Title

Wheel Load Distribution in Concrete Box Girder Bridges

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WHEEL LOAD DISTRIBUTION IN CONCRETE BOX GIRDER BRIDGES

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and
C. MEYER

Report to the Sponsors: Division of Highways, Department of Public Works, State of California, and the Bureau of Public Roads, Federal Highway Administration, United States Department of Transportation.

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OFFICE OF RESEARCH SERVICES
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Structures and Materials Research
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WHEEL LOAD DISTRIBUTION IN CONCRETE BOX GIRDER BRIDGES

by

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to

the Division of Highways
Department of Public Works
State of California
Under Research Technical Agreement
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and
U.S. Department of Transportation
Federal Highway Administration
Bureau of Public Roads

College of Engineering
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1. INTRODUCTION

1.1 Objective

The objective of this investigation was to study the wheel load distribution in concrete box girder bridges subjected to standard design truck loadings. Both simple span and continuous non-skew box girder bridges were considered. The ultimate goal of the investigation was the development of an improved design method for determining wheel load distribution in these bridges.

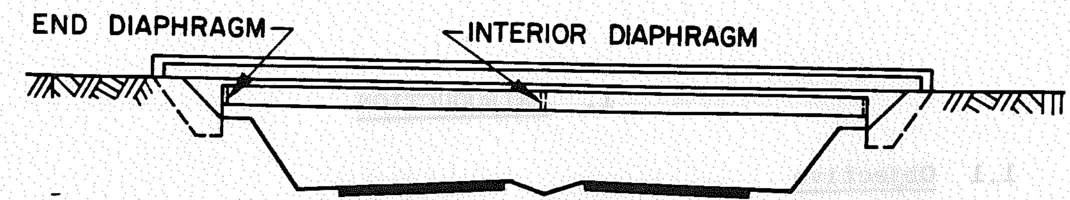
1.2 General Remarks

In recent years, approximately 60% of concrete bridges (computed on the basis of deck area) built in California have been reinforced concrete box girder bridges. These bridges have proven to be economical compared to other types and have found wide usage both as simple span and continuous structures (Fig. 1) primarily in the span ranges between 60 and 100 feet.

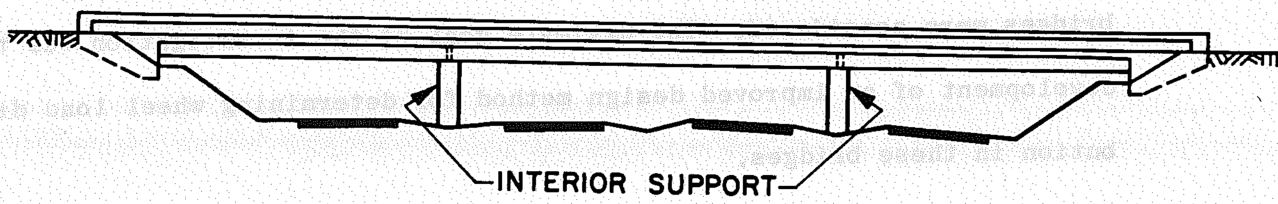
For longer spans, with some recently designed up to 300 feet, post-tensioned box girders have been used extensively. In the past few years, this type of bridge has comprised almost half of all the prestressed highway structures built in California. One set of current plans calls for a segmentally constructed prestressed box girder bridge with a 450 ft. span in Southern California.

The typical cross section of a concrete box girder bridge (Fig. 1c) consists of a top and bottom slab connected monolithically with vertical webs to form a cellular or box-like structure. Transverse diaphragms are placed at the end and interior support sections and in some cases additional interior diaphragms are utilized between supports.

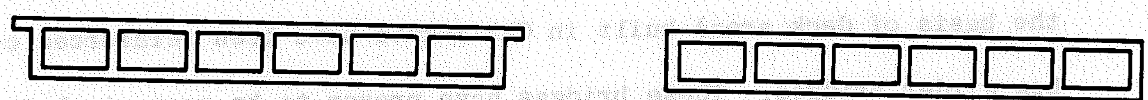
In a previous study [1] of over 200 box girder bridges constructed in California, it was found that the majority of the bridges had a depth-



a) ELEVATION OF TYPICAL SIMPLE SPAN BRIDGE

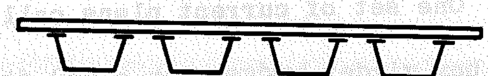


b) ELEVATION OF TYPICAL CONTINUOUS BRIDGE



c) TYPICAL CROSS-SECTIONS WITH AND WITHOUT SLAB OVERHANG

FIG. 1 MULTICELLED BOX GIRDER BRIDGES



a) THIN PLATE STEEL BOXES

b) PRECAST-PRESTRESSED BOXES

FIG. 2 BRIDGES WITH COMPOSITE SLAB OVER INDIVIDUAL BOX GIRDERS span ratio in the .050 to .065 range; a top slab thickness of 6 to 7 inches; a bottom slab thickness of $5\frac{1}{2}$ inches, web thicknesses of 8 inches; and cell widths (center to center spacing of webs) of between 7 and 9 feet. The number of cells, which is directly related to the overall bridge width, ranged from 2 to more than 16, with a preponderance of bridges having from 3 to 9 cells.

Because of the large use of concrete box girder bridges, it is evident that research directed toward improved design methods is desirable. These design methods should be based on rational analytical and experimental studies which can be used to accurately predict the structural response of the bridge to loads, the most important of which are--for the range of span under consideration--the standard design truck loadings specified by the American Association of State Highway Officials (AASHO) [11].

1.3 Previous Studies

The present report is the third in connection with a continuing research program on box girder bridges begun in 1965 at the University of California.

The first report [1] on the analysis of simply supported box girder bridges presented a general method of analysis, based on folded plate theory, for determining displacements and internal forces and moments in multi-celled box girder bridges. Two general computer programs for performing these analyses were described and used to analyze several examples of simple span bridges under a single concentrated load placed at various transverse positions at the midspan section. Results for the percentage of the total midspan moment taken by each girder, the midspan longitudinal stresses; and the midspan transverse and longitudinal slab moments were presented and discussed.

The second report [2] on the analysis of continuous box girder bridges presented three methods for analyzing these bridges. The methods used to perform the analyses were the folded plate method, the finite segment method, and the finite element method. Each of the methods developed was shown to possess certain advantages and disadvantages in particular cases. A detailed comparison of the results obtained by the three methods was made for the general case of a 3-cell box girder bridge under a single eccentric load at midspan. In order to evaluate the effect of continuity, a 3-cell bridge and a 6-cell bridge spanning 60 ft. and having various assumed end support conditions were analyzed under a single concentrated load at two transverse positions at midspan. Results for the percentage of the total moment at midspan and at the supports taken by each girder; the vertical deflections; the longitudinal stresses; and the transverse slab moments were presented and discussed.

A general review of publications and a list of references, up to 1967, on box girder bridges have been given in the first two reports [1] [2].

Of particular interest in connection with the present report are the recent studies made by Mattock at the University of Washington [3] [4] and by Van Horn and his colleagues at Lehigh University [5][6][7][8][9]. Mattock studied the wheel load distribution in composite steel-concrete bridges consisting of individual and separate thin-walled steel boxes with a composite cast-in-place slab deck (Fig. 2a). The design method proposed from these studies formed the basis for a subsequently adopted AASHO specification (1968) for wheel load distribution in these bridges. Van Horn and his colleagues have made similar experimental and analytical studies on bridges consisting of individual and separate pre-cast prestressed hollow box girders covered with a cast-in-place reinforced concrete slab deck, (Fig. 2b). Both of these bridge types differ from the interconnected multicellular box girder bridges considered in this report, (Fig. 1).

1.4 Scope of the Present Investigation

This investigation was concerned with the development of an improved design method for determining the wheel load distribution in multi-celled non-skew box girder bridges.

The basic approach used was to analyze a large number of selected typical box girder bridges to determine analytical values for the maximum number of wheel loads taken by each girder loaded by standard HS 20-44 trucks in critical positions. The typical bridges analyzed included variations in the important parameters affecting wheel load distribution such as span, total width, number of lanes, number of cells, cell width (web spacing), and fixity at the supports. In all, the results from about 120 separate computer analyses of box girder bridges under different loadings were used in the study. Based on these results, a design method for wheel load distribution was developed which is believed to represent more accurately the true design loads to be carried by each girder of a given bridge, than does the present method of design.

2. DESIGN APPROACHES

2.1 Present Method of Design

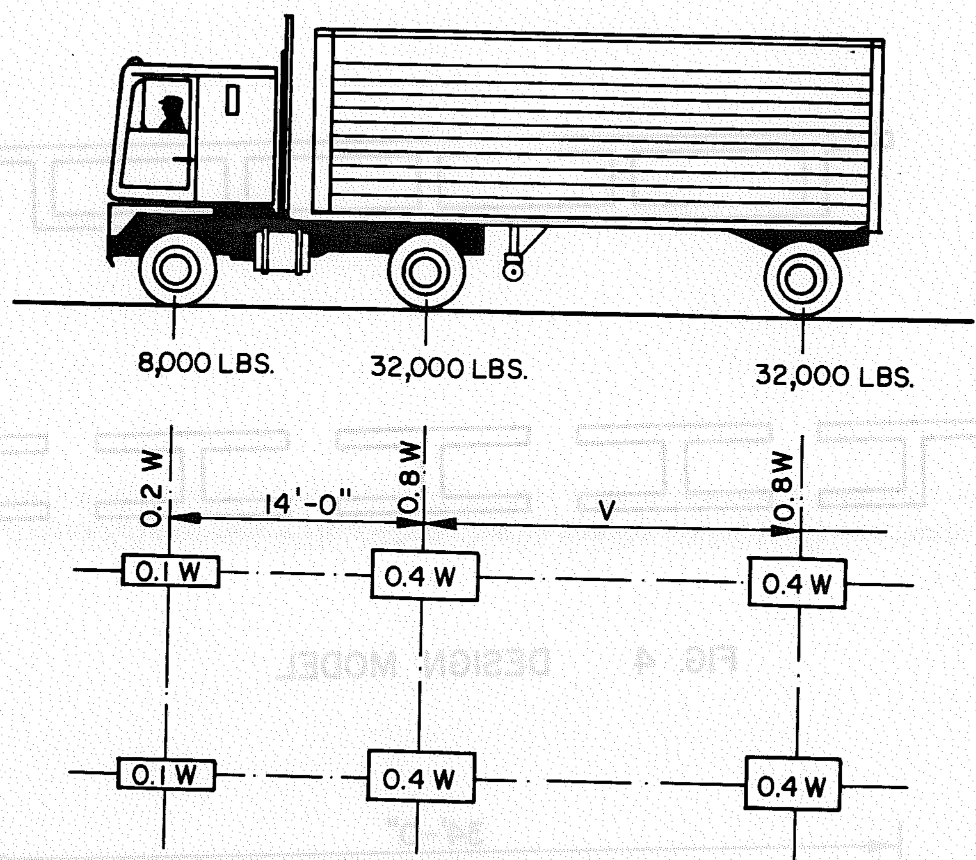
The highway live loadings on bridges specified by AASHO consist of standard trucks or of lane-loads which are equivalent to truck trains. Except for long span box girder bridges the standard truck loadings govern the design rather than the lane loadings. In California, the HS 20-44 standard truck is used for design loadings and is assumed to occupy a width of 10 ft., (Fig. 3). In each span one such truck per traffic lane may occupy any position which will produce the maximum stress. Where maximum stresses are produced in any member by loading any number of traffic lanes simultaneously, the following percentages of the resultant live load stresses are used in view of the improbable coincident of maximum loading:

One or two lanes......100%

Three lanes........... 90%

Four lanes or more.... 75%

A typical box girder cross section which will be used as a design model is shown in Fig. 4. The 1965 AASHO Specifications [11] specify a design procedure in which the bridge is assumed to consist of a number of similar interior I girders plus two exterior girders. Each interior girder consists of a web and a top and bottom flange equal in width to the web spacing while each exterior girder consists of an exterior web with a top flange extending from the midpoint between girder webs to the edge of the cantilever overhang and the bottom flange being equal in width to half of the web spacing (Fig. 4). Each of these girders is designed as an independent member by applying to it a fraction of a single longitudinal line of wheel loads from a standard truck. This fraction, defined



W = COMBINED WEIGHT ON THE FIRST TWO AXLES

V = VARIABLE SPACING – 14 FEET TO 30 FEET INCLUSIVE. IN THIS STUDY ASSUMED TO BE 14 FEET TO PRODUCE MAXMIUM STRESS

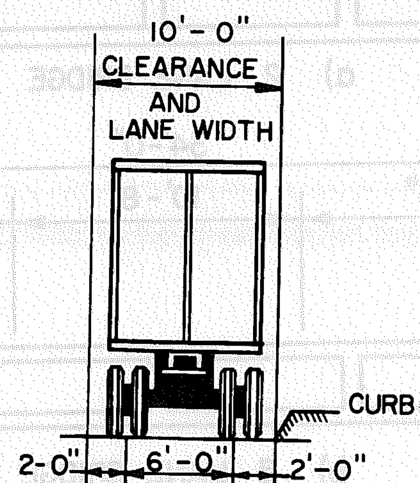


FIG. 3 STANDARD HS 20-44 TRUCK OF THE AMERICAN ASSOCIATION OF STATE HIGHWAY OFFICALS (AASHO)

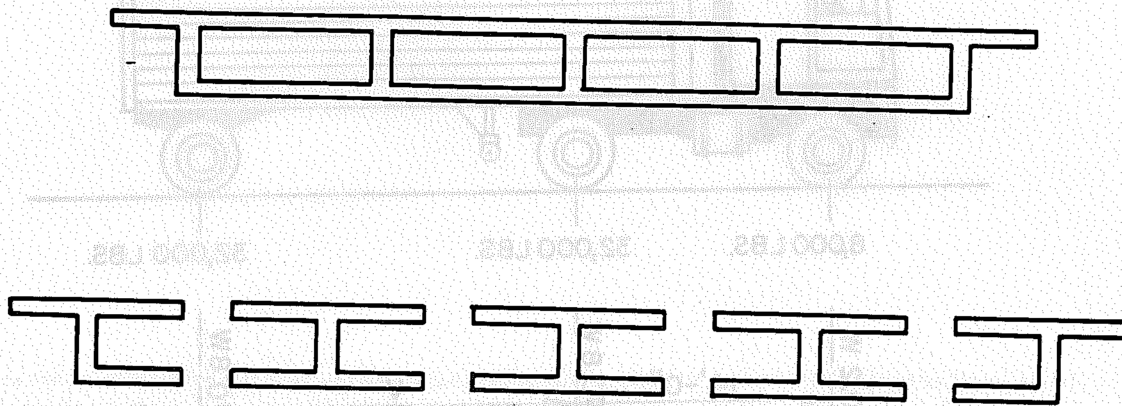
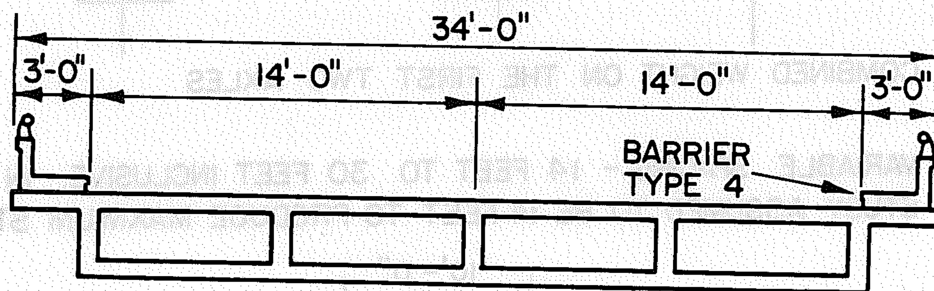
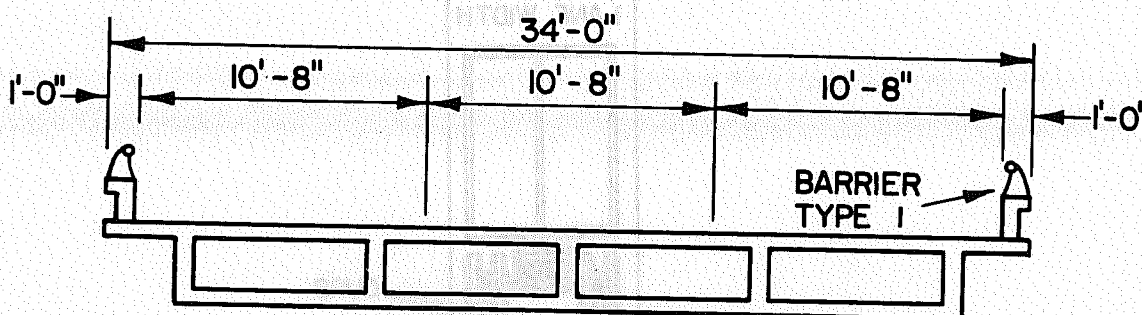


FIG. 4 DESIGN MODEL



a) 2 LANE BRIDGE



b) 3 LANE BRIDGE

FIG. 5 4-CELL EXAMPLE BRIDGE

as the "number of wheel loads, N_{WL}," is determined as follows:

For interior girders,

$$N_{WL} = \frac{S}{7} \qquad (1)$$

For exterior girders,

$$N_{WL} = \frac{W_e}{7} \qquad (2)$$

where N_{WL} = number of wheel loads taken by each girder

S = average girder web spacing in ft. = average box width in ft.

W_e = top slab width in ft. as measured from midpoint between exterior and interior girder webs to outside face of slab overhang; the cantilever overhang is not to exceed S/2.

The State of California used a procedure identical to the above in their design specifications up until December 1967 at which time a change was made. The new method specifies that the entire cross section should be designed as a "whole-width unit," in which the total number of wheel lines applied to the unit shall be:

$$\text{Total } N_{WL} = \frac{\text{overall deck width in ft.}}{7} \qquad (3)$$

In this method no distinction is made between exterior and interior girders and the total moment developed at any section of the "whole-width unit" under the loading given by Eq. (3) is apparently assumed to be constant across the width of the bridge.

It is important to note that in using Eqs. (1), (2) or (3) no reductions are to be made for live loads in more than two lanes, since this is assumed to have been already included in the development of these empirical equations.

Designing bridge structures by means of Eqs. (1), (2) or (3) may introduce some severe incongruities because each is based on a single variable which cannot by itself give an accurate representation of the correct distribution factor for the number of wheel loads to be used for design. While many variables affect this factor, the most important variable ignored in Eqs. (1), (2), and (3) is the number of traffic lanes on the bridge. To illustrate the incongruities in using Eqs. (1), (2) or (3), consider an example of a 4-cell bridge (Fig. 5a) having girder webs spaced at 7 ft., an overall width of 34 ft., and two three foot-barrier curbs, which reduce the usable roadway width to 28 ft. Using Eqs. (1) and (2), each interior girder and each exterior girder should be designed respectively for $N_{WL} = 7.00/7 = 1.00$ and $N_{WL} = 6.5/7 = 0.928$ or a total number of wheel lines for the bridge, $N_{WL} = 3(1.00) + 2(0.928) = 4.856$, which of course is the same as given by Eq. (3) for the whole-width unit.

Since the 28 ft. roadway lies within the 20 to 30 ft. range, the AASHO specification for traffic lanes stipulates that the structure be designed for two lanes or 4 wheel lines.

Suppose, however, that the design of this structure had incorporated California's Type I barrier railing, which has no safety curb and is one foot in width (Fig. 5b). The usable roadway width would then become 32 ft., within the range of 30 to 42 ft. which requires designing for three traffic lanes or 6 wheel lines.

It is readily apparent that if the structure were designed wholly on the basis of Eqs. (1) and (2) or (3), the two-lane structure, which permits 4 wheel lines, but is designed for 4.856 wheel lines, is probably conservatively designed. However, the same structure, with a slightly wider roadway, would still be designed for 4.856 wheel lines, while the traffic lane specification would permit 6 wheel lines to be carried by the structure. Even with the 90% reduction factor for live load in three lanes, the latter design would be non-conservative. Admittedly, an over-riding AASHO specification stipulates that a structure must be designed to carry all the loads which may be imposed thereon, however, this does not excuse the inconsistencies obtained using Eqs. (1), (2), or (3). An improved design method should be developed.

2.2 Basic Concepts and Definitions

In order to make a rational study of the problem of load distribution in a multi-celled continuous box girder bridge subjected to standard truck loadings in critical positions on the bridge, two basic questions must be answered.

1. For a given loading, what is the longitudinal division of the total statical moment between the total positive moment at the midspan section and the total negative moment at the support sections?
2. For a given loading, what is the transverse distribution of the above total positive or negative moments at a section across the cross section or to each independent longitudinal girder?

In a previous report [2] it was shown that the longitudinal division of the total statical moment in box girder bridges is the same as that found in a continuous beam subjected to the same loading. Furthermore, for practical purposes, this is true irrespective of the number of cells in the bridge or of the transverse positions of the truck loads on the bridge deck. Thus only question 2 above, need be further studied.

2.2.1 Transverse Distribution of Total Moment at a Section

Given a continuous or simply supported box girder bridge, the whole cross section will carry a moment at a given section which can be found from a continuous beam analysis. This will be called the "total moment at a section." Each girder of the bridge will carry some fraction of this total moment, called "girder moment", or in normalized form, "moment percentage," which is simply the girder moment divided by the total moment times 100. Thus, all girder moments add up to the total moment at a section, while the moment percentages must add up to 100%.

2.2.2 Load Distribution in Rigid and Flexible Bridges

A "rigid bridge" is herein defined to be a bridge whose cross section does not distort and which deflects uniformly such that a uniform stress distribution across the top and bottom slabs exists. Then each girder moment is proportional to the contribution of that girder's moment of inertia to the total moment of inertia of the bridge cross section.

Assuming that all interior girders have equal dimensions, it is possible to derive a general expression for the ratio of the moment of inertia of an exterior girder to that of an interior girder. Using the notation of Fig. 6, this ratio is as follows:

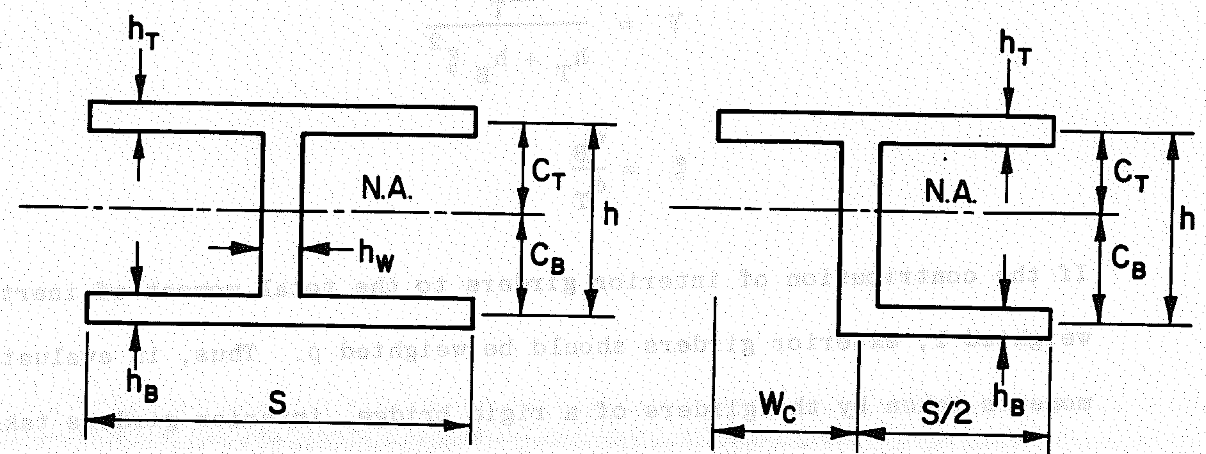
$$\rho = \frac{S + X + Y \cdot W_c}{2S + X} \qquad (4)$$

where

S = box width or girder flange width

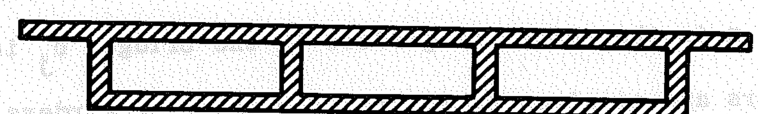
W_c = cantilever overhang

$$X = \frac{2}{3} h_w c_T \frac{1+\xi^3}{h_T+h_B \xi^2}$$

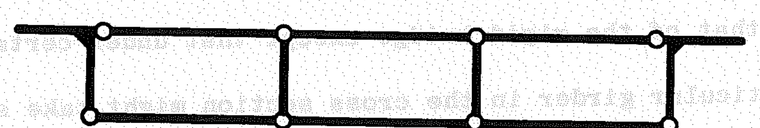


N.A.-NEUTRAL AXIS OF WHOLE BRIDGE CROSS SECTION

FIG. 6 NOTATION FOR INTERIOR AND EXTERIOR GIRDER



RIGID BRIDGE
(PERFECT LOAD DISTRIBUTION)



FLEXIBLE BRIDGE
(NO LOAD DISTRIBUTION)

FIG. 7 RIGID AND FLEXIBLE BRIDGE

$$Y = \frac{2h_T}{h_T + h_B \xi^2}$$

$$\xi = \frac{c_B}{c_T}$$

If the contribution of interior girders to the total moment of inertia are weighted 1, exterior girders should be weighted ρ. Thus, in evaluating the moments taken by the girders of a rigid bridge, interior girders take the moment percentage

$$M_i = \frac{100}{\sum_{j=1}^{n} \rho_j} = \frac{100}{n-2+2\rho} \tag{5}$$

while exterior girders take

$$M_e = \frac{100}{\sum_{j=1}^{n} \rho_j} \cdot \rho = M_i \cdot \rho \tag{6}$$

where n is the number of girders in the bridge, ρ_j is unity for interior girders and as given by Eq. (4) for exterior girders.

The moment percentages obtained for the rigid bridge are defined to be the "average moments" taken by the girders in a given bridge under any loading. It is obvious that the load distribution cannot be better than that of the rigid bridge except that under certain special conditions, a particular girder in the cross section might take slightly less than the average moment (Eq. (5) or (6)), because of constraints imposed on the transverse positions that the trucks can occupy.

A "flexible bridge" is defined to be a bridge without any load distribution among girders, i.e., all girders act independently of the rest of the bridge. This can be achieved by introducing hinges as shown in Fig. 7. The flexible bridge thus represents the case of worst load distribution. All real bridges must therefore fall somewhere in between these two bounds of the rigid and flexible bridges.

2.2.3 Girder Load Concentration Factors

Having the rigid bridge as a well-defined lower bound for load distribution, it is advantageous to normalize characteristic values of load distribution with respect to this bound, i.e., to define a girder load concentration factor α as follows:

$$\alpha = \frac{N_{WL}}{N_{WL}^{r}} \tag{7}$$

where

N_{WL} = maximum number of wheel loads taken by a girder in a given bridge.

N_{WL}^{r} = maximum number of wheel loads taken by the same girder in the same bridge, but with rigid cross section.

It can be seen from Eq. (7) that α is a direct measure of the load distributing properties of a bridge. The closer α is to 1 for all girders in a bridge, the better is the load distribution.

To illustrate some advantages of this concept of the load concentration factor α, consider Figs. 8 and 9. These figures show α as a function of the cell width S for an interior and exterior girder of a flexible bridge, respectively, with the lane width W being restricted to the limits of the practical range, 10 and 16 feet. Although in practice the cell width S takes on values only between about 6 and 10 feet, a larger range is shown in Figs. 8 and 9 to better indicate the general behavior of the variables under consideration.

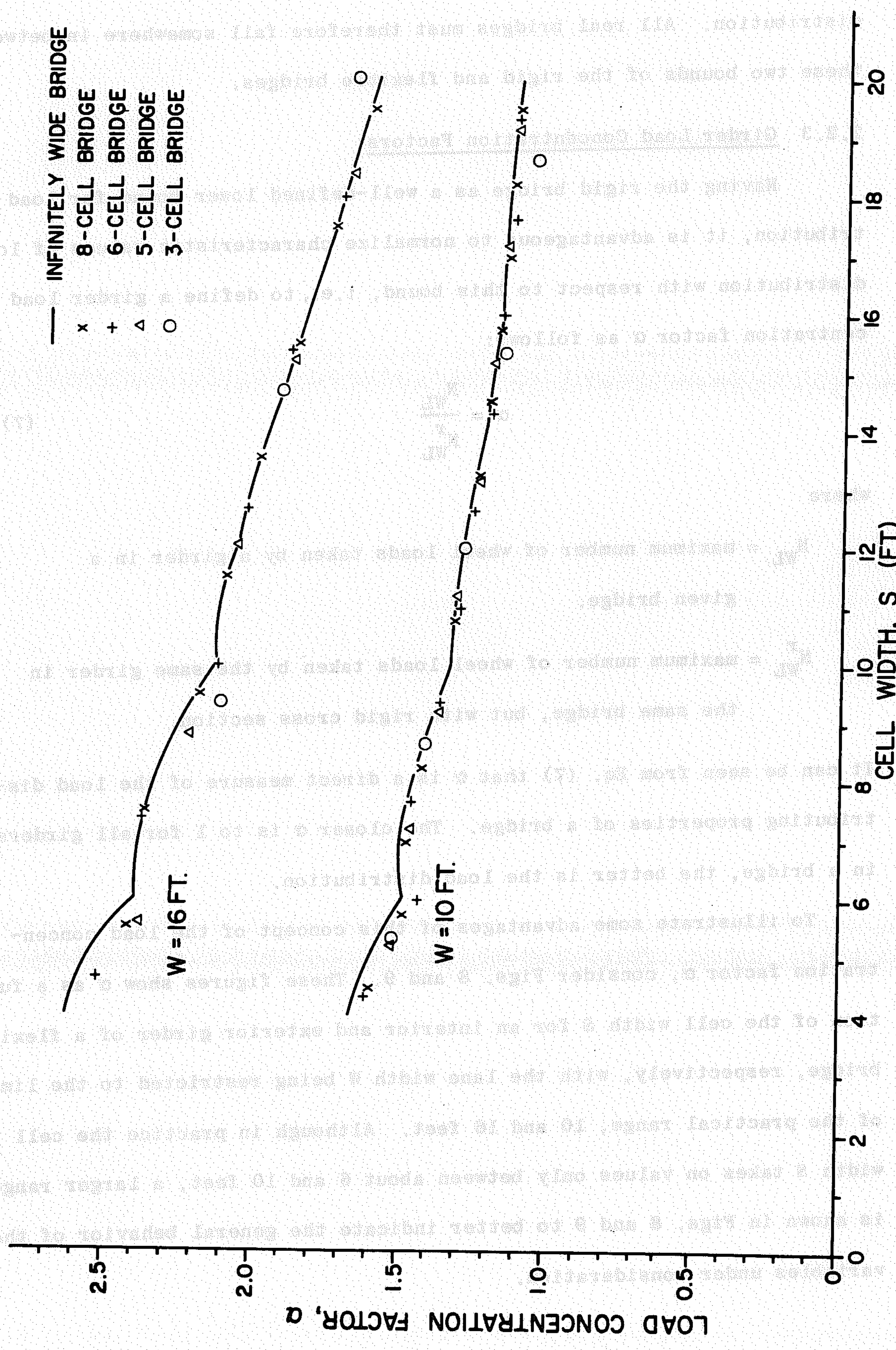


FIG. 8 LOAD CONCENTRATION FACTORS FOR INTERIOR GIRDERS IN FLEXIBLE BRIDGE WITH LANE WIDTH W OF 10 FT. OR 16 FT.

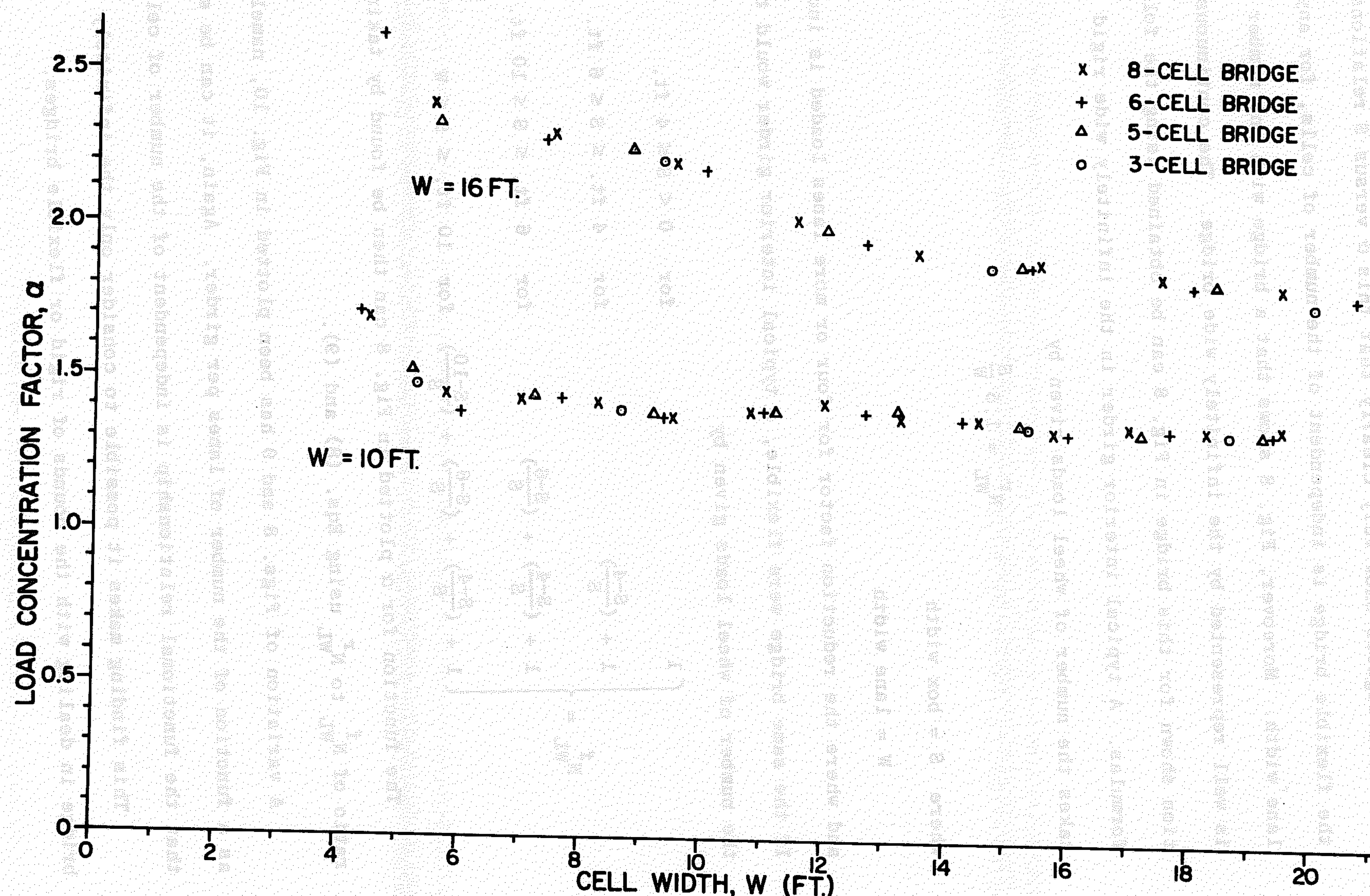


FIG. 9 LOAD CONCENTRATION FACTORS FOR EXTERIOR GIRDERS IN FLEXIBLE BRIDGE WITH LANE WIDTH W OF 10 FT. OR 16 FT.

Both figures show very clearly that this α versus S relationship for the flexible bridge is independent of the number of cells, for any feasible lane width. Moreover, Fig. 8 shows that a bridge with any number of cells is well represented by the infinitely wide bridge. The continuous function shown for this bridge in Fig. 8 can be obtained using the following formulas. A typical interior girder in the infinitely wide rigid bridge takes the number of wheel loads given by

$$N_{WL}^{r} = 1.5\,\frac{S}{W} \tag{8}$$

where S = box width

W = lane width

and where the reduction factor for four or more lanes loaded is included. If the same bridge were flexible, a typical interior girder would take the number of wheel loads given by

$$N_{WL}^{f} = \begin{cases} 1 & \text{for} \quad 0 < S \le 4 \text{ ft.} \\ 1 + \left(\frac{S-4}{S}\right) & \text{for} \quad 4 \text{ ft.} \le S \le 6 \text{ ft.} \\ 1 + \left(\frac{S-4}{S}\right) + \left(\frac{S-6}{S}\right) & \text{for} \quad 6 \text{ ft.} \le S \le 10 \text{ ft.} \\ 1 + \left(\frac{S-4}{S}\right) + \left(\frac{S-6}{S}\right) + \left(\frac{S-10}{S}\right) & \text{for} \quad 10 \text{ ft.} \le S \le W \end{cases} \tag{9}$$

The function for α plotted in Fig. 8 can then be found by taking the ratio of N_{WL}^{f} to N_{WL}^{r} using Eqs. (8) and (9).

A variation of Figs. 8 and 9 has been plotted in Fig. 10, namely α as a function of the number of lanes per girder. Again, it can be seen, that the functional relationship is independent of the number of cells.

This finding makes it possible to consider only the infinitely wide bridge in dealing with the bounds of rigid or flexible bridges.

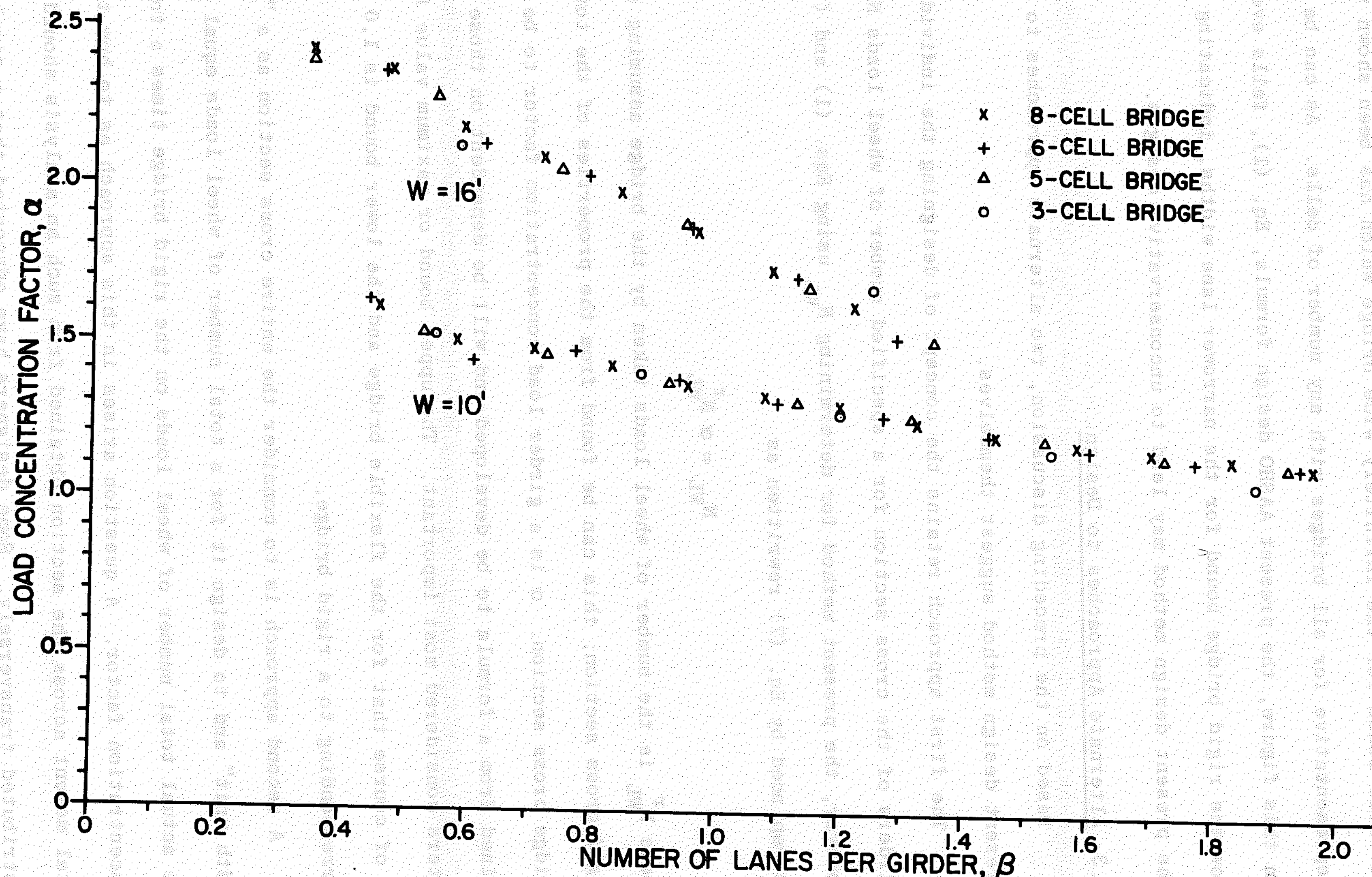


FIG. 10 LOAD CONCENTRATION FACTORS FOR INTERIOR GIRDERS IN FLEXIBLE BRIDGE VERSUS NUMBER OF LANES PER GIRDER

As an immediate application, Fig. 11 shows the bounds for the number of wheel loads for the infinitely wide bridge which has been shown to be representative for all bridges with any number of cells. As can be seen in this figure, the present AASHO design formula, Eq. (1), falls even below the rigid bridge bound for the narrower lane widths, indicating that the present design method may lead to unconservative designs.

2.3 Alternate Approaches to Design

Based on the preceding discussion, two alternate approaches to the present design method suggest themselves.

The first approach retains the concept of designing the individual girders of the cross section for a specified number of wheel loads N_{WL}. However, the present method for determining N_{WL} using Eqs. (1) and (2) is replaced by Eq. (7) rewritten as

$$N_{WL} = \alpha\, N^{r}_{WL} \tag{7a}$$

where N^{r}_{WL} is the number of wheel loads taken by the bridge assuming a rigid cross section; this can be found from the properties of the total bridge cross section. α is a girder load concentration factor to be obtained from a formula to be developed and will be dependent on those parameters considered most important. The upper bound or maximum value for α is of course that for the flexible bridge and the lower bound is 1.0 corresponding to a rigid bridge.

A second approach is to consider the entire cross section as a "whole-width unit" and to design it for a total number of wheel loads equal to the actual total number of wheel loads on the rigid bridge times a total concentration factor. A question arises in this approach as to how the total moment across the section obtained from such an analysis should be distributed transversely. Some designers have advocated that a simple

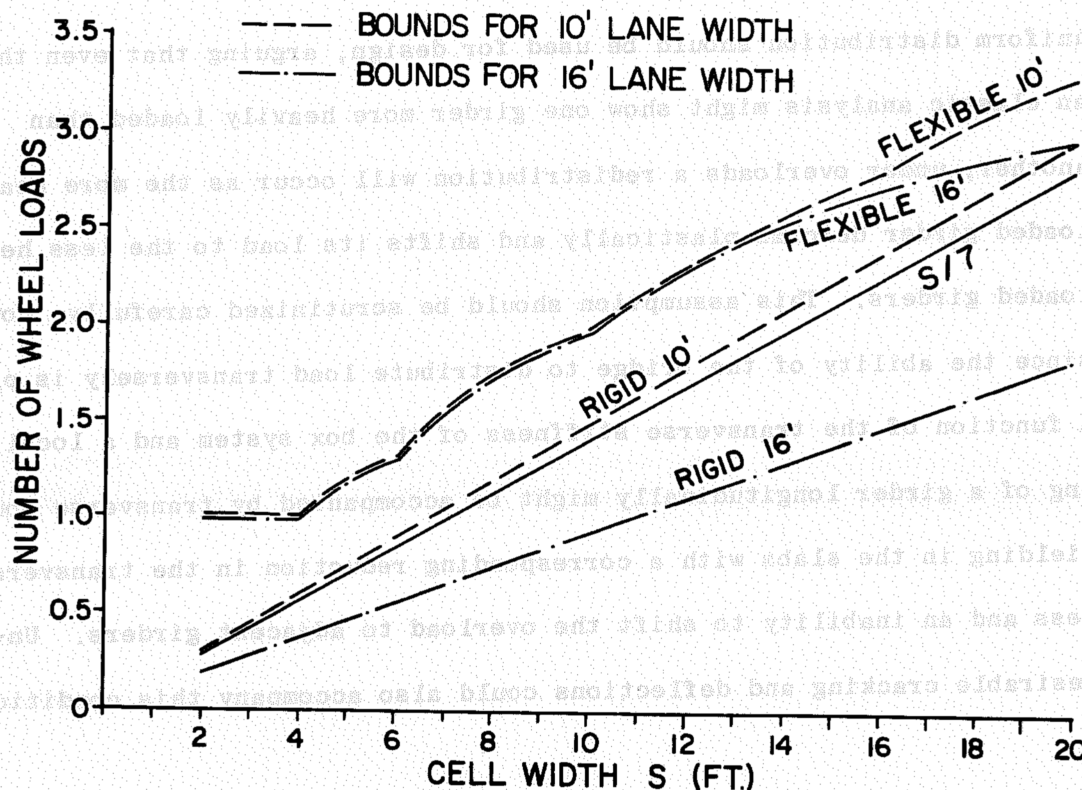


FIG. 11 BOUNDS FOR NUMBER OF WHEEL LOADS OF INFINITELY WIDE BRIDGE COMPARED WITH AASHO-DESIGN FORMULA S/7

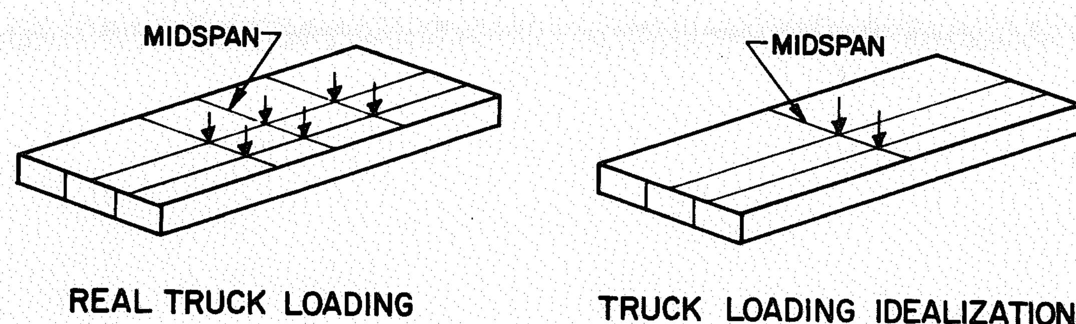


FIG. 12 TRUCK LOADING IDEALIZATION FOR LOAD DISTRIBUTION ANALYSIS

uniform distribution should be used for design, arguing that even though an elastic analysis might show one girder more heavily loaded than another, under overloads a redistribution will occur as the more heavily loaded girder deforms plastically and shifts its load to the less heavily loaded girders. This assumption should be scrutinized carefully, however, since the ability of the bridge to distribute load transversely is primarily a function of the transverse stiffness of the box system and a local yielding of a girder longitudinally might be accompanied by transverse moment yielding in the slabs with a corresponding reduction in the transverse stiffness and an inability to shift the overload to adjacent girders. Undesirable cracking and deflections could also accompany this condition.

3. ANALYTICAL STUDIES

3.1 Introduction

It is the objective of the studies presented in this chapter to investigate the load distribution characteristics of box girder bridges, or specifically, to study the maximum numbers of wheel loads taken by the individual girders of a bridge. For this purpose, a procedure will be described by which these numbers of wheel loads or the girder load concentration factors (see section 2.2.3) can be calculated. In the next chapter, results obtained by this procedure will be used to discuss the important bridge parameters influencing load distribution and to derive then design formulas for the girder load concentration factors.

3.2 Basic Approach

Three computer programs have been used for all of the analyses to be presented in this chapter. All three programs have been extensively described in the two previous reports, [1] and [2].

The first program, MULTPL, analyzes simply supported cellular folded plate structures using the elasticity theory, applying a direct stiffness method of analysis. The Goldberg-Leve equations are used to evaluate plate fixed edge forces, stiffnesses, and final internal forces, moments and displacements. A harmonic analysis with up to 100 non-zero terms of the appropriate Fourier series is used to approximate the loads. For more information see [1].

The second program, MUPDI, is an extension of MULTPL, in that the structure may have interior rigid diaphragms which may or may not be externally supported. Compatibility at these interior diaphragms is accomplished by a force method of analysis. For more information see [1].

The third program, SIMPLA, applies a finite segment method (ordinary theory), so that it may treat folded plate structures having any end boundary conditions and any combination of interior supports. Because the matrix progression method used is very sensitive in numerical calculations, SIMPLA had been written for the IBM 7090/7094 computer (8 digit arithmetic) in double-precision. But the CDC 6400 computer used for the analyses of this report utilizes 14 digits, so that single-precision was found to be sufficient. For more information see [2].

All three programs have been converted to the new computer system of the University of California (1968). However, the necessary program modifications were minor and covered mainly peripheral processing of the computing equipment.

The output of all three programs includes displacements and internal forces anywhere in the structure. Of prime interest are the longitudinal stress resultants at a section, because if they are multiplied by their respective distances to the neutral axis of the bridge and integrated over a specific bridge girder, they yield the moment carried by this girder at that section. The programs MULTPL and MUPDI calculate also the longitudinal plate bending moments, so that it was possible to also consider their small contributions to the girder moments.

The entire stress integration process has been performed for all case studies automatically by appropriate subroutines which have been added to all three programs. Thus it was possible to obtain all girder moments and moment percentages for any bridge and loading at any longitudinal section very rapidly.

For all example bridges for which sufficient data were available, the girder midspan moment percentages have been plotted in influence lines for a unit load moving transversely across midspan such that for example the ordinate at a point x of the influence line for girder j indicates the midspan moment percentage of girder j when a single concentrated load is placed at midspan at the transverse position x.

The next step towards the determination of the maximum numbers of wheel loads taken by individual girders is to combine AASHO standard HS 20-44 trucks as shown in Fig. 3 on the bridge such as to produce a maximum moment in the girder under consideration. However, the available influence lines are valid only for loads placed at any transverse position at midspan. It will be shown later that the distribution of moments among the girders of a given bridge due to a combination of wheel loads placed only at midspan is not much different from the distribution of moments due to real trucks placed such as to produce maximum moments (Fig. 12). Thus the midspan influence lines based on the truck loading idealization of Fig. 12 were used throughout the following analyses resulting in a great saving in computational effort.

In moving the idealized trucks transversely into the most critical positions, the following restrictions had to be observed:

1) Bridge decks measured between curbs may be subdivided into a number of equally wide lanes with widths no less than 10 ft. and not exceeding 16 ft.;

2) The 6 ft. wide standard trucks may be moved only within the lanes allotted to them;

3) Trucks are allowed to come only as close as 2 ft. to the curbs;

4) There must be at least 4 ft. between two adjacent trucks;

5) Any number of lanes may be loaded simultaneously, provided that the load reduction factors listed on page 6 are applied.

If these restrictions are observed, the problem of determining the maximum moment percentages of all girders in a given bridge is a straightforward problem of moving the trucks into the most critical positions. For unit loads, these moment percentages thus obtained, divided by 100, give the maximum numbers of wheel loads that these girders will ever have to take.

Dividing these values by the corresponding values obtained when the same bridge is considered to be rigid, leads to the girder load concentration factors, which are the immediate objective of the analytical studies of this chapter.

3.3 Example Bridges

In selecting example bridges which should represent as comprehensively as possible the majority of bridges which have actually been built in California, over 200 bridges designed by the State had been reviewed in [1] with respect to the major design parameters of span length, overall width, overall depth, depth-to-span ratio, cell width, and number of cells, as well as the use of intermediate diaphragms. On the basis of this study, four typical bridge cross sections had been selected in [1] and are shown again in Fig. 13. The 3-cell and 6-cell bridges have box widths of 9 ft. 4 in., while the 4-cell and 8-cell bridges have box widths of 7 ft. 0 in. Slab thicknesses are constant throughout all the case studies of this report and are top slab, $6\frac{1}{2}$ inches; bottom slab, $5\frac{1}{2}$ inches; webs, 8 inches. Bridge depths are variable, depending on the span, such that the depth-span ratio is kept about constant. These depths and the resulting depth-span ratios are as follows:

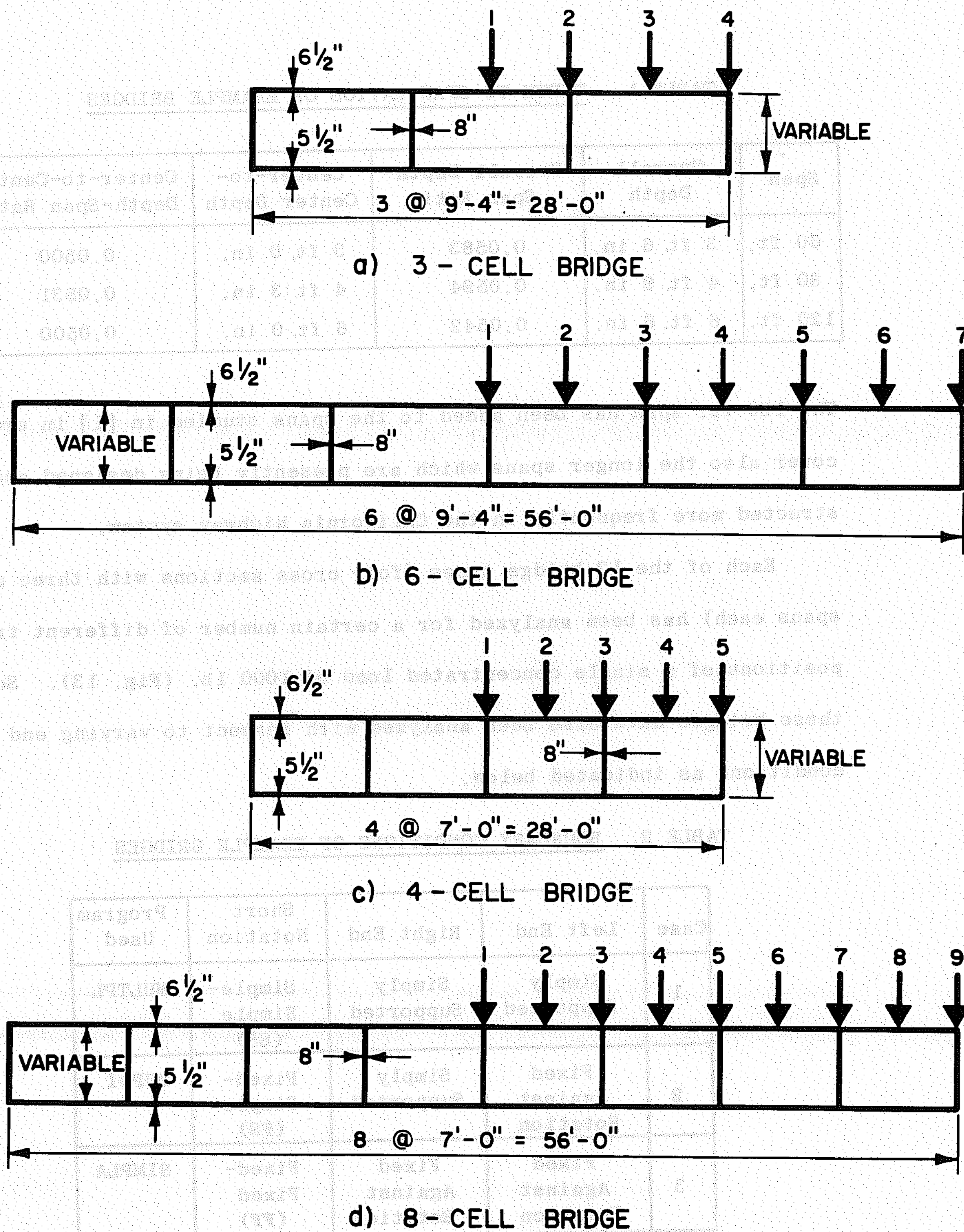


FIG. 13 CENTERLINE DIMENSIONS AND LOAD POSITIONS FOR EXAMPLE BRIDGES

TABLE 1. DEPTH-TO-SPAN RATIOS OF EXAMPLE BRIDGES

Span	Overall Depth	Overall Depth-Span Ratio	Center-to-Center Depth	Center-to-Center Depth-Span Ratio
60 ft.	3 ft. 6 in.	0.0583	3 ft. 0 in.	0.0500
80 ft.	4 ft. 9 in.	0.0594	4 ft. 3 in.	0.0531
120 ft.	6 ft. 6 in.	0.0542	6 ft. 0 in.	0.0500

The 120 ft. span has been added to the spans studied in [1] in order to cover also the longer spans which are presently being designed and constructed more frequently in the California highway system.

Each of the 12 bridge types (four cross sections with three different spans each) has been analyzed for a certain number of different transverse positions of a single concentrated load of 1000 lb. (Fig. 13). Some of these bridges have also been analyzed with respect to varying end boundary conditions as indicated below.

TABLE 2. BOUNDARY CONDITIONS OF EXAMPLE BRIDGES

Case	Left End	Right End	Short Notation	Program Used
1	Simply Supported	Simply Supported	Simple-Simple (SS)	MULTPL
2	Fixed Against Rotation	Simply Supported	Fixed-Simple (FS)	MUPDI
3	Fixed Against Rotation	Fixed Against Rotation	Fixed-Fixed (FF)	SIMPLA

Table 3 gives a summary of all 116 cases that have been run for the studies of this report. The different load positions refer to the midspan sections and are shown in Fig. 13. Many of these example bridges have already been analyzed for the previous reports, [1] and [2].

TABLE 3. CASE STUDIES

Span (ft)		60			80			120		
Boundary Conditions		SS	FS	FF	SS	FS	FF	SS	FS	FF
No. of Cells	Load Case									
3	1	x	x	x	x			x	x	x
	2	x	x	x	x			x	x	x
	3	x	x	x	x			x	x	x
	4	x	x	x	x	x	x	x	x	x
4	1	x	x	x	x			x		
	2	x	x	x	x			x		
	3	x	x	x				x		
	4	x	x	x	x			x		
	5	x	x	x	x			x		
6	1	x	x	x	x			x		
	2	x	x	x	x			x		
	3	x	x	x				x		
	4	x	x	x				x		
	5	x	x	x				x		
	6	x	x	x	x			x		
	7	x	x	x	x	x	x	x	x	x
8	1	x	x	x	x			x		
	2	x			x			x		
	3	x						x		
	4	x						x		
	5	x						x		
	6	x						x		
	7	x						x		
	8	x			x			x		
	9	x	x	x	x			x		

Boundary Conditions: SS - Both ends simply supported.

FS - One end simply supported, the other end fixed against rotation.

FF - Both ends fixed against rotation.

As can be seen in Table 3, influence lines can be constructed for those bridges which have been analyzed for a complete set of load positions, assuming that the influence lines are straight between two adjacent points of load application. As will be seen later, this assumption is justified. Moreover, in most cases the influence lines are even almost perfectly straight between any two adjacent webs. Thus, for example, it was possible to interpolate the missing load case 3 for the 80 ft. span, 4-cell bridge with little error.

All of the bridges thus far mentioned have been extensively studied for the simplified cross sections shown in Fig. 13. But bridges which are actually being built generally have cantilevering slab overhangs as shown in Fig. 5. This modification is in two respects of importance for the analytical model bridges already discussed. Firstly, the bridge deck area is increased such that more lanes may be placed on the bridge, resulting in increased design loads. Secondly, the slab overhang modifies the load-carrying characteristics of the bridge, especially of the exterior girders.

In order to utilize the large amount of information gathered while studying bridges without slab overhangs, a procedure has been developed to modify the results obtained on bridges without overhangs such that they are valid for bridges with slab overhangs. The derivation and validity of this procedure will be shown in Section 3.5.

3.4 Bridges Without Slab Overhangs

The load distributing characteristics of the 18 bridges of Table 4 will be studied in this section. In following the procedure outlined in Section 3.2, two approaches will be used; one is called the "direct stress integration" and the other is designated the "deflection method." The final results obtained using both methods will be compared.

TABLE 4. EXAMPLE BRIDGES WITHOUT SLAB OVERHANGS

Span (ft)	60			80			120		
Boundary Condition	SS	FS	FF	SS	FS	FF	SS	FS	FF
Number of Cells									
3	x	x	x	x			x	x	x
4	x	x	x	x			x		
6	x	x	x				x		
8	x						x		

Boundary Conditions: SS - Both ends simply supported.

FS - One end simply supported, the other end fixed against rotation.

FF - Both ends fixed against rotation.

3.4.1 Direct Stress Integration

The girder moments and moment percentages for any bridge and load case were printed out directly by the computer so that it was possible to plot them immediately in influence lines which indicate the midspan moment percentages of the girders under consideration for a single concentrated load moving at midspan across the bridge. To conserve space, they are not given here, however, they look very similar to the equivalent influence lines for bridges with slab overhangs, shown in Figs. 19-30.

The maximum number of wheel loads that a particular girder in a bridge would ever have to carry was then determined by moving AASHO standard trucks into the most critical positions, while observing the restrictions mentioned in Section 3.2. Possible lane subdivisions are shown in Fig. 14.

However, it should be pointed out that the bridges without slab overhangs are inconsistent analytical models in that the slab overhangs were

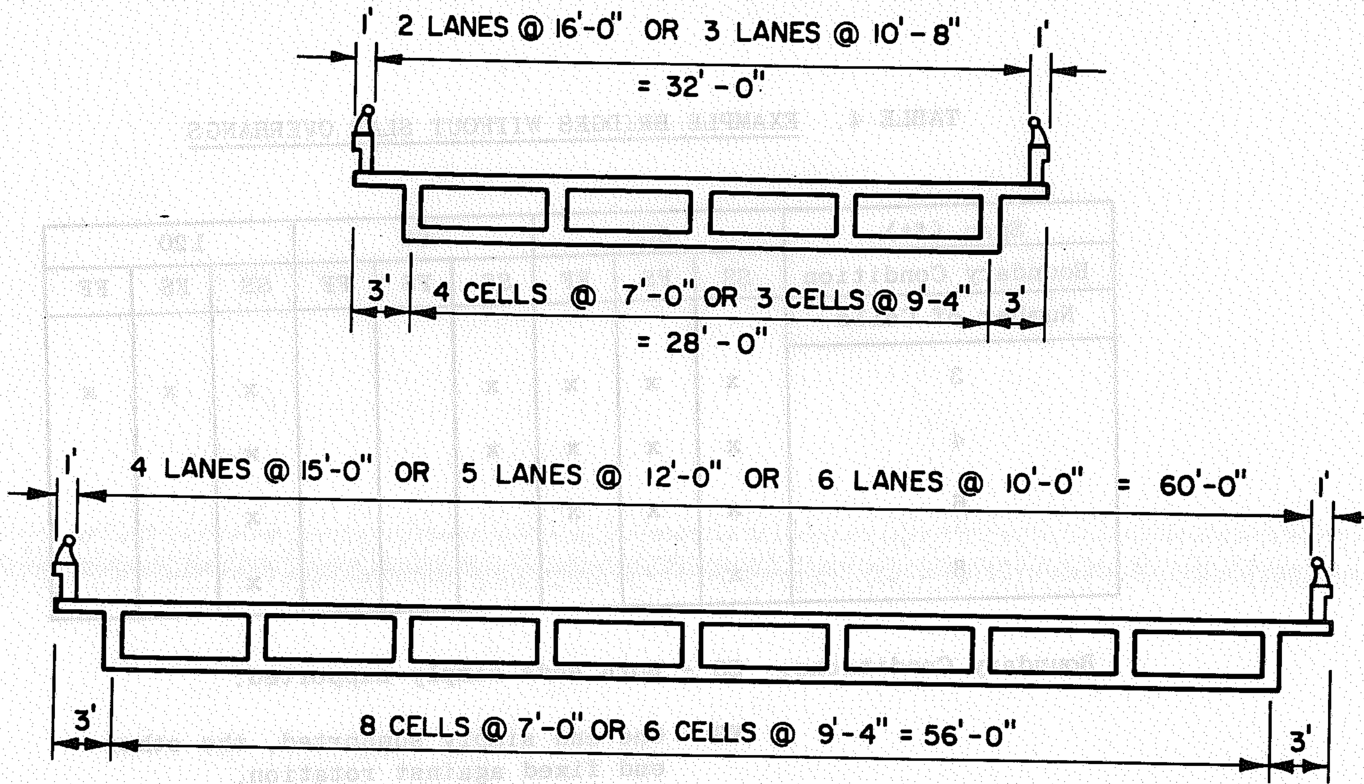


FIG. 14 LANE SUBDIVISIONS OF EXAMPLE BRIDGES

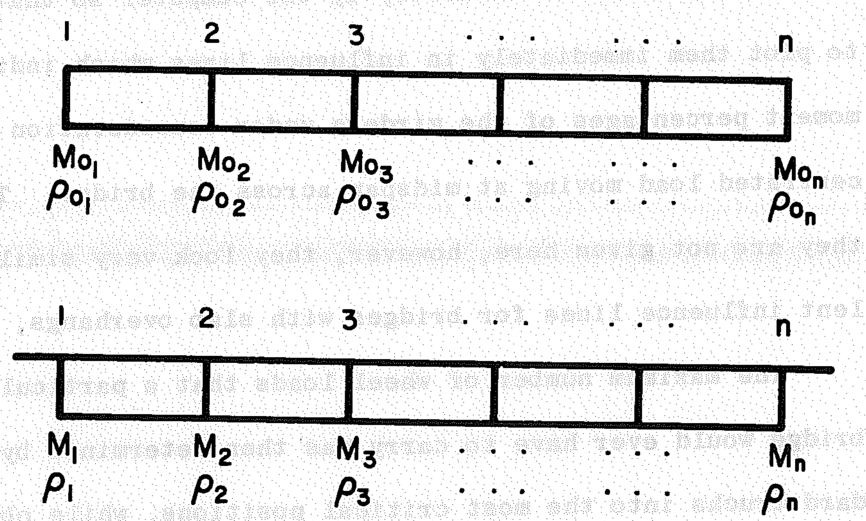


FIG. 15 GIRDER MOMENTS AND ρ-FACTORS OF GENERAL BOX GIRDER BRIDGE WITHOUT AND WITH SLAB OVERHANG

assumed to be present for determination of the available bridge deck width for load placement (so that wheel loads could go as far as over the exterior web), however, the presence of the overhangs for load-carrying capacity was neglected. This inconsistency will be removed in the next section where bridges with slab overhangs are analyzed.

In referring the results thus obtained to upper and lower bounds, the numbers of wheel loads in the corresponding rigid and flexible bridges also had to be calculated.

For this purpose, Table 5 gives the ρ-factors for all example bridges, calculated according to Eq. (4), with $W_c = 0$.

TABLE 5. ρ-FACTORS FOR EXTERIOR GIRDERS OF EXAMPLE BRIDGES WITHOUT SLAB OVERHANGS. FOR INTERIOR GIRDERS, $\rho = 1$.

Span (ft)	60	80	120
3 or 6 cells	0.534	0.547	0.563
4 or 8 cells	0.544	0.560	0.581

As can be seen in Table 5, exterior girders are somewhat more than "half girders" ($\rho = 0.5$), due to the additional half webs.

With these ρ-factors, average moments or moments carried by the girders of a rigid bridge were calculated, using Eqs. (5) and (6).

The flexible bridge was analyzed as outlined in Section 3.2. Similarly the present AASHO design-formulas, Eqs. (1) and (2) were applied directly.

All numbers of wheel loads found from the influence lines were then divided by the corresponding values for the rigid bridge, leading to the desired girder load concentration factors α. The final results are listed in Tables 6 to 9 and are compared with the results of the deflection method to be described below.

3.4.2 Deflection Method

It is proposed that the relative deflection of a girder in a box girder bridge is a measure of the percentage of total moment which this girder carries at the section where the load is applied. This proposition is based on the fact that relative girder deflections are related to relative strains, and these in turn are related to relative longitudinal stresses which yield after integration the relative girder moments. Johnston and Mattock [3] [4], based the main part of an entire experimental research series on this assumption, however, they were dealing with composite box girder bridges with considerably less load distribution than the reinforced concrete box girders of this research. It will be determined in this section if the above proposition holds also for these box girder bridges.

If the midspan deflections of all girders in a given bridge are known, say from experimental measurements, the moment percentages taken by these girders at midspan can be estimated by the following step-by-step method.

1. Calculate the average deflection of the bridge, by dividing the sum of all individual absolute girder deflections by the number of girders, i.e., weighting exterior and interior girders alike;

2. Calculate the relative girder deflections, by dividing the absolute girder deflections by the average bridge deflection found in Step 1;

3. Calculate the average moment taken by interior girders according to Eq. (5), i.e., dividing 100 by the sum of ρ-factors of

all girders in the bridge, ρ being unity for interior girders and as given in Table 5 (or Eq. 4) for exterior girders;

4. Change scale by multiplying the relative girder deflections of Step 2 by the average moment of Step 3;
5. Multiply the exterior girder moments thus obtained with their proper ρ-factors;
6. Normalize all girder moment percentages thus far obtained such that their sum adds up to 100%.

The moments obtained using this approach and those found by direct stress integration, are compared for a few example cases in Figs. 16 and 17. As can be seen in these figures, there may be considerable deviations between corresponding moment values. However, the normalizing process of Step 6 enforces the areas under all moment curves to be constant so that the overestimated regions balance the underestimated regions. This fact in turn has the tendency of reducing the errors of the maximum number of wheel loads taken by an individual girder so that relatively good agreement between the load concentration factors as determined by both methods, might be expected for these cases.

Tables 6-9 are a summary of all girder load concentration factors for the example bridges without slab overhangs, for both, those calculated according to the stress integration method and those found using the deflection method. To illustrate the magnitudes of these α-factors, the upper bounds (flexible bridge) have been listed also, while the lower bounds (rigid bridge) are always 1.0 and therefore not recorded. For comparison, Tables 6-9 show also the α-factors calculated on the basis of the present AASHO design formulas, Eqs. (1) and (2).

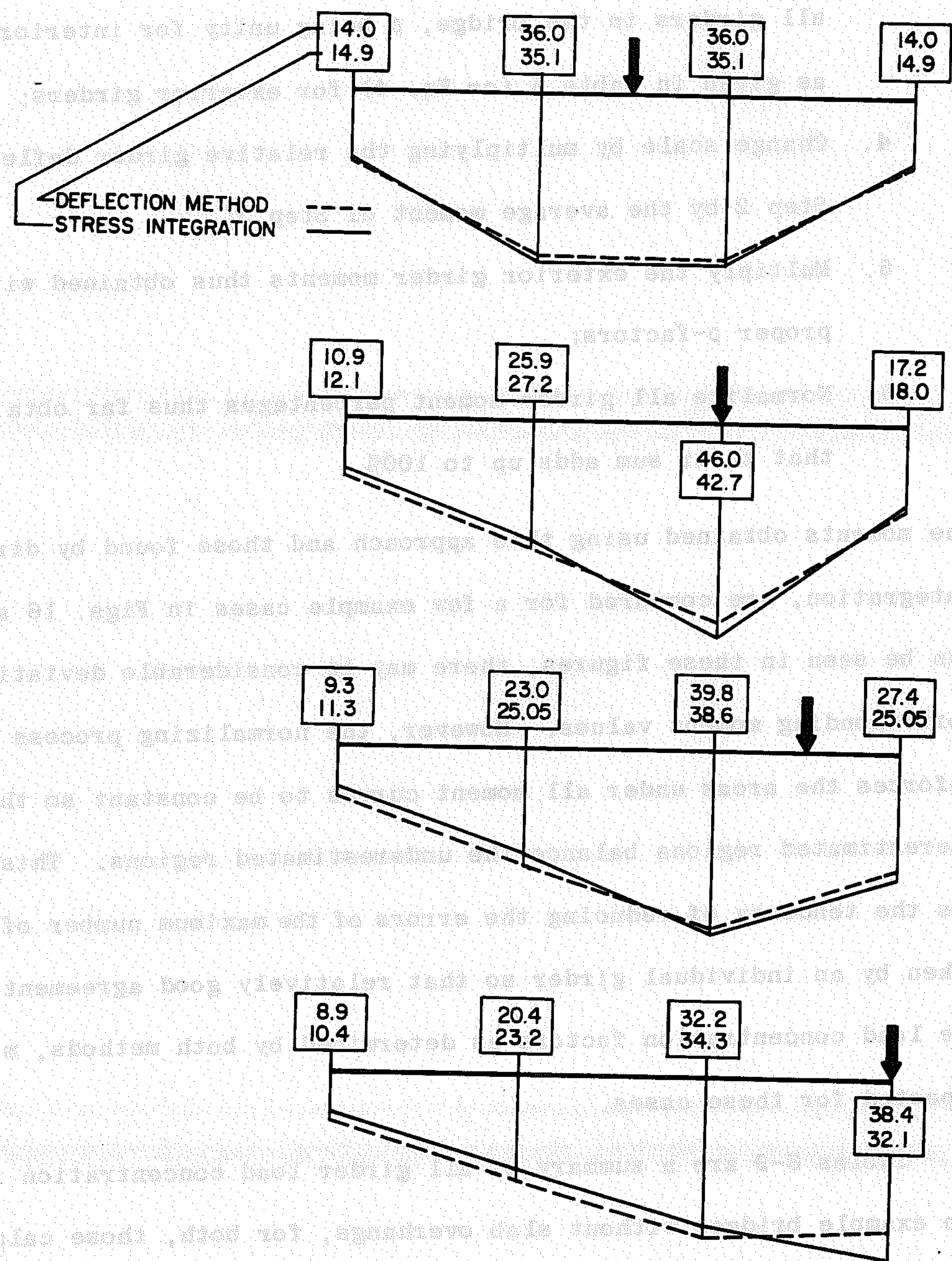


FIG. 16 COMPARISON OF GIRDER MOMENT PERCENTAGES FOR 3-CELL BRIDGE WITH SIMPLE SPAN OF 60 FT. FOR VARIOUS LOAD POSITIONS (MIDSPAN VALUES)

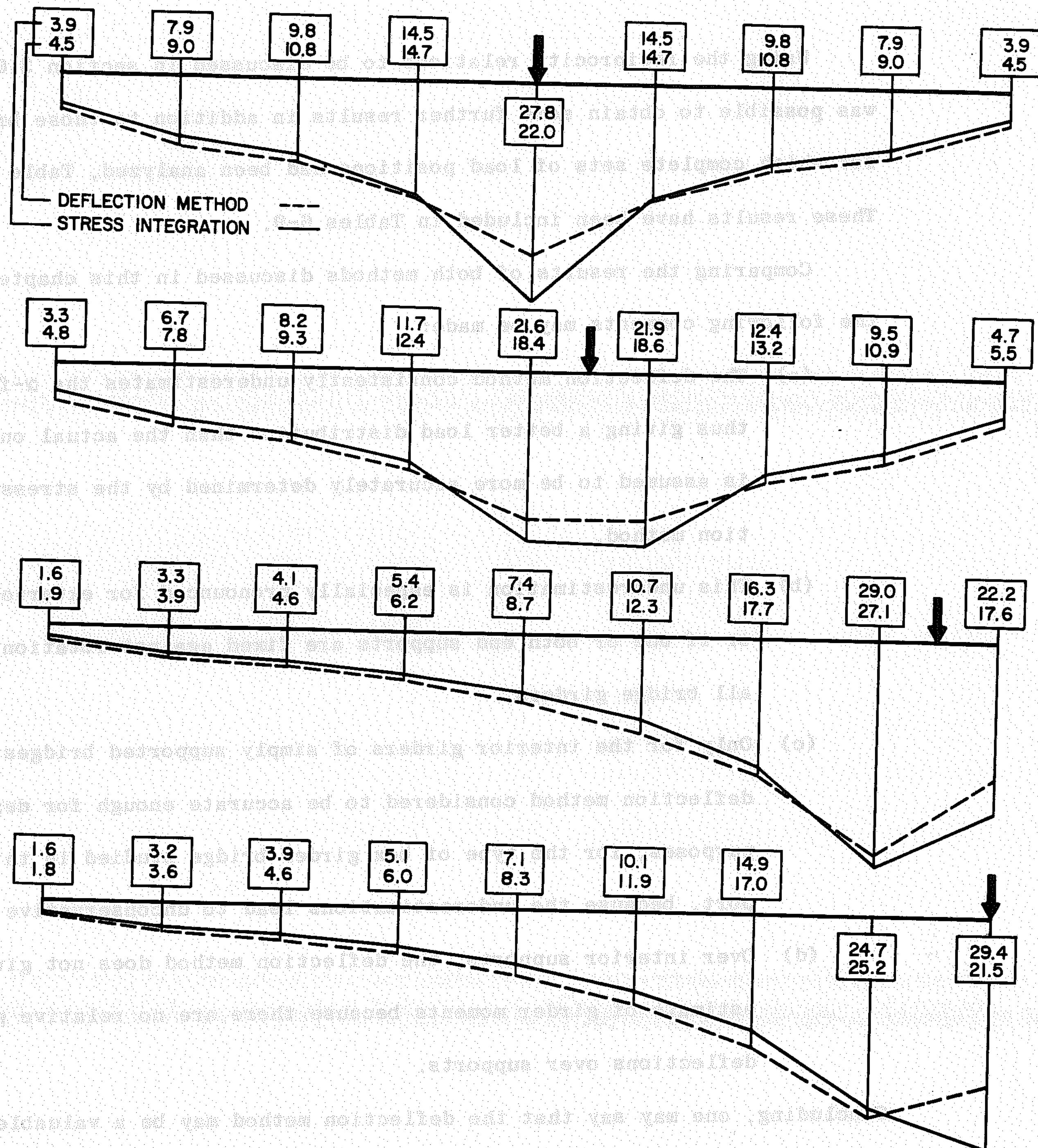


FIG. 17 COMPARISON OF GIRDER MOMENT PERCENTAGES FOR 8-CELL BRIDGE WITH SIMPLE SPAN OF 60 FT. FOR VARIOUS LOAD POSITIONS (MIDSPAN VALUES)

Using the reciprocity relations to be discussed in section 3.6.3, it was possible to obtain some further results in addition to those bridges for which complete sets of load positions had been analyzed, Table 3. These results have been included in Tables 6-9.

Comparing the results of both methods discussed in this chapter so far, the following comments may be made:

(a) The deflection method consistently underestimates the α-factors, thus giving a better load distribution than the actual one, which is assumed to be more accurately determined by the stress integration method.

(b) This underestimation is especially pronounced for exterior girders, or if one or both end supports are fixed against rotation, for all bridge girders.

(c) Only for the interior girders of simply supported bridges is the deflection method considered to be accurate enough for design purposes, for the type of box girder bridge studied in this report, because the underestimations lead to unconservative designs.

(d) Over interior supports, the deflection method does not give any estimate of girder moments because there are no relative girder deflections over supports.

Concluding, one may say that the deflection method may be a valuable tool in visualizing the behavior of various structural systems under loads; it may also offer a simple method for design as long as distributed loads are concerned. But its effect of smoothing out stress concentrations due to wheel loads as they occur in the type of bridges considered here makes it necessary to apply great caution if a general simplified method of design were to be derived from it, especially if short effective spans are concerned.

TABLE 6. α-FACTORS FOR 3-CELL BRIDGES WITHOUT SLAB OVERHANGS

Span (ft.)			60					80	120				
Section			Midspan			Support		Midspan	Midspan			Midspan	
Bound. Condition			SS	FS	FF	FS	FF	SS	SS	FS	FF	FS	FF
Girder	No. of Lanes	a	1.30					1.29	1.28				
R1	2	b	1.11	1.16	1.20	1.10	1.11	1.10	1.07	1.11	1.14	1.01	1.06
		c	1.09					1.07	1.04	1.08	1.13		
		d	1.43	1.43	1.43	1.43	1.43	1.44	1.45	1.45	1.45	1.45	1.45
		e	1.03	1.03	1.03	1.03	1.03	1.03	1.04	1.04	1.04	1.04	1.04
	3	a	1.76					1.75	1.73				
		b	1.01	0.99	0.98	1.00	0.98	1.01	1.00	1.00	1.00	0.98	1.01
		c	1.01					1.01	1.01	1.02	1.02		
		d	1.06	1.06	1.06	1.06	1.06	1.06	1.07	1.07	1.07	1.07	1.07
		e	0.76	0.76	0.76	0.76	0.76	0.76	0.77	0.77	0.77	0.77	0.77
R2	2	a	0.694					0.705	0.720				
		b	1.26	1.49	1.61	1.31	1.39	1.22	1.18	1.27	1.31	1.14	1.14
		c	1.18	1.34	1.52			1.14	1.08	1.14	1.23		
		d	1.96	1.96	1.96	1.96	1.96	1.93	1.89	1.89	1.89	1.89	1.89
		e	0.96	0.96	0.96	0.96	0.96	0.95	0.93	0.93	0.93	0.93	0.93
	3	a	0.940					0.957	0.973				
		b	1.07	1.18	1.28	1.11	1.13	1.06	1.04	1.08	1.11	1.08	1.05
		c	1.04	1.10	1.21			1.03	1.01	1.03	1.05		
		d	1.45	1.45	1.45	1.45	1.45	1.42	1.40	1.40	1.40	1.40	1.40
		e	0.71	0.71	0.71	0.71	0.71	0.70	0.69	0.69	0.69	0.69	0.69

(a) number of wheel loads carried by girder in rigid bridge
(b) α-factors for actual bridge, from stress integration method
(c) α-factors for actual bridge, from deflection method
(d) α-factors for perfectly flexible bridge, upper bound
(e) α-factors, calculated from AASHO design formulas

Note - α-factors for rigid bridge are 1.0

TABLE 7. α-FACTORS FOR 6-CELL BRIDGES WITHOUT SLAB OVERHANGS

Span (ft.)			60					80	120
Section			Midspan			Support		Midspan	Midspan
Bound. Cond.			SS	FS	FF	FS	FF	SS	SS
Girder	No. of Lanes								
C	4	a	0.99					0.985	0.98
		b	1.16	1.31	1.43	1.20	1.34	1.10	1.09
		c	1.11	1.24	1.45			1.08	1.05
		d	1.95	1.95	1.95	1.95	1.95	1.96	1.97
		e	1.34	1.34	1.34	1.34	1.34	1.35	1.36
	5	a	1.235					1.23	1.22
		b	1.08	1.10	1.19	1.06	1.10	1.04	1.03
		c	1.00	1.05	1.17			1.02	1.03
		d	1.56	1.56	1.56	1.56	1.56	1.57	1.58
		e	1.08	1.08	1.08	1.08	1.08	1.08	1.09
	6	a	1.48					1.48	1.47
		b	0.99	0.97	1.00	0.95	0.93	0.98	0.99
		c	0.98	0.97	0.97			0.98	1.00
		d	1.30	1.30	1.30	1.30	1.30	1.30	1.31
		e	0.90	0.90	0.90	0.90	0.90	0.90	0.91
R1	4	a	0.99					0.985	0.98
		b	1.22	1.33	1.47	1.27	1.38		1.11
		c							1.05
		d	1.77	1.77	1.77	1.77	1.77		1.79
		e	1.34	1.34	1.34	1.34	1.34		1.36
	5	a	1.235					1.23	1.22
		b	1.10	1.17	1.24	1.08	1.17		1.05
		c							1.03
		d	1.56	1.56	1.56	1.56	1.56		1.58
		e	1.08	1.08	1.08	1.08	1.08		1.09
	6	a	1.48					1.48	1.47
		b	1.01	0.97	0.99	0.96	0.97		0.99
		c							0.99
		d	1.30	1.30	1.30	1.30	1.30		1.31
		e	0.90	0.90	0.90	0.90	0.90		0.91

TABLE 7 (cont'd). α-FACTORS FOR 6-CELL BRIDGES WITHOUT SLAB OVERHANGS

Span (ft.)			60					80	120
Section			Midspan			Support		Midspan	Midspan
Bound. Cond.			SS	FS	FF	FS	FF	SS	SS
Girder	No. of Lanes								
R2	4	a	0.99					0.985	0.98
		b	1.34	1.49	1.63	1.38	1.49		1.17
		c							1.10
		d	1.95	1.95	1.95	1.95	1.95		1.97
		e	1.34	1.34	1.34	1.34	1.34		1.36
	5	a	1.235					1.23	1.22
		b	1.18	1.28	1.33	1.25	1.32		1.06
		c							1.05
		d	1.56	1.56	1.56	1.56	1.56		1.58
		e	1.08	1.08	1.08	1.08	1.08		1.09
	6	a	1.48					1.48	1.47
		b	1.02	1.11	1.16	1.08	1.15		1.02
		c							1.00
		d	1.30	1.30	1.30	1.30	1.30		1.31
		e	0.90	0.90	0.90	0.90	0.90		0.91
R3	4	a	0.53					0.54	0.55
		b	1.53	1.94	2.12	1.68	1.81	1.40	1.28
		c	1.37	1.62	2.01			1.29	1.15
		d	2.56	2.56	2.56	2.56	2.56	2.52	2.48
		e	1.26	1.26	1.26	1.26	1.26	1.23	1.21
	5	a	0.66					0.673	0.686
		b	1.32	1.65	1.81	1.45	1.58	1.22	1.13
		c	1.22	1.46	1.73			1.13	1.07
		d	2.06	2.06	2.06	2.06	2.06	2.02	1.98
		e	1.01	1.01	1.01	1.01	1.01	1.00	0.98
	6	a	0.79					0.81	0.83
		b	1.12	1.37	1.52	1.22	1.32	1.05	1.03
		c	1.04	1.22	1.45			1.03	1.00
		d	1.72	1.72	1.72	1.72	1.72	1.68	1.64
		e	0.85	0.85	0.85	0.85	0.85	0.83	0.81

(a) numbers of wheel loads carried by girder in rigid bridge
(b) α-factors for actual bridge, from stress integration method
(c) α-factors for actual bridge, from deflection method
(d) α-factors for perfectly flexible bridge, upper bound
(e) α-factors, calculated from AASHO design formulas

Note - α-factors for rigid bridge are 1.0

TABLE 8. α-FACTORS FOR 4-CELL BRIDGES WITHOUT SLAB OVERHANGS

Span (ft.)			60					80	120
Section			Midspan			Support		Midspan	Midspan
Bound. Cond.			SS	FS	FF	FS	FF	SS	SS
Girder	No. of Lanes								
C	2	a	0.98					0.97	0.96
		b	1.10	1.17	1.22	1.06	1.10	1.08	1.07
		c	1.05					1.04	1.03
		d	1.60	1.60	1.60	1.60	1.60	1.62	1.63
		e	1.02	1.02	1.02	1.02	1.02	1.03	1.04
	3	a	1.32					1.31	1.30
		b	0.97	0.96	0.96	0.97	0.98	0.98	0.99
		c	0.99					1.00	1.00
		d	1.05	1.05	1.05	1.05	1.05	1.05	1.06
		e	0.76	0.76	0.76	0.76	0.76	0.76	0.77
R1	2	a	0.98					0.97	0.96
		b	1.09	1.14	1.17	1.13	1.16	1.08	1.06
		c	1.07					1.06	1.03
		d	1.16	1.16	1.16	1.16	1.16	1.18	1.19
		e	1.02	1.02	1.02	1.02	1.02	1.03	1.04
	3	a	1.32					1.31	1.30
		b	1.02	1.02	1.02	1.04	1.04	1.01	1.01
		c	1.02					1.01	1.00
		d	1.19	1.19	1.19	1.19	1.19	1.20	1.21
		e	0.76	0.76	0.76	0.76	0.76	0.76	0.77
R2	2	a	0.533					0.543	0.558
		b	1.29	1.46	1.56	1.31	1.37	1.22	1.17
		c	1.22					1.11	1.08
		d	2.14	2.14	2.14	2.14	2.14	2.10	2.04
		e	0.66	0.66	0.66	0.66	0.66	0.65	0.63
	3	a	0.718					0.734	0.755
		b	1.11	1.18	1.26	1.14	1.15	1.07	1.05
		c	1.06					1.04	1.02
		d	1.59	1.59	1.59	1.59	1.59	1.55	1.51
		e	0.49	0.49	0.49	0.49	0.49	0.48	0.46

(a) number of wheel loads carried by girder in rigid bridge
(b) α-factors for actual bridge, from stress integration method
(c) α-factors for actual bridge, from deflection method
(d) α-factors for perfectly flexible bridge, upper bound
(e) α-factors, calculated from AASHO design formulas

Note - α-factors for rigid bridge are 1.0

TABLE 9. α-FACTORS FOR 8-CELL BRIDGES WITHOUT SLAB OVERHANGS

Span (ft.)			60					80	120
Section			Midspan			Support		Midspan	Midspan
Boundary Cond.			SS	FS	FF	FS	FF	SS	SS
Girder	No. of Lanes								
C	4	a	0.74					0.74	0.735
		b	1.15	1.28	1.45	1.14	1.24	1.11	1.06
		c	1.15	1.20	1.37			1.06	1.04
		d	2.12	2.12	2.12	2.12	2.12	2.13	2.14
		e	1.35	1.35	1.35	1.35	1.35	1.35	1.36
	5	a	0.93					0.93	0.92
		b	1.02	1.03	1.13	1.04	1.06	1.02	1.02
		c	1.03	1.04	1.10			1.01	1.01
		d	1.39	1.39	1.39	1.39	1.39	1.39	1.40
		e	1.08	1.08	1.08	1.08	1.08	1.08	1.09
	6	a	1.11					1.11	1.10
		b	0.99	0.97	0.96	0.98	0.97	1.00	1.00
		c	0.99	0.97	0.96			0.99	1.00
		d	1.43	1.43	1.43	1.43	1.43	1.43	1.43
		e	0.90	0.90	0.90	0.90	0.90	0.90	0.91
R1	4	a	0.74					0.74	0.735
		b	1.18						1.06
		c	1.12						1.04
		d	1.55						1.56
		e	1.35						1.36
	5	a	0.93					0.93	0.92
		b	1.05						1.03
		c	1.03						1.02
		d	1.69						1.71
		e	1.08						1.09
	6	a	1.11					1.11	1.10
		b	0.99						0.99
		c	0.98						0.99
		d	1.43						1.43
		e	0.91						0.91
R2	4	a	0.74					0.74	0.735
		b	1.28						1.14
		c	1.19						1.05
		d	2.12						2.14
		e	1.35						1.36
	5	a	0.93					0.93	0.92
		b	1.05						1.04
		c	1.04						1.01
		d	1.69						1.71
		e	1.08						1.09

TABLE 9 (cont'd). α-FACTORS FOR 8-CELL BRIDGES WITHOUT SLAB OVERHANGS

Span (ft.)			60					80	120
Section			Midspan			Support		Midspan	Midspan
Boundary Cond.			SS	FS	FF	FS	FF	SS	SS
Girder	No. of Lanes	a	1.11					1.11	1.10
R2	6	b	0.99						0.99
		c	0.99						0.99
		d	1.43						1.43
		e	0.91						0.91
R3	4	a	0.74					0.74	0.735
		b	1.31						1.19
		c	1.27						1.07
		d	1.74						1.75
		e	1.35						1.36
	5	a	0.93					0.93	0.92
		b	1.16						1.07
		c	1.11						1.04
		d	1.69						1.71
		e	1.08						1.09
	6	a	1.11					1.11	1.10
		b	1.02						1.01
		c	1.01						1.00
		d	1.43						1.43
		e	0.91						0.91
R4	4	a	0.40					0.41	0.43
		b	1.52	1.86	2.10	1.54	1.79	1.40	1.27
		c	1.38	1.66	1.96			1.30	1.10
		d	2.86	2.86	2.86	2.86	2.86	2.79	2.66
		e	1.25	1.25	1.25	1.25	1.25	1.22	1.16
	5	a	0.50					0.52	0.54
		b	1.29	1.58	1.79	1.37	1.56	1.20	1.12
		c	1.20	1.42	1.69			1.10	1.04
		d	2.29	2.29	2.29	2.29	2.29	2.20	2.12
		e	1.00	1.00	1.00	1.00	1.00	0.96	0.93
	6	a	0.60					0.62	0.64
		b	1.10	1.32	1.49	1.17	1.30	1.07	1.05
		c	1.05	1.21	1.41			1.03	1.00
		d	1.91	1.91	1.91	1.91	1.91	1.84	1.79
		e	0.83	0.83	0.83	0.83	0.83	0.81	0.78

(a) number of wheel loads carried by girder in rigid bridge
(b) α-factors for actual bridge, from stress integration method
(c) α-factors for actual bridge, from deflection method
(d) α-factors for perfectly flexible bridge, upper bound
(e) α-factors, calculated from AASHO design formulas

Note - α-factors for rigid bridge are 1.0

3.5 Bridges With Slab Overhangs

The inconsistency of bridges without slab overhangs mentioned earlier can be removed by including in the bridge analyses the slab overhangs whose existence had been assumed for all cases in subdividing the available road width between curbs into lanes. The 3 ft. overhang assumed in these studies is in fair agreement with current bridge design practice. Usually the overhang thickness tapers, with the average thickness often being about equal to the top slab thickness. Therefore, in the present studies the bridge top slab has been assumed to be of uniform thickness, including the cantilevering overhangs.

In order to utilize all the information gathered in the studies on bridges without slab overhangs, a method has been developed which permits a transformation of these results such that they apply to the same bridges, only with 3 ft. slab overhangs added on both sides.

As mentioned before, in a rigid bridge, the percentage of moment which a girder takes is proportional to its contribution to the total moment of inertia of the bridge. On the basis of this fact, it is proposed that a relative increase of the moment of inertia of a specific girder (relative with respect to the total bridge moment of inertia) is proportional to the relative increase of moment percentage taken by this girder. Thus, if slab overhangs are added to a bridge, the exterior girder moments will increase while the moments taken by interior girders will decrease.

To modify the moment percentages accordingly, consider a box girder bridge without slab overhangs, having n girders (Fig. 15). Let M_{oj} and ρ_{oj} be the moment percentage and ρ-factor of girder j, respectively, and let M_j and ρ_j be the corresponding quantities in the same bridge, but with slab overhangs. Let the relative stiffness factors in the bridge without overhang be defined to be

$$k_{oi} = \frac{\rho_{oi}}{\sum_{j=1}^{n} \rho_j} \qquad i = 1,2,3,\ldots,n \tag{10}$$

and similarly in the bridge with overhang,

$$k_i = \frac{\rho_i}{\sum_{j=1}^{n} \rho_j} \qquad i = 1,2,3\ldots,n. \tag{11}$$

Following the proposition that the percentage of moment taken by a girder is proportional to its relative stiffness in the bridge, a set of redistribution factors will then be defined as

$$r_i = \frac{k_i}{k_{oi}} \qquad i = 1,2,3,\ldots,n \tag{12}$$

with which the modified moment percentages become

$$M_i' = r_i M_{oi} \qquad i = 1,2,3\ldots,n \tag{13a}$$

or in normalized form,

$$M_i = \frac{M_i'}{\sum_{j=1}^{n} M_j'} \cdot 100 \qquad i = 1,2,3\ldots,n \tag{13b}$$

so that their sum adds up to 100%.

For convenience, Table 10 gives the ρ-factors of all example bridges with overhangs, calculated according to Eq. (4), with $W_c = 3.0$ ft.

TABLE 10. ρ-FACTORS FOR EXTERIOR GIRDERS OF EXAMPLE BRIDGES WITH SLAB OVERHANGS. FOR INTERIOR GIRDERS, $\rho = 1$.

Span (ft)	60	80	120
3 or 6 Cells	0.680	0.688	0.700
4 or 8 Cells	0.732	0.743	0.758

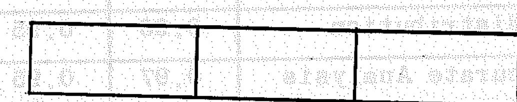
The comparison with Table 3 shows the importance of the 3 ft. overhang for exterior girders. In order to evaluate the accuracy of the moment redistribution method just outlined, the following three example bridges have also been analyzed independently by the computer, using the actual cross sections with slab overhangs, and these "accurate" results have been compared with the approximate moments predicted by the above moment redistribution method.

Case 1: 3 cells, 60 ft. span, load over exterior girder

Case 2: 3 cells, 120 ft. span, load over exterior girder

Case 3: 8 cells, 60 ft. span, load over exterior girder.

For illustration of the redistribution method, case 1 will be computed below:



Girder	L_2	L_1	R_1	R_2	Σ
M_{oi} (%)	8.9	20.4	32.3	38.4	100.0
ρ_o (Table 5)	0.534	1.0	1.0	0.534	3.068
k_{oi} (Eq. 10)	0.174	0.326	0.326	0.174	1.0
ρ (Table 10)	0.680	1.0	1.0	0.680	3.360
k_i (Eq. 11)	0.202	0.298	0.298	0.202	1.0
r_i (Eq. 12)	1.161	0.915	0.915	1.161	
M_i' (Eq. 13a)	10.3	18.6	29.6	44.6	103.1
M_i (Eq. 13b)	10.0	18.0	28.7	43.3	100.0

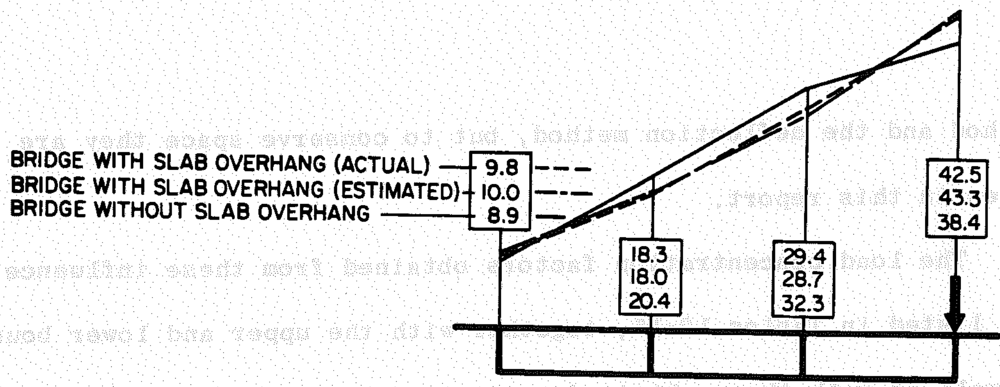
These values and the results for cases 2 and 3 are plotted against the accurate direct computer output in Fig. 18. For comparison, also the moments in the corresponding bridges without slab overhangs are shown. It can be seen that agreement between actual and estimated results is good, especially if the cells are wider (9 ft. 4 in.). Then the 3 ft. slab overhang is of less importance than for bridges with 7 ft. wide cells. Moreover, as in the deflection method of Section 3.4.2, due to the normalizing process, over- and underestimated moments tend to reduce the errors as soon as several trucks are placed on the bridge. In fact, taking as a further example the 4-cell bridge with a 120 ft. span (cell width = 7 ft.), the maximum numbers of wheel loads as predicted by the redistribution method and as analyzed more accurately by the computer for the respective most critical truck positions, compare as follows:

TABLE 11. MAXIMUM NUMBERS OF WHEEL LOADS IN 4-CELL, 120 FT. SPAN BRIDGE, BY REDISTRIBUTION AND ACCURATE ANALYSIS

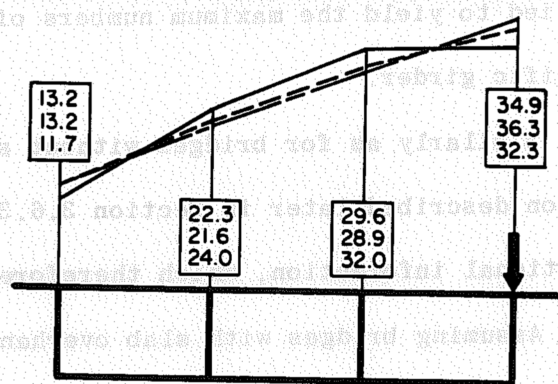
Method Used	Girder		
	C	R1	R2
Redistribution	0.96	0.95	0.78
Accurate Analysis	0.97	0.95	0.75

This agreement is so satisfactory that all further studies in this report will be based on this redistribution method.

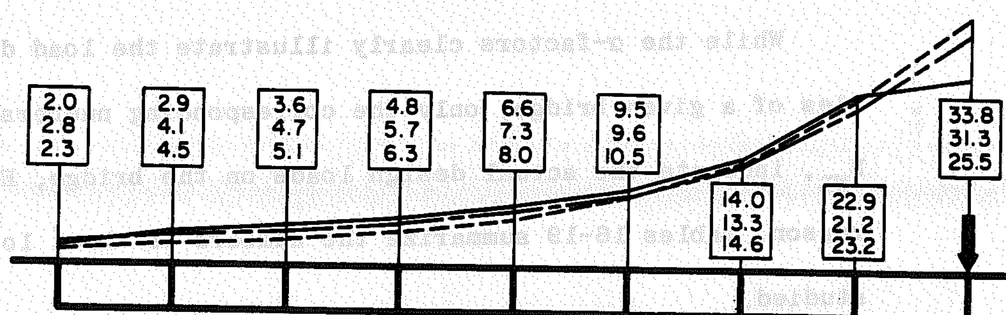
Converting thus all moment percentages for the bridges of Table 4, available from computer output for bridges without overhangs, to those with slab overhangs, leads to the influence lines shown in Figs. 19-30. Similar sets of influence lines have been prepared for the corresponding bridges without slab overhangs, both, using the stress integration



a) 3-CELL BRIDGE WITH SIMPLE SPAN OF 60 FT.
LOAD CASE 4



b) 3-CELL BRIDGE WITH SIMPLE SPAN OF 120 FT.
LOAD CASE 4



c) 8-CELL BRIDGE WITH SIMPLE SPAN OF 60 FT.
LOAD CASE 9

FIG. 18 COMPARISON OF MOMENT PERCENTAGES IN BRIDGES WITH SLAB OVERHANG (MIDSPAN VALUES) method and the deflection method, but to conserve space they are not given in this report.

The load concentration factors obtained from these influence lines are listed in Tables 12-15, together with the upper and lower bounds as well as with those obtained using AASHO design formulas.

In addition, for each case the number of wheel loads for the rigid bridge case is listed by which the corresponding α-factors have to be multiplied to yield the maximum numbers of wheel loads to be carried by this specific girder.

Similarly as for bridges without slab overhangs, the reciprocity relation described later in Section 3.6.3 permitted the evaluation of some additional information, which therefore has been included in Tables 12-15.

Assuming bridges with slab overhangs to be more realistic than ones without, the studies in subsequent chapters will be based on the information presented in Tables 12-15. The influence of the slab overhang on the girder load concentration factors will be discussed in Section 4.7.

While the α-factors clearly illustrate the load distribution properties of a given bridge, only the corresponding numbers of wheel loads, N_{WL}, indicate the actual design loads on the bridge, Eq. (7a). For this reason, Tables 16-19 summarize the numbers of wheel loads for all cases studied.

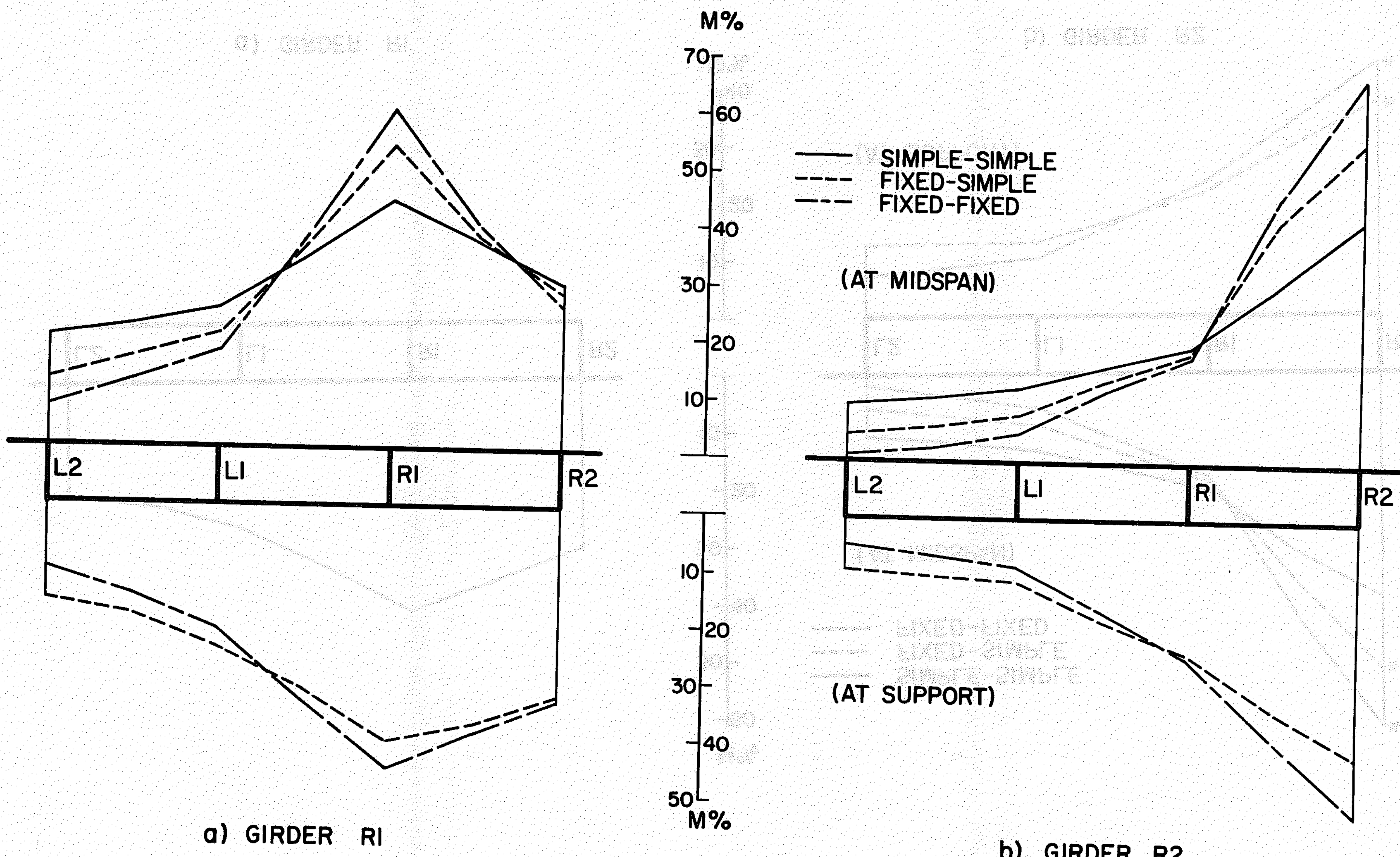
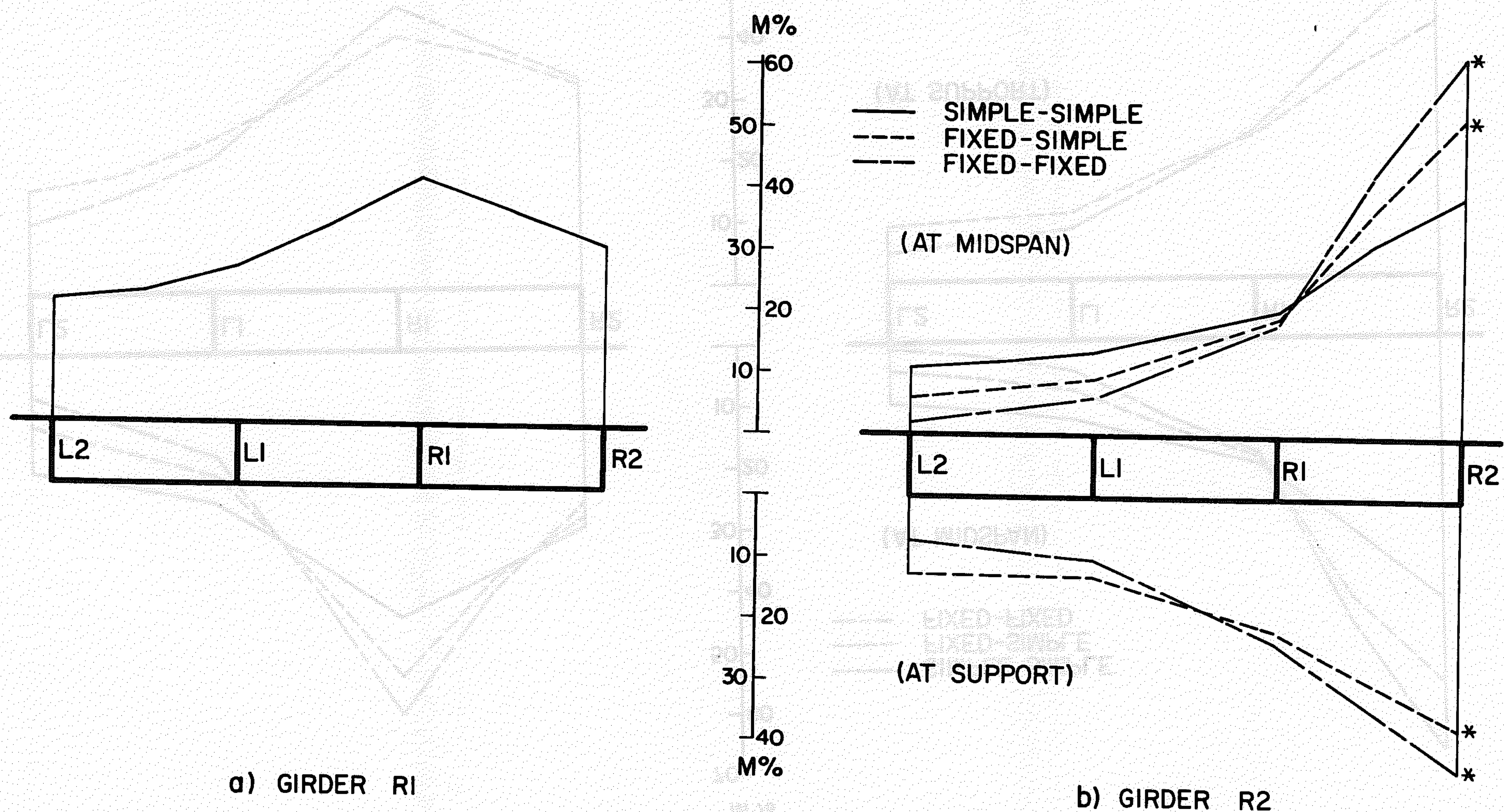


FIG. 19 INFLUENCE LINES FOR MOMENT PERCENTAGES OF 3-CELL BRIDGES WITH 60 FT. SPAN



* CONSTRUCTED WITH THE AID OF THE RECIPROCITY RELATIONSHIP OF SECTION 3.6.3

FIG. 20 INFLUENCE LINES FOR MOMENT PERCENTAGES OF 3-CELL BRIDGES WITH 80 FT. SPAN

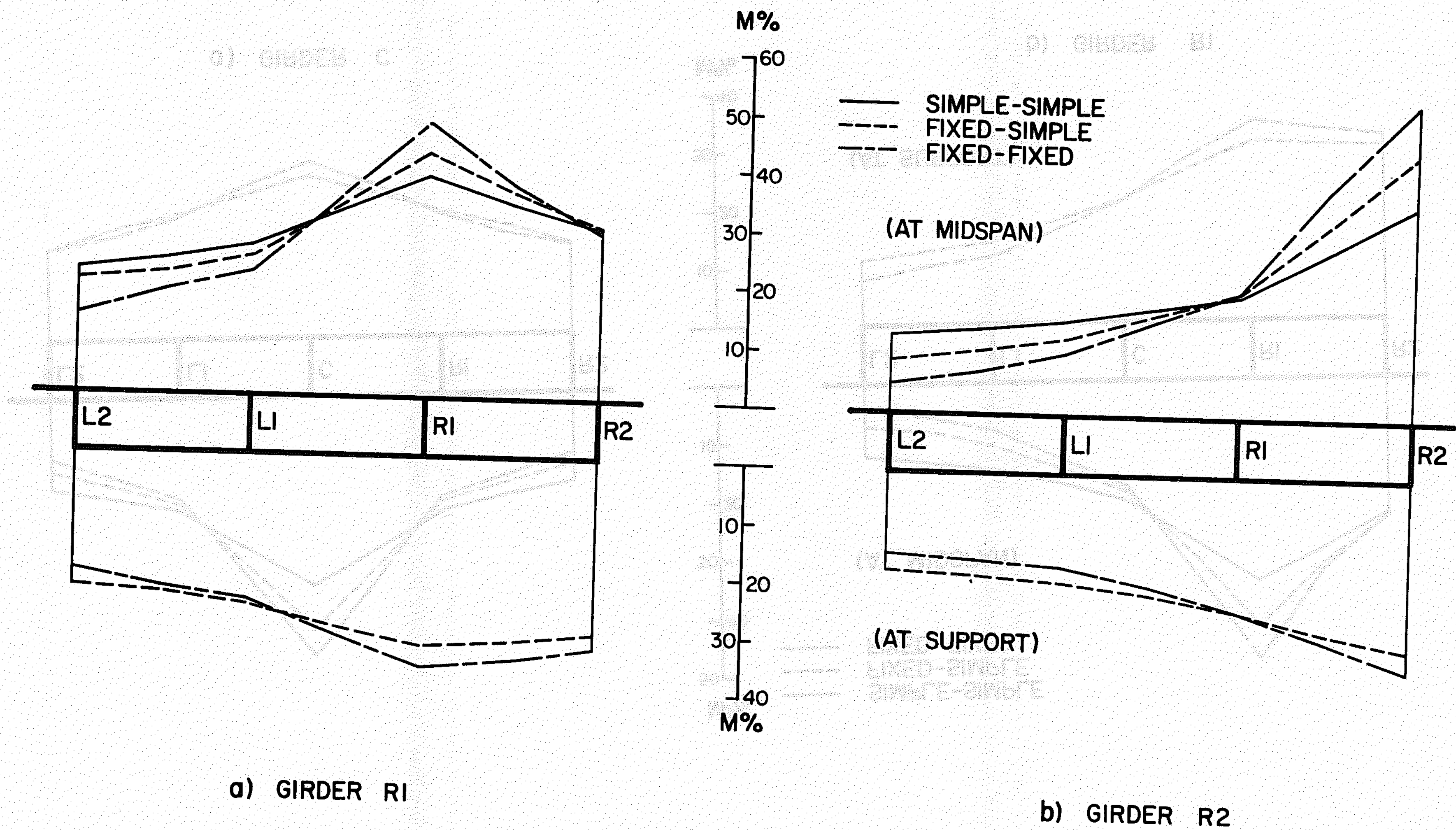


FIG. 21 INFLUENCE LINES FOR MOMENT PERCENTAGES OF 3-CELL BRIDGES WITH 120 FT. SPAN

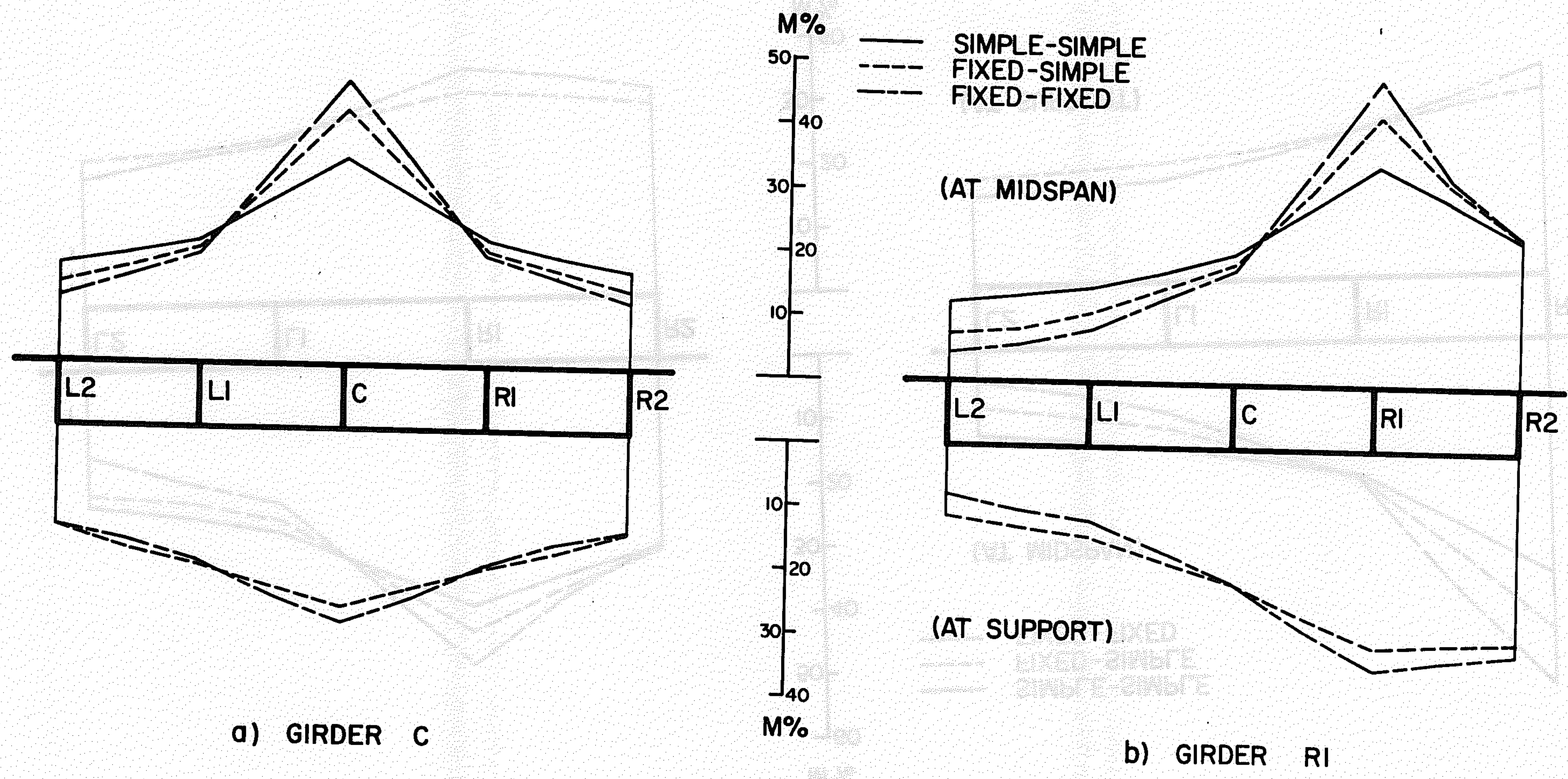
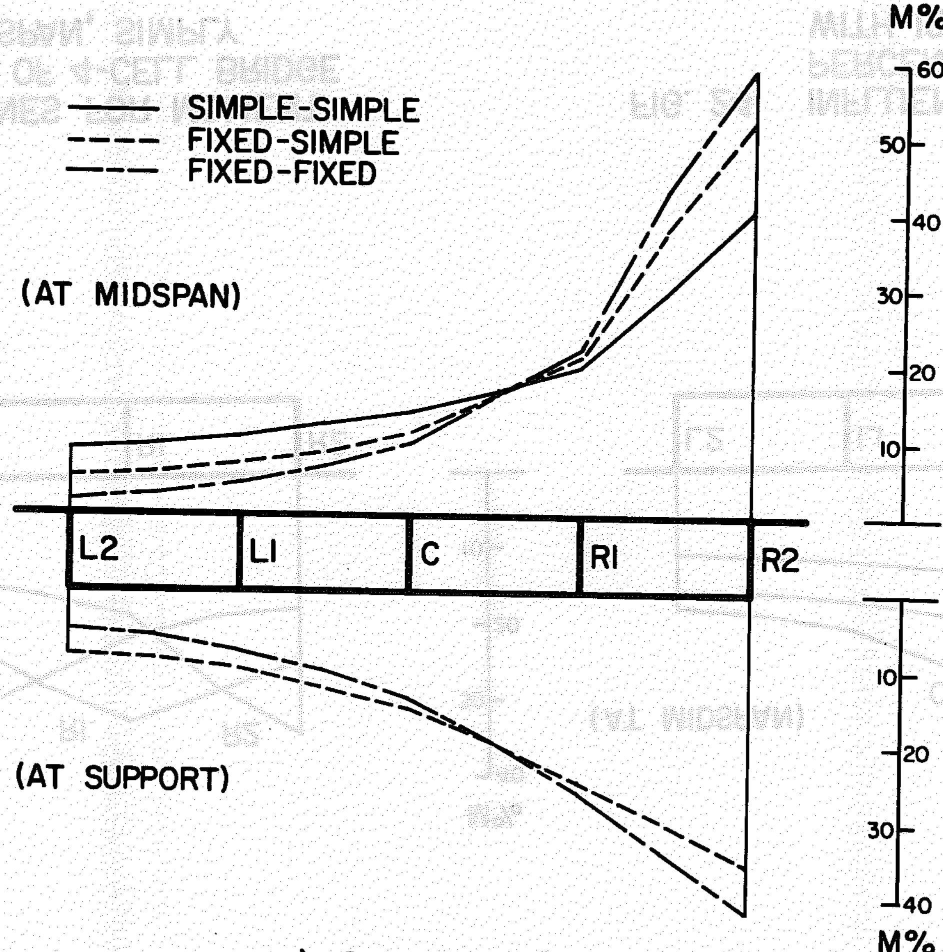


FIG. 22 INFLUENCE LINES FOR MOMENT PERCENTAGES OF 4-CELL BRIDGES WITH 60 FT. SPAN



c) GIRDER R2

FIG. 22 (CONT'D) INFLUENCE LINES FOR MOMENT PERCENTAGES OF 4-CELL BRIDGES WITH 60 FT. SPAN

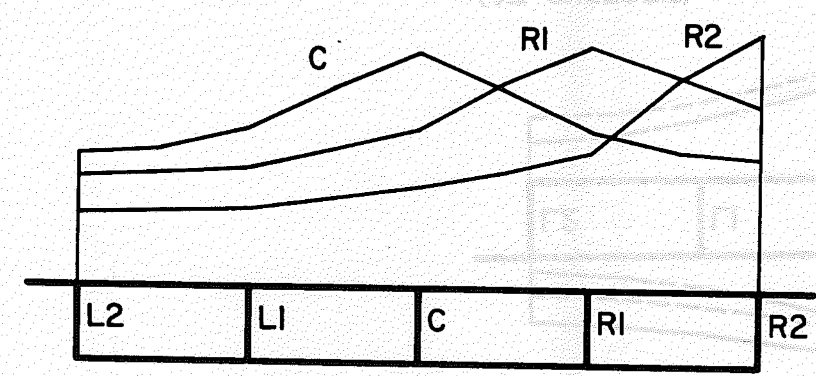


FIG. 23 INFLUENCE LINES FOR MOMENT PERCENTAGES OF 4-CELL BRIDGE WITH 80 FT. SPAN, SIMPLY SUPPORTED

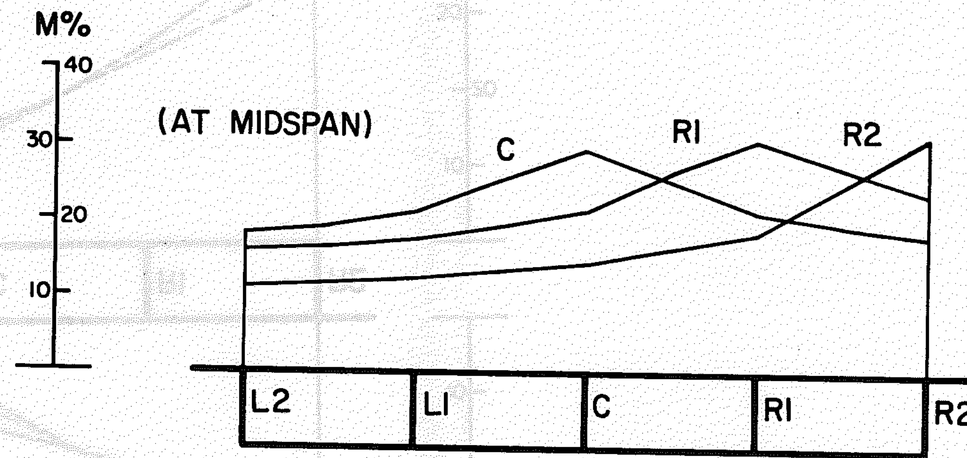


FIG. 24 INFLUENCE LINES FOR MOMENT PERCENTAGES OF 4-CELL BRIDGE WITH 120 FT. SPAN, SIMPLY SUPPORTED

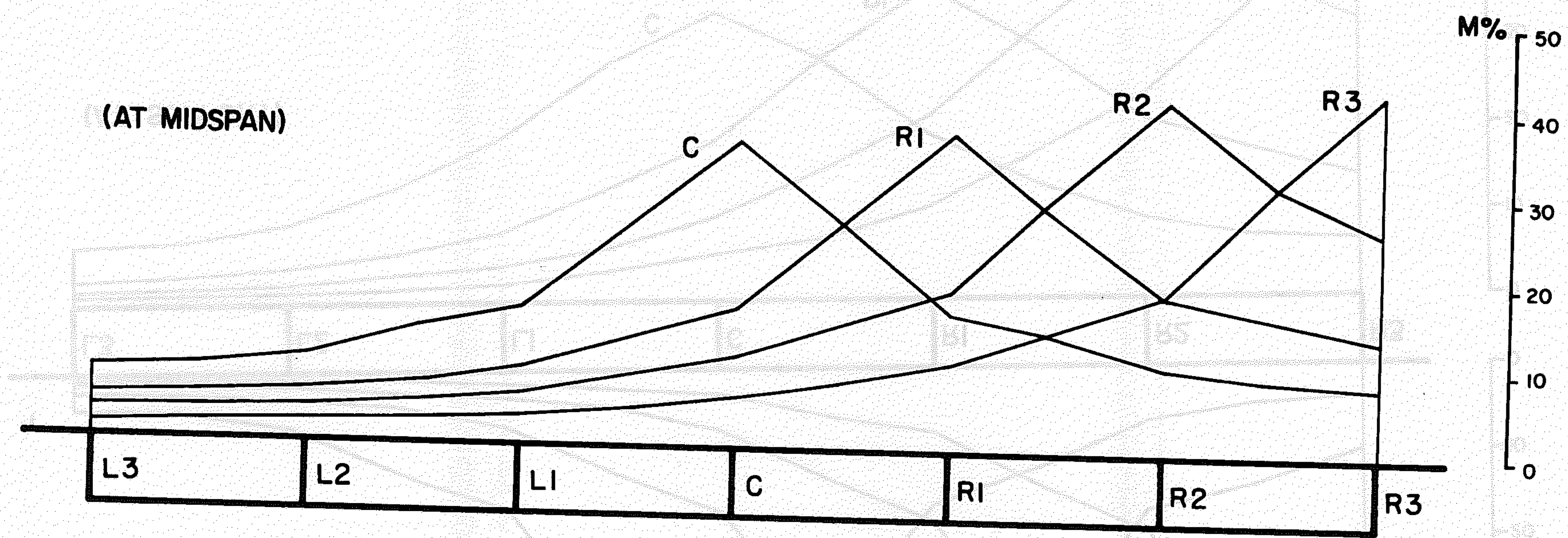


FIG. 25 INFLUENCE LINES FOR MOMENT PERCENTAGES OF 6-CELL BRIDGE WITH 60 FT. SPAN, SIMPLY SUPPORTED

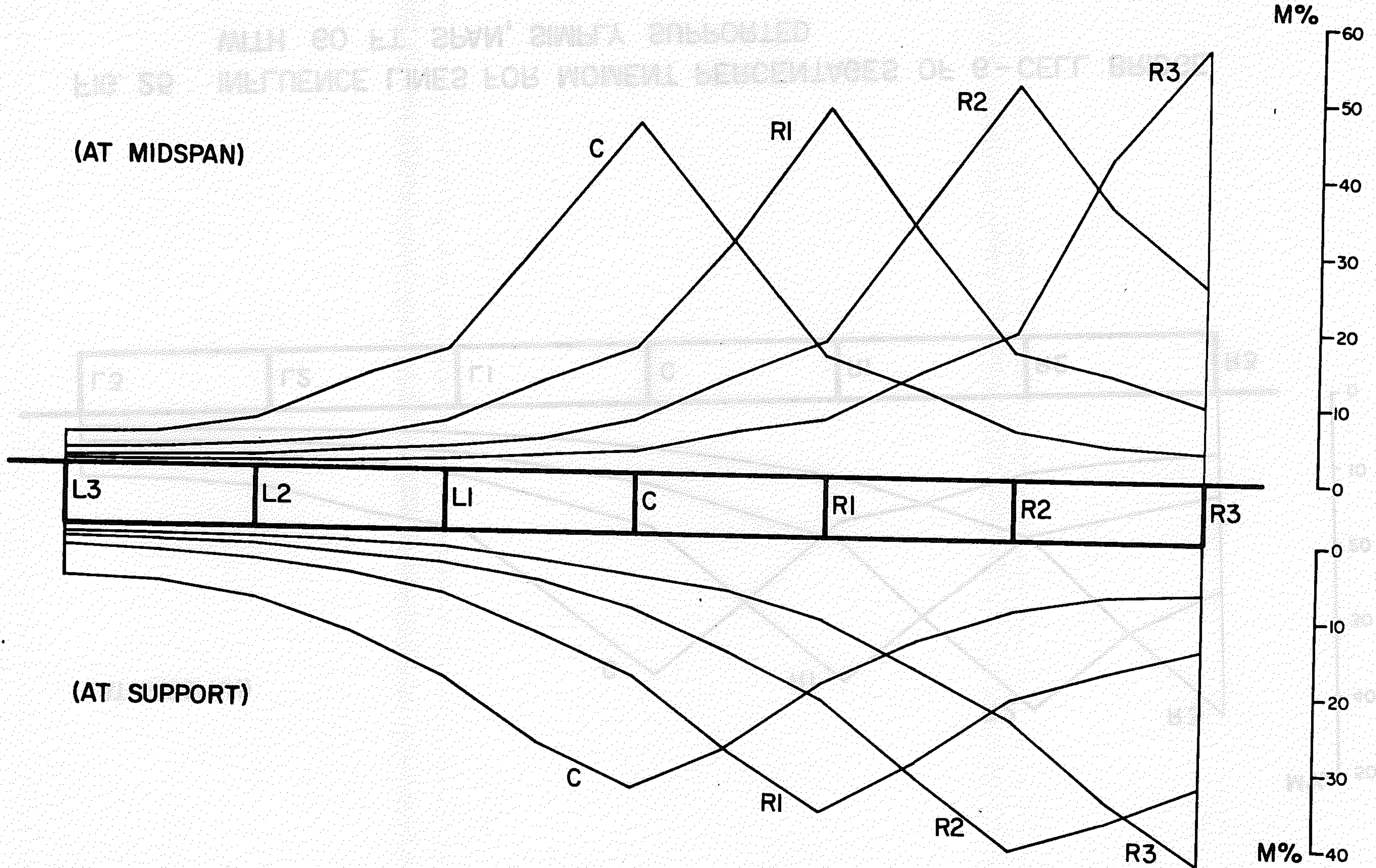


FIG. 26 INFLUENCE LINES FOR MOMENT PERCENTAGES OF 6-CELL BRIDGE WITH 60 FT. SPAN, ONE END FIXED

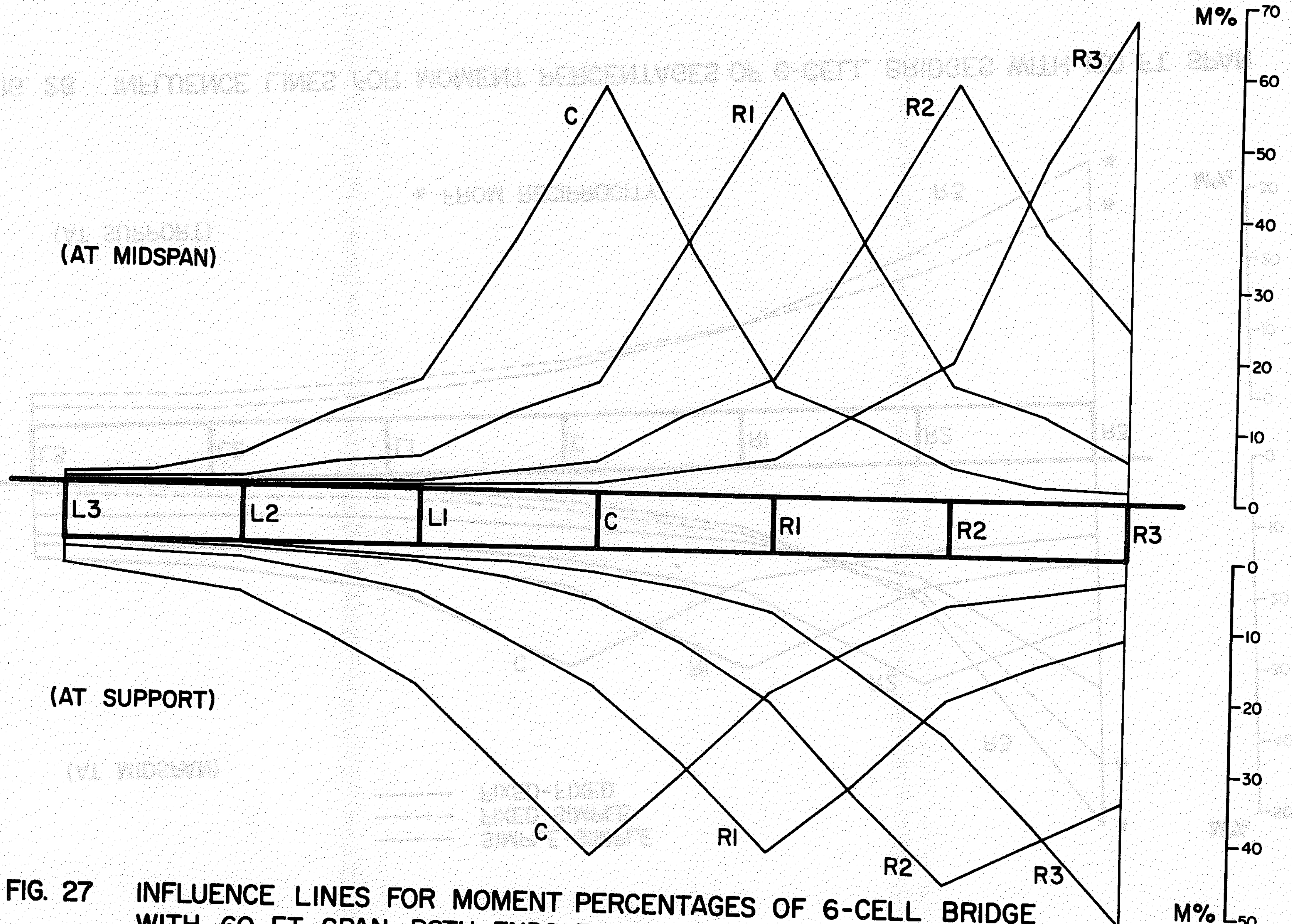


FIG. 27 INFLUENCE LINES FOR MOMENT PERCENTAGES OF 6-CELL BRIDGE WITH 60 FT. SPAN, BOTH ENDS FIXED

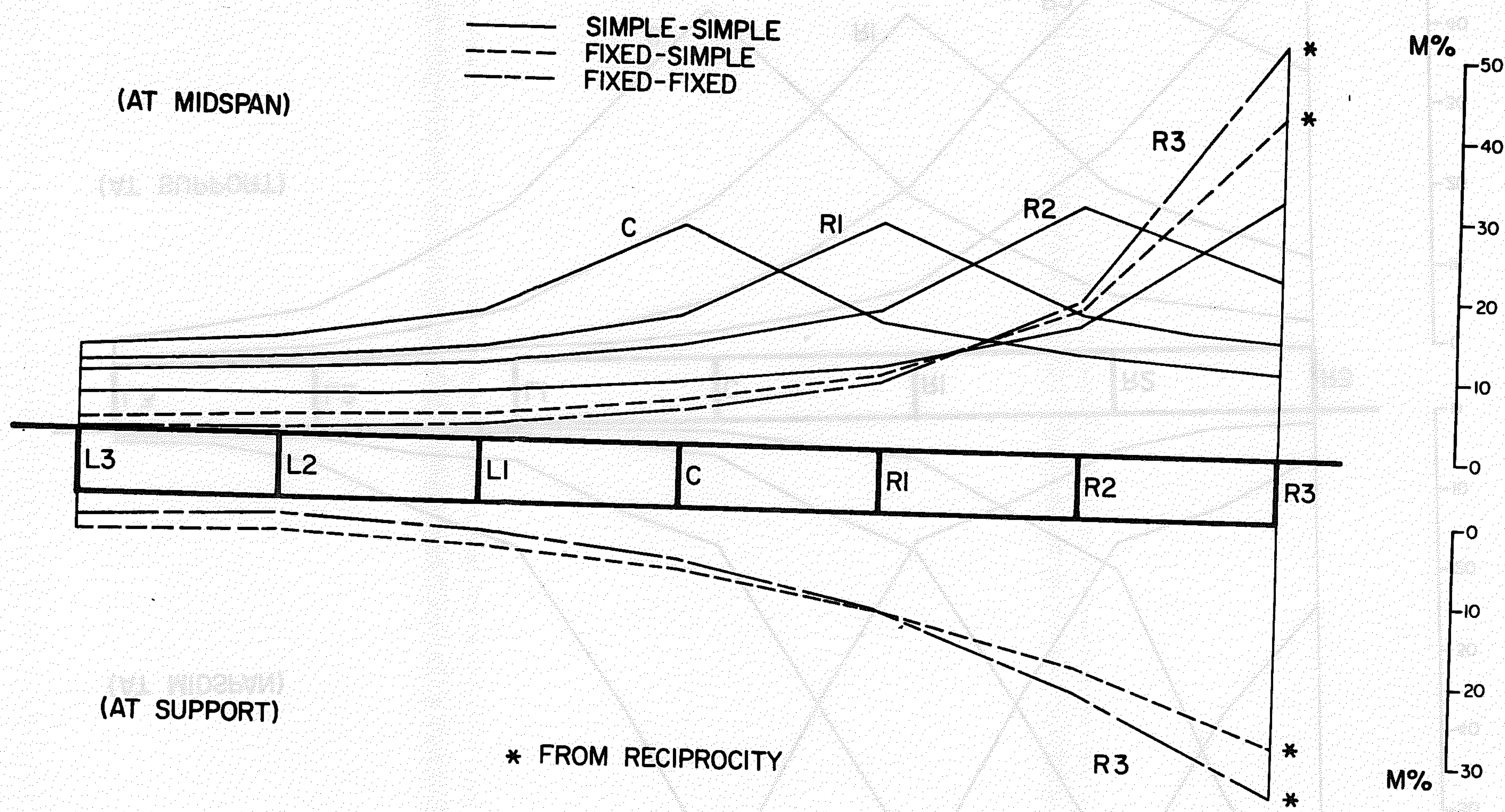


FIG. 28 INFLUENCE LINES FOR MOMENT PERCENTAGES OF 6-CELL BRIDGES WITH 120 FT. SPAN

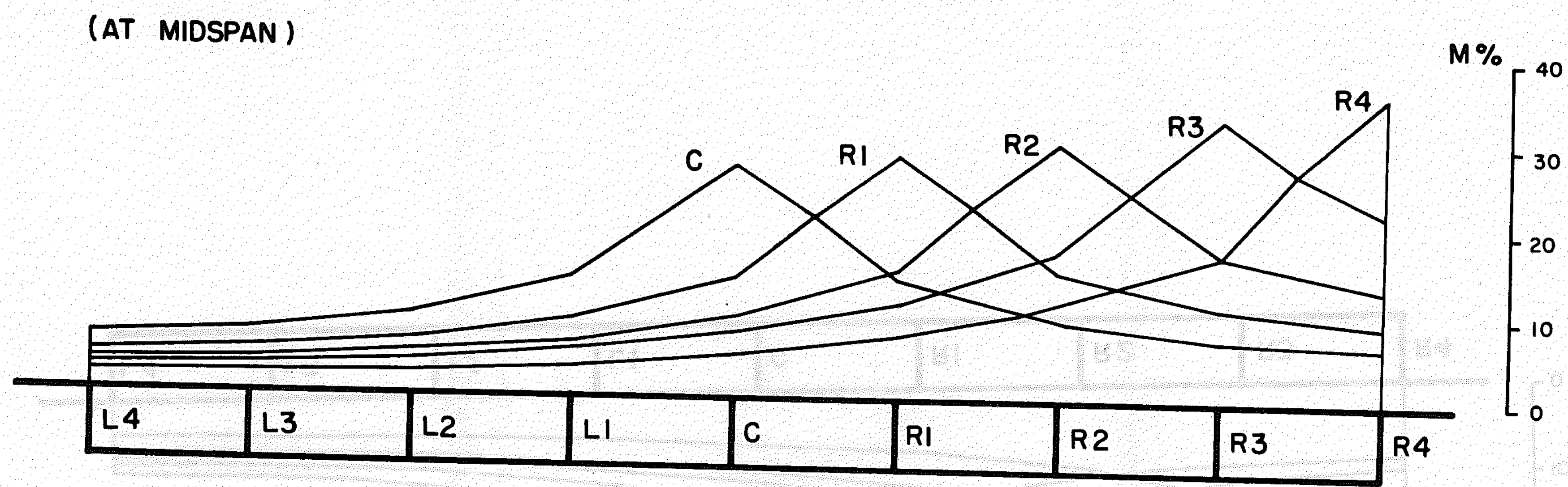


FIG. 29 INFLUENCE LINES FOR MOMENT PERCENTAGES OF 8-CELL BRIDGE WITH 60 FT. SPAN, SIMPLY SUPPORTED

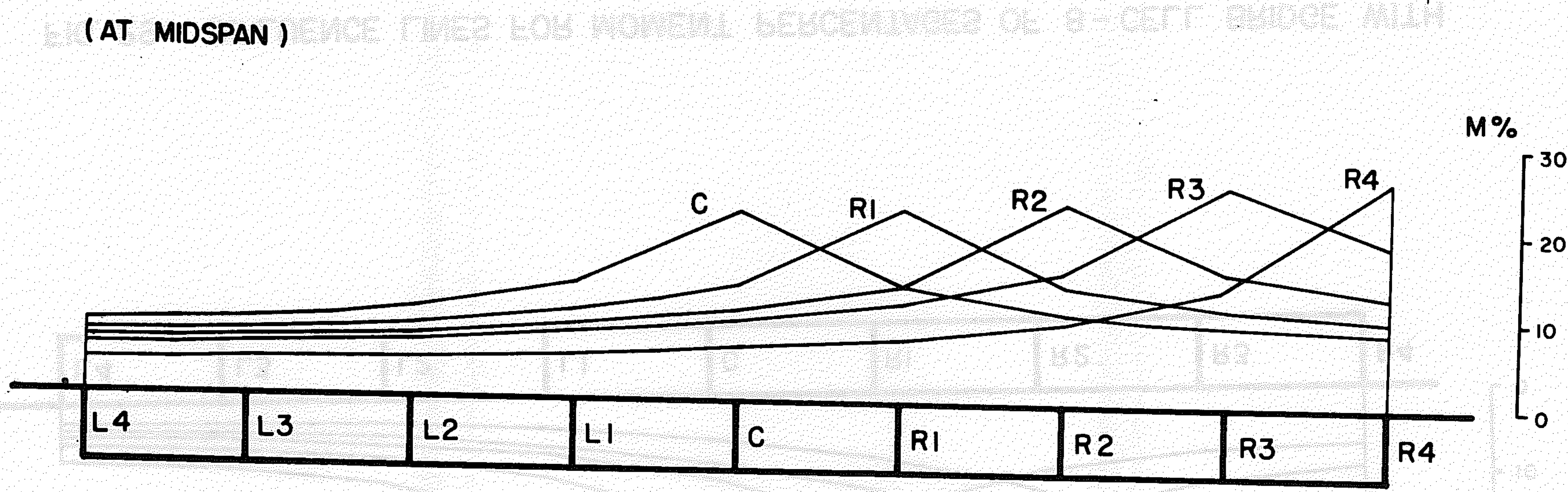


FIG. 30 INFLUENCE LINES FOR MOMENT PERCENTAGES OF 8-CELL BRIDGE WITH 120 FT. SPAN, SIMPLY SUPPORTED

TABLE 12. α-FACTORS FOR 3-CELL BRIDGES WITH SLAB OVERHANGS

Span (ft.)			60					80					120				
Section			Midspan			Support		Midspan			Support		Midspan			Support	
Boundary Cond.			SS	FS	FF	FS	FF	SS	FS	FF	FS	FF	SS	FS	FF	FS	FF
Girder	No. of Lanes	a			1.190					1.186					1.178		
R1	2	b	1.13	1.19	1.25	1.11	1.13	1.11					1.08	1.12	1.15	1.01	1.07
		c	1.56	1.56	1.56	1.56	1.56	1.57					1.58	1.58	1.58	1.58	1.58
		d	1.12	1.12	1.12	1.12	1.12	1.13					1.13	1.13	1.13	1.13	1.13
	3	a			1.608					1.600					1.590		
		b	1.02	1.01	1.01	0.97	1.01	1.01					1.00	1.00	1.00	0.97	0.99
		c	1.16	1.16	1.16	1.16	1.16	1.16					1.17	1.17	1.17	1.17	1.17
		d	0.83	0.83	0.83	0.83	0.83	0.83					0.84	0.84	0.84	0.84	0.84
R2	2	a			0.809					0.816					0.824		
		b	1.23	1.39	1.51	1.23	1.35	1.21	1.31	1.42	1.15	1.23	1.17	1.27	1.36	1.14	1.14
		c	1.68	1.68	1.68	1.68	1.68	1.67	1.67	1.67	1.67	1.67	1.65	1.65	1.65	1.65	1.65
		d	1.36	1.36	1.36	1.36	1.36	1.34	1.34	1.34	1.34	1.34	1.33	1.33	1.33	1.33	1.33
	3	a			1.092					1.100					1.110		
		b	1.05	1.11	1.22	1.06	1.10	1.05	1.07	1.13	1.02	1.05	1.05	1.08	1.10	1.08	1.05
		c	1.24	1.24	1.24	1.24	1.24	1.24	1.24	1.24	1.24	1.24	1.23	1.23	1.23	1.23	1.23
		d	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.99	0.99	0.99	0.99	0.99

(a) number of wheel loads carried by girder in rigid bridge
(b) α-factors for actual bridge, from analyses
(c) α-factors for perfectly flexible bridge, upper bound
(d) α-factors calculated from AASHO design formulas

Note - α-factors for rigid bridge are 1.0

TABLE 13. α-FACTORS FOR 6-CELL BRIDGES WITH SLAB OVERHANGS

Span (ft.)			60					80					120				
Section			Midspan			Support		Midspan			Support		Midspan			Support	
Boundary Cond.			SS	FS	FF	FS	FF	SS	FS	FF	FS	FF	SS	FS	FF	FS	FF
Girder	No. of Lanes																
C	4	a			0.944					0.941					0.938		
		b	1.17	1.32	1.48	1.19	1.33	1.13					1.09				
		c	2.04	2.04	2.04	2.04	2.04	2.04					2.05				
		d	1.41	1.41	1.41	1.41	1.41	1.41					1.42				
	5	a			1.180					1.176					1.171		
		b	1.06	1.12	1.22	1.06	1.14	1.06					1.04				
		c	1.63	1.63	1.63	1.63	1.63	1.63					1.63				
		d	1.13	1.13	1.13	1.13	1.13	1.13					1.14				
	6	a			1.414					1.411					1.407		
		b	0.97	0.97	1.01	0.96	0.98	1.00					1.00				
		c	1.36	1.36	1.36	1.36	1.36	1.36					1.37				
		d	0.94	0.94	0.94	0.94	0.94	0.94					0.95				
R1	4	a			0.944										0.938		
		b	1.25	1.41	1.49	1.32	1.39						1.12				
		c	1.85	1.85	1.85	1.85	1.85						1.87				
		d	1.41	1.41	1.41	1.41	1.41						1.42				
	5	a			1.180										1.171		
		b	1.07	1.17	1.24	1.10	1.16						1.05				
		c	1.63	1.63	1.63	1.63	1.63						1.63				
		d	1.13	1.13	1.13	1.13	1.13						1.14				
	6	a			1.414										1.407		
		b	0.98	0.98	1.03	0.98	0.97						0.99				
		c	1.36	1.36	1.36	1.36	1.36						1.36				
		d	0.94	0.94	0.94	0.94	0.94						0.95				

TABLE 13 (cont'd). α-FACTORS FOR 6-CELL BRIDGES WITH SLAB OVERHANGS

Span (ft.)			60					80					120				
Section			Midspan			Support		Midspan			Support		Midspan			Support	
Boundary Cond.			SS	FS	FF	FS	FF	SS	FS	FF	FS	FF	SS	FS	FF	FS	FF
Girder	No. of Lanes																
R2	4	a	0.944										0.938				
		b	1.32	1.47	1.58	1.37	1.48						1.18				
		c	2.04	2.04	2.04	2.04	2.04						2.05				
		d	1.41	1.41	1.41	1.41	1.41						1.42				
	5	a	1.180										1.171				
		b	1.14	1.24	1.28	1.23	1.30						1.06				
		c	1.63	1.63	1.63	1.63	1.63						1.63				
		d	1.13	1.13	1.13	1.13	1.13						1.14				
	6	a	1.414										1.407				
		b	0.99	1.08	1.12	1.07	1.13						1.00				
		c	1.36	1.36	1.36	1.36	1.36						1.37				
		d	0.94	0.94	0.94	0.94	0.94						0.95				
R3	4	a	0.642					0.648					0.658				
		b	1.48	1.78	1.94	1.59	1.72	1.35	1.60	1.76	1.33	1.39	1.26	1.43	1.61	1.25	1.36
		c	2.11	2.11	2.11	2.11	2.11	2.09	2.09	2.09	2.09	2.09	2.06	2.06	2.06	2.06	2.06
		d	1.70	1.70	1.70	1.70	1.70	1.69	1.69	1.69	1.69	1.69	1.66	1.66	1.66	1.66	1.66
	5	a	0.802					0.809					0.820				
		b	1.29	1.53	1.69	1.39	1.52	1.17	1.35	1.50	1.19	1.33	1.11	1.24	1.35	1.09	1.22
		c	1.69	1.69	1.69	1.69	1.69	1.68	1.68	1.68	1.68	1.68	1.66	1.66	1.66	1.66	1.66
		d	1.36	1.36	1.36	1.36	1.36	1.35	1.35	1.35	1.35	1.35	1.33	1.33	1.33	1.33	1.33
	6	a	0.962					0.971					0.985				
		b	1.10	1.27	1.42	1.19	1.27	1.09	1.13	1.25	1.01	1.11	1.03	1.04	1.14	1.00	1.03
		c	1.41	1.41	1.41	1.41	1.41	1.40	1.40	1.40	1.40	1.40	1.38	1.38	1.38	1.38	1.38
		d	1.14	1.14	1.14	1.14	1.14	1.13	1.13	1.13	1.13	1.13	1.11	1.11	1.11	1.11	1.11

(a) number of wheel loads carried by girder in rigid bridge
(b) α-factors for actual bridge, from analyses
(c) α-factors for perfectly flexible gridge, upper bound
(d) α-factors calculated from AASHO design formulas

Note - α-factors for rigid bridge are 1.0

TABLE 14. α-FACTORS FOR 4-CELL BRIDGES WITH SLAB OVERHANGS

Span (ft.)			60					80	120
Section			Midspan			Support		Midspan	Midspan
Boundary Cond.			SS	FS	FF	FS	FF	SS	SS
Girder	No. of Lanes	a	0.896					0.893	0.885
C	2	b	1.09	1.20	1.22	1.06	1.07	1.09	1.08
		c	1.75	1.75	1.75	1.75	1.75	1.76	1.76
		d	1.12	1.12	1.12	1.12	1.12	1.12	1.13
	3	a	1.21					1.20	1.20
		b	0.93	0.92	0.92	0.93	0.95	0.99	0.99
		c	1.14	1.14	1.14	1.14	1.14	1.15	1.15
		d	0.83	0.83	0.83	0.83	0.83	0.83	0.83
R1	2	a	0.896					0.893	0.885
		b	1.05	1.10	1.14	1.08	1.12	1.06	1.07
		c	1.27	1.27	1.27	1.27	1.27	1.28	1.28
		d	1.12	1.12	1.12	1.12	1.12	1.12	1.13
	3	a	1.21					1.20	1.20
		b	0.96	0.98	0.99	0.99	1.00	1.01	1.01
		c	1.30	1.30	1.30	1.30	1.30	1.31	1.31
		d	0.83	0.83	0.83	0.83	0.83	0.83	0.83
R2	2	a	0.656					0.663	0.671
		b	1.34	1.46	1.55	1.34	1.39	1.24	1.16
		c	1.74	1.74	1.74	1.74	1.74	1.72	1.69
		d	1.41	1.41	1.41	1.41	1.41	1.40	1.38
	3	a	0.885					0.890	0.910
		b	1.15	1.19	1.26	1.16	1.17	1.09	1.05
		c	1.29	1.29	1.29	1.29	1.29	1.28	1.25
		d	1.05	1.05	1.05	1.05	1.05	1.04	1.02

(a) number of wheel loads carried by girder in rigid bridge
(b) α-factors for actual bridge, from analyses
(c) α-factors for perfectly flexible bridge, upper bound
(d) α-factors calculated from AASHO design formulas

Note - α-factors for rigid bridge are 1.0

TABLE 15. α-FACTORS FOR 8-CELL BRIDGES WITH SLAB OVERHANGS

Span (ft.)			60					80	120
Section			Midspan			Support		Midspan	Midspan
Boundary Cond.			SS	FS	FF	FS	FF	SS	SS
Girder	No. of Lanes								
C	4	a	0.710					0.708	0.705
		b	1.16	1.26	1.50	1.03	1.24	1.13	1.11
		c	2.22	2.22	2.22	2.22	2.22	2.22	2.23
		d	1.41	1.41	1.41	1.41	1.41	1.41	1.42
	5	a	0.886					0.885	0.880
		b	1.03	1.05	1.16	1.05	1.05	1.04	1.04
		c	1.46	1.46	1.46	1.46	1.46	1.46	1.47
		d	1.13	1.13	1.13	1.13	1.13	1.13	1.14
	6	a	1.061					1.060	1.056
		b	0.98	0.98	1.05	0.99	1.02	1.00	1.00
		c	1.48	1.48	1.48	1.48	1.48	1.48	1.49
		d	0.94	0.94	0.94	0.94	0.94	0.94	0.95
R1	4	a	0.710					0.708	0.705
		b	1.20						1.09
		c	1.61						1.62
		d	1.41						1.42
	5	a	0.886					0.885	0.880
		b	1.06						1.05
		c	1.78						1.79
		d	1.13						1.14
	6	a	1.061					1.060	1.056
		b	0.99						1.00
		c	1.48						1.49
		d	0.94						0.95
R2	4	a	0.710					0.708	0.705
		b	1.28						1.15
		c	2.22						2.23
		d	1.41						1.42
	5	a	0.886					0.885	0.880
		b	1.05						1.04
		c	1.78						1.79
		d	1.13						1.14
	6	a	1.061					1.060	1.056
		b	0.98						1.00
		c	1.48						1.49
		d	0.94						0.95

TABLE 15 (cont'd). α-FACTORS FOR 8-CELL BRIDGES WITH SLAB OVERHANGS

Span (ft.)			60					80	120
Section			Midspan			Support		Midspan	Midspan
Boundary Cond.			SS	FS	FF	FS	FF	SS	SS
Girder	No. of Lanes								
R3	4	a	0.710					0.708	0.705
		b	1.27						1.16
		c	1.82						1.83
		d	1.41						1.42
	5	a	0.886					0.895	0.880
		b	1.10						1.04
		c	1.78						1.79
		d	1.13						1.14
	6	a	1.061					1.060	1.056
		b	1.00						1.02
		c	1.48						1.49
		d	0.94						0.95
R4	4	a	0.520					0.526	0.534
		b	1.47	1.64	1.85	1.49	1.69	1.39	1.20
		c	2.20	2.20	2.20	2.20	2.20	2.17	2.14
		d	1.79	1.79	1.79	1.79	1.79	1.76	1.74
	5	a	0.649					0.657	0.667
		b	1.27	1.41	1.59	1.30	1.47	1.15	1.07
		c	1.76	1.76	1.76	1.76	1.76	1.74	1.72
		d	1.43	1.43	1.43	1.43	1.43	1.41	1.39
	6	a	0.775					0.788	0.800
		b	1.08	1.18	1.33	1.09	1.23	1.02	0.99
		c	1.47	1.47	1.47	1.47	1.47	1.45	1.43
		d	1.20	1.20	1.20	1.20	1.20	1.18	1.16

(a) number of wheel loads carried by girder in rigid bridge
(b) α-factors for actual bridge, from analyses
(c) α-factors for perfectly flexible bridge, upper bound
(d) α-factors calculated from AASHO design formulas

Note - α-factors for rigid bridge are 1.0

TABLE 16. NUMBERS OF WHEEL LOADS FOR 3-CELL BRIDGES WITH SLAB OVERHANGS

Span (ft.)			60					80					120				
Section			Midspan			Support		Midspan			Support		Midspan			Support	
Boundary Cond.			SS	FS	FF	FS	FF	SS	FS	FF	FS	FF	SS	FS	FF	FS	FF
Girder	No. of Lanes																
R1	2	a	1.19	1.19	1.19	1.19	1.19	1.19					1.18	1.18	1.18	1.18	1.18
		b	1.35	1.42	1.49	1.32	1.35	1.31					1.27	1.32	1.35	1.19	1.26
		c	1.33	1.33	1.33	1.33	1.33	1.33					1.33	1.33	1.33	1.33	1.33
	3	a	1.61	1.61	1.61	1.61	1.61	1.60					1.59	1.59	1.59	1.59	1.59
		b	1.64	1.63	1.62	1.56	1.62	1.62					1.59	1.60	1.59	1.54	1.57
		c	1.33	1.33	1.33	1.33	1.33	1.33					1.33	1.33	1.33	1.33	1.33
R2	2	a	0.81	0.81	0.81	0.81	0.81	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82	0.82
		b	0.99	1.13	1.22	1.00	1.09	0.99	1.07	1.16	0.94	1.00	0.96	1.04	1.12	0.94	0.94
		c	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09
	3	a	1.09	1.09	1.09	1.09	1.09	1.10	1.10	1.10	1.10	1.10	1.11	1.11	1.11	1.11	1.11
		b	1.14	1.21	1.33	1.15	1.21	1.16	1.18	1.24	1.12	1.16	1.16	1.20	1.23	1.20	1.16
		c	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09

(a) number of wheel loads in bridge with rigid cross section
(b) number of wheel loads in actual bridge, from analysis
(c) number of wheel loads, determined by use of present AASHO formulas

TABLE 17. NUMBERS OF WHEEL LOADS FOR 6-CELL BRIDGES WITH SLAB OVERHANGS

Span (ft.)			60					80					120				
Section			Midspan			Support		Midspan			Support		Midspan			Support	
Boundary Cond.			SS	FS	FF	FS	FF	SS	FS	FF	FS	FF	SS	FS	FF	FS	FF
Girder	No. of Lanes																
C	4	a	0.94	0.94	0.94	0.94	0.94	0.94					0.94				
		b	1.10	1.25	1.40	1.12	1.26	1.06					1.03				
		c	1.33	1.33	1.33	1.33	1.33	1.33					1.33				
	5	a	1.18	1.18	1.18	1.18	1.18	1.18					1.17				
		b	1.25	1.33	1.44	1.25	1.35	1.25					1.22				
		c	1.33	1.33	1.33	1.33	1.33	1.33					1.33				
	6	a	1.41	1.41	1.41	1.41	1.41	1.41					1.41				
		b	1.37	1.37	1.44	1.36	1.39	1.41					1.40				
		c	1.33	1.33	1.33	1.33	1.33	1.33					1.33				
R1	4	a	0.94	0.94	0.94	0.94	0.94						0.94				
		b	1.18	1.33	1.40	1.24	1.31						1.05				
		c	1.33	1.33	1.33	1.33	1.33						1.33				
	5	a	1.18	1.18	1.18	1.18	1.18						1.17				
		b	1.27	1.39	1.47	1.30	1.37						1.23				
		c	1.33	1.33	1.33	1.33	1.33						1.33				
	6	a	1.41	1.41	1.41	1.41	1.41						1.41				
		b	1.39	1.39	1.45	1.39	1.37						1.39				
		c	1.33	1.33	1.33	1.33	1.33						1.33				

TABLE 17 (cont'd). NUMBERS OF WHEEL LOADS FOR 6-CELL BRIDGES WITH SLAB OVERHANGS

Span (ft.)			60					80					120				
Section			Midspan			Support		Midspan			Support		Midspan			Support	
Boundary Cond.			SS	FS	FF	FS	FF	SS	FS	FF	FS	FF	SS	FS	FF	FS	FF
Girder	No. of Lanes																
R2	4	a	0.94	0.94	0.94	0.94	0.94						0.94				
		b	1.25	1.39	1.49	1.30	1.30						1.10				
		c	1.33	1.33	1.33	1.33	1.33						1.33				
	5	a	1.18	1.18	1.18	1.18	1.18						1.17				
		b	1.35	1.47	1.51	1.45	1.53						1.24				
		c	1.33	1.33	1.33	1.33	1.33						1.33				
	6	a	1.41	1.41	1.41	1.41	1.41						1.41				
		b	1.40	1.54	1.59	1.51	1.60						1.41				
		c	1.33	1.33	1.33	1.33	1.33						1.33				
R3	4	a	0.64	0.64	0.64	0.64	0.64	0.65	0.65	0.65	0.65	0.65	0.66	0.66	0.66	0.66	0.66
		b	0.95	1.15	1.25	1.02	1.10	0.87	1.04	1.14	0.86	0.90	0.83	0.94	1.06	0.82	0.88
		c	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09
	5	a	0.80	0.80	0.80	0.80	0.80	0.81	0.81	0.81	0.81	0.81	0.82	0.82	0.82	0.82	0.82
		b	1.04	1.23	1.36	1.11	1.22	0.95	1.09	1.21	0.96	1.07	0.91	1.02	1.11	0.89	1.00
		c	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09
	6	a	0.96	0.96	0.96	0.96	0.96	0.97	0.97	0.97	0.97	0.97	0.98	0.98	0.98	0.98	0.98
		b	1.05	1.23	1.37	1.14	1.22	1.06	1.10	1.21	0.98	1.08	1.01	1.02	1.12	0.99	1.01
		c	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09	1.09

(a) number of wheel loads in bridge with rigid cross section
(b) number of wheel loads in actual bridge, from analysis
(c) number of wheel loads, determined by use of present AASHO formulas

TABLE 18. NUMBER OF WHEEL LOADS FOR 4-CELL BRIDGES WITH SLAB OVERHANGS

Span (ft.)			60					80	120
Section			Midspan			Support		Midspan	Midspan
Boundary Cond.			SS	FS	FF	FS	FF	SS	SS
Girder	No. of Lanes								
C	2	a	0.90	0.90	0.90	0.90	0.90	0.90	0.90
		b	0.97	1.07	1.09	0.95	0.96	0.97	0.96
		c	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	a	1.21	1.21	1.21	1.21	1.21	1.20	1.20
		b	1.13	1.11	1.11	1.13	1.15	1.18	1.18
		c	1.00	1.00	1.00	1.00	1.00	1.00	1.00
R1	2	a	0.90	0.90	0.90	0.90	0.90	0.90	0.90
		b	0.95	0.99	1.02	0.96	1.00	0.95	0.95
		c	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	3	a	1.21	1.21	1.21	1.21	1.21	1.20	1.20
		b	1.16	1.19	1.20	1.20	1.21	1.22	1.22
		c	1.00	1.00	1.00	1.00	1.00	1.00	1.00
R2	2	a	0.66	0.66	0.66	0.66	0.66	0.66	0.67
		b	0.88	0.96	1.02	0.88	0.91	0.82	0.78
		c	0.93	0.93	0.93	0.93	0.93	0.93	0.93
	3	a	0.88	0.88	0.88	0.88	0.88	0.89	0.91
		b	1.02	1.05	1.12	1.03	1.03	0.97	0.96
		c	0.93	0.93	0.93	0.93	0.93	0.93	0.93

(a) number of wheel loads in bridge with rigid cross section
(b) number of wheel loads in actual bridge, from analysis
(c) number of wheel loads, determined by use of present AASHO formulas

TABLE 19. NUMBER OF WHEEL LOADS FOR 8-CELL BRIDGES WITH SLAB OVERHANGS

Span (ft.)			60					80	120
Section			Midspan			Support		Midspan	Midspan
Boundary Cond.			SS	FS	FF	FS	FF	SS	SS
Girder	No. of Lanes								
C	4	a	0.71	0.71	0.71	0.71	0.71	0.71	0.70
		b	0.82	0.89	1.06	0.73	0.88	0.80	0.78
		c	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	5	a	0.89	0.89	0.89	0.89	0.89	0.88	0.88
		b	0.91	0.93	1.03	0.93	0.93	0.92	0.91
		c	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	6	a	1.06	1.06	1.06	1.06	1.06	1.06	1.06
		b	1.04	1.04	1.11	1.05	1.08	1.06	1.06
		c	1.00	1.00	1.00	1.00	1.00	1.00	1.00
R1	4	a	0.71						0.70
		b	0.85						0.77
		c	1.00						1.00
	5	a	0.89						0.88
		b	0.94						0.92
		c	1.00						1.00
	6	a	1.06						1.06
		b	1.05						1.06
		c	1.00						1.00
R2	4	a	0.71						0.70
		b	0.91						0.81
		c	1.00						1.00
	5	a	0.89						0.88
		b	0.93						0.91
		c	1.00						1.00
	6	a	1.06						1.06
		b	1.04						1.05
		c	1.00						1.00

TABLE 19 (cont'd). NUMBER OF WHEEL LOADS FOR 8-CELL BRIDGES WITH SLAB OVERHANGS

Span			60					80	120
Section			Midspan			Support		Midspan	Midspan
Boundary Cond.			SS	FS	FF	FS	FF	SS	SS
Girder	No. of Lanes								
R3	4	a	0.71						0.70
		b	0.90						0.82
		c	1.00						1.00
	5	a	0.89						0.88
		b	0.98						0.92
		c	1.00						1.00
	6	a	1.06						1.06
		b	1.04						1.07
		c	1.00						1.00
R4	4	a	0.52	0.52	0.52	0.52	0.52	0.53	0.53
		b	0.77	0.85	0.96	0.77	0.88	0.73	0.64
		c	0.93	0.93	0.93	0.93	0.93	0.93	0.93
	5	a	0.65	0.65	0.65	0.65	0.65	0.66	0.67
		b	0.83	0.92	1.03	0.85	0.96	0.76	0.71
		c	0.93	0.93	0.93	0.93	0.93	0.93	0.93
	6	a	0.78	0.78	0.78	0.78	0.78	0.79	0.80
		b	0.84	0.91	1.03	0.84	0.95	0.80	0.79
		c	0.93	0.93	0.93	0.93	0.93	0.93	0.93

(a) number of wheel loads in bridge with rigid cross section
(b) number of wheel loads in actual bridge, from analysis
(c) number of wheel loads, determined by use of present AASHO formulas

3.6 Longitudinal Variation of Load Distribution Characteristics

3.6.1 Introduction

Before interpreting the data presented in Tables 12-15 for design purposes, it appears to be necessary to study how load distribution characteristics of concrete box girder bridges vary along the span. The first study will deal with the longitudinal variation of load distribution under concentrated loads acting at some fixed position, say at midspan. The next step will be to discuss influence surfaces for box girder bridges which subsequently will be the basis for an exact design approach. Finally, the effect of lumping real truck loadings into midspan loadings as shown in Fig. 12 will be investigated.

3.6.2 Longitudinal Variation of Load Distribution Under Midspan Loads

Consider a box girder bridge with arbitrary end support conditions, subjected to a concentrated load acting at midspan. The transverse load distribution characteristics at midspan where the load is acting have already been discussed extensively and have been presented graphically for example in the influence lines (Figs. 19-30).

But now it is desired to investigate how this distribution varies longitudinally, while the load remains fixed at its midspan position. Figures 31, 32 and 33 introduce as an example the 3-cell bridge with a span of 120 ft. and with the concentrated load acting at midspan over the exterior girder, having the end boundary conditions simple-simple, fixed-simple, and fixed-fixed, respectively. These figures show the individual moment diagrams carried by each of the girders of the bridge considered, for the given load case and boundary conditions.

The following observations can be made from these figures:

(a) At any section of the bridge, the sum of all girder moments

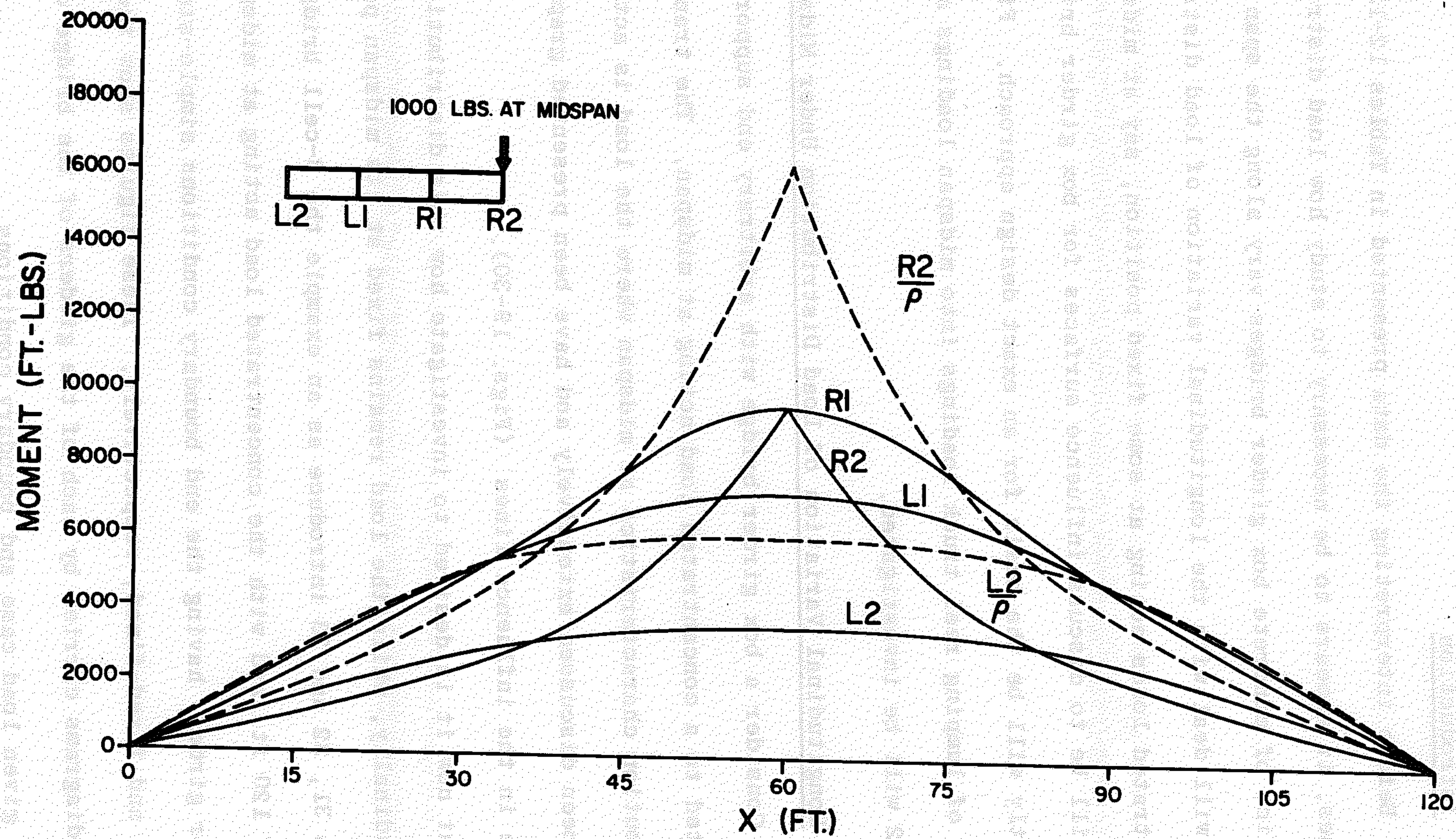


FIG. 31 MOMENT DIAGRAMS FOR THE GIRDERS OF A 3-CELL BRIDGE WITH SPAN OF 120 FT., SIMPLY SUPPORTED

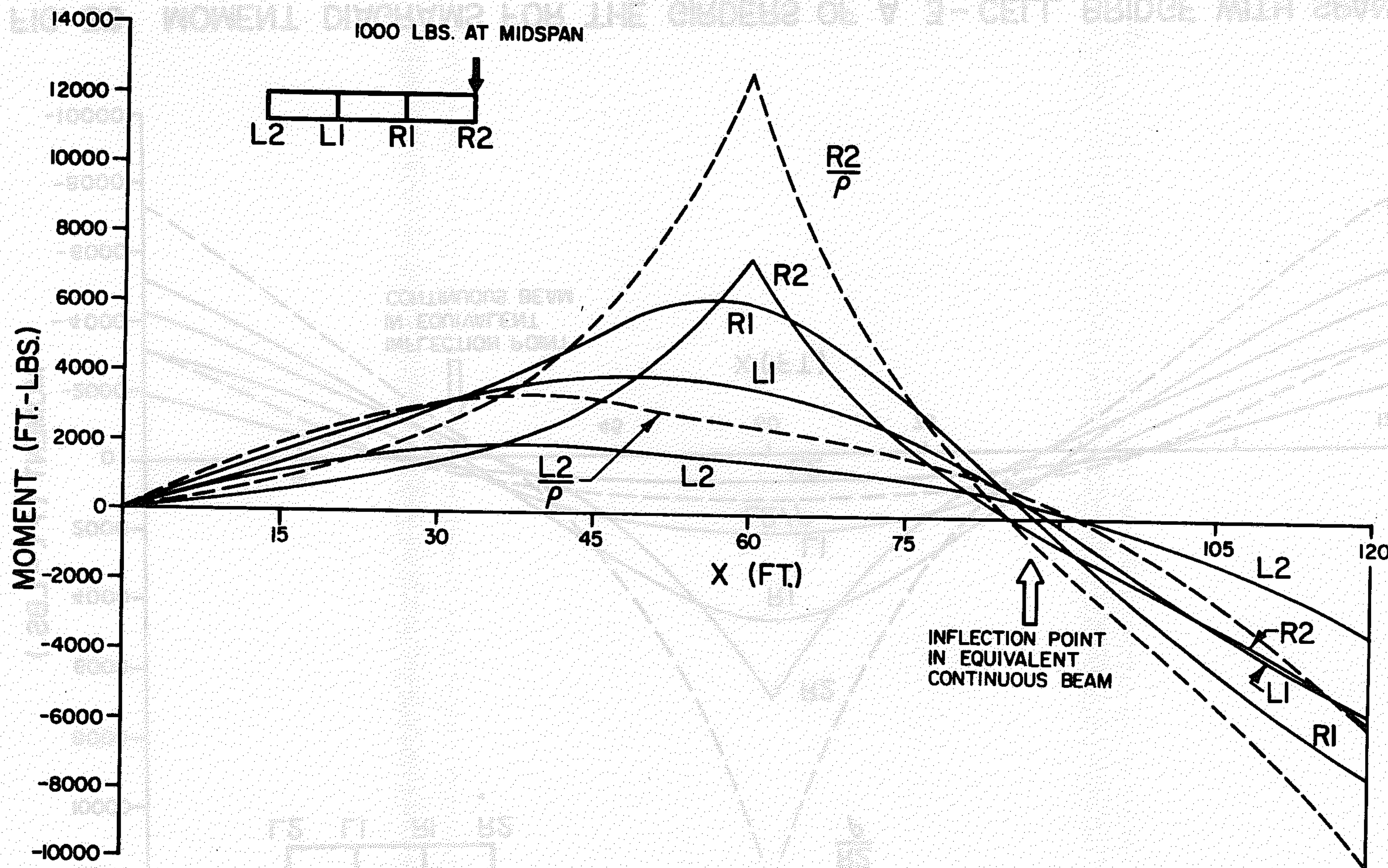


FIG. 32 MOMENT DIAGRAMS FOR THE GIRDERS OF A 3-CELL BRIDGE WITH SPAN OF 120 FT., ONE END SUPPORT FIXED

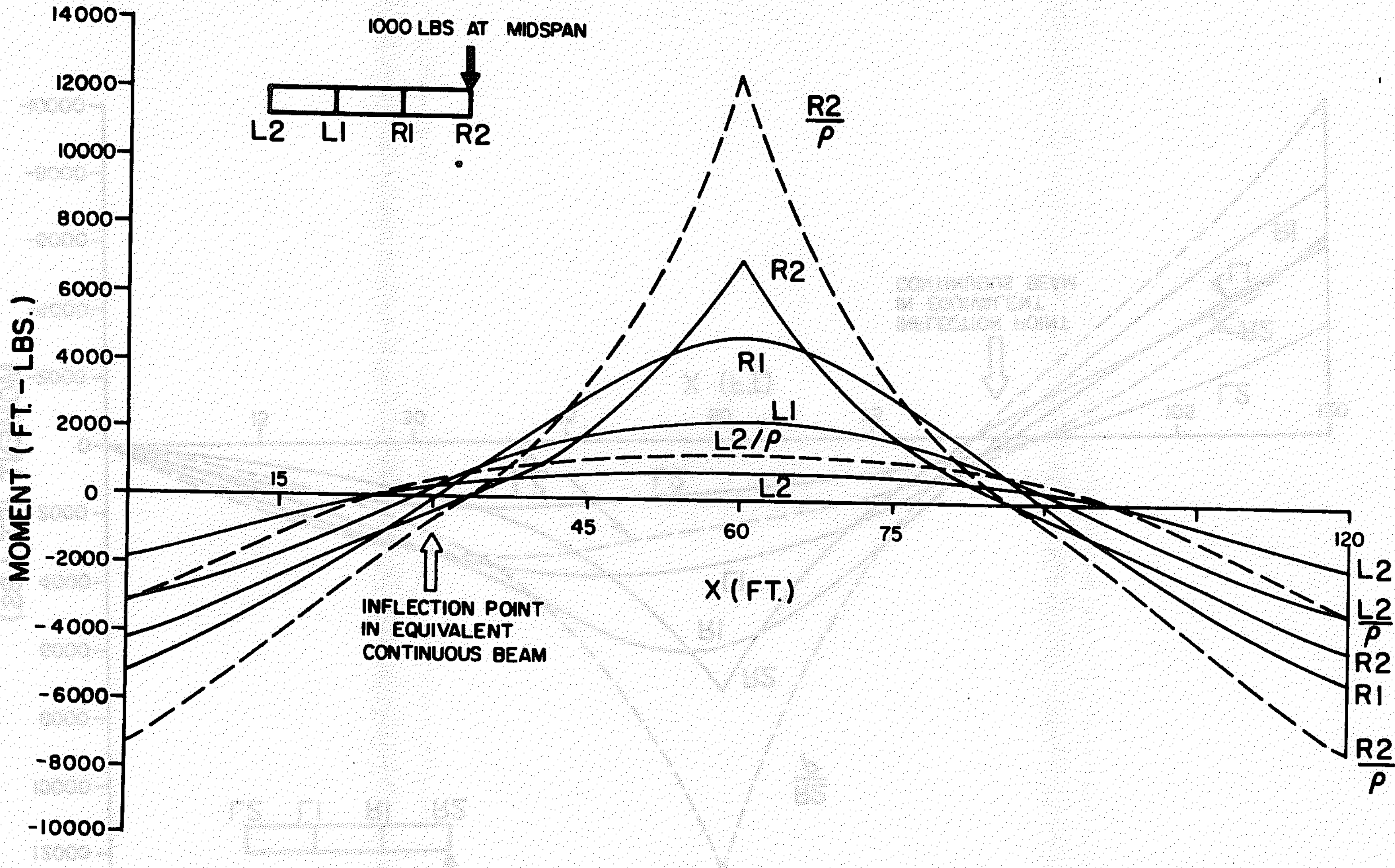


FIG. 33 MOMENT DIAGRAMS FOR THE GIRDERS OF A 3-CELL BRIDGE WITH SPAN OF 120 FT., BOTH END SUPPORTS FIXED

equals the theoretical moment of the equivalent continuous beam at that section within a few percent, which is a check on the methods of analysis used.

(b) In order to make the moments of interior and exterior girders comparable in terms of longitudinal stresses, the latter have been divided by the proper ρ-factor (in this case, $\rho = 0.563$, Table 5). This transformation leads to the dashed lines shown in Figs. 31, 32, 33. The girder on which the load is acting is subjected to the maximum longitudinal stresses and therefore in general to the biggest moment at the section of load application. However, it can be seen that this peak moment decreases very rapidly as the concentrated load is distributed to the rest of the bridge.

(c) There exists a section with almost uniform stress distribution across the whole bridge (for example in Fig. 31 at about $x = 32$ feet). This means that at this section all girders carry approximately those moments which they would carry in a rigid bridge.

(d) Beyond the section of uniform stress and away from the loaded section, a moment redistribution occurs such that girders take slightly more stress (and therefore moment) the further they are away from the loaded girder.

(e) The moment curve in the equivalent continuous beam has in each case a finite slope discontinuity under the load. It can be seen very clearly how this discontinuity in a box girder bridge is entirely picked up by the loaded girder, while the moment curves of all other girders are smooth at the section of load application. This behavior illustrates the fact that the loaded girder forces the rest of the bridge into certain deflections, and since the moments are second derivatives of the deflections, all unloaded girder moment curves are smooth under the load. Similar phenomena in slab-girder bridges have been studied by Newmark [10]. In Fig. 34, girders R2 and R1 of the simply supported bridge of Fig. 31 have been isolated as free bodies. The loading on girder R2 consists of the downward applied concentrated load and an upward distributed reactive force whose magnitude and longitudinal distribution is dependent on the longitudinal and transverse stiffness of the remainder of the bridge to which R2 is connected. The resulting moment diagram for R2 has a peak directly under the load which damps out at a rate which is dependent on the magnitude of the upward reactive force. The downward and upward loading on girder R1 are both distributed loads over the span, however, the downward load again has a greater concentration near midspan than the upward reactive force provided by the stiffness of the remainder of the bridge. This type of load transfer or longitudinal spreading results in less and less concentration of the distributed load near midspan in each succeeding girder and explains why the moment curves get flatter and flatter in Fig. 31 as one proceeds successively from girder R2 to R1 to L1 to L2.

(f) Figs. 32 and 33 show very clearly that each girder has its own point of inflection. The spread of all these points in the longitudinal direction amounts in Fig. 33 to more than 10 feet. In this light it may appear questionable to define an inflection

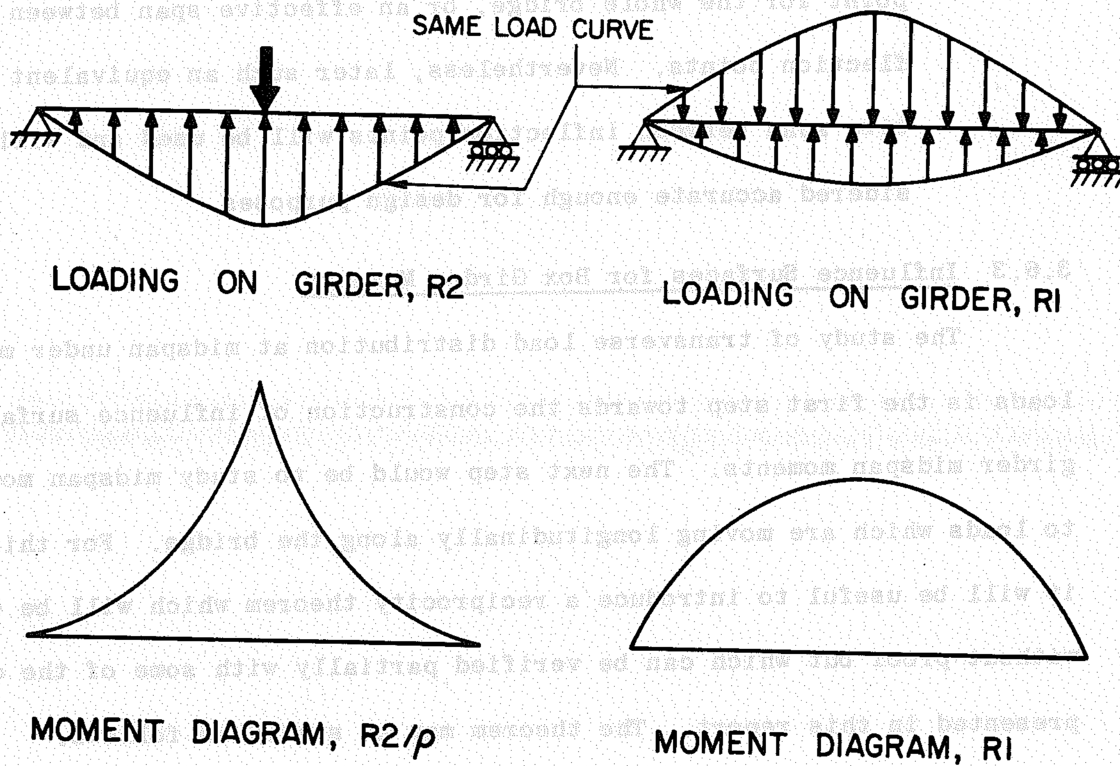


FIG. 34 TYPICAL LOAD TRANSFER FOR 3-CELL BRIDGE

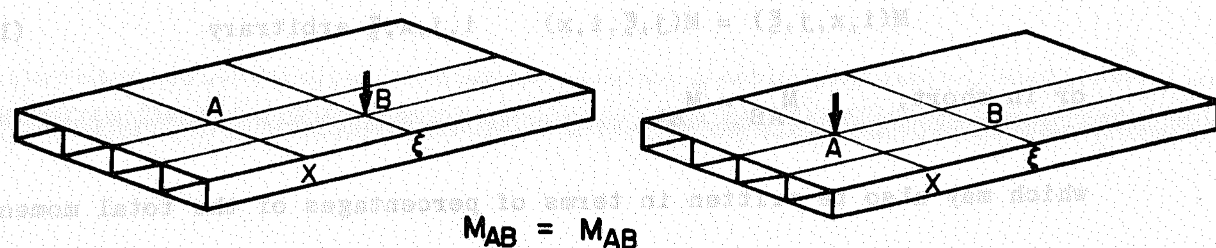


FIG. 35 RECIPROCITY THEOREM FOR GIRDER MOMENTS

point for the whole bridge, or an effective span between inflection points. Nevertheless, later such an equivalent effective span between inflection points will be used and will be considered accurate enough for design purposes.

3.6.3 Influence Surfaces for Box Girder Moments

The study of transverse load distribution at midspan under midspan loads is the first step towards the construction of influence surfaces for girder midspan moments. The next step would be to study midspan moments due to loads which are moving longitudinally along the bridge. For this purpose it will be useful to introduce a reciprocity theorem which will be stated without proof but which can be verified partially with some of the data presented in this report. The theorem may be stated as follows.

Theorem. For a given box girder bridge (Fig. 35), the moment carried by girder i at section x (say point A) due to a unit load on girder j at section ξ (say point B) is equal to the moment carried by girder j at section ξ due to a unit load on girder i at section x.

$$M(i,x,j,\xi) = M(j,\xi,i,x) \qquad i,j,x,\xi \text{ arbitrary} \tag{14}$$

or in short, $M_{AB} = M_{BA}$

which may also be written in terms of percentages of the total moment at a section,

$$M^{\%}_{AB} = M^{\%}_{BA} \tag{15}$$

Both Eqs. (14) and (15) are valid for simply supported bridges for any points A and B. If M or $M^{\%}$ is measured in an exterior girder, it has to be divided by the proper ρ-factor. Thus, if A is a point on an exterior girder and B on an interior girder, Eq. (15) becomes

$$\frac{M^{\%}_{AB}}{\rho} = M^{\%}_{BA} \tag{16}$$

For bridges with one or both end supports fixed against rotation, Eq. (14) is no longer valid, however Eq. (15) still holds, except for roundoff errors which average about 2%.

There are two corollaries of the above theorem. While Eqs. (14) and (15) reflect symmetry of the bridge system in both directions, x and y, one corollary specializes in the symmetry in y only (transversely) while the other one deals with symmetry in x only (longitudinally).

Corollary 1. Considering moments and loads only at a given transverse section x, the moment carried by girder i due to a unit load on girder j equals the moment carried by girder j due to a unit load on girder i.

$$M(i,x,j,x) = M(j,x,i,x) \qquad i,j,x \text{ arbitrary} \tag{17}$$

or in short, $M_{ij} = M_{ji}$

or in terms of moment percentages,

$$M^{\%}_{ij} = M^{\%}_{ji} \tag{18}$$

Both Eqs. (17) and (18) are true for box girder bridges with any end boundary conditions. If exterior girders are involved, their moments or moment percentages have to be again modified as in Eq. (16). Taking x at the midspan section, Eq. (18) can directly be verified using the moment influence lines (Figs. 19-30). A relationship similar to Eq. (17), applied to slab-girder bridges, has been stated by Newmark [10].

Corollary 2. Considering moments carried only by girder i and loads on girder j, the moment carried by girder i at section x due to a unit load on girder j at section ξ equals the moment carried by girder i at section ξ due to a unit load on girder j at section x.

$$M(i,x,j,\xi) = M(i,\xi,j,x) \qquad i,j,x,\xi \text{ arbitrary} \tag{19}$$

or in short,

$$M_{x\xi} = M_{\xi x}$$

or in terms of moment percentages,

$$M^{\%}_{x\xi} = M^{\%}_{\xi x} \tag{20}$$

While Eq. (20) is valid for any box girder bridge, Eq. (19) is true only if the bridge is simply supported.

With the help of the above reciprocity relations it is very easy to construct influence surfaces for girder moments. Specifically, these influence surfaces are constructed as follows:

(a) Simply supported box girder bridges.

It is desired to obtain the influence surface for the moment of girder i at x. Place a unit concentrated load on girder i at x and analyze the bridge. Plot for each girder the moment diagram carried by that girder. If i is an interior girder, the exterior girder moments first have to be divided by the proper ρ-factor. If i is an exterior girder, all interior girder moments first have to be multiplied with the proper ρ-factor as given by Eq. (4). By virtue of Eq. (14), the result is the influence surface for the moment of girder i at x.

(b) Box girder bridges with one or both ends fixed.

It is desired to obtain the influence surface for the moment of girder i at x. Proceed as in (a), but using moment percentages rather than absolute moments. Multiply each ordinate thus obtained by the total statical moment which the whole bridge carries at section x and which can be found from an equivalent continuous beam analysis. The result is the influence surface for the moment of girder i at x.

Proof: It is desired to obtain M_{AB}, i.e., the moment at point A due to a unit load at B, where A is given and B is arbitrary. From Eq. (15), $M^{\%}_{AB} = M^{\%}_{BA}$. But by definition, $M^{\%}_{AB} = M_{AB}/M_{A}$, where M_A is the moment taken by all girders at section A. From this it follows that $M_{AB} = M^{\%}_{AB} \cdot M_A = M^{\%}_{BA} \cdot M_A$.

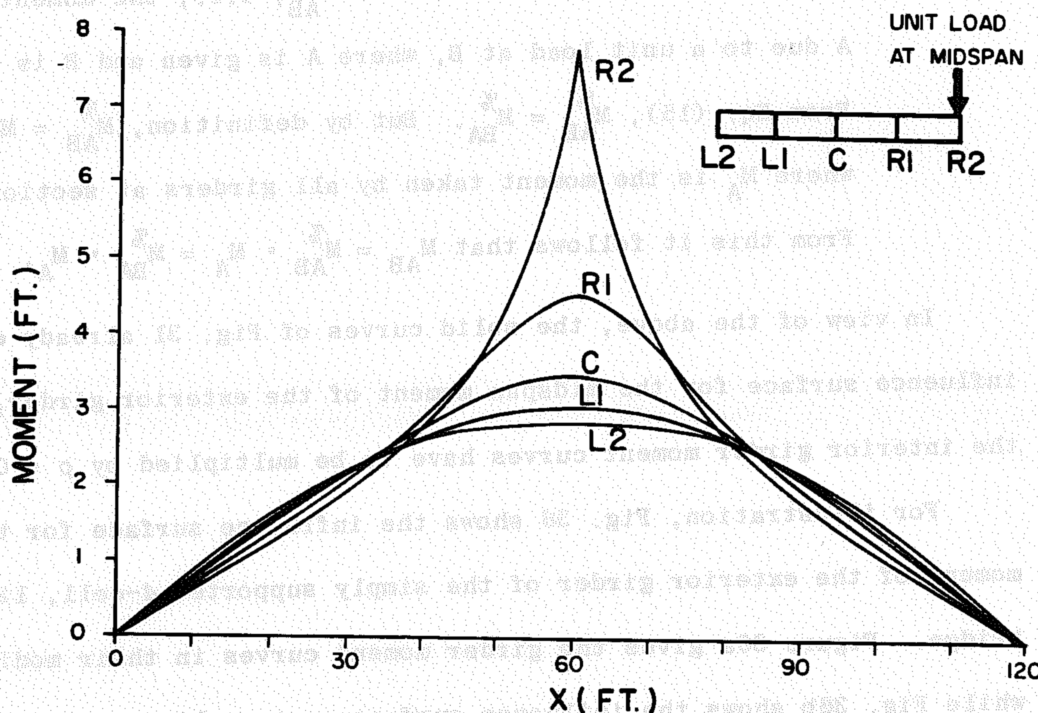
In view of the above, the solid curves of Fig. 31 already describe the influence surface for the midspan moment of the exterior girder, except that the interior girder moment curves have to be multiplied by $\rho \doteq 0.563$.

For illustration, Fig. 36 shows the influence surface for the midspan moment of the exterior girder of the simply supported 4-cell, 120 ft. span bridge. Figure 36a gives the girder moment curves in their modified form, while Fig. 36b shows the influence surface contour lines. Similarly, Fig. 37 gives the influence surface for the quarterspan moment of the exterior girder in the same bridge. The accuracy of the influence surfaces for this simply supported bridge, which have been constructed as outlined above has been checked for numerous points by independent separate analysis and the differences never exceeded 1%.

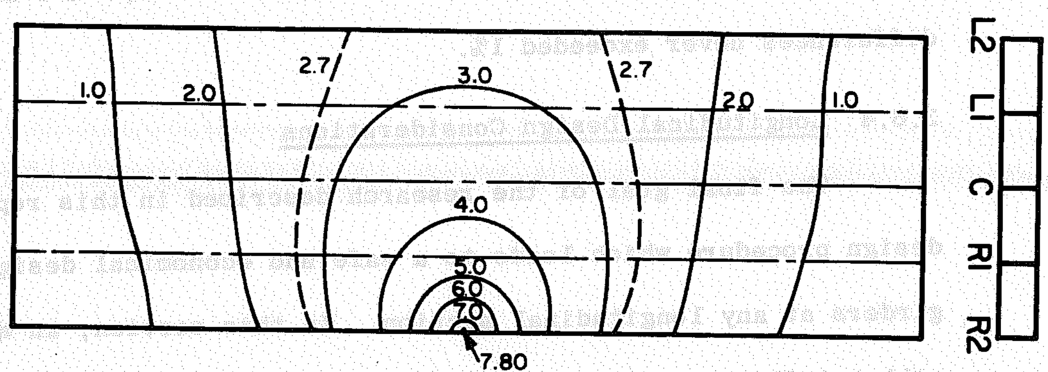
3.6.4 Longitudinal Design Considerations

The final goal of the research described in this report is a general design procedure which leads to a safe and economical design of all bridge girders at any longitudinal section. In this section, an exact approach will briefly be outlined, followed by an approximate approach whose assumptions will be discussed.

In a hypothetical one-girder bridge, the problem of finding the maximum moment envelope is a one-dimensional one, and the solution is straightforward by moving a truck into the respective most critical position.

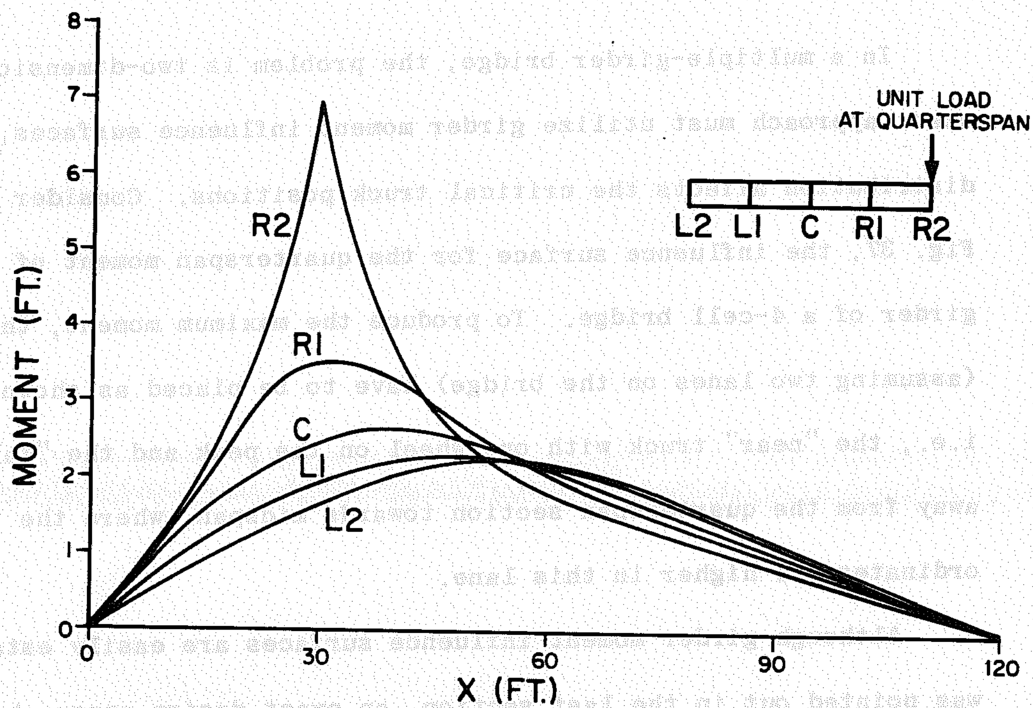


a) MODIFIED GIRDER MOMENT DIAGRAMS DUE TO A UNIT LOAD OVER GIRDER R2 AT X = 60.0 FT.

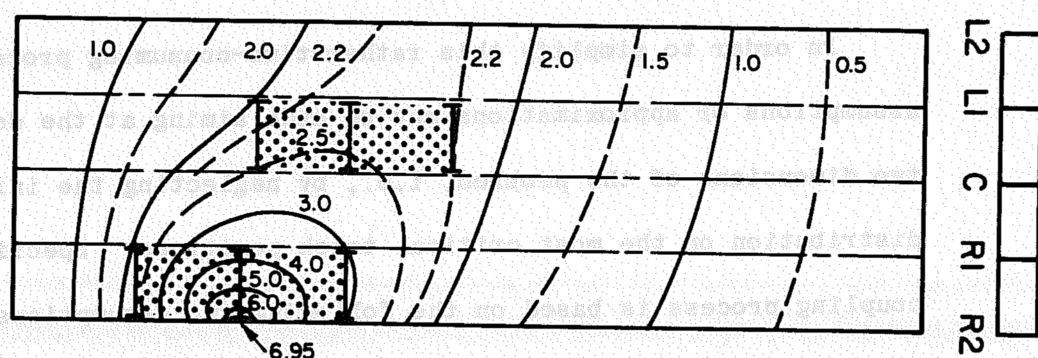


b) CONTOUR LINES

FIG. 36 INFLUENCE SURFACE FOR MIDSPAN MOMENT OF EXTERIOR GIRDER OF 4 - CELL BRIDGE, SIMPLY SUPPORTED OVER 120 FT. SPAN



a) MODIFIED GIRDER MOMENT DIAGRAMS DUE TO A UNIT LOAD OVER GIRDER R2 AT X = 30.0 FT.



b) CONTOUR LINES

FIG. 37 INFLUENCE SURFACE FOR QUARTERSPAN MOMENT EXTERIOR GIRDER OF 4-CELL BRIDGE, SIMPLY SUPPORTED OVER 120 FT. SPAN.

In a multiple-girder bridge, the problem is two-dimensional and the exact approach must utilize girder moment influence surfaces, because load distribution affects the critical truck positions. Consider for example Fig. 37, the influence surface for the quarterspan moment of the exterior girder of a 4-cell bridge. To produce the maximum moment, the two trucks (assuming two lanes on the bridge) have to be placed as shown in Fig. 37, i.e., the "near" truck with one wheel on the peak and the "far" truck moved away from the quarterspan section towards midspan, where the influence ordinates are higher in this lane.

Although girder moment influence surfaces are easily established as was pointed out in the last section, an exact design approach requires a great number of influence surfaces (for each girder for several transverse sections), and in each case the trucks have to be moved by trial and error into the most critical positions.

In order to simplify this rather time-consuming process, certain assumptions or approximations may be made aiming at the decoupling of the two dimensions of the problem, i.e., by neglecting the influence of load distribution on the most critical truck positions. Specifically, this decoupling process is based on the following two assumptions:

(a) If loads are acting at a section other than midspan, then the distribution of moments among the girders at this section is not much different from the distribution of moments at midspan under midspan loads.

(b) If real trucks are placed such as to produce maximum midspan moments in certain girders, then the distribution of moments among the girders at midspan is not much different from the distribution due to equivalent truck loadings with all wheel lines lumped at midspan according to Fig. 12.

For a check of assumption (a), the moment distribution at midspan under midspan loads has been compared with that at quarterspan under quarterspan loads, and the results are shown in Fig. 38. As can be seen in this figure, agreement is fairly good and could be expected to be much better if the loading becomes more uniform.

Assumption (b) is much more severe because it neglects the distribution which wheel loads not acting at midspan undergo before their influence is felt at midspan. Although the absolute influence of these loads which are not acting at midspan is smaller than that of midspan loads, higher α-factors are assumed for the former ones, resulting in an over-conservative design as illustrated qualitatively in Fig. 39.

In order to investigate the errors introduced by assumption (b), several bridges have been analyzed with real standard trucks on the bridge, and the resulting maximum numbers of wheel loads are compared in Table 20 with those resulting from the idealized truck loadings.

TABLE 20. ERRORS DUE TO TRUCK IDEALIZATION

Example Bridge	3 Cells with 2 Lanes				8 Cells with 5 Lanes	
Span and Boundary Conditions	60 ft. Simple-Simple		120 ft. Fixed-Fixed		60 ft. Simple-Simple	
Girder	R1 (Inter.)	R2 (Exter.)	R1 (Inter.)	R2 (Exter.)	R3 (Inter.)	R4 (Exter.)
N_{WL} From Real Trucks	1.32	0.93	1.33	1.01	0.97	0.744
N_{WL} From Idealized Trucks	1.35	0.99	1.35	1.12	0.98	0.826
Percentage Error	+2.3%	+6.4%	+1.5%	+10.9%	+1.0%	+11.0%

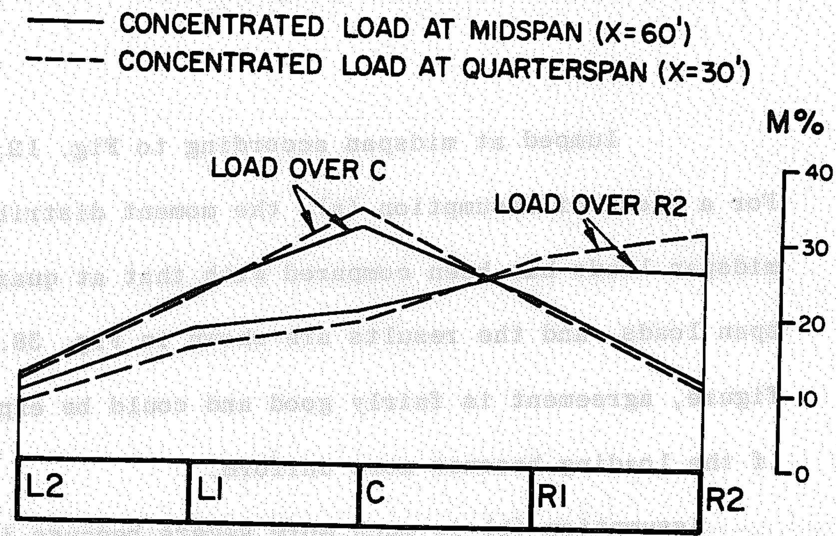


FIG 38 COMPARISON BETWEEN MOMENT PERCENTAGES AT MIDSPAN UNDER MIDSPAN LOADS AND AT QUARTER-SPAN UNDER QUARTERSPAN LOADS

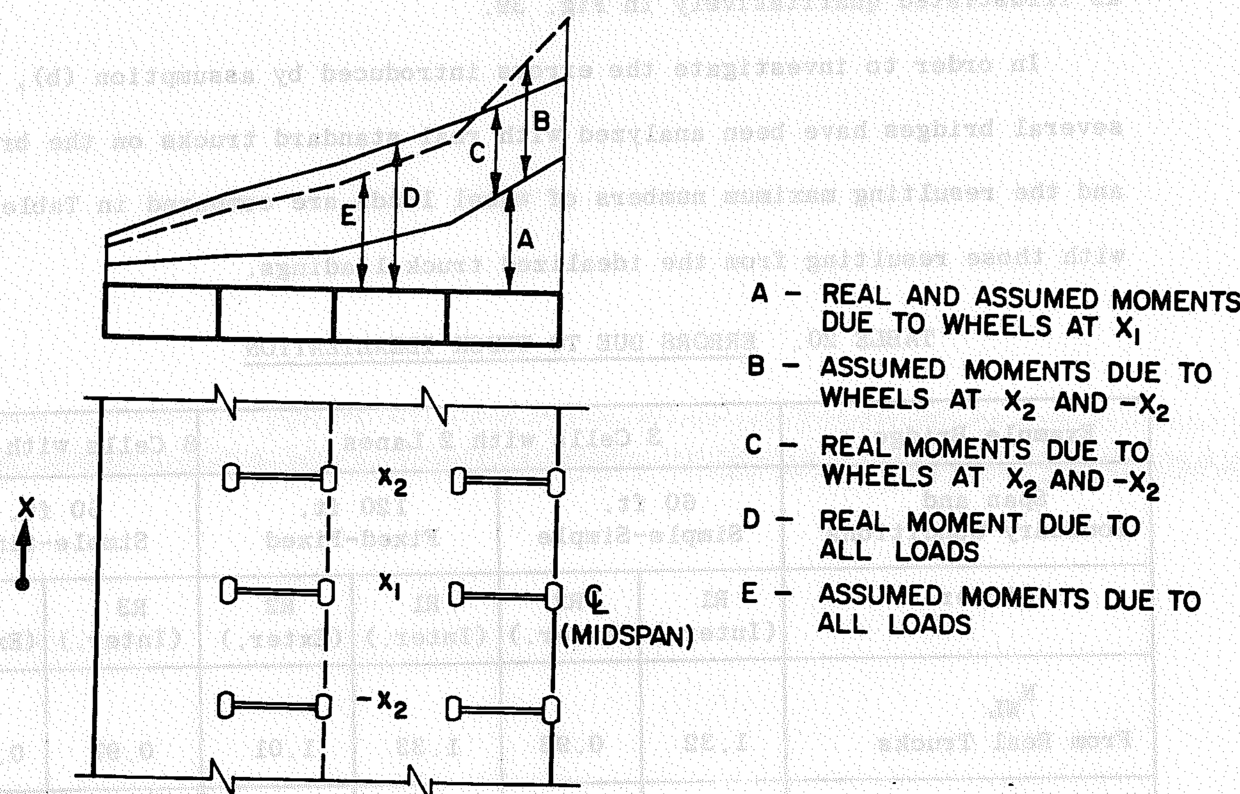


FIG. 39 OVERESTIMATION OF MOMENTS DUE TO WHEEL LOADS NOT ACTING AT MIDSPAN

As can be seen from these results, the errors for interior girders are in all cases negligibly small. However, for exterior girders, especially for narrower cell widths, the overestimation may be considerable, although on the conservative side.

At least half of the latter error is due to the extrapolation method with which slab overhangs were taken into account, as described in Section 3.5. For bridges with narrower cell widths it was found that this extrapolation method does not yield highly accurate results under eccentric loading, and tended to overestimate the actual numbers of wheel loads taken by the exterior girder.

The rest of the error is due to the localized effect of the peak stresses as illustrated in Fig. 39. The moment peak due to an eccentric load damps out very fast (Figs. 31, 32, 33) and when its influence arrives at a section 14 feet away (as is the assumed distance between wheel axles of standard trucks), the peak moment is almost completely distributed onto other bridge girders, independently of the span and width of the bridge. Bridges having cells with narrower widths are faster in this load distributing process than those with wider cells, due to their higher transverse stiffness.

Concluding, it may be said that the truck loading idealization upon which the main work of this report is based, is permissible within engineering accuracy for a simplified design method, especially if interior girders are concerned. For exterior girders, the truck loading idealization leads to a consistent overestimation which in a few cases may go as high as 11%, so if desired, reduction factors might be applied to the α-factors when applied to loads which are more than a certain distance away from the section at which a critical moment is being determined.

4. DISCUSSION OF RESULTS

4.1 Introduction

The objective of this chapter is to furnish the necessary information for deriving a safe and economical design method for box girder bridges which is simple in its use.

The actual bridge behavior is a very complex problem, subject to the influence of a large number of parameters. In this chapter, this actual bridge behavior will be studied by investigating the influence of the design variables considered most important, each separately as far as possible, in order to prepare the basis for a design procedure to be presented in Chapter 6, which will take account of most of these design parameters. In order to deal with the more realistic bridges with slab overhangs, the following studies will all be based on the girder load concentration factors given in Tables 12-15. The influence of the slab overhang on these results will be studied in Section 4.7.

4.2 Exterior Versus Interior Girders

In returning to the load concentration factors for bridge girders, α, a look at the Tables 12-15 reveals a pronounced difference between interior and exterior girders, the α-factors for the latter girders being in all of the cases studied much higher than those for the former ones. However, their actual design loads, N_{WL}, summarized in Tables 16-19, are smaller than those for interior girders, as one would expect.

The reason for the higher α-factors for exterior girders lies in their much less uniform moment influence lines, because loads placed over interior girders can be distributed to two sides.

Figures 40-43, which will be discussed in detail in the next section, show the α-factors and N_{WL}-values for some simply supported bridges as a function of the number of traffic lanes per girder. In these figures, it can be seen how the exterior girders (marked with asterisks) are clearly distinct from the interior girders.

Thus, one might propose design formulas separately for interior and exterior girders as a continuation of present AASHO design practice. But one could also attempt to make a design formula generally applicable to exterior as well as interior girders.

4.3 Number of Traffic Lanes per Girder

This important parameter will for the purpose of short reference be denoted by β, i.e.,

$$\beta = \frac{N_L}{\sum_{j=1}^{n} \rho_j} = \frac{N_L}{n-2+2\rho} \tag{21}$$

where

N_L = number of lanes

n = number of girders

$$\rho_j = \begin{cases} 1 & \text{for interior girders} \\ \rho & \text{for exterior girders} \end{cases}$$

β-values for the cases studied within this report are listed in Table 21.

TABLE 21. NUMBER OF LANES PER GIRDER, β, FOR EXAMPLE BRIDGES

Number of Cells	3		4		6			8		
Number of Lanes	2	3	2	3	4	5	6	4	5	6
β-Values	0.592	0.888	0.446	0.669	0.627	0.784	0.941	0.471	0.590	0.707

These β-values vary slightly with the span because of variations in the ρ-factors, but Table 21 lists the averages.

For some simply supported bridges, the functional relationship between α and β as well as between N_{WL} and β has been plotted in Figs. 40-43. In each figure, bridges of the same cell width are being compared, i.e., 3-cell with 6-cell bridges or 4-cell with 8-cell bridges. For further comparison, the values obtained from the present AASHO design formulas, Eqs. (1) and (2), have also been included.

From the Figures 40-43, the following observations can be made:

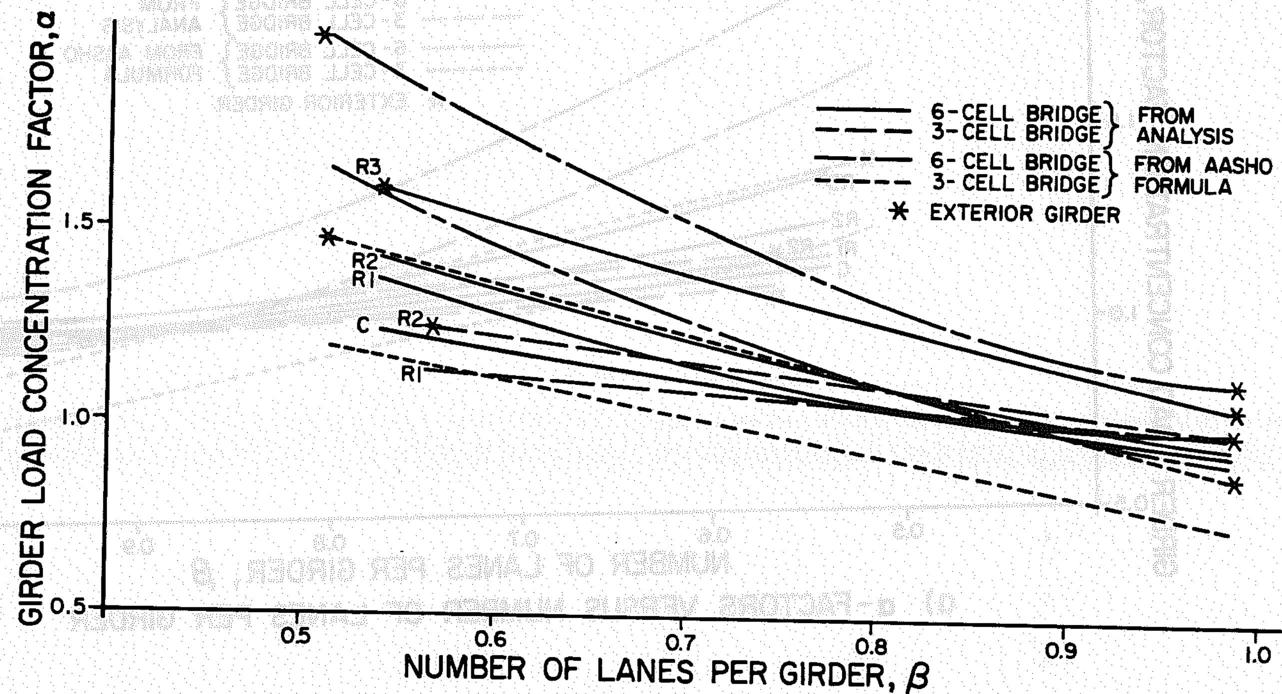
(a) The relative ordinates of the individual girder α-curves for a given bridge remain unchanged through the transformation from α to N_{WL}, because all interior girders of a given bridge have the same rigid bridge multiplier.

(b) In all figures, the importance of the β-parameter is shown very clearly. While the α-factors decrease rapidly with increasing β-values, due to more uniform loading on the bridge, the N_{WL}-values increase with β because more trucks are considered on the bridge.

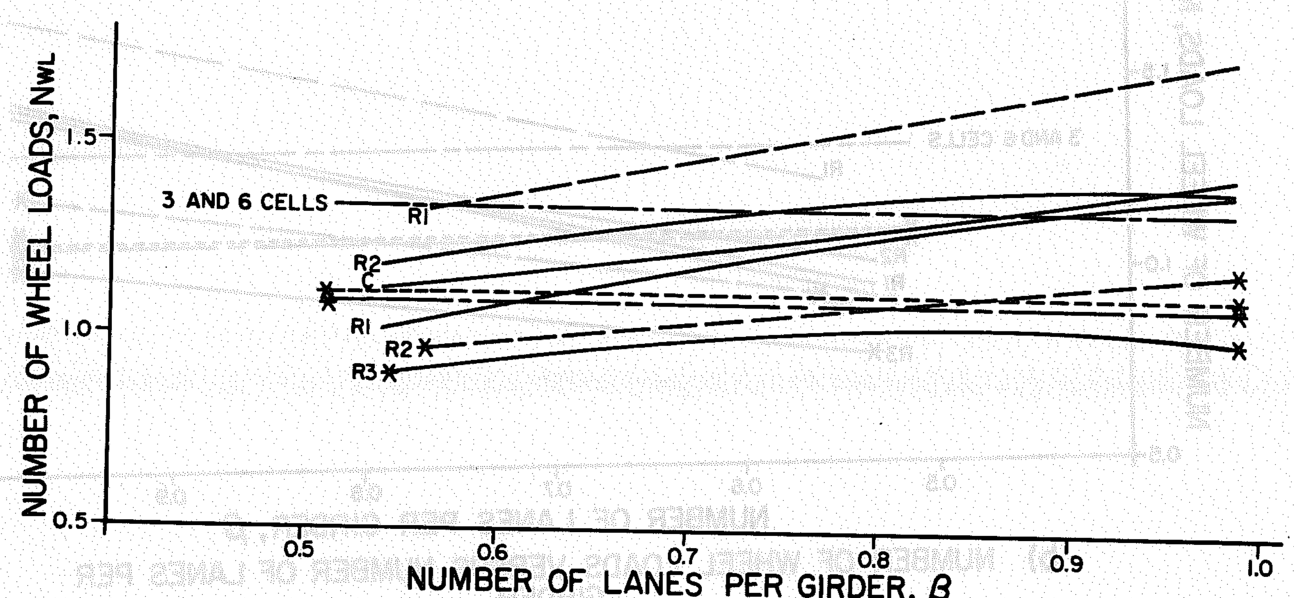
(c) There may exist a considerable range of α- or N_{WL}-values among the individual girders of a given bridge. This range tends to decrease markedly with an increase in β but also with an increase in span.

(d) This general behavior applies to interior girders as well as to exterior girders, irrespectively of the fact that for exterior girders the α-factors are always much higher and the N_{WL}-values much smaller than the corresponding values for interior girders.

(e) The present AASHO design formulas yield N_{WL}-values which are independent of β. For the lower β-range, i.e., wider lane widths or

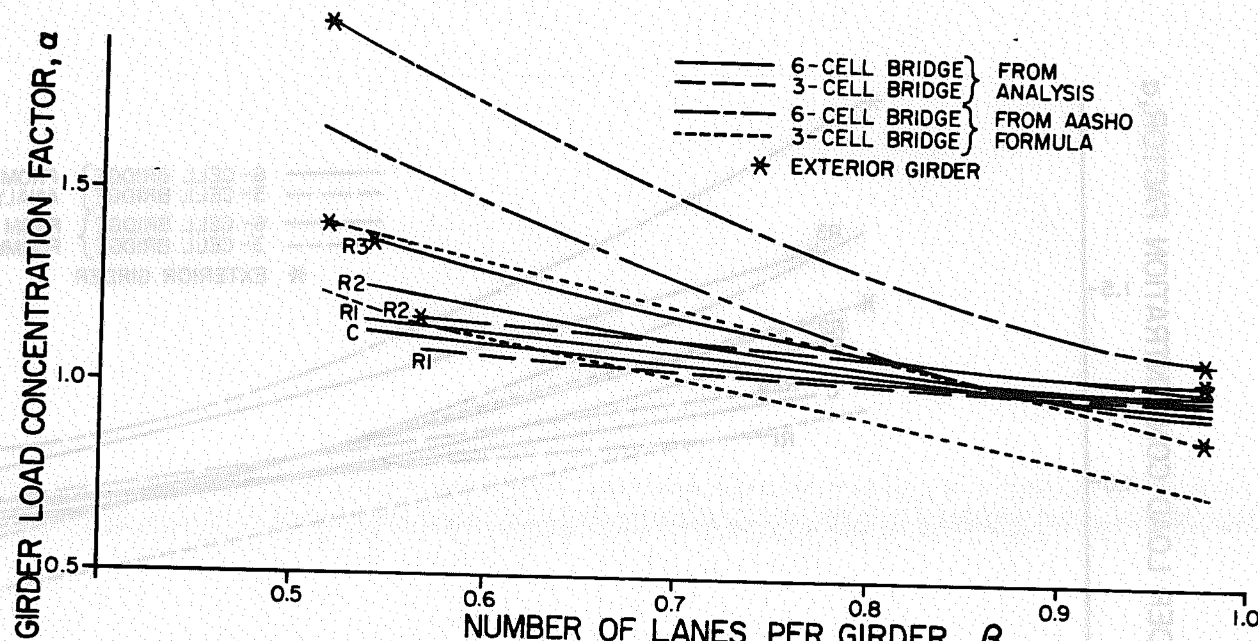


a) α - FACTORS VERSUS NUMBER OF LANES PER GIRDER

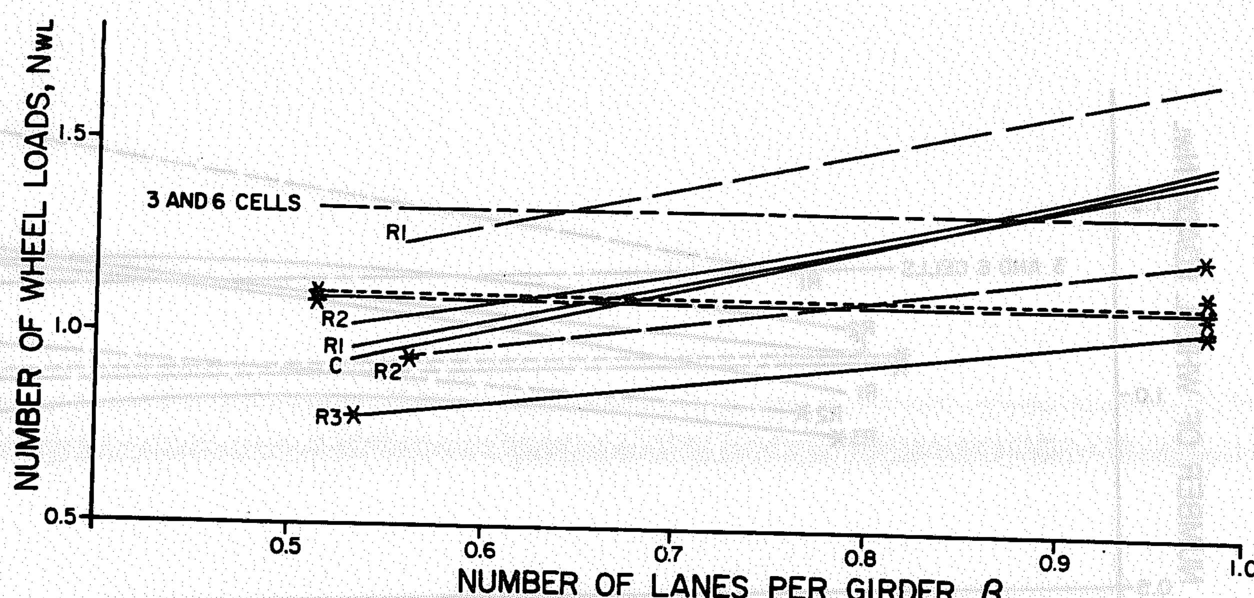


b) NUMBER OF WHEEL LOADS VERSUS NUMBER OF LANES PER GIRDER

FIG. 40 α - FACTORS AND NUMBER OF WHEEL LOADS FOR 3 AND 6-CELL BRIDGES, SIMPLY SUPPORTED OVER 60 FT.



a) α-FACTORS VERSUS NUMBER OF LANES PER GIRDER



b) NUMBER OF WHEEL LOADS VERSUS NUMBER OF LANES PER GIRDER

FIG. 41 α - FACTORS AND NUMBER OF WHEEL LOADS FOR 3 AND 6 - CELL BRIDGES, SIMPLY SUPPORTED OVER 120 FT.

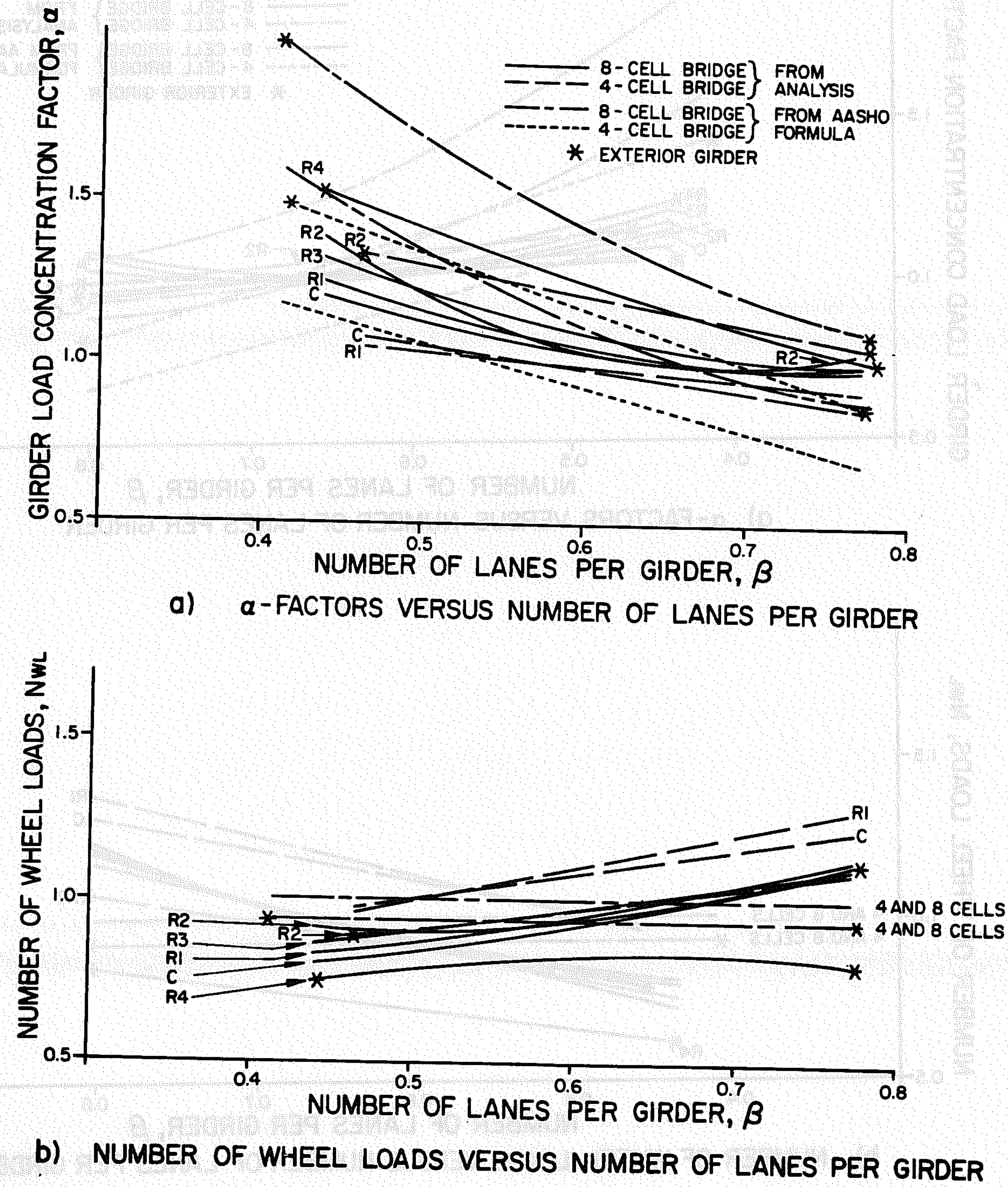
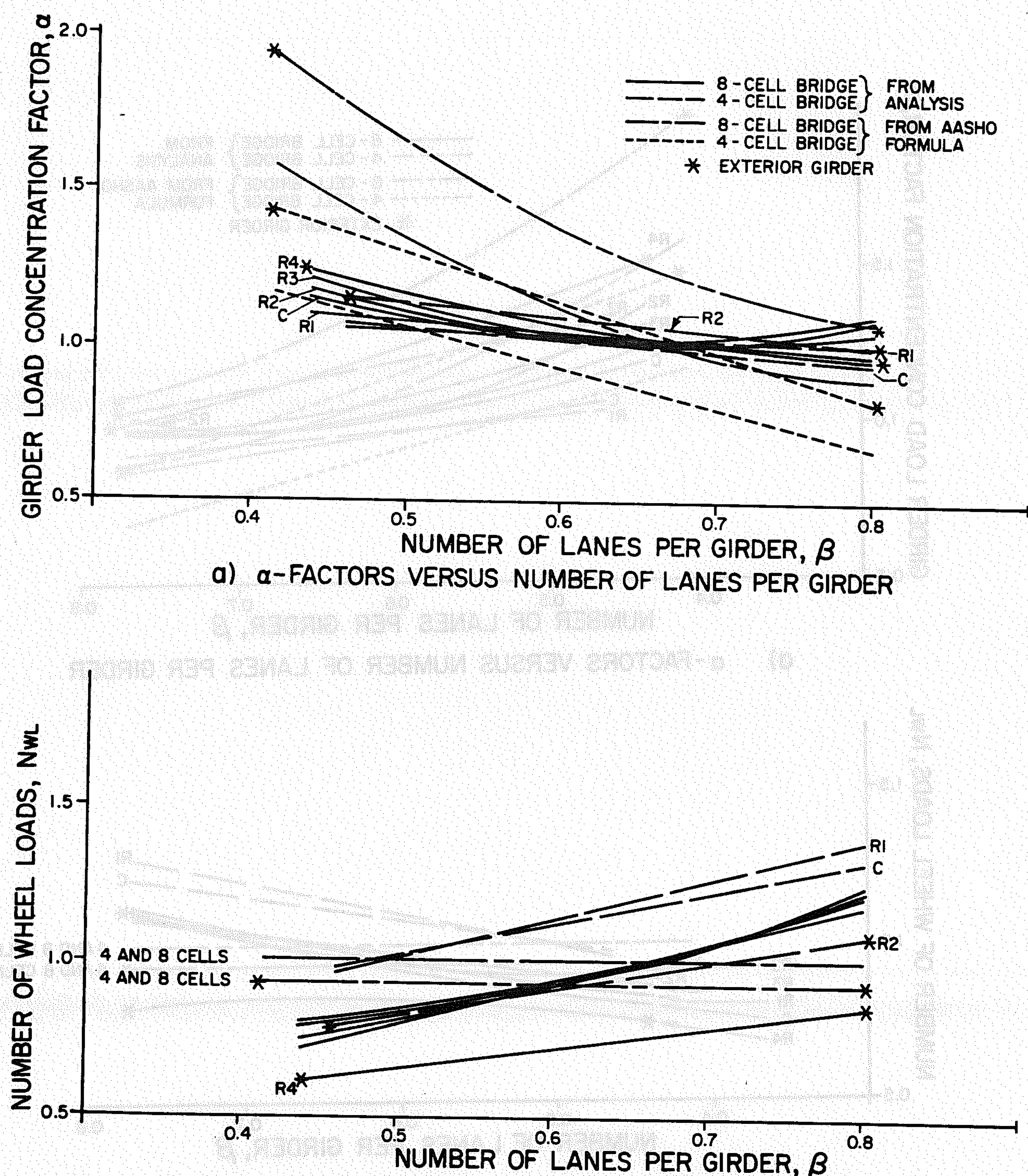


FIG. 42 α-FACTORS AND NUMBER OF WHEEL LOADS FOR 4 AND 8-CELL BRIDGES, SIMPLY SUPPORTED OVER 60 FT.



a) α-FACTORS VERSUS NUMBER OF LANES PER GIRDER

b) NUMBER OF WHEEL LOADS VERSUS NUMBER OF LANES PER GIRDER

FIG. 43 α - FACTORS AND NUMBER OF WHEEL LOADS FOR 4 AND 8 - CELL BRIDGES, SIMPLY SUPPORTED OVER 120 FT.

narrower cell widths, the AASHO formulas tend to overestimate the actual design values considerably. For example, for the 120 ft. 6-cell bridge, at $\beta = 0.627$ (4 lanes), Fig. 41b, the AASHO design formulas would overdesign all girders, including the exteriors, by about 30%. For larger β-values, i.e., narrower lane widths or larger cell widths, the AASHO formulas might become unsafe. For example, for the 120 ft. 3-cell bridge, at $\beta = 0.888$ (3 lanes), Fig. 41b, the AASHO design formulas would underdesign the interior girder by about 20% and the exterior girder by about 6%.

The above observations indicate very clearly the need for a design formula which takes the β-parameter into account. This does not necessarily mean that bridges which were designed according to the AASHO formulas are unsafe. In fact, the higher β-values for which the AASHO formulas are becoming unconservative, are based on 10 ft. wide traffic lanes which are not generally being built in California today, while the cell widths have a feasible range of variation only between about 6 and 9 ft. However, the above finding illustrates that present design practice is overconservative and that greater economy may be achieved by employing a refined method of design.

Summarizing, it may be said that with an increase of β, i.e., the number of traffic lanes per girder, the girder concentration factors decrease rapidly while the maximum numbers of wheel loads to be designed for increase. But a decrease of the α-factors towards unity is a sign of an improved overall load distribution of the bridge towards the optimum of the rigid bridge case.

4.4 Number of Cells

The effect of the number of cells on the girder load concentration factors, or maximum numbers of wheel loads, can be studied by keeping all other variables constant, i.e., changing only the number of cells. For example, combining two 3-cell bridges or two 4-cell bridges to obtain a 6-cell or an 8-cell bridge, respectively, the results can again be studied immediately in Figs. 40-43 in terms of the maximum numbers of wheel loads.

All these figures have the common characteristic with respect to this parameter that, by doubling the number of cells, the N_{WL}-values are reduced very uniformly by a constant amount so that the slopes of the almost straight lines remain about unchanged. For example, in Fig. 41b, the interior girder R1 of the 3-cell, 120 ft. bridge has N_{WL}-values which are very constantly about 0.25 higher than the corresponding values in the 6-cell bridge, although the corresponding α-factors are smaller.

The reason for this behavior lies in the rigid bridge multipliers which are all smaller for the wider bridges because there are always at least 4 traffic lanes assumed with a resulting reduction factor of 0.75. Although the narrower bridges carry approximately the same number of lanes per girder, no reduction factor can be applied.

On the basis of this finding, it can be said that for even wider bridges, the α-factors will increase slightly and then stabilize, while the rigid bridge base remains about unchanged so that the N_{WL}-values do not change significantly for bridges with more than 8 cells.

Concluding, it may be said that the number of cells is of special importance for the N_{WL}-values as well as for the α-factors, at least within the range between 3 and 8, studied in this report, and therefore it has to be considered in a refined design formula.

4.5 Cell Width

Investigating now the influence of the cell width on the load distribution characteristics of box girder bridges, one will immediately conclude by intuition that the load distribution decreases for wider cells, with resulting higher design loads, and this because of two reasons:

(a) The wider the boxes become, the more traffic lanes fall on one girder, resulting in higher maximum numbers of wheel loads to be designed for.

(b) Keeping slab thicknesses constant, the transverse bridge stiffness decreases with wider cells, resulting in worse transverse load distribution and higher design loads.

The first reason is closely associated with the previously discussed β-value, i.e., the number of traffic lanes per girder. In fact, for a fixed β-value, the number of traffic lanes per girder is by definition independent of the cell width. Thus the actual difference between 4- and 8-cell bridges with cell widths of 7 ft. on the one side and 3- and 6-cell bridges with cell widths of 9 ft. 4 in. on the other side, must be studied by keeping β constant.

Under this condition, Figs. 40-43 confirm the intuitive reasoning. While the slopes of the N_{WL} versus β curves are about independent of the cell width, the girders in bridges with 9 ft. 4 in. wide cells have to carry considerably more wheel loads than those in bridges with 7 ft. wide cells. The difference between 4- and 3-cell, 60 ft. bridges is for any β about 0.26 wheel loads and between 8- and 6-cell, 60 ft. bridges, about 0.20 wheel loads. With the longer spans, load distribution improves also for bridges with wider cells so that the above differences decrease to 0.16 and 0.11 wheel loads, respectively, for the 120 ft. spans.

Similar observations hold for the corresponding α-factors, for which however the comparison is not so easy to make because of the curvatures of the α versus β relationships.

Thus, it may be said that wider cell widths lead to much higher α-factors and maximum numbers of wheel loads for which the girders have to be designed. The economy however is not much impaired since less girders have to be built. But still, for all example bridges of this report, summing up the maximum numbers of wheel loads for which all girders of a bridge have to be designed, leads to slightly smaller totals in bridges with 7 ft. wide cells than in bridges with 9ft. 4in. wide cells. For example, the 6-cell, 60 ft. bridge carrying five lanes has to be designed for 3.7% more wheel loads than the corresponding 8-cell bridge.

4.6 Span and End Boundary Conditions

The change of the girder load concentration factors α with varying spans and end boundary conditions is shown for 3-, 4-, and 6-cell bridges in Figs. 44-46, respectively. The end boundary conditions are simulated by defining a theoretical effective span between inflection points, L', of the equivalent continuous beam. Both relations, α versus L and α versus L', are shown in these figures from which the following conclusions may be drawn:

(a) For longer spans, the transverse stiffness-to-longitudinal stiffness ratio of the bridge increases, resulting in better load distribution. Hence, the α-factors of all girders in a given bridge converge with increasing span towards 1, i.e., towards the rigid bridge with perfect load distribution. However, there may be slight oscillations of individual girder curves around 1 due to the complexity of critical truck loading positions.

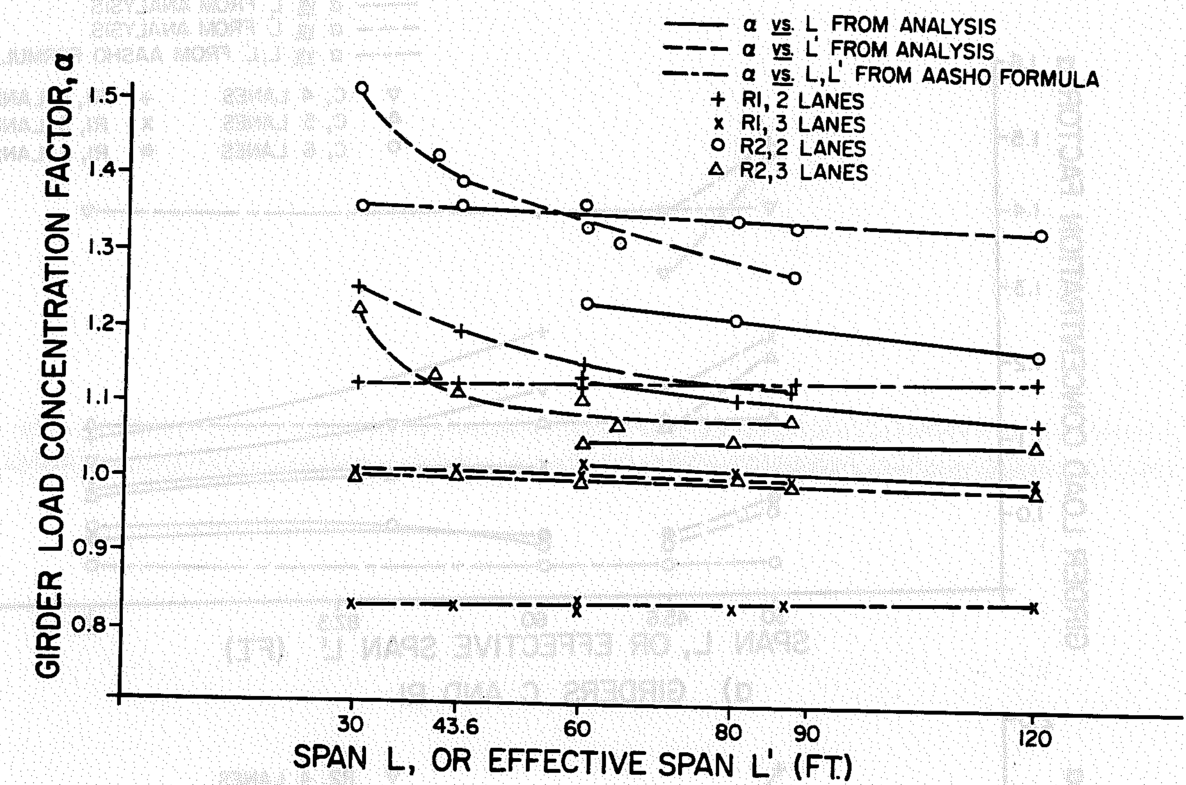


FIG. 44 VARIATION OF α WITH L OR L' FOR 3 CELL BRIDGES

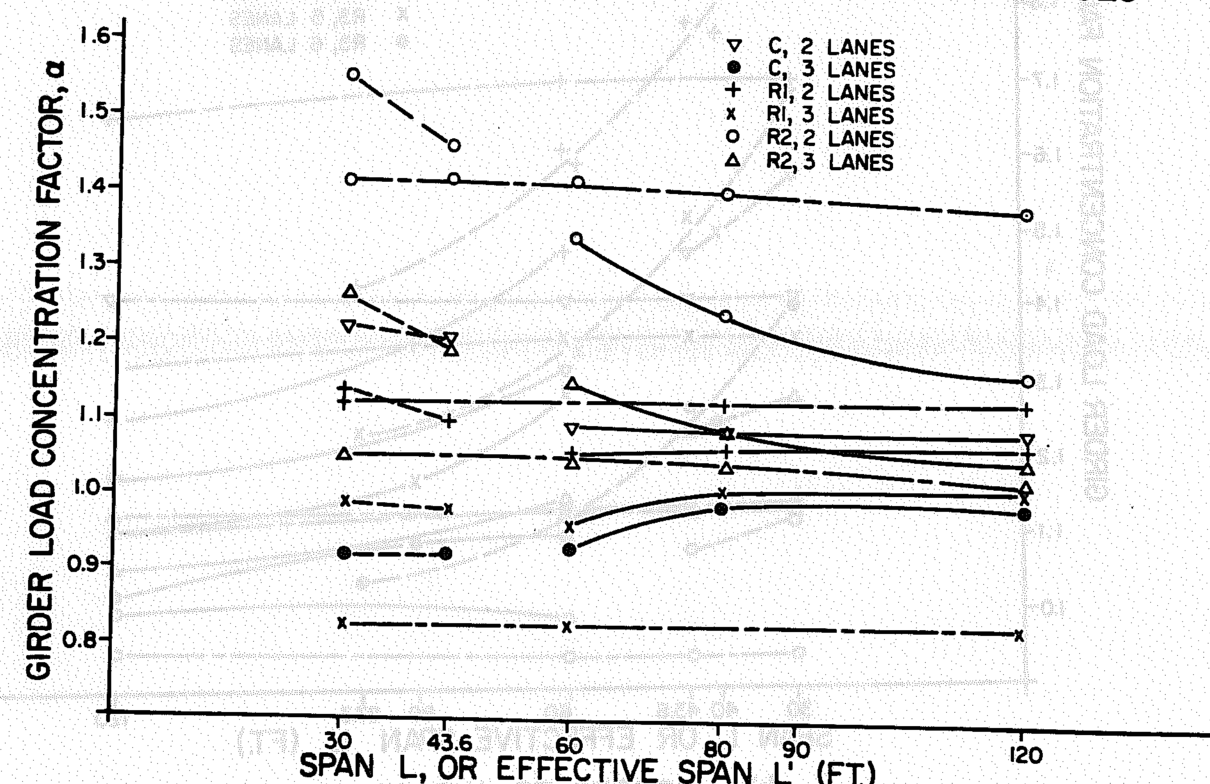


FIG. 45 VARIATION OF α WITH L OR L' FOR 4-CELL BRIDGES

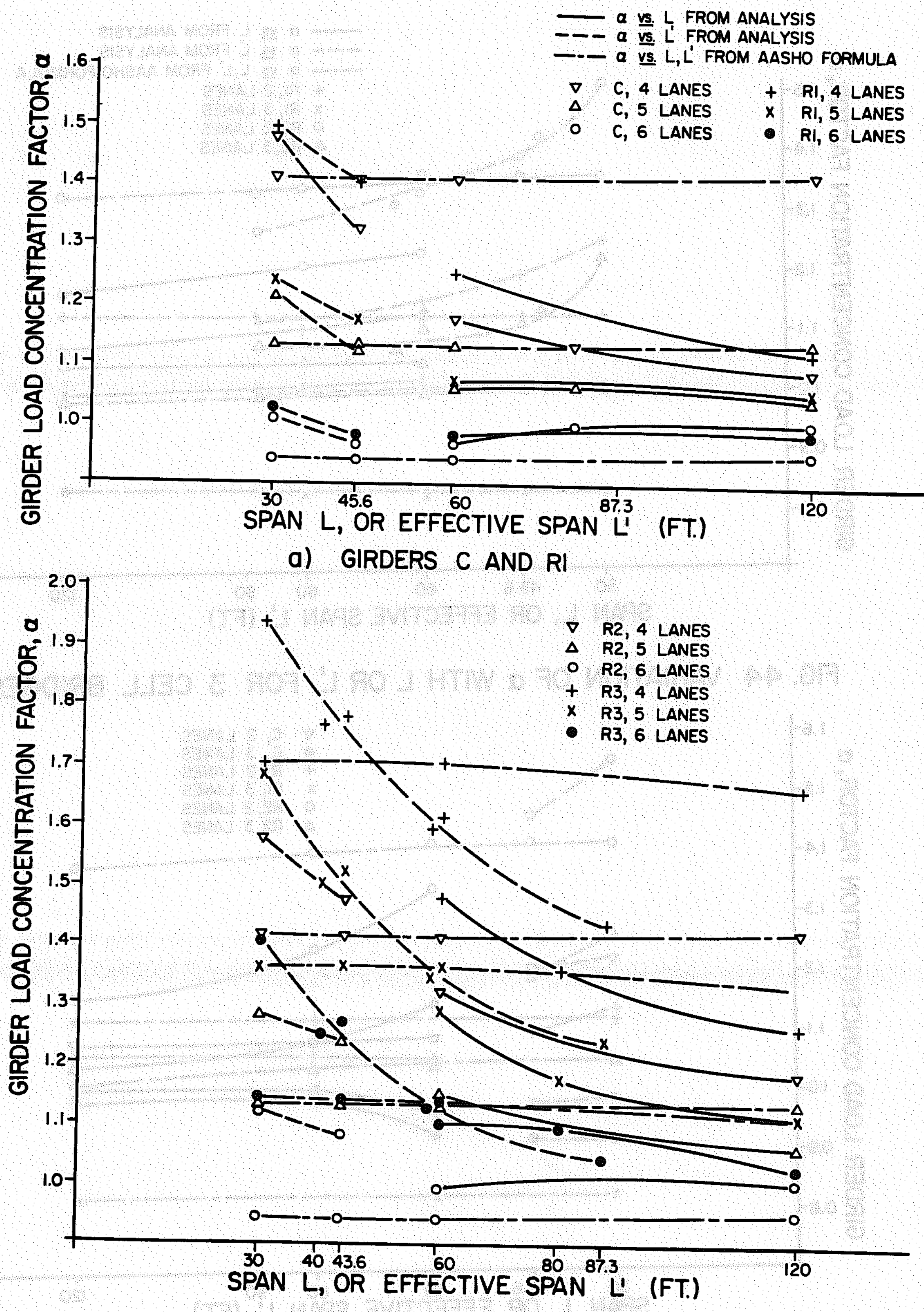


FIG. 46 VARIATION OF α WITH L OR L' FOR 6-CELL BRIDGES

(b) The α versus L' relation is not directly comparable to the α versus L relation, at least for some cases. For example, the α-factor of the exterior girder of a 3-cell bridge with both ends of the 120 ft. span fixed, i.e., with an effective span of 60 ft., is about 10% higher than the equivalent α-factor in a simply supported 60 ft. span bridge. Obviously, the end fixities reduce the effective spans more than assumed, with a resulting worsening of the load distribution. Remembering that each girder has its own point of inflection and therefore its own effective span (see Section 3.6.2), the definition of an inflection point for the whole bridge cannot be very accurate. However, the abovementioned discrepancy of 10% is a maximum, the other discrepancies averaging about 3%.

(c) For comparison, the α-factors resulting from AASHO design formulas have also been plotted in Figs. 44-46. It can be seen that these values are independent of the span and end boundary conditions, the slight decrease with span being due to the change of the ρ-factors and in turn N_{WL} of the rigid bridge case. The neglect of the span and end boundary condition parameters leads to overconservative designs for especially longer spans (coupled with fewer lanes per girders) and to very unconservative designs for short spans, especially if one or both ends are fixed against rotation (coupled with more lanes per girder).

The above observations clearly indicate the need for a design formula which takes into account the span between supports and especially of the end boundary conditions by choosing an equivalent effective span between inflection points. In fact, it must be stressed that the boundary conditions are of much more importance than the clear span between supports.

Increasing the spans beyond 120 feet will lead to more and more uniform load distribution in the bridge. In addition, for these long spans, truck loadings will be replaced by lane loadings, and finally, dead load moments begin to be more and more important until live load moments play only a secondary role in the bridge design so that the question of wheel load distribution becomes less critical.

4.7 Slab Overhang

The effect of a 3 ft. slab overhang on the girder load concentration factors can be immediately studied by comparing the two sets of tables, Tables 6-9 and 12-15. From this comparison, the following observations can be made:

(a) This influence is in general small, decreasing with the longer spans and increasing with the end fixities, i.e., with decreasing effective spans between inflection points. It also increases slightly with the number of cells.

(b) The influence of the slab overhang is in general much more pronounced for exterior girders than for interior girders. The average change in α for exterior girders due to the slab overhang is about 3%, but with maximum values for fixed-fixed bridges going as high as 9%.

(c) Concerning interior girders, the influence of the slab overhang is almost negligible, changing the α concentration factors an average of about 2%, and in only a few cases reaching a maximum of 5-6%.

Keeping these observations in mind, it appears to be justified that the results for bridges having a 3 ft. slab overhang already studied and summarized in Tables 12-15 can be used in developing a generally applicable design method.

Almost all bridges actually being built do have slab overhangs in order to get for little additional cost more roadway area. The effect on the load distributing characteristics of the bridge due to the addition of the 3 ft. overhang has been shown to be generally small. Therefore it is permissible to neglect the influence of the slab overhang width as a parameter on the α-factors, since actual variations from the assumed 3 ft. rarely exceed ± 1 ft.

4.8 Additional Parameter Studies

The parameters so far discussed are believed to be the major factors to be considered in a study on load distribution in box girder bridges. However, the influence of intermediate diaphragms is very important, in fact it is perhaps the most important parameter and will therefore be discussed separately in the next chapter.

There are still some additional parameters which influence the load distribution of bridges, but their range of variation is limited, for example the depth-span ratio or the slab and web thicknesses.

As for the depth-span ratio, an increase will stiffen the bridge longitudinally so that a worse load distribution will be the result. However, the depth-span ratios used for the example bridges of this report (Table 1) are good averages of the feasible range between 0.050 and 0.065 into which most California highway bridges fall [1]. If due to prestress, the depth-span ratio can be dropped far below these averages, then the transverse bridge stiffness becomes very high compared to the longitudinal bridge stiffness so that better load distribution may be expected. For this reason, it might be useful to have a simplified method of taking into account the variation of the depth-span ratio.

Since a variation in the depth-span ratio has basically the same effect as a variation in span, it is proposed that the true span be replaced by an imaginary span such that the bridge under consideration with the given depth obtains the same depth-span ratio as that on which this report is based.

Slab and web thicknesses are also factors which decisively influence load distribution of box girder bridges, but the feasible range of variation is very limited. Web thicknesses of 8 inches are almost standard, as are $5\frac{1}{2}$ inches thick bottom slabs. Very few bridges deviate from these standards [1]. Similarly, the selected top slab thickness of $6\frac{1}{2}$ inches is fairly representative for all bridges built in California, and very few top slabs are thinner than 6 inches or thicker than 7 inches.

Assuming, however, that the web or slab thickness of a bridge under consideration does fall out of the above standard range, then it may be possible to replace the actual cell widths by imaginary ones such that the transverse stiffness of the given bridge with the unusual slab thicknesses becomes about the same as the ones assumed for the example bridges of this report.

In a similar way, slab haunches might be treated. These haunches reduce the effective slab span between webs, resulting in better load distribution. This effect may be simulated by assuming for the load distribution analysis a reduced cell width, using engineering judgment.

4.9 Load Distribution Over Interior Supports

The results of the analyses of Chapter 3 have now extensively been discussed, as far as load distribution characteristics at midspan are concerned. In Section 3.6, also, the longitudinal variation of load distribution has been studied. Therefore there remains to investigate the distribution of wheel loads over the interior supports of continuous box girder bridges. In fact, the high negative moments there make the support sections for design purposes at least as important as the midspan sections.

At midspan, the deflection method mentioned in Section 3.4 was a valuable tool for visualizing the bridge behavior under concentrated loads by studying relative girder deflections. Over interior support on the other hand, no relative girder deflections exist. Nevertheless, the girders are subjected to stresses which may in general be very nonuniform. These stresses are therefore referred to as deformation stresses.

From the longitudinal studies of Section 3.6.2 one might expect that due to moment redistribution the distribution of moments over an interior support may be completely different from that at midspan. However, this is not true. The distribution of moments at midspan and over supports are very similar, the latter distribution being more uniform because the midspan load effects do become distributed somewhat when they arrive at interior supports.

In studying now the α-factors at supports as they are given in Tables 12-15, it will be of main concern to study the improvement of load distribution (convergence of α-factors towards 1) in comparison with the midspan results. In this respect, the following observations can be made:

(a) Except for cases in which the load distribution is already very good at midspan (α-factors very close to 1), the load distribution over supports is always considerably better than at midspan where the loads are acting.

(b) This improvement of load distribution depends very much on the number of lanes on the bridge, or the previously defined β-value, and decreases rapidly with more uniform loadings on the bridge. For example, in the

4-cell, 60 ft. span bridge with only one support fixed, the α-factor of girder C changes from 1.20 at midspan to 1.06 at the support (i.e., 11.7%) if two lanes are on the bridge; and only from 0.92 to 0.93 (i.e., 1.1%) if the bridge is loaded with three lanes.

(c) The percentage reduction of the α-factors over interior supports is almost independent of whether one or both end supports are fixed against rotation, although in the latter case the α-factors themselves are always considerably higher because of the shorter effective span for this case.

(d) As an average, the α-factors at the lower β-values (i.e., 2 lanes on the 3- and 4-cell bridges, 4 lanes on the 6- and 8-cell bridges) are about 10% lower at the support than at midspan. However, in some exceptional cases, where the α-factors are already close to 1 at midspan, there may be only small improvements. In other cases, the improvement may be considerably higher, up to 16% (3-cell, 120 ft. span bridge, with 2 lanes, both ends fixed, girder R2).

(e) Exterior girders which always have the highest midspan α-factors, are also subject to the highest reductions, even in terms of percentages. This demonstrates the localized nature of the stress concentrations in exterior girders under loads.

(f) The above findings are approximately independent of the number of cells (i.e., 3 versus 6 cells) and of the cell width (i.e., 3 versus 4 cells).

From the above observations, it appears to be justified to reduce the α-factors obtained for midspan sections somewhat, if they are to be applied to interior support sections, especially if fewer lanes fall on one girder. However, it might also be good design practice to keep some reserve strength over the supports where the bridge, due to the external indeterminacy, is often subjected to higher stresses than at midspan.

5. INFLUENCE OF INTERMEDIATE DIAPHRAGMS

5.1 General Remarks

Transverse diaphragms in concrete box girder bridges can be classified into support and intermediate diaphragms. The first type serves to provide high transverse rigidity of the bridge over supports to distribute the reactions evenly over the entire width of the bridge. Especially if the bridge is supported by single columns--as is a very common design practice because of the aesthetical appearance--then support diaphragms are absolutely essential for the proper functioning of the bridge.

In this chapter, only the second type of diaphragms will be discussed, whose sole purpose is to improve the load distribution properties of bridges between supports, assuming that rigid support diaphragms are always present. However, as this report is concerned mainly with bridges without diaphragms, in this chapter only more general design considerations concerning the use of diaphragms will be presented.

5.2 Feasibility and Design Considerations

The essential question concerning the use of intermediate diaphragms is if their addition, if at all, is necessary and more specifically, if the reduction in longitudinal girder reinforcement due to any improved load distribution justifies the additional cost of a diaphragm or diaphragms. This question is a matter of feasibility and has to be decided upon by the designer for each bridge anew.

In order to classify and measure a given bridge according to its load distribution characteristics, a quantity may be introduced which might be called the "nonuniformity" because it is the weighted average of the deviations of the α-factors of all bridge girders from the rigid case, i.e.,

$$E_\alpha = \frac{\sum_i |\alpha_i - 1| \rho_i}{\sum_i \rho_i} \qquad (22)$$

Table 22 lists the nonuniformities E_α for all example bridges of this report.

TABLE 22. NONUNIFORMITIES E_α (in %)

Span (ft.)			60					80	120				
Section			Midspan			Support		Midsp.	Midspan			Support	
Boundary Conditions			SS	FS	FF	FS	FF	SS	SS	FS	FF	FS	FF
No. of Cells	No. of Lanes	β											
3	2	0.592	16.7	27.1	35.6	15.9	21.9	15.1	11.7	18.2	23.6	6.3	9.9
	3	0.888	3.2	5.0	9.2	4.2	4.6	2.6	2.1	3.7	4.1	5.0	2.6
4	2	0.446	15.9	24.0	29.2	16.1	19.7	13.3	9.8				
	3	0.669	8.3	8.9	9.4	10.4	6.7	3.7	2.6				
6	4	0.627	30.8	49.4	61.3	37.2	48.0		16.5				
	5	0.784	13.7	26.1	34.6	19.6	27.7		6.5				
	6	0.941	3.5	7.8	13.9	5.9	11.1		1.0				
8	4	0.471	27.8						14.3				
	5	0.590	10.0						4.7				
	6	0.707	2.3						0.7				

Many of the observations made with the α-factors themselves, apply also to the nonuniformities. They decrease rapidly with the number of lanes which are placed on a given bridge, i.e., with more uniform loadings. They increase with decreasing spans and with end fixities, i.e., with decreasing effective spans between inflection points. But for bridges with the same overall width, the E_α's do not greatly depend on the number of cells. Although, for example, there are an average of about 0.2 more lanes on a girder in the 6-cell bridges than in the 8-cell bridges, the corresponding E_α-values for the same span are fairly close, those for bridges with wider cells being slightly higher.

Bridges with very uniform loadings--narrow lane widths or high numbers of lanes per girder--or with large effective spans have E_α- values so small that intermediate diaphragms are hardly justified from a pure load distribution point of view. Even considering transverse slab bending moments, the design will usually be controlled by local bending stresses under wheel loads, compared to which the bending moments resulting from differential girder deflections are small [1]. Therefore, the use of intermediate diaphragms will seldom be controlled by transverse slab bending moments.

5.3 Effect of Intermediate Diaphragms on Load Distribution

In order to study the immediate effect of a rigid midspan diaphragm of 1 ft. thickness on the girder load concentration factors or the numbers of wheel loads carried by the girders, a few additional analytical studies have been made with the computer program MUPDI, the results of which are summarized in Table 23. The bridge studied had a 60 ft. span, 6 cells, and 4 traffic lanes. Values given in Table 23 are for an exterior girder.

TABLE 23. EFFECT OF A MIDSPAN DIAPHRAGM

Section		At Quarterspan, x=15				At Midspan, x=30			
		N_{WL}		α		N_{WL}		α	
With or Without Diaphragm		Without	With	Without	With	Without	With	Without	With
Bound. Cond.	Ideal. Truck Placed at								
Simple-Simple	Midspan	0.71	0.68	1.10	1.07	0.95	0.75	1.48	1.16
Simple-Simple	Quarterspan	0.81	0.80	1.27	1.25	0.71	0.68	1.10	1.06
Fixed-Simple	Midspan	-	0.72	-	1.13	1.15	0.79	1.79	1.23

While the results of Table 23 for the bridge without a diaphragm are based on 2 of the 4 lanes loaded at full capacity, the corresponding bridge with a diaphragm was in all cases loaded with 4 trucks at 75% capacity. These load conditions produced maximum values at midspan in each case. Due to load distribution, results for the quarterspan sections might be slightly increased by choosing different loading arrangements.

Table 23 shows that a midspan diaphragm improves the load distribution of a bridge with high α-factors at midspan tremendously, but only as long as the idealized trucks are acting over the diaphragm. If the loads are acting at quarterspan rather than at midspan where the diaphragm is, the improvement of load distribution due to the diaphragm is only small, both at quarter- and at midspan, due to the high warping stresses developed between the end and support diaphragms. Moreover, using an α-factor obtained for the midspan section--with a diaphragm--to design the whole girder might be unconservative. While for example in the simply supported bridge with a diaphragm, the exterior girder would be designed for 0.75 wheel lines at midspan, Table 23 reveals that the same girder might have to carry 0.80 wheel lines at quarterspan.

Although this error on the unconservative side will in general be small and will become smaller if real standard trucks instead of idealized ones are placed on the bridge, the preceding discussion shows that the matter of diaphragms needs study in each case before α-factors lower than those for a bridge without diaphragms are used in an overall design.

A recent paper by Abdel-Samad, Wright and Robinson [12] contains additional information and studies regarding the effect of diaphragms in box girder bridges.

6. PROPOSED METHODS OF DESIGN

6.1 General Design Considerations

One of the main decisions which a designer has to make is the choice between an accurate, but more time-consuming method of analysis and design, and a simplified, but only approximate method.

The optimum design of a box girder bridge is distinguished in that each of its girders, at any section, is being designed for exactly the maximum moment that will ever be developed at that point. Such a design is very involved because many complete bridge analyses are required for this purpose.

A simplified design is characterized by two features. Firstly, the amount of work involved is reduced to a certain degree, and secondly, the final design is lead more or less away from the optimum design, be this then an overconservative and therefore less economical design, or an underdesigned structure violating safety requirements. But since safety requirements cannot be violated, a simplified method of design will usually lead to overdesigned and therefore less economical structures.

For a complex structural system such as the box girder bridge, there is no simple law describing the numbers of wheel loads carried by a typical interior or exterior girder in terms of the various bridge parameters. In fact, as can be seen very clearly in Figs. 40-43, these design values may vary among individual girders as much as 10%. Designing all bridge girders for the highest number of wheel loads will lead to an overconservative design, and choosing some average value will lead to an economical but not necessarily safe design, unless proper structural means (for example intermediate transverse diaphragms) insure that overloads on any girder are transferred to neighboring girders.

In the light of these general design considerations, two methods of design will be proposed in this chapter. The first one, called "Accurate Method of Design," is based on the influence surface approach and involves considerably more work than the second method, but can lead to any degree of accuracy and therefore economy. The second method, termed "Simplified Method of Design," is an extension of present design practice with which bridge engineers in general are more familiar. Here the design loads are determined with the help of empirical formulas and are in fair agreement with those obtained using a more accurate method of analysis.

6.2 Accurate Method of Design

The accurate determination of internal stresses in a box girder bridge requires for each case a complete bridge analysis, using computer programs such as have been developed within this research sequence [1] [2]. Thus, the method of design outlined in this section will be based on the availability of a computer program, which gives all important internal forces at any point of the structure, due to wheel loads, placed anywhere on the bridge. For convenience, the program may also contain a subroutine for stress integration to yield the bending moment carried by any girder of the bridge at any longitudinal section, such as described in Section 3.2. Furthermore, a standard subroutine for plotting contour lines of influence surfaces will increase the efficiency of this design method considerably.

Once such a computer program is available, the design procedure will be as follows:

(1) Select for each bridge girder a set of points for construction of the maximum moment envelope--for continuous bridges also for minimum moments. Make use of symmetry wherever possible.

(2) Place a unit concentrated load at any point selected in step (1), say over girder i at section x. Analyze the bridge for this loading and record the moment diagrams for all girders. If i is an exterior girder, all interior girder moments have to be scaled with the proper ρ-factor as given by Eq. (4). If i is an interior girder, the exterior girder moments have to be scaled with $1/\rho$.

(3) The moments thus obtained are best arranged by recording them along the girders of a bridge plan view and by interpolating contour lines between them such as shown for example in Figs. 36 and 37. By virtue of the reciprocity relations of Section 3.6.3, the plot giving the modified moments of all girders for this one load condition is the influence surface for the moment of girder i at section x.

(4) Repeat steps (2) and (3) for each of the points selected in step (1).

(5) Find for each influence surface the most critical truck positions, loading any possible combination of lanes and applying the reduction factors of Section 2.1, if more than two lanes are loaded simultaneously. For the critical truck combination, add up the influence surface ordinates to obtain the maximum (or minimum) design moment for the girder under consideration at that section where the unit load was placed.

The design method outlined above leads directly to the critical design moments rather than to maximum numbers of wheel lines falling on individual girders, from which the critical moments would have to be determined in additional steps.

Before this method of design can be applied to bridges with intermediate diaphragms, further theoretical studies would have to be made on the validity of the reciprocity relations of section 3.6.3.

This refined design method is recommended for any unusual structure for which an approximate method of analysis is not reliable. It will also lead to substantial savings in material whenever applied to repetitive standard structures. It requires the availability of a high-speed digital computer and a program such as used for the studies of this research series.

Execution times on the computer may become appreciable in certain cases. Then the cost of this computer time should be weighed against the savings in material expected from a refined method of design. The determination of critical moments using influence surfaces, finally, is a familiar problem for bridge engineers and does not require excessive amounts of work, once the shapes of the influence surfaces are understood.

6.3 Simplified Method of Design

Present design practice treats all interior bridge girders alike, irrespective of the fact that different girders might be subjected to critical moments of different magnitudes. Nevertheless, the design method presented below also still retains this simplification for ease of application.

The key step in this simplified design method is the use of empirical formulas giving the various girder load concentration factors, α, of a bridge in terms of the most important parameters.

From a study of the results presented in this report it has been determined that the load concentration factors for interior girders can be given approximately by

$$\alpha_i = 1 + (\bar{\beta} - C)(\bar{L} + 0.033)(\bar{N}_c - 0.6) \qquad (23)$$

where $\bar{\beta} = \dfrac{1.37}{0.14 + \sqrt{\beta}}$

$$\bar{C} = 0.709 + \frac{5}{S} - 0.006\ N_c$$

$$\bar{L} = \frac{18.5}{L'-0.1(L-L')+3.16}$$

$$\bar{N}_c = (0.0966\ S - 0.21)N_c$$

with
L = span between supports, in feet

L' = span between inflection points of equivalent continuous beam (=L, if simply supported), in feet

β = number of lanes per girder, Eq. (21)

S = cell width or web spacing, in feet

N_c = number of cells

The α-factors for exterior girders are then approximated by

$$\alpha_e = A\ \alpha_i \qquad (24)$$

where $A = 1.44 - 0.225\ \dfrac{L'}{L} - \dfrac{1.3L-0.2L'}{750}$

α_i = interior girder concentration factor, given by Eq. (23)

Equations (23) and (24) were derived empirically to approximate as closely as possible the bridge behavior as it is reflected in Tables 12-15. To illustrate the degree of accuracy of these formulas, Tables 24 and 25 compare the α-factors of interior and exterior girders, respectively, as they are listed in Tables 12-15 with those calculated according to Eqs. (23) and (24). The error percentages also listed in Tables 24 and 25 indicate that the general agreement is good enough for design purposes. For interior and exterior girders, respectively, the average absolute errors are 1.8% and 5.2%, the highest overestimations, + 5.2% and + 9.0%, and the largest underestimations, -4.5% and -7.0%. The relatively high

TABLE 24. COMPARISON BETWEEN α_i FACTORS FROM EQ. (23) AND CRITICAL α-FACTORS FOR INTERIOR GIRDERS FROM TABLES 12-15

Span (ft.)			60			80	120		
Boundary Conditions			SS	FS	FF	SS	SS	FS	FF
Number of Cells	Number of Lanes								
3	2	a	1.13	1.19	1.25	1.11	1.08	1.12	1.15
		b	1.13	1.18	1.26	1.10	1.07	1.10	1.15
		c	0.0	-0.8	+0.8	-0.9	-0.8	-1.8	0.0
	3	a	1.02	1.01	1.01	1.01	1.00	1.00	1.00
		b	1.02	1.03	1.04	1.02	1.01	1.02	1.02
		c	0.0	+2.0	+3.0	+1.0	+1.0	+2.0	+2.0
4	2	a	1.09	1.20	1.22	1.09	1.08		
		b	1.12	1.17	1.24	1.10	1.07	1.09	1.13
		c	+2.7	-2.5	+1.6	+0.9	-0.9		
	3	a	0.96	0.98	0.99	1.01	1.01		
		b	1.01	1.02	1.02	1.01	1.01	1.01	1.01
		c	+5.2	+4.1	+3.0	0.0	0.0		
6	4	a	1.32	1.47	1.58		1.18		
		b	1.31	1.42	1.62	1.24	1.17	1.23	1.34
		c	-0.8	-3.4	+2.5		-0.8		
	5	a	1.14	1.24	1.28		1.06		
		b	1.15	1.21	1.30	1.12	1.09	1.11	1.17
		c	+0.9	-2.4	+1.6		+2.8		
	6	a	0.99	1.08	1.12		1.00		
		b	1.03	1.04	1.07	1.03	1.02	1.02	1.04
		c	+3.0	-3.7	-4.5		+2.0		
8	4	a	1.27				1.16		
		b	1.28	1.38	1.55	1.22	1.16	1.21	1.31
		c	+0.8				0.0		
	5	a	1.10				1.05		
		b	1.13	1.18	1.26	1.10	1.07	1.10	1.14
		c	+2.7				+1.9		
	6	a	1.00				1.02		
		b	1.02	1.03	1.04	1.02	1.01	1.01	1.02
		c	+2.0				-1.0		

(a) Critical α-factors for interior girders, from Tables 12-15

(b) α_i-factors calculated according to Eq. (23)

(c) Percentage errors of (b) with respect to (a)

TABLE 25. COMPARISON BETWEEN α_e-FACTORS FROM EQ. (24) AND FOR EXTERIOR GIRDERS FROM TABLES 12-15

Span (ft.)			60			80	120		
Boundary Conditions			SS	FS	FF	SS	SS	FS	FF
Number of Cells	Number of Lanes								
3	2	a	1.23	1.39	1.51	1.21	1.17	1.27	1.36
		b	1.27	1.39	1.55	1.21	1.11	1.20	1.31
		c	+3.3	0.0	+2.6	0.0	-5.1	-5.5	-3.7
	3	a	1.05	1.11	1.22	1.05	1.05	1.08	1.10
		b	1.14	1.21	1.28	1.12	1.05	1.11	1.16
		c	+8.6	+9.0	+4.9	+6.7	0.0	+2.8	+5.4
4	2	a	1.34	1.46	1.55	1.24	1.16		
		b	1.26	1.39	1.53	1.21	1.11	1.19	1.28
		c	-6.0	-4.8	-1.3	-2.4	-4.3		
	3	a	1.15	1.19	1.26	1.09	1.05		
		b	1.14	1.21	1.26	1.11	1.05	1.10	1.15
		c	-0.9	+1.7	0.0	+1.8	0.0		
6	4	a	1.48	1.78	1.94		1.26	1.43	1.61
		b	1.48	1.68	2.00	1.36	1.22	1.34	1.52
		c	0.0	-5.6	+3.1		-3.2	-6.3	-5.6
	5	a	1.29	1.53	1.69		1.11	1.24	1.35
		b	1.29	1.43	1.60	1.23	1.13	1.21	1.33
		c	0.0	-6.5	-5.3		+1.8	-2.4	-1.5
	6	a	1.10	1.27	1.42		1.03	1.04	1.14
		b	1.16	1.23	1.32	1.13	1.06	1.11	1.18
		c	+5.5	-3.1	-7.0		+2.9	+6.7	+3.5
8	4	a	1.47	1.64	1.85	1.39	1.20		
		b	1.44	1.64	1.91	1.34	1.20	1.32	1.49
		c	-2.0	0.0	+3.2	-3.6	0.0		
	5	a	1.27	1.41	1.59	1.15	1.07		
		b	1.27	1.40	1.56	1.21	1.11	1.20	1.30
		c	0.0	-0.7	-1.9	+5.2	+3.7		
	6	a	1.08	1.18	1.33	1.02	0.99		
		b	1.15	1.22	1.28	1.11	1.05	1.10	1.16
		c	+6.5	+3.4	-3.8	+8.8	+6.1		

(a) α-factors for exterior girders, from Tables 12-15

(b) α_e-factors calculated according to Eq. (24)

(c) Percentage errors of (b) with respect to (a)

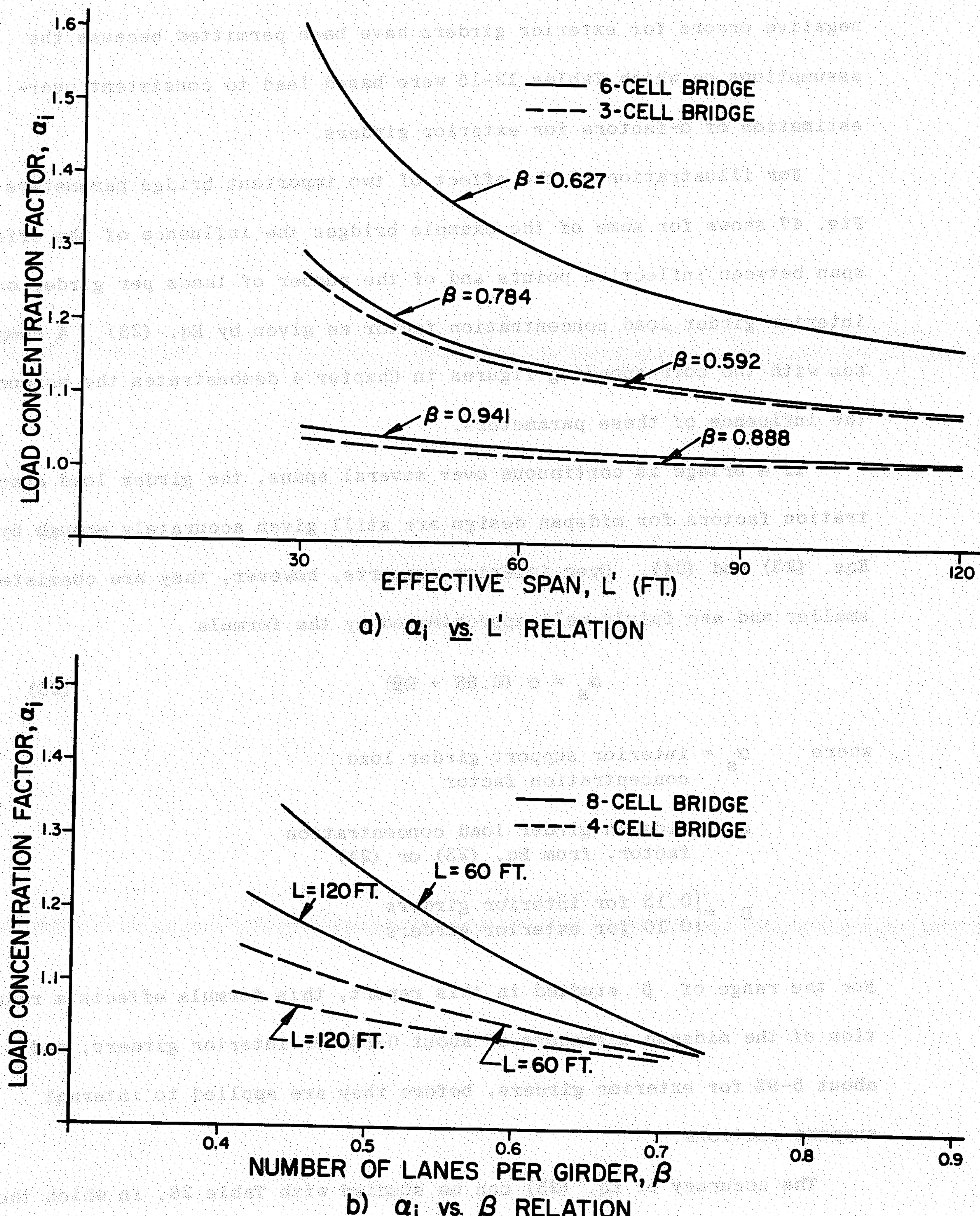


FIG. 47 INFLUENCE OF EFFECTIVE SPAN AND NUMBER OF LANES PER GIRDER ON α_i- FACTOR OF DESIGN FORMULA (23)

negative errors for exterior girders have been permitted because the assumptions on which Tables 12-15 were based lead to consistent over-estimation of α-factors for exterior girders.

For illustration of the effect of two important bridge parameters, Fig. 47 shows for some of the example bridges the influence of the effective span between inflection points and of the number of lanes per girder on the interior girder load concentration factor as given by Eq. (23). A comparison with the corresponding figures in Chapter 4 demonstrates the essence of the influence of these parameters.

If a bridge is continuous over several spans, the girder load concentration factors for midspan design are still given accurately enough by Eqs. (23) and (24). Over interior supports, however, they are consistently smaller and are fairly well approximated by the formula

$$\alpha_s = \alpha \ (0.86 + B\beta) \tag{25}$$

where α_s = interior support girder load concentration factor

α = midspan girder load concentration factor, from Eq. (23) or (24)

$$B = \begin{cases} 0.15 \text{ for interior girders} \\ 0.10 \text{ for exterior girders} \end{cases}$$

For the range of β studied in this report, this formula effects a reduction of the midspan α-factors of about 0-8% for interior girders, and about 5-9% for exterior girders, before they are applied to internal support sections.

The accuracy of Eq. (25) can be studied with Table 26, in which the α-factors for interior support sections are compared with those listed in Tables 12-15. The average absolute error for interior girders is 2.9%, ranging from -5.3 up to +5.3%. For exterior girders, the average absolute

TABLE 26. COMPARISON BETWEEN α-FACTORS OVER INTERIOR SUPPORTS, FROM EQ. (25) AND CRITICAL α-FACTORS FROM TABLES 12-15

Span (ft.)			60						120					
Boundary Conditions			Fixed-Simple			Fixed-Fixed			Fixed-Simple			Fixed-Fixed		
No. of Cells	Girder	No. of Lanes	a	b	c	a	b	c	a	b	c	a	b	c
3	Int.	2	1.11	1.12	+0.9	1.13	1.19	+5.3	1.01	1.04	+3.0	1.07	1.09	+1.9
		3	0.97	1.02	+5.2	1.01	1.03	+2.0	0.97	1.01	+4.1	0.99	1.01	+2.0
	Ext.	2	1.23	1.28	+4.1	1.35	1.42	+5.2	1.14	1.10	-3.5	1.14	1.20	+5.3
		3	1.06	1.15	+8.5	1.10	1.21	+10.0	1.08	1.05	-2.8	1.05	1.10	+4.8
4	Int.	2	1.08	1.08	0.0	1.12	1.15	+2.7		1.01			1.05	
		3	0.99	0.97	-2.0	1.00	0.97	-3.0		0.97			0.97	
	Ext.	2	1.34	1.26	-6.0	1.39	1.39	0.0		1.08			1.16	
		3	1.16	1.12	-3.4	1.17	1.17	0.0		1.02			1.07	
6	Int.	4	1.37	1.36	-0.7	1.48	1.54	-4.1		1.17			1.28	
		5	1.23	1.18	-4.1	1.30	1.27	-2.3		1.08			1.14	
		6	1.07	1.04	-2.8	1.13	1.07	-5.3		1.02			1.04	
	Ext.	4	1.59	1.55	-2.5	1.72	1.84	+7.0	1.25	1.24	-0.8	1.36	1.40	+2.9
		5	1.39	1.34	-3.6	1.52	1.50	-1.3	1.09	1.13	+3.7	1.22	1.25	+2.5
		6	1.19	1.17	-1.7	1.27	1.26	-0.8	1.00	1.06	+6.0	1.03	1.12	+8.7
8	Int.	4		1.27			1.43			1.11			1.21	
		5		1.11			1.18			1.03			1.07	
		6		0.99			1.00			0.97			0.98	
	Ext.	4	1.49	1.49	0.0	1.69	1.73	+2.4		1.20			1.35	
		5	1.30	1.29	-0.8	1.47	1.43	-2.7		1.10			1.19	
		6	1.09	1.14	+4.6	1.23	1.19	-3.3		1.03			1.08	

(a) Critical α-factors over supports from Tables 12-15

(b) α_s-factors calculated with Eq. (25)

(c) Percentage errors of (b) with respect to (a)

error is 3.6%, varying between -6.0 and +10.0%.

Based on these basic design formulas (Eqs. (23), (24), and (25)), the procedure for a simplified design method can be summarized as follows.

(1) Locate the neutral axis of the complete bridge cross section.

(2) Calculate the moments of inertia of an exterior and a typical interior girder about the neutral axis of the bridge found in step (1), and form the ratio

$$\rho = \frac{I_{ext}}{I_{int}} \tag{26}$$

where I_{ext} = moment of inertia of exterior girder about bridge neutral axis

I_{int} = moment of inertia of interior girder about bridge neutral axis.

Interior and exterior girders are then assigned the relative stiffnesses 1.0 and ρ, respectively. An explicit formula for ρ has been given by Eq. (4) in Chapter 2.

(3) Determine the total number of wheel loads on the bridge, N_T, using reduction factors for multiple lane loadings.

$N_T = 4.0$ for bridge with 2 lanes, both lanes loaded @ 100%

$N_T = 5.4$ for bridge with 3 lanes, 3 lanes loaded @ 90%

$N_T = 1.5\ N_L$ for bridge with $N_L \geq 4$ lanes, N_L lanes loaded @ 75%

Find also the number of lanes per girder, as given by Eq. (21), i.e.,

$$\beta = \frac{N_L}{\sum_{j=1}^{n} \rho_j} \tag{21}$$

(4) Calculate the numbers of wheel loads taken by a typical interior and by an exterior girder in a rigid bridge, using the formulas

$$N_i^r = \frac{\rho_i}{\sum_j \rho_j} N_T = \frac{1}{n-2+2\rho} N_T \qquad (27)$$

$$N_e^r = \frac{\rho_e}{\sum_j \rho_j} N_T = \rho\, N_i^r \qquad (28)$$

where N_i^r, N_e^r = number of wheel loads taken by a typical interior (exterior) girder in a rigid bridge

N_T = total number of wheel loads on bridge, from step (3)

$\rho_j = \begin{cases} 1 & \text{for interior girders} \\ \rho & \text{for exterior girders, as given by Eq. (4) or (26)} \end{cases}$

(5) Calculate for exterior and interior girders the load concentration factors using Eqs. (23) and (24), and if the bridge is continuous, find the support α-factors thereafter with the help of Eq. (25).

(6) Find the maximum number of wheel loads for a typical interior girder for midspan and support sections according to

$$N_{WL_i} = \alpha_i \cdot N_i^r \qquad (29)$$

where α_i and N_i^r are given by Eqs. (23) or (25) and (27), respectively. For exterior girders, the maximum numbers of wheel loads for midspan and support sections are found similarly,

$$N_{WL_e} = \alpha_e \cdot N_e^r \qquad (30)$$

with α_e and N_e^r being given by Eqs. (24) or (25) and (28), respectively.

(7) Analyze a single independent girder as a simple or continuous beam as the case may be and construct maximum and minimum moment envelopes for a typical interior girder and for an exterior girder, by moving a standard truck with N_{WL} = 1.0 wheel lines into the

respective most critical girder positions.

(8) To obtain final design moment envelopes multiply maximum ordinates by N_{WL} found by Eq. (29) or (30) for midspan sections, and multiply minimum ordinates by N_{WL} found by Eq. (29) or (30) for support sections.

Concluding, it might be appropriate to summarize once more the limitations of the simplified method of design just outlined, and the assumptions on which the calculation of the girder load concentration factors α was based.

(1) AASHO-standard trucks were approximated by idealized one-axle vehicles, resulting in moderate overestimations of interior girder α-factors, and in considerable overestimations of exterior girder α-factors. Equation (24) was designed to lessen this error somewhat.

(2) α-factors were calculated for midspan sections and were assumed to be valid anywhere along the span in the positive moment region. This assumption was also shown to be a source of moderate over-estimation of the α-factors for both, exterior and interior girders.

(3) The extrapolation method taking slab overhangs into account was based on the assumption that moments of individual girders are proportional to their relative stiffnesses. It was shown that this assumption introduced errors which were not always on the conservative side. However, the errors due to assumptions (1) and (2) are more significant and on the safe side.

(4) All interior girders have the same dimensions. However, if the variations are small, say not to exceed 10%, it is recommended that average values be used, as the present AASHO design formula

does in determining the average web spacing.

(5) All interior girders are designed for the loads on the most heavily loaded girder, resulting in varying degrees of reserve strength of less loaded girders.

(6) The simplified design method is valid only for the range of variables studied in this report, i.e.,

number of cells, N_c	- from 3 to 8
cell width, S	- from 7' to 9'-4"
lane width, W_L	- from 10' to 16'
span L, or effective span, L'	- from 30' to 120'

Interpolations between cases studied in this report are permissible without objections. Extrapolations outside the cases studied should be used with caution, with the exception that extrapolations to longer spans than those studied are permissible as discussed in Section 4.6.

The above design method has been developed for bridges without intermediate diaphragms. If diaphragms are used, this simplified design method based on Eqs. (23) and (24), if not modified, will lead to more conservative designs, because then neighboring girders of the most heavily loaded girders are forced to carry a greater share of the total load so that all critical α-factors are reduced somewhat. However, if advantage of this phenomenon were to be taken by reducing the α-factors for analysis, this should be done with caution, since the findings of Chapter 5 indicated that the advantage of the load distributing effect of diaphragms is of importance only in the vicinity of diaphragm sections.

6.4. Example

For demonstration of the two proposed design methods, and for comparison of their results with the present AASHO design method, a 3-cell

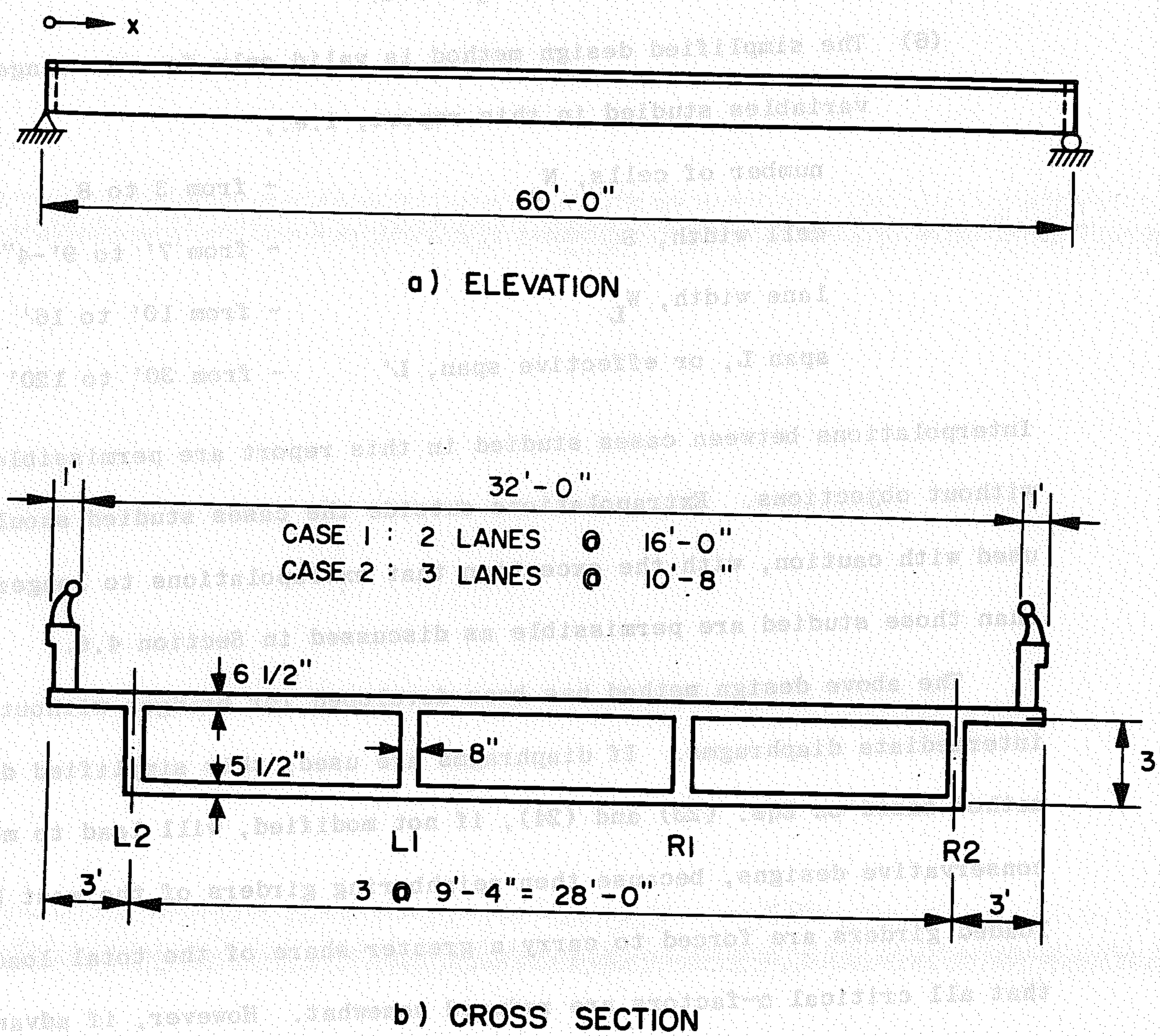
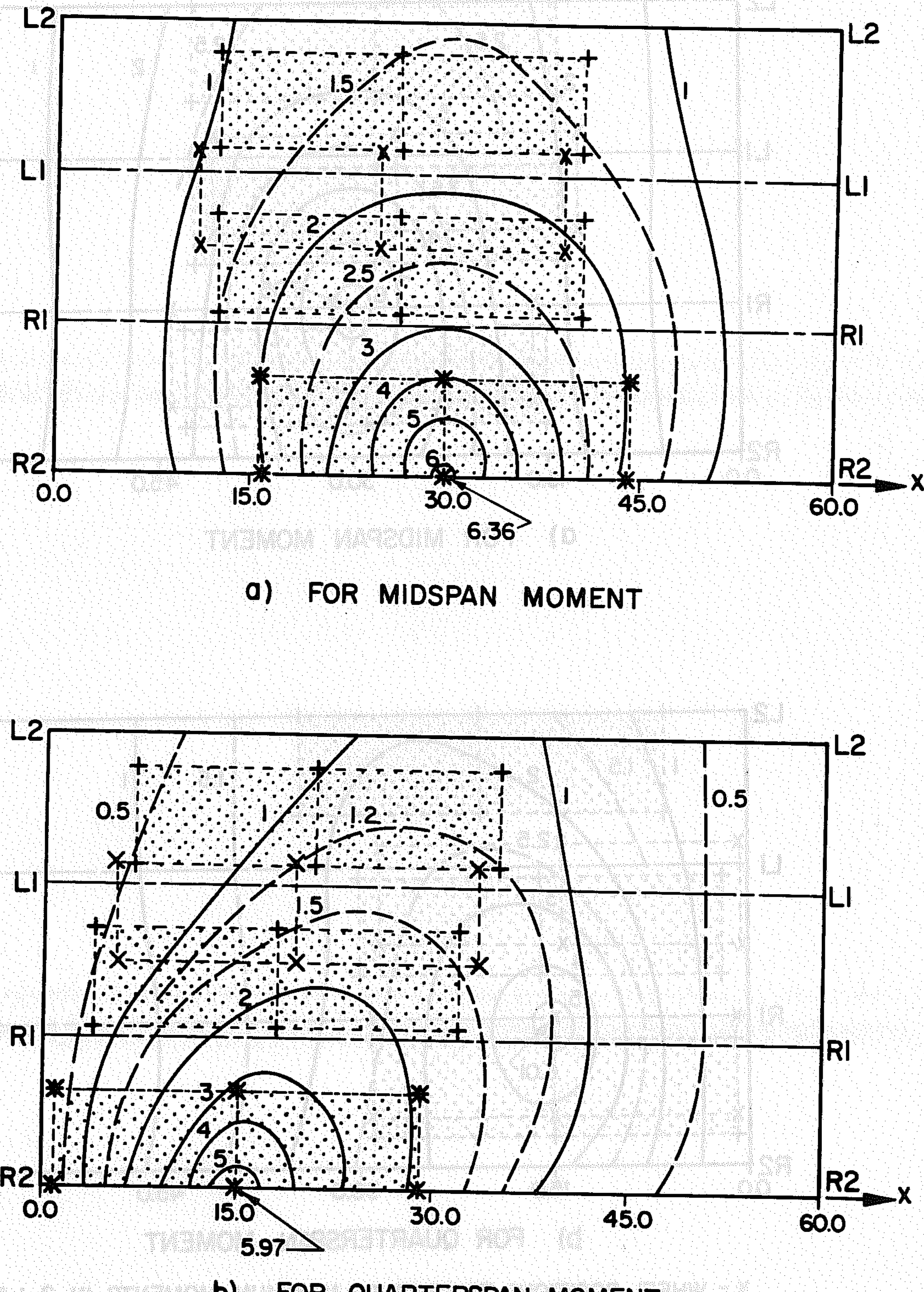


FIG. 48 EXAMPLE BRIDGE FOR COMPARISON OF DIFFERENT DESIGN METHODS



X- WHEEL POSITIONS TO PRODUCE MAXIMUM MOMENTS IN 2-LANE BRIDGE
+- WHEEL POSITIONS TO PRODUCE MAXIMUM MOMENTS IN 3-LANE BRIDGE

FIG. 49 INFLUENCE SURFACES FOR GIRDER R2 OF EXAMPLE BRIDGE

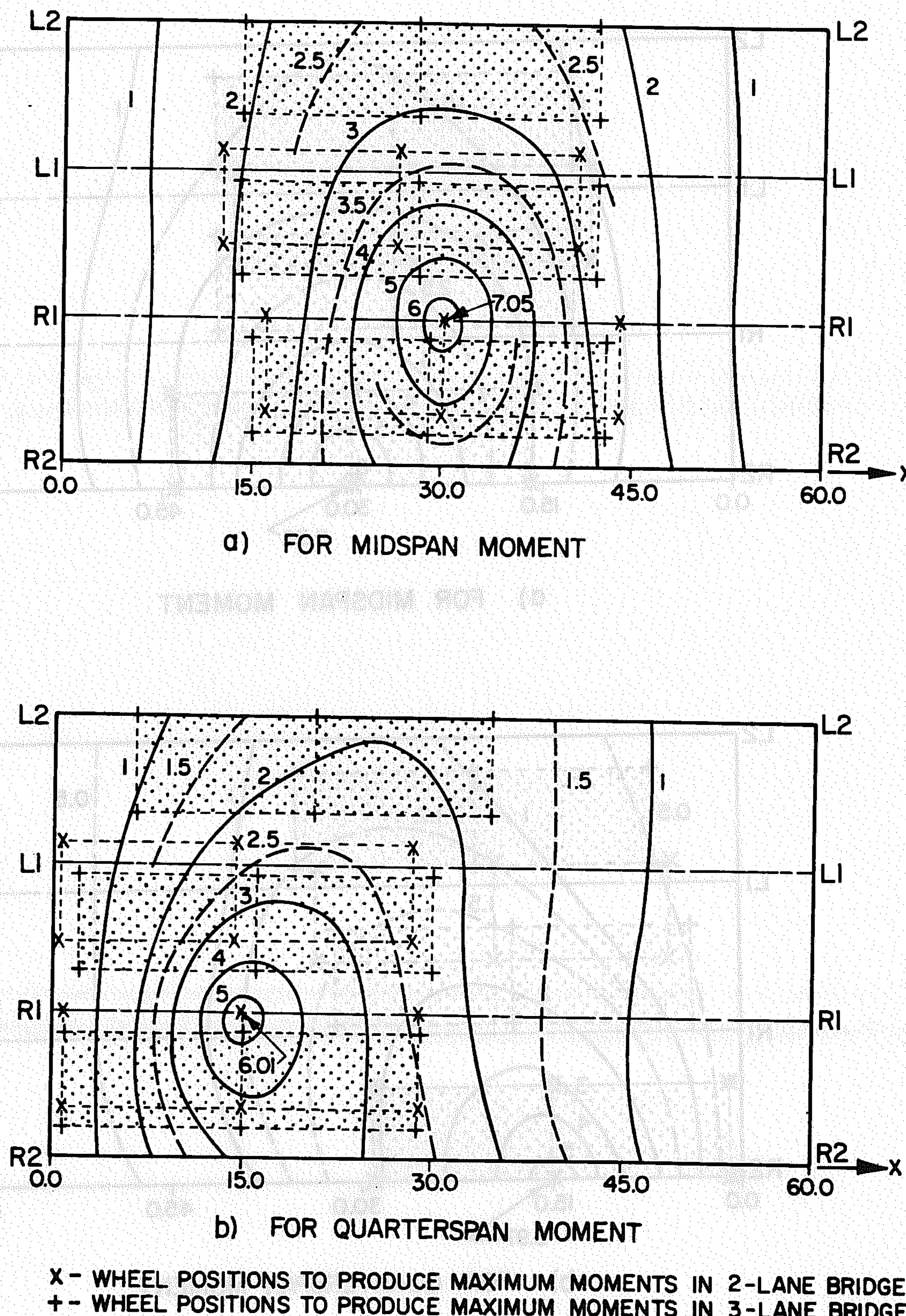


FIG. 50 INFLUENCE SURFACES FOR GIRDER R1 OF EXAMPLE BRIDGE

bridge, simply supported over 60 ft. (Fig. 48) will be analyzed as a design example, applying the various methods.

6.4.1 Accurate Design Method

(1) Because of double symmetry of the bridge, select the midspan and quarterspan points of girders R1 and R2 to approximate the maximum moment envelopes for these two girders.

(2) and (3) The moments in the four bridge girders at various sections, due to each of the four load conditions, are directly output from the computer and are listed in Table 27. Modifications due to the ρ-factor, which is 0.680 for this example bridge (see Table 10), are also listed in Table 27.

(4) The modified girder moments listed in Table 27 are plotted in the influence surfaces (Figs. 49 and 50).

(5) Critical truck positions are shown in Figs. 49 and 50 for the case that 2 lanes of 16 feet width are placed on the bridge or 3 lanes of 10 feet 8 inch width. Detail calculations for determination of critical moments are given in Table 28.

6.4.2 Simplified Method of Design

(1) and (2) Neutral axis lies 1.713 ft. above bottom slab center line. $\rho = 0.680$ is found from Table 10.

(3) Total number of wheel loads on 2-lane bridge, $N_{T2} = 4.0$

Total number of wheel loads on 3-lane bridge, $N_{T3} = 6.0\ (0.9) = 5.4$

Number of lanes per girder,

$$\beta = \frac{2}{0.68+1.0+1.0+0.68} = 0.595 \qquad \text{for 2 lanes}$$

$$= 0.595\ (3/2) = 0.893 \qquad \text{for 3 lanes}$$

(Compare these values with the averages of Table 21).

TABLE 27. GIRDER MOMENTS FOR EXAMPLE BRIDGE, FIG. 48

Unit Load at	Moment of Girder	Moment at x = 7.5	15.0	18.75	22.5	26.25	28.125	30.0
R2 Midspan x=30	L2	0.607	1.092	1.262	1.379	1.446	1.463	1.468
	L1	1.054	1.958	2.301	2.547	2.688	2.723	2.734
	L1•ρ	0.716	1.331	1.565	1.732	1.828	1.851	1.859
	R1	1.304	2.647	3.291	3.880	4.274	4.371	4.402
	R1•ρ	0.887	1.800	2.238	2.638	2.906	2.973	2.994
	R2	0.775	1.787	2.473	3.427	4.700	5.498	6.360

Unit Load at	Moment of Girder	Moment at x = 7.5	11.25	13.125	15.0	16.875	18.75	22.5	30.0	37.5	45.0
R2 Quarter-Span x=15	L2	0.357	0.529	0.612	0.693	0.769	0.841	0.965	1.092	1.021	0.775
	L1	0.885	1.252	1.411	1.554	1.680	1.788	1.939	1.958	1.661	1.174
	L1•ρ	0.602	0.851	0.960	1.057	1.142	1.216	1.318	1.331	1.130	0.802
	R1	2.005	2.763	3.033	3.229	3.355	3.408	3.304	2.637	1.865	1.175
	R1•ρ	1.363	1.879	2.062	2.195	2.281	2.318	2.247	1.793	1.268	0.799
	R2	2.352	3.864	4.786	5.968	4.971	4.236	3.122	1.774	1.048	0.601

Unit Load at	Moment of Girder	Moment at x = 7.5	15.0	18.75	22.5	26.25	28.125	30.0
R1 Midspan x=30	L2	0.693	1.288	1.515	1.679	1.776	1.799	1.807
	L2/ρ	1.020	1.895	2.229	2.470	2.612	2.647	2.658
	L1	1.095	2.167	2.675	3.126	3.456	3.456	3.576
	R1	1.089	2.285	2.994	3.868	5.043	5.821	7.053
	R2	0.853	1.722	2.144	2.521	2.793	2.867	2.891
	R2/ρ	1.255	2.533	3.154	3.708	4.109	4.217	4.253

Unit Load at	Moment of Girder	Moment at x = 7.5	11.25	13.125	15.0	16.875	18.75	22.5	30.0	37.5	45.0
R1 Quarter-Span x=15	L2	0.584	0.828	0.934	1.028	1.111	1.180	1.277	1.288	1.095	0.779
	L2/ρ	0.860	1.218	1.373	1.512	1.634	1.736	1.879	1.895	1.611	1.146
	L1	1.459	2.053	2.278	2.445	2.553	2.602	2.554	2.167	1.667	1.131
	R1	2.262	3.726	4.642	6.010	4.911	4.267	3.352	2.285	1.606	1.043
	R2	1.292	1.797	1.981	2.113	2.193	2.222	2.145	1.722	1.229	0.778
	R2/ρ	1.900	2.643	2.914	3.109	3.226	3.268	3.155	2.533	1.808	1.144

TABLE 28. CALCULATION OF DESIGN MOMENTS OF EXAMPLE BRIDGE

Girder	At	2-Lane Bridge		3-Lane Bridge		
		Truck 1	Truck 2	Truck 1	Truck 2	Truck 3
R1	Midspan x=30	16(7.05) 16(4.86) 16(2.64) 16(2.65) 4(2.64) 4(2.65)	16(4.57) 16(3.32) 16(2.83) 16(2.53) 4(1.89) 4(2.00)	16(6.10) 16(4.55) 16(2.86) 16(2.73) 4(2.32) 4(2.45)	16(5.25) 16(3.65) 16(2.79) 16(2.60) 4(2.00) 4(2.08)	16(3.05) 16(2.64) 16(2.37) 16(2.16) 4(1.78) 4(1.92)
	Σ	296.4	227.6	278.9	245.0	178.3
	2 Trucks 3 Trucks	296.4 + 227.6=<u>524.0</u>^{1k}		278.9 + 245.0 = 523.9^{1k} (523.9 + 178.3) 0.9 = <u>632.0</u>^{1k}		
	Quarter-span x=15	16(6.01) 16(3.83) 16(2.58) 16(2.42) 4(0.29) 4(0.29)	16(3.44) 16(2.24) 16(2.22) 16(2.35) 4(0.12) 4(0.09)	16(5.36) 16(3.58) 16(2.59) 16(2.45) 4(0.29) 4(0.29)	16(4.13) 16(2.64) 16(2.25) 16(2.16) 4(0.53) 4(0.41)	16(2.28) 16(1.80) 16(1.80) 16(1.76) 4(1.04) 4(0.76)
	Σ	239.8	164.8	226.0	182.6	129.4
	2 Trucks 3 Trucks	239.8 + 164.8=<u>404.6</u>^{1k}		226.0 + 182.6 = 408.6^{1k} (408.6 + 129.4) 0.9 = <u>484.2</u>^{1k}		
R2	Midspan x=30	16(6.36) 16(4.14) 16(2.00) 16(1.96) 4(2.00) 4(1.96)	16(2.30) 16(2.10) 16(1.82) 16(1.66) 4(1.17) 4(1.00)	16(6.36) 16(4.14) 16(2.00) 16(1.96) 4(2.00) 4(1.96)	16(2.81) 16(2.25) 16(2.14) 16(1.79) 4(1.50) 4(1.25)	16(1.84) 16(1.56) 16(1.50) 16(1.26) 4(1.08) 4(0.95)
	Σ	247.2	134.8	247.2	154.8	106.7
	2 Trucks 3 Trucks	247.2 + 134.8=<u>382.0</u>^{1k}		247.2 + 154.8 = 402.0^{1k} (402.0 + 106.7) 0.9 = <u>457.8</u>^{1k}		
	Quarter-span x=15	16(5.97) 16(3.25) 16(1.94) 16(1.91) 4(0.30) 4(0.25)	16(1.76) 16(1.41) 16(1.24) 16(1.18) 4(0.62) 4(0.42)	16(5.97) 16(3.25) 16(1.94) 16(1.91) 4(0.30) 4(0.25)	16(2.37) 16(1.63) 16(1.39) 16(1.43) 4(0.69) 4(0.40)	16(1.22) 16(1.20) 16(1.10) 16(0.98) 4(0.51) 4(0.37)
	Σ	211.2	93.6	211.2	113.5	75.5
	2 Trucks 3 Trucks	211.2 + 93.6=<u>304.8</u>^{1k}		211.2 + 113.5 = 324.7^{1k} (324.7 + 75.5) 0.9 = <u>360.2</u>^{1k}		

(4) Number of wheel loads taken by rigid bridge girders,

$$N_i^r = \frac{1}{0.68+1.0+1.0+0.68} N_T = 0.298 \; N_T$$

For 2-lane bridge, girder R1, $N_i^r = 0.298 \; (4) = 1.192$

girder R2, $N_e^r = 0.68 \; (1.192) = 0.811$

For 3-lane bridge, girder R1, $N_i^r = 0.298 \; (5.4) = 1.609$

girder R2, $N_e^r = 0.68 \; (1.609) = 1.092$

(5) α-factor for girder R1, from Eq. (23),

$$\alpha_{i_2} = 1 + \left[\left(\frac{1.37}{0.14+\sqrt{.595}}\right) - \left(0.709 + \frac{5}{9.33} - 0.006(3)\right)\right]\left[\left(\frac{18.5}{60+3.16}\right) + 0.033\right] \times$$

$$\times \left[\left((0.0.0966) \; (9.33) - 0.21\right) (3) - 0.6\right]$$

$= 1.13$ for 2-lane bridge

For the 3-lane bridge, replace $\beta = 0.595$ by 0.893, leading to $\alpha_{i_3} = 1.02$.

For girder R2, Eq. (24) gives

$$\alpha_{e_2} = \left[1.44 - 0.225 \; (1.0) - \left(\frac{1.3-0.2}{750}\right) (60)\right] 1.13$$

$= 1.27$ for 2-lane bridge

$$\alpha_{e_3} = \left[1.44 - 0.225 \; (1.0) - \left(\frac{1.3-0.2}{750}\right) (60)\right] 1.02$$

$= 1.14$ for 3-lane bridge

For comparison, the corresponding α-factors from Table 12 are

$\alpha_{i_2} = 1.13$; $\alpha_{i_3} = 1.02$; $\alpha_{e_2} = 1.23$; $\alpha_{e_3} = 1.05$

(6) Maximum number of wheel loads,

for 2-lane bridge, girder R1, $N_{WL} = 1.13 \; (1.192) = 1.349$

girder R2, $N_{WL} = 1.27 \; (0.811) = 1.030$

for 3-lane bridge, girder R1, $N_{WL} = 1.02\ (1.609) = 1.640$

girder R2, $N_{WL} = 1.14\ (1.092) = 1.246$

(7) Moving one wheel line, consisting of two concentrated loads of 16^K and one load of 4^K, spaced at 14 feet, along the midspan (x=30) and quarterspan (x=15) moment influence lines of a simply supported beam with a span of 60 feet, leads to the critical moments

$$M^o_{x=30} = 16\ (15) + 16\ (8) + 4\ (8) = 400^{1K}$$

$$M^o_{x=15} = 16\ (11.25) + 16\ (7.8) + 4\ (0.9) = 308.4^{1K}$$

Multiplying these moments with the number of wheel lines obtained in step (6), yields for the 2-lane bridge

$$M_{R1,30} = 1.349\ (400) = 540^{1K};\ M_{R1,15} = 1.349\ (308.4) = 416^{1K}$$

$$M_{R2,30} = 1.030\ (400) = 412^{1K};\ M_{R2,15} = 1.030\ (308.4) = 318^{1K}$$

and for the 3-lane bridge,

$$M_{R1,30} = 1.640\ (400) = 656^{1K};\ M_{R1,15} = 1.650\ (308.4) = 506^{1K}$$

$$M_{R2,30} = 1.246\ (400) = 498^{1K};\ M_{R2,15} = 1.246\ (308.4) = 384^{1K}$$

The corresponding moments for the case that the α-factors from Table 12 had been used instead of Eqs. (23) and (24), are

540^{1K}, 416^{1K}, 396^{1K}, 306^{1K} for the 2-lane bridge

656^{1K}, 506^{1K}, 456^{1K}, 352^{1K} for the 3-lane bridge.

6.4.3 Present AASHO Design Method

Using the present AASHO design formulas for comparison, no distinction is made between the 2-lane and thd 3-lane bridge. Thus one gets for the number of wheel loads

for girder R1, $N_{WL} = \frac{S}{7} = \frac{9.33}{7} = 1.331$

for girder R2, $N_{WL} = \frac{We}{7} = \frac{3+4.667}{7} = 1.095$

and the critical moments become

$$M_{R1,30} = 1.331\ (400) = 532.4^{\prime K};\quad M_{R1,15} = 1.331\ (308.4) = 411^{\prime K}$$

$$M_{R2,30} = 1.095\ (400) = 438^{\prime K};\quad M_{R2,15} = 1.095\ (308.4) = 338^{\prime K}$$

6.4.4 Discussion

All design moments so far obtained are summarized in Table 29. Also listed there are the percentage errors of the various methods with reference to the accurate method.

TABLE 29. COMPARISON OF DESIGN MOMENTS IN FT. KIPS FOR EXAMPLE BRIDGE, USING DIFFERENT METHODS OF DESIGN

Design Method			Accurate		Simplified Eqs.(23)&(24)		Simplified (Table 12)		AASHO Des. Form.	
No. of Lanes	Gir-der	At	Des. Mom.	Error%	Des. Mom.	Error%	Des. Mom.	Error%	Des. Mom.	Error%
2	R1	Mdsp	524	-	540	+3.1	540	+3.1	532	+1.5
		Qtrsp	405	-	416	+2.7	416	+2.7	411	+1.5
	R2	Mdsp	382	-	412	+7.9	396	+3.7	438	+14.7
		Qtrsp	305	-	318	+4.3	306	+0.3	338	+10.8
3	R1	Mdsp	632	-	656	+3.8	656	+3.8	532	-15.8
		Qtrsp	484	-	506	+4.5	506	+4.5	411	-15.1
	R2	Mdsp	458	-	498	+8.7	456	-0.4	438	-4.4
		Qtrsp	360	-	384	+6.7	352	-2.2	338	-6.1

As can be seen in Table 29, the proposed simplified design method does yield fairly good results. Only the exterior girder is somewhat overdesigned at midspan, because Eq. (24) overestimates the α-factors given in Table 12. As Table 25 reveals, this discrepancy for the example bridge studied in this section, is one of the few extremes.

A refinement of design Eq. (24) would lead towards the results obtained by using the α-factors from Table 12, and agreement with the accurate method of design would be excellent.

A design using the AASHO-formulas would overdesign the exterior girder considerably, if 2 lanes were to be placed on the bridge. However, if the bridge were to carry 3 traffic lanes, the AASHO formulas result in both, exterior and interior girders being underdesigned. This result affirms the conclusion drawn from Fig. 40, that for higher β-values, the AASHO-formula leads to underdesigned structures.

7. CONCLUSIONS

The concrete box girder bridge has proven to be an economical structural system to satisfy the needs of a modern highway system. While its use is therefore widespread, an accurate analysis is very complex and feasible only with the aid of a high-speed digital computer.

The method of design presently being used is oversimplified. The determination of wheel loads on a bridge girder is based only on the web spacing of the girders, neglecting all other important bridge parameters such as the number of lanes, number of cells, span and end boundary conditions. It has been shown that this might lead to underdesigned structures in cases where narrower traffic lanes are used or effective spans are small, while conversely for wider traffic lanes and longer effective spans overdesigned structures may occur.

The widespread use of this bridge type makes it mandatory to have a method of design available by means of which the real structural behavior is better described, with the result that at any point in the bridge, stress or strength requirements are met satisfactorily with the greatest possible economy.

In this investigation, previous analytical studies have been used to study the characteristics of wheel load distribution in straight box girder bridges, both simply supported and continuous, in terms of the major design variables. As a measure of load distribution, the concept of a girder load concentration factor has been introduced, which is a magnification factor of the number of wheel loads carried by a given girder of a rigid bridge whose cross section can only deflect uniformly. These load concentration factors were calculated from the actual number of wheel loads taken by the bridge girders using influence lines for girder moment percentages under concentrated loads moving across the bridge at midspan which were determined by accurate computer analyses. Extensive studies have been made to determine how these girder load concentration factors are influenced by the number and width of cells, by the number of lanes, the span and continuity to adjacent spans, by the longitudinal position considered (midspan or interior support), and by a slab overhang.

It has been shown that the present design concept of treating each individual bridge girder independently of the rest of the bridge, may lead to satisfactory designs provided that the fractions of wheel lines for which the girders are being designed, are determined more accurately than at the present. Instead of basing the calculation on the cell width only, the other aforementioned parameters should be considered also.

Empirical formulas based on the above parameter studies have been derived in this report which permit a more accurate calculation of the number of wheel loads for which an individual bridge girder should be designed than does the present method of design.

As an alternative, an accurate design method has been presented which is based on the determination of girder moment influence surfaces. While the simplified method of design is easy to use, the accurate design method is lengthy, although it can be automated to a great extent by interaction with a digital computer.

The reports published to date in the research on box girder bridges being conducted at the University of California have dealt with straight, non-skew box girder bridges. At present, analytical and experimental studies are being made on skew box girder bridges and additional studies on curved box girder bridges are being initiated.

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