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DESIGN CONDITIONS FOR FAIL SAFE QUENCHING OF A HIGH CURRENT DENSITY SUPERCONDUCTING SOLENOID WITH A SHORTED SECONDARY CIRCUIT

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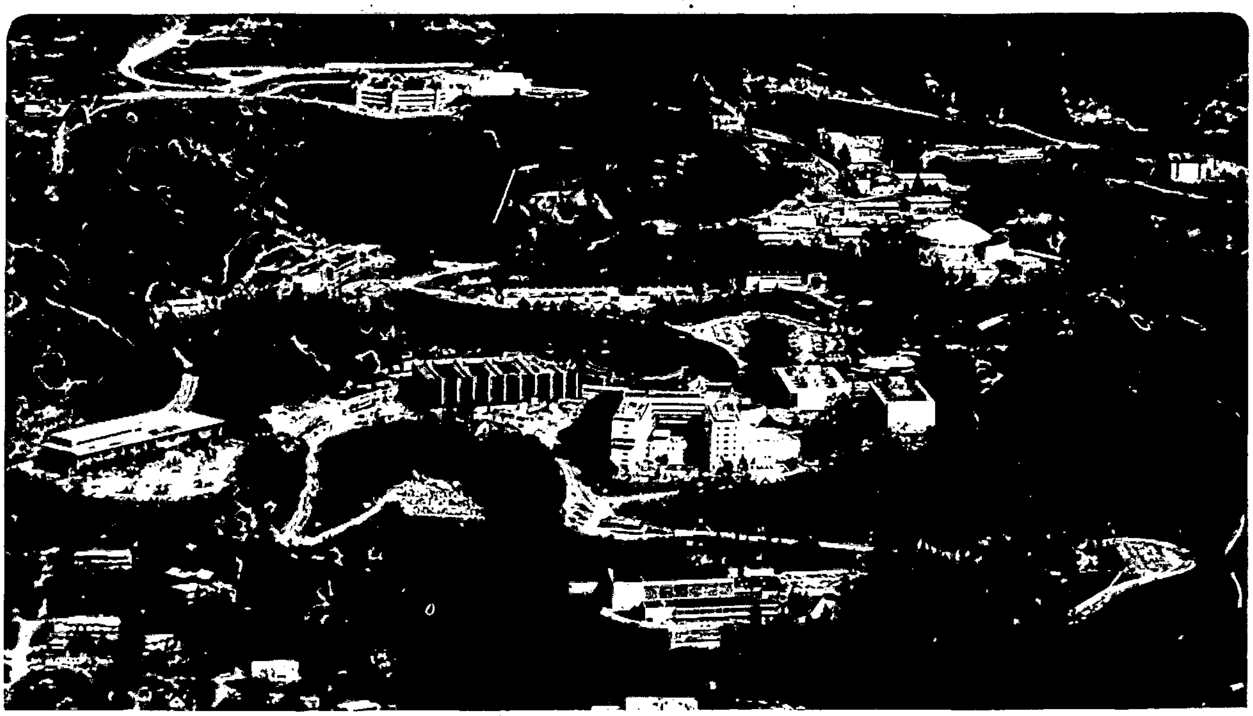

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DISCLAIMER

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DESIGN CONDITIONS FOR FAIL SAFE QUENCHING OF A HIGH CURRENT DENSITY SUPERCONDUCTING SOLENOID WITH A SHORTED SECONDARY CIRCUIT

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NOMENCLATURE

The symbols used in this report are listed and defined below. In most cases the symbols are defined again as they are used in the text. The units corresponding to the symbol definitions are the standard SI electrical and mechanical units. The electrical system utilized is a rationalized MKS system with the permeability of vacuum set equal to $\mu_o = 4\pi \times 10^{-7}$ Hm^{-1}.

In general, superscripts are not used with the exception of t^*, t^{**}, F_2^*, F_2^{**}, a', and b' which are defined here and in the text. Subscripted symbols are divided into two categories. They are: the letter subscripts which define the unit and the numerical subscript unit which defines which circuit the unit applies to. In general, a subscript of 1 applies to the superconducting coil and subscript of 2 applies to the secondary circuit which is the source of quench back. In most cases, standard symbol notation is used so that one can guess what the symbol is by looking at the equation and reading the text.

Symbol	Meaning of the Symbol	Units
a	long dimension of a rectangular superconductor (see Fig. 6)	m
a'	long face dimension of a rectangular superconductor (see Fig. 6)	m
a_1	average radius of a superconducting solenoid coil	m
a_2	average radius of a solenoidal shorted secondary circuit which induces quench back	m
A_{c1}	cross-section area of the super-conductor plus matrix material	m^2
A_{c2}	cross-section area of the secondary circuit material which induces quench back	m^2
b	short dimension of a rectangular superconductor (see Fig. 6)	m
b'	short face dimension of a rectangular superconductor (see Fig. 6)	m
B	magnetic induction	T
$\dot{B}$	time rate of change of magnetic induction	Ts^{-1}
B_{c1}	lower critical magnetic induction of a superconductor	T
B_{c2}	upper critical magnetic induction of a superconductor	T
c1	insulation thickness between the short faces of a rectangular conductor (see Fig. 6)	m
c2	insulation thickness between the long faces of a rectangular conductor (see Fig. 6)	m
c3	insulation thickness between the turns of round superconductor (see Fig. 6)	m

Symbol	Meaning of the Symbol	Units
c4	insulation thickness between the superconducting coil and the quench back secondary circuit	m
C(T)	volume specific heat as a function of temperature	$Jm^{-3}K^{-1}$
C_{sc}	volume specific heat of the superconducting material	$Jm^{-3}K^{-1}$
d1	length of half of a coil turn	m
d2	coil width perpendicular to the turn direction	m
d3	coil thickness perpendicular to the turn direction	m
d_f	superconducting filament diameter	m
d_s	mean distance between superconducting filaments in a multifilamentary superconductor	m
D_w	round conductor wire diameter (see Fig. 6)	m
E_o	stored magnetic energy in the magnet before the start of a quench (at a current i_o)	J
E(t)	electric field in the conductor as a function of time t	Vm^{-1}
F(T)	integral of specific heat per unit volume over electrical resistivity as a function of temperature T as defined by Eq. (1)	$A^2m^{-4}s$
$F_1(T)$	F(T) integral applied to the superconductor circuit as a function of T	$A^2m^{-4}s$
$F_2(T)$	F(T) integral applied to the secondary circuit which induces quench back as a function of T	$A^2m^{-4}s$

Symbol	Meaning of the Symbol	Units
F_2^*	change in integral $F_2(T)$ needed to make the secondary circuit hot enough to induce quench back (see Eq. (13))	A^2m^{-4}
F_2^{**}	F integral difference similar to F_2^* as defined by Eqs. (30a) and (30b)	A^2m^{-4}
$g_1(a_1,\ell_1)$	solenoid inductance function as a function of a_1 and ℓ_1 which applies to the superconducting coil (for an iron bound solenoid, $g_1=1$)	
$g_2(a_2,\ell_1)$	solenoidal inductance function as a function of a_2 and ℓ_2 which applies to the shorted secondary circuit quench-back is induced from (for an iron bound solenoid $g_2=1$)	
$G(T)$	integral of specific heat per unit volume times electrical resistivity as a function of temperature T as defined by Eq. (3)	V^2m^{-2}
$H(T)$	enthalpy per unit volume	Jm^{-3}
$H_m(T)$	enthalpy per unit mass	Jkg^{-1}
ΔH_2	enthalpy change per unit volume in the secondary circuit needed to induce quench back	Jm^{-3}
ΔH_{SC}	enthalpy change per unit volume in the superconductor needed to drive it normal $\Delta H_{SC} = \Delta H_S + \Delta H_M$	Jm^{-3}
ΔH_M	enthalpy change per unit volume due to change in magnetization of the superconductor (see Eq. (21b))	Jm^{-3}
ΔH_S	sensible enthalpy change per unit volume needed to drive the superconductor normal	Jm^{-3}
ΔH_i	change in enthalpy per unit volume in the installation between the superconductor and the quench back circuit needed to induce quench back	Jm^{-3}

Symbol	Meaning of the Symbol	Units
i_o	initial current in the superconducting coil circuit before the quench starts	A
i_1	superconducting coil circuit current	A
i_2	current in the secondary circuit which induces quench back	A
$J_1(t)$	current density in the overall section of the superconductor	Am^{-2}
$J_2(t)$	current density in the overall section of the secondary circuit which induces quench back	Am^{-2}
$J_c(B,T)$	superconductor critical current density (superconductor only) as a function of magnetic induction B and temperature T	Am^{-2}
J_o	starting current density in the overall superconductor section	Am^{-2}
k_i	thermal conductivity of the insulation between superconducting turns	$Wm^{-1}K^{-1}$
k_{in}	thermal conductivity of the insulation between the superconductor and the shorted secondary circuit which causes quench back	$Wm^{-1}K^{-1}$
k_{sc}	thermal conductivity of the superconducting material	$Wm^{-1}K^{-1}$
ℓ_1	superconducting solenoid length	m
ℓ_2	shorted secondary circuit solenoid length (this circuit causes quench back)	m
ℓ_{tp}	superconductor twist pitch	m
L	Lorentz number ($L=2.45\times10^{-8}\ w\Omega K^{-2}$)	$W\Omega K^{-2}$
L_1	superconducting coil self inductance	H
L_2	shorted secondary circuit which causes quench back self inductance	H

Symbol	Meaning of the Symbol	Units
L_m	self inductance of the m^{th} circuit	H
m_{active}	mass of the superconductor plus all shorted secondary circuits which make a significant contribution to quench protection	kg
M_{12}	the mutual inductance between circuits 1 and 2	H
N_1	number of turns in the superconducting coil	
N_2	number of turns in the secondary circuit which induces quench back	
N_C	number of coils in a multiple super-conducting coil system (N_C is the number of coils on a single pair of leads)	
r	normal metal to superconductor ratio	
R(t)	coil resistance as a function of time t	Ω
R_1	coil resistance	Ω
R_2	secondary circuit, which causes quench back, resistance	Ω
R_m	resistance of the m^{th} circuit	Ω
R_C	the resistance of a single one of N_C coil when the superconductor is just above its critical temperature	Ω
R_{ex}	the resistance of an external resister across the terminals of the super-conducting coil	Ω
R_{o1}	coil resistance constant for a one dimensional quench	Ωm^{-1}
R_{o2}	coil resistance constant for a two dimensional quench	Ωm^{-2}

Symbol	Meaning of the Symbol	Units
R_{o3}	coil resistance constant for a three dimensional quench	Ωm^{-3}
t	time	s
$t1$	time to propagate a quench one half a turn length	s
$t2$	time to propagate a quench from turn to turn a distance d2 (the coil width)	s
$t3$	time to propagate a quench from turn to turn a distance d3 (the coil thickness)	s
t^*	the shorter of t1 or t2	s
t^{**}	the longer of t1 or t2	s
t_H	time to heat the insulation and superconductor from the quench back element	s
t_Q	time to bring the quench back element from temperature T_o to temperature T_Q	s
t_{QB}	calculated quench back time	s
t_{QBR}	required quench back time for safe passive quenching of the magnet	s
t_{QR}	time t_Q required for safe passive quenching of the magnet	s
t_{SO}	time required to switch on an active quench protection system	s
T	temperature	K
T_C	superconductor critical temperature (it is a function of current density and magnetic induction)	K
T_M	hotspot temperature in the superconductor which is allowed (usually 300 K)	K
T_o	starting temperature of the superconductor before the quench starts	K

Symbol	Meaning of the Symbol	Units
T_Q	temperature in the secondary circuit which will induce quench back in the superconducting coil (usually 10 K)	K
T_S	temperature in the superconductor circuit if the coil becomes completely normal at time zero (also known as T_{S1}	K
T_{S1}	temperature in the superconductor circuit if the coil becomes completely normal at time zero (also known as T_S)	K
T_{S2}	temperature in the secondary circuit from which quench back was induced after a time of infinity	K
$T_{S3} \rightarrow T_{Sm}$	temperature of other secondary circuits 3 through m at the end of the quench	K
v_L	quench velocity along the superconductor (v_L is primarily a function of J and B	ms^{-1}
V	material volume in a circuit	m^3
α	turn to turn quench velocity ratio in direction d2 (to get quench velocity in this direction multiply α by v_L)	
β	turn to turn quench velocity ratio in direction d3 (to get quench velocity in this direction multiply β by v_L)	
γ	turn to turn quench velocity ratio in a direction perpendicular to a round wire conductor	
Γ	gamma function for turn to turn quench velocity defined by Eq. (32a)	$Wm^{-1}K^{-1}$
ε	one minus the coupling coefficient between the superconducting coil and the secondary circuit defined by Eq. (11e)	
$\rho(T)$	electrical resistivity as a function of temperature T (this applies only to materials in the normal state)	Ωm

Symbol	Meaning of the Symbol	Units
ρ_1	electrical resistivity of the superconductor matrix material	Ωm
ρ_2	electrical resistivity of the secondary circuit material in the circuit which induces quench back	Ωm
τ_1	time constant for the superconducting coil circuit as it goes normal (defined by Eq. 11a)	s
τ_2	time constant for the secondary circuit which induces quench back (see Eq. (11b)	s
τ_m	time constant of the m^{th} circuit (see Eq. 15b)	s
τ_L	a coupled circuit long time constant defined by Eq. (11c)	s
τ_s	a coupled circiut short time constant defined by Eq. (11d)	s
μ_o	permeability of vacuum ($\mu_o = 4\pi\times10^{-7}$ Hm^{-1})	Hm^{-1}

1. Introduction

Most of the work on large superconducting magnets has been based on the concept of cryostability. Most of the superconducting magnets with stored energies above 3MJ use cryostabilized conductors. The concept of cryostability implies that there is sufficient helium in direct contact with the superconductor to insure good heat transfer in order to keep instabilities in the superconductor from driving the magnet normal.[1]

Because large intrinsically stable superconducting magnets have been considered risky until recently, many magnets builders would not consider this approach when building large magnets. The use of high current density superconductors (current densities well above the cryostability limit of 10^8 Am^{-2}) offers a number of advantages in large superconducting magnets including: reduced magnet cold mass and size; increased access to the device requiring the magnetic field; more efficient helium cooling with enhanced cryogenic safety; and reduced construction cost.

The use of high current density conductors (with matrix current densities above 1.5 x 10^8 Am^{-2}) requires careful attention to problems in (a) quench protection, (b) magnetic strain and training, and (c) cooling system design. The design of a large adiabatically stable magnet is quite different from the design of either a large cryostable magnet or small high current density magnets. Quench protection becomes very important when one builds large high density superconducting magnets. This problem is addressed in this report.

A superconducting magnet will quench safely if its energy is removed fast enough to insure that the integral of current density squared with time in the superconductor is small enough that the hot spot temperature in the superconductor below some value (say 300K). As the stored energy of the superconducting magnet increases, the time constant for current decay will also increase in most cases at a constant current density in the conductor. As the superconductor current density increases, the time constant for current decay must get smaller in order to keep the hot spot temperature low enough. As the stored energy of these magnets increases, the conventional solution is to use an <u>active quench protection</u> system to protect the coil by reducing the time constant for current decay.[2] The conventional approach is to discharge the coil through an external resistor. If the current density in the coil is not too high or if the stored energy of the coil is not too high, this approach will work. The conventional active quench protection system has limits which are usually bounded by an $E_o J_o^2$ limit of around 10^{23} JA^2m^{-4}.[3] (Note E_o is the magnet stored energy dumped into the protection resistor and J_o is the conductor matrix current density.) See Fig. 1.

The Lawrence Berkeley Laboratory has built and tested high current density magnets which use closely coupled shorted secondary circuits as an integral part of the magnet quench protection system.[4] Using these circuits the EJ^2 limit in the superconductor could be increased up to values of 10^{25} JA^2m^{-4}. (This limit is often determined by mechanical stress rather than quench protection.[5]) The role of the closely coupled shorted secondary circuits is as follows:

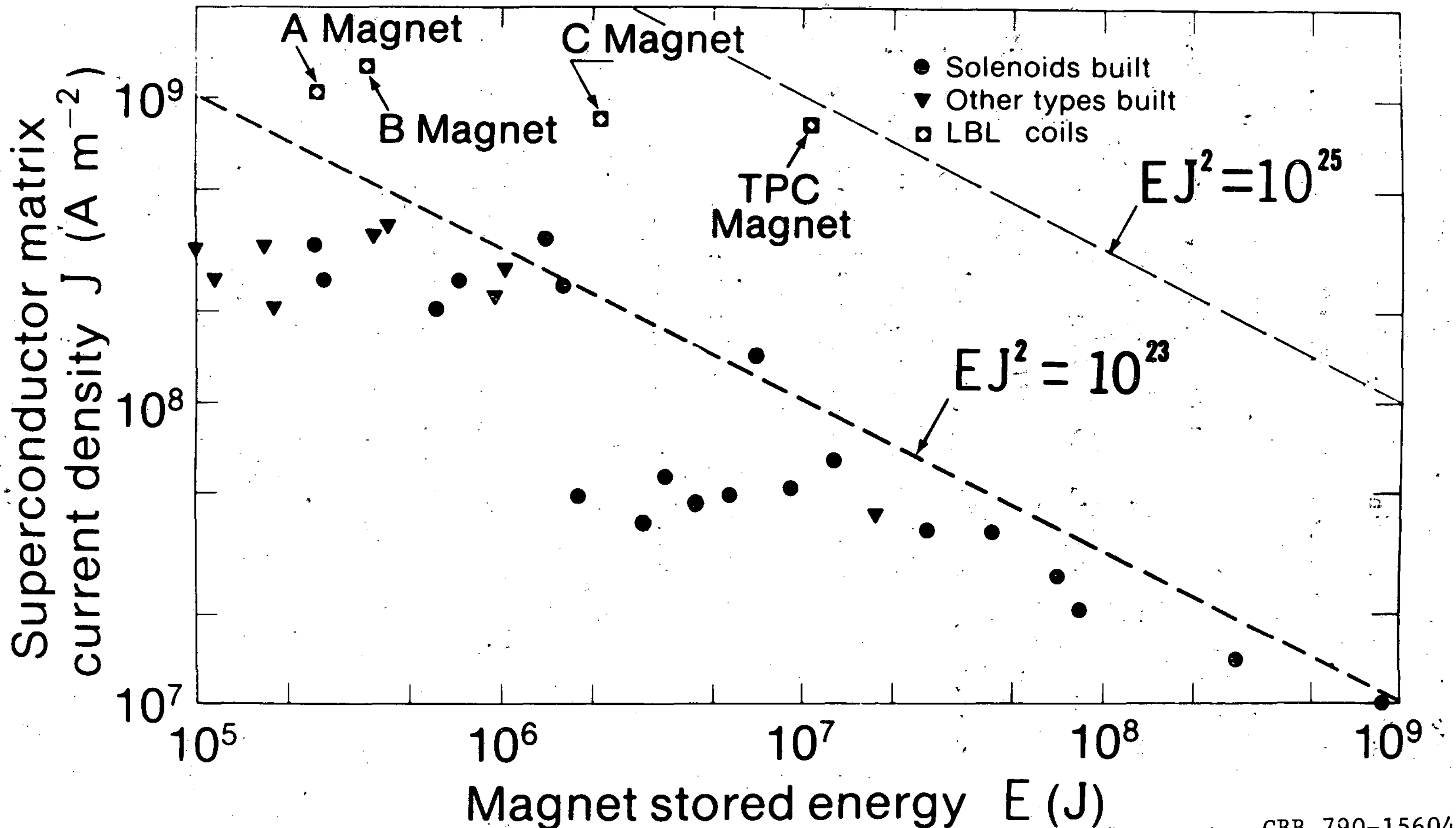

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Fig. 1. The superconductor current density (J) and magnet stored energy (E) for a number of superconducting magnets. (Note: Most of the magnets have EJ^2 limits of less than 10^{23} JA^2m^{-4}. The LBL magnets have shorted secondary circuits.)

1) Current is shifted from the superconductor to the secondary circuits inductively, which reduces hot spot temperature in the superconductor.
2) The secondary circuits can absorb a substantial amount of the magnet stored energy during the quench.
3) Since the rate of flux decay is slower than the rate of current decay in the superconducting coil, the voltages in the magnet system during a quench are reduced over the same superconducting coil without secondary circuits.
4) The shorted secondary circuits permit some unusual types of external active quench protection to be used (i.e., the varistor resistor or the center tap discharge systems).
5) The shorted secondary circuits causes the whole coil to become normal in a time which is smaller than would occur by ordinary normal region propagation within the coil.[4,6] This phenomena will be referred to as "quench back".[3,5]

Before continuing our discussion of the shorted secondary circuit concept, it is useful to point out this concept was thought of before the LBL experiments. Maddock and James[2] discussed the concept of a coupled secondary circuit and its usefulness in protecting cryostable magnets. The paper concluded that the shorted secondary circuit material would be better used if it were combined with the primary circuit. This conclusion is probably correct in cryostable magnets which are protected by an ordinary resistor, but we believe this conclusion does not hold in high current density magnets where quench back occurs. Maddock and James did not look at some of the unusual active quench protection systems studied by LBL (i.e., the varistor resistor.) The shorted secondary circuit improves the performance of the varistor resistor quench protection system whether or not quench back is present (All that is required is good coupling between the primary and secondary circuits.)

The role of well coupled shorted secondary circuits in the active quench protection of high current density magnets is described in a number of LBL publications (see references 3,7,8,9 and 10). This report deals with the concept of protecting large high current density magnets without using active external quench protection circuits. In other words, this reports suggests how one might design coils which can be quenched in a fail safe way using passive elements which are part of the coil itself. (It should be pointed out that one might well have an active quench protection system in order that hot spot temperatures be reduced.)

One approach to passive quench protection of large high current density superconducting magnets is to use the phenomena of quench back from one or more secondary circuits built into the magnet to drive the entire coil normal fast enough to insure safe quenching. At LBL, we found experimentally that a one meter diameter solenoid could be self protected even when the current density in winding was as high as 1.17×10^9 Am^{-2}.[4,5,9] All that was required was a closely coupled 1100 aluminum bore tube. As magnet size and stored energy grow, the design of the secondary circuit system becomes more important. In large magnets, such as the TPC magnet, a simple 1100 aluminum bore tube is not good enough to fully insure passive quench protection at full design current.

The theory developed in this report is a simplified closed form theory which is similar to a larger theory which has been under development at LBL for over a year.[11] The central element of this theory is quench back and the integral of J squared with time put into the coil hot spot while the coil is turning completely normal. This report shows the essential elements involved in the thermal quench back process. Using this report, one should be able to make a first cut design of a large high current density magnet which has shorted secondary circuits. The equations are simple enough to permit hand calculations.

2. The Quench Back Time Required in order to Insure Safe Passive Quench Protection

There is a maximum quench back time (the time it takes the whole magnet to go normal through quench back) which determines the limit of fail safe quenching. If the actual quench back time for a given magnet design is less than this maximum quench back time, the quench will always be safe without active quench protection.

The quench back time required for fail safe quenching is a function of the maximum allowable hot spot temperature T_M, the normal metal to superconductor ratio r of the superconductor, the starting current density J_o over the whole superconductor plus matrix, and the temperature the superconductor matrix would achieve if the entire coil where to turn normal instantaneously T_{S1}.

Using adiabatic theory[12] which assumes no heat transfer out of the superconductor matrix one finds that

$$F_1(T_M)-F_1(T_{S1}) = \int_{T_{S1}}^{T_M} \frac{C(T)}{\rho(T)}\, dT = \frac{r+1}{r} \int_0^{t_{QBR}} J_1(t)^2 dt \quad , \qquad (1)$$

where, C(T) is the superconductor specific heat per unit volume as a function of temperature T; $\rho(T)$ is the superconductor matrix resistivity as a function of T; $J_1(t)$ is the current density in the superconductor plus the matrix as a function of time t. t_{QBR} is the maximum required quench back time in order to achieve safe quenching without an active protection system. Equation 1 assumes that none of the magnet stored energy is dissipated resistively from time t=0 to $t=t_{QBR}$. (This assumption minimizes the difference between $F_1(T_M)$ and $F_1(T_{S1})$).

Using the assumption that the current density in the superconductor does not change between t=0 and $t=t_{QBR}$ (this is a conservative assumption) one can estimate a lower limit value for t_{QBR} using the following equation:

$$t_{QBR} = \frac{F(T_M)-F(T_{S1})}{(\frac{r+1}{r})\, J_o^2} \,, \qquad (2)$$

where J_o is the starting current density in the superconductor plus it's matrix; r is the normal metal to superconductor ratio for the superconducting wire;

$$F_1(T_M) = \int_0^{T_M} \frac{C(T)}{\rho(T)}\, dT \;; \qquad (2a)$$

and

$$F_1(T_{S1}) = \int_0^{T_{S1}} \frac{C(T)}{\rho(T)}\, dT \;. \qquad (2b)$$

The temperature T_M (the hot spot temperature) is arbitrarily chosen depending on the type of damage to the coil one wants to prevent. In most cases one chooses a value of T_M around 300K. The value of T_{S1} is a function of the mass and material contained in the superconducting coil and the shorted secondary circuits.

In the full theory which has been developed at LBL the value of T_{S1} can be found by solving the following integral equation for T_{S1}

$$G_1(T_{S1}) = \int_0^{T_{S1}} \rho(T)C(T)dT = \frac{r}{r+1} \int_0^{\infty} E(t)^2 dt \;, \qquad (3a)$$

where $\rho(T)$, r, and C(T) are previously defined for the superconductor matrix, E(t) is the electrical field in the superconducting material which has been turned normal. The value of E(t) is very much a function of the various time constants of the secondary and primary circuits as the coil energy is deposited in those windings when the coil quenches.

There are equations which are similar to Eq. (3a) that calculates $G(T)$ and T_S for each of the active circuits in the magnet. At the end of the quench, each circuit will have a different temperature T_S. Also at the end of the quench, the thermal energy deposited in all of the circuits must equal the initial energy stored in the magnet.

$$E_o = V_1(H_1(T_{S1}) - H_1(T_o)) + V_2(H_2(T_{S2}) - H_2(T_o)) + \ldots \qquad (3b)$$

where E_o is the initial energy stored in the magnetic field given by Eq. (4b). $H_1(T_{S1})$ is the enthalpy per unit volume of the material in the coil circuit at a temperature T_{S1}; $H_2(T_{S2})$ is the enthalpy per unit volume first secondary circuit at a temperature T_{S2}; and so on. $H_1(T_o)$, $H_2(T_o)$, and so on are the enthalpy per unit volume of the materials in the circuits at the initial temperature T_o. V_1, V_2 and so on are the metal volumes in the circuits. Calculation of T_{S1}, T_{S2}, and so on are not easy to make using a hand calculator, therefore a simplifying assumption is made.

If one assumes that the final temperature in the primary superconductor circuit and all of the active secondary circuits is the same at the end of a quench when the coil is driven completely normal, an estimate of T_S is easy to make. One should find the mass of each secondary circuit which has a low enough resistivity to absorb a significant portion of the magnet energy. The average enthalpy of the circuits at the end of the quench can be determined by

$$H_m(T_S) \approx \frac{E_o}{m_{active}} , \tag{4a}$$

where

$$E_o = \frac{L_1}{2} i_o^2 , \tag{4b}$$

where L_1 is the self inductance of the magnet. i_o is the coil current; E_o is the magnet stored energy; m_{active} the active mass of the coil and $H(T_S)_m$ the enthalpy gain per unit mass from T_o(4.5K) to T_S. One can determine T_S given $H_m(T_S)$ using Fig. 2[13,14] if one knows the fraction of the coil mass which is copper and which is aluminum. An approximate value for T_S can be determined graphically using Fig. 2. (In most cases, the assumption that all the circuits end up at the same temperature is not a bad one. During quenching of the A and B coils, the coil temperature away from the hot spot and the bore tube temperature were the same within a few degrees.)[5]

Once T_{S1} has been determined, the value of $F(T_{S1})$ can be determined from Fig. 3 for various copper matrix superconductors. Figure 3 gives $F(T_{S1})$ as a function of T_{S1} and the residual resistivity ratio RRR of the matrix material in the superconducting wire. A table at the bottom of Fig. 3 gives $F(T_M)$ for various RRR at values of T_M = 300K and T_M = 400K. It is interesting to note that

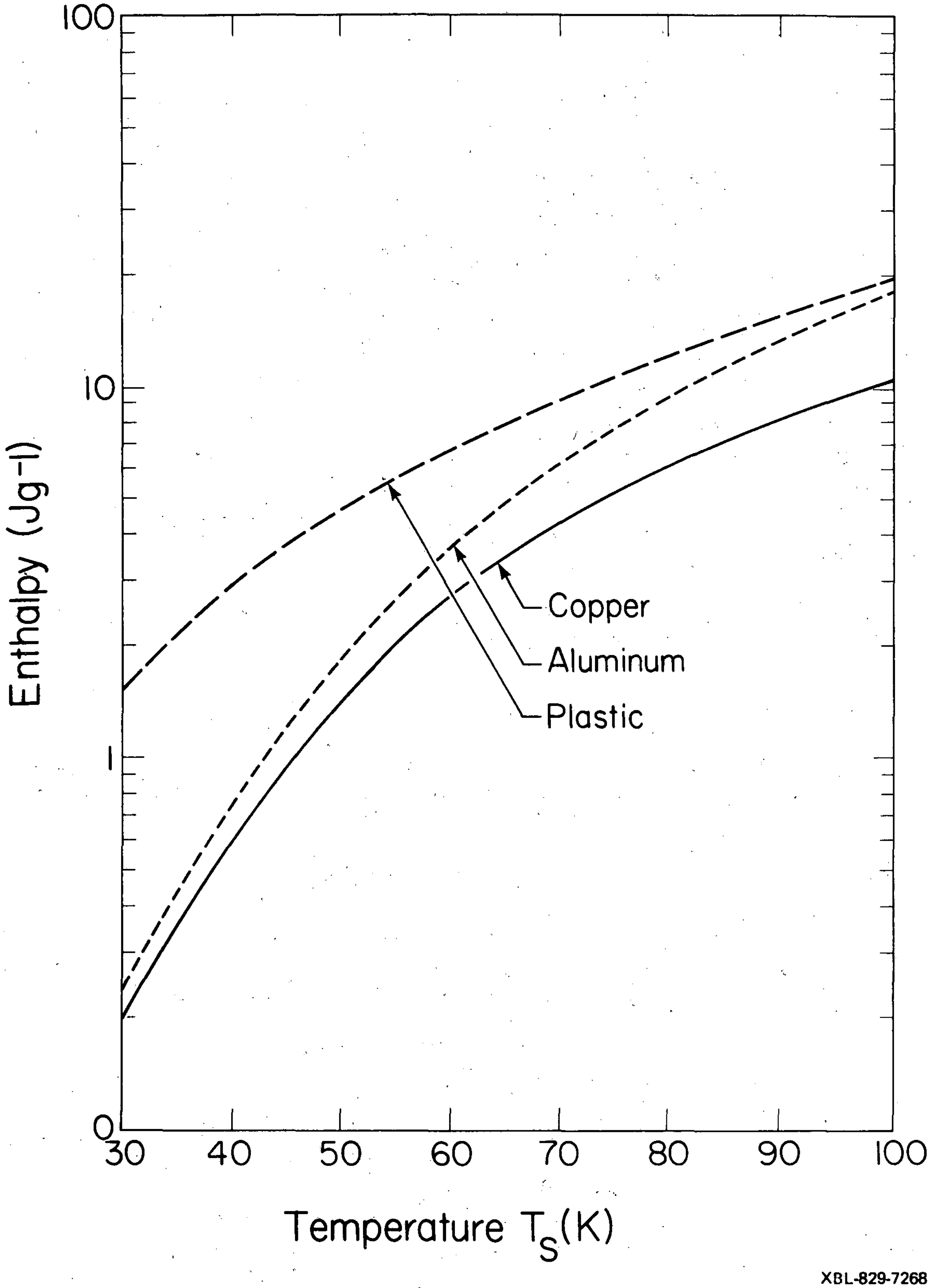

Fig. 2. Entalpy per unit mass a function of temperature for various materials used in construction of high current density superconducting coils. (If $H_m(T_s)$ is known from Eq. (4a) one can find T_s when one knows the portion of the various active materials.)

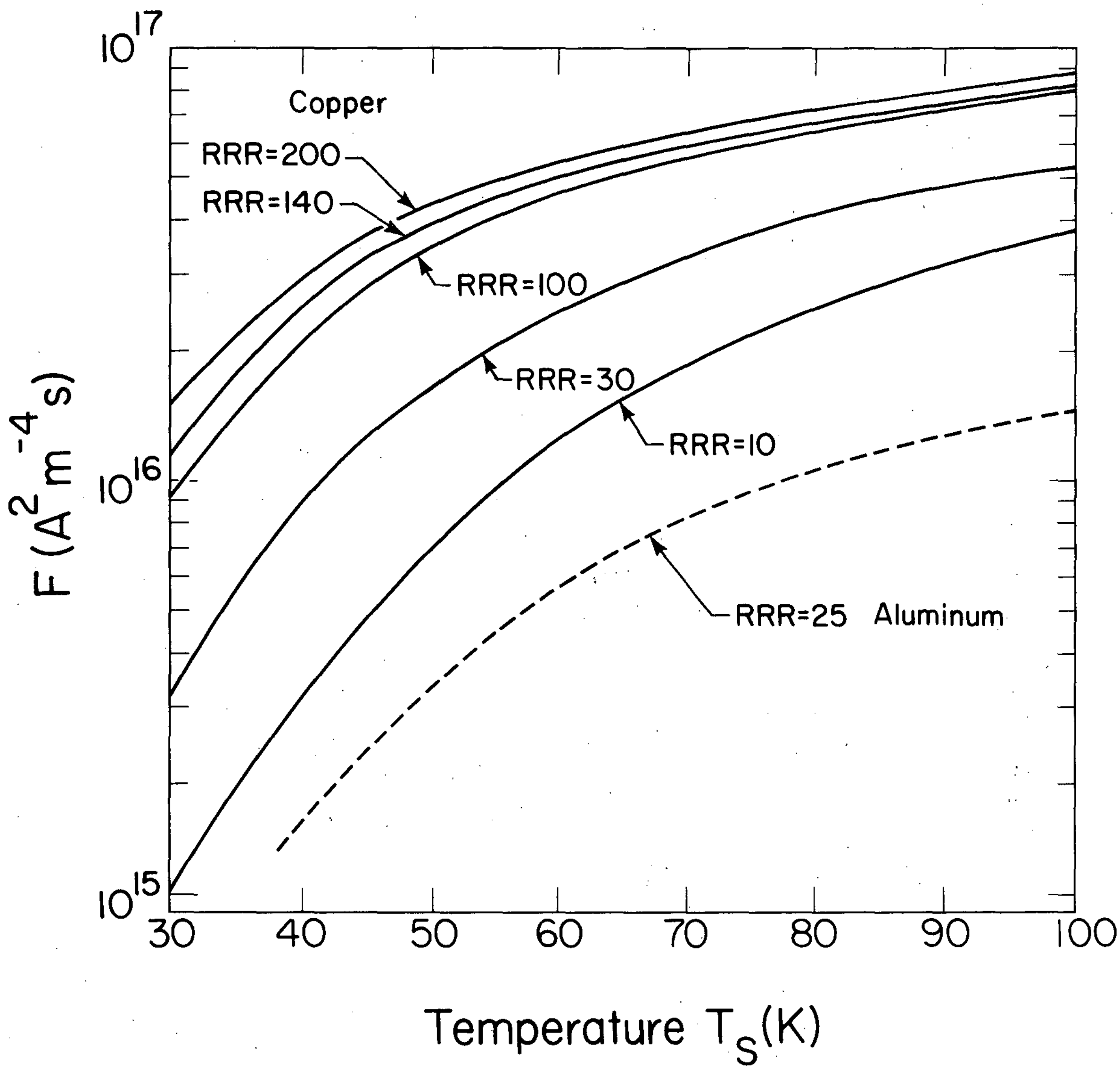

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Fig. 3. Value of F at temperature T_s for copper at various residual resistivity ratios (RRR) and aluminum at RRR = 25.

The value $F(T_m)$ for T_M = 300 K and 400 K for copper at various RRR and aluminum at RRR = 25.

		$F(T_M)$	
Material	RRR	T_M = 300 K	T_M = 400 K
Copper	10	10.5×10^{16}	12.7×10^{16}
Copper	30	11.9×10^{16}	13.6×10^{16}
Copper	100	14.5×10^{16}	16.2×10^{16}
Copper	140	14.8×10^{16}	16.5×10^{16}
Copper	200	15.4×10^{16}	17.9×10^{16}
Aluminum	25	4.0×10^{16}	4.7×10^{16}

$F(T_M)-F(T_{S1})$ is nearly constant for all values of residual resistivity ratio from 10 to 1000 as long as $T_{S1} \geq 65K$.

In large coils one should set the value of T_{S1} to 100K or less. If T_M is set to 300K, and T_{S1} is set to 100K, the maximum quench back time allowed takes the following form for copper based superconductors with a matrix residual resistivity ratio of 10 to 1000;

$$t_{QBR} \approx \frac{6.7 \times 10^{16}}{\left(\frac{r+1}{r}\right) J_o^2} \quad . \qquad (5)$$

Reducing T_{S1} from 100K to 80K increases t_{QBR} by about 16 percent.

3. The Quench Back Process in General

In the previous section it was stated that if the time for quench back t_{QB} is less than t_{QBR}, the high current density magnet would quench without hot spot problems not using external quench protection. This statement is true regardless of the size of the magnet or the current density in the superconductor cross-section. (Note: voltages within the coil could be a problem.) There is only a limited ability to increase t_{QBR} for a given starting current density J_o in the superconductor cross-section. An understanding of the quench back process and the factors which make it work gives one a clear understanding on how to control t_{QB} so that it is less than t_{QBR}.

Quench back in superconducting magnets has two causes:[5,15] 1) normal regions are induced by heat transfer from other parts; we call this form of quench back "thermal quench back." 2) Normal regions can also be induced by A.C. losses in the superconductor due to changing magnetic fields; we call this form of quench back "magnetic quench back". Thermal quench back has been the dominant form of quench back observed in the various LBL magnet tests. Magnetic quench back has been observed in a number of our tests when a variator has been put across the magnet leads. It is suspected that this may be the dominant form of quench back in the Nb_3Sn tape solenoids built by IGC.

Thermal quench back is the form of quench back which is dominant at low currents in a magnet. Thermal quench back has two important time periods associated with it. The first time period is the time in which the secondary circuit heats up to a temperature of around 10°K. The second time period is the time for heat to flow from the secondary circuit to the superconductor. At low currents in a coil, the dominant time period is the time to heat the secondary circuit. As the current density in the superconductor gets higher, the second time period for heat transfer becomes more important. The first time period is a function of the matrix resistivity in the superconductor, the resistivity of the secondary circuit directly adjacent to the superconductor circuit, and the change of enthalpy per unit volume needed to heat the secondary circuit to say 10°K. The second time period for heat transfer is a function of the thickness of the superconductor layer adjacent to the secondary circuit and the thickness of the electrical insulation between the secondary circuit and the superconductor.

Magnetic quench back can become important at high currents if the magnet is constructed to take magnetic quench back into account. Since heat is applied to the superconductor directly through a.c. loss,[15,16,17] there is no heat transfer time period associated with heat flow through an insulator layer. Magnetic quench back is controlled by the superconductor matrix material resistivity, the secondary circuit material resistivity and geometric factors in the coil design. Unlike thermal quench back, the amount and distribution of the secondary circuit material is important for magnetic quench back. The mechanism for magnetic quench back is discussed in Ref. 15. It will not be discussed further here because the experimental data concerning magnetic quench back is spotty at best. At this time, magnetic quench back is not well enough understood to permit its use in the design of a fail safe coil which can be quenched at full current without an active quench protection system.

4. Growth of the Normal Region in the Coil before Quench Back

Normal region growth in a coil starts out three dimensional. In most cases which are of interest to those of us who build magnets with shorted secondary circuits, the normal region growth becomes two dimensional, then one dimensional. When a magnet system has multiple coils, one of those coils may go completely normal before the other coils go normal through quench back.

When quench propagation is three dimensional the growth of coil resistance with time is;[18]

$$R(t) = R_{o3}\,\alpha\,\beta\,V_L^3\,t^3 \quad , \tag{6}$$

when $0 \leq t \leq t3$ where $R(t)$ is the coil resistance, t is time; R_{o3} is the three dimensional resistance constant; α and β are transverse to longitudinal propagation ratios, which are defined later in this section; V_L is the propagation velocity of a quench along the superconducting wire.

t3 represents the time for quench propagation across the thickness of the coil d3. If β is the transverse to longitudinal velocity ratio in the coil thickness direction. (Note: it is assumed that the coils are thin or if they are thick they are broken up into thin sections in order to achieve close coupling with the secondary circuits.) Thus t3 is defined as follows:

$$t3 = \frac{d3}{\beta\, V_L} \quad , \tag{6a}$$

For many of the cases of interest t3 is a relatively short time. (For example: in the TPC magnet t3 is about 2 percent of time that a normal region propagates around the coil, which is the time where two dimensional quench propagation becomes one dimensional quench propagation.) Since in most of the cases of interest t3 is much smaller than the next lowest time of propagation for one of the other dimensions, one can often safely assume that

$$R(t) \approx 0 \quad , \tag{6b}$$

when $0 < t < t3$.

When quench propagation is two dimensional the growth of the magnet resistance is;

$$R(t) = R(t3) + R_{o2}\ \alpha\ V_L^2\ (t-t3)^2 \quad , \tag{7}$$

when $t3 \leq t \leq t^*$ where t^* is the shorter of two times t1 or t2 which are defined as follows

$$t1 = \frac{d1}{V_L} \quad , \tag{7a}$$

and

$$t2 = \frac{d2}{V_L} \quad , \tag{7b}$$

where d1 = half of the length of one turn, (In the case of a solenoid this would be half the circumference) and d2 is the width of the coil (In the case of a solenoid this would be the length of the solenoid.) Note: that d2 is defined as the maximum length which when applied yields the most conservative solution. Using the above definition for d1 and d2 one can define a value for R_{o2}

$$R_{o2} = \frac{N_1 \rho_1 \frac{r+1}{r}}{d2\ A_{c1}} \quad , \tag{7c}$$

where N_1 is the number of turns in the coil; (if there is more than one coil in a system of coils N_1 is the number turns in a single coil.) ρ_1 is the resistivity of the matrix material of the superconductor at a temperature just above critical temperature; r is the normal metal to superconductor ratios in the superconducting wire; d2 is the coil width; and A_{c1} is the cross sectional area of the superconductor (matrix material plus superconductor).

The quench propagation becomes one dimensional at t = t*. If t* = t1 the growth of the magnet resistance takes the following form

$$R(t) = R(t1) + R_{o1}\ \alpha\ V_L(t-t1). \tag{8}$$

If t* = t2 the growth of the magnet resistance takes the following form:

$$R(t) = R(t2) + R_{o1}\ V_L(t-t2)\quad . \tag{9}$$

Note Eq. (8) applies when a quench propagate along a magnet turn faster than it does across the coil width (the TPC magnet is an example of this type of magnet). Equation (9) applies when the propagation across the width of the coil is faster than along a turn. (A large diameter small cross-section coil might be an example of this type of magnet.) It should be noted that there is one special case of a magnet which has one dimensional propagation which does not fit either Eqs. (8) or (9). This is the ultra pure aluminum stabilized thin solenoid. The turn to turn propagation velocity can be so slow that all propagation is along the wire going around and around. This special case is not discussed here.

The value of R_{o1} depends on which equation describes the one dimensional quench propagation. If Eq. (8) describes the resistance growth $t^* = t1$ then;

$$R_{o1} = \frac{2d1\ N_1 \rho_1 \frac{r+1}{r}}{d2\ A_{c1}} , \tag{8a}$$

where d1, d2, N_1, ρ_1, r and A_{c1} are previously defined. If Eq. (9) describes the resistance growth, $t^* = t2$ then;

$$R_{o1} = \frac{2N_1 \rho_1 \frac{r+1}{r}}{A_{c1}} , \tag{9a}$$

again N_1, ρ_1, r, and A_{c1} are previously defined.

Before leaving this study of normal region growth, it useful to point out that Eqs. (7), (8), and (9) assume that once the coil has gone normal its resistance doesn't change. Early in the quench, before a significant portion of the magnet energy is dumped into the coils or the magnet secondary circuits this is true. If one uses Eqs. (7), (8), and (9) as equations which describe the quench back phenomena, a longer quench back time will result. (In this study that is a good thing.)

The quench velocity along the wire V_L is complicated to calculate. A theory for this calculation is given in Ref. (19). Otherwise one should measure the velocity V_L. The functional form for the quench velocity along a niobium titanium in copper wire at low fields is shown in Fig. 4. Figure 4 shows that there is dependence of longitudinal quench velocity V_L on the current density J in the

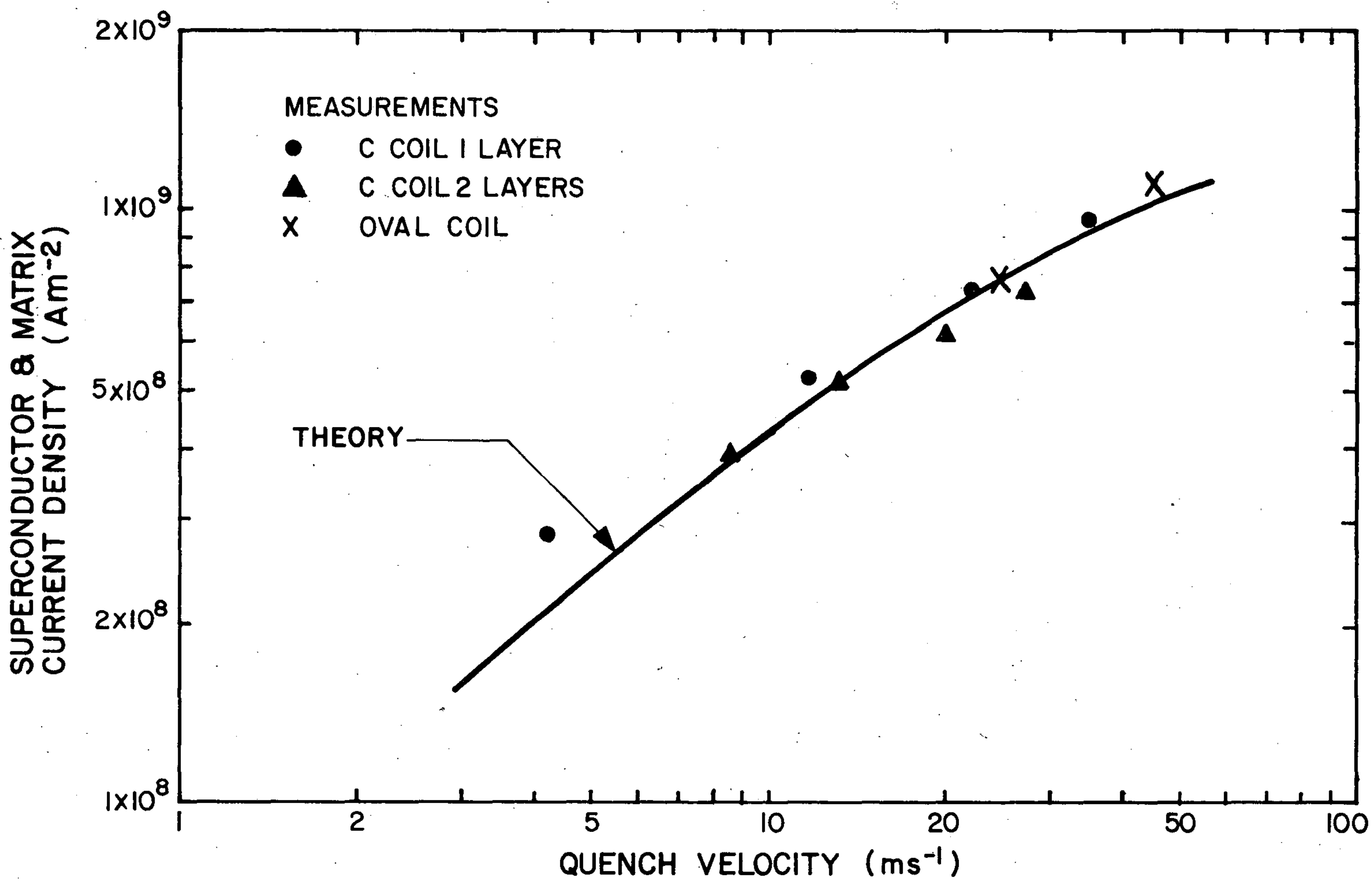

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Fig. 4. A theoretical calculation of quench velocity along the wire as a function superconductor plus matrix current density. Points are measurements in C coil and an oval coil using the C coil conductor which is 1.5 mm ϕ with r = 1.8. The induction at the conductor is about 1.0 Tesla.

superconductor. This dependence varies from $J^{1.4}$ at low current densities to $J^{2.1}$ at high current densities. In general, the longitudinal quench velocity goes up with magnetic field for a given current density (see Fig. 5).[20] To first order, there is little dependence of V_L upon r or ρ_1. Increased temperature speeds up V_L as does increased magnetic field.

The ratio of transverse to longitudinal quench velocity α and β can be expressed approximately using the following expressions, which are derived from equations given in Ref. (18);

$$\alpha \approx \left[\frac{\rho_1 k_i}{L\ T_c}\ \frac{a}{c1}\ \frac{b'}{b}\ \frac{r+1}{r}\right]^{1/2} , \tag{10a}$$

and

$$\beta \approx \left[\frac{\rho_1 k_i}{L\ T_c}\ \frac{b}{c2}\ \frac{a'}{a}\ \frac{r+1}{r}\right]^{1/2} , \tag{10b}$$

where k_i is the thermal conductivity of the insulation; ρ_1 is the resistivity of matrix material; r is the normal metal (matrix) to superconductor ratio; L is the Lorentz number ($L = 2.45\text{x}10^{-8}\ \Omega WK^{-2}$) T_c is the critical temperature of the superconductor (use $T_C = 7$). The values of a, a', b, b', c1 and c2 are dimensions of the superconductor and its insulation as defined in Fig. 6. If the conductor is round and evenly spaced from like conductors, the ratio of transverse to longitudinal quench velocity r takes the following form;

$$\alpha = \beta = \gamma \approx 0.7\left[\frac{\rho_1 k_i}{L\ T_c}\ \frac{D_w}{c3}\ \frac{r+1}{r}\right]^{1/2} , \tag{10c}$$

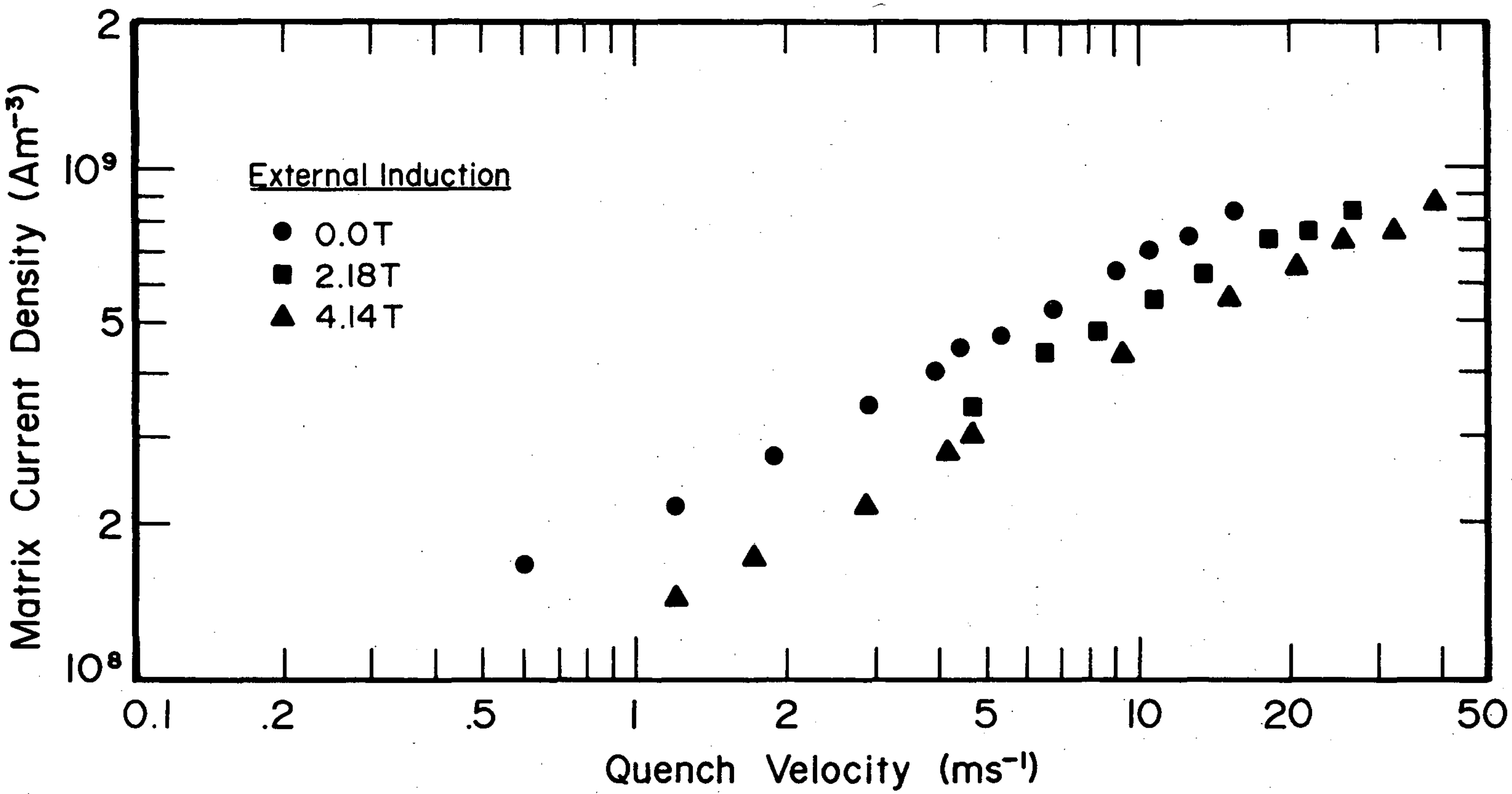

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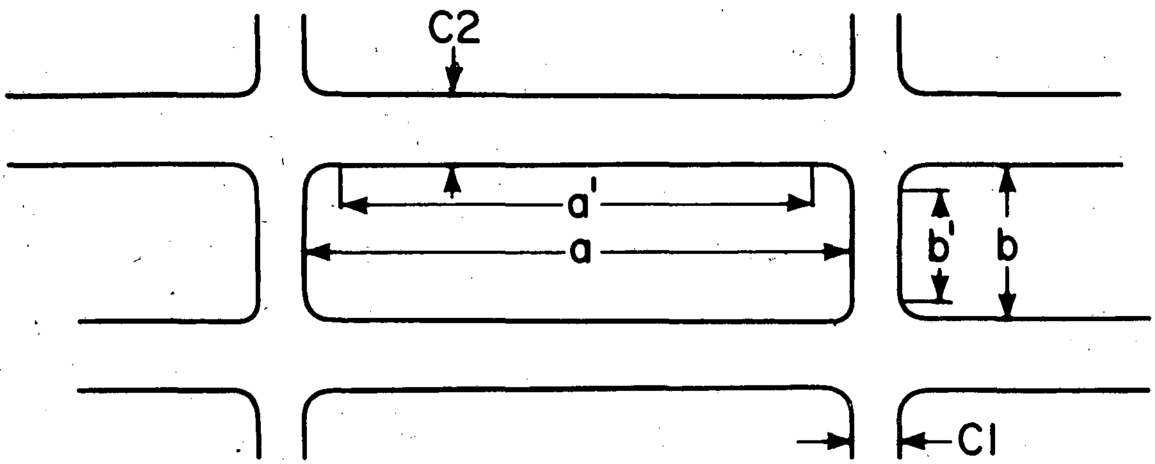
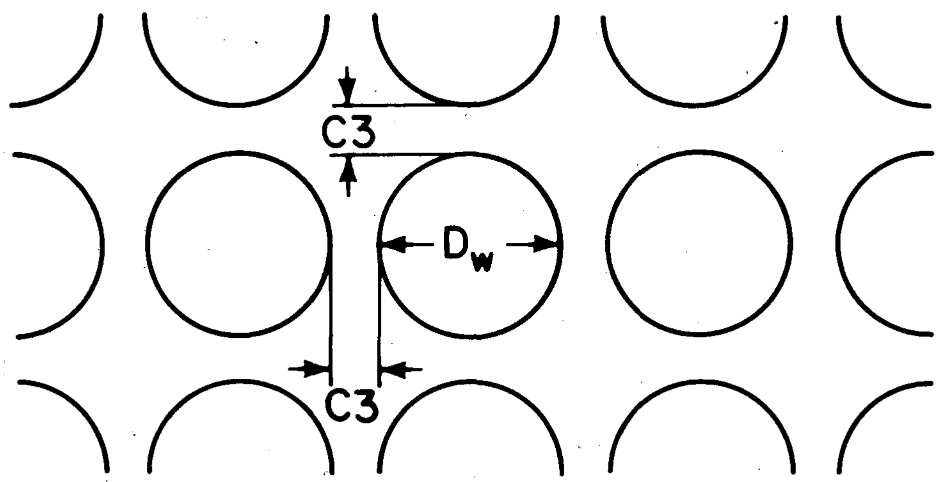
Fig. 5. Measured quench velocity along the superconductor versus overall current density in the superconductor plus matrix material and magnetic induction. This data was taken by M. Scherer and P. Turowski of KFK Karlsruhe in 1976 (see Ref. 20).



a. Rectangular conductor package



b. Evenly spaced round conductor package

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Fig. 6. The dimensions of superconductor and insulation between conductors which are used in Eqs. (10a), (10b) and (10c).

where c3 is the thickness of the insulation between wires and D_w is the diameter of the wire. (See Fig. 6). (Note: In most cases Eq. 10a, 10b, and 10c yield a good approximation of the velocity ratio, but care must be taken when configurations are different than those shown in Fig. 6.)

5. Shift of the Current from the Primary Circuit to the Secondary Circuit

The development of the coupled equations which describe how the current shifts from the primary circuit to the secondary circuits is described elsewhere. I start with the assumption that the coupling between all circuits is good (at least 95 percent). This simplifies the coupled equations considerably. The second simplification is to assume that the magnet system consists of two closely coupled circuits (it is not too difficult to extend it to three.)

When the coupling is good, the solution takes the following form:[4]

$$\begin{aligned} i_1 &= \frac{i_o}{(\tau_L-\tau_s)}\left[(\tau_1-\tau_s)e^{-t/\tau_L} + (\tau_L-\tau_1)e^{-t/\tau_s}\right] , \\ i_2 &= \frac{M_{12}\, i_o}{L_2(\tau_L-\tau_s)}\left[(\tau_2+\tau_s)e^{-t/\tau_L} - (\tau_2+\tau_s)e^{-t/\tau_s}\right] , \end{aligned} \tag{11}$$

where τ_1, τ_2, τ_L, τ_s, are defined as follows

$$\tau_1 = \frac{L_1}{R_1} \quad , \tag{11a}$$

$$\tau_2 = \frac{L_2}{R_2} \tag{11b}$$

$$\tau_L \approx \tau_1 + \tau_2 \quad , \tag{11c}$$

$$\tau_s \approx \frac{\varepsilon \; \tau_1 \tau_2}{\tau_1 + \tau_2} \quad , \tag{11d}$$

$$\varepsilon = 1 - \frac{M_{12}^2}{L_1 L_2} \quad , \tag{11e}$$

where L_1 is the inductance of the primary circuit (the superconducting coil); L_2 is the inductance of the secondary circuit; R_1 is the resistance of the primary circuit given in Eqs. (6), (7), (8), and (9); R_2 is the resistance of the secondary circuit; and M_{12} is the mutual inductance between the two circuits. In this theory before quench back occurs, L_1, L_2, M_{12}, and R_2 are assumed to be constant.

i_1 and i_2 can be calculated directly from Eq. (11), or when τ_s is small and when it is early in the quench before much energy has been absorbed by the coil or the secondaries a simplified form can be used;

$$\begin{aligned} i_1 &= \frac{\tau_i}{\tau_1 + \tau_2} \, i_o \quad , \\ i_2 &= \frac{\tau_2}{\tau_1 + \tau_2} \, \frac{N_1}{N_2} \, i_o \quad . \end{aligned} \tag{12}$$

6. <u>The Time Required for the Secondary Circuit to Reach the Quench Back Temperature</u>

Let us define t_Q as the time needed to get the secondary circuit warm enough for quench back to occur. Let us define this temperature as T_Q. For a magnet made with Niobium Titanium conductor T_Q = 10K. If the magnet is made from Nb_3Sn conductor T_Q is from 12 to 18K depending on the field in the conductor. If one applies Eq. (1) to the secondary circuit one can define an F* which is the change in F required to heat the secondary circuit from the initial temperature T_o (about 4.5K) to T_Q

$$F_2^* = F_2(T_Q)-F_2(T_o) = \int_0^{t_Q} J_2(t)^2 dt \quad , \tag{13}$$

where

$$F_2(T_Q) = \int_0^{T_Q} \frac{c2(T)}{\rho_2(T)} dT = \frac{H_2(T_Q)}{\rho_2} \quad , \tag{13a}$$

$$F_2(T_o) = \int_0^{T_o} \frac{c2(T)}{\rho_2(T)} dT = \frac{H_2(T_o)}{\rho_2} \quad , \tag{13b}$$

where $H_2(T)$ is the volume enthalpy of the secondary circuit material as a function of temperature T; $\rho_2(T)$ is the resistivity of the secondary circuit material as a function of temperature; $J_2(t)$ is the current density in the secondary circuit material as a function of time t and t_Q is time needed to get the secondary circuit warm.

Equation (13) may be rewritten such that F_2* is a function of the enthalpy charge ΔH_2 of the secondary circuit material between T_o and T_Q and the resistivity of the material in the secondary circuit ρ_2. At temperatures below 18K, ρ_2 is a constant for most materials which could be used in a secondary circuit thus $\rho_2(T) = \rho_2$ for all T from 0 to 18K. Thus we find that

$$F_2^* = \frac{\Delta H_2}{\rho_2} = \int_0^{t_Q} J_2(t)^2 dt \quad , \tag{14a}$$

where

$$\Delta H_2 = H_2(T_Q) - H_2(T_o) \quad . \tag{14b}$$

Using Eqs. (14) and (12) (I assume close coupling and not much energy loss resistive heating) one gets the following form for the heating of the secondary circuit to the quench back temperature T_Q

$$F_2^* = \frac{\Delta H_2}{\rho_2} = \int_0^{t_Q} \left(\frac{\tau_2}{\tau_1+\tau_2} \frac{N_1}{N_2} \frac{i_o}{Ac2} \right)^2 dt \quad , \tag{15}$$

where τ_1, τ_2, N_1, N_2, and i_o have been previously defined from Eq. (12) and A_{c2} is the cross-sectional area of the secondary circuit which causes quench back. Equation (15) can be used even when more than one secondary circuit is in the magnet structure. The $\tau_1+\tau_2$ term is replaced by a τ_L which is defined as

$$\tau_L \approx \sum_1^N \tau_m \quad , \tag{15a}$$

where

$$\tau_m = \frac{L_m}{R_m} \quad . \tag{15b}$$

For magnet structures which are of interest, τ_m is constant with time for all values of m except 1.

When the coupling is not good enough to insure a low value of τ_s, Eq. (15) can be replaced by

$$F_2^* = \frac{\Delta H_2}{\rho_2} = \int_0^{t_Q} \left(\frac{i_2}{Ac2}\right)^2 dt \quad , \tag{16}$$

where i_2 comes from Eq. (11). In both Eqs. (15) and (16), the value of t_Q is the time to heat the secondary circuit to a temperature T_Q (high enough to cause quench back).

In Eqs. (15) or (15a) the value of τ_1 can be found using the following relationships

$$\tau_1 = \frac{L_1}{R_{o3}\ \alpha\ \beta\ V_L^3\ t^3} \quad , \tag{17a}$$

when $0 \leq t \leq t3$ τ_1 is generally very large because R(t) is usually quite small in the time region.

$$\tau_1 = \frac{L_1}{R(t3)+R_{o2}\ \alpha\ V_L^2(t-t3)^2} \quad , \tag{17b}$$

when $t3 \leq t \leq t^*$, and R_{o2} is defined by Eq. (7c)

$$\tau_1 = \frac{L_1}{R(t1)+R_{o1} \alpha V_L(t-t1)} , \quad (17c)$$

when $t^* \leq t \leq t^{**}$, $t^* = t1$, and R_{o1} is defined by Eq. (8a):

$$\tau_1 = \frac{L_1}{R(t2)+R_{o1} V_L(t-t2)} , \quad (17d)$$

when $t^* \leq t \leq t^{**}$, $t^* = t2$ and R_{o1} is defined by Eq. (9a). Note t^{**} is defined as the longer of the two times t1 and t2. If there is more than one coil and $t > t^{**}$ the value of τ_1 to first order can be defined

$$\tau_1 = \frac{L_1}{R_C} , \quad (17e)$$

where R_C is the resistance of the one coil when it is completely normal.

The growth of the normal region in the coil proceeds according to Eqs. (15) or (16) until quench back occurs at time t_{QB}. t_{QB} is greater than t_Q.

$$t_{QB} \approx t_Q + t_H , \quad (18)$$

where t_H is a time period associated with the time heat needs to flow from the secondary circuit to heat up the superconductor.

Table 1 shows the enthalpy of various materials as a function of temperature from 4.5 to 15K.[13,14] This table can be used to calculate the value of ΔH_2 given in Eqs. (14), (15), and (16).

Table 1. Enthalpy of various materials as a function of temperature from 4.5 to 15°K.

Temperature	Enthalpy ($J \cdot m^{-3}$) Aluminum*	Copper*	Nb-Ti+	Epoxy**
4.5	1620	1600	8800	9000
5.0	2110	2230	11300	13000
6.0	3270	3830	17200	24000
7.0	4860	6330	24900	40000
8.0	7020	9790	34700	62000
10.0	13200	21400	--	120000
12.0	22400	40100	--	200000
15.0	48600	95200	--	360000

*See Reference 14 for tables and charts of enthalpy.

+See Reference 22 for method of enthalpy calculation.

**Estimate--good data is hard to find.

The resistivity of the secondary circuit material ρ_2 is a function of the material, its purity, and the magnetic field it is in. This also applies to ρ_1 the superconductor matrix material resistivity. Table 2 shows the resistivity at 15K of various coppers and aluminums at zero field and 1.0 T. As a general rule, one can use the zero field resistivities for calculating quench back time. The result will nearly always yield a quench back time which is longer than if one uses a value corrected for magneto-resistance.

7. The Time Required to Transfer Enough Heat into the Superconductor to Drive it Normal

Equation (18) suggests there is a time required to transfer heat to the superconductor once the secondary circuit has reached a temperature above the superconductor critical temperature in a Eq. (18), this time is defined at t_H.

The time period for heat transfer can be represented by the sum of two terms. The first is the time required to pass heat through insulation between the shorted secondary and the superconductor to heat up the superconductor to the critical point. The second is the time required to heat up the insulation to an intermediate temperature between that of the shorted secondary and the superconductor. The time required to transfer heat to the superconductor is approximately;

$$t_H \approx \frac{b\ c4\ \Delta H_{SC}}{k_{in}\ \Delta T} + \frac{(c4)^2 \Delta H_i}{k_{in}\ \Delta T}, \tag{19}$$

Table 2. The resistivity of coppers and aluminums at 15K as a function of residual resistivity and magnetic field

Material	RRR*	Resistivity Ωm B=0.0 T	Resistivity Ωm B=1.0 T
Copper	10	1.55×10^{-9}	1.6×10^{-9}
	30	5.17×10^{-10}	5.8×10^{-10}
	100	1.55×10^{-10}	2.0×10^{-10}
	300	5.17×10^{-11}	1.0×10^{-10}
Aluminum	14	1.8×10^{-9}	1.8×10^{-9}
	25	1.0×10^{-9}	1.1×10^{-9}
	100	2.45×10^{-10}	3.5×10^{-10}
	300	8.12×10^{-11}	1.3×10^{-10}
	1000	2.45×10^{-11}	5.0×10^{-11}

* $RRR = \dfrac{\rho(273K)}{\rho(4.2K)}$ = residual resistivity ratio

where t_H is the time required to transfer heat into the superconductor to drive it normal, b is the superconductor thickness, c4 is the insulation thickness, k_{in} is the insulation thermal conductivity between the shorted secondary and the coil,[21] and ΔT is the temperature difference between the shorted secondary at time t_Q and the starting temperature T_o (in other words, $\Delta T = t_Q - T_o$). ΔH_{SC} is the enthalpy change needed to drive the superconductor normal, and ΔH_i is the average enthalpy change in the insulation layer. ΔH_i can be approximated as follows:

$$\Delta H_i \approx \frac{H(T_Q)-H(T_o)}{2} \quad , \tag{20}$$

where for thin insulation $H(T_Q)$ and $H(T_o)$ can be found for epoxy plastic in Table 1. When the insulation is thicker ΔH_i is lower because of glass in it.

The calculation of ΔH_{SC} is more difficult because the enthalpy change includes a sensible (i.e., temperature dependent) term and a term due to magnetization of the superconductor

$$\Delta H_{SC} = \Delta H_S + \Delta H_M \quad , \tag{21}$$

where

$$\Delta H_S = \frac{r(\Delta H_{Cu})+\Delta H_{Nb-Ti}}{1+r} \tag{21a}$$

and

$$\Delta H_M = \frac{B_{c1}B_{c2}}{2\mu_o(1+r)} \left\{1 - \frac{1}{J_c(0)} - \left[\frac{T}{T_C(0)}\right]^2\right\} \left\{1 + 3\left[\frac{T}{T_C(0)}\right]^2 - \frac{J_c}{J_c(0)}\right\} \quad , \tag{21b}$$

where ΔH_S is the sensible enthalpy change and ΔH_M is the enthalpy change due to collapse of magnetization in the wire. r is the normal metal-to-superconductor ratio, ΔH_{Cu} is the enthalpy change from T_o to T_C for the normal substrate metal, and ΔH_{Nb-Ti} is the sensible enthalpy change in Nb-Ti from T_o to T_C.[22]

Derivation of Eq. (21b) is found in Ref. (22). The symbol definition in Eq. (21b) is as follows: B_{c1} is the lower critical induction at 0°K for the Type II superconductor (B_{c1} is about 0.014 T for Nb-Ti); B_{c2} is the upper critical induction at 0°K for the Type II superconductor (B_{c2} is about 14.0 for Nb-Ti); μ_o is the permeability of air ($\mu_o = 4\pi \times 10^{-7}$); J_c is the current density in the matrix; $J_c(0)$ is the critical current density in the matrix at zero temperature; T is the operating temperature (we have called it T_o in other equations); $T_C(0)$ is the critical temperature at zero induction and current density ($T_C(0)$ for Nb-Ti is about 9.5°K); and r is the copper-to-superconductor ratio in the matrix.

Tables 3 and 4 give values of ΔH_i, k_{in}, and ΔH_{SC} which can be applied in Eq. (19) to calculate the heat transfer time t_H. Both tables assume that the starting temperature T_o is 4.5°K. Equation (19) has interesting implications. The thickness of the insulation c4 has a large effect on the time required to drive the superconductor normal once the secondary circuit has heated up.

Table 3. Enthalpy change ΔH_i and average thermal conductivity k_i as a function of the temperature difference ΔT of the heated shorted secondary and the initial temperature $\Delta T = T_Q - T_o$ and the thickness of the insulation.

Thickness	ΔT	$k_i (Wm^{-1}K^{-1})$	$\Delta H_i (Jm^{-3})$
0.2 mm	2	0.045	16000
0.1 Formvar	5	0.051	60000
0.1 Epoxy	10	0.063	200000
0.5 mm	2	0.119	15000
0.1 Formvar	5	0.127	55000
0.4 Epoxy-glass	10	0.136	180000
1.0 mm*	2	0.130	14000
0.1 Formvar	5	0.139	50000
0.9 Epoxy-glass	10	0.148	160000

*For insulation thicker than 1.0 mm use the same values as for 1.0 mm

Table 4. Enthalpy change ΔH_{SC} as a function of magnetic induction in the material and the copper-to-superconductor ratio r.
(Note: $T_o = 4.5°K$)

	$\Delta H_{SC} (J \cdot m^{-3})$**		
Induction	r = 1.0	r = 1.8	r = 2.5
0.5	24500	19300	16800
1.0	12100	9300	7900
1.5	8200	6300	5300
1.65	6900	4400	3700

*The material is copper based Nb-Ti multifilamentary superconductor

**These values apply for a magnet similar to the TPC magnet.

8. Determination of the Final Temperatures in the Magnet at the End of the Quench

There are at least three important temperatures of at the end of the quench process in a magnet with shorted secondary circuits. They are; 1) the hot spot temperature of the superconductor T_M, 2) the temperatures that a quench backed part of the coil comes to at the end of the quench T_{S1}, and 3) the final temperature that the secondary circuits come to at the end of the quench T_{S2}, T_{S3} and so on. The assumption used to calculate a maximum required quench back time t_{QBR} was based on the notion that $t_{S1} = t_{S2} = t_{S3}$ and so on. Once one knows the the characteristics of the circuits and the coupling coefficients, one can calculate T_M, T_{S1}, T_{S2} and so on.

To illustrate the technique, let's look at a simple circuit represented by a quench superconducting coil and its shorted secondary circuit. Equations (11) can be used to calculate i_1 and i_2 for short periods of time. One must note the τ_1 and τ_2 are functions of temperature and time. One must adjust τ_L and τ_s accordingly. If the problem is broken up into time steps a computer can calculate all of the relevant parameters in a reasonable way. As a result, $i_1(t)$ and $i_2(t)$ throughout the quench can be calculated.

Using Eqs. (11) and (17) before quench back one can follow the quench back process and calculate i_1 and i_2 from $t=0$ to $t=t_{QB}$ when t is greater than t_{QB} both the primary and secondary circuits (both are in the normal state) heat up. The resistance of those circuits increase with increasing temperature (τ_1 and t_2 will go down) in general the values of T_M, T_{S1} and T_{S2} can be found from:

$$F(T_M) = \int_0^{T_M} \frac{C1(T)}{\rho_1(T)}\, dT = \frac{r+1}{r} \int_0^{\infty} J_1(t)^2 dt \quad , \tag{22}$$

where

$$J_1(t) = \frac{i_1(t)}{A_{c1}} \quad , \tag{22a}$$

and

$$F(T_{S1}) = \int_0^{T_{S1}} \frac{C1(T)}{\rho_1(T)}\, dT = \frac{r+1}{r} \int_{t_{QB}}^{\infty} J_1(t)^2 dt \quad , \tag{23}$$

and

$$F(T_{S2}) = \int_0^{T_{S2}} \frac{C2(T)}{\rho_2(T)}\, dT = \int_0^{\infty} J_2(t)^2 dt \quad , \tag{24}$$

where

$$J_2(t) = \frac{i_2(t)}{A_{c2}} \quad . \tag{24a}$$

T_{S1} and T_{S2} can also be calculated using an equation which is similar to Eq. (3).

9. An Approximate Calculation of t_Q

Solving Eq. (16) for t_Q is an involved process which does not lend itself to a closed form solution. One can make a first order estimate of t_Q if several assumptions are made. They are:

1) The current in the primary circuit drops only a little before quench back occurs. This assumption is often valid when $i_1 > 0.7\ i_o$.

2) The velocity of quench propagation along the wire does not change with time. Quench velocity is determined by J_o.

3) The energy dissipated in the coil and its secondaries is small before quench back (small compared to the total magnetic energy stored in the coil).

4) The coupling is good so that t_{QB} is large compared to τ_s ($t_{QB} \gg \varepsilon\tau_2$).

5) $\tau_2 \ll \tau_1$ during most of the quench back process so that

$$J_2(t) = \frac{1}{A_{c2}} \frac{\tau_2}{\tau_1} \frac{N_1}{N_2} i_o \quad . \tag{25}$$

6) L_1, L_2 and M_{12} are constant and the physical dimension of the circuits are nearly the same.

The method of derivation of t_Q is nearly the same as the one used in Ref. (15). This method is extended here to include cases with linear quench propagation and a constant resistance case (where one coil in a string has gone normal). The equation for determining quench back time is dependent on whether quench propagation is three-dimensional, two-dimensional, one-dimensional or constant R. One must do the calculation first in order to find out what regime one is in.

When one is trying to design a coil with fail safe quenching, one knows the quench back time required t_{QBR} in order to insure fail safe operation. Since the required quench back time is known, a required value of t_Q is also known. The required value for t_Q for fail safe quenching known as t_{QR} can be estimated as follows:

$$t_{QR} \approx t_{QBR} - t_H \quad , \tag{26}$$

where t_{QBR} is given by Eq. (2), (a conservative value is given by Eq. (5)) and t_H is given by Eq. (19). When one is designing a fail safe coil, one simply assumes (as a first quess) that t_Q is less than or equal to t_{QR}. To determine the quench propagation regime compare t_{QR} with t3, t2 and t1.

If t_{QR} is less than t3 (see Eq. (6a)), the quench propagation is three dimensional. This case is highly unlikely because in order to have close coupling between primary and secondary circuits t3 must be small. If t_{QR} is less than t* (the smaller of t1 or t2 as defined by Eqs. (7a) and (7b), the quench propagation is quadratic. An approximate equation for quadratic quench propagation (Eq. (16) in Ref. 15) is as follows

$$t_Q = \frac{t3}{3} + \left[\frac{5\,F_2^*}{\left(\dfrac{L_2\, R_{o2}\, N_1\, \alpha\, V_L^2\, i_o}{L_1\, R_2\, N_2\, A_{c2}} \right)^2} \right]^{1/5} \quad , \tag{27}$$

where F_2* is determined using Eq. (14), L_1 is the primary circuit inductance, L_2 is the secondary circuit inductance; N_1 is the number of turns in the primary circuit. N_2 is the number of turns in the secondary circuit, R_2 is the resistance of the secondary circuit, V_L is the quench velocity along the wire (see Ref. 19); i_o is the starting current in the coil; and A_{c2} is the cross-sectional area of the secondary circuit. The term R_{o2} is defined by Eq. (7c) and α is defined by Eq. (10a) or (10c).

When t_{QR} is greater than t*, the quench propagation is one dimensional. The one dimensional quench propagation equations can take two forms depending on whether t* = t2 or t* = t1. This depends on whether one dimensional propagation is along the superconductor or from turn to turn perpendicular to the direction of the conductor.

When t* = t1 one dimensional propagation is from turn to turn. (This form is important in the TPC magnet.) The equation for t_Q takes the following approximate form;

$$t_Q \approx \frac{t1}{2} + \left[\frac{3\,F_2^*}{\left(\dfrac{L_2\,R_{o1}\,N_1\,\alpha\,V_L\,i_o}{L_1\,R_2\,N_2\,A_{c2}}\right)^2}\right]^{1/3} , \qquad (28)$$

where F_2*, L_1, L_2, N_1, R_2, V_L, i_o, A_{c2} and α are defined as before. R_{o1} is defined using Eq. (8a).

When t* = t2 one dimensional propagation is along the wire (large coils with relatively small cross-sections would have this property.) The equation for t_Q takes the following form:

$$t_Q \approx \frac{t2}{2} + \left[\frac{3\,F_2^*}{\left(\dfrac{L_2\,R_{o1}\,N_1\,V_L\,i_o}{L_1\,R_2\,N_2\,A_{c2}}\right)^2}\right]^{1/3} , \qquad (29)$$

where F_2^*, L_1, L_2, N_1, R_2, V_L, i_o, A_{c2} are previously defined, and R_{o1} is defined using Eq. (9a).

Equations (27), (28), and (29) apply in a majority of cases. When the magnet consists of a system of N_C coils where one coil may become completely normal before quench back drives the other coils normal, a fourth form of the t_Q equation will apply. This applies when t_{QR} is greater than t** and t** is defined as the longer of the two times t1 or t2 (note: t** is always greater than t*.) When t_{QR} is greater than t^{**}, constant resistance quench back occurs. The equation for t_Q takes the following form;

$$t_Q \approx t^{**} + \frac{F_2^{**}}{\left(\frac{L_2\ R_1\ N_1\ i_o}{L_1\ R_2\ N_2\ A_{c2}\ N_C}\right)^2} \quad , \tag{30}$$

where L_1, L_2, N_1, N_2, R_2, i_o, and A_{c2} have been previously defined. R_1 is the resistance of the lentine primary circuit when all of the coils just turned normal. (The resistance of one coil in a system with N_C coils when its just above the critical point is $R_c = R_1/N_C$.) F_2^{**} is defined as follows depending on the value of t*. When t* = t1

$$F_2^{**} = F_2^* - \left(\frac{L_2\ R_{o1}\ N_1\ \alpha\ V_L\ i_o}{R_2\ L_1\ N_2\ A_{c2}\ N_C}\right)^2 \ \frac{t^{**\ 3}}{3} \quad , \tag{30a}$$

where R_{o1} is defined by Eq. (8a) when t* = t2

$$F_2^{**} = F_2^{*} - \left(\frac{L_2\ R_{o1}\ N_1\ V_L\ i_o}{R_2\ L_1\ N_2\ A_{c2}\ N_C}\right)^2 \ \frac{t^{**\,3}}{3} \quad , \tag{30b}$$

where R_{o1} is defined by Eq. (9b). Note that the other variables in Eqs. (30a) and (30b) have been defined previously.

Equations (27), (28), (29), and (30) give value for quench back time which in most cases are conservative. One must have make sure that the quench propagation regime is understood in order to apply the proper model. Equations (27) through (30) do not describe the dynamics of the quench back process. In the next section these equations will be further simplified so that t_Q becomes a function of basic material properties such as ΔH_2, ρ_2 and ρ_1. Once that has been done, one can understand the quench process well enough to design the superconductor and the secondary circuits so that fail safe quenching of high current density coils will result.

10. Quench Back Time t_{QB} as a Function of Basic Material Parameters and Current Density in the Coil

Equations (27) through (30) can be used to find t_Q and when combined with Eq. (19) t_{QB}. A better understanding of the quench back process requires further modification of these equations. Most superconducting magnets which have used the shorted secondary concept, have been relatively thin and are bounded by iron poles. As a result, L_1 and L_2 can be simply stated as can R_2 the resistance of the secondary circuit[23,24]

$$L_1 \approx \frac{\mu_o \ \pi \ a_1^2 \ N_1^2}{\ell_1} \ g_1(a_1,\ell_1) \quad , \tag{31a}$$

$$L_2 \approx \frac{\mu_o \ \pi \ a_2^2 \ N_2^2}{\ell_2} \ g_2(a_2,\ell_2) \quad , \tag{31b}$$

$$R_2 \approx \frac{\rho_2 \ 2\pi \ a_2 \ N_2}{A_{c2}} \quad , \tag{31c}$$

where a_1 is the average radius of the superconducting coil; a_2 is the average radius of the secondary circuit coil; ℓ_1 is the length of the superconducting coil; ℓ_2 is the length of the secondary circuit coil; N_1 is the number of turns in the superconducting coil; N_2 is the number of turns in the shorted secondary; ρ_2 is the resistivity of the secondary circuit material; and A_{c2} is the cross-sectional are of the secondary circuit; $g_1(a_1,\ell_1)$ and $g_2(a_2,\ell_2)$ are geometric inductance factors. $g_1(a_1,\ell_1)$ and $g_2(a_2,\ell_2)$ are one when the solenoid is bounded by unsaturated iron poles.

Since the coupling between the primary and secondary circuit is good $a_1 \approx a_2$ and $\ell_1 \approx \ell_2$. Therefore the geometric factor g_1 and g_2 are to first order, equal to one another. Thus Eq. (31a) and (31b) can be used for L_1 and L_2 in the general case with no iron. Because the coupling is good, the g_1 and g_2 terms are divided out.

The value of F_2* is given by Eq. (14). F_2* is a function of nothing but ΔH_2 and ρ_2. The turn to turn to longitudinal quench velocity ratio α found in Eqs. (27) and (28) can be substituted using Eq. (10a). The value of R_{o2} in Eq. (27) is found by substitution from

Eq. (7c); the value of R_{o1} in Eq. (28) is found by substitution from Eq. (8a); and the value of R_{o1} in Eq. (29) is found by substitution from Eq. (9a).

When one makes the substitutions given above into Eq. (27), one finds that this equation will take the following form:

$$t_Q \approx \frac{t3}{3} + \left[\frac{5\ \Delta H_2\ \rho_2}{\dfrac{a_2^2\ \ell_1^2\ \rho_1^3\ \left(\frac{r+1}{r}\right)^3\ J_o^2\ k_i\ a\ b'\ V_L^4}{\ell_2^2\ a_1^4\ 4\pi^2\ d_2^2\ L\ T_c\ cl\ b}} \right]^{1/5} . \qquad (32)$$

One can simplify the equation above further by applying the following approximations

$$\begin{aligned} a_1 &\approx a_2 \\ \ell_1 &\approx \ell_2 \\ 4\pi^2 &\approx 40 \\ \Gamma &\approx \frac{k_i ab'}{clb} . \end{aligned} \qquad (32a)$$

For a solenoid magnet d_2 is the length of a coil ℓ_1; and d_1 is the πa_1 half a turn length where a_1 is the coil radius. L is the Lorentz number ($L = 2.45 \times 10^{-8}\ W\Omega K^{-2}$). The temperature used is the critical temperature T_c. For the sake of this approximation, one can use $T_c = 7K$. Thus Eq. (32) takes the following form:

$$t_Q \approx \frac{t3}{3} + \left[\frac{1400\ \Delta H_2\ \rho_2\ L\ a_1^2\ \ell_1^2}{\rho_1^3\ \left(\frac{r+1}{1}\right)^3\ \Gamma\ V_L^4\ J_o^2} \right]^{1/5} . \qquad (33)$$

Equations (28) and (29) can be manipulated in the same way as Eq. (27) was. I will leave this manipulation to the reader. When this manipulation has been done on Eq. (28), one gets the following form;

$$t_Q \approx \frac{t1}{2} + \left[\frac{21\ \Delta H_2\ \rho_2\ L\ \ell_1^2}{\rho_1^3\ \left(\frac{r+1}{r}\right)^3\ \Gamma\ V_L^2\ J_o^2}\right]^{1/3} . \tag{34}$$

When this manipulation has been done on Eq. (29), one gets the following form;

$$t_Q \approx \frac{t2}{2} + \left[\frac{30\ \Delta H_2\ \rho_2\ a_1^2}{\rho_1^2\ \left(\frac{r+1}{r}\right)^2\ J_o^2\ V_L^2}\right]^{1/3} \tag{35}$$

Note: when a round wire is used instead of a rectangular wire, the value of Γ used in Eq. (33), (34), and (35) takes the following form

$$\Gamma \approx 0.5\ \frac{k_i\ D_w}{C_3} \tag{32b}$$

We find from Eqs. (33), (34), and (35) that quench back in a solenoid is a function of ρ_2, the resistivity of the secondary material, ΔH_2 the enthalpy change in the secondary needed to induce quench back, ρ_1 the resistivity of the matrix material in the superconductor, r the matrix material to superconductor ratio in the conductor, J_o the starting current density in the conductor, and V_L the quench velocity along the wire at a current density J_o (note: V_L is a function of J_o as well. From Fig. 4, one can use $V_L \approx 2.65 \times 10^{-15}\ J_o^{1.8}$.) The quench back time t_Q is also a function of various coil geometric factors and where applicable it is also a function of Γ, the constant which determines the turn to turn quench propagation.

For the sake of completeness let's look at Eq. (30) when one coil in a magnet which consists of N_C coils becomes normal. The quench propagates in the one coil until it becomes completely normal. Equation (36) gives the time to quench back if quench back occurs after the one coil has become completely normal. The equation for quench back which is equivalent to Eq. (30) is as follows:

$$t_Q \approx t^{**} + \frac{\rho_2^2 \, N_C^2 \, F_2^{**}}{\rho_1^2 \left(\frac{r+1}{r}\right)^2 J_o^2} , \tag{36}$$

where ρ_1, ρ_2, J_o and r are previously defined N_C is the number of coils in the string which has a total inductance L_1 and F_2** is defined as follows depending on the value of t*. When t* = t1

$$F_2^{**} = \frac{\Delta H_2}{\rho_2} - \frac{\rho_1^3 \left(\frac{r+1}{r}\right)^3 \Gamma \, V_L^2 \, J_o^2 \, t_o^3}{21 \, \ell_1^2 \, \rho_2^2 \, L \, N_C^2} , \tag{36a}$$

where ρ_1, ρ_2, r, Γ V_L, J_o, t_o, ℓ_1, N_C, ΔH_2, and L are previously defined. When t* = t2

$$F_2^{**} = \frac{\Delta H_2}{\rho_2} - \frac{\rho_1^2 \left(\frac{r+1}{r}\right)^2 V_L^2 \, J_o^2 \, t_o^3}{30 \, a^2 \, \rho^2 \, N^2} , \tag{36b}$$

where ρ_1, ρ_2, V_L, J_o, t_o a_1, N_C, and ΔH_2 have been previously defined.

11. The Role of Quench Back in the Quench Protection of a Magnet with an External Resistor

Quench protection with an external resistor has been studied for the last twenty years.[2,12] In 1977, the effect of a shorted secondary circuit on quench protection with a resistor or varistor was presented.[3,8,9] The shorted secondary circuit closely coupled to the primary circuit reduces size of the external resistor and the voltage across the leads when the resistor is switched into the circuit. If the resistor is replaced by a thyrite varistor, the peak voltage is reduced further.[7,10] The previous theory dealing with external quench protection ignores quench back. Quench back can have a profound effect on the sizing of an external resistor. The value of the external resistor and the peak voltage across the leads are reduced substantially when quench back is considered.

Equation (22) can be used to design quench protection system based upon a resistor being put across the leads of the magnet. If there is no shorted secondary circuit, the value of $F(T_M)$ (see Eq. (22)) can be calculated as follows;

$$F(T_M) = \frac{r+1}{r} J_o^2 \left(\frac{\tau_1}{2} + t_{SO}\right) , \tag{37}$$

where

$$\tau_1 = \frac{L_1}{R_{ex}} , \tag{37a}$$

where r, J_o, and L_1 are previously defined. R_{ex} is the resistance of an external resistor t_{SO} is the time required to detect the quench and switch the resistor into the primary circuit.

In most cases, t_{SO} is small so one can estimate the resistance of the external resistor as follows;

$$R_{ex} \geq \frac{r+1}{r} J_o^2 \frac{L_1}{2F(T_M)} \quad . \tag{38}$$

$F(T_M)$ is defined as in Eq. (22) where T_M is the maximum allowable temperature at the hot spot in the coil.

If a shorted secondary circuit which is closely coupled to the primary circuit is used, the equation describing the calculation of $F(T_M)$ when quench back is not considered goes as follows;

$$F(T_M) = \frac{r+1}{r} J_o^2 \left[\frac{\tau_1^2}{2(\tau_1+\tau_2)} + \frac{\tau_s}{2} + t_{SO} \right] \quad , \tag{39}$$

where

$$\tau_1 = \frac{L_1}{R_{ex}} \quad \text{and} \quad \tau_2 = \frac{L_2}{R_2} \quad , \tag{39a}$$

where L_2 and R_2 are previously defined. τ_s is defined by Eq. (11d). One can solve Eq. (39) to achieve a value of R_{ex} to yield the desired value of $F(T_M)$ and T_M.

The value of R_{ex} in Eq. (39) is smaller than that calculated by Eq. (38) for the same value $F(T_M)$. When quench back is considered, the value of R_{ex} goes down considerably. Using Eq. (30), setting $t^{**} = 0$, setting $F_2^{**} = F_2^*$, and substituting R_{ex} for R_1/N_C one can find the value of R_{ex} necessary to insure a quench back time t_{QBR}. This R_{ex} is;

$$R_{ex} \approx \left[\frac{L_1 \; R_2 \; N_2 \; A_{c2}}{L_2 \; N_1 \; i_o} \; \frac{F_2^*}{t_{QR}} \right]^{1/2} \quad , \tag{40}$$

where L_1, L_2, R_2, N_2, N_1, i_o, A_{c2}, and F_2* are previously defined. t_{QR} is the required value of t_Q needed to achieve the required quench back time t_{QBR}

$$t_{QR} = t_{QBR} - \left(t_H + \frac{\tau_s}{2} + t_{SO}\right) \quad , \tag{40a}$$

where $t_{SO} < 0.05\ t_{QR}$ and τ_s is defined by Eq. (11d). Since the value of R_{ex} is expected to be small, $\tau_s = \varepsilon\tau_2$ where ε is defined by Eq. (11e). If there is sufficient material in the primary circuit then τ_2 may be quite small because the secondary circuit does not have to be a large energy absorber. One can use the full form for τ_s and adjust τ_1 so that $\tau_s < \tau_{QR}$. A good rule of thumb for designing the resistance R_{ex} is to make

$$t_{QR} \approx 0.5\ (t_{QBR} - t_H) \quad . \tag{40b}$$

For a typical solenoid which is closely coupled to its secondary circuit $a_1 \approx a_2$ and $\ell_1 \approx \ell_2$. Using the proceeding approximations, one manipulates Eq. (40) into a functional form ρ_2, ΔH_2, J_o, t_{QR} and geometric parameters a_1 and N_1. The functional form of Eq. (40) is as follows;

$$R_{ex} \approx \frac{2\pi\ a_1\ N_1}{A_{c1}\ J_o} \left[\frac{\Delta H_2\ \rho_2}{t_{QR}}\right]^{1/2} \quad . \tag{41}$$

Table 5 compares calculated values of R_{ex} for superconducting solenoid under the following conditions: 1) a magnet without a shorted secondary circuit, 2) a magnet with a shorted secondary circuit with a τ_2 of 7.5 seconds, 3) a magnet with a shorted secondary which caused quench back to the primary circuit. The time constant for this circuit is 7.5 seconds. The following constants apply to the TPC like solenoid given in Table 5.

J_o = 6.5×10^8 Am^{-2}

i_o = 2300 A

r = 1.8

N_1 = 1772

A_{c1} = 3.54×10^{-6} Am^{-2}

t_{QBR} = 0.120

t_H = 0.08

$F(T_M)$ = 1.47×10^{17} RRR = 140 copper

T_M = 300 K

ΔH_2 = 9.36×10^4 Jm^{-3} copper 4.5 to 15K seems to fit data better

ρ_2 = 1.73×10^{-10} Ωm copper RRR = 90 AVC

τ_s is small in all cases but Eq. (41)

t_{QR} = 0.025 s Eq. (40) use in Eq. (41)

Table 5 shows that the shorted secondary circuits reduce the value of the external resistor R_{ex} even when quench back is not considered. A varistor resistor was also included in Table 5. The calculations were made using methods outlined in Ref. 10. The varistor resistor calculation may not be conservative but the effect is a large one. When quench back is considered, R_{ex} is reduced to a low value. For the sake of conservatism, I would double the value calculated for R_{ex} (Heat transfer may be important and it may have a negative effect on quench back when R_{ex} is low.)

Table 5. The resistance of an external quench protection resistor and the peak voltage across that resistor in order to insure safe quenching of a TPC like magnet.

	R_{ex} (Ω)	V_{max} (volts)
Magnet coil without a secondary circuit normal resistor	10.3	23700
Magnet coil without a secondary circuit varistor resistor b = 0.2*	7.36**	16900
Magnet coil with secondary circuit normal resistor no quench back	2.25	5170
Magnet coil with secondary circuit varistor no quench back b = 0.2*	0.33**	760
Magnet coil with secondary circuit normal resistor quench back considered	0.134+	309

*See Ref. 10 for the method of calculation when a varistor is used.

**Starting value of varistor resistance at i_o = 2300 A.

+It would be recommended the R_{ex} be increased by a factor of two. The voltage across the resistor would also be doubled.

Equation (41) and the calculations from that equation has an important effect on how one views end tap quench protection on the TPC magnet. When quench back is considered, voltages required for safe quenching are greatly reduced (see Table 5) and a varistor resistor appears to be no longer necessary. As a result, the major argument against end tap quench protection (high voltages which persist for a long time) goes away.

Equation 40 or 41 can also be used to advantage when large high current density magnets are run in persistent mode. Two examples are given in Refs. 25 and 26. The value of R_{ex} calculated by Eq. (40) or (41) is the minimum allowable resistance for a persistent switch when the switch is open (the resistance of the persistent switch when it is closed should be about 10^{-9} ohms.)

It is suggested that when the reader trys a calculation of R_{ex} using Eqs. (40) or (41), he should check to see that a correct value of t_{QR} is being used. Equation (40b) may yield too large a value of t_{QR}. t_{QR} can be calculated iteratively using the form for τ_s given by Eq. (11d) (remember Eq. (11d) is not valid unless $\varepsilon < 0.1$). The result may be a much smaller value for t_{QR} than would be indicated by Eq. (40b).

12. Calculation of Quench Back in the TPC Magnet Investigation of Basic Parameters on Quench Back Time

The theory for quench back has been used to calculate quench back time for the proposed new TPC magnet.[27] Various parameters are changed to demonstrate how the theory works. The following cases are presented in this section:

1) The base line case
   a) 9.55 mm thick 1100 aluminum bore tube RRR=25
   b) 2 mm thick layer of copper with RRR=200 before magnet-resistance is considered
   c) the superconductor with a copper to superconductor ratio of 1.8 and a matrix RRR = 140 before magneto-resistance is considered.
2) A case without a layer of copper cladding. The superconductor is the same as the base line case. The bore tube has an RRR=25
   a) 9.5 mm thick 1100-0 bore tube
   b) 25.4 mm thick 1100-0 bore tube
3) The effect of insulation thickness between the cladding and the coil
   a) 0.2 mm insulation
   b) 0.5 mm insulation
   c) 1.0 mm insulation (the base line case)
   d) 1.5 mm insulation

4) The effect of the cladding resistivity. This case is like the base line case except the copper cladding is moved outside the coil. Magneto-resistivity is not a factor.

a) copper RRR = 20 outside the coil

b) copper RRR = 200 outside the coil

c) copper RRR = 2000 outside the coil

5) The effect of the superconductor matrix resistivity. All other aspects are like the base case

a) superconductor matrix RRR = 140 (base case)

b) superconductor matrix RRR = 50

c) superconductor matrix RRR = 10

Case 5b and 5c are calculated using Eq. (34) and Eq. (33).

6) The effect of copper to superconductor ratio in the conductor. All other aspects are like the base line case.

a) copper to superconductor ratio r = 1.0

b) copper to superconductor ratio r = 1.8 (base case)

c) copper to superconductor ratio r = 2.6

It is hoped that the six test cases, which apply to the new TPC magnet, will show the effect of various magnet design parameters on t_{QB} and t_{QBR}. A comparison of the base line case with two layers of copper cladding with the cases without cladding explains the reasons for choosing the copper cladding. In addition the case with the bore tube alone shows that bore tube thickness does not have a marked effect on the process of quench back. Large changes in cladding resistivity and in the resistivity of the superconductor matrix material are not possible in the new TPC maagnet. It is hoped that this section will demonstrate how changing these resistivities will affect the quench back performance of future superconducting magnets with shorted secondary circuits.

a) The base line case for the TPC magnet.

The comparisons in this section are all made with the base line case. The TPC magnet coil and secondary circuits are 2.18 m in diameter and 3.3 m long. The maximum current in the TPC magnet is 2300 A. The maximum field is 1.5 T. In all cases, where applicable magneto-resistance is considered.[13,28,29]

In order to determine whether quench propagation is linear, quadratic or cubic one must know the values of t1, t2, and t3 given by Eqs. (7a), (7b), and (6a) respectively. This means that one should know the velocity of quench along the wire V_L. (See Figure 4) In the TPC magnet, V_L is the velocity in the circumferential direction. The turn to turn velocity ratios α (the turn to turn velocity ratio along the length of the coil) and β (the turn to turn velocity ratio through the coil thickness) are calculated using Eqs. (10a) and (10b). Note: V_L and ρ_1 which are used to calculate t1, t2 and t3 are functions of the coil current.

The parameters of the TPC magnet needed to calculate t1, t2, and t3 using Eqs. (7a), (7b), and (6a) are as follows:

$d1 = \pi a_1 = 3.424$ m

(a_1 is the coil radius of 1.09 m)

$d2 = \ell_1 = 3.3$ m

(ℓ is the coil length)

$d3 = 1.1 \times 10^{-3}$ m

$a = 3.6$ mm

$a' = 3.1$ mm

$b = 1.0$ mm

$b' = 0.5$ mm

$c1 = 0.05$ mm

$c2 = 0.1$ mm

$k_i = 0.036$ $Wm^{-1}K^{-1}$

$r = 1.8$

$L = 2.45 \times 10^{-8}$ ΩK^{-2}

$T_c = 7$ K

ρ_1 = (given in the table)

from Eqs. (10a) and (10b)

$$\alpha = 3430 \ \rho_1^{1/2} \quad ,$$

$$\beta = 1680 \ \rho_1^{1/2}$$

Table 6 gives the values of α and β as a function of magnet current for the TPC magnet. Also given in this Table are the values of ρ_1 based on a starting matrix RRR = 140. ρ_1 includes magneto-resistance. V_L is included in the Table. The estimated value for V_L is based on measurements of V_L in the c magnet and the basic quench velocity theory given in Ref. 19. From α, β, d1, d2, and d3, and V_L one can calculate the quench propagation times t1, t2 and t3 which are given in Table 6.

The calculation of t_{QBR} is done using Eq. (2). We have chosen a hot spot temperature T_M of 300 K. We assume that all active elements in the TPC magnet will reach the same final temperature T_S if the entire coil goes normal at time zero. The active circuits for the TPC magnet have the following masses; superconducting coil mass 360 kg, copper cladding coil mas 360 kg, and the 1100 aluminum bore tube RRR = 25 mass 580 kg. One can estimate T_S using Fig. 2 when one knows the magnet stored energy as a function of current i_o. (Assume there is 740 kg of copper and 580 kg of aluminum in the active circuits.) Once T_s and T_M have been found, one can calculate $F(T_S)$ and $F(T_M)$ using Fig. 3.

Table 6. Longitudinal quench velocity V_L, superconductor matrix resistivity ρ_1, quench velocity ratios α and β and quench propagation times t1, t2, and t3 as a function of TPC magnet current i_o.

i_o	V_L	ρ_1*	t1	α	t2	β	t3
500	1.8	1.18×10^{-10}	1.90	0.037	49.15	0.018	0.034
800	3.9	1.25×10^{-10}	0.88	0.038	22.65	0.019	0.015
1100	6.4	1.32×10^{-10}	0.54	0.039	13.84	0.019	0.009
1400	9.1	1.40×10^{-10}	0.38	0.041	8.94	0.020	0.006
1700	12.0	1.50×10^{-10}	0.29	0.042	6.55	0.021	0.005
2000	16.0	16.3×10^{-10}	0.21	0.044	4.71	0.021	0.003
2300	20.0	1.75×10^{-10}	0.17	0.045	3.64	0.022	0.003

*Includes the effect of magneto-resistance at zero field RRR = 140 copper has a resistivity of 1.12×10^{-10} Ωm at 10°K.

Table 7. The parameters needed to calculate t_{QBR} for the TPC magnet base case.

i_o (A)	J_o (Am^{-2})	E_o (J)	T_S	$F(T_M)-F(T_S)$ ($A^2m^{-4}s$)	t_{QBR} (s)
500	1.41×10^8	0.53×10^6	35	1.31×10^{17}	4.24
800	2.26×10^8	1.36×10^6	45	1.15×10^{17}	1.45
1100	3.11×10^8	2.58×10^6	55	1.03×10^{17}	0.68
1400	3.95×10^8	4.18×10^6	63	0.95×10^{17}	0.39
1700	4.80×10^8	6.17×10^6	70	0.89×10^{17}	0.25
2000	5.65×10^8	8.53×10^6	77	0.84×10^{17}	0.17
2300	6.50×10^8	11.28×10^6	84	0.78×10^{17}	0.12

*T_M = 300 K, r = 1.8.

Since the quench back time t_Q is in general greater than t_1, the one-dimensional equation (Eq. (34)) can be used to calculate t_Q. The value of tl can be found in Table 6. The following values can be substituted into Eq. (34)

$$\Delta H_2 = 21600 \text{ Jm}^{-3}$$

$$L = 2.45 \times 10^{-8} \text{ W}\Omega\text{K}^{-2}$$

$$\Gamma = 1.296 \text{ Wm}^{-1}\text{K}^{-1}$$

$$\left(\frac{r+1}{r}\right) = 3.76 \qquad r = 1.8$$

$$\ell_1 = 3.3 \text{ m}$$

Table 9 shows the calculated values of t_Q (for the cladding outside and inside the coil). The calculated value of t_H (See sub-section C) is also given in Table 9. t_{QB} and t_{QBR} are also presented in Table 9. Figure 7 shows a plot of t_{QB} and t_{QBR} versus current for the base line case of the TPC magnet. As long as t_{QB} is less than t_{QBR} the magnet quenches safely without an external active quench protection system.

b) The TPC magnet without copper cladding quench back from the bore tube alone

Equation (34) can be used to calculate t_Q. The quench back element is 1100-0 aluminum RRR = 25 in which there is almost no magneto-resistance. All other parameters except ΔH_2 and ρ_2 are unchanged.

$$\Delta H_2 = 11600 \text{ Jm}^{-3}$$

$$\rho 2 = 10^{-9} \ \Omega\text{m}$$

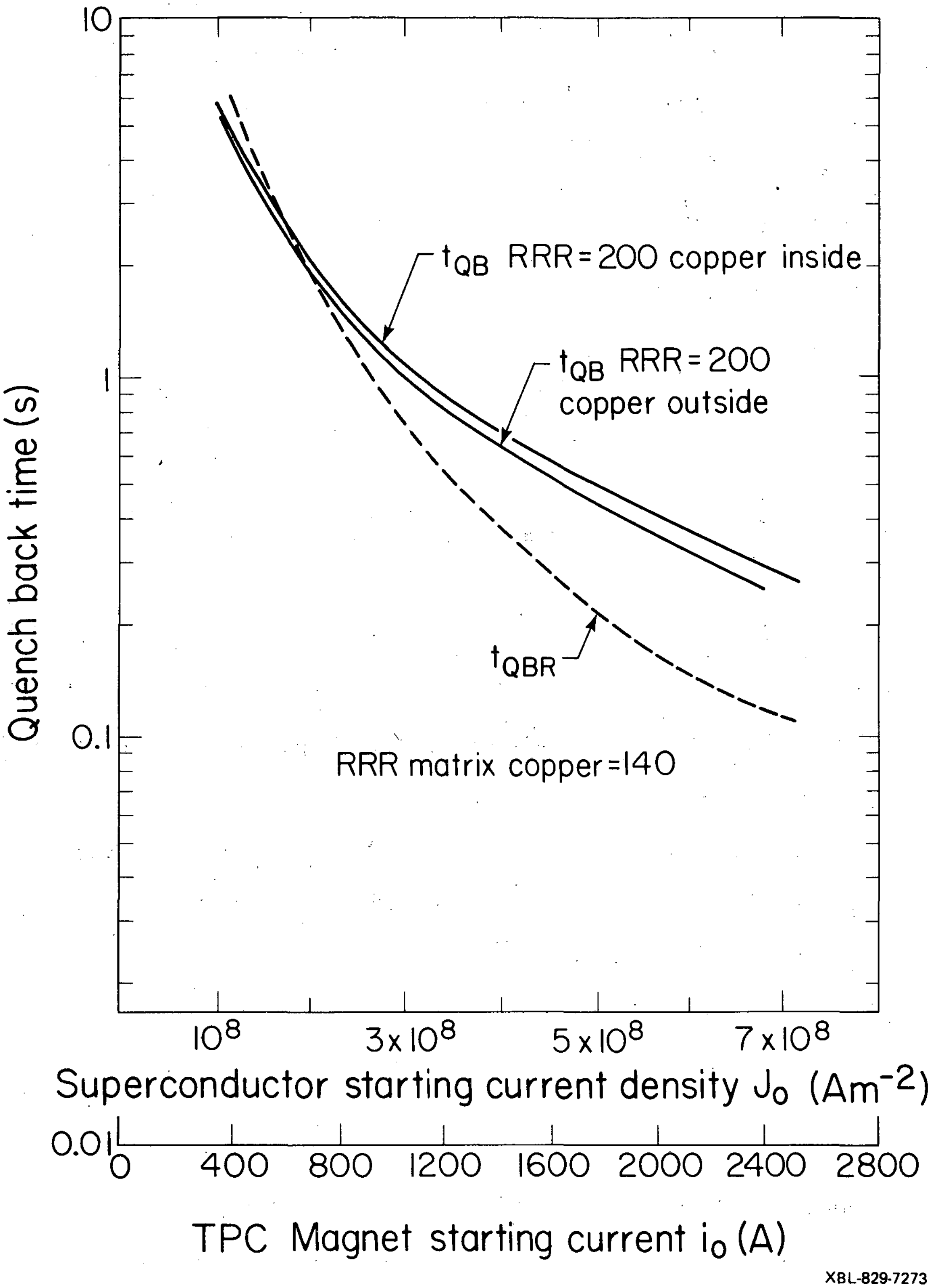

Fig. 7. Predicted quench back time t_{QB} required quench back time t_{QBR} for the TPC Magnet as a function of magnet current and superconductor current density (see Table 9).

Table 8. The parameters needed to calculate t_Q using Eq. (34).

i_o	J_o (Am^{-2})	V_L ms^{-1}	ρ_1 (Ωm)*	ρ_2 (Ω_m) inside**	outside**
500	1.41×10^8	1.8	1.18×10^{-10}	0.9×10^{-10}	0.78×10^{-10}
800	2.26×10^8	3.9	1.25×10^{-10}	1.0×10^{-10}	0.78×10^{-10}
1100	3.11×10^8	6.4	1.32×10^{-10}	1.1×10^{-10}	0.78×10^{-10}
1400	3.95×10^8	9.1	1.40×10^{-10}	1.2×10^{-10}	0.78×10^{-10}
1700	4.80×10^8	12.0	1.50×10^{-10}	1.37×10^{-10}	0.78×10^{-10}
2000	5.65×10^8	16.0	1.63×10^{-10}	1.54×10^{-10}	0.78×10^{-10}
2300	6.50×10^8	20.0	1.75×10^{-10}	1.71×10^{-10}	0.78×10^{-10}

*The superconductor matrix copper has an RRR = 140 magneto-resistance is considered.

**The copper cladding has a RRR = 200. The cladding inside has magneto-resistance considered. The cladding outside is at zero field.

Table 9. The calculated values of t_Q, t_H, and t_{QB} for the base line case. t_{QBR} is compared with t_{QB}.

i_o (Al)	t_Q (s) inside	t_Q (s) outside	t_H^* (s)	t_{QB} (s) inside	t_{QB} (s) outside	t_{QBR}^{**} (s)
500	3.71	3.58	0.105	3.82	3.69	4.24
800	1.62	1.52	0.100	1.72	1.62	1.45
1100	0.93	0.86	0.095	1.03	0.96	0.68
1400	0.63	0.57	0.090	0.72	0.66	0.39
1700	0.46	0.41	0.084	0.54	0.49	0.25
2000	0.34	0.29	0.082	0.42	0.37	0.17
2300	0.25	0.22	0.081	0.33	0.30	0.12

*See subsection c in the section for the calculation of t_H.

**When t_{QBR} is greater than t_{QB}, the magnet will quench safely without external quench protection. According to this calculation, the cross over point is at about 700 A (See Fig. 7).

The other major difference in this case is t_{QBR}. Two thicknesses of the aluminum bore tube are used for comparison. Bore tube thickness does not effect t_{QB} but it does t_{QBR}. The two thicknesses are 9.53 mm (3/8 inch) and 25.4 mm (1 inch). Table 10 shows the parameters needed to calculate t_{QBR}. Table 11 shows t_{QB} and compares it with t_{QBR}. Figure 8 shows a plot of t_{QB} and t_{QBR} versus magnet current. Note: From Table 11 and Fig. 8, there is no safe region for quenching of the TPC magnet when it has the bore tube alone as a quench back element. (t_{QBR} is always less than t_{QB}).

c) The effect of electrical insulation thickness c4 between the superconducting coil and the quench back element. A calculation of t_H

The thickness of the electrical insulation between the shorted secondary circuit element causing quench back and the superconducting coil has an effect on t_{QB}. This effect comes from t_H and t_H only. At low currents in the coil, the effect is not a strong one when the coil current density is above 5 x 10^8 Am^{-2}, the value of t_H may be a substantial fraction of t_{QB}. Table 12 shows the effect of insulation thickness c4 on t_H at various currents in the TPC magnet.

Table 12 shows that t_H grows quickly with insulation thickness. At high currents the thickness of the insulation between the superconductor and the quench back element has a marked effect on quench back time t_{QB}. From Table 9, one can see that t_H is one quarter of the predicted quench back time t_{QB} at a current of 2300 A.

Table 10. The parameters needed to calculate t_{QBR}.

i_o (A)	Thickness = 9.53 mm T_s	$F(T_M)-F(T_S)$	t_{QBR}	Thickness = 25.4 mm T_S	$F(T_M)-F(T_S)$	t_{QBR}
500	38	1.26×10^{17}	4.08	32	1.35×10^{17}	4.37
800	48	1.11×10^{17}	1.40	40	1.22×10^{17}	1.54
1100	57	1.00×10^{17}	0.66	47	1.13×10^{17}	0.75
1400	66	0.92×10^{17}	0.38	52	1.07×10^{17}	0.44
1700	75	0.86×10^{17}	0.24	58	1.00×10^{17}	0.28
2000	85	0.78×10^{17}	0.16	65	0.93×10^{17}	0.19
2300	92	0.73×10^{17}	0.11	72	0.88×10^{17}	0.13

T_M = 300 K, r = 1.8, see Eq. (2).

Table 11. The calculated values of t_Q, t_H and t_{QB} for the case with an 1100-0 bore tube as a quench back element. t_{QBR} is compared with t_{QB}.

i_o (A)	t_Q	t_H*	t_{QB}	t_{QBR}** 9.5 mm	t_{QBR}** 25.4 mm
500	5.95	0.105	6.06	4.08	4.37
800	2.50	0.100	2.60	1.40	1.54
1100	1.39	0.905	1.49	0.66	0.75
1400	0.91	0.090	1.00	0.38	0.44
1700	0.64	0.084	0.72	0.24	0.28
2000	0.45	0.082	0.53	0.16	0.19
2300	0.32	0.081	0.40	0.12	0.13

*See subsection c in this section for the calculation of t_H.

**Note: t_{QBR} is less than t_{QB} for all values of current in the table. This suggests that there is no region where failsafe quenching can occur (See Fig. 8).

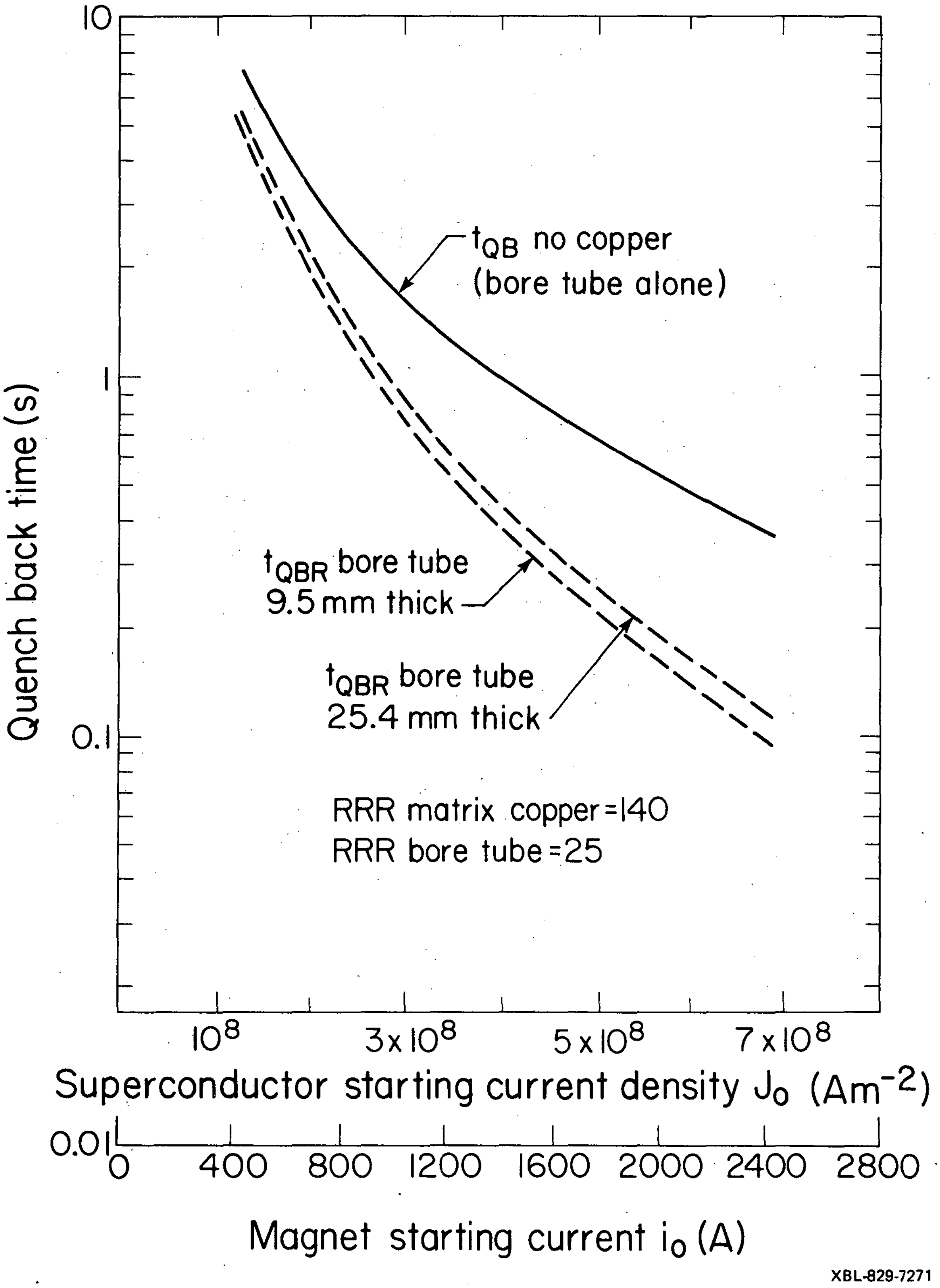

Fig. 8. Predicted quench back time t_{QB} and required quench back time t_{QBR} for the TPC magnet without a copper quench back coil as a function of magnet current and superconductor current density. (Note: Quench back is caused by the bore tube which is assumed to be RRR = 25 aluminum. Two bore tube thicknesses are included.) (See Table 11).

Table 12. The time required to transfer heat across the insulation t_H as a function of magnet current i_o and insulation thickness c4.

i_o (A)	thermal time constant t_H (s)			
500	0.027	0.040	0.105	0.211
800	0.024	0.037	0.100	0.204
1100	0.021	0.034	0.095	0.196
1400	0.018	0.031	0.090	0.188
1700	0.016	0.029	0.084	0.181
2000	0.015	0.026	0.082	0.178
2300	0.014	0.027	0.081	0.176
c4 (mm)	0.2	0.5	1.0	1.5
k^*_{in} ($Wm^{-1}K^{-1}$)	0.051	0.127	0.139	0.139

*k_{in} is given in Table 3. ΔH_i used in Eq. (19) is also given in Table 3.

d) the effect of changing the resistivity of the secondary circuit material

Table 9 shows the calculated quench back time for the TPC magnet from a shorted secondary circuit made of RRR = 200 copper. This table gives the value of the quench back time with this circuit inside and outside the superconducting coil. The difference between the two cases is ρ_2 the average resistivity of the secondary circuit (see Table 8). This sub-section presents a systematic study of the effect of ρ_2 on quench back time (see Eq. (34)).

The calculation illustrated in this sub-section use the same values of ΔH_2, L, Γ, r, ρ_1, J_o, V_L and ℓ_2 is the base line case, the rebuilt TPC magnet. Only ρ_2 is changed. In Table 13 and Fig. 9 , one can compare the effect of secondary circuit resistivity ratios RRR = 20 (ρ_2 = 7.8 x 10^{-10}), RRR = 200 (ρ_2 = 0.78 x 10^{-10}) and RRR = 2000 (ρ_2 = 0.078 x 10^{-10}) with one another. In all three comparison cases, (RRR = 20, RRR = 200, RRR = 2000), the copper is located outside the coil so that magneto-resistance in the circuit is not a factor.

Table 13 shows that reducing ρ_2 does reduce the quench back time. An RRR = 20 cladding circuit has no safe region where the TPC magnet can be quench. At an RRR of 200, the magnet will safely self quench up to a current of 770 A. When the cladding resistance ratio RRR = 2000, the safe quench current is around 1200 A.

Table 13. Quench back time as a function of copper cladding resistivity at various magnet currents.

i_o	base case RRR = 200 (inside)	copper cladding outside RRR = 20	copper cladding outside RRR = 200	copper cladding outside RRR = 2000	t_{QBR}
500	3.82	6.72	3.69	2.28	4.24
800	1.72	2.87	1.52	1.04	1.45
1100	1.03	1.65	0.96	0.64	0.68
1400	0.72	1.10	0.66	0.46	0.39
1700	0.54	0.79	0.49	0.35*	0.25
2000	0.42	0.57	0.37	0.28*	0.17
2300	0.33	0.45	0.30	0.23*	0.12

*Calculated using Eq. (34), Eq. (33) may yield a slightly lower value. In these cases $t_Q < t1$ but not by very much.

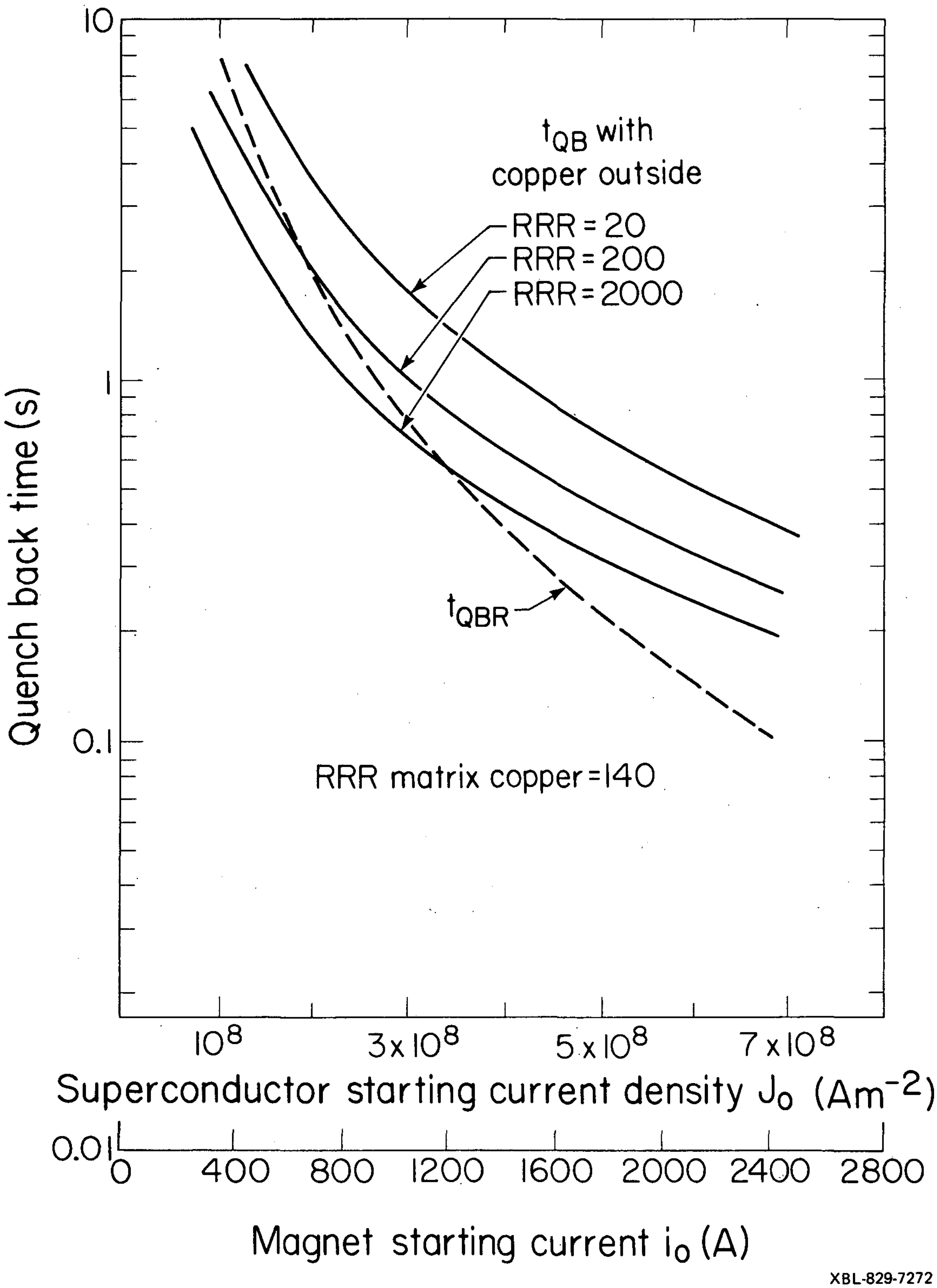

Fig. 9. Predicted quench back time t_{QB} and required quench back time t_{QBR} for the TPC magnet as a function of magnet current or superconductor current density and the residual resistance ratio (RRR) of the copper quench back coil. (Note: The copper coil is assumed to be outside the superconducting coil so that magneto-resistance can be neglected.) (See Table 13).

Figure 9 shows a plot of t_{QB} and t_{QBR} versus magnet current for various resistivity ratio secondary circuits.

Increasing the cladding resistance ratio (decreasing resistivity ρ_2) is not a very effective way of reducing quench back time. One can see in the next sub-section that increasing the superconductor matrix resistivity ρ_1 is far more effective in reducing quench back time.

e) the effect of changing the resistivity of the superconductor matrix material

When one looks at Eqs. (33), (34), and (35) it is clear that the matrix resistivity ρ_1 has a greater effect on quench back time than does the resistivity of the secondary circuit material ρ_2 which induces quench back. The value of t_{QBR} is not the same as for the base line case. $F(T_S)$ and $F(T_M)$ are both smaller when the resistivity of the superconductor matrix is increased. As a result, t_{QBR} is not the same as in the base line case. Table 14 shows how t_{QBR} is calculated for a magnet with a superconductor matrix resistivity ratio RRR = 10 and RRR = 50.

Table 15 shows calculations of the quench back time as a function of magnet current and the resistivity ratio of the copper in the superconductor matrix. Table 15 values were calculated using the two-dimensional propagation equation (Eq. (33)). When the superconductor matrix copper resistivity ratio is 10, one can see that t_{QB} is less than t1. Thus it is appropriate to use Eq. (33). When the resistivity ratio is 50 t_{QB} is greater than t1 but not by very much.

Table 14. The calculation of t_{QBR} as a function of superconductor matrix resistivity ratio.

i_o (A)	T_S (K)	$F(T_M) - F(T_S)$ RRR = 10*	$F(T_M) - F(T_S)$ RRR = 50**	t_{QBR} (s) RRR = 10	t_{QBR} (s) RRR = 50	t_{QBR} (s) RRR = 140†
500	35	1.03×10^{17}	1.25×10^{17}	3.34	4.05	4.24
800	45	1.00×10^{17}	1.13×10^{17}	1.26	1.43	1.45
1100	55	0.95×10^{17}	1.03×10^{17}	0.63	0.68	0.68
1400	63	0.91×10^{17}	0.95×10^{17}	0.38	0.39	0.39
1700	70	0.86×10^{17}	0.89×10^{17}	0.24	0.25	0.25
2000	77	0.82×10^{17}	0.84×10^{17}	0.17	0.17	0.17
2300	85	0.78×10^{17}	0.78×10^{17}	0.12	0.12	0.12

*For copper RRR = 10, $F(T_M) = 1.05 \times 10^{17}\ A^2m^{-4}s$

**For copper RRR = 50, $F(T_M) = 1.33 \times 10^{17}\ A^2m^{-4}s$

†For copper RRR = 140, $F(T_M) = 1.48 \times 10^{17}\ A^2m^{-4}s$

Table 15. Quench back time as a function of current and the resistivity of the superconductor matrix material, ρ_1.

i_o (A)	ρ_1 (Ωm) RRR = 10	ρ_1 (Ωm) RRR = 50	ρ_2 (Ωm) RRR = 200	t_{QB} (s) RRR = 10	t_{QB} (s) RRR = 50
500	1.51×10^{-9}	3.1×10^{-10}	0.90×10^{-10}	0.90	2.01
800	1.51×10^{-9}	3.3×10^{-10}	1.00×10^{-10}	0.46	0.97
1100	1.52×10^{-9}	3.5×10^{-10}	1.10×10^{-10}	0.30	0.61
1400	1.53×10^{-9}	3.7×10^{-10}	1.20×10^{-10}	0.23	0.43
1700	1.55×10^{-9}	3.9×10^{-10}	1.37×10^{-10}	0.19	0.33
2000	1.57×10^{-9}	4.1×10^{-10}	1.54×10^{-10}	0.16	0.27
2300	1.59×10^{-9}	4.3×10^{-10}	1.71×10^{-10}	0.15	0.22

*Equation (33) was used to calculate t_Q.

Table 16 compares calculation of quench back time using both Eqs. (33) and (34). When RRR = 50, there is not much difference between the use of Eq. (33) and Eq. (34). Figure 10 is a plot of t_{QB} and t_{QBR} versus magnet current for various superconductor matrix resistivity ratios. (RRR = 10 uses Eq. (33), RRR = 50 uses Eq. (34) and RRR = 140 uses Eq. (34).

Increasing the resistivity of the superconductor matrix appears to be an attractive way to reduce quench back time in order to insure safe quenching without active quench protection. The major question is whether a reduced resistivity superconductor matrix has a negative effect on superconductor stability.

f) the effect of changing the conductor copper to superconductor ratio

Changing the ratio of normal metal to superconductor r in the superconductor affects the quench back time t_{QB} and the required quench back time t_{QBR}. When one looks at Eqs. (33), (34) and (35) it is clear that normal metal to superconductor ratio r will have a marked effect on t_Q. It is also clear from Eq. (2) that t_{QBR} is affected as much or more by r than is t_Q. To first order, r does not affect the velocity of quench velocity V_L. Therefore, to first order, tl is not affected by changes in r.

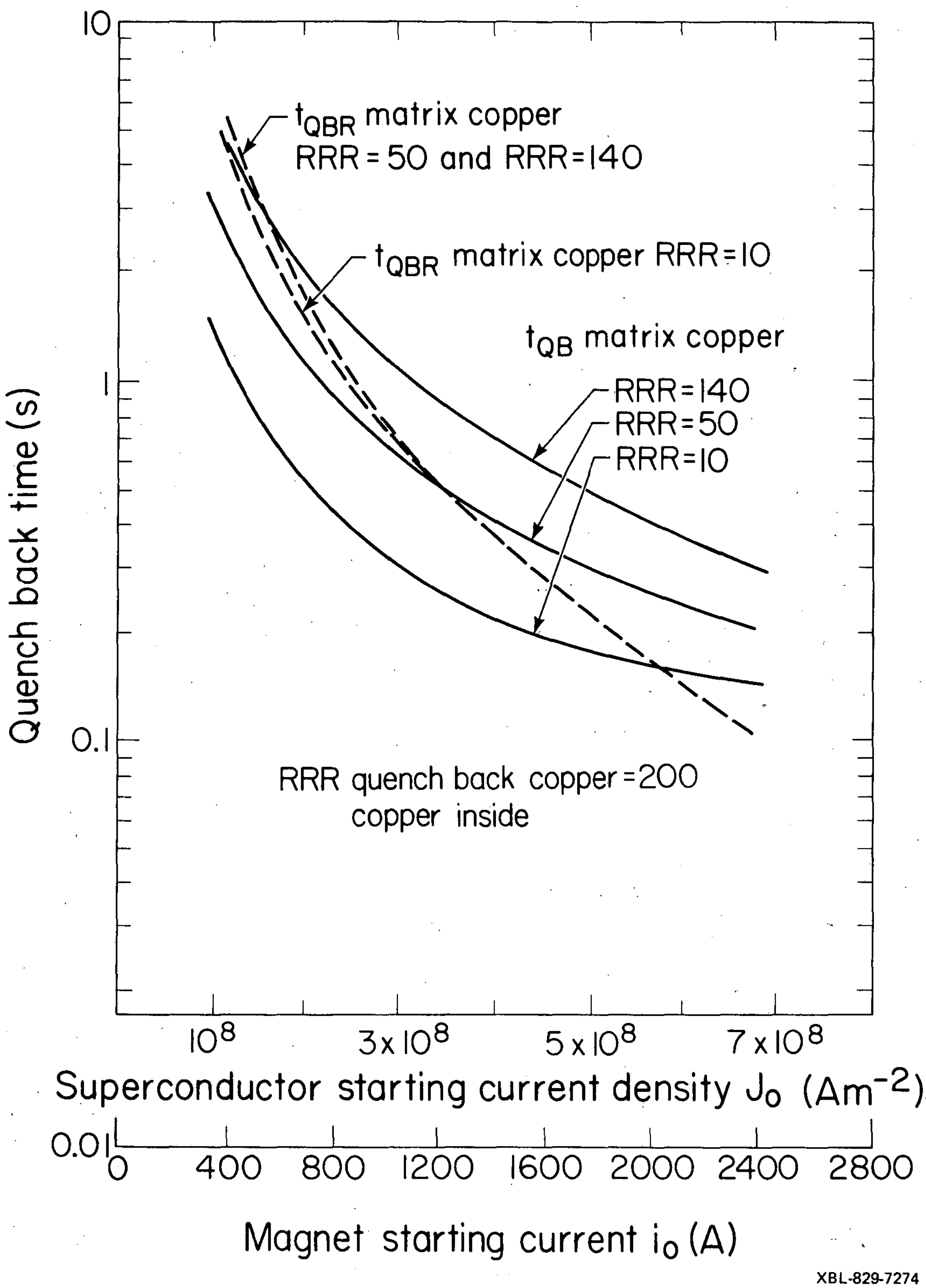

Fig. 10. Predicted quench back time t_{QB} and required quench back time t_{QBR} for the TPC magnet as a function of magnet current or superconductor current density and the residual resistance ratio (RRR) of the matrix copper in the superconductor (see Tables 14 and 16). (Note: Eq. (33) was used for the RRR = 10 and RRR = 50 curves and Eq. (34) was used for the RRR = 140 curve.)

Table 16. A comparison of quench back time calculated using Eqs. (33) and (34).

i_o (A)	t_{QB} (s) RRR = 10 Eq. (33)	t_{QB} (s) RRR = 10 Eq. (34)	t_{QB} (s) RRR = 50 Eq. (33)	t_{QB} (s) RRR = 50 Eq. (34)	t_{QB} (s) RRR = 140 Eq. (34)
500	0.90	1.27	2.01	2.11	3.82
800	0.46	0.64	0.97	0.97	1.72
1100	0.30	0.42	0.61	0.63	1.03
1400	0.23	0.32	0.43	0.45	0.72
1700	0.19	0.26	0.33	0.35	0.54
2000	0.16	0.22	0.27	0.29	0.42
2300	0.15	0.18	0.22	0.23	0.33

Table 17 shows a comparison of quench back time t_{QB} and required quench back time t_{QBR} with current in the coil and copper to superconductor ratio in the superconductor. As r is decreased, both t_{QB} and t_{QBR} are decreased. The required quench back time t_{QBR} goes down faster than the calculated time for quench back t_{QB}. As a result, reducing r will reduce the current up to which safe quenching can occur. The reasons for this are t_{QBR} is changed directly, but t_1 and t_H which are a part of t_{QB} are not affected very much by changes in r. (A decrease of r will increase t_H slightly).

It can be concluded that maximizing the copper to superconductor ratio is a good thing in terms of fail safe quench protection through thermal quench back. Increasing r will, in many cases, result in better conductor stability. The other side of the coin is that more superconductor (a lower r) means a greater safety margin against fluctuations of conductor temperature. (As an example, the TPC magnet is expected to have a critical temperature of about 6.2K. If the copper to superconductor ratio is increased from 1.8 to 2.6, the critical temperature will be reduced to about 5.5K).[30]

13. The Effect of Superconductor Matrix Resistivity on Superconductor Stability

From the previous section, it appears that increasing the resistivity of the superconductor matrix ρ_1 is a good way to reduce quench back time so that fail safe quenching can occur at higher current densities. The major drawback of this approach is the effect that increasing

Table 17. Quench back time as a function of the conductor copper to superconductor ratio at various magnet currents.

i_o (A)	r = 1.0 t_{QB}	r = 1.0 t_{QBR}*	r = 1.8 t_{QB}	r = 1.8 t_{QBR}*	r = 2.6** t_{QB}	r = 2.6** t_{QBR}*
500	3.12	3.30	3.82	4.24	4.04	4.76
800	1.46	1.13	1.72	1.45	1.86	1.63
1100	0.88	0.53	1.03	0.68	1.11	0.76
1400	0.62	0.30	0.72	0.39	0.77	0.44
1700	0.47	0.19	0.54	0.25	0.58	0.28
2000	0.37	0.13	0.42	0.17	0.45	0.19
2300	0.29	0.09	0.33	0.12	0.35	0.14

*When t_{QBR} is greater than t_{QB}, the magnet will quench safely without external quench protection. According to this calculation the safe current for quenching is:

630 A when r = 1.0 (50% superconductor)
700 A when r = 1.8 (35.7% superconductor)
750 A when r = 2.6 (27.7% superconductor)

**This case would decrease the margin to safety in the TPC magnet considerably.

ρ_1 might have on the stability of the conductor. In other words, does increasing the resistivity of the superconductor matrix material cause the magnet to quench spontaneously.

In general, stability of a superconductor is a measure of its ability to return spontaneously to the superconducting state after the passage of a disturbance. Stability and stabilization mean different things to different people, so it is useful to describe it in general terms before applying stability criteria to our particular case. There are two forms of stabilization which are used in the literature to describe the process of insuring stable performance of a superconducting device. They are: 1) cryogenic stability where the conductor may become partially or wholly normal without the device quenching, and 2) intrinsic stability where the superconductor itself is stable against disturbances which may drive the superconductor partially or wholly normal. The best of all worlds is to have both types of stability present in the device simultaneously. Unfortunately, high current density devices, such as the TPC magnet, do not have very much cryostability. Therefore, cryostability is not an important factor in the design of a high current density device. (It might be useful to point out that old TPC magnet had a degree of cryostability at low currents, but cryostability is not a factor in the design of that magnet.)

Since cryostability is not an important factor in the design of high current density (superconductor plus matrix current densities above 1.5×10^8 Am^{-2}) magnets, it will not be considered further. This section will deal with the concept of the "intrinsic stability" of the superconductor itself. (It should be noted that the term intrinsic stability is out of use. It was used to describe adiabatic or flux jump stability). "Intrinsic stability", in the context of this usage, is an all encompassing term used to describe a number of types of conductor stabilization. These include: 1) adiabatic stability of single filaments and multifilamentary superconductor, 2) self field stability, and 3) dynamic stability. In general, one wants to have a conductor which is stable in all three modes simultaneously.

The concept of adiabatic stability comes from the concept of the critical current model which says that a superconductor in an electric field must carry its critical current.[31] (Another way to say this is that a superconductor has a resistivity which is proportional to the electric field.) The critical current is a function of temperature and magnetic induction. The energy stored within a superconducting filament due to circulating currents (at critical current density) is proportional to filament diameter d_f squared and critical current density in the superconductor $J_c(B,T)$ squared. Flux jump instability will occur when the shield magnetic induction from the center of the edge of the filament is of the order of 0.25 Tesla.[32] According to the paper by Wilson et al.,[33] the superconductor will be stable if the following conditions prevail:

$$d_f J_c \leq \left[\frac{\pi^2}{4\mu_o} C_{SC} \left[J_c^{-1} \left(\frac{dJ_c}{dT} \right) \right]^{-1} \right]^{1/2} \qquad (42)$$

where d_f is the filament diameter, J_c is the critical current in the superconductor, μ_o is the permeability of air ($\mu_o = 4\pi \times 10^{-7}$ Hm^{-1}), C_{SC} is the specific per unit volume for the superconductor and

$$\left[J_c^{-1}\left(\frac{d\ J_c}{dT}\right)\right]^{-1} \approx \frac{T_c}{2} \quad (42a)$$

many authors set $T_c/2 = 4$. The specific heat of the superconductor is somewhat complicated to calculate because it contains both a term for specific heat of the normal state and the magnetization term for specific heat.[34] Wipf suggests using a specific heat of 4.2×10^3 Jm^{-3} K^{-1} at 4.2K.[35] Using this value of the superconductor specific heat and a T_o of 4K then the following form of Eq. (42) emerges:

$$J_c\ d_f \leq 1.82 \times 10^5\ Am^{-1} \quad (43)$$

Other researchers have suggested a range of values for the right hand side of Eq. (43). These values range from the 1.5×10^5 Am^{-1} suggested by Smith and Lewin[16] in 1967 to 4×10^5 Am suggested by Collings.[32] Equation (43) suggests that the minimum filament diameter for the TPC magnet conductor be 61 μm when the superconductor $J_c = 3 \times 10^9$ Am^{-2} (at 2T and 4.2K).

Wilson[33] and Hart[36] have both suggested that stability could be increased by increasing the specific heat of the superconducting material C_{SC} and by increasing T_c. A conductor which as a positive

dJ_c/dT derivative will be stable regardless of the filament diameter. Efforts toward increasing C_{SC} and T_c have not met with a great deal of success. The same can be said for the making the dJc/dT derivative positive. The general rule is that the higher the critical current density is in the superconductor, the smaller the filaments must be. This is precisely why a small filament superconductor was chosen for the TPC Magnet.

Multifilamentary conductors are not flux jump stable unless the filaments are transposed within the superconductor. Up to a certain size a simple twist is sufficient to insure that the conductor is stable against circulating currents from filament to filament. The twist pitch required for stable operation of a multifilamentary conductor is a function of the matrix material resistivity and the rate of magnet flux change the conductor sees. (The minimum twist pitch is about 5 times the mean diameter of the conductor matrix.) In general, a higher resistivity matrix material will result in a more stable conductor for given rate of magnetic flux change. For a typical multifilamentary conductor the twist pitch ℓ_{tp} should be:

$$\ell_{tp} < \left[\frac{8\ \rho_1\ J_c\ df}{B}\left(\frac{1}{r+1}\right)^{1/2}\frac{ds}{d_s+d_f}\right]^{1/2} \qquad (44)$$

where ℓ_{tp} is the critical twist pitch which causes doubling of the superconductor a.c. loss in a magnetic induction which is changing at the rate B=dB/dt. ρ_1, J_c, r and d_f are previously defined. d_s is average distance between filaments. For a densely packed conductor with round filaments

$$d_s \approx \left[\frac{(r+1)\pi}{4}\right]^{1/2} \qquad (44a)$$

The twist pitch can be longer if superconductor matrix resistivity is high. Twist pitch affected by the superconductor critical current and the filament diameter. The conductor for slowly charging d.c. magnets can have twist pitches in excess of 100 mm.

Twisting does not insure stability against self field effects.[37] Self field is not a problem if the magnetic induction difference from one side of a conductor to the other side of the conductor is less than 0.5T. There are a number of criteria which govern self field instability if the induction difference from side to side is greater than 0.5T. The conductor will be stable against self field effects if: 1) the minimum conductor dimension is about 2 mm or less,[38] 2) the filament diameter must be smaller than is indicated by Eq. (43). The larger the matrix the smaller the filaments should be. Increasing the matrix to superconductor ratio enhances self field stability.

The last form of stability which is important on the superconductor level is dynamic stability. The principle behind dynamic stability is that thermal diffusivity of matrix material must be much greater than the thermal diffusion of the superconductor. On the other hand, the magnetic diffusivity of the superconductor must be much larger than the magnetic diffusivity of the matrix material.[39,40] For effective dynamic stability, the filaments of superconductor must be relatively small and the matrix material should have a reasonably high thermal conductivity (the matrix material should have a low electrical resistivity.) According to the dynamic stability criteria given by Wilson et al;[33]

$$d_f \, J_c \;\leq\; \left[\frac{8 \; k_{SC} \; r}{\rho_1} \left[J_c^{-1} \left(\frac{dJ_c}{dt} \right) \right]^{-1} \right]^{1/2} \tag{45}$$

where

$$\left[J_c^{-1} \left(\frac{dJ_c}{dT} \right) \right]^{-1} \approx \frac{T_c}{2} \approx 4 \text{ K} \tag{42a}$$

and where J_c is the superconductor critical current density; d_f is the filament diameter; k_{SC} is the thermal conductivity of the superconductor; ($k_{SC} \approx 0.1$ wm^{-1} K^{-1}) ρ_1 is the resistivity of the matrix material; and r is the matrix material to superconductor ratio.

Equation 45 takes a form which is very similar to Eq. (43). When the matrix material is made from good copper, the product of $J_c d_f$ in Eq. (45) is the same order as the $J_c d_f$ product in Eq. (43). The resistivity of the superconductor matrix material affect the dynamic stability. If this resitivity ρ_1 is increased, the filament diameter must be decreased.

Dynamic stability is the only form of stability which is directly affected by the resistivity of the superconductor matrix. (Self field stability may be affected indirectly.) If the conductor is made with good copper ($\rho_1 = 1.5 \times 10^{-10}$ Ωm, RRR = 100). The filament diameter adiabatic and dynamic stability are nearly the same. (For the TPC magnet conductor is a $d_f \approx 60$ μm). Since the TPC magnet conductor has a filament diameter of around 26 μm, the resistivity of the matrix can be increased to 9.5×10^{-10} Ωm (RRR ≈ 16) without sacrificing dynamic stability at $J_c = 3 \times 10^9$ Am^{-2}. The authur would not advocate increasing the matrix material resistivity to this extent without experimentally varifying the material stability within an epoxy impregnated structure.

14. Concluding Comments

The theory presented in this report can be used to design large high current density superconducting magnets which are passively quench protected using a shorted secondary circuit. In order to have effective passive quench protection, the theory suggests that the quench back time t_{QB} be less than a certain minimum quench back time t_{QBR}. This minimum quench back time is a function of the maximum allowable hot spot temperature T_M, the magnetic energy stored E_o, the mass of the active elements of the coil, the current density J_o at the start of the quench, and the copper to superconductor ratio.

Only thermal quench back is considered in this report. Thermal quench back time t_{QB} has two parts to it. They are the time to heat the secondary circuit t_Q and the time needed to transfer that heat to the superconducting coil t_H. The theory for thermal quench back has been simplified so that closed form equations result. Two forms of the equations for calculating the time to heat the secondary circuit t_Q are presented. The first form, given in Section 9 if this report, gives t_Q in terms of basic magnet and circuit parameters such as L_1, L_2, R_2, R_0, N_1, N_2, α, V_L, i_o, A_{c2} and F_2. (See the nomenclature for the meaning of these terms.) The second form, given in section 10 of this report, gives t_Q in terms of basic material parameters in the primary and secondary circuits such as ρ_1, ρ_2, ΔH_2, and r as well as coil geometric factors a_1 and ℓ_1 and the current density J_o in the conductor. The second form of the equations is particularly useful when one designs a high current density magnet.

Thermal quench back is a function of the product of ΔH_2 ρ_2, ρ_1, r, J_o, V_L and Γ, as well as the physical dimensions of the magnet. Thermal quench back time is not a function of the amount material in the circuit the coil quenches back from. (This is true to first order.) Reducing r and increasing J_o will reduce quench back time, but they are counter productive because they reduce the required quench back time even more. For a given magnet geometry, the only effective ways of reducing the quench back time t_{QB} are to reduce the $\Delta H_2 \rho_2$ product, increase ρ_1, or increase the turn to turn normal region propagation velocity by increasing Γ.

When the quench back theory given in this report was applied to the new TPC magnet (see Section 12), the following conclusions could be drawn: 1) increasing the bore tube thickness, without copper cladding, does not reduce the quench back time t_{QB} nor does it significantly increase the required quench back time t_{QBR}. 2) Copper cladding appears to reduce quench back time over the cases with the bore tube alone. The copper cladding will, in theory, permit fail safe quenching of the TPC magnet to currents of 700 A. 3) Decreasing the resistivity of the copper cladding will decrease quench back time and increase the fail safe quenching current. The use of ultra pure aluminum is justified but is difficult to obtain with the proper insulation. Increasing the resistivity of the superconductor matrix material will, according to the theory, reduce the quench back time. Increasing the matrix resistivity changes stability parameters in the superconductor. It was decided that the TPC magnet was not the place to test the effect of increasing the matrix material resistivity.

The theory suggests that increasing matrix material resistivity ρ_1 is the most promising approach for insuring that a magnet will quench in a fail safe manner. One can increase the matrix resistivity either by cold working the material or by adding a small amount of another metal to the material. In most cases, the desired resistivity for the matrix material (almost always this material is copper) is between 3×10^{-10} Ωm and 10^{-9} Ωm. Unfortunately, increasing matrix resistivity does affect the stability of the conductor. If the superconductor filaments diameter is less than 25 μm, one should be able to achieve adiabatic and dynamic stability with a matrix resistivity as high as 10^{-9} Ωm. It is suggested that further experimental work should be pursued before the concept of increasing the resistivity of the superconductor matrix material is applied to large high current density magnets which use thermal quench back as a primary means of quench protection.

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