfolio summary

Consumer Goods		Utility		Basic Materials	
ULVR.L	0.37612	AGL.AX	0.59375	AAL.L	-0.21998
AAPL.MX	0.30785	AQN.TO	-0.03586	CHAMBLFER.NS	0.18719
ABF.L	-0.08085	CNA.L	0.33619	DEEPAKFER.NS	0.08192
AEP.L	0.49109	CESC.BO	-0.22167	DHANUKA.BO	0.29952
ADM	-0.16029	NTPC.NS	-0.37603	DUPP.PA	-0.14567
Healthcare		**Industrial Goods**		Expected Return	0.04421
ABT.L	-0.37682	BOY.L	-0.02706		
ACAD	0.07868	ACC.BO	0.12973		
ACHN	0.10063	CAT	0.42189	Standard Deviation	0.06819
APC.V	-0.22148	DE	-0.56243		
ALNY	0.14760	ESCORTS.NS	-0.12404		

tion of this optimal portfolio is shown by Table 4.2.

Single Index Model (denoted by SIM) With the risk-free rate to be $R_f = 0.001$, an optimal portfolio is built based on the single index model. The allocation proportions, the expected return, and the standard deviation of this optimal portfolio is shown by Table 4.3.

Constant Correlation Model (denoted by CCM) With the risk-free rate to be $R_f = 0.001$, an optimal portfolio is built based on the constant correlation model. The allocation proportions, the expected return, and the standard deviation of this optimal portfolio is shown by Table 4.4.

Multigroup Model (denoted by MGM) With the risk-free rate to be $R_f = 0.001$, an optimal portfolio is built based on the multigroup model. The allocation proportions, the expected return, and the standard deviation of this optimal

Table 4.3: Single index model: optimal portfolio summary

Consumer Goods		Utility		Basic Materials	
ULVR.L	0.16809	AGL.AX	0.15390	AAL.L	-0.13734
AAPL.MX	0.20237	AQN.TO	0.02342	CHAMBLFER.NS	0.01406
ABF.L	0.32474	CNA.L	0.12024	DEEPAKFER.NS	-0.01914
AEP.L	0.06751	CESC.BO	-0.08011	DHANUKA.BO	0.19602
ADM	-0.09586	NTPC.NS	-0.12137	DUPP.PA	0.06629
Healthcare		**Industrial Goods**		Expected Return	0.0258
ABT.L	0.10961	BOY.L	0.05787		
ACAD	0.01788	ACC.BO	0.07548		
ACHN	0.04772	CAT	0.00560	Standard Deviation	0.04780
APC.V	-0.01589	DE	-0.13366		
ALNY	-0.02163	ESCORTS.NS	-0.02579		

portfolio is shown by Table 4.5.

4.3 Portfolio Evaluation

4.3.1 General Analysis

Using the allocations proportions of each portfolio calculated by historical returns from January 2008 to December 2012, I calculate the historical returns of the five portfolios from January 2013 to December 2015, which are demonstrated in Figure 4.1 along with the market index S&P 500. In Figure 4.1, the equal allocation portfolio, optimal portfolio of classical Markowitz model, optimal portfolio of single index model, optimal portfolio of constant correlation model, optimal portfolio of multigroup model, and market index are denoted by EA, CMM, SIM, CCM, MGM, and market respectively.

Table 4.4: Constant correlation model: optimal portfolio summary

Consumer Goods		Utility		Basic Materials	
ULVR.L	0.23567	AGL.AX	0.20023	AAL.L	-0.14996
AAPL.MX	0.34560	AQN.TO	0.00057	CHAMBLFER.NS	-0.01821
ABF.L	0.53738	CNA.L	0.08196	DEEPAKFER.NS	-0.07260
AEP.L	0.08523	CESC.BO	-0.16227	DHANUKA.BO	0.34836
ADM	-0.30437	NTPC.NS	-0.32543	DUPP.PA	0.09380
Healthcare		**Industrial Goods**			
ABT.L	0.06028	BOY.L	0.08332	Expected Return	0.04055
ACAD	0.01965	ACC.BO	0.09625		
ACHN	0.06113	CAT	0.04483	Standard Deviation	0.08006
APC.V	-0.07457	DE	-0.05438		
ALNY	-0.07749	ESCORTS.NS	-0.05499		

Figure 4.2 plots the 25 stocks and the five portfolios we build in the last section in an expected value versus risk diagram. From Figure 4.2 we can that the multigroup model has the highest expected return along with medium level of risk; the constant correlation models has about the same risk as the multigroup model, but a lower expected return; the classical Markowitz model has a medium expected return with the highest risk; the single index model and the equal allocation portfolio have rather low expected returns and risks.

Figure 4.3 plots the cumulative returns of the optimal portfolios and the market index. Here we use the market index as a benchmark and the roughly-designed equal allocation portfolio as a contrast. The four well-optimized portfolios, CMM, SIM, CCM, and MGM, are supposed to outperform both the market and the EA. From Figure 4.3 we can see that all the four well-optimized portfolios outperforms the market and the EA, and EA outperforms the market. The classical Markowitz

Table 4.5: Multigroup model: optimal portfolio summary

Consumer Goods		Utility		Basic Materials	
ULVR.L	0.30601	AGL.AX	0.05419	AAL.L	-0.08803
AAPL.MX	0.32790	AQN.TO	-0.05395	CHAMBLFER.NS	0.01659
ABF.L	0.52800	CNA.L	-0.04683	DEEPAKFER.NS	-0.02177
AEP.L	0.13523	CESC.BO	-0.16583	DHANUKA.BO	0.35032
ADM	-0.14021	NTPC.NS	-0.31419	DUPP.PA	0.14293
Healthcare		**Industrial Goods**			
ABT.L	0.15043	BOY.L	0.02387	Expected Return	0.03663
ACAD	0.03082	ACC.BO	0.01273		
ACHN	0.06706	CAT	-0.02085		
APC.V	-0.02924	DE	-0.14189	Standard Deviation	0.06769
ALNY	-0.01813	ESCORTS.NS	-0.10516		

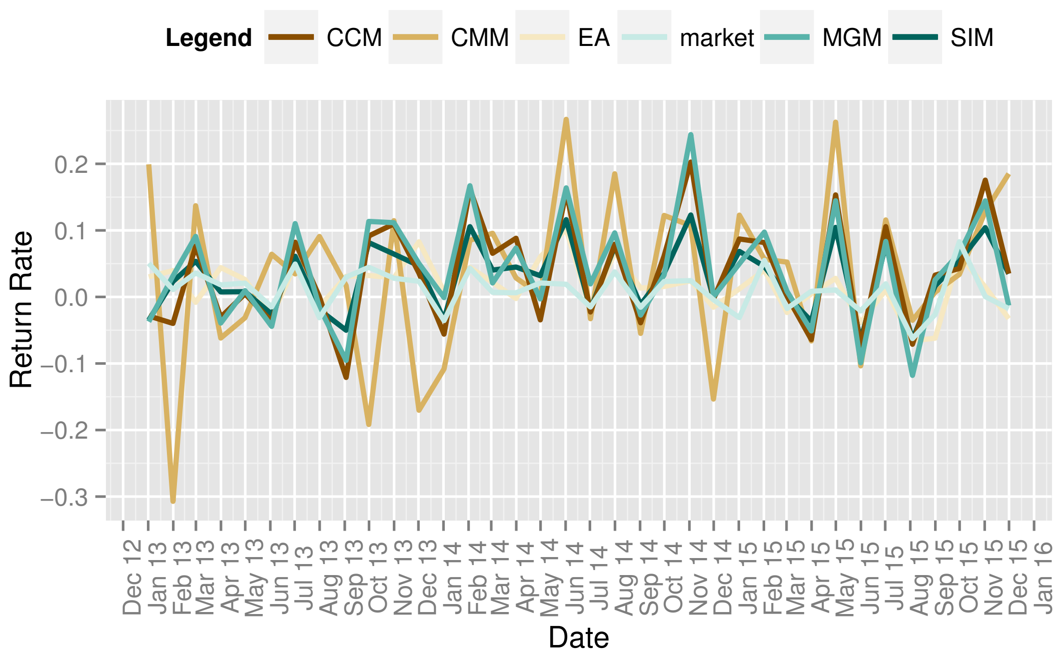
model remains below the market and the equal allocation portfolio for a long time but eventually grows beyond them. This evidence verifies the effectiveness of the four portfolio optimizing models; and even that of the EA which is simply supported by the idea of diversification.

Among the four well-optimized portfolios, MGM outperforms all the others; CCM is next in rank below MGM, followed by SIM and CMM. CMM, as mentioned above, goes beyond EA in the last minute, and appears to be the worst model.

Based on the observations above, I think the the multigroup model has the best performance. It is not surprising, as stocks we select are naturally grouped into 5 different sectors, which meets the assumption of the multigroup model. I will apply different measures to evaluate the portfolio performance and test my idea.

Before proceeding to applying the portfolio performance measures, I test the normality of the portfolios returns first. Figure 4.4 plots the densities of the five portfolios and the market index; Figure 4.5 to 4.10 gives the Q-Q plots of the five

Figure 4.1: Historical returns of the five portfolios



portfolios and the market index. From Figure 4.5 we can find that EA, market, and CCM have nearly symmetrical distributions, while CMM, MGM, and SIM are left-skewed. According to the Q-Q plots none of the return data appears to be normally distributed, which is confirmed by the Shapiro-Wilk test results. Therefore besides some classical performance measures that assume normality, I also try measures that work with non-normality.

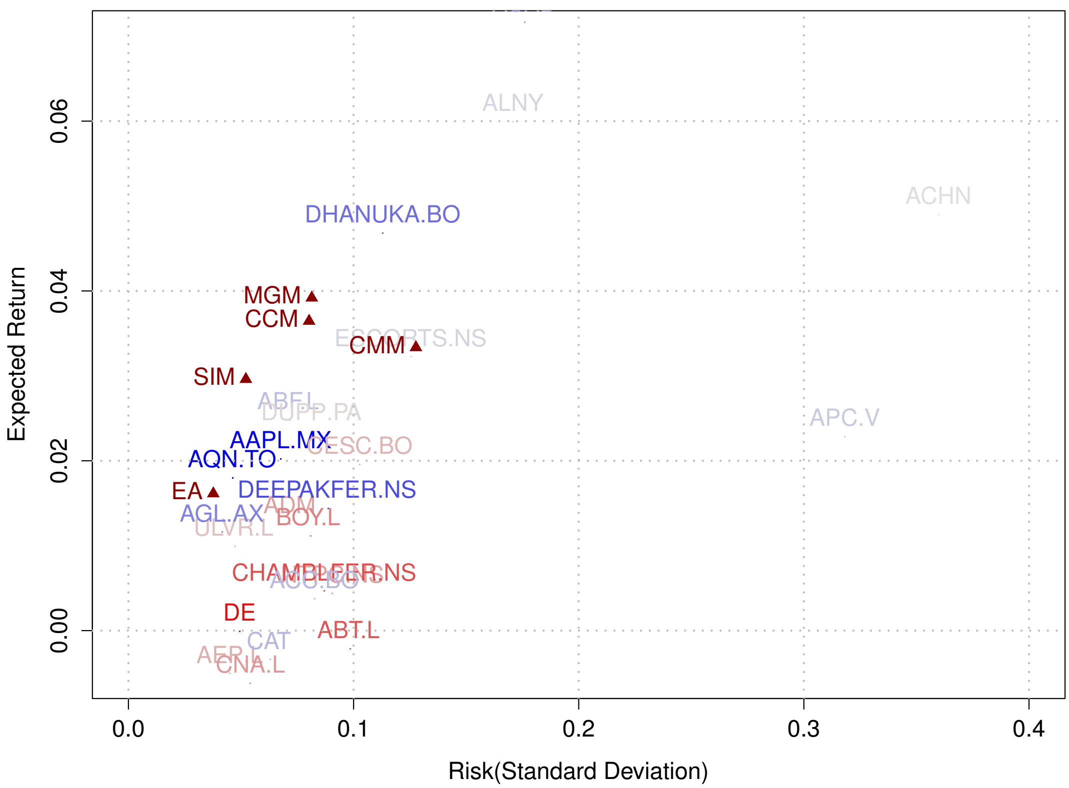
4.3.2 Portfolio Performance Measures

I apply 11 performance measures to the five portfolios. The values of the measures corresponding to different portfolios are organized in Table 4.6.

Table 4.6: Portfolio Performance Measures

		EA	CMM	SIM	CCM	MGM
Sharpe ratio		0.40140	0.25332	0.54836	0.44184	0.46897
Adjusted Sharpe ratio		1.46358	0.84781	2.25423	1.80457	1.85284
Sharpe ratio based on VaR ($\alpha = 5\%$)		0.32668	0.17082	0.50425	0.39104	0.43707
Sharpe ratio based on CVaR ($\alpha = 5\%$)		0.24995	0.14290	0.37396	0.27972	0.30205
Treynor ratio		0.01705	0.05074	0.03538	0.03706	0.02991
Treynor-Black ratio		0.22244	0.20330	0.43806	0.34012	0.34690
omega ratio (market index as benchmark)		1.81067	1.58301	3.07498	2.44187	2.85415
Gini ratio		0.01173	0.01534	0.02871	0.02870	0.03305
Jensen's alpha		0.00582	0.02566	0.02012	0.02541	0.02478
Treynor-Mazuy Market Timing measure	alpha	0.01070	0.02934	0.02864	0.03556	0.03644
	beta	0.96464	0.69636	0.95728	1.13408	1.47727
	gamma	-4.73026	-3.60803	-9.12157	-10.86752	12.27421
Merton-Henriksson Market Timing Measure	alpha	0.01454	0.02620	0.03817	0.04686	0.04687
	beta	1.27110	0.63305	1.65285	1.96071	2.29633
	gamma	0.63850	0.00734	-1.40532	-1.67084	-1.69646

Figure 4.2: Expected returns and risks of stocks and portfolios



4.3.3 Performance Evaluation

Table 4.6 includes the results of all the portfolio performance measures that help us evaluate in depth. Below is the ranking of the portfolios by each measure, from the best to the worst:

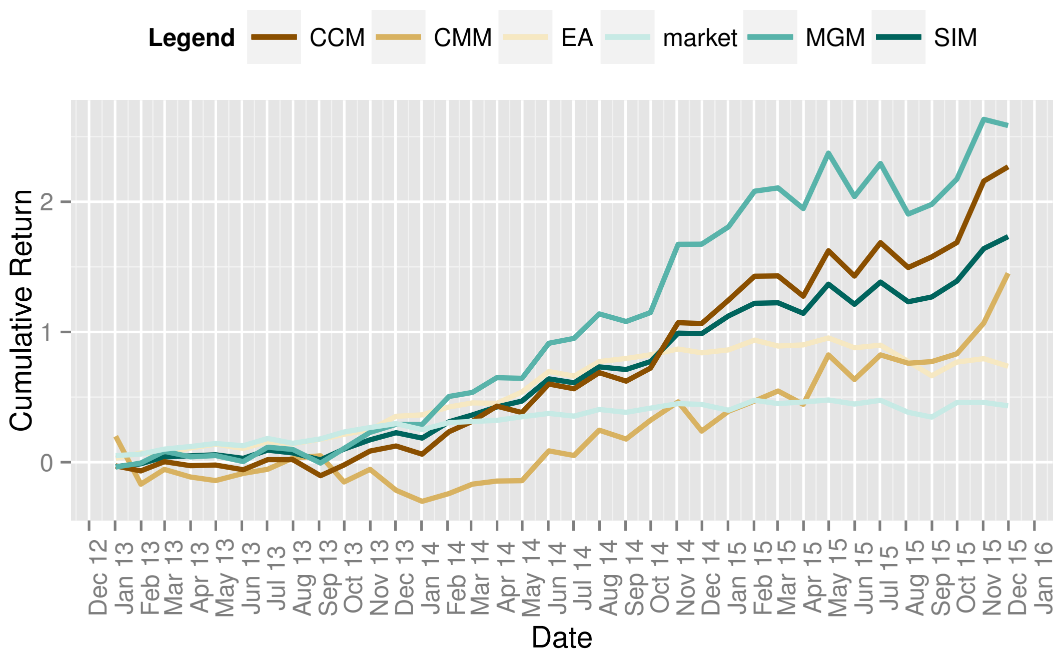
Sharpe ratio SIM, MGM, CCM, EA, CMM.

Adjusted Sharpe ratio SIM, MGM, CCM, EA, CMM.

Sharpe ratio based on VaR SIM, MGM, CCM, EA, CMM.

Sharpe ratio based on CVaR SIM, MGM, CCM, EA, CMM.

Figure 4.3: Cumulative returns of the five portfolios and the market



Treynor ratio CMM, CCM, SIM, MGM, EA.

Treynor-black ratio SIM, MGM, CCM, EA, CMM.

Omega ratio SIM, MGM, CCM, EA, CMM.

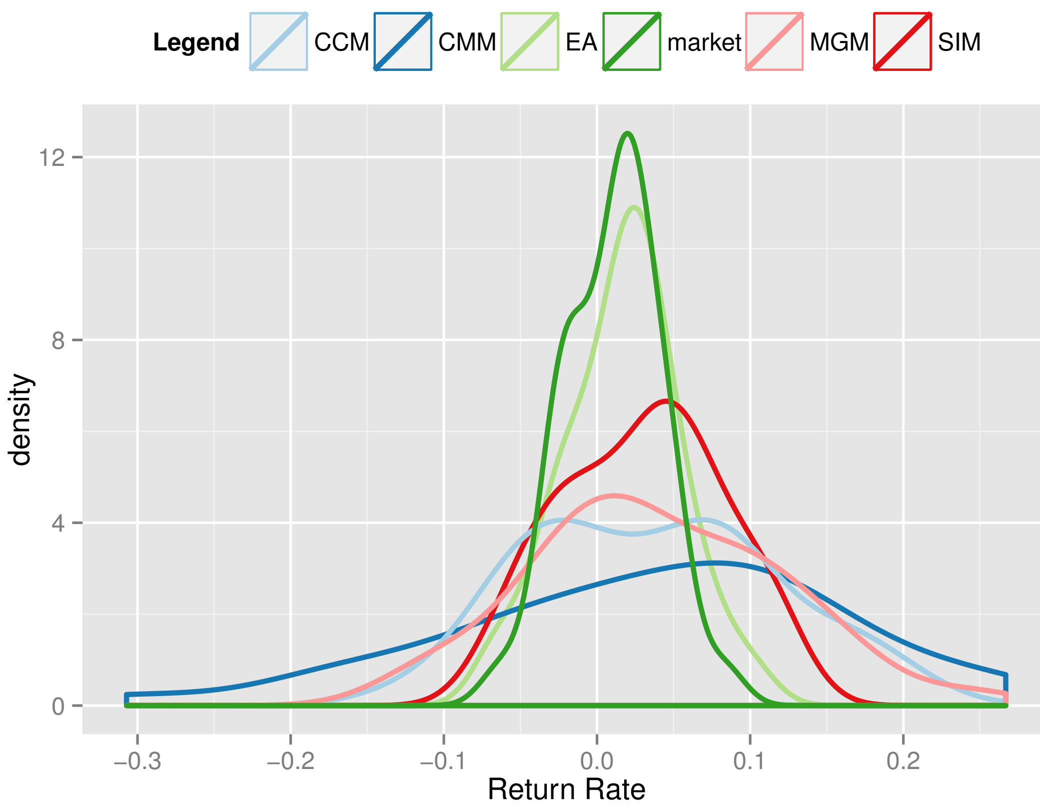
Gini ratio MGM, SIM, CCM, EA, CMM.

Jensen's alpha CMM, CCM, MGM, SIM, EA.

Treynor-Mazuy market timing All the gammas are negative, indicating no market timing ability. The ranking of alpha is: MGM, CCM, CMM, SIM, EA.

Merton-Henriksson market timing All the gammas are non-zero, indicating market timing ability. The ranking of alpha is: MGM, CCM, SIM, CMM, EA.

Figure 4.4: Densities of portfolio returns



From the ranking above we can find that Sharpe ratio, adjusted Sharpe ratio, Sharpe ratio based on VaR, Sharpe ratio based on CVaR, Treynor-black ratio and omega ratio provide the same ranking. Gini ratio is slightly different from them by exchanging the rank of MGM and SIM. The results of these measures corroborate my findings in the general analysis; we can conclude that they are effective measures. On the other hand, the Treynor ratio and Jensen's alpha provide completely different ranking compared with the others. They rank the CMM portfolio with poorest performance to be the best, and the best MGM, SIM portfolios to be the worst. We conclude that they are ineffective measure.

Ruling out the ineffective measures and leaving the special market timing measures

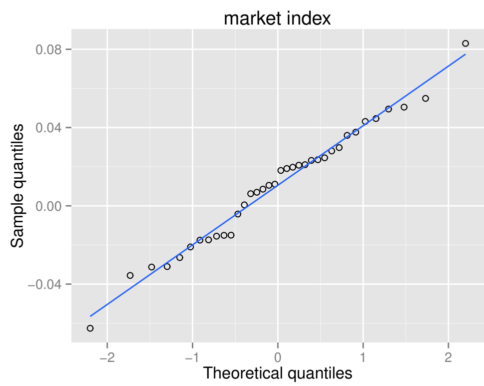


Figure 4.5: Q-Q plot of market

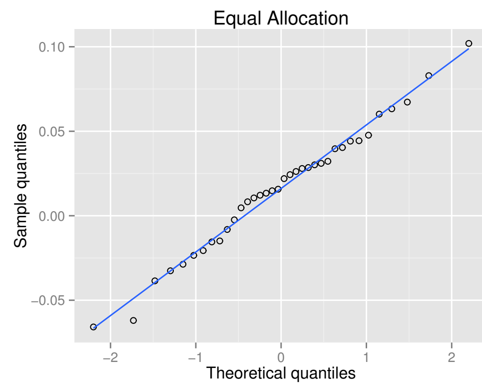


Figure 4.6: Q-Q plot of EA

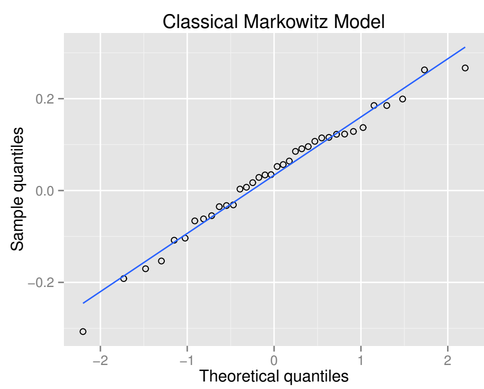


Figure 4.7: Q-Q plot of CMM

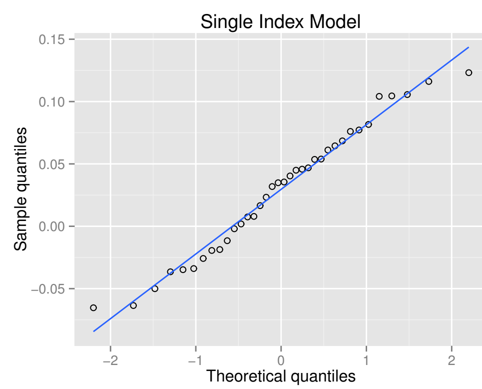


Figure 4.8: Q-Q plot of SIM

alone, we grade the portfolios from point 5 to point 1 as they are ranked from the highest to the lowest. Then the portfolio scores from the highest to the lowest are: SIM, MGM, CCM, EA, CMM.

We can see that the majority of effective measures rank SIM higher than MGM because they consider the risk-aversion of investors. Compared with MGM that has the highest expected return and a medium level of risk, investors would prefer SIM with the lowest expected return but also the lowest risk among the four well-optimized portfolios.

An exception is Gini ratio: it does not take risk-aversion into account. It simply calculates the dispersion, and assign equal weights to the differences between any

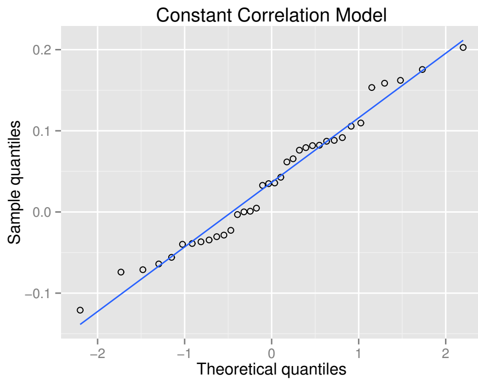


Figure 4.9: Q-Q plot of CCM

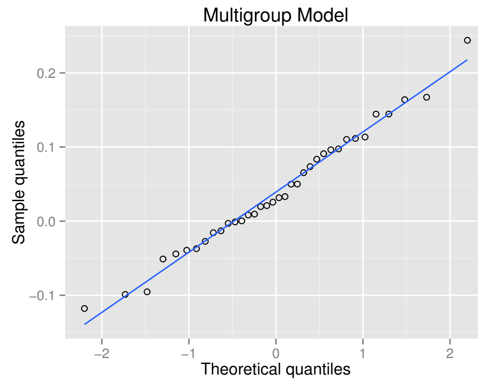


Figure 4.10: Q-Q plot of MGM

two return rates. Thus Gini ratio does not worry about the rather high risk of MGM and puts it in the first place.

As for the market timing ability, the two market timing measures provide opposite results. Since Goetzmann et al. [2000] show that Merton-Henriksson market timing measure gives weak results if it is applied to monthly results instead of a daily timer, and the historical return data used in this thesis is on a monthly basis, I tend to trust the result of Treynor-Mazuy market timing measure, which indicates no market timing ability for any of the five portfolios.

CHAPTER 5

Conclusion and Future Work

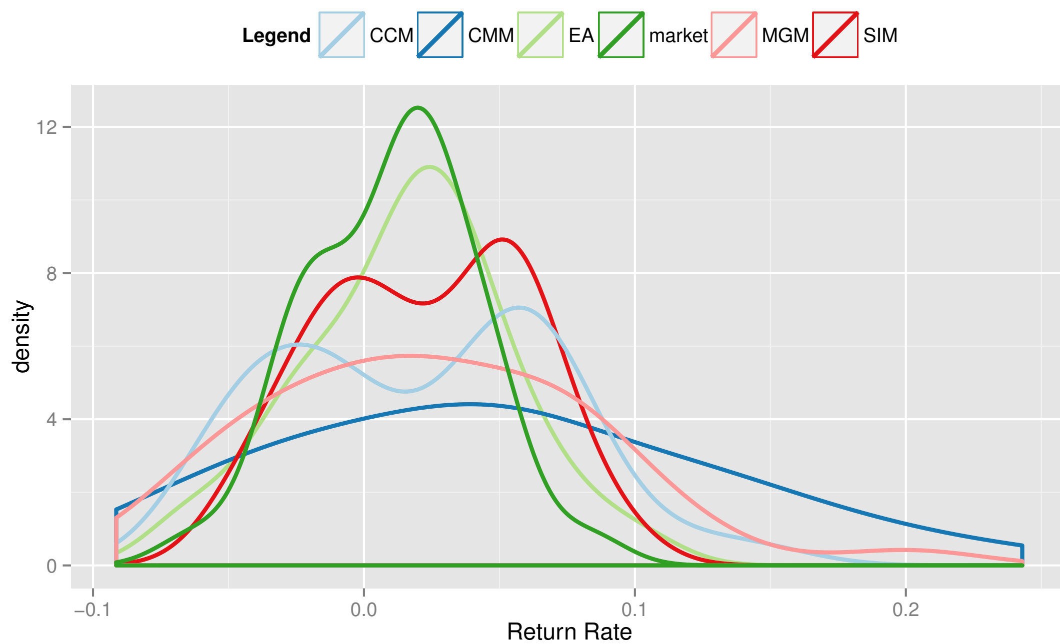
In the thesis we discuss five portfolio optimizing models and four categories of portfolio performance measures. Based on the historical monthly return of 25 selected stocks and the market index S&P 500 from January 2008 to December 2015, we construct optimal portfolios based on classical Markowitz model, single index model, constant correlation model, and multigroup model, with an equal allocation portfolio as contrast.

We evaluate the performance of these portfolios, utilizing eleven measures from three categories, and analyze the portfolio performance as well as the effectiveness of evaluation measures. The single index model and the multigroup model work the best: if the investors' are risk averse, they may prefer the single sindex model; if they do not concern about losses more than gains, they may prefer the multigroup model.

The classical Markowitz works the worst, and it even underperforms the equal allocation portfolio. The constant correlation model does not have a good performance, the reason of which may be a mismatch between the model's assumptions and the reality. I think the reason is that stocks from the same sector do not share a constant correlation; it is confirmed through calculating the covariance matrix of the stocks.

As for the performance evaluation measures, Sharpe ratio, adjusted Sharpe ratio, Sharpe ratio based on VaR, Sharpe ratio based on CVaR, Treynor-black ratio, omega ratio, and Gini ratio are verified to be effective, while the Treynor ratio and Jensen's alpha work badly on ranking the portfolio performances. However, we can not claim that they are useless measures, since the choice of the most appropriate measure depends on the investment environment involved.

Figure 5.1: Densities of portfolio returns: removing the data during market crash



For example, the stock market crash from January 2008 to February 2009 caused by the global financial crisis significantly impacts the result of the empirical study. After removing the data during the market crash, I optimize the portfolios again and obtain quite another result.

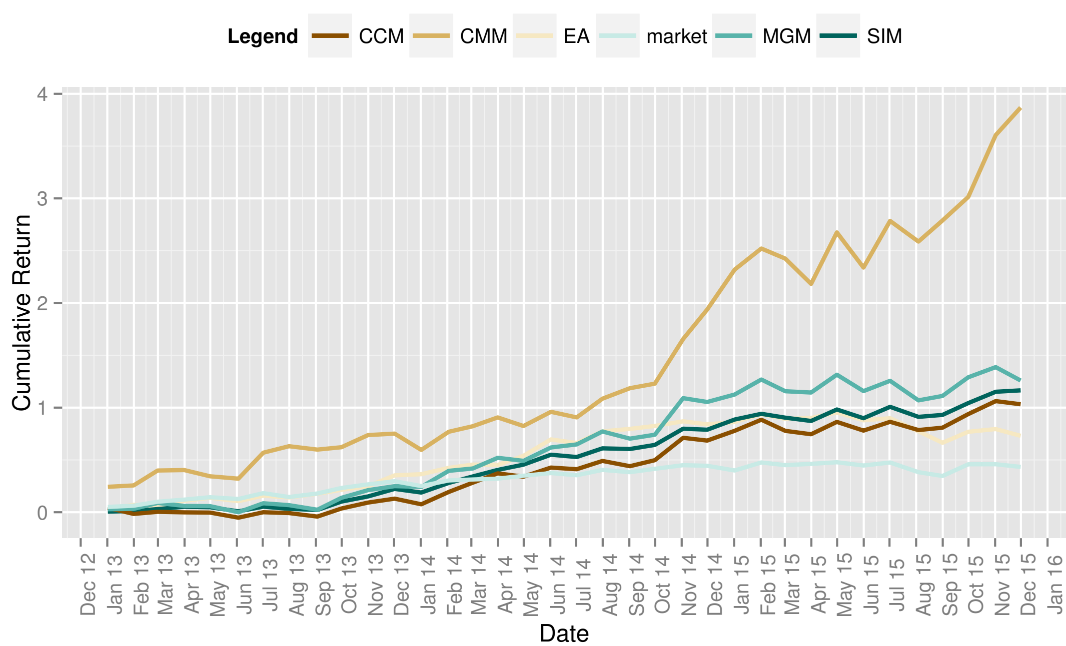
In this case, all the portfolio returns have right-skewed distributions, as shown by Figure 5.1. Compared with Figure 4.4, we can see that the market crash creates a long tail on negative returns, making the original distributions left-skewed. When we remove the part of data during the crash, the long tail disappears and distributions become right-skewed.

Figure 5.2 plots the cumulative historical returns of portfolios and the market; we observe that the classical Markowitz model has an overwhelming superiority

Table 5.1: Portfolio Performance Measures: removing the data during market crash

		EA	CMM	SIM	CCM	MGM
Sharpe ratio		0.39737	0.57392	0.56540	0.39111	0.38774
Adjusted Sharpe ratio		1.44992	2.45336	2.39645	1.60177	1.54799
Sharpe ratio based on VaR ($\alpha = 5\%$)		0.32185	0.60265	0.52514	0.33171	0.36394
Sharpe ratio based on CVaR ($\alpha = 5\%$)		0.24681	0.39643	0.39210	0.24093	0.23792
Treynor ratio		0.01688	0.04287	0.02850	0.02086	0.01902
Treynor-Black ratio		0.21665	0.47508	0.44945	0.23720	0.22231
omega ratio (market index as benchmark)		1.78542	3.74307	2.64240	1.91029	2.17044
Gini ratio		0.01154	0.04927	0.02239	0.01482	0.01732
Jensen's alpha		0.00569	0.03551	0.01350	0.01001	0.01059
Treynor-Mazuy Market Timing measure	alpha	0.01057	0.03904	0.01840	0.01149	0.01381
	beta	0.96745	1.14393	0.83010	0.97604	1.28040
	gamma	-4.71729	-2.87367	-4.91534	-0.61226	-2.341731
Merton-Henriksson Market Timing Measure	alpha	0.01439	0.03998	0.02341	0.01225	0.013457
	beta	1.27312	1.26230	1.19860	1.02864	1.32156
	gamma	-0.63681	-0.27503	-0.74676	-0.10416	-0.13210

Figure 5.2: Cumulative returns of the five portfolios and the market: removing the data during market crash

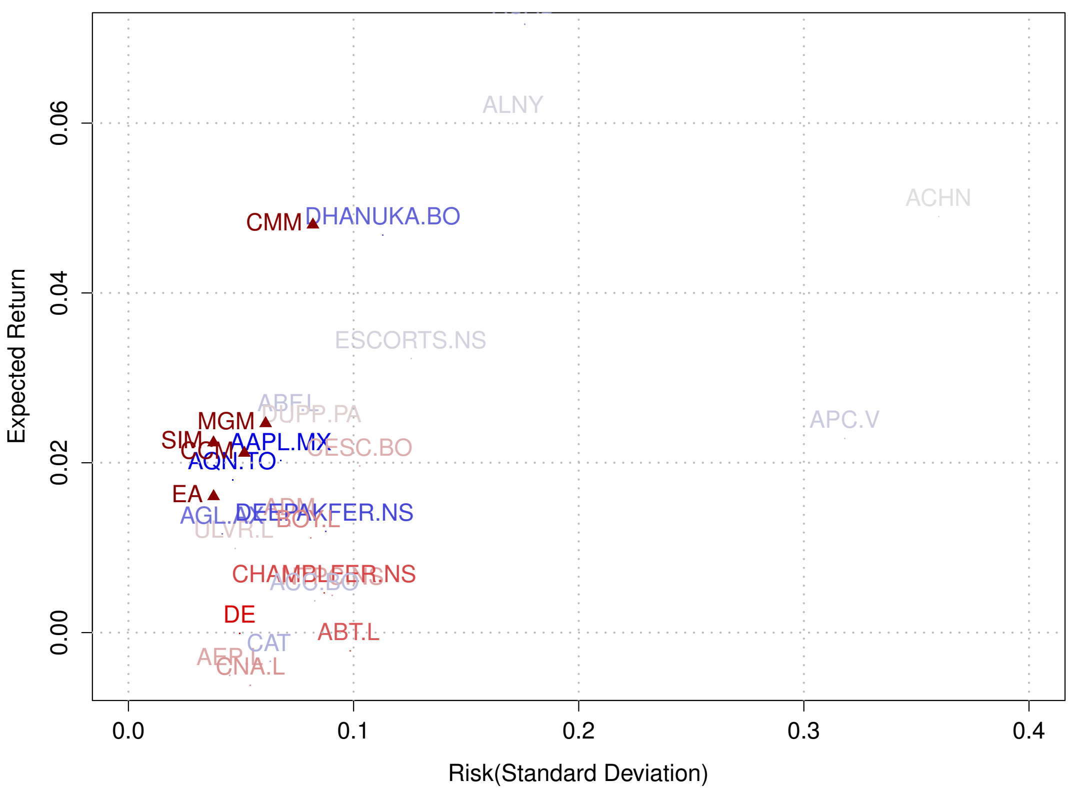


over the others. Figure 5.3 shows that the SIM, CCM and MGM portfolios have similar expected returns and risks, while the CMM portfolio exhibits extremely high expected return and risk, doubling those of the other portfolios.

Table 5.1 shows that all the measures share a coherent ranking of models, namely all the measures are effective. The classical Markowitz model has the best performance, and the single index model is ranked the second, followed by the multi-group model and the constant correlation model. The ranks of MGM and CCM differ among different measures, but they are always worse than CMM and SIM. In this example, by removing the data during the market crash we obtain totally different results of portfolio performance and effective portfolio evaluation measures. What stays the same is the result of market timing measures; they still indicate no market timing ability for any of the portfolios.

This thesis have several limitations. First, none of the optimal portfolios has good market timing. Second, the thesis only discusses the classical models and

Figure 5.3: Expected returns and risks of stocks and portfolios: removing the data during market crash



measures. Third, the entire thesis lies on a probabilistic frame work; all the risk measures are probabilistic in nature, not structural. In a structural model, the system components and their relationships are modeled in Monte Carlo simulations. A change in component X will cause the change of component Y, the change on Y then causes changes on Z, and so on. Probabilistic models, such as all the models in this thesis, just consider the likelihood of losses, but does not analyze the underlying structure that triggers the losses.

In the future we can apply some newly issued optimizing models and performance evaluation measures, especially structural models. It may help us design better portfolio strategies with good market timing.

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