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Author

Swanson, W.P.

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W.P. Swanson

August 1985

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Neutron Dose Equivalent at Electron Storage Rings

William P. Swanson

Lawrence Berkeley Laboratory
University of California
Berkeley, California 94720

ABSTRACT

Simple assumptions are used to predict the average dose equivalent from giant-resonance neutrons near a beam of circulating electrons. The methodology is derived assuming uniform beam loss around a circular ring and numerical results are given for a proposed set of storage ring parameters. Comparison is made to the two limiting cases of a point source and an infinite line source of photoneutrons. The dose equivalent at 1 m from the ring is calculated as a function of concrete shielding thickness and uncertainties are discussed. By simple scaling, the method can be applied to any electron storage ring or circular accelerator, or, with suitable modification of the source term, to any device that approximates an isotropic ring source.

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William P. Swanson

Lawrence Berkeley Laboratory
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Berkeley, California 94720

In the energy regime within which most electron storage rings operate (a few hundred MeV to tens of GeV), the potentially harmful radiation is primarily caused by two sources: bremsstrahlung when the circulating beam is lost from orbit, and giant-resonance photoneutrons produced in the resulting electromagnetic cascade. Of these, the bremsstrahlung dose is harder to treat analytically, owing to its forward-peaked angular distribution. However, it is shown below that the neutron dose equivalent can be predicted simply and with sufficient accuracy that radiation shielding can be designed with reasonable confidence.

Ring Parameters Assumed

The following parameters correspond to those anticipated for an existing electron storage ring, Aladdin, constructed at the Physical Sciences Laboratory of the University of Wisconsin at Madison (Row81, Sym84):

$E_0 = 1 \text{ GeV}$ Energy of the circulating electrons

$I = 1 \text{ A}$ Average current

$R_0 = 14.5 \text{ m}$ Average radius of the ring*

$f = 3.3 \text{ MHz}$ Rotation frequency of the beam

The above parameters are equivalent to a stored energy of

$$W = 288 \text{ J.}$$

* It should be noted that the Aladdin ring is not circular but more resembles a square with rounded corners. For simplicity, a circular ring is assumed in this discussion.

In the picture developed below one imagines this amount of energy stored in the beam and subsequently lost in the stainless-steel vacuum pipe. The neutron fluence and dose equivalent calculated then correspond to one complete cycle of "store and dump".

Neutron Source Term and D-E Conversion Factor

The production of photoneutrons in electromagnetic cascades initiated by electrons has been calculated by Swanson (Swa78, Swa79a) and the resulting yields have been largely confirmed by measurement [see Table 2 of Swa78, Ste83, Yan84]. An important observation is that, for the energy range relevant to this report, the yield of neutrons from a given material is proportional to electron beam power, for any E_0 above about twice the energy of the giant-resonance peak. The calculations of Swa79a for Fe for $E_0 = 1$ GeV give

$$Y(\text{Fe}) = 8.18 \times 10^8 \text{ neutrons J}^{-1},$$

where the unit "J" refer to the total amount of energy, W , carried by the incident electron beam onto a thick sample of the material. For comparison, the yields from Al, Cu and Pb are:

$$Y(\text{Al}) = 6.20 \times 10^8 \text{ neutrons J}^{-1}, \text{ and}$$

$$Y(\text{Cu}) = 11.8 \times 10^8 \text{ neutrons J}^{-1},$$

$$Y(\text{Pb}) = 21.3 \times 10^8 \text{ neutrons J}^{-1},$$

which represent differences of -24%, +44% and +160% from $Y(\text{Fe})$ at 1 GeV, respectively. The photoneutron yield for Fe drops from its value at 1 GeV by only 7% at $E_0 = 100$ MeV. Iron is used in the present example because of the presence of a stainless-steel vacuum pipe and magnet yokes; the other materials (Al, Cu, Pb), because of their common usage, serve for comparison. Aluminum can be considered representative of concrete [$A = 27$ for Al, as compared to an effective atomic number $A_E \approx 21$ for concrete]. If another material than Fe is used, the results can be appropriately scaled.

The total neutron yield from a beam dump into iron is, then:

$$\begin{aligned} Q &= 8.18 \times 10^8 W = 8.18 \times 10^8 \times 288 \text{ J} \\ &= 2.36 \times 10^{11} \text{ [neutrons per dump]}, \end{aligned} \quad (1)$$

where W is the total energy stored in the electron beam in Joules.

For conversion from neutron fluence to dose equivalent, we adopt the values from ICRP-21 (ICR73, Table 4). The spectrum of photoneutrons from the giant photoneutron resonance resembles that of a fission spectrum (Swa79b, pp. 71-75) and the average energy varies relatively little with the atomic number of the medium in which the electromagnetic cascade occurs. For the present calculation, an average neutron energy of 2 MeV is assumed. The corresponding conversion factor from ICRP-21 is:

$$\begin{aligned} & 7.0 \text{ neutrons cm}^{-2} \text{ s}^{-1} \text{ per mrem h}^{-1} \\ & = 2.52 \times 10^8 \text{ neutrons m}^{-2} \text{ mrem}^{-1}. \end{aligned}$$

Dose Equivalent at Ring Center

The dose equivalent at ring center is particularly simple to calculate and, in many cases, easily accessible to measurement. Furthermore, its value is insensitive to the actual deviations from a uniform beam loss around the ring. The neutron fluence at a distance d from a *point* electron loss in an Fe medium is:

$$\phi = \frac{Q}{4\pi d^2} [\text{neutrons m}^{-2}]. \quad (2)$$

As the center is equidistant from every point of the ring, we obtain for the neutron fluence at the ring center ($d = R_0$):

$$\begin{aligned} \phi_c &= Q/[4\pi R_0^2] = \frac{8.18 \times 10^8 \times 288 \text{ J}}{4\pi(14.5\text{m})^2} \\ &= 8.92 \times 10^7 [\text{neutrons m}^{-2}] \end{aligned} \quad (3)$$

Utilizing the above conversion factor, we obtain

$$H_c = \frac{8.92 \times 10^7 \text{ neutrons m}^{-2}}{2.52 \times 10^8 \text{ neutrons m}^{-2} \text{ mrem}^{-1}} = 0.354 \text{ mrem.} \quad (4)$$

for the neutron dose equivalent at the ring center.

Dose Equivalent in the Median Plane

The neutron dose equivalent at any point in the plane of the ring (median plane) is easily obtained from equations (2 - 4) by substituting the average value of the inverse-squared-distance from a uniform circular source (see Appendix I) for d^{-2} :

$$\left(\frac{1}{d^2}\right) = \left| \frac{1}{R^2 - R_0^2} \right| = \left| \frac{1}{(R + R_0) S} \right| \quad (5)$$

where R is the distance from the ring center to the point in question and $S = |R - R_0|$. For $R \approx R_0$ we may use the approximation

$$\left(\frac{1}{d^2}\right) \approx \left| \frac{1}{2R_0 S} \right| \quad (6)$$

Substituting values of $\overline{d^{-2}}$ obtained from equation (5) for a range of distances, S , leads directly to the values shown in Fig. 1. The neutron dose equivalent is plotted as a function of distance, S , measured radially from any point on the ring, both inwards (towards the ring center) and outwards. The most general interpretation of these ring curves is that, for an arbitrary (non-uniform) beam loss distribution, they give the neutron dose equivalent *averaged* around any circle concentric with the electron ring. If uniform beam loss is assumed, these curves show the actual (unaveraged) values at the indicated distances from the ring.

For comparison, Fig. 1 also shows the neutron dose equivalent as a function of distance from a hypothetical infinite line source having the same strength as the ring in terms of neutrons per unit length. This curve has a strict S^{-1} behavior.

As expected, the values for both ring curves of Fig. 1 merge and very closely approach the behavior of the infinite line source for small values of the distance, S . In fact, as equation (5) predicts, both ring curves obey an inverse-distance (not inverse square) law for S small compared to the ring radius. At a distance about equal to the ring radius, the dose equivalent deviates noticeably from a pure inverse-distance behavior; the curve inside the ring is at a minimum, as symmetry would require. As one proceeds outward from the ring by the same amount, $S \approx R_0$, the dose equivalent has deviated from a pure inverse-distance law by about 34% and then continues to roll over to become nearly inverse *square* for distances greater than several times R_0 . This can be seen by comparison with the uppermost curve of Fig. 1 which shows the neutron dose equivalent for a *point-loss* of the entire stored beam at distance S . Comparison of this curve with the ring curves clearly shows the relationship between the average neutron dose equivalent and the maximum possible produced by an accidental point loss. This difference amounts to a factor of 30 at $S = 1$ m but

diminishes rapidly with S . Not coincidentally, the curve for point loss intersects precisely with the value at the ring center.

Neutron Dose Equivalent After Concrete Shielding

Figure 2 shows the neutron dose equivalent in the median plane at 1 m from the ring assuming various thicknesses of intervening concrete. These results are computed by the summation of small increments ($\Delta \theta = 1^\circ$) around the ring, assuming uniform beam loss on the circumference. The geometrical model assumes that the inner surface of the concrete wall is contiguous with the ring. Each contribution is attenuated by its proper attenuation in the concrete, using a tenth-value layer of $\text{TVL} = 92 \text{ g cm}^{-2}$ (Fas84). For a concrete density of 2.35 g cm^{-3} , this corresponds to 39.2 cm.

It is evident from Fig. 2 that there is a change in slope in the effective attenuation coefficient. This occurs because the first shielding layers strongly diminish the contributions of sources farther removed from the point of measurement, because of their larger slant ranges through the shielding. Thereafter the proximal arc source is attenuated in a manner which more resembles the attenuation of a point source of neutrons.

Circumferential Distribution of Dose-Equivalent Contributions

Figure 3 shows the circumferential distribution of dose contributions to point P at distance $S = 1 \text{ m}$ outside the ring, for chosen shielding thicknesses. Units are mrem J^{-1} , and the abscissa scale is such that $\theta = 0$ when the source is closest to P. The meaning of this graph is that a *point* loss of W Joules, anywhere around the ring (angle θ), will contribute a dose at point P of the amount read on the ordinate scale. For an extended beam loss, one must integrate over theta, and that is what has been done in the preceding section, with the assumption of a uniformly distributed loss.

For no shielding, the dose at P is dominated by a single large peak, symmetrical about $\theta = 0$. As shielding is added (lower curves of Fig. 3), one sees that the peak itself is attenuated by some factor, depending on the shielding thickness, and contributions from across the ring ($\theta = 180^\circ$) are attenuated by approximately the same factor, whereas contributions near $\pm 20^\circ$ are severely attenuated, owing to their large slant distances through the shielding. All curves are symmetrical about 0° , as one would expect from the assumed isotropy of the source.

Accuracy

To the extent that the photoneutron source can be considered a *uniform* isotropic ring source, and is unperturbed by such external influences as shielding, room scattering, or neutron multiplication by materials near the ring, the major uncertainties in the results of Figs. 1 and 2 are estimated as follows:

Uncertainty in neutron yield for Fe	$\pm 20\%$
Uncertainty in correct value of DE-conversion factor	$\pm 15\%$
Error resulting if the major neutron producing medium is not pure Fe, but rather a combination of Cu, Fe and Al (concrete)	$\pm 29\%$
<hr/>	
Combined uncertainty (in quadrature)	$\pm 38\%$

For the arguments and results presented here to be valid, it must be assumed that the cascade is contained within the material of the vacuum pipe itself or in shielding material in close proximity; The case in which the electrons are dumped into a thin-walled pipe so that the electromagnetic cascade continues out into the room to produce photoneutrons in far-off corners would considerably lower and flatten the curves of Fig. 1.

In most cases the beam is not lost uniformly from orbit, but losses are concentrated in regions of high dispersion or of maximum amplitude in the betatron oscillations; more neutron shielding would be needed in such regions. However, the maximum needed anywhere should not exceed an amount determined by the uppermost curve of Fig. 1 for a point loss of the stored beam.

Possible shielding by objects near the beam line (especially magnets) is not considered here. Iron magnets are not good absorbers of giant-resonance neutrons, and their major effect would be to rescatter the neutron fluences incident on them. As a first approximation for radiation-protection planning, their shielding effect can be considered as neutral. Another effect which can significantly alter the neutron fluence, is the scattering from the walls of the room. McCall et al. have published an empirical formula which estimates the additional fluence

due to fast neutrons scattered around a concrete room of area A (A is the total for floor, ceiling and 4 walls combined) containing a neutron source of strength Q neutrons (McC79; also see Eis82):

$$\phi_{\text{scat}} = 5.4 \frac{Q}{A} \quad (7)$$

As an example, assume the area of the room to be 20 times the area of the ring: $A = 20 \times \pi R_0^2 = 13210 \text{ m}^2$. This would lead to a rather uniform contribution of scattered fast neutrons of:

$$\frac{\phi_{\text{scat}}}{\phi_c} = \frac{5.4Q / [20\pi R_0^2]}{Q / [4\pi R_0^2]} = 1.08, \quad (8)$$

relative to the fluence of direct neutrons at the ring center, ϕ_c . This would therefore approximately double any measurement obtained at the ring center and add about 15% to the fluence measured at 1 m from the ring (inside or out). This contribution (approximate) is indicated as the horizontal line in Fig. 1.

Implications for Radiation Protection

For purposes of discussion, the arbitrary reference distance of 1 m from the ring is chosen to assess the radiation protection needs.

- (1) The bremsstrahlung dose equivalent far exceeds the average neutron dose equivalent and will dominate the shielding (Swa85). It is very probable that an adequate shield for bremsstrahlung will be more than adequate for the neutrons if concrete is used. However, if bremsstrahlung is shielded primarily by non-hydrogenous materials such as Pb or Fe, the neutrons may not be adequately attenuated.
- (2) In itself, the unshielded neutron dose equivalent is marginal; if one cycle of "store and dump" is executed ten times each 40-hour work week, the ICRP limit equivalent to 100 mrem/week would be just reached (at 1 m). However, in their Guidance on Maintaining Exposures to As Low as Reasonably Achievable, the U. S. Department of Energy (DOE) states as a design objective: "... onsite personnel levels less than one-fifth of the permissible ... limits ..." (DOE81). Therefore, at least some *concrete* shielding is advisable because of the neutrons; metal shielding (e.g., Pb or Fe) is not effective against giant-resonance neutrons.

- (3) At the site boundary (assume 50 m) the *direct* neutron dose equivalent is less than the value at 1 m by a factor of 200, not taking into account any difference in elevation. At 1000 cycles of "store and dump" (fills) per year this would be:

$$\left[0.020 \text{ mrem / fill} \right] \times \left[1000 \text{ fills / year} \right] = 20 \text{ mrem / year} \quad (9)$$

direct neutron dose equivalent with no neutron shielding. This is to be compared with the proposed revision of DOE radiation standards for protection of the public which "... incorporate a provision for Headquarters concurrence for anticipated routine operations that may result in estimated exposures exceeding 25 mrem/year to any member of the public" (DOE84). This requirement appears to be satisfied even without shielding, as far as the neutron dose is concerned. An adequate shield against bremsstrahlung which contains at least some concrete would greatly improve upon this.

- (4) These predictions are for the *average* neutron dose equivalent; regions of the ring where higher beam losses are expected should have increased shielding.
- (5) The maximum possible accident is easily estimated from the point-loss prediction shown by the uppermost curve of Fig. 1.

Applicability to other Isotropic Ring Sources

The manner of scaling to the parameters of other electron rings or circular accelerators (synchrotron, betatron) is quite obvious from the above equations (2 - 5). It is implicit that one should scale by the total stored beam energy, W . Loosely speaking, the neutron dose equivalent "near the ring" is proportional to W / R_0 and inversely proportional to S ; if the ring radius, R_0 , is not too different from 14.5 m, one can scale the results of Fig. 1 approximately by the inverse radius [equation (6)]. For points not too far from the ring (i. e., less than half of R_0 , one can scale as the inverse distance, S , and for large distances (several times R_0), inverse *square* would be appropriate. Direct use of equation (5) is almost as easy as the approximate scaling.

The method outlined above can be adapted, with appropriate modifications to the source strength, to predict radiation doses of any kind produced by a uniform isotropic ring source. As an example, the method could be used to study the fluence or dose-equivalent of evaporation neutrons produced by a proton or heavy-ion accelerator.

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Fig. 1 Neutron dose equivalent in the median plane as a function of distance, S , from an unshielded electron ring of radius $R_0 = 14.5$ m. Upper heavy curve: inwards towards ring center; Lower heavy curve: outwards from ring. Dose equivalents from a point source (for the same total number of electrons stopped) and from an infinite line source (for the same number of electrons lost *per unit length* as for the ring) are also shown. Dashed line indicates approximate contribution from scattered fast neutrons. Energy lost by dumped beam is 288 J, corresponding to 1.8×10^{12} electrons at 1 GeV, circulating at 3.3 MHz ($I = 1A$).

Fig. 2. Neutron dose equivalent in the median plane, at $S = 1$ m from the ring, as a function of concrete thickness. Ring and beam parameters are same as for previous figure. Density of concrete assumed is 2.35 g cm^{-3} . Dashed line illustrates slope corresponding to $\text{TVL} = 39 \text{ cm}$ (arbitrary normalization).

Fig. 3. Neutron dose equivalent at $S = 1$ m from ring, per J lost at any point on the ring circumference, as a function of angle subtended at ring center. $R_0 = 14.5$ m. Curves are labeled to show 0.0, 0.1 and 0.5 m of concrete shielding.

APPENDIX I: Average inverse distance-squared from a ring source.

Imagine a point of observation, P, at distance R from the origin, O, and a source point, Q, on the periphery of a circle of radius R_0 centered at O. The angle between OP and OQ is θ and the distance QP is called d. For every choice of R, R_0 and θ , the law of cosines gives:

$$d^2 = [R^2 + R_0^2 - 2RR_0 \cos\theta] = a + b \cos\theta, \quad (I-1)$$

where $a = R^2 + R_0^2$ and $b = -2RR_0$. The average inverse-squared distance is found by averaging over θ , implying equal weight is given to every source point Q (uniform circular beam loss).

$$\overline{d^{-2}} = \frac{1}{2\pi} \int_0^{2\pi} d^{-2} d\theta = \frac{1}{2\pi} \int_0^{2\pi} [a + b\cos\theta]^{-1} d\theta. \quad (I-2)$$

This form can be found in standard tables of integrals and gives:

$$\overline{d^{-2}} = \frac{1}{2\pi} \left[\frac{2}{\sqrt{a^2 - b^2}} \arctan \left[\frac{\sqrt{a^2 - b^2} \tan(\theta/2)}{a + b} \right] \right]_0^{2\pi} \quad (I-3)$$

In general, the angle given by the arctan is multiple-valued, and one must carefully observe the quadrant corresponding to the integration limits. The lower limit clearly gives zero because $\tan(\theta/2) = 0$ when $\theta = 0$. At the upper limit, $\theta = 2\pi$, we again find zero for the argument of the arctan. As the result of integration must be non-zero, the only sensible value for the arctan is π . That is, $\theta/2$ has progressed continuously from 0 through $\pi/2$ to π as the circle is completed, forcing the angle given by the arctan to take on values covering the same range. (This becomes self-evident if one evaluates the integral with $b = 0$.)* The result is

$$\overline{d^{-2}} = \frac{1}{2\pi} \frac{2\pi}{\sqrt{a^2 - b^2}} = [a^2 - b^2]^{-1/2} \quad (I-4)$$

To evaluate, we expand:

$$[a^2 - b^2] = R_0^4 + 2R_0^2 R^2 + R^4 - 4R^2 R_0^2 \quad (I-5)$$

*Alternatively, one can avoid this ambiguity and obtain the identical result by averaging over the range $\theta = 0$ to π .

$$= R_0^4 - 2R_0^2 R^2 + R^4 = [R_0^2 - R^2]^2$$

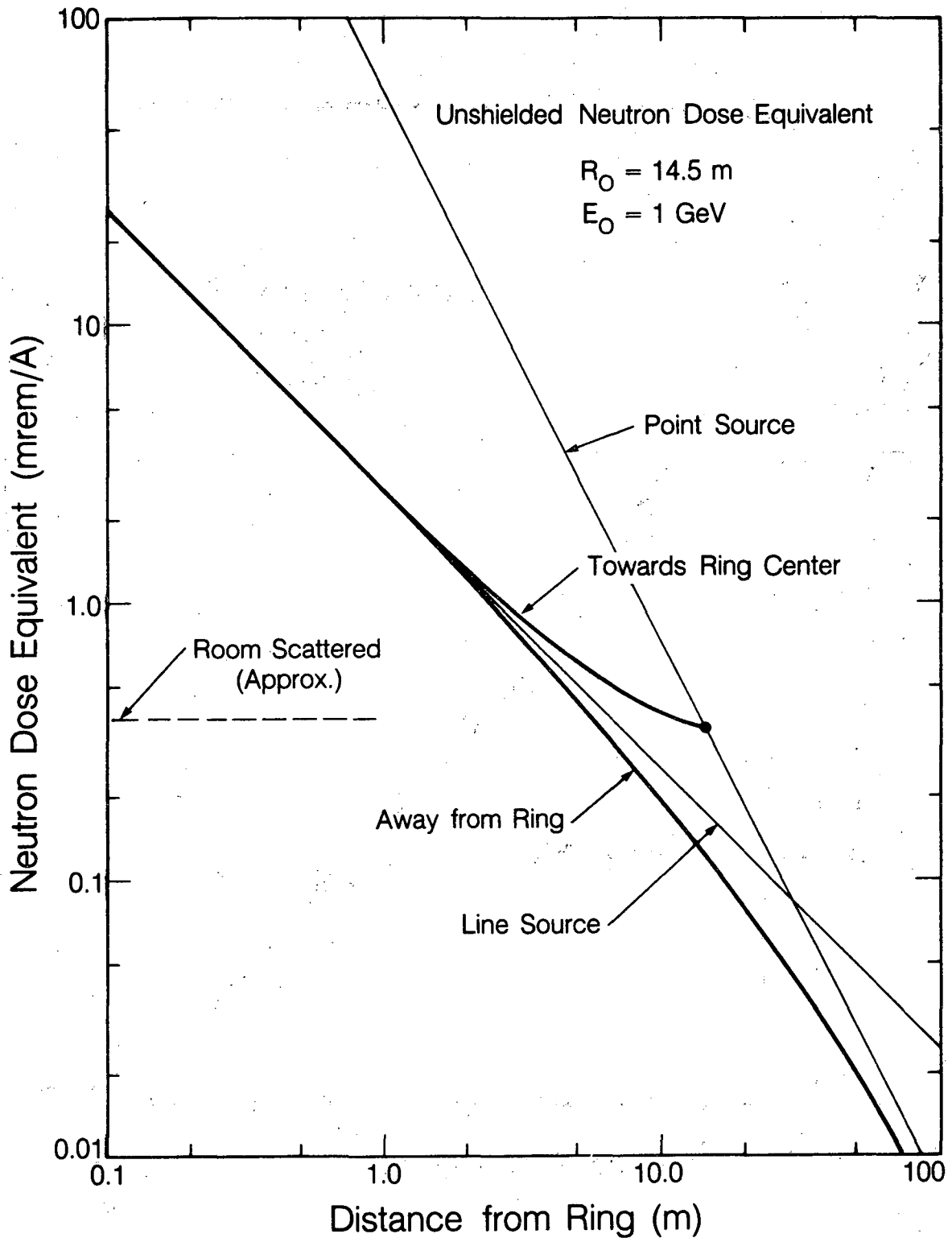
Taking the positive root gives the desired result:

$$\overline{d^{-2}} = [R_0^2 - R^2]^{-1} \quad (I-6)$$

for the average inverse-square distance from any point on the median plane, P, to every point on the circle.

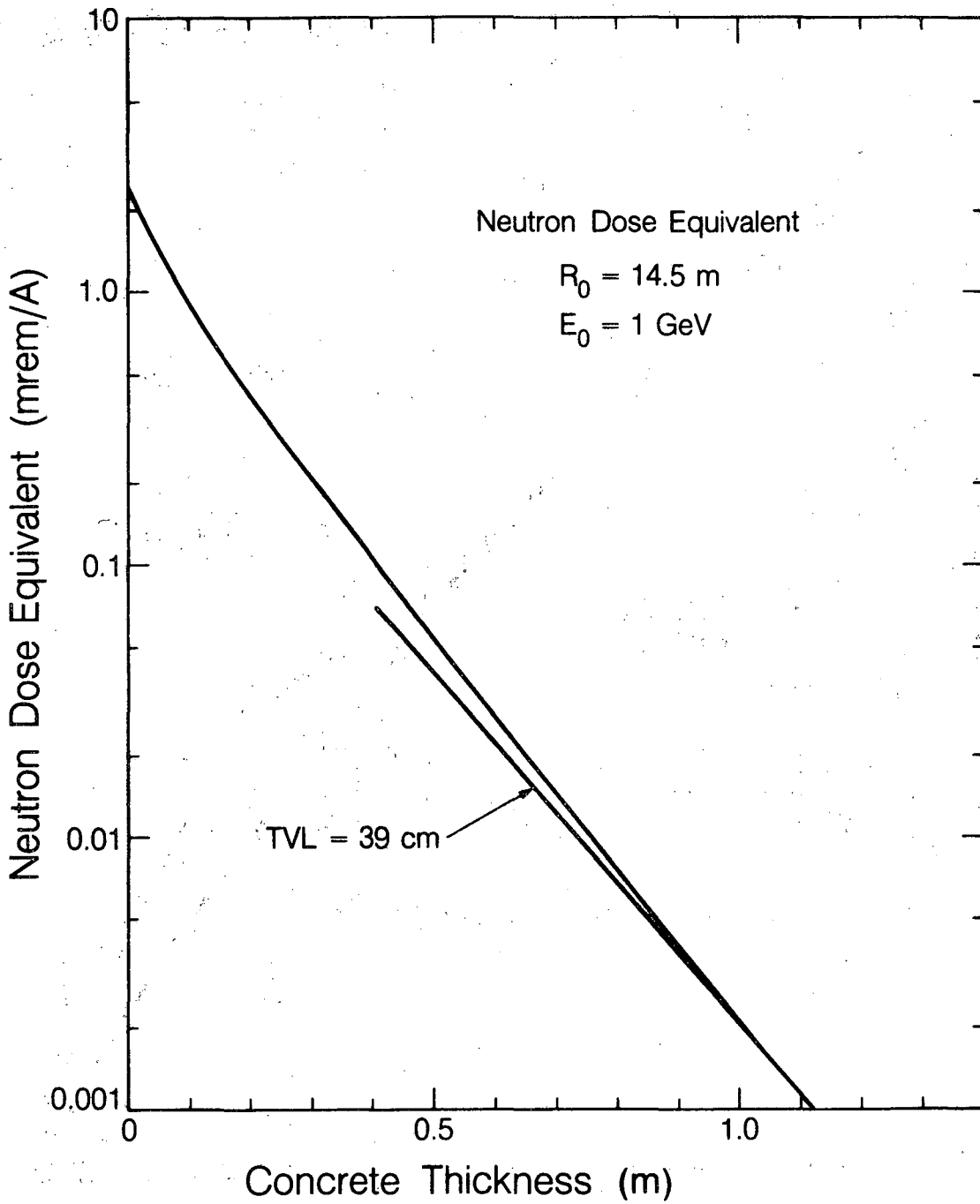
For points at a vertical distance, Z, above or below the median plane, the average inverse-square distance can be found by a generalization of a in the preceding derivation: $a = R_2 + R_0^2 + Z^2$ in equation (I-1). Then, using equation (I-4), $\overline{d^{-2}}$ becomes:

$$\overline{d^{-2}} = [(R^2 - R_0^2)^2 + 2Z^2 (R^2 + R_0^2) + Z^4]^{-1/2} \quad (I-7)$$



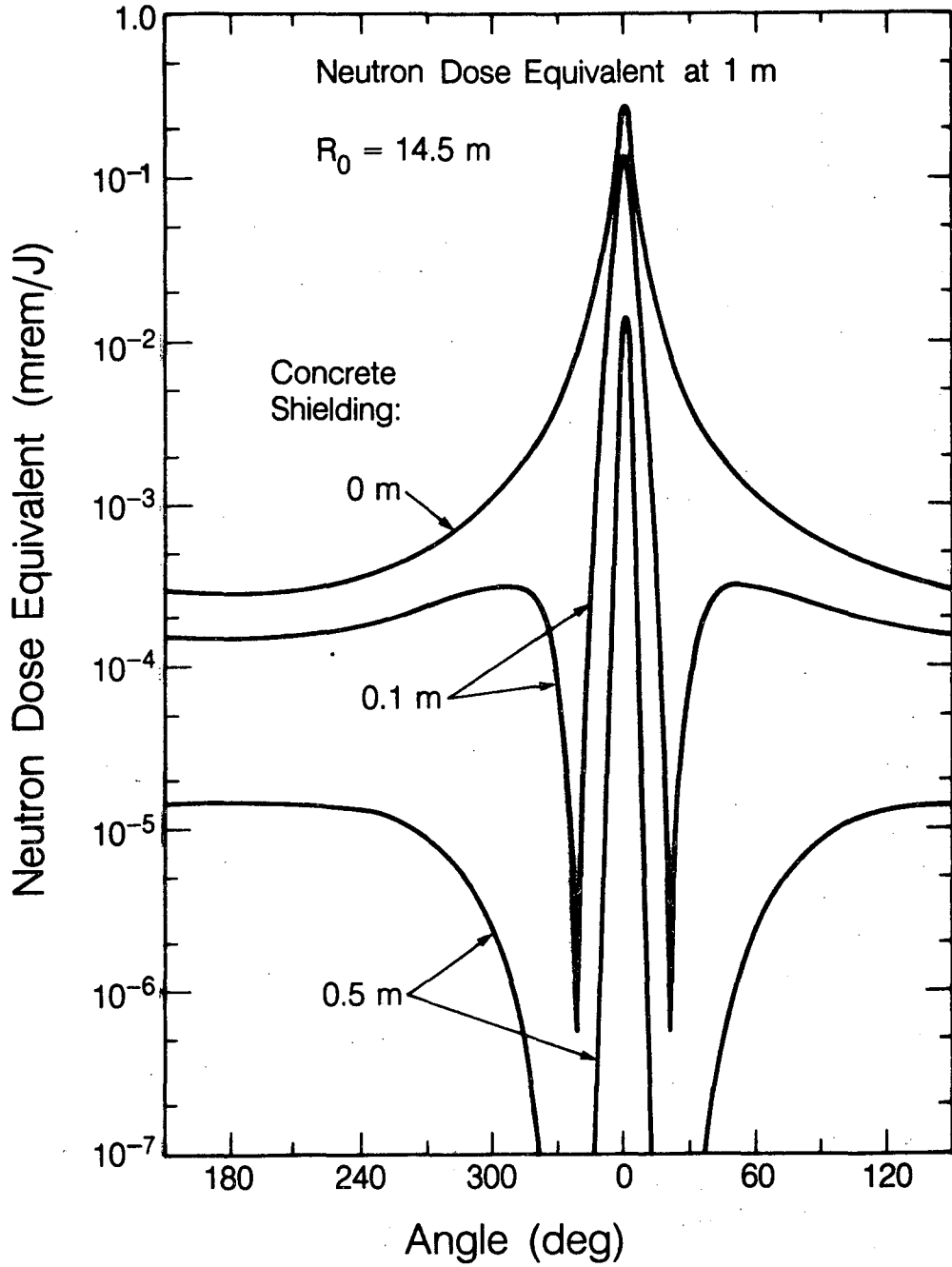
XBL 854-9319

Fig. 1



XBL 854-9310

Fig. 2



XBL 854-9311

Fig. 3

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