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Recent experiments at high energies have indicated that the width of the diffraction peak in the elastic cross section is considerably smaller in $\bar{p}p$ scattering than in pp scattering.[1,2] On the other hand, the total cross section for $\bar{p}p$ is greater than than for pp.[3] We have then the qualitative feature that the larger total cross section is associated with the narrower diffraction peak. The purpose of this letter is to investigate whether this feature may be understood in terms of a Regge pole model for high-energy scattering. We find that, because all Regge exchanges give a positive coefficient for the residue function in the contribution to the imaginary part of the $\bar{p}p$ amplitude, this feature can be understood only if some residue functions are allowed to be negative.

(I) We consider first the three-pole model of Hadjioannou, Phillips, and Rarita in which only helicity nonflip amplitudes are considered:[4]

$$
\begin{aligned}
A_{\bar{p}p} &= P + P' - \omega \\
A_{pp} &= P + P' + \omega \\
\frac{d\sigma}{dt} &= |A|^2 \\
\sigma_T &\propto \operatorname{Im} A(t = 0) .
\end{aligned}
\tag{1}
$$

We have used the trajectory symbols to denote the contributions to the amplitude which are of the form

$$\alpha(t)[2\alpha(t) + 1]\zeta \; b(t) \; E^{\alpha(t)},$$

where E is the laboratory energy, t the squared momentum transfer, and ζ is given by

$$\zeta = \tau i - \cot[\pi \alpha(t)/2] \qquad \text{for even signature,}$$

$$\zeta = \tau i - \tan[\pi \alpha(t)/2] \qquad \text{for odd signature.}$$

The j parity τ is $+1$ for even signature, -1 for odd. Since $\sigma_T(pp)$ is constant, we have

$$\alpha_{P'}(0) = \alpha_{\omega}(0) ,$$

$$b_{P'}(0) = b_{\omega}(0) = \text{const}[\sigma_T(\bar{p}p) - \sigma_T(pp)]/E^{1-\alpha_{P'}(0)} .$$

If $b_{\omega}(t)$ is positive definite this model cannot explain the fact that

$$d\sigma_{\bar{p}p}/dt < d\sigma_{pp}/dt \qquad (\text{for } -t > +0.2) .$$

We have verified this in detail by machine computations. We were not able to achieve a χ^2 less than 1000 in this model; there are 63 data points. The result may be understood by the following argument: if $(\text{Re } A)^2 < (\text{Im } A)^2$ the P-ω cross term in $d\sigma/dt$ is positive for $\bar{p}p$ and negative for pp. Requiring $\alpha_P'(0) < 0.5$, as inferred from the pion proton data,[5] implies $(\text{Re } A)^2 > (\text{Im } A)^2$ only if there is a large ω contribution to Re A arising because $\alpha_{\omega}(0)$ $[= \alpha_{P'}(0)]$

is near 1. The ω contribution then dominates Re A, since $\alpha_P(0) = 1$, and $(\mathrm{Re}\ A)^2$ is the same for pp or $\bar{p}p$.

(II.) We consider next the extension of the three-pole model to include helicity flip amplitudes. Wagner has shown, by use of the factorization theorem, that the differential elastic cross section due to trajectories with $\tau\pi = +1$ (π is the parity; this condition is satisfied for P, P', and ω) can be written[6]

$$\frac{d\sigma}{dt} = \sum_{i,j} A_{ij}\, \zeta_i\, \zeta_j^*\, E^{\alpha_i+\alpha_j-2} [b_{i+}(t)\, b_{j+}(t) - t b_{i-}(t)\, b_{j-}(t)]^2$$

where $A_{ji} = A_{ij} = -1$ for $i = P, P'$, $j = \omega$ and $A_{ij} = 1$ otherwise for pp scattering; and $A_{ij} = 1$ for $\bar{p}p$ scattering; all other factors are the same for pp and $\bar{p}p$. The "Regge couplings" $b_{i\pm}(t)$ are the square roots of the residue functions, $[2\alpha_i(t) + 1]\alpha_i(t)\, b_i(t)$. If $b_{\omega\pm}$ are real, the same argument used in (I) shows that the inclusion of helicity flip does not change the result of (I).

(III.) If we relax the positive definiteness requirement on $b_\omega(t)$ in (I), or equivalently, allow $b_{\omega\pm}$ in (II) to become imaginary, the narrower $\bar{p}p$ peak may be explained. We have obtained a fit to the data of Ref. 1 by taking for $b_\omega(t)$ the form

$$b_\omega(t) = b_{P'}(0) \left[(1+g)e^{a_1 t} - g\, e^{a_2 t} \right] \tag{2}$$

with $a_2 < a_1$, and for $b_P(t)$ and $b_{P'}(t)$ the form

$$b(t) = b(0)e^{at} . \tag{3}$$

The exponential form in (2) and (3) is merely a convenient parametrization of the decrease of the residue function in a small region of momentum transfer. For $\alpha_i(t)$ we have taken the form

$$\alpha_i(t) = -1 + [1 + \alpha_i(0)]^2 [1 + \alpha_i(0) - \alpha_i'(0)t]^{-1} ,$$

which has the property of containing the least amount of curvature consistent with $\alpha_i(t)$'s being a Herglotz function.[7] Our fit to the data thus involves nine parameters: g, a_1, a_2 a_P, $a_{P'}$, $\alpha'_P(0)$, $\alpha'_{P'}(0)$, $\alpha'_\omega(0)$, and $\alpha_{P'}(0)$, two of which, $\alpha'_P(0)$ and $\alpha'_{P'}(0)$, may be taken from πp data $(\alpha'_P(0) \approx \alpha'_{P'}(0) \approx 0.3)$.[5] We have found that the data may be fitted for values of the last four parameters in a wide range so that, in practice, only five parameters are necessary. In Figs. 1 and 2, we show the results, for the pp and $\bar{p}p$ cross sections, of minimizing χ^2 for a particular choice of the last four parameters. The variation of $b_\omega(t)$ needed to fit the data is very rapid $[b_\omega(-0.2) \approx -b_\omega(0)/2]$, and somewhat hard to understand since b_ω is expected to obey a dispersion relation with only a right-hand cut (starting at $4m_\pi^2$). There is, however, an alternative interpretation of the result. The second term in Eq. (2) may be the residue from the ϕ trajectory if $\alpha_\phi(t) \approx \alpha_\omega(t)$ for the range of t under discussion. Whichever interpretation is adopted, it is necessary for a residue function to become negative.

(IV.) For the sake of completeness, we consider contributions from other possible trajectories having charge conjugation number, C , equal to -1 . Three types of trajectory are possible.[8,9]

(a) Other trajectories with $C = \tau = \pi = -1$ add to (not subtract from) ω .[10] The conclusion of (III) that some residue function must become negative is therefore not altered if some trajectory of this type, other than ω (and ϕ), is important. (b) Making use of the helicity amplitudes (ϕ_1) defined by Goldberger et al.,[11] we have that a trajectory with $C = \tau = -\pi = -1$, such as an axial vector meson, contributes equally to ϕ_2 and ϕ_4 .[12] Since the P contributes with equal magnitude and opposite sign to ϕ_2 and ϕ_4 , no difference between $\bar{p}p$ and pp can arise from such a trajectory. (c) A trajectory with $C = \pi = -\tau = -1$ might be of importance, since it contributes to the nonhelicity-flip amplitudes ϕ_1 and ϕ_3 proportionally to $-4t$ and $+t$, while P contributes in the ration 2:1. Such a trajectory, however, would also require a negative residue function to narrow the $\bar{p}p$ peak relative to the pp peak.

If an argument can be found to preclude negative Regge pole residue functions, the explanation of the pp and $\bar{p}p$ diffraction peaks will, presumably, involve Regge cuts.

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FOOTNOTES AND REFERENCES

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10. This conclusion is independent of the isotopic spin of the trajectories since, for pp scattering, both $T = 0$ and $T = 1$ trajectories contribute equally and with the same sign, while the contribution to $\bar{p}p$ is obtained from the pp contribution by multiplying by C.

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12. The contributions to ϕ_i , $i = 1,\cdots,5$, may be obtained from the results of reference 8 for the amplitudes $\overline{G}_i$ by means of the asymptotic form of a crossing matrix which (taking into account that contributions to $\overline{G}_2$ and $\overline{G}_4$ are, at most, like $s^{\alpha-2}$, and to $\overline{G}_3$, at most, like $s^{\alpha-1}$) is

$$(\phi) \approx s^{-1/2} \begin{pmatrix} 0 & 0 & -4s & 0 & 2 \\ -t & 0 & 0 & 0 & -t \\ 0 & 0 & s & 0 & 1 \\ -t & 0 & 0 & 0 & t \\ 0 & 0 & 0 & 0 & \sqrt{t} \end{pmatrix} .$$

FIGURE CAPTIONS

Fig. 1. Differential cross section for elastic pp scattering as a function of $\log_{10} s$. The data are from reference 1. The curves were computed for $a_P = 3.65$, $a_{P'} = 8.40$, $a_1 = 6.94$, $a_2 = -0.161$, and $g = 0.517$; $\alpha_\omega(0) = 0.5$, $\alpha'_P = \alpha'_{P'} = \alpha'_\omega = 0.34$ were kept fixed to give a χ^2 equal to 60.0 for 63 data points (48 for pp and 15 for $\bar{p}p$).

Fig. 2. Differential cross section for elastic $\bar{p}p$ scattering as a function of $-t$. The data are from reference 1. The curves are for the same parameters as in Fig. 1. The dashed lines give pp scattering for comparison.

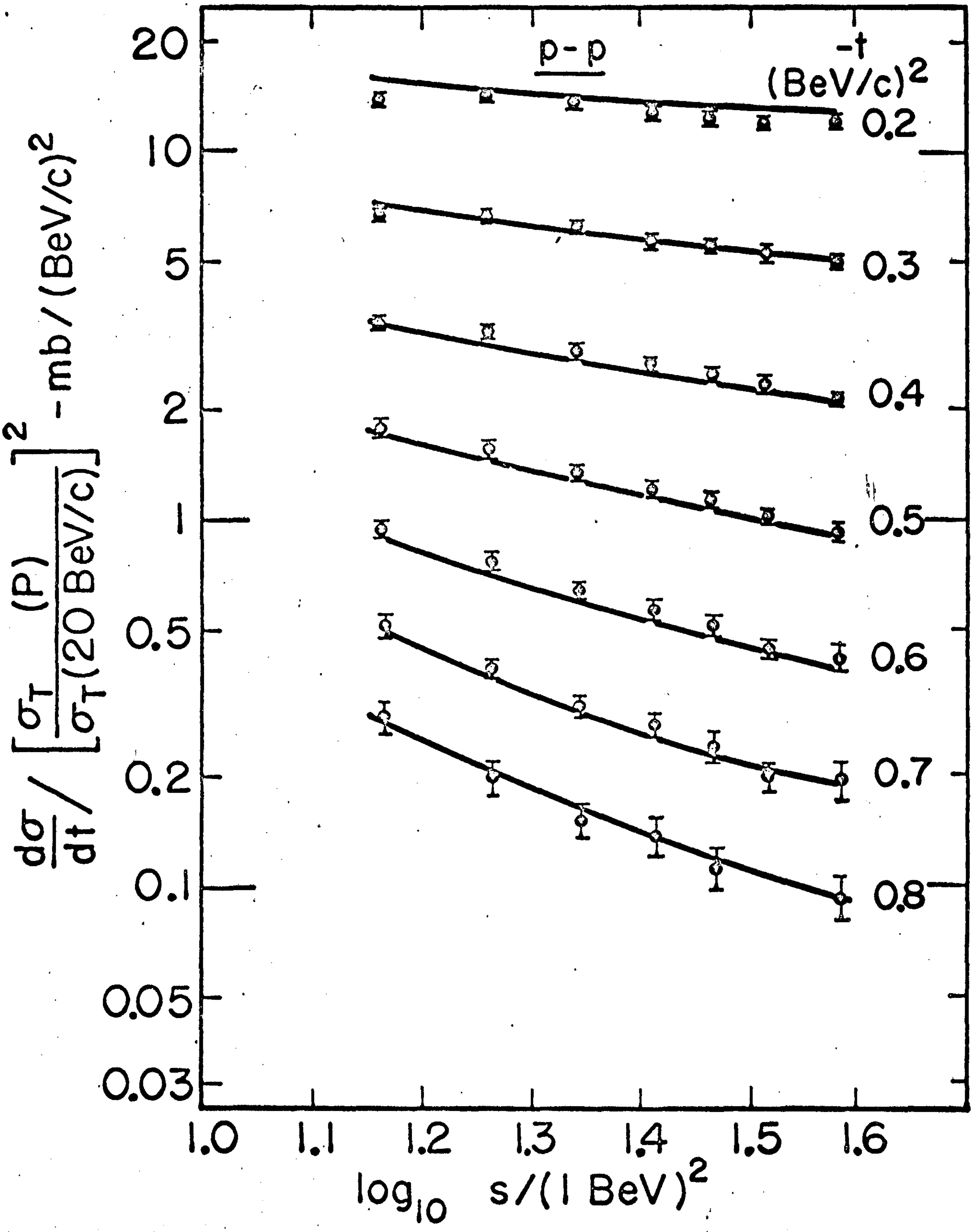

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Fig. 1

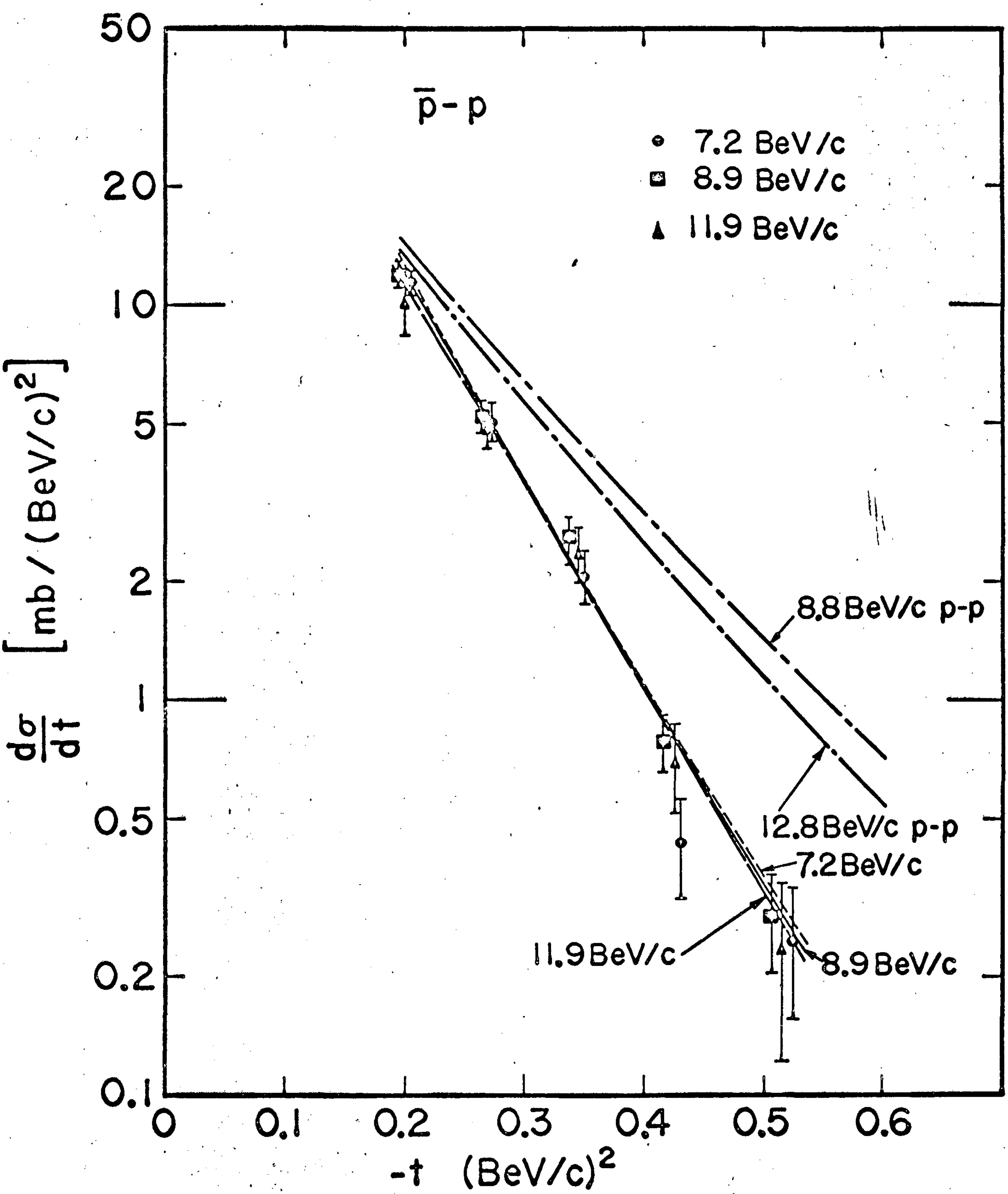

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Fig. 2.