Calibration of Large-Scale Economic Models of the Agricultural Sector and Reliable Estimation of Regional Derived Demand for Water

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Abstract

Given the importance of regional response in physical and economic terms to water resource planners, large scale regional models are a widely used component of water resource management. Given the absence of a sufficiently rich data base to estimate cost functions by econometric approaches, linear constrained optimization models have been extensively used to derive normative estimates of former response to water policy.

A long standing problem in linear models is the inevitable trade-off between the precision of calibration of the model and the constrained nature of the solution. Often model results significantly depart from empirical reality with deleterious effects on policy. This research shows that a nonlinear cost function formulation overcomes the constraint problem when the function parameters are estimated from actual farmer responses.

The Positive Quadratic Programming (PQP) theory developed was applied to the California Agricultural and Resources Model (CARM) and used to estimate demand functions for irrigation water by region.

Justification and Objectives

Allocation of water resources is increasingly performed indirectly through economic incentives rather than by direct physical mandates. The desirability and efficiency of economic policy instruments depends on the relative responsiveness of water uses to changes in economic incentives. This responsiveness is summarized in the form and estimation of the "derived demand" function for water. Since agriculture is the major water user in the Western States, the ability to accurately estimate regional agricultural demand functions for water limits the current capabilities of economic incentives as a regional water policy instrument.

The ability to estimate water demand functions is currently limited both by data and methodology.

The data problem stems from both its absence in a sufficiently disaggregated form and the lack of past observations at water prices that have future policy relevance. Thus traditional derived demand estimation by econometric methods must be presently discounted.

An alternative methodology based on available farm management data sources is to construct normative programming models of regional farm production which yield the demands for irrigation water. However, the linearity of these models leads to the problem of excessive constraints which consequently limits the policy value of the resulting derived demands.

The objective of this research project is simple. To develop and demonstrate an alternative to linear programming models that

(a) closely calibrates with actual farmer actions and thus is believable

(b) Is not tightly constrained and thus is able to respond to future policy scenarios.

The methodology is demonstrated by an application to an already existing regional programming model of California agricultural production.

Methodology Review and Theory

Since the introduction of linear programming for economic analysis, it has been recognized that the linear constraint set implies Leontief linear production technology. In this section, a common situation is specified in which the cost functions which satisfy the first order conditions for profit maximization differ from those resulting from linear production functions. The positive quadratic programming (PQP) specification is based on the discrepancy between the linear cost function and the cost function implied by the farmer's actions. In addition, the PQP specification is shown to be consistent with the first and second order conditions for production in the "rational" region of a production function.

Specifying a multi-output linear programming problem as

(1) $\text{Max} \quad r^T x$

Subject to $Ax \leq b$

where x is an n x 1 vector of outputs, r an n x 1 vector of net returns, b an m x 1 vector of inputs, and A an m x n matrix of linear production function coefficients.

The optimal solution of k outputs $\bar{x}$ will be associated with the optimal basis matrix B and the vector of constraining resources $\bar{b}$ as:

(2) $B\bar{x} = \bar{b}$

$B = kxk \quad \bar{x} = kx1 \quad \text{and} \quad \bar{b} = kx1, \; k \leq m$

It follows directly from the linear independence of B that the vector dimension of optimal LP outputs is equal to the number of binding constraints at the optimum which has an upper bound of m.

In microeconomic studies of farms, the number of empirically justifiable constraints are comparatively few. Land area and soil type is clearly a constraint, as is water in some irrigated regions. Crop contracts and quotas, building capacities, breeding stock, finance, managerial skills, and perennial crops are others. However, it is rare that some other traditional constraints such as labor, machinery, or crop rotations are truly restricting to short-run production decisions. These inputs are limiting, but only in the sense that once exceeded, the cost per unit output increases due to overtime, increased probability of disease, or machinery failure.

In contrast, the sectoral or regional model has greater constraint aggregation and fewer empirically justifiable constraints. However, the dominant arguments of less suitable soil types and less experienced management used as crop acreage is expanded on a regional basis to provide an intuitive basis for increasing regional cost functions. The empirical situation in which POP is an appropriate technique is when the number of crop outputs that the farmers actually produce exceeds the number of truly inflexible short-run constraints on factor inputs. We think that the majority of representative farm and regional programming models fall into this category. If the farmers are producing more crops than the number of binding constraints, they must be producing the more profitable crops at a level where the marginal expected profit is zero and the profit function for that crop conditional on the binding constraints has an interior solution. To reiterate, if farmers are observed to produce ℓ crops but there are only k ($k < \ell$) real constraints binding at the optimum, then farmers must expect ℓ-k unconstrained interior solutions for the most profitable crops. This in turn implies that the expected profit function must be concave in the region of the optimum output of ℓ-k crops.

This conclusion is based on the assumption of optimizing behavior, inherent in all programming models, and Muth's [1961] concept of rational expectations in which "expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory."

Since the model is at the microeconomic level, theory tells us that the demand function is reasonably assumed perfectly elastic. There are some special cases where regional and seasonal specialization could cause some price effect, but, given the collective nature of the effect, a rational individual will not act on it. The revenue is linear in output and thus the concavity of the expected profit function in output must be contained in the expected cost function for those crops with interior solutions, hereafter termed nonmarginal crops. Thus the cost function for a particular nonmarginal crop can be expressed as a function of the output level.

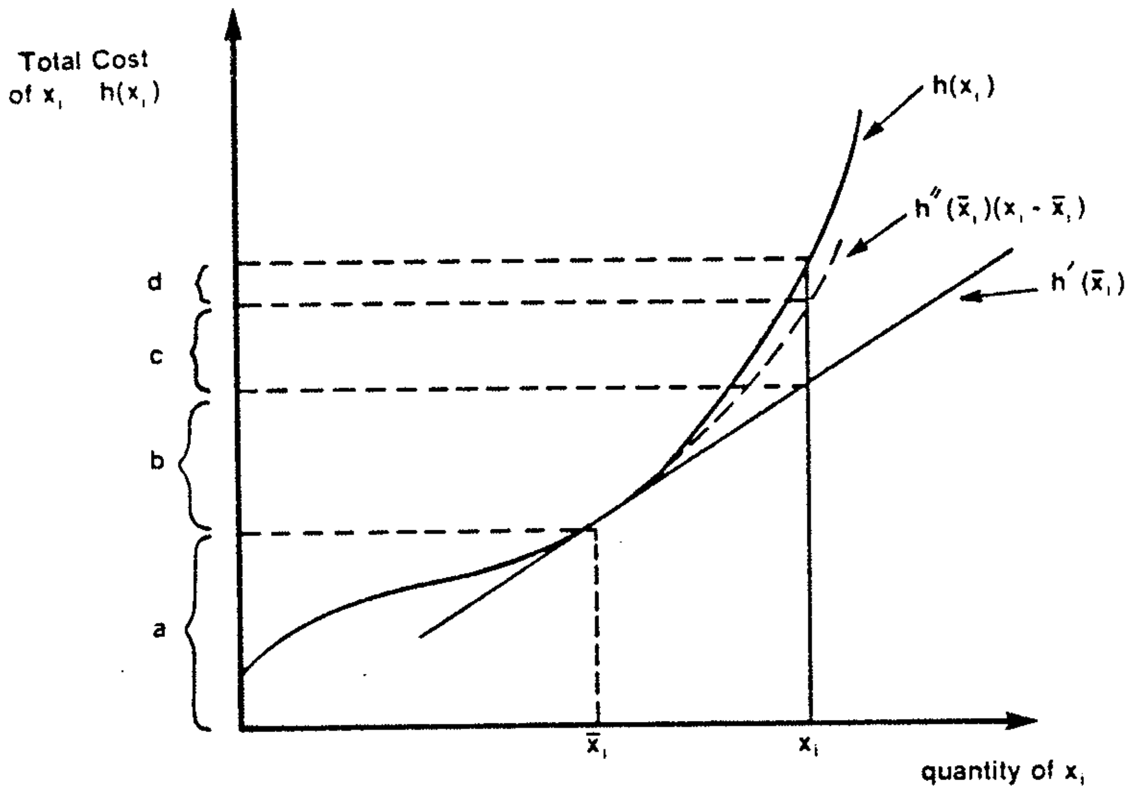
(3) $$c_i = c_i(x_i) \qquad i = 1 \ldots (\ell-k)$$

Using a Taylor's series expansion around the output level that minimizes expected average variable cost for the region $(\overline{x}_i)$, the total cost of producing x_i units of crop i can be decomposed into four parts (Figure 1).

(4) $$c_i(x_i) = c(\overline{x}_i) + c'(\overline{x}_i)\ (x_i-\overline{x}_i) + c''(\overline{x}_i)(x_i-\overline{x}_i)^2 + r_3.$$

The first term on the right hand side is the expected fixed cost of producing crop i at the level $\overline{x}_i$. The second linear term is the cost of unit increases

Figure 1
Taylor Series Expansion of the Cost Function



a = $h(\bar{x}_i)$ — fixed cost

b = $h'(\bar{x}_i)\,(x_i - \bar{x}_i)$ — Linear Variable Cost Term

c = $h''(\bar{x}_i)\,(x_i - \bar{x}_i)^2$ — Quadratic Variable Cost Term

d = r_3 — remainder term of higher orders in output i and is the linear variable cost of producing output i. In the traditional LP production models, the first fixed cost term may or may not be included in the objective function since it does not affect the optimal solution. The linear variable cost coefficients $\nabla_x c(\overline{x}_i)$ are obtained from farm management surveys, experimental data, and farm interviews. However, the third quadratic term cannot be zero for the (ℓ-k) nonmarginal crops which have interior solutions. The two most persuasive theoretical reasons for this positive definite quadratic term are, first, the decreasing returns observed in the rational region of production of conventional production functions and, second, the costs of risk caused by changes in the output mix. The microeconomic implications of these alternative theories will be explored later, but for a moment the pragmatic analyst will recognize that a large amount of usually unobtainable data would be needed to estimate the individual components of the quadratic cost term.

Taking a positive approach to the model, the theoretical basis of the cost is desirable but not essential to short-run analysis of changing comparative advantage resulting from specified policy shifts. The quadratic cost term implicit in the observed production pattern of farmers is accordingly termed the implicit cost component.

Invariably the second order expansion will not capture the true nature of the cost function, and the remainder term r_3 contains the omitted higher order terms. However, the implicit cost considerably improves the model in that it enables the nonmarginal crops to be at interior solutions and the full range of crops actually produced by farmers to be represented by the model without the introduction of specious constraints that distort policy analysis. The quadratic implicit cost specification has the advantage that it can be easily estimated from dual values in the standard linear program and solved by readily available quadratic programs. These two steps are shown in the following section.

While the second order expansion is an approximation, the production function or input requirement set implied by the linear quadratic specification is consistent with the class of production functions that exhibit decreasing marginal productivity over some range, whereas the linear programming specification is not. Thus the PQP specification is not an ad hoc addition to the traditional programming specification, but a cost specification that is consistent with the combined effects of risk aversion and decreasing returns to scale in the production set.

The most common specification that yields a cost of risk that is quadratic in output levels is the mean variance approach based on Freund [1956]. There have been many modifications and applications of the mean variance concept which generally improves the diversification and reality of model output, but has not led to claims of complete validation or precise predictions.

Wicks [1978] shows that linear specifications of alternative risk formulations do not yield good predictive results. Weins [1976] used the Kuhn Tucker conditions and the resulting duals to estimate an aggregate risk aversion coefficient, but his results were hampered by the need for a single risk aversion coefficient implicit in approaches that specify risk as the only nonlinear effect on the regional or individual revenue function.

From the duality properties of cost and production functions the properties of the production function implied by the quadratic cost function can be deduced. Using Varian's [1978] proposition of an elasticity of scale measure

$$(5) \quad e(b) = \frac{df(Sb)}{ds} \frac{S}{f(b)}$$

where S is a scale parameter and f(b) is the implied production function. If b^* is the cost minimizing set of inputs for an output level x_i and input prices w_i then Varian shows that

$$(6) \quad e(b^*) = \frac{AC(x_i)}{MC(x_i)}$$

If $e(b^*)$ is less than one, the production technology exhibits decreasing returns to scale, and a unity value for $e(b^*)$ indicates constant returns to scale. Thus, the locally increasing average cost function used in the POP specification implies production in the "rational" range of the production function. In this context the typical farm or regional unit is defined by its truly fixed inputs (e.g., land, etc.), and the variable inputs are differentiated by cropping activity. Consequently, fertilizer applied to wheat is considered independent of fertilizer applied to cotton, and expansion of the cotton acreage results in the application of increased variable inputs to the fixed farm or regional resources.

There are many reasons why the decreasing returns to scale for cropping activities cannot be expected to be equal across regions. Soils and expected climatic conditions will vary, as will the structure and scale of the representative farm. The heterogeneity of farm types could change the expected returns to scale as could the technology embodied in customary regional farming practices. Ultimately, regional variability is an empirical question which is answered for us by the degree of regional crop specialization observed in practice.

It seems that in recognizing the existence of increasing expected average activity cost functions and the change of these functions across cropping activities and regions, the PQP specification is a closer approximation to neoclassical micro theory than the conventional linear specification on which it is based.

Calibration and Solution of the P.Q.P. Problem

In this section we prove that the dual values of a linearly constrained problem (LP or QP) provide the coefficient values of a quadratic term, which, when added to the LP objective function, results in an unconstrained problem that has an optimal solution identical to the constrained LP problem. This result is then used to illustrate how an LP problem which requires additional constraints to realize the empirically observed output levels can be reformulated as a quadratic program that only contains the true fixed resource constraints, but exactly reproduces the vector of constrained and unconstrained output levels observed in the calibration period.

To reiterate, the PQP approach uses the information contained in the empirical observations of crop acreages actually grown, to derive a quadratic cost term. The cost function now satisfies the unconstrained profit maximizing conditions for nonmarginal crops at the output levels that farmers chose on the average in the district. That is, the equilibrium marginal cost that results from the PQP approach is the one that rational profit maximizing farmers would have expected in that year and region in order to have decided on the acreages that they did.

The PQP approach is related to the penalty function approach to programming solutions with nonlinear constraints (S.U.M.T) (Fiacco and McCormick [1968]); however, the economic problem has two important differences. First, the penalty function approach uses nonlinear costs in the objective function to approximate the effect of nonlinear constraints. Whereas in PQP the artificial calibration constraints are used to impute the real, but unknown costs.

Second, sequential unconstrained minimization techniques (S.U.M.T.) use arbitrarily high penalty costs to achieve the constraints, while the PQP implicit cost is based on marginal conditions and only equals the constraint for the calibration year or years. In the microeconomic problem, we have shown that the additional calibration constraints needed to produce reasonable results for the more profitable nonmarginal crops are approximations to compensate for the absence of a specific nonlinear cost term in the objective function. The calibration constraints are approximating the marginal conditions and will undoubtedly change under different policy scenarios, thus representing them by constraints greatly reduces the policy value of results from these models. If the policy scenario dictates an increase in the comparative profitability of a given nonmarginal crop in a region, the the calibration constraints will restrict expansion of the crop acreage and consequent policy prescriptions will be determined by arbitrary constraint relaxation by the analyst. A formal extrapolative method for constraint relaxation is found in "Recursive Programming," Day [1962]. The fundamental hypothesis of the Day approach is that the rate of response to comparative advantage is determined by historical extrapolation rather than the degree of change in comparative advantage. In times of rapid change for the agricultural sector, this would seem to be a difficult assumption to substantiate.

Empiricial validation of programming models requies that the analyst has observations on the regional output levels for one or more years. The central thrust of this paper is that this source of empirical data is most usefully used not to constrain the final model, but to estimate the missing quadratic term in the cost function for each nonmarginal crop activity in each region in a way that is consistent with the linear data in the model and the truly binding resource and management constraints. Fortunately, this can be achieved by a straightforward two step procedure.

The following theorem proves that if linear transformations of the optimal dual values associated with the binding calibration constraints are used as the coefficients in a quadratic cost term, the resulting optimal solution to the quadratic program without any calibration constraints will be identical to the fully constrained linear program. That is, the transformed dual variables are the optimal estimates of the quadratic cost coefficients that achieve the observed interior solutions. The term estimate is used generally, since most programs are calibrated against a single year's data which results in a deterministic onto mapping from the set of empirical acreages to the set dual values.

Given a time series of base runs and resulting calibration duals the optimal expected implicit cost can be estimated by two alternative methods. For small dimension base run models, the mean implicit cost can be estimated endogenously by a simultaneous self dual specification. Where the latter approach is precluded by model dimensions or the length of the time series, a time varying stochastic parameter approach (Duncan and Horn [1972]) can be employed to estimate the systematic change in the expected implicit costs. This analysis will be addressed in a subsequent paper.

The P.Q.P. Theorem

We define two problems P1 and P2 whose constraint structure is shown for two nonmarginal activities in Figures 2 and 3, respectively. The revenue component of the objective function f(x) can be thought of as linear or nonlinear. Problem P1 is the usual specification with a set of empirically justified resource constraints b and an equality calibration constraint for each regional crop activity observed. The right hand side of the calibration set is the actual acreages $\tilde{x}$ plus a small but critical perturbation factor ε, without which the Fritz John constraint qualification (Aoki [1971]) is violated (unless there are no binding resource constraints).

Problem P2 has the same (nx1) vector of possible activities x, the same revenue function f(x) and the true resource constraints $Ax \leq b$, but the offending calibration constraints have been removed and a concave but unknown function of calibration constraint set has been added to the objective function.

Problem P1. Max $f(x)$ $(f(\cdot): R^n \rightarrow R^1)$

(7) Subject to $Ax \leq b$

$$Ix = \tilde{x} + \varepsilon \qquad x \geq 0$$

where x = nx1 b = mx1. Rewriting in Lagrangian form and representing the two sets of binding constraints by the vector functions $g_1(x)$ and $g_2(x)$, we have:

(8) $$\text{Max } L(x, \lambda_1, \lambda_2) = f(x) - \lambda_1 g_1(x) - \lambda_2 g_2(x).$$

Define an arbitrary concave vector function of the calibration constraint set $g_2(x)$ as $h(g_2(x))$.

Figure 2 L.P. Problem P1

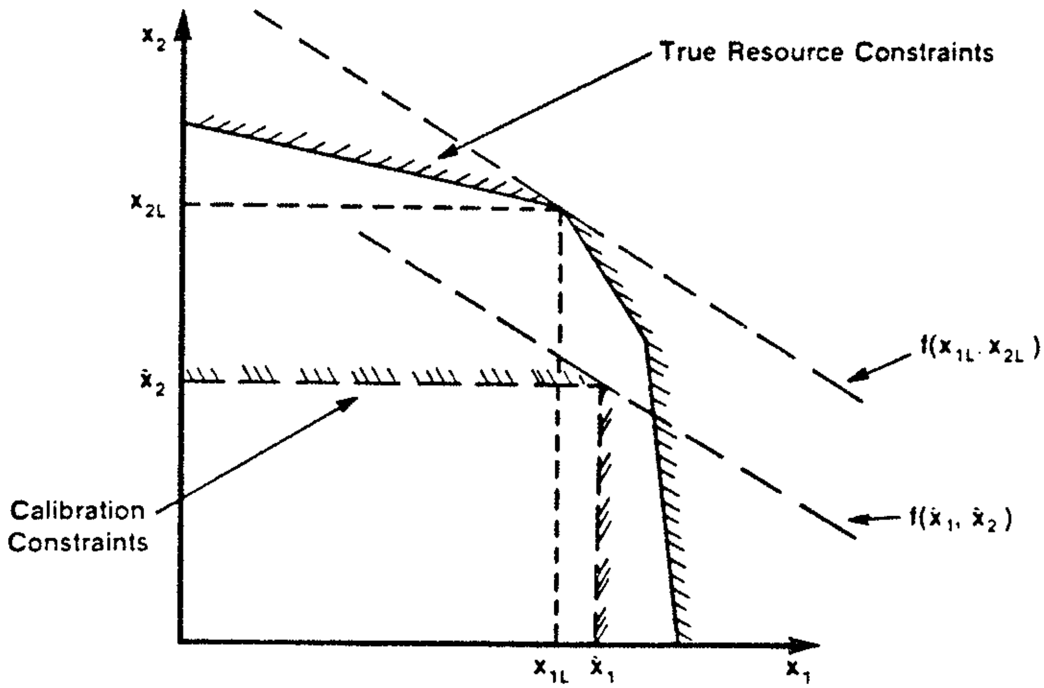
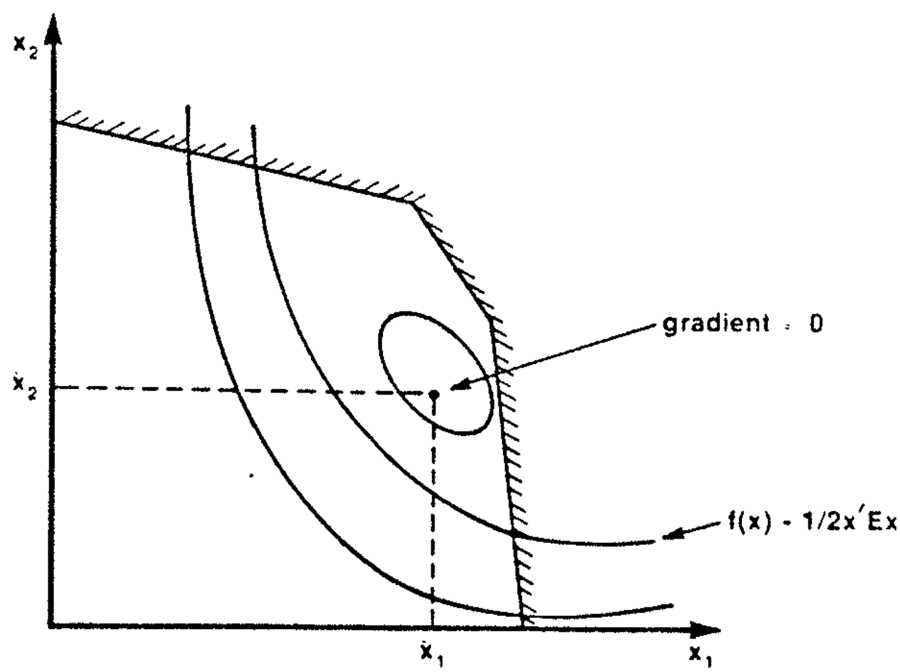


Figure 3 PQP Problem P2



<u>Problem P2.</u> Max $f(x) + h(g_2(x))$

(9) Subject to $Ax \leq b \qquad x \geq 0$

<u>Proposition</u>

If the optimal solution to P1 is characterized by x^*, λ_1^*, λ_2^*, the problem P2 has an optimal solution x° equal to x^* if and only if

(10) $\nabla_g h(g_2(x^\circ)) = -\lambda_2^*$

<u>Necessity</u>

Since f(x) is concave and continuous

(11) $x^* = x^\circ \Longrightarrow \nabla_x f(x^*) = \nabla_x f(x^\circ)$

Defining the Jacobian matrix of the set of constraint vectors $g_1(x)$ with respect to x as $J_x(G_1)$

The first order conditions for P1 require that:

(12) $\nabla_x f(x^*) = J_{x^*}(G_1)^T \lambda_1^* + J_{x^*}(G_2)^T \lambda_2^*$ 1/

The first order conditions for P2 are:

(13) $\nabla_x f(x^\circ) = -J_{x^\circ}(G_2)^T \nabla_g h(g_2(x^\circ)) + J_{x^\circ}(G_1)^T \lambda_1^*$

Equating (13) and (12) implies that:

(14) $J_{x^\circ}(G_2)^T \nabla_g h(g_2(x^\circ)) = - J_{x^*}(G_2)^T \lambda_2^*$

1/The notation $\nabla_x f(x^*)$ denotes the gradient function of f(x) with respect to the vector x at the optimal values x^*, and T superscript denotes the transpose.

Since the calibration constraint function $g_2(x)$ is linear, the Jacobian is constant and satisfaction of the constraint qualification implies that $J_x(G_2)^{-1}$ exists, therefore

(15) $\nabla_g h(g_2(x^\circ)) = -\lambda_2^* \text{ if } x^\circ = x^*$

Sufficiency

Substituting (11) into (14) and equating (13) to (14), the two revenue function gradients are equal at their respective optimal solutions.

(16) $\nabla_x f(x^\circ) = \nabla_x f(x^*)$

Since f(x) is continuous and concave, equality of the gradients implies equality of their arguments.

(17) $\therefore x^\circ = x^* \quad \text{if } \nabla_g h(g_2(x^\circ)) = -\lambda_2^*$

Implementation of the P.O.P. Approach

Empirical implementation of positive programming is achieved in two stages. The first stage starts with the data and specification of a conventional LP (or QP) problem. The linear cost part of the Taylor series expansion (4) $\nabla_x c(\bar{x})$ is incorporated as a vector of costs in the revenue function f(x). The actual regional crop acreages are increased by a small perturbation ε say (.005) $\tilde{x}$ and are formulated as equality constraints. The constrained LP problem is now run to obtain the dual values on the calibration constraints for the nonmarginal crops. The ε perturbation of the calibration constraint right hand side ensures that relevant resource constraints will be binding on the marginal crops in the basis. The absence of a quadratic cost coefficient for the marginal crops is not a problem as they are constrained by the active resource constraints.

Given the vector of dual values from P1 for the nonmarginal crops they are multiplied by the negative reciprocal of the observed acreage $\tilde{x}_i$ and used as the diagonal coefficients of the quadratic cost function in problem P2. Problem P2 is then solved for the optimal base period solution. The principle steps are:

a Given a standard LP or QP and the vector of actual acreage grown $\tilde{x}$. Perturb $\tilde{x}$ by ε and add the equality calibration constraints.

b Run problem P1. If $\tilde{x}$ is $\ell x1$ $(\ell<n)$ problem P1 will result in $k,(k<m)$ binding resource constraints and $\ell-k$ values of $\lambda_{2i}{}^*$ corresponding to the binding calibration constraints.

c From the Taylor series expansion (4) we know that the function $h[g(x°)]$ is quadratic in $(x-\bar{x})$. Therefore, $h[g(x°)]$ has the form $1/2(\tilde{x}-\bar{x})^T E(\tilde{x}-\bar{x})$ where E is a $\ell x \ell$ positive semidefinite matrix. By the PQP theorem

(18) $\nabla_g h[g(x°)] = -\lambda_2{}^* = E(\tilde{x} - \bar{x})$

Given the minimal data set $\tilde{x}$, cross cost effects are restricted to zero, and thus for the single period calibration case considered here E is a diagonal matrix with nonzero elements e_{ii} where:[2]

(19) $e_{ii} = -\lambda_i{}^*/(\tilde{x}_i-\bar{x})$

corresponding to the nonmarginal cropping activities.

[2]With a larger time series on $\tilde{x}$ the full matrix E with cross effects can be estimated.

d Using the values e_{ii}, the problem P2 is specified as

$$(20)\quad \text{Max} \quad f(x) - 1/2x'Ex$$

$$\text{Subject to} \quad Ax \leq b \quad x \geq 0$$

The problem P2 calibrates exactly with the base year vector $\tilde{x}$ without spurious constraints and is available for policy analysis in the knowledge that the model response will be determined by economic comparative advantage and resource constraints that have a clearly demonstrated empirical basis.

Estimation of Water Demands from the C.A.R.M. Model

Californian farmers are assumed to have the best knowledge of local production effects. They are certainly aware of the variations in climatic conditions which affect productivity and thus evaluate the expected value of their profit function with a weighted average estimator of yield per acre. Similarly, farmers base their price expectations on the past series of prices. As stated in a more general way by Nerlove [42] p. 129.:

> "[it is assumed] that economic agents base their forecast on past values of the variable and that they optimize their forecast given knowledge of some specification of the mechanism generating the mechanism of the time series"

Based on these premises, the CARM yield coefficients $\bar{\eta}$ are set equal to their 6-year average. (A further study would be required to define the optimal order of the yield moving average.) The regional equilibrium condition can accordingly be stated as:

$$(21)\quad \underset{\underline{x}}{\text{Max:}} \quad \Phi(\underline{\bar{\eta}}\ \underline{\acute{z}},\ \underline{p}_{t-1},\ \underline{b})$$

<u>subject to all constraints</u>

where:

- $\underline{\bar{\eta}}$ vector m:1 of expected yield
- $\underline{z}$ vector m:1 of crop acreage
- $\underline{p}_{t-1}$ vector n:1 of lagged price
- $\underline{b}$ vector m:1 of input resource.

Perennial acreage is fixed over several years. Thus, in a single period equilibrium, perennial acreage is not specified as an optimizing variable. For this reason, CARM regional perennial levels are kept constrained to their regional base year acreage in the Positive Quadratic Programming runs. Besides, constraining perennial crops reduces the dimensionality of the quadratic program.

The first step for CARM model calibration consists in computing the gradient of the approximated objective function at the points defined by empirically measured acreage. The constrained base run for calibration contains 66 nonlinear variables. The constraint matrix has dimensions of 1109:1145 with 9,028 nonzero elements. As it is desired to construct a continuous rather than stepped cost function, only one soil type is specified. The computer code used for the computation is MINOS, a large scale in-core nonlinear optimization model developed at the Stanford Optimization Laboratory by Murtagh and Saunders [41]. It took 1175 iterations to solve the problem.

The computed values of the constraints gradients (the duals on calibration constraints) provide a stringent test of the model consistency. Negative (downward) gradients of the concave objective function are contradictory with the profit-maximization and regional specialization assumptions.

In CARM, correction of minor misspecifications of the linear costs yielded positive gradient values of the objective function at the calibration points. The implicit regional cost functions are computed neglecting the second-order cross-derivative coefficients of the Hessian and are introduced into the model objective function. After calibration, the model has a total of 327 nonlinear variables.

Regional derived demands for irrigation water depend simultaneously on a great number of variables, including demands for agricultural products, supplies of other input factors, substitution among products or input factors and relative economic advantage among regions. Estimation of the regional derived demands for irrigation therefore necessitates a full information approach and the system has to be modeled as a multi-input and multi-output sectorial equilibrium. Given the size of the problem, its complexity and the limited data base available, only the programming approach appears to be feasible at a reasonable cost. Linear programs do not model the partial equilibrium conditions while existing quadratic models cannot disaggregate results at the regional level.

To determine the regional derived demand functions for surface water in California, parametric programming is applied to the PQP model solution in the CARM regions corresponding to the Central Valley in California, where most of the agricultural production takes place (Figure 4). By modifying the regional cost of surface water in the regions, the quantity of water used by competitive farmers and the equilibrium regional acreage of crops are computed for the base year 1978. The first scenario is one in which surface water costs increase simultaneously in all parts of the state due to an increase in energy costs. As a first approximation, the impact of an energy price

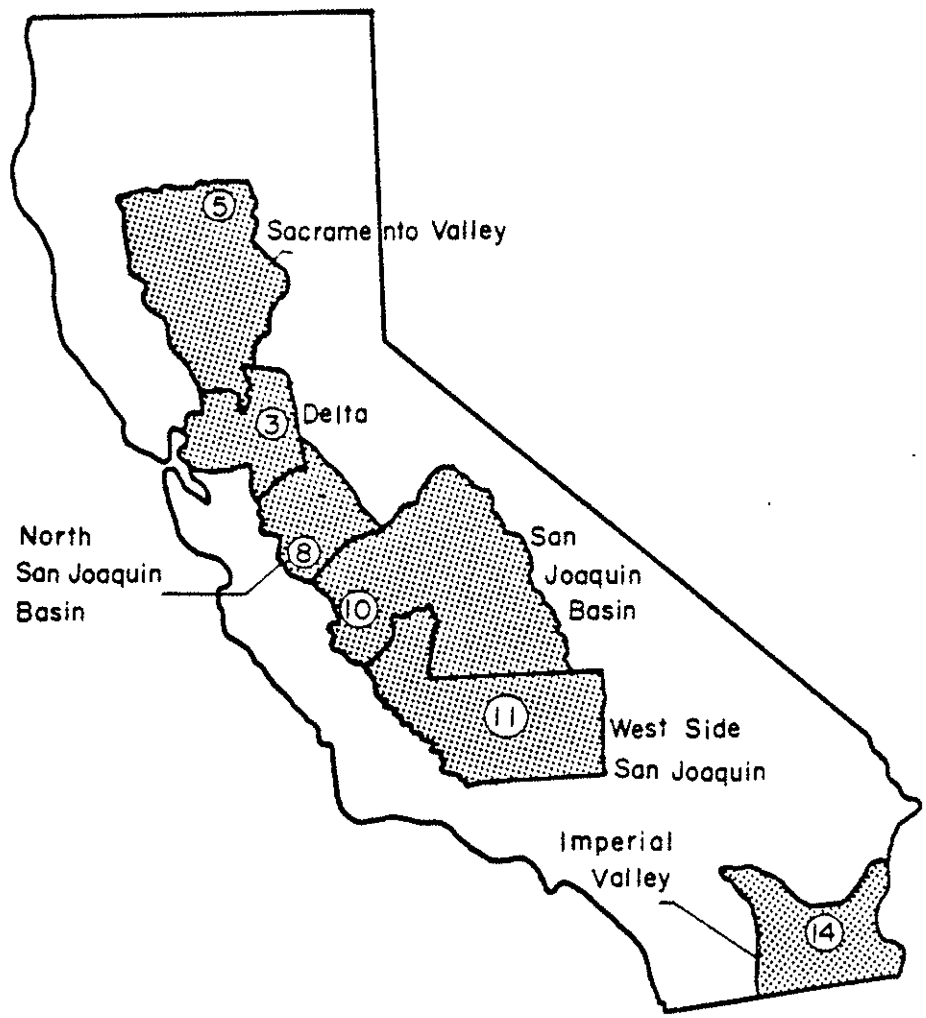


FIG. 4. STUDY AREA FOR SURFACE WATER DEMANDS

increase is assumed to be proportional to the current regional cost. The regional costs of surface water are accordingly increased by 50 percent, 100 percent, 150 percent, and 200 percent. Ground water costs are kept constant but quantities pumped are allowed to increase by 15 percent above 1978 levels to account for substitution between surface and ground water. In Table 1, the corresponding costs are displayed for the four parametric runs.

The corresponding short-run PQP derived demand for irrigation water in the CARM regions of the Central Valley are displayed in Figures 5 to 8. The response of the different regions are quite different, ranging from completely inelastic response (region 8) to more elastic responses (regions 5 and 11). This disparity stresses the importance of a regional analysis. The corresponding short-run elasticities are computed for three cost ranges and are given in Table 2.

Existing empirical estimates of water demand elasticities are based on the parameterization of nonequilibrium models. By contrast, the PQP program is a sectorial equilibrium approach in which cross-sectional and interregional relevant information is used to estimate the demands for surface water. Therefore, comparison of existing empirical elasticities estimates with the regional PQP elasticies is difficult, since they are not established on the same basis. For completeness and illustration, the PQP regional elasticities are nevertheless compared with two other sets of elasticities computed recently in the San Joaquin Valley by Shumway [53] and Howitt _et al._ [24] (Table 3). In order to do so, only the water costs of the studied region are increased while other regional water costs are held constant. Shumway's results are projections of elasticities for 1980 computed for the west side of the San Joaquin Valley with a statewide linear programming model. The

Table 1. Costs of Surface Irrigation Water for the Parametric Runs

	surface water cost $ [1978]				
increase	--	50%	100%	150%	200%
region					
1	2.0	3.0	4.0	5.0	6.0
2	5.3	8.0	10.7	13.3	16.0
3	6.0	9.0	12.0	15.0	18.0
4	6.0	9.0	12.0	15.0	18.0
5	7.0	10.5	14.0	17.5	21.0
6	8.0	12.0	16.0	20.0	24.0
7	8.0	12.0	16.0	20.0	24.0
8	1.0	1.5	2.0	2.5	3.0
9	8.0	12.0	16.0	20.0	24.0
10	7.0	10.5	14.0	17.5	21.0
11	16.1	24.2	32.2	40.3	48.3
12	93.8	140.6	187.5	234.8	281.3
13	18.0	27.0	36.0	45.0	54.0
14	6.0	9.0	12.0	15.0	18.0

REGION II

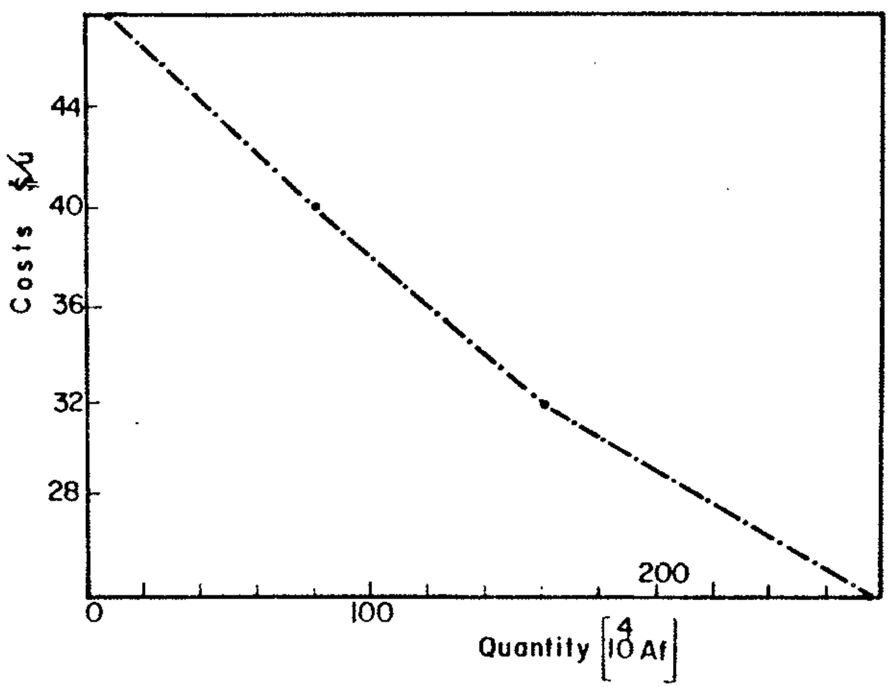


FIG. 5. DERIVED DEMAND FOR SURFACE WATER
WEST SIDE SAN JOAQUIN

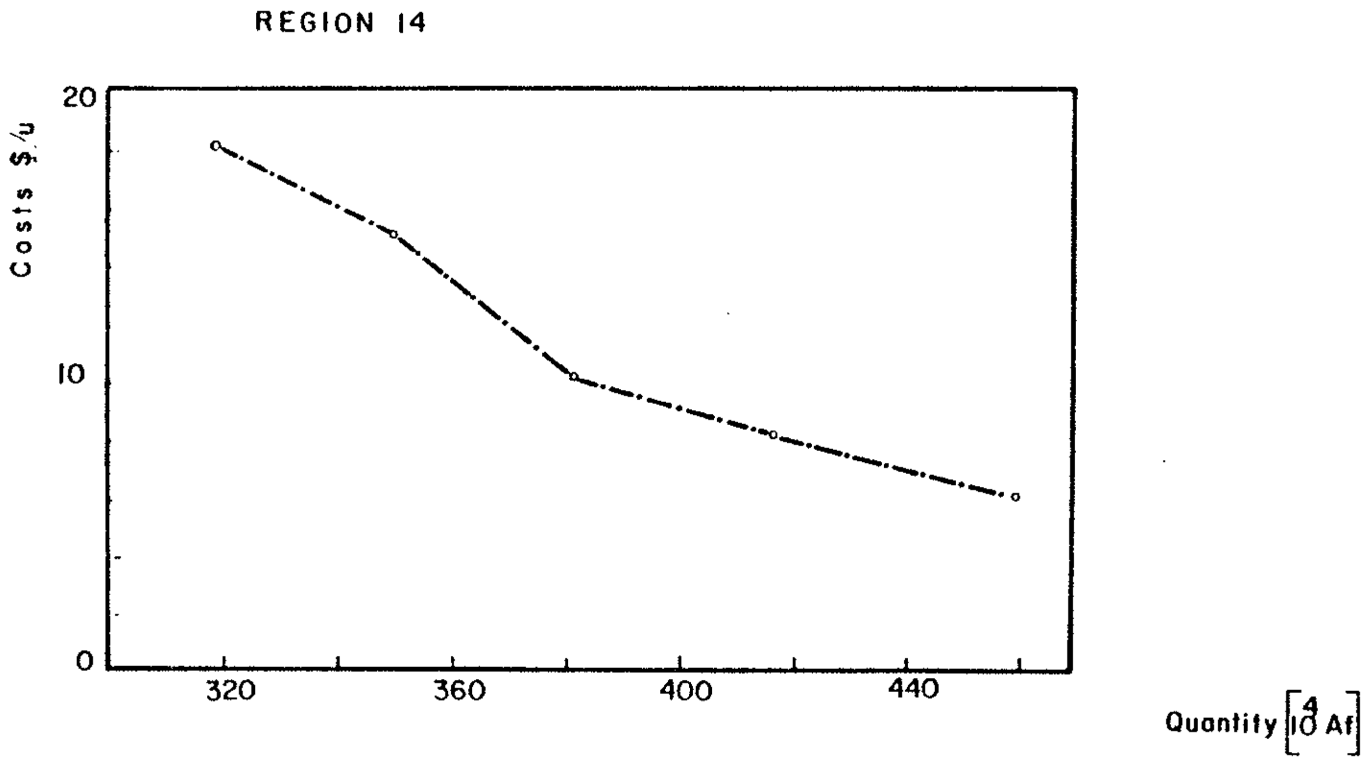


FIG. 6. DERIVED DEMAND FOR SURFACE WATER IMPERIAL VALLEY

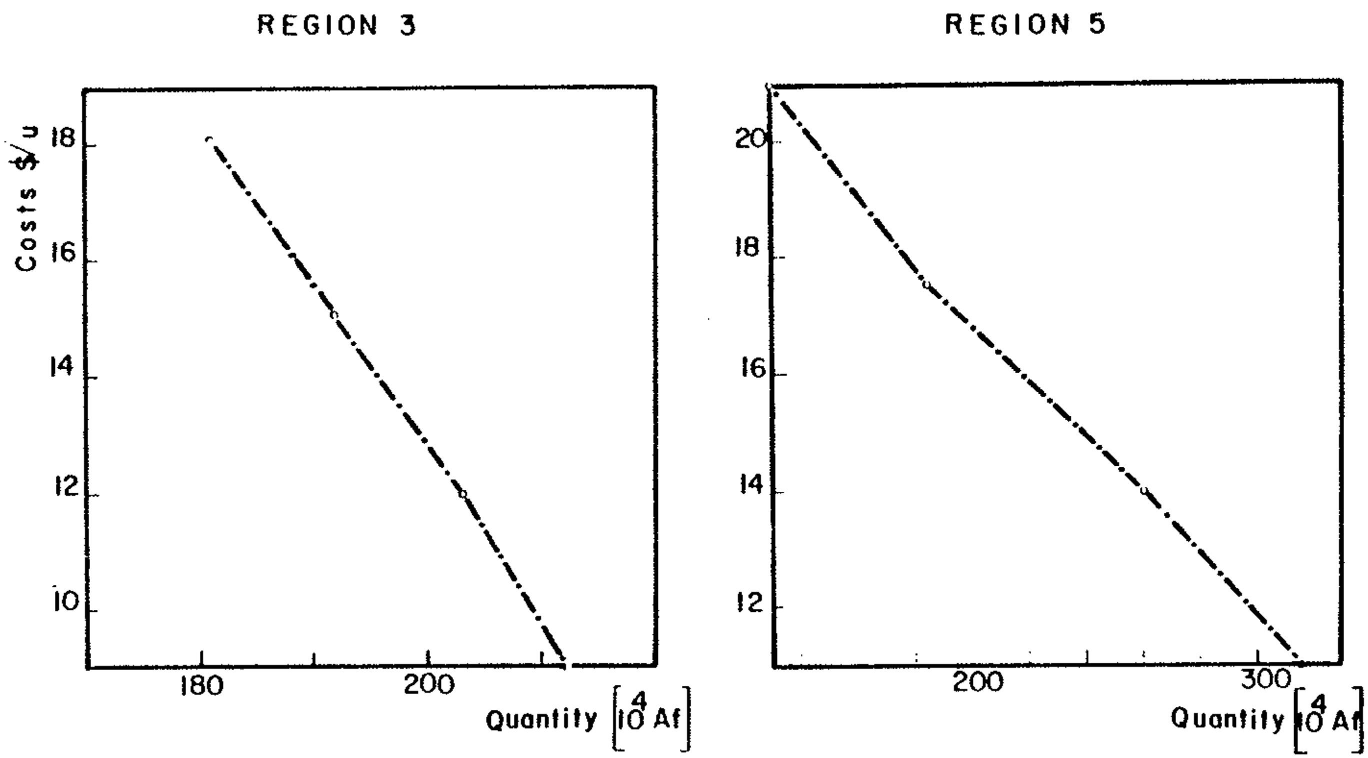


FIG. 7. DERIVED DEMANDS FOR SURFACE WATER DELTA AND SACRAMENTO VALLEY

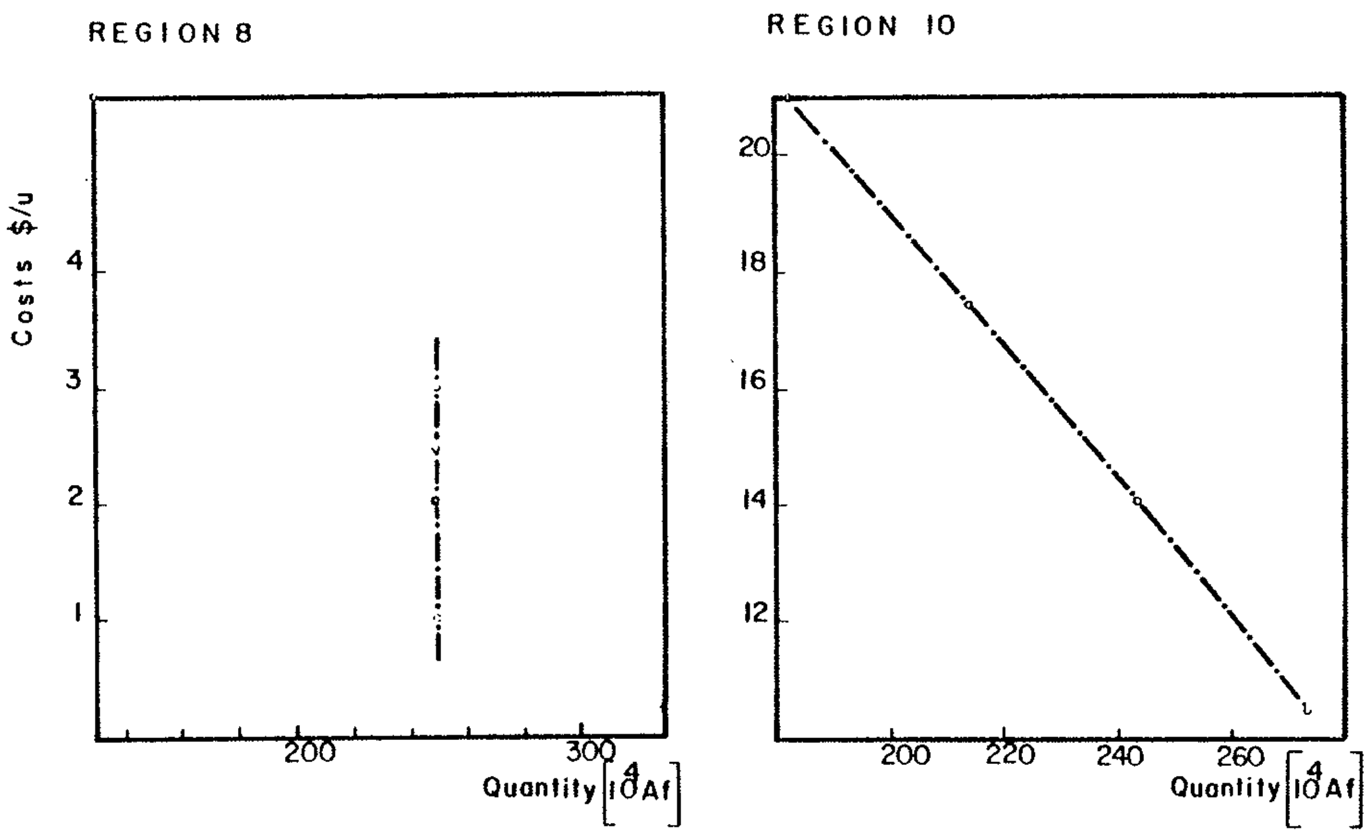


FIG. 8. DERIVED DEMANDS FOR SURFACE WATER SAN JOAQUIN BASIN

TABLE 2. Short-run Price Elasticities of Surface Water Central Valley of California

CARM region		price elasticity base year [1978]		
cost increase		50-100%	100-150%	150-200%
3	Sacramento Valley	-0.12	-0.27	-0.38
5	Delta	-0.81	-1.33	-2.31
8	North San Joaquin	-0.00	-0.00	-0.00
10	San Joaquin	-0.40	-0.60	-0.88
11	West Side San Joaquin	-1.77	-3.00	-10.00
14	Imperial Valley	-0.31	-0.41	-0.54

Table 3. Comparison of POP Price Elasticity With Other Empirical Estimates in West Side San Joaquin (Region 10)

	nonequilibrium models		equilibrium models
	Shumway (LP)	Howitt *et al.* (QP)	(PQP)
	West side San Joaquin	San Joaquin	Region 11 + 10
price range $ [1978]			
15. - 25.	-0.62		-1.08[1]
25. - 30.	-0.71		
30. - 40.	-1.21	-1.50	

[1]Weighted average value

estimates by Howitt et al. are computed with quadratic model of the San Joaquin Valley and represent aggregate values.

The PQP regional elasticities are in quantitative agreement with the two sets of aggregate estimates. However, the PQP elasticities are regional values and thus represent a significant improvement on currently available aggregate estimates.

The demonstration of the potential of PQP is not complete without illustrating the importance of the partial equilibrium approach. In Figure 9, two different water demands are drawn for CARM region 14 (Imperial Valley). The first curve displays the water demand of the Imperial Valley with respect to a change in water cost, all other regional water costs being kept constant at the base year level. The second curve represents the water demand in the Imperial Valley with respect to a change in water cost when all water costs in the State vary proportionally to their base year level. The important difference between the two curves is caused by interregional effects. The conclusion of the analysis is clear. Change in water costs of other regions significantly impacts on the response of the Imperial Valley. This fact stresses the importance of the PQP partial equilibrium approach for modeling the California agricultural sector and its resources by regions.

Conclusions

The PQP estimates displayed on Table 2 could be improved and refined by additional research. However, to our knowledge, they are the first regional elasticity estimates to be derived from a statewide nonlinear equilibrium model. Thus, they already have a significant policy value and represent a good starting basis for State Agency pricing policy. For example,

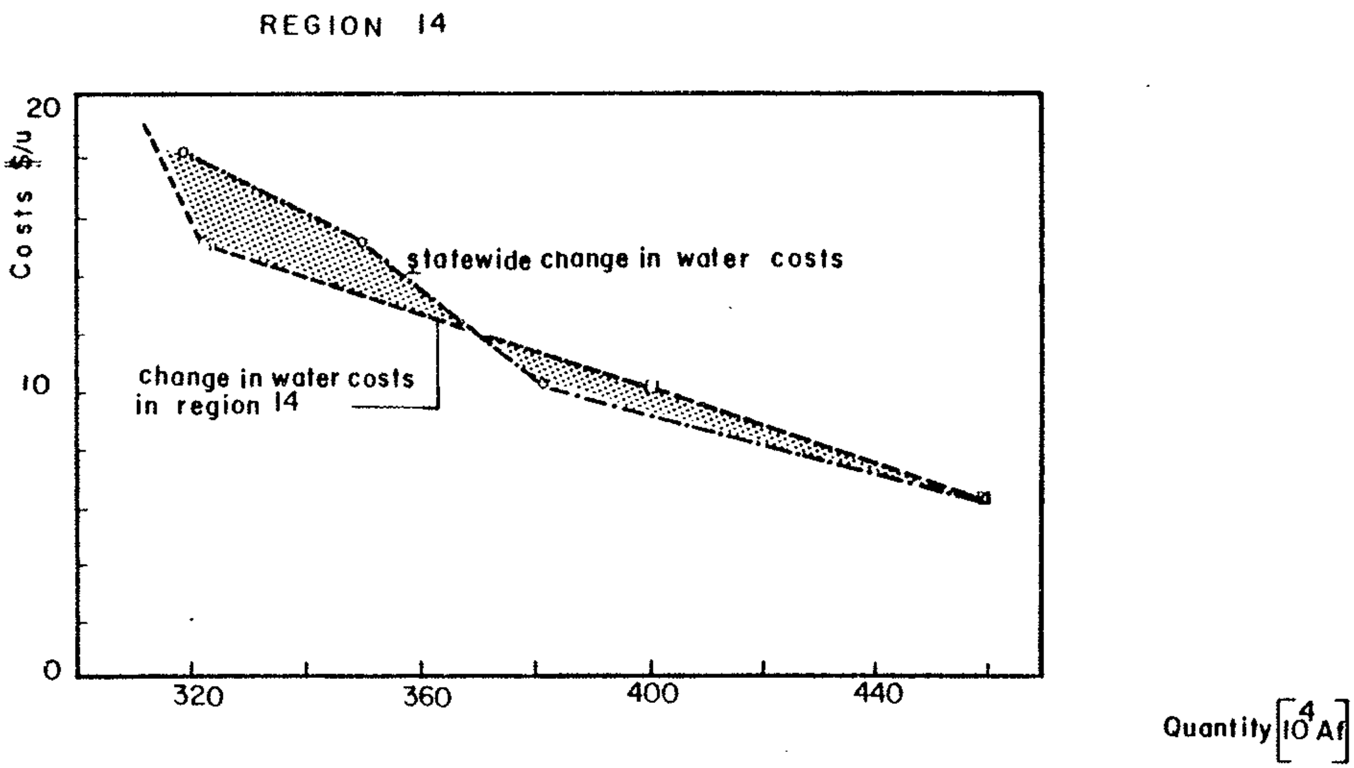


FIG. 9. ILLUSTRATION OF INTERREGIONAL EFFECTS ON SURFACE WATER DEMAND - IMPERIAL VALLEY

the PQP approach may be particularly useful for the implementation of the water transfers recently advocated in the California legislature (Katz bill AB 3941).