

# Lawrence Berkeley National Laboratory

## Recent Work

### Title

INTRINSIC QUADRUPOLE MOMENTS OF DEFORMED NUCLEI

### Permalink

<https://escholarship.org/uc/item/5rd679c2>

### Authors

Glendenning, N.K.  
Sawicki, J.

### Publication Date

1960-08-01

UNIVERSITY OF  
CALIFORNIA

*Ernest O. Lawrence*

*Radiation*

*Laboratory*

TWO-WEEK LOAN COPY

*This is a Library Circulating Copy  
which may be borrowed for two weeks.  
For a personal retention copy, call  
Tech. Info. Division, Ext. 5545*

BERKELEY, CALIFORNIA

## **DISCLAIMER**

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

To be published  
Conference on  
Ontario, Canada

MCP 9303

UNIVERSITY OF CALIFORNIA  
Lawrence Radiation Laboratory  
Berkeley, California

Contract No. W-7495-eng-48

**INTRINSIC QUADRUPOLE MOMENTS OF DEFORMED NUCLEI**

**N. K. Glendenning and J. Sawicki**

**August 1960**

INTERNATIONAL CONFERENCE ON NUCLEAR STRUCTURE

AUGUST 29 - SEPTEMBER 3

1960

Secretary: Dr. L. G. Elliott  
Atomic Energy of Canada Limited  
Chalk River, Ontario, Canada

RESEARCH CONTRIBUTION

Proposed Session No. No. 6 (Pairing Force and Nuclear Structure)

Title: Intrinsic Quadrupole Moments of Deformed Nuclei

Authors: N. K. Glendenning and J. Sawicki

Institution: Lawrence Radiation Laboratory

Address: University of California, Berkeley, California

Abstract:

Intrinsic quadrupole moments of deformed nuclei are calculated on a particle picture as a function of  $Z$  and the deformation  $\delta$ . Effects of the pairing force are examined. The single-particle levels are those of the Nilsson Model, but configuration mixing between major shells  $N' = N \pm 2$  must be included. Observed moments are accounted for by slightly smaller deformations than implied by the uniform-charge model.

## INTRINSIC QUADRUPOLE MOMENTS OF DEFORMED NUCLEI

Norman K. Glendenning  
Lawrence Radiation Laboratory  
University of California  
Berkeley, California

and

Jerzy Sawicki  
Department of Physics  
University of California  
Berkeley, California

July 12, 1960

Quadrupole moments of deformed nuclei have always been discussed in terms of the uniformly charged spheroid which establishes a connection between the quadrupole moment and the nuclear deformation.<sup>1</sup> Since our knowledge of nuclear deformations rests principally on this connection, it is of interest to investigate the effects of the particle structure of nuclei on their quadrupole moments.

In this work we adopt a particle picture of the nucleus in calculating the quadrupole moments. The  $Z$  protons of the nucleus are assumed to move in a spheroidal well of fixed deformation and the interparticle interactions are approximated by the pairing force, introduced first to account for superfluidity by Bardeen, Cooper, and Schrieffer (BCS).<sup>2</sup>

Assume that a certain single-particle Hamiltonian representing the independent-particle motion in nuclei can be solved. Represent the quantum numbers of the eigenstates by  $\nu$  and the eigenvalues by  $E_\nu^0$ . Then in terms of these eigenvalues, Migdal<sup>3</sup> has shown how to compute the single-particle density matrix  $\rho_{\nu\nu'}$ , corresponding to particles moving in the single-particle potential and interacting through the pairing force. If there is some additional perturbation  $V$  on the single-particle Hamiltonian, then the density matrix, up to first order in  $V$ , is

$$\rho_{\nu\nu'} = \rho_{\nu\nu'}^0 + \rho_{\nu\nu'}^1 \quad (1)$$

$$\rho_{\nu\nu'}^0 = \frac{1}{2} \left( 1 - \frac{\epsilon_\nu}{E_\nu} \right) \delta_{\nu\nu'} \quad (2)$$

$$\rho_{\nu\nu'}^1 = \frac{\epsilon_\nu \epsilon_{\nu'} - E_\nu E_{\nu'} - \Delta^2}{2E_\nu E_{\nu'} (E_\nu + E_{\nu'})} \quad V_{\nu\nu'} = b_{\nu\nu'} V_{\nu\nu'} \quad (3)$$

where

$$\epsilon_\nu = E_\nu^0 - \lambda, \quad E_\nu = \sqrt{\epsilon_\nu^2 + \Delta^2} \quad (4)$$

and  $\lambda$  and  $\Delta$  are the chemical potential and energy gap of the BCS theory.

We have computed the intrinsic quadrupole moment of a certain model nucleus comprising  $Z$  protons moving in a permanently deformed potential well, and interacting through a pairing force.<sup>4</sup> Because the deformation is assumed time-independent the results can have quantitative meaning for real nuclei only in the strongly deformed regions. Nevertheless the model may have at least qualitative meaning for weakly deformed nuclei near closed shells. For the basic states in Eqs. (1) - (4) we take the eigenvalues and eigenfunctions of Nilsson and Mottelson<sup>5</sup> which result from an exact diagonalization of their Hamiltonian, except that only matrix elements that are diagonal in the oscillator quantum number  $N$  are included. That is, the deformation-dependent part of their Hamiltonian is

$$V_D = V_0 \delta_{NN'} \quad (5)$$

$$V_0 = \frac{2}{3} M \omega_0^2 Q_{20}$$

It is, however, essential to include the effects on the quadrupole moment of the nondiagonal matrix elements of  $V_0$ . This can be done by treating the nondiagonal part,

$$V_{ND} = V_0 (1 - \delta_{NN'}), \quad (6)$$

as the perturbation  $V$  in Eq. (3). Then the quadrupole moment is

$$Q_0 = Q_D + Q_{ND}, \quad (7)$$

$$Q_D = \sum_{N \Omega \alpha} \left( 1 - \frac{E_{N \Omega \alpha}^0 - \lambda}{[(E_{N \Omega \alpha}^0 - \lambda)^2 + \Delta^2]^{1/2}} \right) \langle N \Omega \alpha | Q_{20} | N \Omega \alpha \rangle, \quad (8)$$

$$Q_{ND} = -\frac{4}{3} \delta M \omega_0^2 \sum_{N \Omega \alpha \alpha'} b(N \Omega \alpha; N+2, \Omega \alpha') \times |\langle N \Omega \alpha | Q_{20} | N+2, \Omega \alpha' \rangle|^2, \quad (9)$$

where  $b$  is defined in Eq. (3),  $Q_{20}$  is the single-particle quadrupole moment operator, and the basic states  $\nu = N \Omega \alpha$  are defined by Nilsson.<sup>5</sup>

The results of our calculation are displayed in the figure and in the table. The quantity  $Q_0/ZR^2$ , which for the uniform-charge model is a constant  $(4/58(1 + \delta/2))$  is found to decrease monotonically with increasing  $Z$  through the rare earth region. In the presence of the pairing force  $Q_0/ZR^2$  is a smooth function of  $Z$ . For small deformation ( $\delta \leq 0.1$ ) it shows shell-closure effects as the pairing force is reduced. However, at the deformations appropriate to the rare earth region, the pairing force has very little effect on the calculated quadrupole moment.

As indicated in the table, our values of the deformation  $\delta$  for the lower rare earths, deduced by a comparison of the experimental quadrupole moments with our calculated values, are slightly smaller than those deduced via the uniform-charge model.



Table I

Intrinsic quadrupole moments  $Q_0 = Q_D + Q_{HD}$  (in  $10^{-24}$  cm<sup>2</sup>) for some strongly deformed nuclei, compared with the observed moments  $Q_{obs}$ . The deformation,  $\delta$ , deduced from this comparison is compared with the deformation implied by analyzing the experimental values of  $Q_0$  with  $Q = 4/5 ZR^2 \delta (1 + \delta/2)$  for the uniform-charge model.

Z	A	Calculated $Q_0$			$Q_{obs}$	$\delta$ (this calculation)	$\delta$ (uniform model)
		$\eta = 2$	$\eta = 4$	$\eta = 6$			
12	24	0.35	0.63	0.94	0.45	0.24	0.33
62	150	3.25	6.60	10.11	3.62	0.13	0.17
62	152	3.29	6.71	10.26	5.82	0.20	0.26
64	154	3.41	6.85	10.58	5.75	0.20	0.28
64	156	3.45	6.90	10.72	6.75	0.23	0.28
64	158	3.44	6.91	10.67	7.35	0.25	0.30
64	160	3.45	6.94	10.71	7.59	0.25	0.30
66	160	3.60	7.10	11.07	6.65	0.22	0.26
66	162	3.63	7.13	11.17	7.14	0.23	0.28
66	164	3.62	7.16	11.16	7.48	0.24	0.29
68	164	3.76	7.35	11.41	7.07	0.23	0.26
68	166	3.78	7.39	11.46	7.52	0.24	0.28
68	168	3.78	7.41	11.50	7.55	0.24	0.28
68	170	3.80	7.44	11.55	7.34	0.23	0.27
70	172	3.91	7.72	11.79	7.42	0.23	0.26
72	176	3.95	7.93	12.10	7.25	0.22	0.24
72	178	3.98	7.98	12.15	6.88	0.21	0.23
72	180	3.98	7.99	12.20	6.60	0.20	0.23
74	182	3.98	8.15	12.49	6.45	0.19	0.21
74	184	3.99	8.19	12.55	6.15	0.18	0.20

## REFERENCES

1. R. J. Blin-Stoyle, *Revs. Modern Phys.* 28, 75 (1956).
2. J. Bardeen, L. N. Cooper, and J. R. Schrieffer, *Phys. Rev.* 108, 1175 (1957); S. T. Belyaev, *Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd.* 31, No. 11 (1958).
3. A. B. Migdal, *Nuclear Phys.* 13, 655 (1959).
4. For full details see: N. K. Glendenning and J. Sawicki, *Nuclear Phys.* (to be published).
5. S. G. Nilsson, *Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd.* 29, No. 16 (1955); B. R. Mottelson and S. G. Nilsson, *Hgl. Danske Videnskab. Selskab, Mat.-fys. Skrifter* 1, No. 8 (1959).

FIGURE CAPTION

The quantity  $Q_0/ZR^2$ , computed on the particle model of this article, is shown as a function of  $Z$  for several values of the deformation. For the uniform-charge model  $Q_0/ZR^2$  has the constant values 0.346, 0.219, 0.103 for the deformations  $\eta = 6, 4, 2$  respectively.

