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Design and Applications of High Performance Piezoelectric Micromachined Ultrasonic
Transducers (pMUTs)

By

Sedat Pala

A dissertation submitted in partial satisfaction of the
requirements for the degree of

Doctor of Philosophy

in

Engineering - Mechanical Engineering

in the

Graduate Division

of the

University of California, Berkeley

Committee in charge:

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Professor Kristofer Pister

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Design and Applications of High Performance Piezoelectric Micromachined Ultrasonic Transducers (pMUTs)

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by

Sedat Pala

Abstract

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Doctor of Philosophy in Mechanical Engineering

University of California, Berkeley

Professor Liwei Lin, Chair

Micromachining technologies have enabled exciting applications in a small form-factor for ultrasonic transducers in medical imaging, range finder, human machine interface, and more. With the advantages in low size, weight, power and cost micromachined ultrasonic transducers are potential alternatives to bulk ultrasonic transducers. This work studies design of high performance piezoelectric micromachined ultrasonic transducers (pMUTs) for several applications.

A free-clamped boundary pMUT has been proposed, which results in more than 16x higher electromechanical coupling coefficient and 100% electrode coverage area as compared with those of conventional clamped-clamped boundary pMUTs with only 67% electrode coverage area. The cavity in the free-clamped boundary design is also utilized as an acoustic resonator to increase 10-100x larger bandwidth as compared with those of conventional pMUTs with clamped-clamped boundary. Furthermore, a two-term mode shape approach for the conventional clamped pMUTs is presented to result in an improved lumped element model for less than 0.5% error in the intrinsic and extrinsic properties of pMUT including equivalent circuit parameters, input impedance, velocity, displacement, bandwidth, directivity, and the on-axis pressure in the near and far fields.

First, an ultrasound-induced airborne haptic device via pMUTs is demonstrated by modulating the high frequency ultrasound into a low frequency patterns to stimulate mechanoreceptors inside the human skin. Results show that the human haptic sensations from volunteers can achieve 80% and 76% identification accuracy for the switching on/off and Morse code patterns, respectively. Second, structures with curved surfaces are studied for two properties: (1) the signal attenuation due to the curved surface; and (2) the radius-of-curvature (RoC) measurement by using the time-of-flight scheme. Results show good matches between the measured and simulated attenuation at 25 mm away due to the curvature as 7 dB. Good RoC measurement accuracies for 3D-printed curved surfaces have also been achieved up to 200 mm at a distance 5 cm away. Lastly, a phase-shift cancellation scheme is proposed to reduce the ring down period and increase axial resolutions. Results show the ring down period is shortened by 13% and 7.5% and the axial distance resolution is increased by 30% and 25%, for a single element and an 80-element array, respectively.

To my parents
Fatma & Bahattin

Your endless love, support and patience made this possible...

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The only real chance one can have in this world is having a good teacher at young ages. I was the luckiest person on the planet meeting my high school physics teacher Sedef Halilbeyoglu Akin and my undergraduate degree teacher Prof. Huseyin Vural. I believe dreams are the most significant indicators of what one can achieve, and they taught me how to follow my dreams. I could not thank them enough. Also, I'd like to thank Prof. Tayfun Akin and Prof. Kivanc Azgin for introducing me the field of MEMS and creating a unique MEMS research environment.

I have met many brilliant researcher during my Ph.D. at Berkeley. First, I am thankful to Zhichun Shao for helping me in pMUT microfabrication. I'd like to thank Yande Peng, Ryuichi Arakawa, Fan Xia, Yue Liang, and Ben Eovino for their collaboration in my research. I'd also like to thank rest of the Lin Lab for their friendship and useful discussions: Peisheng He, Ruiqi Guo, Alexander Alvara, Xining Zang, and Heya Kaakeh.

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I'd like to put a little philosophy into this by answering the fundamental question: what is happiness? Although this might be a strong argument in this age, I believe happiness must be the production. Then the engineering model of the happiness would be:

$$H = \frac{\partial S}{\partial t} \quad \begin{array}{l} H : \text{happiness} \\ t : \text{time} \\ S : \text{anything: idea, science, art, friendship, love, human, pMUT, ...} \end{array}$$

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Chapter 1

Introduction

The field of sound has been investigated for millennia in many fields such as art, science, and communication. For the last three centuries, there have been many major improvements in understanding sound as it requires computational techniques and tools both in the frequency and time domain. Since then, many researchers have contributed in the field such as Rayleigh, Lamb, Stokes, Helmholtz, Alexandre Graham Bell, Edison, and more [1]. The last century was the most interesting times in acoustic research as it introduces many applications in underwater and air environments, and communications with advanced computation tools like finite element methods.

Within the last fifty years, the most advancement in acoustics has been in rapid rate of miniaturization which introduced many exciting applications including medical imaging, range finders in air, and non-destructive metrology. One interesting observation is that, the electro acoustic transducers do not obey Moore's law [2] that is size reduction cannot be realized without sacrificing some performances. In general, reducing the size compromises the total acoustic pressure output. Within the last thirty years, advances in Micro-Electro-Mechanical Systems (MEMS) technologies have opened up a new era in acoustics transducers, especially ultrasonic transducers. With the strong need in miniaturization and performance trade-off, the acoustic transducer design has never been more important.

The sound, also ultrasound, is a propagating wave in the elastic media such as air and water, and results pressure changes. The energy is transferred from one particle in the media to another by random collisions. For example, there are 8.3×10^9 collisions in a cubic meter volume of air at the room temperature [1]. These collisions result in a pressure which is also known as atmospheric pressure. For a sinusoidal wave, the pressure and displacement at a point have the same frequency; however, they have the 90° phase shift. The acoustic pressure is the incremental pressure variation on top of the ambient pressure. The last important consideration is the heat transfer during these collisions. As the pressure increases in a fixed volume, the temperature rises. Therefore, it is important to investigate if the propagating soundwave causes temperature difference and heat transfer between specific regions. As an example, under the normal atmospheric conditions the speed of travel of a thermal diffusion is about 1.5m/s at 10kHz [3]. The time for a half wave is 0.00005 seconds in which thermal wave travels 75 μm . For a same frequency acoustic wave, the half wavelength is 17150 μm . There is more than three orders of magnitude in difference; therefore, this heat propagation can be neglected at these frequencies. However,

if the frequency of ultrasound increases significantly, like tens of MHz, it is important to include thermal effects in the analysis.

1.1 Ultrasonic Transducers

There are three types of ultrasonic transducers (UTs); bulk, capacitive micromachined ultrasonic transducers (cMUTs), and piezoelectric micromachined ultrasonic transducers (pMUTs) as shown in Figure 1.1. Among those, Bulk UTs (shown in Figure 1.1a) has prominent acoustic pressure output property compared to those in its micromachined counterparts [4-7]. The Bulk UTs utilize the thickness mode which makes them difficult to use in different frequency ranges. The effective electromechanical coupling coefficient of Bulk UTs is higher since they utilize high performance piezoelectric materials such as PZT [8]. However, one of the biggest challenges is the impedance mismatch of the Bulk UTs and the medium especially air. To overcome this challenge, matching layers are preferred between the transducer and the medium, which makes the total dimensions even bigger.

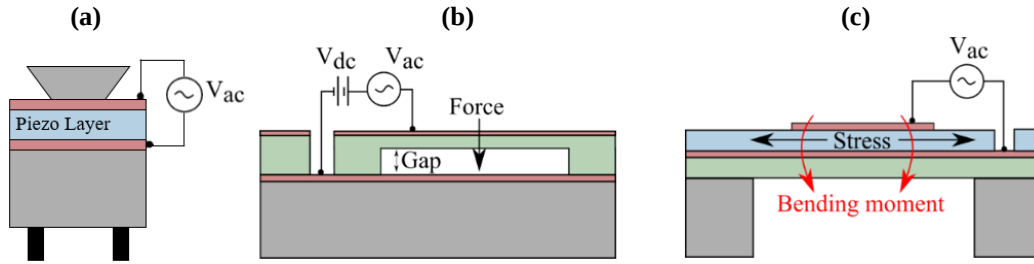


Figure 1.1: Three types of ultrasonic transducers and excitations; (a) Bulk ultrasonic transducer, (b) cMUT, and (c) pMUT

Given these challenges, there is a strong need for a better impedance matching structure. At this point, the micromachined transducers are the potential alternatives since they utilize the flexural mode instead of thickness mode. In the case of cMUTs (shown in Figure 1.1b), the diaphragm, also called the elastic layer, is suspended with a capacitive gap. The elastic layer is bent with a force which is created by the electrostatic action. In general, cMUTs are driven with a small AC signal on top of a high DC signal. The reason is that the electromechanical coupling coefficient is a function of the capacitance change with respect to the displacement in the gap direction. As the DC voltage increases, the coupling gets better. Another issue related with cMUT is the pull-in limitation. Only 2/3 of the gap can be utilized since the pull-in effect occurs at the last 1/3 of the gap distance. This is important if there is a non-uniformity in the gap thickness due to variations in fabrication process. To overcome this, the collapse mode [9-11] is introduced in which a high DC voltage is applied to include pull-in condition, and the center of the diaphragm is always in contact with the bottom electrode.

Among the transduction mechanisms for micromachined transducers, capacitive [12] and piezoelectric [9] mechanisms have been used widely. Devices based on piezoelectric transduction have prominent futures with the following advantages: no need to have a bias voltage, possible large displacements from the flexural mode transducers, and the ease the of fabrication process without a well-controlled thin gap [13]. In the case of pMUTs (shown

in Figure 1.1c), the piezoelectric layer is sandwiched between two electrodes and placed on top of an elastic layer. The center line of the piezoelectric layer is off centered with the neutral axis of the whole diaphragm such that any stress in the piezoelectric layer creates a bending moment with respect to the neutral axis. This stress is created by the piezoelectric actuation. Due to prominent futures of pMUTs, there has been great effort to realize high performance ultrasonic transducers. In general, performance enhancement studies for pMUTs can be summarized in two approaches: (1) optimal geometric designs such as the collapse mode [11] in capacitive MUTs (cMUTs) or curved [14], ring [15], free [16] and pinned [17] boundary, bimorph [18] structures in piezoelectric MUTs (pMUTs); and (2) high piezoelectric effect thin film materials such as PZT [8] and AlN [19].

1.2 pMUT Design Considerations and Performance Metrics

Although there are different device geometries, circular shape diaphragm is the mostly preferred one due to the mechano-acoustic transduction mechanism. Specifically, under the same conditions, including surface area, thickness, applied force, and boundary, circular diaphragms have the largest mean displacement of the fundamental mode as compared to those of other shapes such as triangular, rectangular, pentagon, etc. Some of the prior efforts to increase the bandwidth and piezoelectric transduction efficiency includes a curved diaphragms [20], multiple electrodes [21], multiple active layers [18], the ring shape geometry [15], a resonant coupler [22], and the pinned-boundary [23].

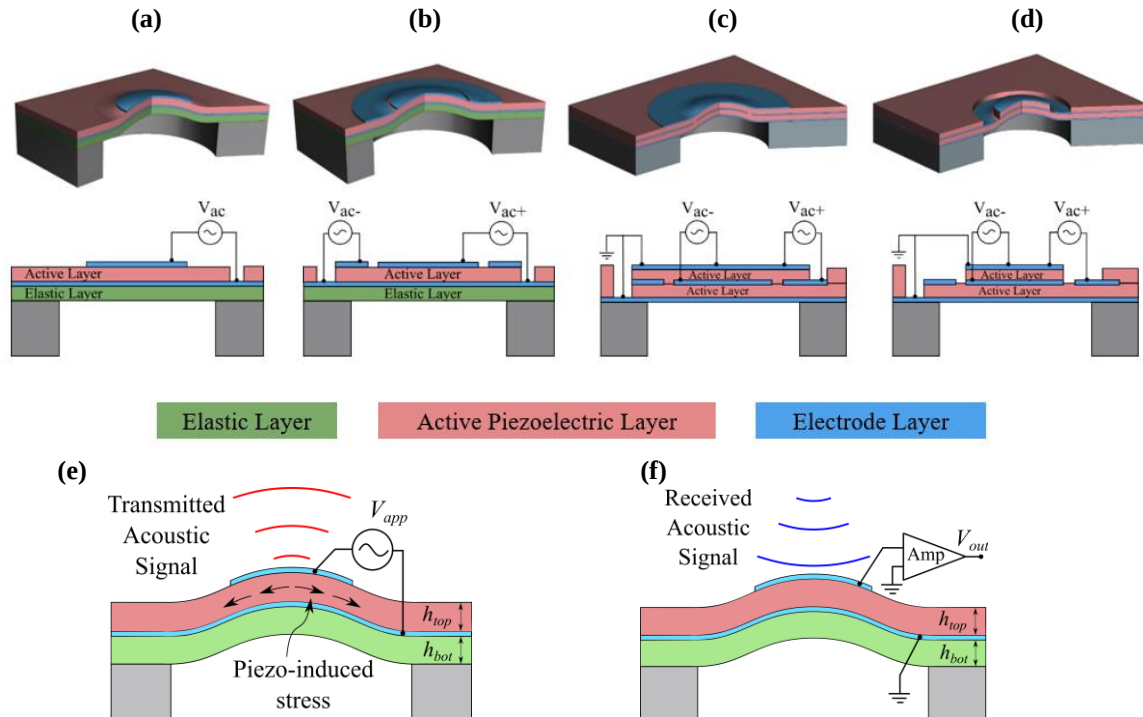


Figure 1.2: 3-D illustrations of different types of pMUT designs and their respective excitation schemes. (a) unimorph, single electrode, clamped boundary, (b) unimorph, double electrode, clamped boundary, (c) bimorph, dual electrode, clamped boundary, and (d) bimorph, dual electrode, pinned-clamped boundary. (e) The transmitter mode also known as the drive-mode, with an AC voltage applied on one of the electrodes while the other one is grounded. (f) The receiver mode also called as the sense-mode with externally applied pressure which bends the diaphragm and creates internal stress in the piezoelectric layer.

The traditional pMUT design is given in Figure 1.2a with a single active layer and electrode pair. With the introduction of the outer electrode having a 180° phase excitation with respect to the inner circular-shape electrode (Figure 1.2b), the acoustic pressure output can have a 2x increase. Next, the inactive elastic layer is replaced with another active piezoelectric layer as seen in Figure 1.2c. With a clever choice of electrode placement, the performance can have a 2x increase. Lastly, instead of using a clamped boundary for both layers, the top layer boundary is released to mimic the pinned boundary condition and enable a higher compliance as shown in Figure 1.2d. The inflection point is also pushed towards to the clamped boundary so that larger region can be used in pinned case. This resulted in another performance increase.

There are two modes of operation in pMUTs: transmitter and receiver. In the transmitter mode (Figure 1.2e) which is also known as the drive-mode, an AC voltage is applied on one of the electrode-pairs while the other electrode is grounded. This potential difference creates the piezo-induced stress in the piezoelectric layer. The neutral axis of the stacked layers is designed to be placed at a distance from the mid surface of the piezoelectric layer. By doing so, the piezo-induced stress creates a bending moment at the neutral axis. Displacement due to the bending creates acoustic signal in the surrounding medium. On the other hand, the acoustic wave hits to the diaphragm surface in the receiver mode (Figure 1.2f) also called the sense-mode. Externally applied pressure bends the diaphragm and creates internal stress in the piezoelectric layer, which generates the electric field. In general, ultrasonic transducers work in the pulse-echo mode which is the combination of transmitter and receiver modes. Time difference between transmitted and received acoustic signals is also called the time of flight (ToF), which is a measure of the distance between the transducer and reflecting surface.

The maximum acoustic pressure is obtained when the diaphragm surface velocity is maximized for the maximum signal-to-noise (SNR) ratio. A system with the low mechanical resistance will have very close frequencies for the maximum velocity and the maximum displacement frequencies and it is the resonance frequency. The mechanical structure explained above is generally described by a single degree of freedom mass-spring-damper system which has a representative frequency response given in Figure 1.3a. If the maximum displacement is considered, high quality factors, Q , are favored. However, the acoustic power output also depends on the radiation impedance of the structure. The radiation impedance has two components: resistive and reactive parts. Power flows through the resistive component, while the reactive component stores the power. Therefore, a system with high resistive and low reactive component is favored in ultrasonic devices. Hence, there is a balance between these two desired features: the high acoustic power and high surface displacement.

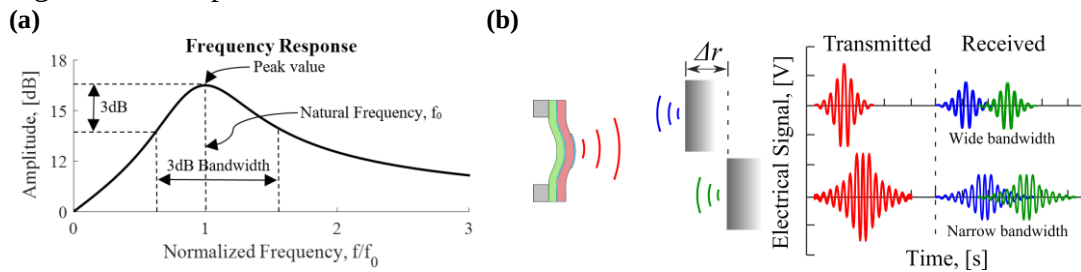


Figure 1.3: Ultrasonic transducer characterizations. (a) Frequency response illustration. (b) Pulse-echo experiments for wide (top) and narrow (bottom) bandwidth transducers. The two echoes from closely placed objects are clearly resolved for wide bandwidth transducer; whereas, they are overlapped for narrow bandwidth transducer.

Another design consideration is the bandwidth which affects the mechano-acoustic transduction efficiency and range resolution. As shown in Figure 1.3b, the pulse generated by a narrow band transducer is broadened with corresponding echoes. The two echoes generated by two objects having distance Δr between them can be overlapped and unresolved in the narrow bandwidth case (Figure 1.3b bottom graph). On the other hand, the same two object can be resolved in the wide band transducer. Devices with bandwidth result in high the losses and some of the loss mechanisms include thermo-elastic damping, anchor loss, and loss into the acoustic medium. Among these, the loss due to acoustic radiation is desired since causes the power to flow from the mechanical domain to acoustical domain. This is also called the mechano-acoustic transduction efficiency. The characteristic mechanical impedance of the transducer should match to the characteristic mechanical impedance of the medium for high efficiency. The bandwidth consideration has a strong correlation with the quality factor, Q . As stated before, high quality factors are desired for large displacement amplitudes at the resonance and high bandwidth is desired for the large power flow. However, bandwidth is inversely proportional to the quality factor and a low quality factor is desired for pMUTs for the high transduction efficiency as it must be based on a large radiation resistance rather than any other mechanical losses.

As a last important design consideration, electromechanical coupling coefficient is widely used for transducer performance measures. It is defined by a real, positive, dimensionless number. Among the many physical definitions of electromechanical coupling coefficient, it is often defined as a change in the stiffness or capacitance as the response to the change in electrical or mechanical boundary conditions. As such, it is possible to measure the electromechanical coupling coefficient with the impedance measurement to be explained in the upcoming sections. It highly depends on material properties, specifically on piezoelectric coefficient and it also depends on the geometry specifically stress-strain distribution in the diaphragm. Previously, Mason has defined the electromechanical coupling coefficient as a portion of the total input electrical energy stored in the mechanical domain form for a static voltage [24]. Nevertheless, the efficiency and electromechanical coupling coefficient should be defined clearly to prevent any misleading conclusions. Efficiency is the ratio of power output to the power input and the electromechanical coupling coefficient is the ratio of converted energy to the stored energy.

1.3 Dissertation Organization

This dissertation presents the design of high performance piezoelectric micromachined ultrasonic transducers (pMUTs) for several applications which are summarized in Figure 1.4. It starts with an intuitive improved lumped element modeling of clamped boundary structures. With the intuitive modelling scheme, a free boundary structure is introduced for the first time with both ultra-high bandwidth and electromechanical coupling coefficient as compared to those of its traditional counter parts such as clamped or free boundaries and other diaphragm designs such as ring or rectangular shapes. Next, ultrasound-induced airborne haptics via pMUTs is introduced to provide a low footprint, low power, and low cost solution for the mobile haptics in augmented reality (AR), virtual reality (VR), and gaming applications. Next, the effect of curvature on the echo amplitude and phase is explained. The effect on the amplitude analysis resulted in an qualitative and quantitative

analyses of attenuation due to reflector curvature and the phase analysis resulted in non-contact radius of curvature measurement method. Lastly, a phase-shifted excitation scheme is proposed to reduce the blind zone regions during the time-of-flight measurement with better axial resolutions.

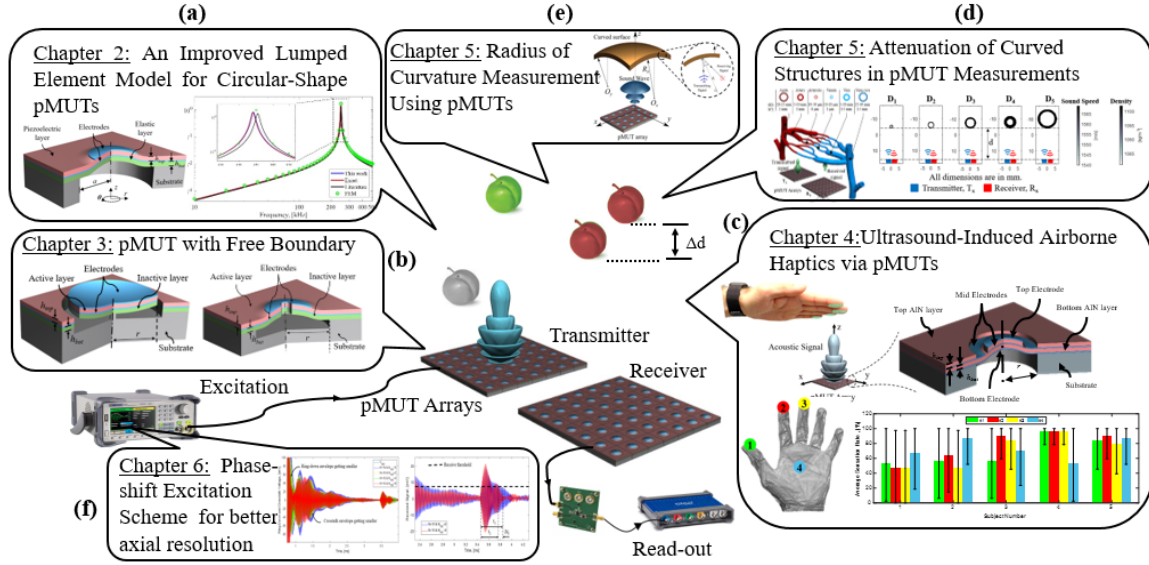


Figure 1.4: Summary of the chapters in this dissertation as shown in boxes. At the center, traditional transmitter and receiver pMUT arrays are shown with reflective objects for at blind zone (grey apple), measureable distances (green apple), and axial resolution (closely placed red apples). (a) Illustration of the improved lumped element model explained in Chapter 2. (b) The inner and outer edge free boundary pMUTs in Chapter 3. (c) Ultrasound-induced airborne haptics illustration explained in Chapter 4. (d) The blood pressure measurement case study with reflector curvature attenuation, (e) a non-contact radius of curvature measurement illustration explained in Chapter 5. (f) Illustration of phase-shifted excitation explained in Chapter 6.

Chapter 2 presented an improved, two-term mode shape model for the circular-shape pMUT (Figure 1.4a). The proposed model has been compared with the literature of the exact solution and finite element method simulations. The errors in all these parameters are below 0.5% with respect to those in the exact solution. In addition, the reductions of error in these parameters are 5x to three orders of magnitude as compared to those in the literature. It is found that the correct model of the deformation shape is critical for the model accuracy in many intrinsic and extrinsic parameters such as individual equivalent circuit parameters, input impedance, frequency, bandwidth, quality factor, directivity, and the on-axis pressure in the near and far fields. The approximate energy-based method can be utilized to obtain accurate mode shapes and parameters for different shapes and types of ultrasonic transducers, such as rectangle and ring geometries, dual electrode, and bimorph structures.

In Chapter 3, a pMUT with the free boundary condition is introduced and compared with those of conventional designs with the clamped boundary. The structure has a diaphragm with the free boundary at the outer edge sitting on a central post (Figure 1.4b). The free edge mode shape has been derived analytically and an assumed mode is proposed to ease the computational complexity in the analyses. An equivalent circuit model is constructed with derived lumped parameters. Increased mechanical capacitance and enhanced strain field distribution result in 16x higher electromechanical coupling coefficient as compared to that of a traditional clamped boundary pMUT. Furthermore, the

acoustic cavity underneath the free boundary structure is utilized as an acoustic resonator to increase the coupling and bandwidth. In a case study, the bandwidth of the free boundary pMUT is more than two orders of magnitude higher than that of a fixed boundary pMUT. The free edge is also helpful to release the internal residual stresses due to the thin film deposition process.

Chapter 4 introduces an ultrasound-induced airborne haptic interface device by using the pMUT array (Figure 1.4c). By emitting the ultrasonic waves at the natural frequency of 109.4 kHz, a local maximum acoustic pressure at 950 Pa is measured 15 mm away from the transducer array. The pulse width modulation scheme is utilized to convert the high frequency signals to a low frequency pattern at 100 Hz with 50% duty cycle in the haptic perception experiments under an applied amplitude of 12 V_p. The radiation force is calculated as 0.4 mN which is reported as the minimum force required to stimulate human skin and it is found that the average sensation delay time of five different volunteer is 2.3 seconds. Two types of demonstration experiments are designed and conducted: switching on/off and Morse code transmission to achieve the average identification accuracy rates of 80% and 76%, respectively. These results imply that the proposed concept has a promising potential to be used as high intensity, low power, and small footprint haptic perception interfaces in various practical fields, such as in human machine interface, medical, entertainment, and AR/VR applications.

In Chapter 5, structures with curved surfaces are studied for two properties: (1) the signal attenuation magnitude due to the curved surface (Figure 1.4d); and (2) the radius-of-curvature measurement by using the time of flight scheme (Figure 1.4e). A pMUT array is 3 mm × 3 mm in size for a total of 39 × 39 elements made of diaphragms with 1 μm-thick AlN piezoelectric layer and 5 μm-thick Si elastic layer. By emitting the ultrasound wave with 12 V_p in amplitude and three sine wave cycles at frequencies of 2-10 MHz, the attenuations on curved surfaces with the radius of curvatures of 1-6 mm have been measured in the mineral oil environment. The echo signals from a surface 25 mm away are captured by a receiver array placed next to the transmitter. For the artificial blood vessels with dimensions similar to arteries and veins, the measured attenuation is 7 dB which is in a good agreement with simulations. These results indicate that the signal to noise ratio has a room of ~10 dB for improvements by selecting appropriate reflecting surface in applications such as blood pressure measurement. In the radius of curvature measurements using a PMUT array, experiments are conducted in air using 3D printed surfaces having RoCs of 20-2000 mm placed 5 cm away the pMUT array. The extracted RoC values have a good agreement with the fabricated samples up to 200 mm in RoC. The resolutions can be further enhanced by optimizing the axial resolution of the transducers. These results imply that the proposed concept can be used to measure surface curvature inside an opaque media where optical systems cannot be used such as beams and columns inside buildings, and implants inside human bodies.

In Chapter 6, a 180° phase-shift excitation scheme is proposed to reduce the ring down period for the operation of ultrasonic transducers with higher axial resolution (Figure 1.4f). The fabricated devices have an 80-element array with circular diaphragms of 415 μm in radius to result in a resonant frequency of 160 kHz. By shrinking the ring down period, the

blind region is shortened by 13% and 7.5% for the single element and 80-element array, respectively. In addition, the axial resolution of the system is increased by 30% and 25% for the single element and the array structure, respectively. These results suggest that the proposed excitation pattern has a promising potential to be used in the pulse echo time-of-flight measurement applications of ultrasonic systems.

Chapter 2

An Improved Lumped Element Model for Circular-Shape pMUTs

In general, performance enhancement studies for pMUTs can be summarized in two approaches: (1) optimal geometric designs such as collapse mode [11] in capacitive MUTs (cMUTs) or curved [14], ring [15], free [16] and pinned [17] boundary, bimorph [18] structures in piezoelectric MUTs (pMUTs); and (2) high piezoelectric effect thin film materials such as PZT [8] and AlN [19]. In parallel to the performance enhancement efforts, an accurate analytical model is needed to gain a better understanding of the physical mechanism for improved structures. As stated in [25], the purpose of computation is to have insights and not just pure numbers. Although constructing a better model takes time, it can be used continuously [1]. Hence, there has been great efforts to construct improved models focusing on transduction mechanisms [10], mechanical structures [14], and complete physics analyses [26]. However, these models are mostly complicated with advanced functions which result in impractical equivalent circuit models.

This chapter starts with the analytical solution and a two-term mode shape approach is followed in three domains: mechanical, electrical, and acoustic. An approximate energy-based method, Rayleigh-Ritz, is utilized to predict more accurate mode shapes and the Lumped elements and equivalent circuit model are presented with comparisons of literature and exact solution for several key parameters.

2.1 pMUT Structure and Design Considerations

The first and most important parameter in ultrasonic transducers is the pressure outputs. As mentioned earlier, there are two main approaches for improvements: material and geometric properties. The later has been investigated deeply such as curved [14], ring [15], rectangular [27], free [16] boundary, pinned [17] boundary, bimorph [18] structures in piezoelectric pMUTs. Most of the work in the literature is in circular geometry due to high pressure output requirement.

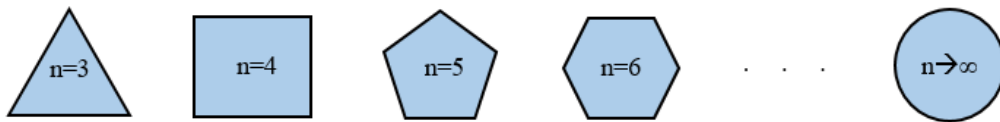


Figure 2.1: The surface geometries of mechanical structures. n is the number of boundary.

In Figure 2.1, the possible surface geometries are shown as the number of edges ranging from three (triangle), to infinity (circle). With the same surface area and other conditions (thickness, applied force, boundary), the circular diaphragm has the largest mean normalized displacement (fundamental mode) over the surface area as compared with those of triangular, rectangular, pentagon, ... etc. diaphragms as shown Figure 2.2a.

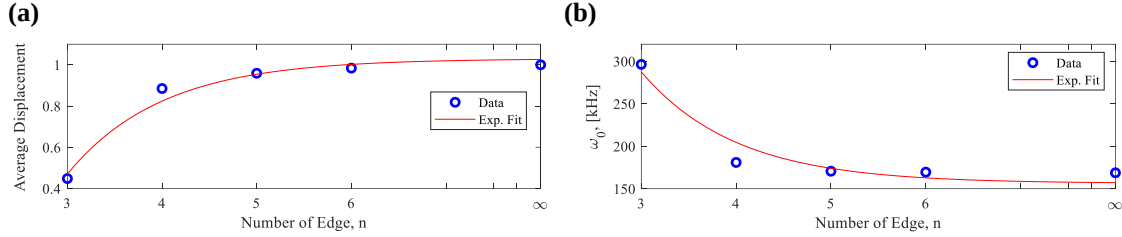


Figure 2.2: (a) The mean normalized displacement; (b) the natural frequency of different geometric shapes with the same surface area, thickness, and boundary conditions.

Since the surface area is constant for each geometric shape, the velocity is obtained by the frequency (shown in Figure 2.2b) times the displacement. In general, the pMUT diaphragm composes of multiple material layers; piezoelectric, elastic, and electrode layers as shown in Figure 2.3a. The piezoelectric layer is sandwiched between two electrodes in order to utilize the 31-mode piezoelectric actuation.

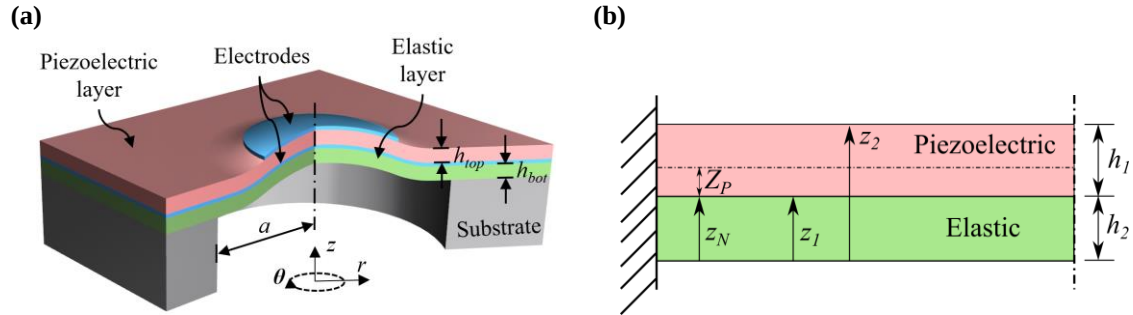


Figure 2.3: (a) 3-D schematics of a clamped, circular-shape, and deformed diaphragm showing the coordinate system used in the mechanical analysis. (b) Cross sectional view of the vibrating diaphragm with geometric parameters in the thickness direction.

Starting with the mechanical domain, it is a stack of disks clamped at the outer edge and free to vibrate in transverse direction. The mechanical excitation is achieved by the moment created by piezo-induced stress inside the piezoelectric layer which is off centered to neutral axis. The electrical domain composes of a clamped capacitance formed by the two electrodes on each side of the piezoelectric layer. An important observation is that, the surface area has partially electroded. The reason is the summation of piezo-induced strains accumulates to zero over the whole surface area. Lastly, the vibrating diaphragm disturbs the medium generating acoustic waves. The medium and the interface between the structure and medium are the acoustic domains.

All in all, there are three energy domains: electrical, mechanical, and acoustical. These domains and coupling between them are analyzed separately in the upcoming sections. An energy based intuitive method is followed in order to obtain each lumped parameters more accurately in equivalent circuit analysis.

2.2 Free Vibrations of Clamped Disk

A plate is defined by two surfaces which have small distances apart as the thickness of the plate. Thin plate theory applies for the structures having the ratio of diameter to thickness of greater than 20. Although it is widely investigated [28-31], the equation of motion (EOM) for transverse vibration of a clamped diaphragm is derived for the sake of completeness in this section. Equilibrium approach is used to derive EOM. In basic terms, the disparity in length scales enables to work in the two dimensions for a three-dimensional structure and displacements of each point can be described by the displacement of the mid-plane also known as neutral plane. In more advanced terms, classical plate theory also known as Kirchooff-Love plate theory assumes that [30]:

1. Thickness of the plate is small compared to its other two dimensions.
2. The neutral plane of the undeformed plate remains neutral plane after the deformation which implies that the neutral plane does not undergo in-plane deformations.
3. The transverse displacement component of the mid-surface is small compared to the thickness of the plate.
4. Plane sections normal to the mid-surface of the undeformed plate remains normal to the mid-surface after the deformation. This implies that transverse shear stresses in the thickness direction are neglected.

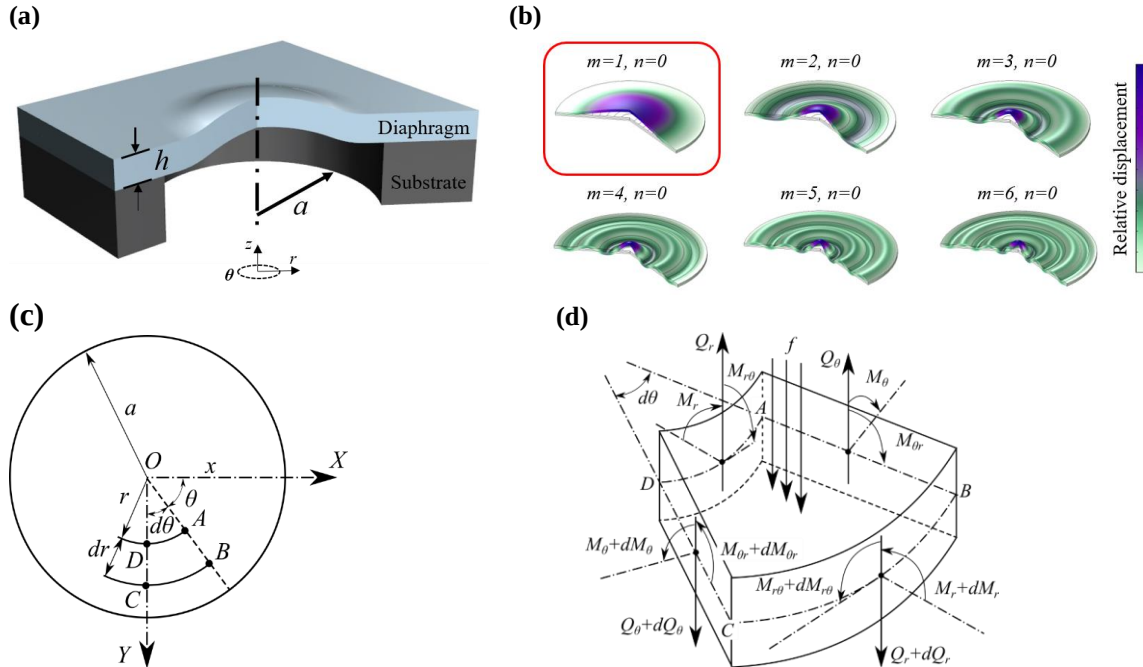


Figure 2.4: (a) 3-D schematics of a single layer with clamped boundary conditions, circular-shape, and deformed diaphragm used in the mechanical analysis. (b) Axisymmetric mode shapes of the circular diaphragm showing the number of the circular nodal lines, m , and radial nodal lines, n . (c) Top view of the circular diaphragm showing the infinitesimal element ABCD. (d) The force, Q , and moment, M , resultants with externally applied force intensity, f on the infinitesimal element ABCD in three dimension indicating the neutral axis at thickness the center.

Figure 2.4a shoes the 3-D schematics of a single layer with clamped boundary conditions, circular-shape, and deformed diaphragm. For the free vibration, a single layer is modelled but later the piezoelectric and elastic stacked layer structures with electrodes

are included. In Figure 2.4b, only the axisymmetric mode shapes are shown. The mode-of interest is the first (fundamental) mode having only one circular nodal line at the clamped edge and zero radial nodal line, i.e., $m=1, n=0$. Since the free body diagram (FBD) of the plate are given as force and moment resultant, one should notice that the given $f(r, \theta, t)$ is the intensity of the externally distributed force.

The transverse normal strain is neglected as it is small compared to other stress terms (stress, σ_{zz} , in z direction shown in Figure 2.4c). In addition, there will be no resulting forces in r and θ directions (shown in Figure 2.4a) and the flexural deformation of the plate is considered as [32]:

$$\int_{-h/2}^{h/2} \sigma_{rr} dz = 0, \quad \int_{-h/2}^{h/2} \sigma_{\theta\theta} dz = 0 \quad (2.1)$$

The stress-strain relationships and coordinate transformations are given in Appendix A. These in-plane stresses create bending moment around mid-surface; however, net forces at the mid-surface due to these stresses are zero. It should be noted that, the normal (σ_{rr} and $\sigma_{\theta\theta}$) and transverse shear stresses (σ_{rz} and $\sigma_{\theta z}$) are assumed to vary linearly and parabolic along the thickness, respectively.

Dynamic equilibrium equations are written to derive equation of motion (EOM) using the infinitesimal incremental approach. Writing force balance equation in the z direction for force and moment resultants (Appendix A) gives the most general form of equation of motion of a transversely vibrating thin plate without any dissipative elements as [30]:

$$D\nabla^4 w(r, \theta, t) + \Lambda \frac{\partial^2 w(r, \theta, t)}{\partial t^2} = f(r, \theta, t) \quad (2.2)$$

where $w(r, \theta, t)$ is the transverse displacement; ∇^4 is the biharmonic operator given in Appendix A, $f(r, \theta, t)$ is the externally applied force, D is the transverse flexural rigidity; and Λ is the surface density given as [30, 31]:

$$D = \frac{Y}{1-\nu^2} \int_{-h/2}^{h/2} z^2 dz = \frac{Yh^3}{12(1-\nu^2)} \quad (2.3)$$

$$\Lambda = \rho h$$

in which ρ is the layer density; h is the layer thickness; Y is the Young's modulus of the layer; and ν is Poisson's ratio of the layer.

2.2.1 Exact solution for Free Vibration of a Circular Plate

The undamped free vibration of the circular plate is obtained by equating the external force, $f(r, \theta, t)$, to zero which is the right side of EOM given in Eq.(2.2):

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \quad (2.4)$$

The transverse displacement can be written in terms of spatial and time components, which are also known as separation of variables:

$$w(r, \theta, t) = W(r, \theta)T(t) \quad (2.5)$$

and the time dependent component is given by a harmonic response:

$$T(t) = e^{j\omega t} \quad (2.6)$$

where j is the imaginary number, ω is the oscillation frequency. Inserting equation (2.5) and (2.6) into (2.4) yields:

$$\nabla^4 W(r, \theta) - k^4 W(r, \theta) = 0 \quad (2.7)$$

where

$$k^4 = \frac{\rho h \omega^2}{D} \quad (2.8)$$

Then the separation of variables is applied one more time for the displacement as multiplication of two functions: radial and angular components. Step by step solution is given in Appendix A. The general form of solution is [30, 31]:

$$W(r, \theta) = [C_m J_m(kr) + D_m Y_m(kr) + E_m I_m(kr) + F_m K_m(kr)] [A_n \cos(n\theta) + B_n \sin(n\theta)] \quad (2.9)$$

where J_n and Y_n are the Bessel's functions of the first and second kind, respectively; I_n and K_n are the modified Bessel's functions of the first and second kind; n is an integer describing angular periodicity of 2π ; and the coefficients $A_n \dots F_m$ are constant coefficients to be determined by the boundary conditions. For clamped circular disk, these boundary conditions are zero displacement and zero slope at the clamped edge with the radius a in this case, that is:

$$W(a, \theta) = 0 \quad (2.10)$$

$$\frac{\partial W(a, \theta)}{\partial r} = 0 \quad (2.11)$$

In addition to these boundary conditions, it is trivial that all the displacements should be finite throughout the plate. The Bessel's functions of second kind are infinite when the argument is zero, which is at the point where $r = 0$. Then the natural modes of vibration for a w_{mn} are given by [30, 31]:

$$w_{mn}^1(r, \theta, t) = \cos(n\theta) [J_m(kr)I_m(ka) - J_m(ka)I_m(kr)] [A_{mn}^1 \cos(\omega_{mn}t) + B_{mn}^1 \sin(\omega_{mn}t)] \quad (2.12)$$

$$w_{mn}^2(r, \theta, t) = \sin(n\theta) [J_m(kr)I_m(ka) - J_m(ka)I_m(kr)] [A_{mn}^2 \cos(\omega_{mn}t) + B_{mn}^2 \sin(\omega_{mn}t)] \quad (2.13)$$

where Lastly, the most general solution is [30, 31]:

$$w(r, \theta, t) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} [w_{mn}^{-1}(r, \theta, t) + w_{mn}^{-2}(r, \theta, t)] \quad (2.14)$$

This solution has been vastly investigated and it is possible to find k values for several combinations of m and n values. Some of them are;

$$\begin{aligned} k_{10}a &= 3.196 & k_{20}a &= 4.611 & k_{30}a &= 5.906 \\ k_{11}a &= 6.306 & k_{21}a &= 7.799 & k_{31}a &= 9.197 \\ k_{12}a &= 9.439 & k_{22}a &= 10.958 & k_{32}a &= 14.402 \end{aligned} \quad (2.15)$$

The exact solution to the axisymmetric modes of the undamped, free vibration of clamped diaphragm is obtained as [28]:

$$W_{0n}(r) = J_0(k_{0n}r)I_0(k_{0n}a) - J_0(k_{0n}a)I_0(k_{0n}r) \quad (2.16)$$

and the fundamental mode shape is obtained with $n=0$ [33]:

$$W_{00}(r) = J_0(k_{00}r)I_0(k_{00}a) - J_0(k_{00}a)I_0(k_{00}r) \quad (2.17)$$

However, from the designer perspective, the general solution given in equation (2.15) is not practical due to the fact that it is difficult to work with Bessel's functions. That is why, another method is used to obtain more compact mode shape representations using simple functions such as polynomials.

2.2.2 Rayleigh-Ritz Solution for Free Vibration of a Circular Plate

The Rayleigh Ritz method is an energy based, approximate method for vibration problems, which usually gives sufficiently enough accurate solutions for the small number of lowest modes. This method is suitable for acoustic applications, since ultrasonic transducers are excited at the fundamental mode. The generated acoustic power is proportional to the mean velocity of the vibrating surface. Hence, the lowest mode which has the highest mean velocity over the surface is generally selected in the mechano acoustic transduction. This method is built upon Rayleigh method, which is stated as that the vibration frequency of a conservative system (no dissipating elements and non-conservative forces) has a stationary value in the neighborhood of a natural mode which is, in fact, a minimum value in the neighborhood of the fundamental natural mode [31].

The shape of the deformation is approximated by trial functions which are selected among the admissible set satisfying geometric boundary conditions. Boundary conditions are classified in two groups: geometric and natural boundary conditions. The geometric boundary conditions are resulted from pure geometric compatibility like displacement and slope. On the other hand, Natural boundary conditions are resulted from force balance like force and moments.

In the Rayleigh – Ritz method, the number of trial functions and assumed mode shapes, are used in the calculation. One can calculate several natural frequencies and mode shapes

at once. In this method, the number of trial functions (p) is equal to the dimensions of the square stiffness and mass matrices. It will be equal to the number of eigenvalues obtained from the eigenvalue problem constructed using these matrices in the eigenvalue problem [34]. As the number of assumed mode shapes increases, accuracy of the mode shapes and natural frequencies starting from lowest to highest increases. The most significant improvement in accuracy is on the first natural frequency. Effect of the improvement decreases from the first to the last mode shape of interest [34].

One should notice that, this method always gives larger values for the natural frequencies due to additional constraints and high stiffness imposed by incorrect assumption of modes. As a rule of thumb, the number of trial functions is selected as the twice the number of interested mode shape [35].

The deflection of the plate is symmetrical about the center, hence the deflection pattern is a function of only radial coordinate, r . In addition to axial symmetry, some of the terms vanishes due to fixed boundary conditions [36]. The potential and kinetic energy expressions are derived in polar coordinates as follow [29, 36, 37]:

$$V = \frac{D}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right)^2 r dr d\theta \quad (2.18)$$

$$T = \frac{\rho h}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial w}{\partial t} \right)^2 r dr d\theta \quad (2.19)$$

The vibration frequency of a conservative system (no dissipating elements and non-conservative forces) has a stationary value in the neighborhood of a natural mode which is, in fact, a minimum value in the neighborhood of the fundamental natural mode [31]. In other words, energy is conserved in the system having no dissipative elements, i.e.,

$$V_{\max} = T_{\max} \quad (2.20)$$

where $V_{\max}$ and $T_{\max}$ are the maximum potential and kinetic energies, respectively. The kinetic energy is a function of the square of the velocity, which can be separated in spatial and time dependent terms as [30]:

$$T_{\max} = \omega^2 T_{\max}^* \quad (2.21)$$

where $T_{\max}^*$ is the maximum kinetic energy solely dependent on spatial coordinates, and ω is the frequency of vibration. Rayleigh's quotient is defined as the ratio of the maximum potential and kinetic energies and given by [36]:

$$R = \omega^2 = \frac{V_{\max}}{T_{\max}^*} \quad (2.22)$$

The assumed mode shape is:

$$W = W_0 \sin(\omega t) \quad (2.23)$$

where W_0 is function of r only, since the deflection pattern in mode-of-interest is symmetric about the center. The assumed mode should satisfy all the boundary conditions which are given in equations (2.10) and (2.11) Then;

$$W_0 = \sum_{i=1}^p c_i \phi_i(r) \quad (2.24)$$

where $c_1, c_2, \dots, c_p$ are unknown constants, also called Ritz constants, $\phi_i(r)$ is the trial function, and p is the number of terms used. The coefficients $c_1, c_2, \dots, c_p$ are selected to minimize the Rayleigh's quotient given in equation (2.22).

$$\frac{\partial R}{\partial c_i} = 0, \quad i = 1, 2, \dots, p \quad (2.25)$$

The maximum energy equations can be discretized by mass and stiffness matrices:

$$V_{\max} = \frac{D}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial^2 W_0}{\partial r^2} + \frac{1}{r} \frac{\partial W_0}{\partial r} \right)^2 r dr d\theta = \frac{1}{2} \sum_{i=1}^p \sum_{j=1}^p c_i c_j k_{ij} = \frac{1}{2} \bar{c}^T [K] \bar{c} \quad (2.26)$$

$$T_{\max}^* = \frac{\rho h}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial W_0}{\partial t} \right)^2 r dr d\theta = \frac{1}{2} \sum_{i=1}^p \sum_{j=1}^p c_i c_j m_{ij} = \frac{1}{2} \bar{c}^T [M] \bar{c} \quad (2.27)$$

In the maximum energy equations given in equations (2.26) and (2.27), $[K]$ and $[M]$ are stiffness and mass matrices, respectively and $\bar{c}$ is constant vector composed of unknown constant coefficients. Rayleigh quotient is given in equation (2.22) and R is obtained as a function of unknown coefficients.

$$R(c_1, c_2, \dots, c_p) = \frac{V_{\max}(c_1, c_2, \dots, c_p)}{T_{\max}(c_1, c_2, \dots, c_p)} \quad (2.28)$$

The minimum value of Rayleigh's quotient can be found by equating the rate of change of R with respect to constant coefficients to zero.

$$\begin{aligned} \frac{\partial R}{\partial c_i} &= \frac{T_{\max}^* \left(\frac{\partial V_{\max}}{\partial c_i} \right) - V_{\max} \left(\frac{\partial T_{\max}^*}{\partial c_i} \right)}{T_{\max}^{*2}} = 0, \quad i = 1, 2, \dots, p \\ \frac{1}{T_{\max}^*} \left[\left(\frac{\partial V_{\max}}{\partial c_i} \right) - \frac{V_{\max}}{T_{\max}^*} \left(\frac{\partial T_{\max}^*}{\partial c_i} \right) \right] &= 0, \quad i = 1, 2, \dots, p \\ \frac{1}{T_{\max}^*} \left(\frac{\partial V_{\max}}{\partial c_i} - \lambda^{(p)} \frac{\partial T_{\max}^*}{\partial c_i} \right) &= 0, \quad i = 1, 2, \dots, p \end{aligned} \quad (2.29)$$

where $\lambda^{(p)}$ is the p^{th} eigenvalue of the eigenvalue problem. Moreover, $\lambda^{(p)}$ is defined as Rayleigh's quotient and it is equal to ω^2 . Equations (2.29) can be written in the matrix form as well, which gives the correlation between stiffness and mass distributions given in equations (2.26) and (2.27).

$$\frac{\partial R}{\partial \bar{c}} = \frac{1}{T_{\max}^*} \left(\frac{\partial V_{\max}}{\partial \bar{c}} - \lambda^{(p)} \frac{\partial T_{\max}^*}{\partial \bar{c}} \right) = \bar{0} \quad (2.30)$$

where

$$\frac{\partial V_{\max}}{\partial \bar{c}} = \bar{c}^T [K] \quad \text{and} \quad \frac{\partial T_{\max}^*}{\partial \bar{c}} = \bar{c}^T [M] \quad (2.31)$$

Then equation (2.29) is written in the matrix form as follow:

$$[[K] - \lambda^{(p)} [M]] \bar{c} = \bar{0} \quad (2.32)$$

which is an eigenvalue problem of order p . Well known nontrivial solution of this eigenvalue problem gives the natural frequencies of the plate.

$$\omega_i = \sqrt{\lambda_i^{(p)}}, \quad i = 1, 2, \dots, p \quad (2.33)$$

After finding natural frequencies of the plate, the vector of Ritz coefficients, $\bar{c}_i$, corresponding to i^{th} natural frequency should be determined by solving the eigenvector part of the eigenvalue problem.

$$[[K] - \lambda_i^{(p)} [M]] \bar{c}^{(i)} = \bar{0} \quad (2.34)$$

Then the mode shape of i^{th} symmetric mode is given as:

$$W_0^{(i)} = \sum_{j=1}^p c_j^{(i)} \phi_j(r) \quad (2.35)$$

Eigenvalues are the Rayleigh quotients for each mode and corresponding eigenvectors are the Ritz coefficient vectors. The two boundary conditions for the clamped disk are the geometric boundary conditions and they are given as zero displacement and zero slope at the clamped edge. Therefore, the trial functions are selected among the set given by:

$$\phi(r) = c_p \left[1 - \left(\frac{r}{a} \right)^2 \right]^{p+1} \quad p = 1, 2, \dots, n \quad (2.36)$$

As stated before, the number of terms is selected to be twice of the number of modes-of-interest. In this work, fundamental mode which is the lowest mode is interested. So, $p = 2$ is selected. Then the assumed approximate mode shape is:

$$W_0 = c_1 \left[1 - \left(\frac{r}{a} \right)^2 \right]^2 + c_2 \left[1 - \left(\frac{r}{a} \right)^2 \right]^3 \quad (2.37)$$

where a is the radius of circular diaphragm; c_1 and c_2 are Ritz coefficients. Following the explained procedure with assumed mode shape given in equation (2.29) to construct the eigenvalue problem, the problem is solved (detailed derivation is given in Appendix B) with results:

$$\lambda = \begin{bmatrix} 104.3877 \\ 1854.01 \end{bmatrix} \quad (2.38)$$

and the natural frequencies of first two axisymmetric mode shapes is:

$$\omega_1 = \frac{10.217}{a^2} \sqrt{\frac{D}{\rho h}} \quad \text{and} \quad \omega_2 = \frac{43.0582}{a^2} \sqrt{\frac{D}{\rho h}} \quad (2.39)$$

and the corresponding Ritz coefficients are:

$$c^{(1)} = \begin{bmatrix} 3.067 \\ 1 \end{bmatrix} \quad \text{and} \quad c^{(2)} = \begin{bmatrix} -0.8384 \\ 1 \end{bmatrix} \quad (2.40)$$

Then the assumed mode shapes for the first two axisymmetric vibration modes are:

$$W_0^{(1)} = 3.067 \left[1 - \left(\frac{r}{a} \right)^2 \right]^2 + \left[1 - \left(\frac{r}{a} \right)^2 \right]^3 \quad (2.41)$$

$$W_0^{(2)} = -0.8384 \left[1 - \left(\frac{r}{a} \right)^2 \right]^2 + \left[1 - \left(\frac{r}{a} \right)^2 \right]^3 \quad (2.42)$$

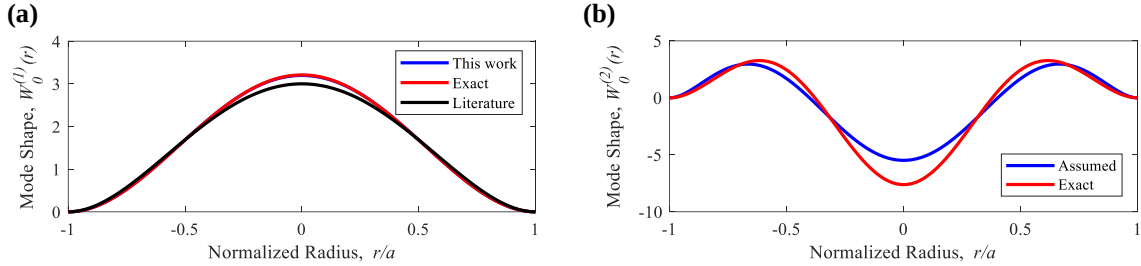


Figure 2.5: Mode shape comparison obtained by this work (equation (2.41)) and exact solution (equation (2.17)) for the first mode (a) and second mode (b).

The first two mode shapes of the axisymmetric transverse vibration of the circular diaphragm are given in Figure 2.5a and Figure 2.5b. Although both solutions give sufficiently enough accuracy, working with basic functions such as polynomials is preferred rather than advance functions such as Bessel functions.

2.3 Derivation of Lumped Element Parameters

Analyzing a system in three dimensional domains is complicated and computationally demanding. Hence, engineers have developed methods to simplify these analyses using several methods including lumped parameters. In the case of piezoelectric ultrasonic transducers, the energy is transferred from electrical to mechanical and mechanical to acoustical domain in one dimension. As stated previously, several assumptions are done in order to obtain mode shapes of a circular plate, which boils down to analyze energy transfer only in a single dimension. In addition to advantages in analysis, the lumped models help to understand the design parameters.

The procedure explained in [38, 39] is followed in order to derive Hamilton's Principle equations. Energy methods are utilized as in the case of Rayleigh – Ritz method to derive the lumped parameters in equivalent circuit. Hamilton's principle is stated as [37]:

$$\delta \int_{t_0}^{t_1} L dt + \int_{t_0}^{t_1} \delta W dt = 0 \quad (2.43)$$

where L is the Lagrangian containing energy terms for conservative forces and δW is the variational virtual work done by non-conservative forces. For the case of piezoelectric transduction, Lagrangian is defined as kinetic energy minus the electrical enthalpy. The electrical enthalpy which is function of strain and electrical fields is defined as [39]:

$$H(S, E) = \int_V \left(\frac{1}{2} S^T c^E S - S^T e E - \frac{1}{2} E^T \epsilon^S E \right) dV \quad (2.44)$$

where superscript T stands for transpose, superscript E stands for constant electric field, superscript S stands for constant strain field, and V is the volume. The electrical enthalpy defined as an integral over the volume. One should note that, the kinetic energy is the function of velocity and the electrical enthalpy is the function of displacement and applied voltage, implicitly. Then the variations of these terms are given as:

$$\begin{aligned} \delta T &= \frac{\partial T}{\partial \dot{w}} \delta \dot{w} \\ \delta H &= \frac{\partial H}{\partial w} \delta w + \frac{\partial H}{\partial V_e} \delta V_e \end{aligned} \quad (2.45)$$

Work done on the structure composes of two terms due to external force and external voltage. The electrical work done by the charge Q in an electrical potential has an opposite sign to the work done by an external force. That is;

$$\delta W = F \delta w - Q \delta V_e \quad (2.46)$$

Inserting Lagrangian and virtual work done given in equations (2.45) and (2.46), respectively into the Hamilton principle in equation (2.43) yields:

$$\int_{t_0}^{t_1} \left\{ \left[F - \frac{\partial H}{\partial w} - \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{w}} \right) \right] \delta w - \left(Q + \frac{\partial H}{\partial V_e} \right) \delta V_e \right\} dt = 0 \quad (2.47)$$

Since the variations are arbitrary, these two terms should be zero desperately in order to satisfy the equation for any variation. Then the governing equations are:

$$F - \frac{\partial H}{\partial w} - \frac{d}{dt} \left(\frac{\partial T}{\partial \dot{w}} \right) = 0 \quad (2.48)$$

$$Q + \frac{\partial H}{\partial V_e} = 0 \quad (2.49)$$

Since the governing equations contain mechanical and electrical domains, and the coupling between them, the analysis is also separated in these two domains. Later, the acoustic domain is also added for complete analysis of pMUT physics.

2.3.1 Mechanical Domain

The lumped elements in mechanical domains are mass, spring, and damping, which are obtained using the mode shape derived in previous section. The obtained mode shape is normalized in order to have unity at the maximum displacement that is the center point in the circular diaphragm. The exact and approximate mode shapes are given in earlier sections and the mode shape used in the literature is [38, 40, 41]:

$$W_0^{(1)} = 3 \left[1 - \left(\frac{r}{a} \right)^2 \right]^2 \quad (2.50)$$

Table 2.1 compares the natural frequency and normalized mode shape calculations for the fundamental mode obtained from the exact solution, the literature, and this work. In the natural frequency calculations, error made in literature with respect to the exact solution is 1.14%; however, this error is dropped to 0.03% in this work, which corresponds to the enhancement of more than one order of magnitude. While both results are good enough for engineering purposes, this work provides better approximations.

Table 2.1: Comparison of natural frequency and mode shape calculations for the fundamental mode of a clamped circular plate obtained from exact solution, literature, and this work

Solution	Frequency, ω_0 , [rad/s]	Mode Shape, $W_0(r/a)$
Exact [26]	$\frac{10.21}{a^2} \sqrt{\frac{D}{\Lambda}}$	$J_0 \left(3.196 \left(\frac{r}{a} \right) \right) - \frac{J_0(3.196)}{I_0(3.196)} I_0 \left(3.196 \left(\frac{r}{a} \right) \right)$
Literature [41]	$\frac{10.33}{a^2} \sqrt{\frac{D}{\Lambda}}$	$\left[1 - \left(\frac{r}{a} \right)^2 \right]^2$
This Work	$\frac{10.22}{a^2} \sqrt{\frac{D}{\Lambda}}$	$0.754 \left[1 - \left(\frac{r}{a} \right)^2 \right]^2 + 0.246 \left[1 - \left(\frac{r}{a} \right)^2 \right]^3$

* The parameters in the equations are explained in the text.

Normalized mode shapes obtained from the exact solution, literature and this work are given in Figure 2.6a for comparison. Errors made in the literature and this work with respect to the exact solution are given in Figure 2.6b. Although inner electrodes are generally used for the pMUT cell, the maximum errors are indicated for both inner and outer electrode regions. The errors in this work are decreased more than one order of magnitude as compared to those of the literature.

The lumped parameters can be expressed with respect to any point over the displaced surface. For different points, each lumped parameter will be different however, their relative values will not change. In other words, the ratio of the parameters does not change, even though parameters themselves change. The lumped elements are derived at the maximum displacement point (center of the diaphragm, $r=0$). Starting with the kinetic energy given in equation (2.19):

$$T = \int_V \frac{1}{2} \rho \left(\frac{dw}{dt} \right)^2 dV = \frac{1}{2} L_m \left[\left(\frac{dw}{dt} \right) \Big|_{r=0} \right] \quad (2.51)$$

$$m_m = \frac{\rho h \int W_0^2 dA}{\left(W_0|_{r=0}\right)^2} \quad (2.52)$$

where A is the area and W_0 is the assumed mode. Then, it can be simplified as:

$$m_m = 0.184 \rho \pi a^2 h \quad (2.53)$$

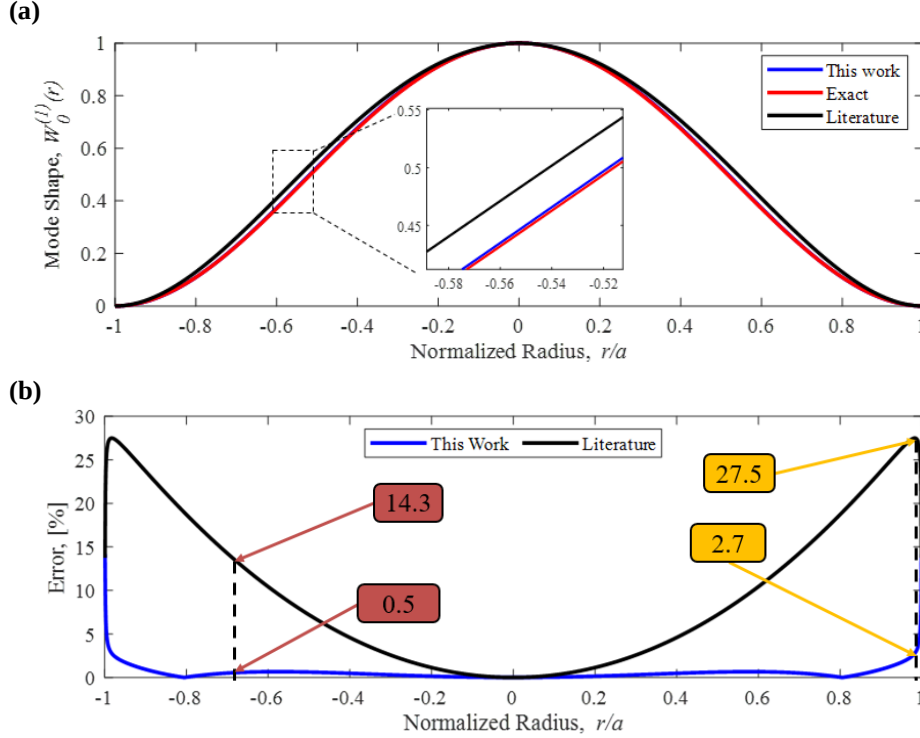


Figure 2.6: Fundamental mode shape and errors made in approximate calculations versus normalized diaphragm radius. (a) The normalized mode shapes for fundamental mode obtained from the exact solution, literature, and this work. (b) Comparison of errors in the mode shape calculation for both the literature and this work with respect to the exact solution. The maximum error percentage at the inner (red color) and outer (orange color) electrode regions are indicated.

For the mechanical capacitance that is the stiffness, the electrical enthalpy terms which is given in equation (2.44) is computed with all components. Note that thin plate theory assumes that all the shear stresses are zero and plane stress conditions. The material is assumed to be transversely isotropic since most of the piezoelectric materials has an in plane isotropy. The electrical field is applied only in the transverse direction and all other electric fields are assumed to be zero which makes $E^T = [0 \ 0 \ E_3]$ as a column vector. Then the reduced form of electrical enthalpy is written as [39, 42-44]:

$$H(S, E) = \int_V \left\{ \frac{1}{2} [S_{rr} \ S_{\theta\theta} \ S_{zz}] \begin{bmatrix} c_{11}^E & c_{12}^E & c_{13}^E \\ & c_{11}^E & c_{13}^E \\ & & c_{33}^E \end{bmatrix} \begin{bmatrix} S_{rr} \\ S_{\theta\theta} \\ S_{zz} \end{bmatrix} - [S_{rr} \ S_{\theta\theta} \ S_{zz}] \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ & e_{11} & e_{13} \\ & & e_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix} - \frac{1}{2} [0 \ 0 \ E_3] \begin{bmatrix} \epsilon_{11}^S & \epsilon_{12}^S & \epsilon_{13}^S \\ & \epsilon_{11}^S & \epsilon_{13}^S \\ & & \epsilon_{33}^S \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ E_3 \end{bmatrix} \right\} dV \quad (2.54)$$

This complex matrix multiplication can be simplified with the structural specific assumptions. First assumption is the plane stress assumption in the diaphragm.

$$S_{zz} = \frac{T_{zz}}{c_{33}^E} - \frac{c_{13}^E}{c_{33}^E} (S_{rr} + S_{\theta\theta}) + d_{33} E_3 = \frac{e_{33}}{c_{33}^E} E_3 - \frac{c_{13}^E}{c_{33}^E} (S_{rr} + S_{\theta\theta}) \quad (2.55)$$

and the stiffness constant for a transversely isotropic material can be expressed by Young's Modulus, Y , and Poisson's ratio, ν , as follow [45]:

$$c_{11}^E = \frac{Y(1-\nu)}{(1-2\nu)(1+\nu)}, \quad c_{12}^E = -\frac{Y\nu}{(1-2\nu)(1+\nu)}, \quad c_{13}^E = \frac{Y\nu}{(1-2\nu)(1+\nu)} \quad (2.56)$$

where these $c_{11}^E, c_{12}^E, c_{13}^E$ are elements of compliance matrix. Last assumption is the effective piezoelectric constant:

$$e_{31,f} = e_{31} - \frac{c_{13}^E}{c_{33}^E} e_{33} \quad (2.57)$$

Then the electrical enthalpy is calculated as:

$$H(S, E) = \int_V \left\{ \left[\frac{1}{2} \frac{Y}{1-\nu^2} (S_{rr}^2 + 2\nu S_{rr} S_{\theta\theta} + S_{\theta\theta}^2) \right] - [e_{31,eff} (S_{rr} + S_{\theta\theta}) E_3] - \left[\frac{1}{2} (\epsilon_{33}^s - 2d_{31} e_{31,eff}) E_3^2 \right] \right\} dV \quad (2.58)$$

Each term in equation (2.58) corresponds to energy domains and the coupling between them: mechanical and electrical. The first term is solely dependent on the strains which will be used to derive mechanical capacitance or stiffness parameter. The last term is dependent only on electrical potential which will be used to derive electrical capacitance or clamped capacitance. Lastly, the term in between is dependent on both strain and electrical potential which will be used to derive the turns ratio of the transformer. This term defines the coupling between mechanical and electrical domains. A compact way to derive these terms is given at [46].

For the derivation of mechanical capacitance, it is better to remember that the major mechanism for the deformation and so energy storage is bending and the membrane stretching action is assumed to be negligible. The first term given in equation (2.58) is equivalent to the potential energy given in equation (2.18) for the clamped circular disks [36]. Hence, strain energy can be calculated using the strain – displacement relations given in Appendix A as:

$$H_{strain} = \frac{1}{2} \int_V \frac{Y}{1-\nu^2} z^2 \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right)^2 dV = \frac{1}{2} k_m (w|_{r=0})^2 \quad (2.59)$$

Then the mechanical capacitance is expressed as:

$$k_m = C_m^{-1} = \frac{D \int_A \left(\frac{\partial^2 W_0}{\partial r^2} + \frac{1}{r} \frac{\partial W_0}{\partial r} \right)^2 dA}{(W_0|_{r=0})^2} \quad (2.60)$$

where again A is the area and W_0 is the assumed mode. Then for the assumed mode given in equation (2.41), the mechanical capacitance is calculated as:

$$k_m = 19.227\pi \frac{D}{a^2} \quad (2.61)$$

For a quick check, the natural frequency of the mode-of-interest is found using the equivalent parameters as:

$$\omega = \sqrt{\frac{k_m}{m_m}} = \sqrt{\frac{1}{L_m C_m}} = \frac{10.22}{a^2} \sqrt{\frac{D}{\rho h}} \quad (2.62)$$

which is found as very close to one found with using Rayleigh-Ritz method. The error is less than 0.1% which is due to the rounding of numbers. If one uses a higher number of significant figures, results will be identically the same.

Mechanical damping is assumed to be small, and it is dominated by the real part of acoustic radiation impedance ($\text{Re}(Z_a) \gg b_m$) [47]. Therefore, the mechanical damping is neglected in the analysis.

2.3.2 Electrical Domain

The last term in equation (2.58) is purely electrical and it is used to derive lumped parameter for electrical capacitance. At this point one should notice that, this capacitance occurs only between three electrodes; hence, electrode area must be used in the derivation and the fringing effects are neglected:

$$H_{\text{electrical}} = \int_V \frac{1}{2} (\epsilon_{33}^s - 2d_{31}e_{31,f}) E_3^2 dV = \frac{1}{2} C_e V_e^2 \quad (2.63)$$

where C_e is the electrical capacitance and V_e is the potential difference between electrodes. The equivalent electrical capacitance is calculated as:

$$C_e = (\epsilon_{33}^s - 2d_{31}e_{31,f}) h_p A_e E_3^2 \quad (2.64)$$

where A_e is the area underneath the electrodes; E_3 is the electrical field in the transverse direction; and h_p is the piezoelectric layer thickness. The free electrical capacitance, C_f is:

$$C_f = \frac{\epsilon_{33}^s A_e}{h_p} \quad (2.65)$$

then:

$$C_e = (1 - k_{31}^2) C_f \quad (2.66)$$

where k_{31}^2 is the coupling coefficient, which is measure of how effectively electrical energy is converted to mechanical energy. It is a material specific value and a measure of 31-mode piezoelectric actuation.

$$k_{31}^2 = \frac{2d_{31}e_{31,f}}{\epsilon_{33}^T} \quad (2.67)$$

The last component of equivalent circuit is the coupling between mechanical and electrical domains. In the circuit representation, it is described by the transformer having a turns ratio of η .

At this point, the electric field outside the electrodes is assumed to be zero, as the fringing effects are neglected. The term at the middle in (2.58) describes the coupling between displacement and electrical field. The strain – displacement relations yields:

$$\begin{aligned} H_{coupling} &= -\int_V e_{31,f} z \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) E_3 dV \\ &= -e_{31,f} E_3 \int_{z_p-h_p-z_N}^{z_p-z_N} z dz \int_{A_e} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) dA_e \end{aligned} \quad (2.68)$$

where A_e is electrode area; z_p is the distance from the bottom surface of the layer stack to the top surface of the piezoelectric layer; z_N is the distance from bottom of the layer stack to the neutral axis; and h_p is the thickness of the piezoelectric layer. It is noted that z_p defines the distance from neutral axis of whole layer stack to the midplane of the piezoelectric layer, in other words, it is the moment arm.

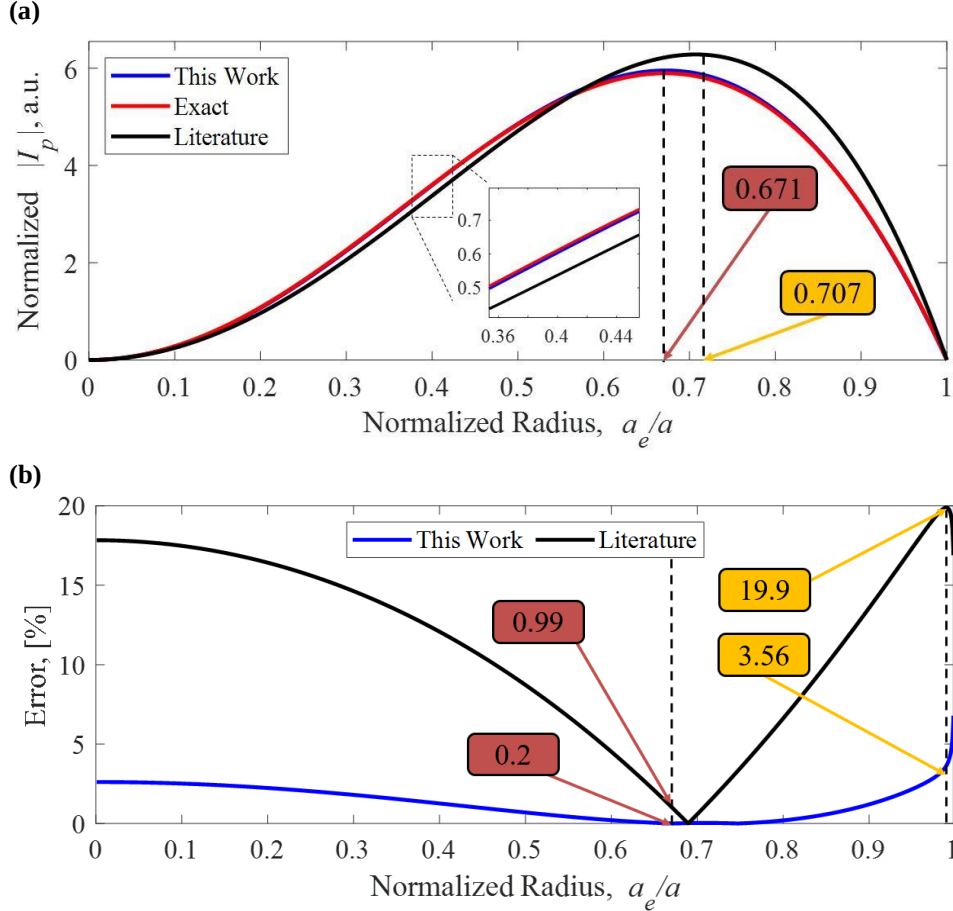


Figure 2.7: (a) Normalized sum of the strains, I_p with respect to the ratio of the inner electrode radius to the diaphragm radius for the exact solution, literature and this work. (b) Comparison of errors in the literature and this work with respect to the exact solution. The maximum error percentage at the inner electrode and outer electrode regions are indicated in red color and orange color boxes.

$$Z_p = \frac{1}{h_p} \int_{z_p - h_p - z_N}^{z_p - z_N} z dz \quad (2.69)$$

$$I_p = - \int_{A_e} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) dA_e \quad (2.70)$$

where I_p is the sum of the strains. Note that sum of the strains is limited with the electrode area since the coupling only happens in this region. It correlates how well coupled a mode shape to the electrode layout for the assumed mode given in equation (2.41) and a circular electrode having same center with the diaphragm and radius a_e .

In Figure 2.7a, I_p values from the literature and this work are given as a function of the normalized radius that is the ratio of inner electrode radius to the diaphragm radius. In general, ~60% radius ratio is utilized in literature [18, 41]. However, the true maximum coupling is achieved when the ratio is 67.1%, i.e., the inner electrode radius equals ~2/3 of the diaphragm radius. Using the mode shape used in literature, the ratio is found as 70.7%. Errors made in the literature and this work with respect to exact solution are given in Figure 2.7b. The error made in this work is about 5x smaller than the error made in the literature.

Lastly in electrical domain, electrical and mechanical domains are coupled with an ideal transformer having the turns ratio obtained from the coupling enthalpy as:

$$\begin{aligned} H_{coupling} &= \eta V_e \left(w \Big|_{r=0} \right) = -e_{31,f} V_e Z_p I_p \\ \eta &= -e_{31,f} Z_p I_p \end{aligned} \quad (2.71)$$

2.3.3 Acoustic Domain

In the acoustic domain, a vibrating diaphragm disturbs the medium and creates an acoustic field which can be described by the acoustic pressure. In this analysis, the vibrating diaphragm is placed in a medium that is considered to be infinitely large such that the boundaries need not to be considered and it is assumed to be homogenous, isotropic, and non-viscous. This acoustic field can be described by pressure.

The generated pressure field is given by the Rayleigh's integral [1, 48-51]:

$$p(r', \theta', z) = \frac{i \rho_m c_m k}{2\pi} \iint_A \left(\frac{\partial w(r, \theta, t)}{\partial t} \right) \frac{e^{-ikR}}{R} dA \quad (2.72)$$

where ρ_m is the density of the medium; c_m is the acoustic velocity in the medium; k is the wave number given as ω/c_m ; and R is the distance vector between the point N in the medium and the differential element on the vibrating surface as shown in Figure 2.8a. The acoustic field created by the vibrating surface is divided into two regions: the near and far field. In the near field, energy transfer between the vibrating mass and the surrounding medium takes place in both directions. Hence, the acoustic medium does not travel away from the surface.

Although this near field does not affect the far field solution, it plays an important role in the mechano-acoustic efficiency of the transducer. The term of wave propagation is defined in the far field away from the source. The transferred acoustic power is proportional

to the radiation resistance of the medium. Therefore, the impedance at the surface which is also called the radiation impedance is important and it is given by the pressure at the surface divided by the surface velocity. The pressure at the surface is given by Rayleigh's integral; however, this time the point of interest, N , is on the surface (Figure 2.8b).

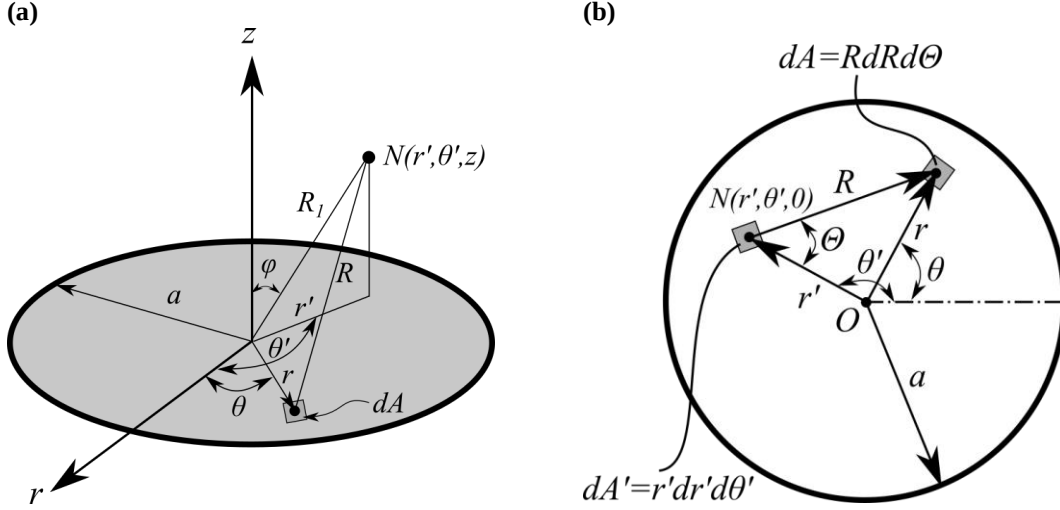


Figure 2.8: Coordinate system used in the acoustic domain. (a) Point $N(r', \theta', z)$ is defined in the acoustic medium as a point of interest and pressure at point N generated by an infinitesimal area on the surface. (b) Coordinate system used in the calculation of the radiation impedance describing the pressure on an infinitesimal area on the surface generated by another infinitesimal area on the same surface ($z=0$).

The velocity profile is obtained using the displacement profile derived in the mechanical domain analysis, and the differential elements are shown in Figure 2.8b. The transformation relation between the coordinates are given as [48]:

$$\begin{aligned} r &= \sqrt{r'^2 + R^2 - 2r'R\cos(\Theta)} \\ R_{\max} &= r'\cos(\Theta) + \sqrt{a^2 - r'^2 \sin^2(\Theta)} \end{aligned} \quad (2.73)$$

where $R_{\max}$ is the maximum value for a given pair of (r', Θ) . The limits of the Rayleigh's integral in the R coordinate is from 0 to $R_{\max}$. The acoustic impedance is given as the ratio of the force on a point divided by the velocity of the surface [48, 50]:

$$z_a = \frac{F_a}{v_a} = \frac{\iint_A p(r, 0) dA}{v_a} = \frac{\rho_m c_m}{\pi a^2} \left(1 - \frac{J_1(2ka)}{ka} + j \frac{K_1(2ka)}{ka} \right) \quad (2.74)$$

where J_1 and K_1 are the Bessel and Struve functions of first order. Similar to mechanical domain, these advance functions are difficult to work and non-intuitive. The acoustic radiation impedance is simplified as:

$$z_a = R_a + jX_a = \frac{\rho_m c_m}{\pi a^2} (r_a + jx_a) \quad (2.75)$$

where R_a is the resistive and X_a is the reactive term. Power in the mechanical domain is transferred to the acoustical domain by propagating through the resistive term. In addition, r_a and x_a are dimensionless resistive and reactive terms, respectively, and they are more convenient to use for the comparison purposes.

The wave propagation for the piston vibrator is in the z direction as shown in Figure 2.9a that is away from the vibrating surface. In the literature, the piston vibrator approximation shown in Figure 2.9a is mostly utilized in which the velocity of the vibrating surface is only in the z direction and its amplitude is uniform. On the other hand, the surface velocity in the mode shape approach as shown in Figure 2.9b of the resulting force on the adjacent medium particles has both z and r direction (shown in Figure 2.9c). The force in the z direction is desired which creates the wave propagating in the medium as power flow into the medium. However, the force in r direction is not desired because it moves the particle in the horizontal direction back and forth, which is also defined as the power storage.

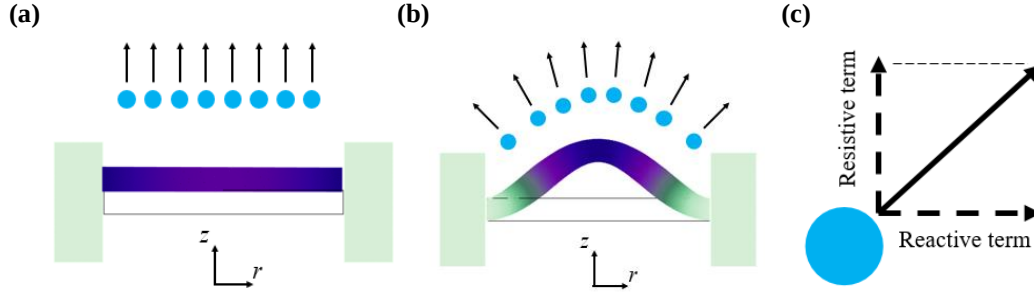


Figure 2.9: Velocity distribution over the plate thickness and applied force on the medium particles shown as blue dots in (a) piston vibrator, (b) mode shape approximation. (c) Zoomed-in image for a single particle in mode shape approximation showing the resistive and reactive components of the resultant forcing.

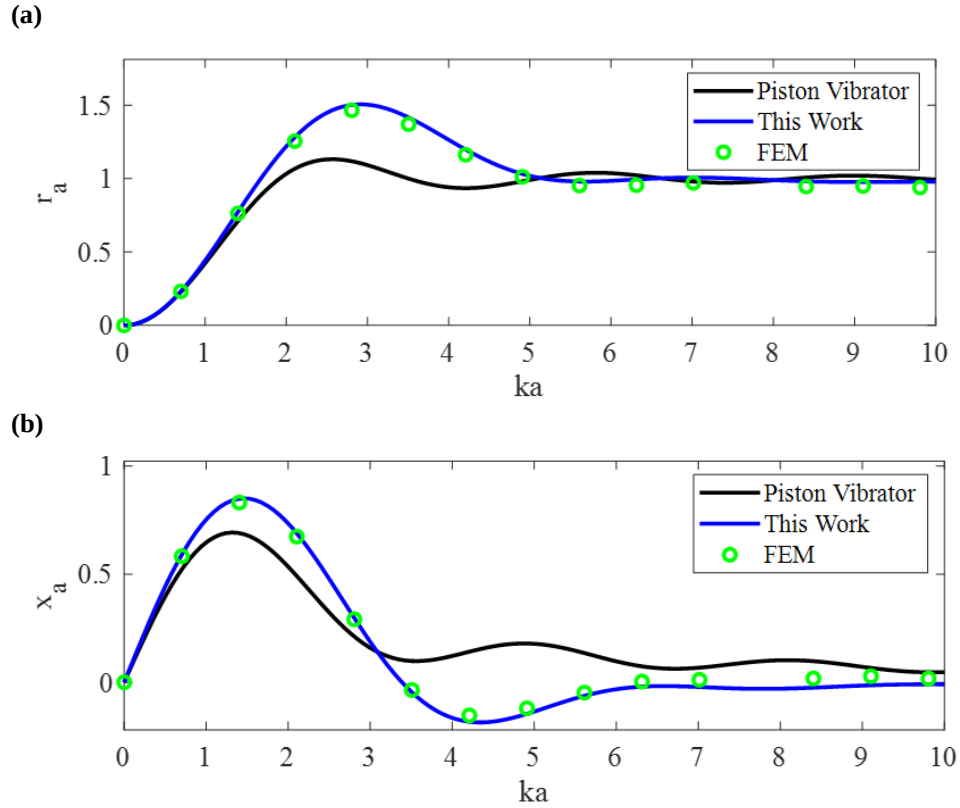


Figure 2.10: Comparison of the dimensionless resistance, r_a , and reactance, x_a , for a piston vibrator, this work, and FEM simulations as a function of wave number multiplied by the radius of the circular diaphragm (ka).

Figure 2.10a and Figure 2.10b show the comparison of the dimensionless resistance, r_a , and reactance, x_a , from the literature (a piston vibrator), this work, and Finite Element Simulation (FEM). It is found that there is an approximate frequency after which, both resistance and reactance values are close in all three solutions [52]. In general, ultrasonic transducers work in the low $ka (<< 1)$ values where k is the wave number and a is the radius in which all solutions converge. In this region, the piston vibrator and mode shape approaches behave similarly and the wavelength is large enough to assume that the plate behaves as a point source and the dimensionless resistance, r_a , can be simplified as [26, 52]:

$$r_a \approx \frac{(ka)^2}{2} \quad (2.76)$$

and the dimensionless reactance, x_a , can be simplified as [26, 52]:

$$x_a \approx ka \quad (2.77)$$

Table 2.2 gives the detailed numerical values for small ka for the dimensionless resistive and reactance components of the radiation acoustic impedance in the piston vibrator, exact solution, this work and FEM results.

Table 2.2: Comparison values of normalized resistive and reactive radiation impedance components for piston vibrator, this work, exact solution and finite element simulation results.

$\mathbf{r_a}$					$\mathbf{x_a}$			
ka	Piston Vibrator	This Work	Exact Solution [26]	FEM Results	Piston Vibrator	This Work	Exact Solution [26]	FEM Results
0.1	0.0056	0.0056	0.0050	0.0040	0.0846	0.0930	0.0933	0.0924
0.2	0.0204	0.0204	0.0199	0.0196	0.1678	0.1849	0.1855	0.1837
0.3	0.0446	0.0447	0.0445	0.0443	0.2485	0.2746	0.2753	0.2723
0.4	0.0785	0.0790	0.0785	0.0780	0.3250	0.3606	0.3617	0.3576
0.5	0.1202	0.1214	0.1213	0.1209	0.3967	0.4424	0.4435	0.4386
0.6	0.1698	0.1725	0.1723	0.1723	0.4620	0.5184	0.5200	0.5138
0.7	0.2262	0.2310	0.2308	0.2307	0.5203	0.5881	0.5900	0.5817
0.8	0.2877	0.2956	0.2960	0.2946	0.5713	0.6512	0.6529	0.6424
0.9	0.3541	0.3666	0.3668	0.3634	0.6129	0.7056	0.7080	0.6964
1.0	0.4234	0.4419	0.4424	0.4379	0.6463	0.7522	0.7547	0.7434
1.5	0.7737	0.8514	0.8532	0.8397	0.6796	0.8495	0.8530	0.8339
2.0	1.0326	1.2198	1.2233	1.1960	0.5347	0.7342	0.7281	0.7144
2.5	1.1307	1.4487	1.4538	1.4118	0.3233	0.4782	0.4818	0.4616
3.0	1.0922	1.5041	1.5106	1.4589	0.1596	0.1878	0.1907	0.1826
3.5	1.0013	1.4187	1.4262	1.3739	0.0990	-0.0427	-0.0405	-0.0326
4.0	0.9414	1.2646	1.2758	1.2280	0.1221	-0.1640	-0.1628	-0.1426
4.5	0.9456	1.1163	1.1246	1.0901	0.1663	-0.1811	-0.1805	-0.1562
5.0	0.9913	1.0195	1.0280	0.9990	0.1783	-0.1348	-0.1341	-0.1132

2.4 Equivalent Circuit Model

Equivalent circuit models are powerful tools when working in multi-physics as in the case of piezoelectric ultrasound transducers [24]. As explained before, there are three energy domains and couplings in the transduction path: electrical, mechanical, and acoustical. Instead of analyzing the structures in three dimensional manners using partial differential equations which can be computationally demanding and physically non-intuitive, they can be analyzed in simplified representations as an electrical equivalent circuit. In addition to being computational friendly, electrical circuit models enable designer to see the energy and power flow not only in the electrical domain but also in all physical domains [53]. In the equivalent circuit analysis, lumped parameters are used as circuit elements in each domain. Most general electro-mechano-acoustic equivalent circuit is given in Figure 2.11a for ultrasonic transducers.

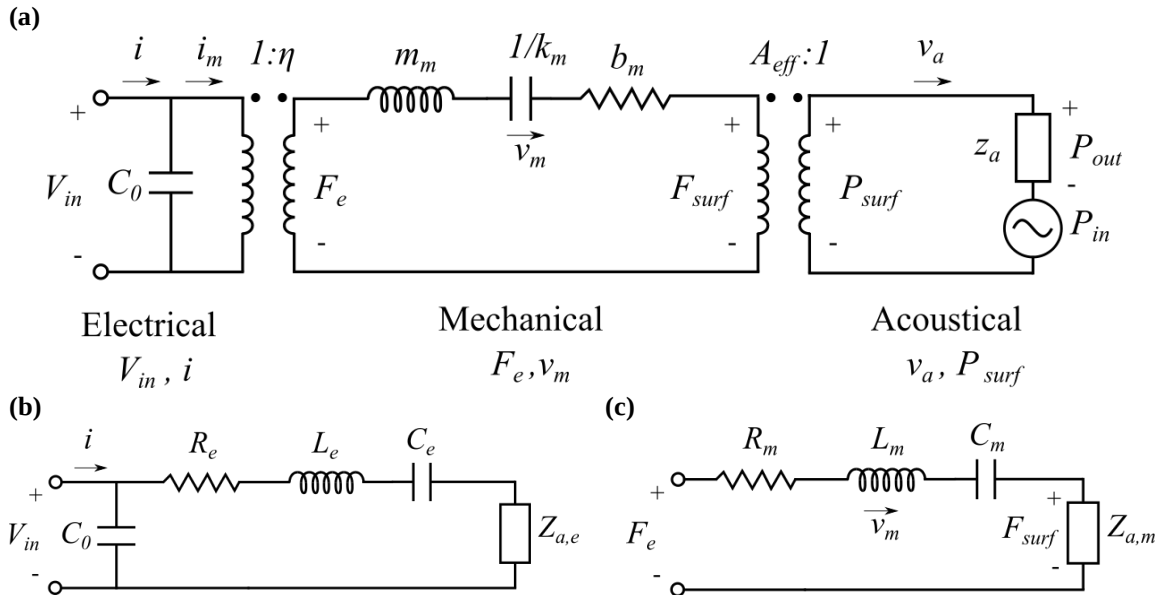


Figure 2.11: (a) Most general form of the equivalent circuit showing all three physical domains separately. Each domain, corresponding effort, and flow variables are given with ideal transformers. (b) Butterworth–Van dyke (BVD) representation of the most general equivalent circuit together with acoustic domain impedance shown separately. All domains are transformed into the electrical domain. (c) Equivalent circuit used in the velocity and displacement calculations. All domains are transformed into the mechanical domain.

Each physical domain and corresponding effort and flow variables are given. In the electrical domain, an electrical voltage V_{in} is the effort variable and defines the potential difference between electrodes and current, i , which is the flow variable. C_0 is the electrical clamped capacitance also called as feedthrough capacitance. Electrical and mechanical domains are coupled with an ideal transformer having the turns ratio of η . The transformer turns ratio is defined as the ratio of effort variables in the coupled domains:

$$\eta = \frac{F_e}{V_{in}} \quad (2.78)$$

In the mechanical domain, the generated piezoelectric force, F_e , is the effort variable and the velocity of the vibrating mass, v_m , is the flow variable. The symbols m_m , k_m , and

b_m are equivalent mechanical mass, stiffness, and damping coefficient, respectively. Similarly, mechanical and acoustical domains are coupled with another ideal transformer having the turns ratio of A_{eff} . This is also a well-known relation between the force and pressure.

$$A_{eff} = \frac{F_{surf}}{P_{surf}} \quad (2.79)$$

In the acoustical domain, the surface pressure, P_{surf} , is the effort variable and the volumetric velocity, v_a , is the flow variable. The total pressure at the surface equals the summation of the output pressure of the transducer, P_{out} , and the incident pressure in the medium, P_{in} . The variable, z_a , represents the acoustic radiation impedance of the vibrating surface in a medium. In the transmitter mode, the incident pressure, P_{in} , is set to be zero. Hence, the output pressure, P_{out} , equals the surface pressure, P_{surf} . Table 2.3 gives the comparison of each lumped parameters for the exact solution, literature, and this work.

Table 2.3: Comparison values of normalized resistive and reactive radiation impedance components for piston vibrator, this work, exact solution, and finite element simulation results.

Parameter	Exact Solution	Literature	This Work
Clamped capacitance*, C_0	$0.449 \frac{\epsilon^s_{33} \pi a^2}{h_p} (1 - k_{31}^2)$	$0.49 \frac{\epsilon^s_{33} \pi a^2}{h_p} (1 - k_{31}^2)$	$0.449 \frac{\epsilon^s_{33} \pi a^2}{h_p} (1 - k_{31}^2)$
Turns Ratio, η	$2\pi(0.938)e_{31,f} Z_p$	$2\pi e_{31,f} Z_p$	$2\pi(0.947)e_{31,f} Z_p$
Mass, m_m	$0.183\pi^2 \Lambda$	$0.2\pi^2 \Lambda$	$0.184\pi^2 \Lambda$
Stiffness, k_m	$19.073\pi \frac{D}{a^2}$	$21.333\pi \frac{D}{a^2}$	$19.226\pi \frac{D}{a^2}$
Damping **, b_m	Neglected	Neglected	Neglected
Effective Surface Area, A_{eff}	$0.312A_{tot}$	$\frac{1}{3} A_{tot}$	$0.313A_{tot}$
Acoustic radius ***, a_c	$0.558a$	$\frac{1}{\sqrt{3}} a$	$0.559a$
Radiation Impedance ****, z_a	$\frac{\rho_m c_m}{A_{eff}} (r_a + jx_a)$	$\frac{\rho_m c_m}{A_{eff}} (r_a + jx_a)$	$\frac{\rho_m c_m}{A_{eff}} (r_a + jx_a)$

* : Electrode coverage ratio is taken as 0.67, 0.7, and 0.67 for exact, literature, and this work, respectively.

** : Mechanical damping is small, assumption: $\text{Re}(Z_a) \gg b_m$, [47]

*** : The given mode shapes in Table 2.1 are normalized such that the maximum displacement over the surface are is unity.

**** : The values of r_a and x_a can be obtained either integration of the given integral in equation (2.74) or as given in Table 2.2 for respective solution.

2.5 Finite Element Model (FEM) Simulations

A good finite element analysis starts with both an understanding of the physics of the system that is to be analyzed and a complete description of the geometry of the system. In addition, all the physics that are relevant to the problem, the materials properties, the loads,

the constraints should also be defined. In Figure 2.12, the geometries, physical domains, boundary conditions are shown and the FEM simulations are conducted using COMSOL Multiphysics v5.3.

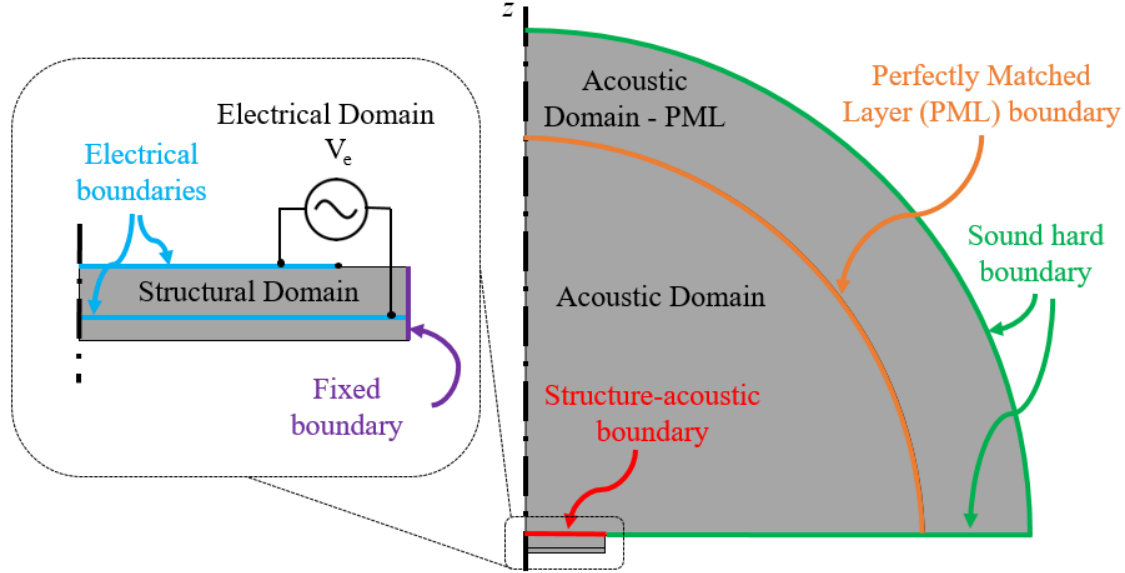
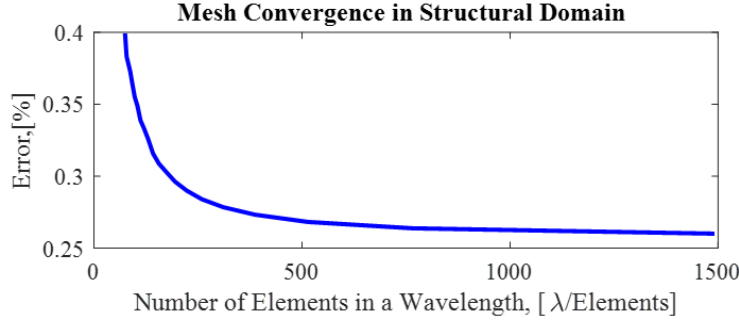


Figure 2.12: Geometry, physical domains, boundary conditions in 2D with axisymmetric axis used FEM simulations. The physical domains are the structural domain, electrical domain, and acoustic domain. The structural domain has fixed boundary at the outer edge: the top layer of the structural layer and the piezoelectric layer sandwiched between electrodes. The outer most boundary is sound hard boundary, and there is PML layer to mimic infinitely large domain.

The pMUT modelled as 2D axisymmetric structure knowing that the mode-of-interest is also axisymmetric. The structural domain composes of two layers stacked as piezoelectric and elastic layer. The piezoelectric layer is sandwiched between two electrodes. The electrodes defined as the surface without any thickness because their relative thin thickness compared to other dimensions. The solid mechanics, i.e. mechanical, domain is coupled with electrical domain via piezoelectric effect multiphysics. the solid mechanics domain is coupled with the acoustic domain via the acoustic-structure boundary. Since the analytical modelling assumes an infinitely large domain, that is no reflections in the medium, a perfectly matched layer (PML) covering the acoustic domain is added. The simulations are done in the frequency domain. The PML layer effectively attenuates the incoming waves so that the reflected wave has no impact on the results. Although there are several ways to determine the PML thickness, a thickness providing at least 100dB attenuation is used in this study. The frequency domain solution is done to validate the results of the analytical model and the material properties, geometric values, domain specific constants, and applied electric voltage are given in Table 2.5.

One of the important questions in FEM simulations is how to choose a mesh type and size. The main concerns are the computational power in hand, computational time, and correct description of the physics. In general, researchers have used tetrahedral elements because they fit very well to arbitrary shaped geometries with their simple computation requirement. In addition, tetrahedral meshes are better in physics behaving in isotropic manner such as sound wave propagation. Regarding with the mesh size, a fine enough mesh size to give accurate answers is preferred, but not too fine, as that would lead to an impractical solution time. One common method is the adaptive mesh refinement which lets the software improve the mesh size based on the defined physics in COMSOL Multiphysics.

(a)



(b)

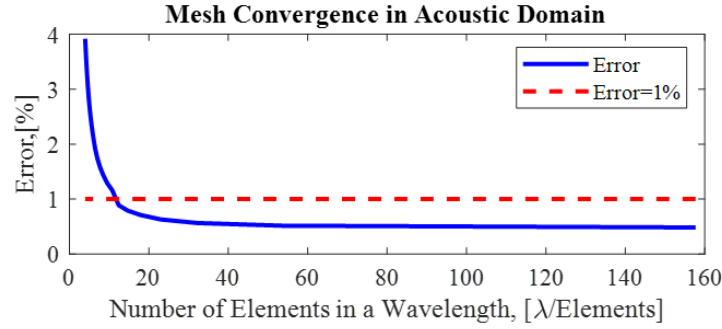


Figure 2.13: Mesh convergence study results in (a) the solid mechanics domain, and (b) the acoustic domain. The variable is selected to be the number of elements (meshes) per wavelength of the propagating wave. Results are compared as the percentage error with respect to previous mesh size solution.

A rule of thumb in the literature is to use mesh size which enables three elements (meshes) per wavelength. The mesh convergence study results are given in Figure 2.13. The variable is selected to be the number of elements (meshes) per wavelength of the propagating wave. The results are compared as the percentage error with respect to the solution from the previous mesh size. Mesh convergence study results for solid mechanics domain are given in Figure 2.13a and those for acoustic domain are given in Figure 2.13b. Overall, ten elements per wavelength mesh size is selected in order to have error less than 1%. The simulation results are compared with the analytical method in the next section.

2.6 Comparison of Results

In order to compare the results, the reduced electrical equivalent circuit, also known as Butterworth–Van dyke (BVD) representation of the pMUT is given in Figure 2.11b. The terms, R_e , L_e , and C_e represent the electrical resistance, inductance, and capacitance, respectively, and $Z_{a,e}$ represents the acoustical impedance transformed into the electrical domain. The equivalent circuit used in the surface average velocity and displacement calculations is given Figure 2.11c, where R_m , L_m , and C_m are the mechanical resistance, inductance, and capacitance, respectively, and $Z_{a,m}$ represents the acoustical impedance transformed into the mechanical domain. All these transformed electrical and mechanical domain parameters are given in Table 2.4.

Table 2.4: Equivalent circuit parameter transformed into electrical and mechanical domains

Electrical Domain	Mechanical Domain
$L_e = \frac{m_m}{\eta^2}$	$L_m = m_m$
$C_e = \frac{\eta^2}{k_m}$	$C_m = \frac{1}{k_m}$
$R_e = \frac{b_m}{\eta^2}$	$R_m = b_m$
$Z_{a,e} = \frac{A_{eff}^2 Z_a}{\eta^2}$	$Z_{a,m} = A_{eff}^2 Z_a$

In all results, the specific and simplest case of pMUT is the single electrode and unimorph structure in the air medium. The piezoelectric material is selected to be AlN which is one of the most commonly used piezoelectric material [54]. In order to ease the complexities, the elastic layer is also selected to be AlN. All material properties, dimensions and medium properties are given in Table 2.5.

Table 2.5: Summary of the dimensions and material properties for comparison study

Parameter	Value
AlN Modulus of elasticity [55], Y	348 GPa
AlN Poisson's Ratio [55], ν	0.3
AlN Density [40, 55, 56], ρ	3300 kg/m ³
AlN Dielectric constant [55], ϵ_r	10.26
Piezoelectric coefficient [40, 56, 57], $e_{31,f}$	-1.02 C/m ²
Layer thickness, h_1 and h_2	1 μ m
Radius, a	200 μ m
Air density [58], ρ_m	1.23 kg/m ³
Speed of sound in air [58], c_m	343 m/s

Frequency response, on-axis pressure at small and large ka values, single element input impedance, bandwidth, quality factor, and directivity of a single pMUT cell are compared. First, the equivalent input impedance is obtained from the equivalent circuit given in Figure 2.11b as follow:

$$Z_{in} = \frac{V_{in}}{i} = \left[sC_0 + \frac{\eta^2 s}{s^2 m_m + s A_{eff}^2 Z_a + k_m} \right]^{-1} \quad (2.80)$$

where s is the frequency parameter as $j\omega$. The input impedance comparison is shown in Figure 2.14. There is a constant error of 24% in the literature in contrast to an error of 0.005% in this work with respect to the exact solution. This means the error in this work is more than 3 order of magnitude smaller.

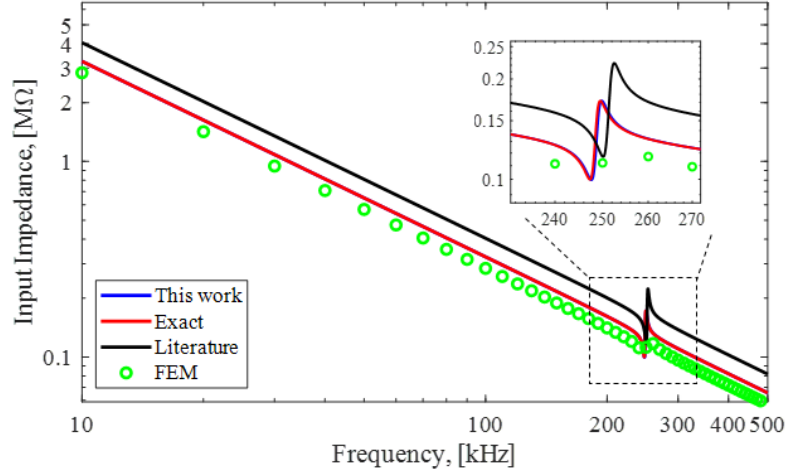


Figure 2.14: Electrical input impedance comparison for this work, exact solution, literature, and FEM. Zoomed-in image shows the region around the resonance of the diaphragm.

The velocity and displacement of the diaphragm are obtained using the equivalent circuits given in Fig. 2.12c. The velocity and displacements are given as:

$$v_m = \frac{F_e}{Z_{mechanical}} = \frac{\eta V_{in} s}{s^2 m_m + s A_{eff}^2 z_a + k_m} \quad (2.81)$$

$$u_m = \frac{\partial w_m}{\partial t} = j \omega w_m$$

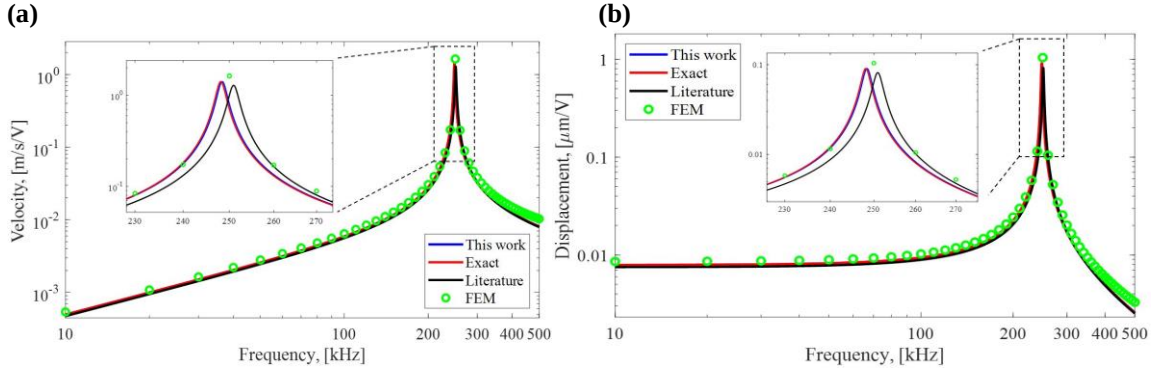


Figure 2.15: (a) Velocity and (b) and displacement comparisons for this work, exact solution, and literature. The zoomed-in images show the regions around the mechanical resonance.

Figure 2.15a and Figure 2.15b show the velocity and displacement outputs versus the frequency, respectively, between this work, the literature, exact solution, and FEM. As explain in the mechanical domain analysis, the error made in the literature for the natural frequency calculation has about 1.14% in error. This error drops to 0.03% in this work with respect to the exact solution. This corresponds to an enhancement of more than one order of magnitude. While both errors are small and good enough to be used in engineering purposes, this work provides much better accuracy for the natural frequency. The overall velocity error in the literature is around 4.7% and it drops to 0.16% in this work. This implies the error is reduced for almost 30x in this work and similar improvement is also observed in the displacement results.

In this study, it is assumed that the dominating damping (B_m) is coming from the acoustic radiation impedance on the surface due to the air medium. Hence, mechanical damping factors including thermomechanical damping, anchor loss, etc. are neglected ($b_m = 0$) and the damping factor is simplified as [40]:

$$B_m = 2 \operatorname{Re}(Z_{a,m}) = 2 \rho_m c_m \pi a_c^2 r_a \quad (2.82)$$

where the multiplier 2 comes from the fact that medium is on both sides of the diaphragm. The transducer bandwidth (BW) and quality factor (Q) are given as:

$$BW = \frac{1}{2\pi m_m} \frac{B_m}{\pi m_m} = \frac{1}{\pi} \frac{\rho_m c_m \pi a_c^2 r_a}{m_m} \quad (2.83)$$

$$Q = \frac{\sqrt{m_m k_m}}{B_m} = \frac{\sqrt{m_m k_m}}{2 \rho_m c_m \pi a_c^2 r_a}$$

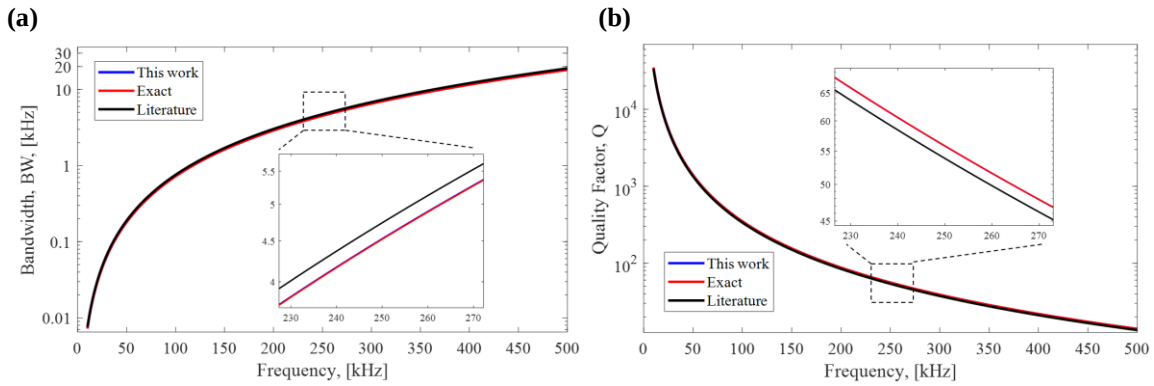


Figure 2.16: (a) Bandwidth, and (b) quality factor comparisons for this work, the exact solution, and literature. The zoomed-in images show the regions around the mechanical resonance. Results of this work and the exact solution are very close such that the blue color line (this work) is not visible in the figure.

Bandwidth and quality factor comparisons between this work, the literature, and exact solution are given in Figure 2.16a and Figure 2.16b, respectively. The bandwidth, BW, error in the literature with respect to the exact solution drops from 4.9% to a value of 0.17% in this work. This corresponds to the 30x improvement. Similarly, the quality factor error drops from 3.5% to 0.04%.

The directionality is defined as the ratio of the far field pressure at a specific angle, ϕ as shown in Fig. 2.9a to the on-axis pressure at $\phi = 0^\circ$. Using the pressure equation given in equation (2.72), the directionality, $D(\phi)$, is derived as:

$$D(\phi) = \frac{\iint_A \left(\frac{\partial w(r, \theta, t)}{\partial t} \right) \frac{e^{-ikR}}{R} dA}{\iint_A \left(\frac{\partial w(r, \theta, t)}{\partial t} \right) dA} \quad (2.84)$$

where the vector distance between the infinitesimal element on the surface and the point in medium, R , is:

$$R^2 = R_1^2 - r^2 - 2R_1r \sin(\varphi)\cos(\theta) \quad (2.85)$$

$$R \approx R_1 - r \sin(\varphi)\cos(\theta), \quad R_1 \gg a$$

Figure 2.17 shows the result of the directionality for different ka values, i.e., frequencies. For the very low $ka \ll 1$ values, all approaches have good approximation for the directionality. As the ka values increases, the error between the literature and the exact solution increases, while results in this work follows the exact solution more closely. It is noted that the piston vibrator approximation is used in literature whereas the two-term mode shape is used in this study.

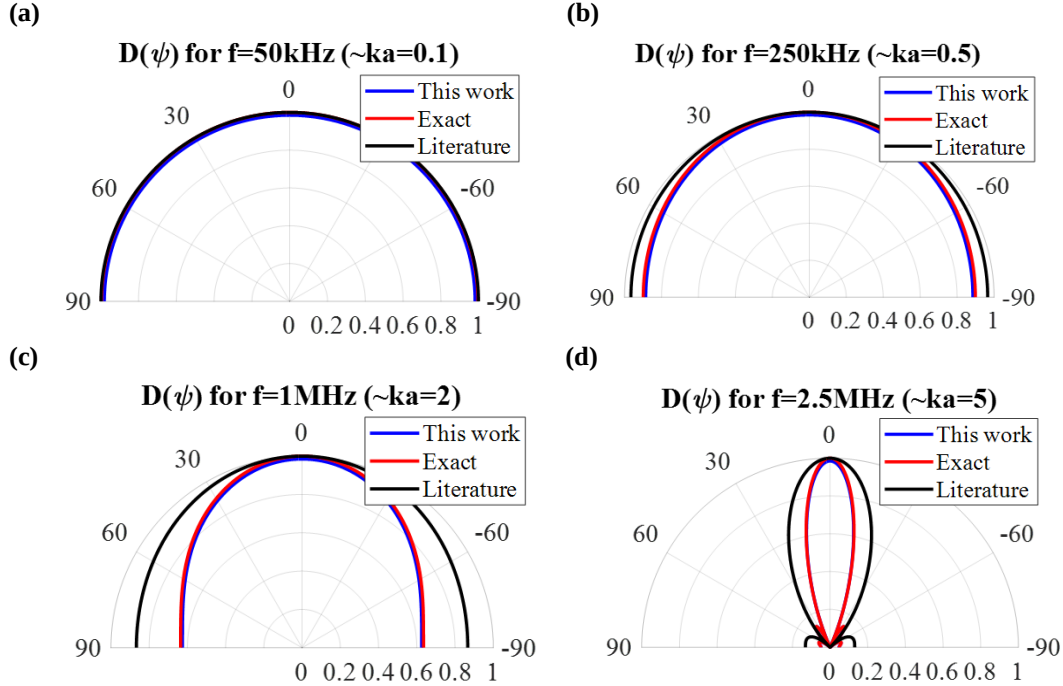


Figure 2.17: (a-d) The directivity comparison for this work, the exact solution, and literature. The directivity is shown for different frequencies, i.e., ka values.

Lastly, the on-axis pressure is calculated by setting $\varphi = 0^\circ$ in the pressure calculation equation and the Rayleigh's integral and the sectorial distance, R , become [1, 48-51]:

$$p(r', \theta', z) = \frac{i\rho_m c_m k}{2\pi} \iint_A \left(\frac{\partial w(r, \theta, t)}{\partial t} \right) \frac{e^{-ikR}}{R} dA \quad (2.86)$$

$$R = \sqrt{R_1^2 + r^2}$$

Figure 2.18a shows the on-axis pressure as a distance from the surface at the resonance frequency. In this region, i.e., small ka values, all approaches capture similar behaviors for the near and far field pressure. Figure 2.18c shows the error of the on-axis pressure from the literature is around 15% in the near field and 7% in the far field. In this work, these errors drop to 0.34% and 0.38% in the near field and far field, respectively. This indicates an improvement of more than 20x. Similarly, Figure 2.18b shows on-axis pressure at high ka values. In the near field, the literature results have the error of more than 80 and this work has only 0.05% error as shown in Figure 2.18d, that is more than 4 order of magnitude improvement in the near field. All approaches capture the far field on-axis pressure

behaviors for high ka values and the error in the literature is around 7% and it decreases to 0.38% in this work, which corresponds to about 20x improvement.

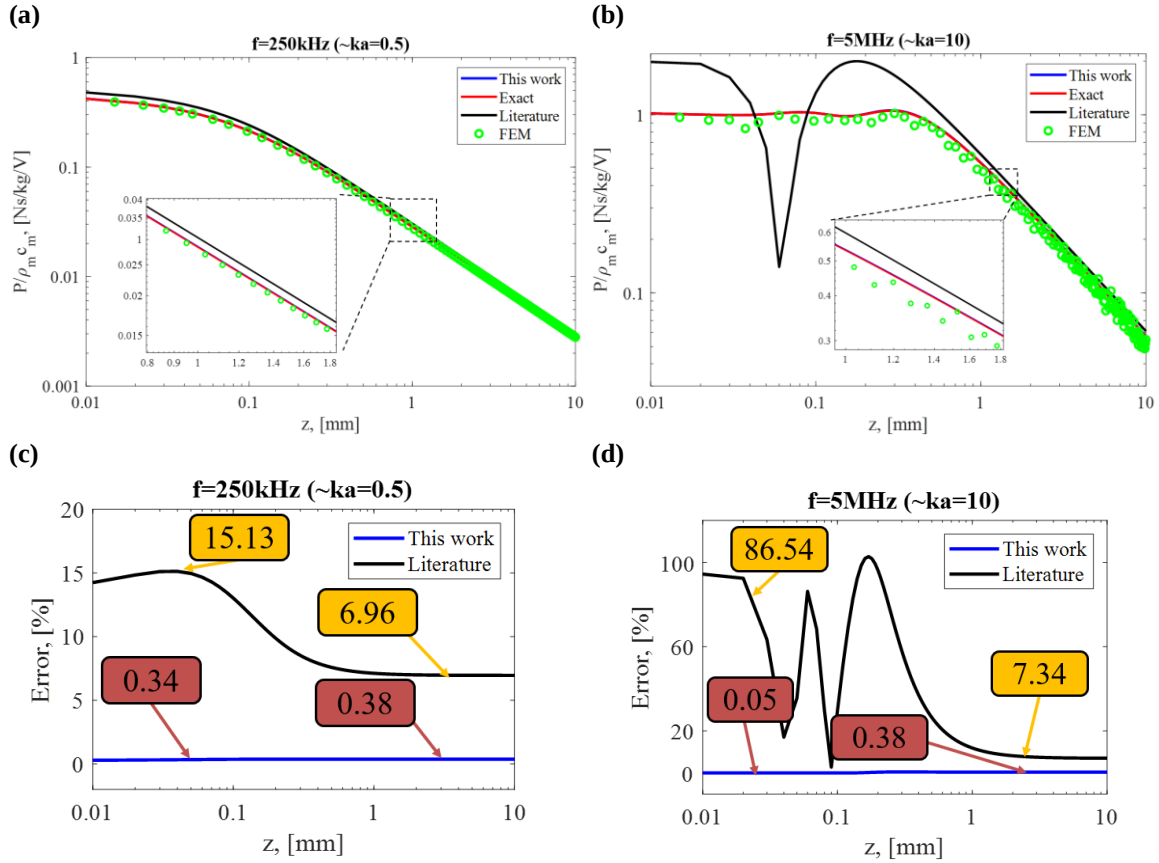


Figure 2.18: On-axis pressure at a distance z from the diaphragm surface at: (a) 250 kHz corresponding to a small ka value ~ 0.5 ; and (b) 5 MHz corresponding to a large ka value ~ 10 . The error of the on-axis pressure for: (c) 250 kHz and (d) 5 MHz of the vibration frequency of the diaphragm. The near and far fields errors are separately indicated in both frequencies.

2.7 Fundamentals of Array Design

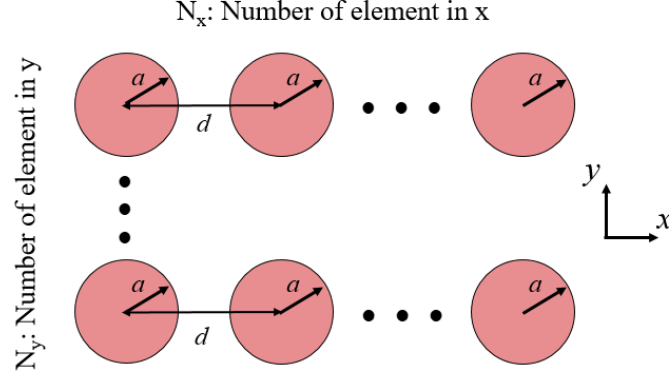
Having an array of ultrasonic transducers brings many capabilities such as beam forming and steering for both transmitting and receiving applications [56, 59-61]. In Figure 2.19a, the 2D array structure is shown for circular pMUTs. The important parameters are the number of elements in each direction and inter element distance.

The directivity given in equation (2.84) can be simplified with the steering angle as [48, 62]:

$$D(\theta) = \frac{\sin\left[\pi \frac{Nd}{\lambda} (\sin(\theta_s) - \sin(\theta))\right]}{N \sin\left[\pi \frac{d}{\lambda} (\sin(\theta_s) - \sin(\theta))\right]} \quad (2.87)$$

where N is the number of elements, d is the inter-element distance, λ is the wavelength, θ_s is the steering angle, and θ is the angular direction shown in Figure 2.19b. Note that θ_s is zero for receive beam forming applications.

(a)



(b)

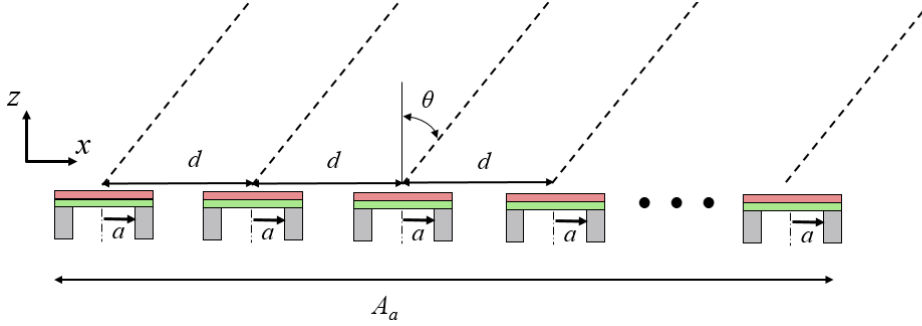


Figure 2.19: (a) Illustration of 2D array structure composed of circular pMUTs. N_x and N_y are the number of element in each direction, a is the pMUT radius, and d is the inter-element distance. (b) Illustration of 1D array structure showing the beam angle.

The term A_a is the aperture of the transducer array and it is defined by [40]:

$$A_a = \frac{Nd}{\lambda} \quad (2.88)$$

and a general approximation for the full width half max (FWHM) of the beam can be obtained as [40]:

$$\theta_b \approx \frac{50.67}{A_a} = 50.67 \frac{\lambda}{Nd} \quad (2.89)$$

The directivity or ultrasonic beam geometry depends on four parameters; number of elements, inter element distance, wavelength, and phase angle. These parameters other than the phase angle are design parameters; therefore, they are determined in the design stage. In Figure 2.20, the effect of each design parameter is shown. Furthermore, the phase angle can be adjusted with drive circuitry by applying the phase difference (time delay) to each element which is called beam forming and steering in the literature [63].

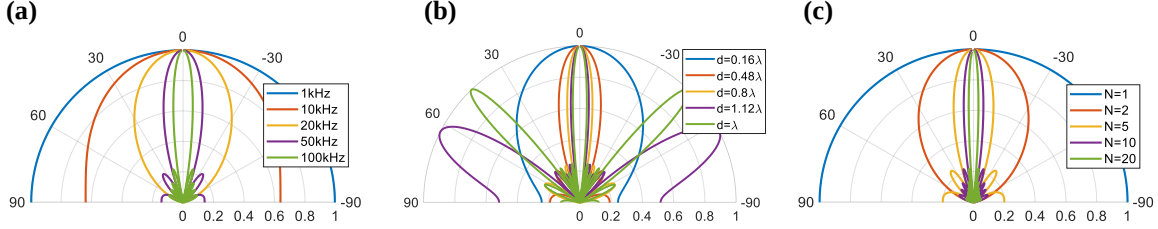


Figure 2.20: (a) The effect of operation frequency; i.e. wavelength, while keeping the number of element and inter-element distance constant. (b) The effect of inter-element distance while keeping the number of element and wavelength constant. (c) The effect of the number of element while keeping the inter-element distance and wavelength constant.

For a given transducer, the small frequency response is omnidirectional. As the drive frequency increases, so as ka value, the sound beam forms and the beam width decreases as shown in Figure 2.20a. The second parameter is the distance between each element. As shown in Figure 2.20b, the beam width gets smaller, i.e., acoustic wave is more directional as the inter element increases. However; it is important to note that after certain distance, there are secondary beams formed which may create misreading for pulse echo method. The last parameter is the number of element (Figure 2.20c). The high number of element can result in better beam width. Overall, it can be summarized as that the large aperture is desired in order to have better beam performances.

Analytically, each lobe is created when the directionality function in Eq. (2.87) has a node, i.e., roots making this equation zero. It can be obtained by making the numerator to zero:

$$\sin\left[\pi \frac{Nd}{\lambda}(\sin(\theta_s) - \sin(\theta))\right] = 0 \quad (2.90)$$

$$\pi \frac{Nd}{\lambda}(\sin(\theta_s) - \sin(\theta)) = m\pi$$

where m is integer of value $\pm 1, \pm 2, \pm 3, \dots$ and not multiple integer of N in order to have nonzero denominator. Then the roots are:

$$\theta = \sin^{-1}\left(\sin(\theta_s) - \frac{m\lambda}{Nd}\right) \quad (2.91)$$

The first zero crossing happens when $m = \pm 1$ and the width of the main lobe is:

$$\theta_b = \sin^{-1}\left(\sin(\theta_s) + \frac{\lambda}{Nd}\right) - \sin^{-1}\left(\sin(\theta_s) - \frac{\lambda}{Nd}\right) \quad (2.92)$$

where $0 \leq \theta_b \leq \pi$. A more quantitative analysis on the beam performance is given as the normalized main lobe sharpness and the amount of side-leaking energy. More specifically, main lobe sharpness is defined as the beam directivity in the steering direction. The main lobe sharpness factor is given as [62]:

$$q = \frac{1}{\pi} \left[\sin^{-1}\left(\sin(\theta_s) + \frac{\lambda}{Nd}\right) - \sin^{-1}\left(\sin(\theta_s) - \frac{\lambda}{Nd}\right) \right] \quad (2.93)$$

A small q means a better directivity of the main lobe since it is sharply defined. An example case of $\theta_s = 0, N = 12, d = 0.5\lambda$ is plotted in Figure 2.21 which provides similar conclusion with Figure 2.20 in the quantitative manner.

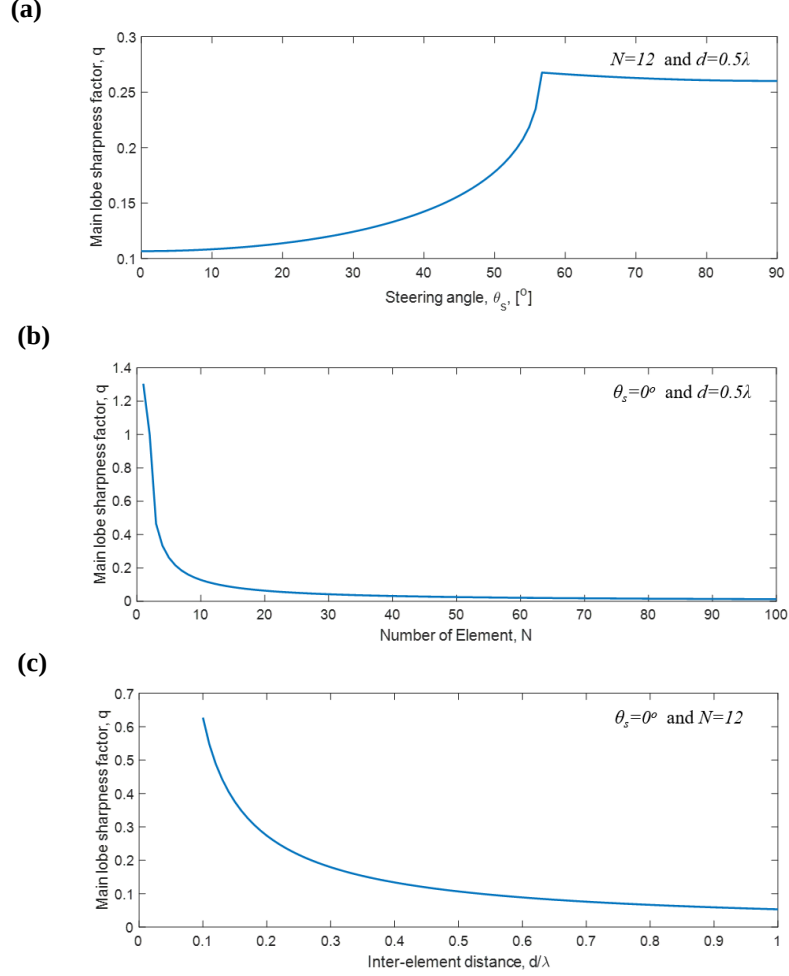


Figure 2.21: Main lobe sharpness factor for an array as a function of: (a) steering angle, (b) number of elements, and (c) inter-element distance.

The last analysis is the secondary or grating lobes. As the inter-element distance increases, secondary lobes are created at the $N - 1$ zero crossing positions which may have amplitude as big as the main lobe as shown in Figure 2.20b. In order to prevent this, the directivity function should have only one zero crossing, i.e., $m = \pm 1$ and $N = 1$ [64].

$$\theta_{g1} = \sin^{-1} \left(\sin(\theta_s) - \frac{\lambda}{d} \right) \quad (2.94)$$

If the following condition is satisfied, there will be no real solution.

$$\sin(\theta_s) - \frac{\lambda}{d} < 1 \quad (2.95)$$

The critical case is obtained at the upper limit of steering angle ($\theta_{s,\max}$) as [64]:

$$\beta = \frac{d_{cr}}{\lambda} = \frac{1}{1 + \sin(\theta_{s,\max})} \quad (2.96)$$

$\theta_{s,\max}$ is also called as maximum steerable angle.

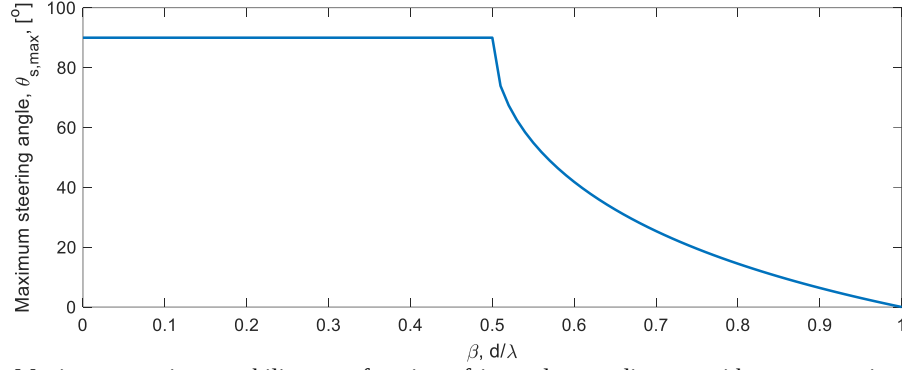


Figure 2.22: Maximum steering capability as a function of inter-element distance without any grating or side lobes created.

Figure 2.22 shows the maximum steering angle capability of the array without creating any grating lobes or side lobes which has an amplitude as comparable as that of the main lobe. One should note that, at the critical inter-element distance, there might be a portion of the grating lobe in the directivity analysis. Hence, it is recommended to have inter-element distance smaller than the critical value. In order to completely eliminate the grating lobes, $m = N - 1$ is used knowing that there is $N - 2$ many zero crossing points between the main lobe and the grating lobe [64]:

$$\beta_{\max} = \frac{d_{\max}}{\lambda} = \frac{1}{1 + \sin(\theta_{s,\max})} \frac{N - 1}{N} \quad (2.97)$$

It is concluded that, for a sufficiently large number of elements, the inter-element distance should be selected as the half of the wavelength in order to steer the beam at 90° . In Figure 2.23, the maximum steering capability as a function of maximum inter-element distance is given for different number of elements.

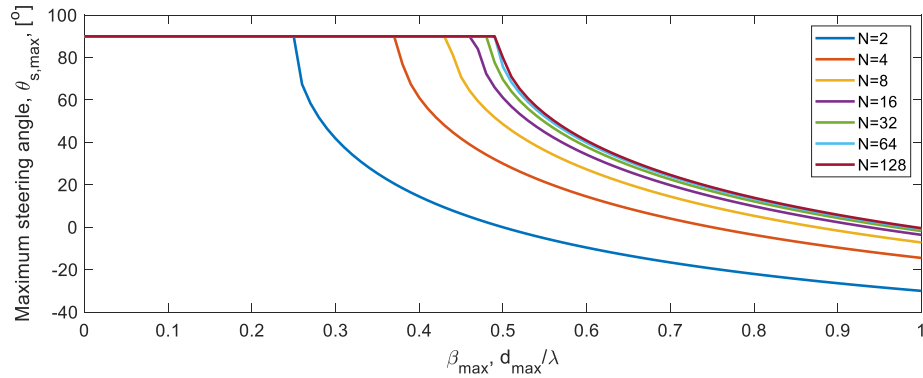


Figure 2.23: Maximum steering capability as a function of maximum inter-element distance is given for different number of elements

The conclusion aligns well with the literature that the inter-element distance is the half of the wavelength [65]. Since micromachined ultrasonic transducers has large number of elements due to the advantage of miniaturization, having the inter-element distance as the half wavelength enables 90° beam steering capability without introducing and grating lobes.

2.8 Conclusions

An improved, two-term mode shape approximation for the circular-shape pMUT has been demonstrated. The small signal electro-mechano-acoustic equivalent model can analyze many parameters such as individual equivalent circuit parameters, input impedance, frequency, bandwidth, quality factor, directivity, and the on-axis pressure in the near and far fields. The proposed model has been compared with the literature, exact solution and finite element method simulations. The errors in all these parameters have decreased below 0.5% as compared to those of exact solution. In addition, enhancements in these parameters compared to the literature error ranges from 5x to three orders of magnitude. We found that the correct modelling of the deformation shape is critical for the model accuracy. The approximate energy-based method can be utilized to obtain more accurate mode shapes and parameters for different types of ultrasonic transducers such as rectangle, ring geometries and dual electrode, bimorph structures.

Chapter 3

Piezoelectric Micromachined Ultrasonic Transducers (pMUT) with Free Boundary

The performance enhancement studies of pMUTs can be divided into two approaches; material and design. Material properties are key factors for piezoelectric transducers [66] and there is a great amount of efforts in the material enhancement studies [8, 18, 67-69]. On the other hand, the design of the transducer is limited by the transduction mechanism. In general, there are two transduction mechanisms; capacitive and piezoelectric types. Some of the piezoelectric micromachined transducers (pMUT) properties such as zero bias voltage, large surface displacement, ease of fabrication without a precision gap control, and linear transduction make them attractive in many cases.

Although there are different geometries, circular geometry is the mostly preferred due to the mechano-acoustic transduction mechanism. Specifically, under the same conditions, including surface area, thickness, applied force, and boundary, circular diaphragms have the largest mean displacement of the fundamental mode as compared to other shapes such as triangular, rectangular, pentagon, etc. Some of the prior efforts to increase the bandwidth and piezoelectric transduction efficiency includes a curved diaphragms [20], multiple electrodes [21], multiple active layers [18], the ring shape geometry [15], a resonant coupler [22], and the pinned-boundary [23].

In this chapter, a pMUT with the free boundary condition is introduced for the first time and compared with those of conventional designs with the clamped boundary. The acoustic cavity underneath the free boundary structure is utilized as an acoustic resonator to increase the coupling and bandwidth by matching frequencies of mechanical and acoustical resonances. The free edge mode shape is derived analytically and an assumed mode is proposed to ease the computational load in analyses. An equivalent circuit model is constructed and lumped parameters are derived. Radiation impedance analysis is investigated and the frequency responses of both free and clamped boundaries are provided.

3.1 Free boundary pMUT Structure

The design composed of a vibrating disk with free edges, supported at the center with a post with the defined active and elastic layers shown in Figure 3.1a and Figure 3.1c. The vibrating disk composes of the active piezoelectric layer sandwiched between two electrodes on an elastic layer. The main difference as compared to the traditional design as shown in the previous chapter is the free edges.

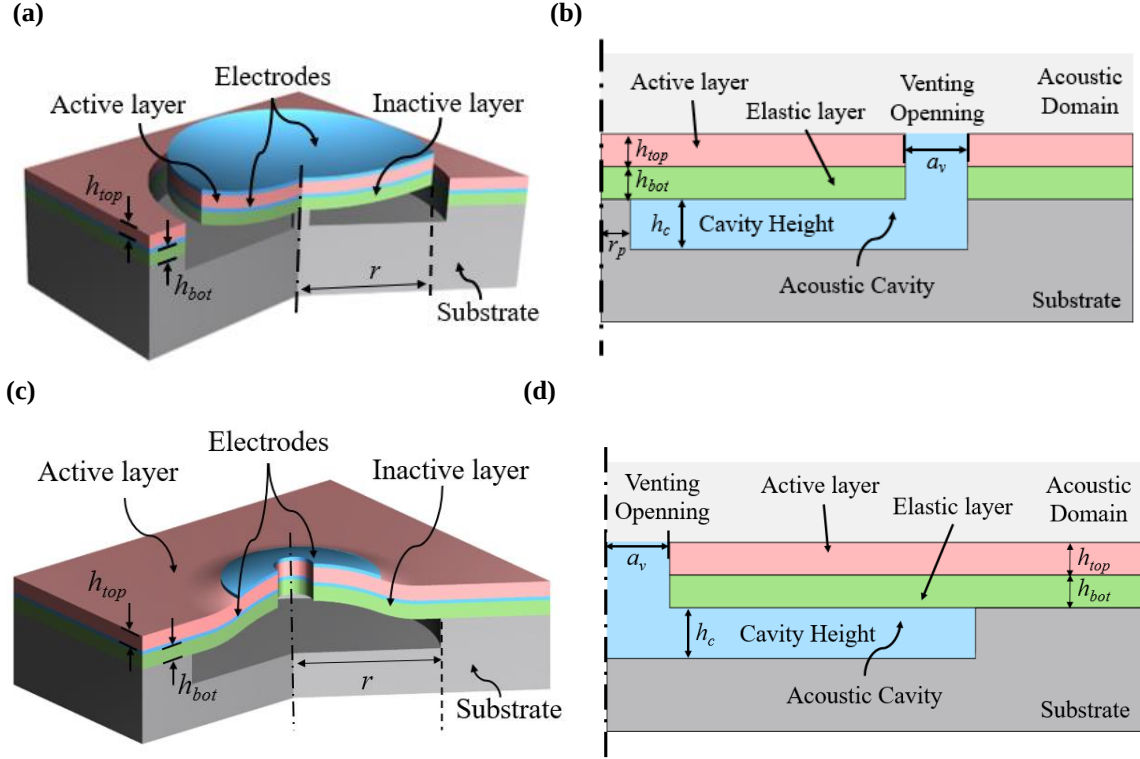


Figure 3.1: (a) The 3-D schematics of a single layer, outer-edge free boundary condition, circular-shape diaphragm with a central post support used in the mechanical analysis. (b) Cross-sectional view. (c) The axisymmetric cross sectional view of a pMUT with a venting hole at the center. (d) Cross-sectional view for (c).

There is an acoustic cavity created underneath the disc (Figure 3.1). Acoustic cavity behaves as Helmholtz resonator with the acoustic mass, stiffness and damping to create a different acoustic impedance on the backside. As the disc vibrates, the acoustic cavity is also excited. With the proper design, the acoustic resonance of the cavity and the mechanical resonance of the free circular plate can be matched resulting in a wide bandwidth response. In addition, the free-edge structure has higher electromechanical coupling factor due the low mechanical capacitance as compared with the same diaphragm dimensions with the clamped boundary. Another advantages of using a hanged structure is to reduce the potential internal stresses built up during the thin film deposition process such that structures are expected to be easily fabricated for low frequency devices. In this work, the outer edge free case is analyzed and the same procedures are applicable for the inner edge free case. One should note that the free edge either can be at the outer edge with the supporting post structure at the center (Figure 3.1b) or at the inner edge with a venting hole at the center (Figure 3.1d) The three energy domains; electrical, mechanical, and acoustical are investigated separately and combined with the coupling between them in this work. In contrary to clamped pMUT case, the effect of the acoustic cavity on the acoustic impedance is also analyzed.

3.2 Free Vibrations of Free Disk

The classical plate theory assumptions used in Chapter 2 are valid in the case of the free disk. The thin plate theory applies for the structures with the ratio of diameter to thickness or more than 20. Although it is widely investigated [28-31], equation of motion (EOM) for transverse vibration of a free diaphragm is derived for the sake of completeness in this section. Again, the equilibrium approach is used to derive EOM.

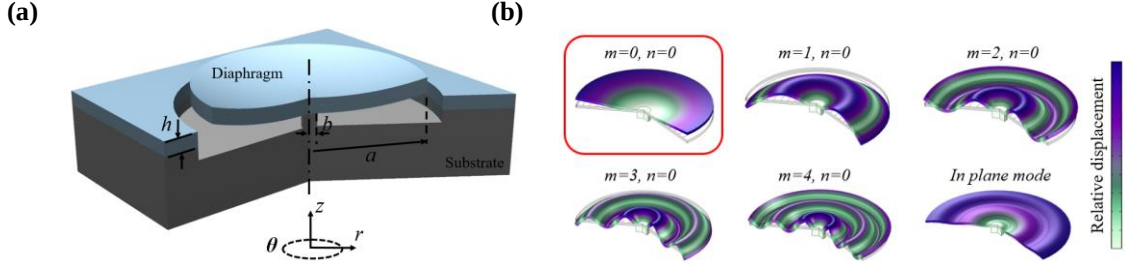


Figure 3.2: (a) 3-D schematics of a single layer, free boundary condition, circular-shape diaphragm with the central support post used in the mechanical analysis. (b) The axisymmetric mode shapes of the diaphragm with the number of circular nodal lines, m , and radial nodal lines, n . The fundamental mode-of-interest is shown with the red box.

In Figure 3.2a, the 3-D schematics of a single layer, free boundary condition, circular-shape diaphragm with the central support post is shown with the coordinate system used in the analysis. For the free vibration, the single layer is modeled while the piezoelectric and elastic stacked layers with electrodes are included in later analyses. In Figure 3.2b, only axisymmetric mode shapes are shown. The mode-of interest is the first (fundamental) mode having zero circular and radial nodal lines, i.e., $m=0, n=0$. Although the post radius, b , has a finite value due to practical reasons, it is safe to assume that $b/a < 0.1$ in the mechanical analysis which can be easily met in micromachined structures. The same approach in Chapter 2 is followed.

Dynamic equilibrium equations are written to derive the equation of motion (EOM) using the infinitesimal incremental approach. Writing the force balance equation in the z direction for force and moment resultants (Appendix A) gives the most general form of equation of motion of a transversely vibrating thin plate without any dissipative elements as [30]:

$$D\nabla^4 w(r, \theta, t) + \Lambda \frac{\partial^2 w(r, \theta, t)}{\partial t^2} = f(r, \theta, t) \quad (3.1)$$

where $w(r, \theta, t)$ is the transverse displacement; ∇^4 is the biharmonic operator given in Appendix A; $f(r, \theta, t)$ is the externally applied force; D is the transverse flexural rigidity; and Λ is the surface density given as [30, 31]:

$$D = \frac{Y}{1-\nu^2} \int_{-h/2}^{h/2} z^2 dz = \frac{Yh^3}{12(1-\nu^2)} \quad (3.2)$$

$$\Lambda = \rho h$$

in which ρ is the layer density; h is the layer thickness; Y is the Young's modulus of the layer; and ν is Poisson's ratio of the layer.

3.2.1 Exact solution for Free Vibration of a Circular Plate

The undamped free vibration of the circular plate is obtained by equating the external force, $f(r, \theta, t)$, to zero which is the right side of EOM given in (2.2):

$$D\nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \quad (3.3)$$

With the separation of variables technique, the EOM can be written as:

$$\nabla^4 W(r, \theta) - k^4 W(r, \theta) = 0 \quad (3.4)$$

where

$$k^4 = \frac{\rho h \omega^2}{D} \quad (3.5)$$

The separation of variables is applied one more time for the displacement as multiplication of two functions: radial and angular components. Step by step solution is given in Appendix A. The general form of solution is [30, 31]:

$$W(r, \theta) = [C_m J_m(kr) + D_m Y_m(kr) + E_m I_m(kr) + F_m K_m(kr)] [A_n \cos(n\theta) + B_n \sin(n\theta)] \quad (3.6)$$

Where m is number of nodal lines; J_m and Y_m are the Bessel's functions of the first and second kind, respectively; I_m and K_m are the modified Bessel's functions of the first and second kind; n is an integer describing angular periodicity of 2π ; and the coefficients $A_n \dots F_m$ are constant coefficients to be determined by the boundary conditions. For the circular disk with free boundary condition with zero displacement and zero slope at the edge with a radius, b , and zero shear and moment at the free edge around the central post, a , that is [70]:

$$W(b, \theta) = 0 \quad (3.7)$$

$$\frac{\partial W(b, \theta)}{\partial r} = 0 \quad (3.8)$$

$$\frac{\partial^2 W(a, \theta)}{\partial r^2} + \frac{\nu}{r} \frac{\partial W(a, \theta)}{\partial r} + \frac{\nu}{r^2} \frac{\partial^2 W(a, \theta)}{\partial \theta^2} = 0 \quad (3.9)$$

$$\frac{\partial}{\partial r} \left(\frac{\partial^2 W(a, \theta)}{\partial r^2} + \frac{1}{r} \frac{\partial W(a, \theta)}{\partial r} + \frac{1}{r^2} \frac{\partial^2 W(a, \theta)}{\partial \theta^2} \right) + \frac{1-\nu^2}{r^2} \frac{\partial^2}{\partial \theta^2} \left(\frac{\partial W(a, \theta)}{\partial r} - \frac{W(a, \theta)}{r} \right) = 0 \quad (3.10)$$

Note that the boundary conditions at the center is specific to the structure proposed here as supported at the center by a post. In addition to these boundary conditions, it is trivial that all the displacements should be finite throughout the plate. The matrix determinant

obtained from applying boundary conditions in equations (3.7)-(3.10) is solved in order to obtain natural frequency of the free plate.

The Bessel's functions of second kind are infinite when the argument is zero, which is at the point where $r=0$. Therefore, for zero nodal circle mode shapes, $m=0, n=0$, the Bessel and modified Bessel functions are approximate as given in Appendix C. This solution has been vastly investigated and it is possible to find k_{mn} values for several combinations of m and n values for a given Poisson's ratio, ν . In this study $m=0$, $n=0$, and $\nu=0.3$ are used. Some of $k_{mn}a$ values for $b/a < 0.1$ are [70]:

$$k_{00}a = 1.937 \quad k_{10}a = 4.573 \quad k_{20}a = 7.79 \quad k_{30}a = 10.94 \quad (3.11)$$

For the mode-of-interest, $k_{00}a = 1.937$, the constant coefficients, $C_m \dots F_m$, are:

$$\begin{bmatrix} C_0 \\ D_0 \\ E_0 \\ F_0 \end{bmatrix} = \begin{bmatrix} -1 \\ 2.9825 \\ 0.6542 \\ 2.9825 \end{bmatrix} \quad (3.12)$$

The constant coefficients are normalized so that the maximum displacement on the free plate is unity. The normalized constant coefficients, $C_0' \dots F_0'$, are:

$$\begin{bmatrix} C_0' \\ D_0' \\ E_0' \\ F_0' \end{bmatrix} = \begin{bmatrix} -0.2627 \\ 0.7835 \\ 0.1719 \\ 0.7835 \end{bmatrix} \quad (3.13)$$

and the fundamental mode shape is obtained as [70]:

$$W_{00}(r) = C_0' J_0' \left(k_{00}a \left(\frac{r}{a} \right) \right) + D_0' Y_0' \left(k_{00}a \left(\frac{r}{a} \right) \right) + E_0' I_0' \left(k_{00}a \left(\frac{r}{a} \right) \right) + F_0' K_0' \left(k_{00}a \left(\frac{r}{a} \right) \right) \quad (3.14)$$

where J_0' , Y_0' , I_0' , and K_0' are approximation of Bessel functions given in Appendix C. From the designer perspective, the general solution given in equation (3.14) is not practical due to the fact that it is difficult to work with Bessel's functions. Another method is used to obtain more compact mode shape representations using simple functions such as polynomials.

3.2.2 Rayleigh-Ritz Solution for Free Vibration of a Circular Plate

Similar to Chapter 2, the energy based approximate method is utilized to get compact and intuitive mode shapes. In addition to axial symmetry, some of the terms vanishes due to fixed boundary conditions [36]. The potential and kinetic energy expressions are derived in polar coordinates as follow [29, 36, 37]:

$$V = \frac{D}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right)^2 - 2(1-\nu) \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial r} \right) r dr d\theta \quad (3.15)$$

$$T = \frac{\rho h}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial w}{\partial t} \right)^2 r dr d\theta \quad (3.16)$$

One should note that the potential energy term is different for clamped and free edge cases. The four boundary conditions for the free disk placed on a post at the center are the geometric boundary conditions. They are given as zero displacement and zero slope at the edges of the central post and zero shear and moment at the edge of the free boundary. Therefore, the two trial functions are selected among the set satisfy the geometric boundary conditions as:

$$\phi_1(r) = c_1 \left(\frac{r}{a} \right)^3 \quad \text{and} \quad \phi_2(r) = c_2 \left(\frac{r}{a} \right)^2 \ln \left(\frac{r}{a} \right) \quad (3.17)$$

The assumed approximate mode shape is:

$$W_0 = c_1 \left(\frac{r}{a} \right)^3 + c_2 \left(\frac{r}{a} \right)^2 \ln \left(\frac{r}{a} \right) \quad (3.18)$$

where a is the radius of the circular diaphragm; c_1 and c_2 are Ritz coefficients. The explained procedure with assumed mode shape given in equation (3.18) is used to construct the eigenvalue problem. It is solved (detailed derivation is given in Appendix B for free boundary case) as:

$$\lambda = \begin{bmatrix} 14.5457 \\ 1041.56 \end{bmatrix} \quad (3.19)$$

and the natural frequencies of first two axisymmetric mode shapes is:

$$\omega_1 = \frac{3.814}{a^2} \sqrt{\frac{D}{\rho h}} \quad \text{and} \quad \omega_2 = \frac{32.273}{a^2} \sqrt{\frac{D}{\rho h}} \quad (3.20)$$

and the corresponding Ritz coefficients are:

$$c^{(1)} = \begin{bmatrix} -0.51 \\ 1 \end{bmatrix} \quad \text{and} \quad c^{(2)} = \begin{bmatrix} 0.2336 \\ 1 \end{bmatrix} \quad (3.21)$$

The Ritz coefficients are normalized so that the maximum displacement in the respective mode shape is unity. The normalized assumed mode shapes for the first two axisymmetric vibration modes are:

$$W_0^{(1)} = \left(\frac{r}{a} \right)^3 - 1.961 \left(\frac{r}{a} \right)^2 \ln \left(\frac{r}{a} \right) \quad (3.22)$$

$$W_0^{(2)} = \left(\frac{r}{a} \right)^3 + 4.281 \left(\frac{r}{a} \right)^2 \ln \left(\frac{r}{a} \right) \quad (3.23)$$

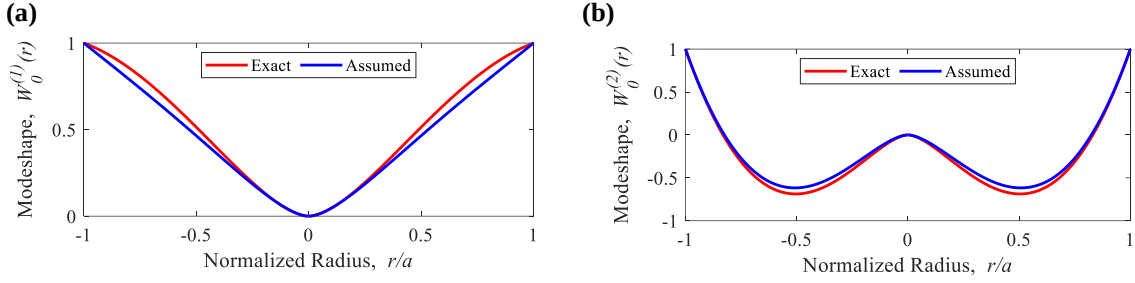


Figure 3.3: Mode shape comparison obtained by exact and two term approximate solutions for: (a) first mode that is also mode-of-interest, and (b) second mode.

It is possible to find different approximate mode shapes as given in [16]. Depending on the choice of trial functions, the degree of error changes. The first two mode shapes of the axisymmetric transverse vibration of the circular diaphragm are given in Figure 3.3a and Figure 3.3b. Although both solutions give good accuracy, working with basic functions is preferred rather than advance functions such as Bessel functions.

3.3 Derivation of Lumped Element Parameters

Analyzing a system in three dimensional domains is complicated and computationally demanding. Hence, engineers have developed methods to simplify these analyses using several methods including lumped parameters. In the case of piezoelectric ultrasonic transducers, the energy is transferred from electrical to mechanical and mechanical to acoustical domain in one dimension. As stated previously, several assumptions are done in order to obtain mode shapes of a circular plate, which boils down to analyze energy transfer only in the single dimension, which are explained in detail in Chapter 2.

3.3.1 Mechanical Domain

The lumped elements in mechanical domains are mass, spring, and damping, which are obtained using the mode shape derived in previous section. The obtained mode shape is normalized in order to have unity at the maximum displacement that is at the center point in the circular diaphragm. The free structure will be compared to clamped case given that there is not much literature on this structure. Table 3.1 compares the natural frequency and normalized mode shape calculations for the fundamental mode for free and clamped boundary conditions obtained from the exact solution and the two term approximate solutions. One should note that, at the center, $r=0$, the mode shape solutions in free boundary is undetermined. The error made in natural frequency calculation using the approximate solution for free boundary is 1.55% compared to the exact solution. Another important result is that free boundary has much smaller natural frequency than the clamped case for the same radius. As an intuition, free boundary has a bigger compliance, i.e., smaller stiffness, which results in smaller natural frequency.

Table 3.1: Comparison of natural frequency and mode shape calculations for the fundamental mode of a free and clamped circular plate obtained from the exact solution and two-term approximate solutions

Solution		Frequency, ω_0 , [rad/s]	Mode Shape, $W_0(r/a)$
Free	Exact [70]	$\frac{3.75}{a^2} \sqrt{\frac{D}{\Lambda}}$	$C_0 'J_0 \left(k_{00} a \left(\frac{r}{a} \right) \right) + D_0 'Y_0 \left(k_{00} a \left(\frac{r}{a} \right) \right) + E_0 'I_0 \left(k_{00} a \left(\frac{r}{a} \right) \right) + F_0 'K_0 \left(k_{00} a \left(\frac{r}{a} \right) \right)$
	Approximate	$\frac{3.81}{a^2} \sqrt{\frac{D}{\Lambda}}$	$\left(\frac{r}{a} \right)^3 - 1.961 \left(\frac{r}{a} \right)^2 \ln \left(\frac{r}{a} \right)$
Fixed	Exact [26]	$\frac{10.21}{a^2} \sqrt{\frac{D}{\Lambda}}$	$J_0 \left(3.196 \left(\frac{r}{a} \right) \right) - \frac{J_0(3.196)}{I_0(3.196)} I_0 \left(3.196 \left(\frac{r}{a} \right) \right)$
	Approximate [41]	$\frac{10.22}{a^2} \sqrt{\frac{D}{\Lambda}}$	$0.754 \left[1 - \left(\frac{r}{a} \right)^2 \right]^2 + 0.246 \left[1 - \left(\frac{r}{a} \right)^2 \right]^3$

* The parameters in the equations are explained in the text.

Normalized mode shapes obtained from the exact solution and two-term approximate approach are given in Figure 3.4a for comparison. The error made in the mode shape in the approximate solution with respect to the exact solution is given in Figure 3.4b. The maximum error is 9.75%. An important note is that the mode shape is not defined at the center, $r=0$, which is the supporting post location and is assumed to be rigid. In Figure 3.4c, comparison of mode shapes for the fundamental mode of a free and clamped circular plate are given for the exact solution and two - term approximate solutions.

The lumped elements are derived at the maximum displacement point (free edge of the diaphragm, $r=a$). Starting with the kinetic energy given in equation (2.19):

$$T = \int_V \frac{1}{2} \rho \left(\frac{dw}{dt} \right)^2 dV = \frac{1}{2} L_m \left[\left(\frac{dw}{dt} \right) \Big|_{r=a} \right] \quad (3.24)$$

$$m_m = \frac{\rho h \int_A W_0^2 dA}{\left(W_0 \Big|_{r=a} \right)^2} \quad (3.25)$$

where A is the area and W_0 is the assumed mode. Then, it can be simplified as:

$$m_m = 0.481 \rho \pi a^2 h \quad (3.26)$$

For the derivation of mechanical capacitance, the major mechanism for the deformation is bending and the membrane stretching action is neglected. The potential energy given in equation (3.15) can be expressed as:

$$V = \frac{D}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right)^2 - 2(1-\nu) \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial r} \right) r d\theta dr = \frac{1}{2} k_m \left(w \Big|_{r=a} \right)^2 \quad (3.27)$$

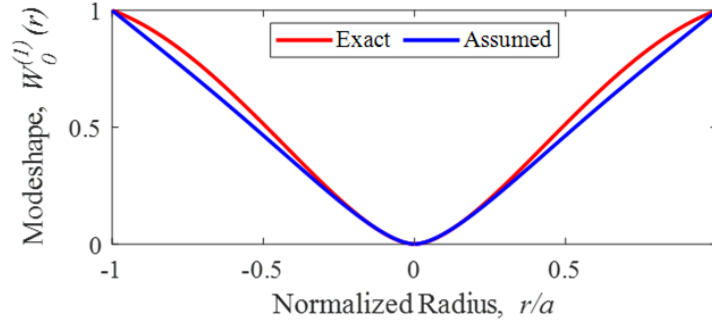
The mechanical capacitance is expressed as:

$$k_m = C_m^{-1} = \frac{D \int_A \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right)^2 - 2(1-\nu) \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial r} \right) dA}{(w_0|_{r=a})^2} \quad (3.28)$$

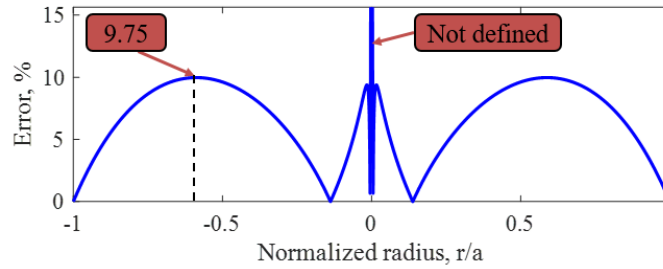
where A is the area and w_0 is the assumed mode. For the assumed mode in equation (2.41), the mechanical capacitance is calculated as:

$$k_m = 7.001\pi \frac{D}{a^2} \quad (3.29)$$

(a)



(b)



(c)

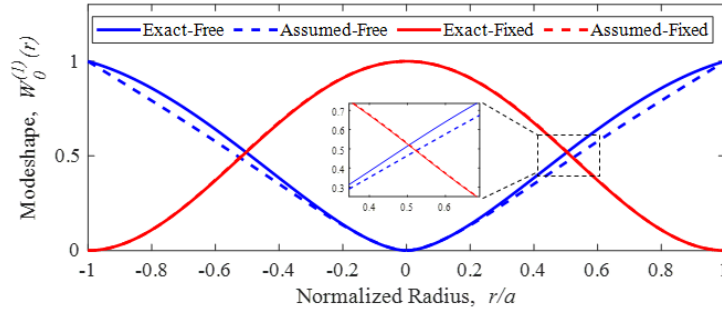


Figure 3.4: (a) Fundamental mode shape, and (b) errors made in approximate calculations versus normalized diaphragm radius for free boundary structure. (a) Comparison of mode shapes for the fundamental mode of a free and clamped circular plate obtained from the exact solution and two-term approximate solutions.

For a quick check, the natural frequency of the mode-of-interest is found using the equivalent parameters as:

$$\omega = \sqrt{\frac{k_m}{m_m}} = \sqrt{\frac{1}{L_m C_m}} = \frac{3.81}{a^2} \sqrt{\frac{D}{\rho h}} \quad (3.30)$$

which is found to be very close to the one using the Rayleigh-Ritz method. The error is around 1.5% which is due to additional stiffness added by the incorrect assumptions of mode shapes. As compared to the clamped case, the natural frequency is much smaller for that is the free boundary case. This is also an intuitive result giving that the free boundary is more compliant than the clamped boundary.

Mechanical damping is assumed to be small and is dominated by the real part of acoustic radiation impedance ($\text{Re}(Z_a) \gg b_m$) [47]. Therefore, the mechanical damping is neglected in the analysis.

3.3.2 Electrical Domain

The capacitance in the diaphragm occurs only between the electrodes; hence, electrode area must be used in the derivation. Neglecting the fringing effects, the electrical enthalpy is given as:

$$H_{\text{electrical}} = \int_V \frac{1}{2} (\epsilon_{33}^s - 2d_{31}e_{31,f}) E_3^2 dV = \frac{1}{2} C_e V_e^2 \quad (3.31)$$

where C_e is the electrical capacitance and V_e is the potential difference between electrodes. The equivalent electrical capacitance is calculated to be:

$$C_e = (\epsilon_{33}^s - 2d_{31}e_{31,f}) h_p A_e E_3^2 \quad (3.32)$$

where A_e is the area underneath the electrodes, i.e., the electrode area; E_3 is the electrical field in the transverse direction; and h_p is the thickness of piezoelectric layer. The free electrical capacitance, C_f is:

$$C_f = \frac{\epsilon_{33}^s A_e}{h_p} \quad (3.33)$$

and

$$C_e = (1 - k_{31}^2) C_f \quad (3.34)$$

where k_{31}^2 is the coupling coefficient. It is the measure of how effectively the electrical energy is converted to mechanical energy. It is a material specific value and a measure of the 31-mode piezoelectric actuation.

$$k_{31}^2 = \frac{2d_{31}e_{31,f}}{\epsilon_{33}^T} \quad (3.35)$$

The last component of the equivalent circuit is the coupling between mechanical and electrical domains. In the circuit representation, it is described by the transformer having a turns ratio of η . The coupling is between mechanical strains and electric field across the piezoelectric layer. The strain field in the free boundary case is different from the clamped case. Hence, the coupling enthalpy term is expressed as:

$$\begin{aligned} H_{coupling} &= -\int_V e_{31,f} z \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right)^2 - 2(1-\nu) \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial r} \right) E_3 dV \\ &= -e_{31,f} E_3 \int_{z_p-h_p}^{z_p} z dz \int_{A_e} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right)^2 - 2(1-\nu) \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial r} \right) dA_e \end{aligned} \quad (3.36)$$

where A_e is electrode area; z_p is the distance from the bottom surface of the layer stack to the top surface of the piezoelectric layer; z_N is the distance from bottom of the layer stack to the neutral axis; and h_p is the thickness of the piezoelectric layer. It is noted that Z_p defines the distance from neutral axis of whole layer stack to the midplane of the piezoelectric layer, in other words, it is the moment arm.

$$Z_p = \frac{1}{h_p} \int_{z_p-h_p}^{z_p} z dz \quad (3.37)$$

$$I_p = -\int_{A_e} \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right)^2 - 2(1-\nu) \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial r} \right) dA_e \quad (3.38)$$

where I_p is the sum of the strain. It is noted that the sum of the strain is limited with the electrode.

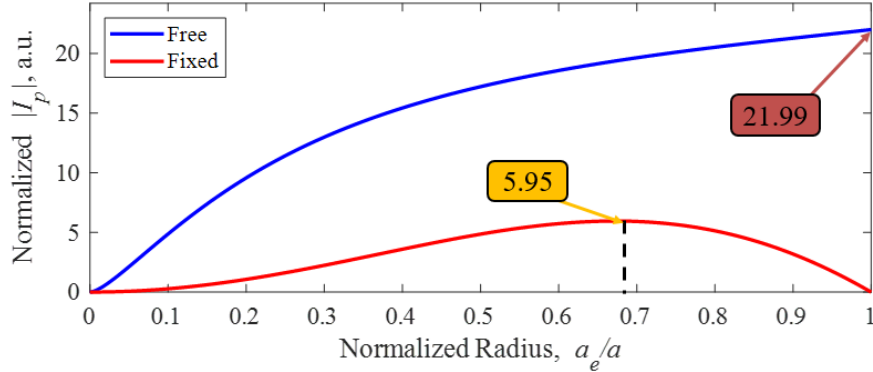


Figure 3.5: Normalized sum of the strain, I_p with respect to the ratio of the electrode to the diaphragm radius for the free and fixed boundary cases

The lumped element for turns ratio in the equivalent circuit is:

$$\underbrace{\eta V_e}_{F_{piezo}}(w|_{r=a}) = H_{coupling} = e_{31,f} V_e Z_p I_p \quad (3.39)$$

This result in $I_p = 21.99$ at $r = a$;

$$\eta = 2\pi(3.5)e_{31,f}Z_p \quad (3.40)$$

This is one of the advantages of having a free boundary, that is the whole surface can be electrode as a result of having zero saddle point in the diaphragm. The structure with free boundary has a prominent advantage without any saddle point in the mode shapes. In other words, the strain vector inside the deformed shape does not change signs which makes possible the electrode coverage ratio to be 100%. Whereas, clamped boundary has a saddle point at the 0.67a position [71], which limits the electrode coverage ratio to be 67%. In Figure 3.5, the comparison of the sum of the strain between clamped and free boundary cases is shown for a circular electrode having the same center and radius a_e . The maximum value of sum of the strains, I_p , for the free boundary case is about 3.7 times larger than that of the fixed boundary case. This enhancement is proportionally reflected to the turn-ratio in the equivalent circuit. The turn-ratio in the free boundary structure is 3.7 times larger than that of fixed boundary structure.

3.3.3 Acoustic Domain: Top Surface

The front side acoustic process model is similar to radiation from a baffled source in the infinite medium which is assumed to be homogenous, isotropic, and non-viscous. A vibrating diaphragm disturbs the medium and creates an acoustic field which can be described by pressure. Similar to the clamped case, the acoustic impedance is given as the ratio of the force on a point divided by the velocity of the top surface [48, 50]:

$$z_{a,f} = \frac{F_a}{v_a} = \frac{\iint_A p(r,0) dA}{v_a} \quad (3.41)$$

where F_a is the surface force and v_a is the surface velocity. The acoustic radiation impedance is simplified as:

$$z_{a,f} = R_a + jX_a = \frac{\rho_m c_m}{\pi a^2} (r_a + jx_a) \quad (3.42)$$

where R_a is the resistive and X_a is the reactive term. In addition, r_a and x_a are dimensionless resistive and reactive terms, respectively, and they are more convenient to use for the comparison purposes. The resistive part is the damping in acoustic domain, and the imaginary part is the added mass as:

$$R_a = \frac{\rho_m c_m}{\pi a^2} r_a \quad \text{and} \quad M_a = \frac{\rho_m c_m}{\pi a^2} \frac{x_a}{\omega} \quad (3.43)$$

Figure 3.6a and Figure 3.6b show the comparison of dimensionless resistance, r_a , and reactance, x_a , results for free and fixed boundary conditions obtained from the approximate solution and Finite Element Simulation (FEM). It is found that there is an approximate frequency after which, both resistance and reactance values are close in all

solutions [52]. In general, ultrasonic transducers work in the low $ka(<<1)$ values where k is the wave number and a is the radius in which all solutions converge. In this region, the fixed pMUT and free pMUT have similar behavior. In this region, the wavelength is large enough to assume that the plate behaves as a point source and the dimensionless resistance, r_a , can be simplified as [26, 52]:

$$r_a \approx \frac{(ka)^2}{2} \quad (3.44)$$

and the dimensionless reactance, x_a , can be simplified as [26, 52]:

$$x_a \approx ka \quad (3.45)$$

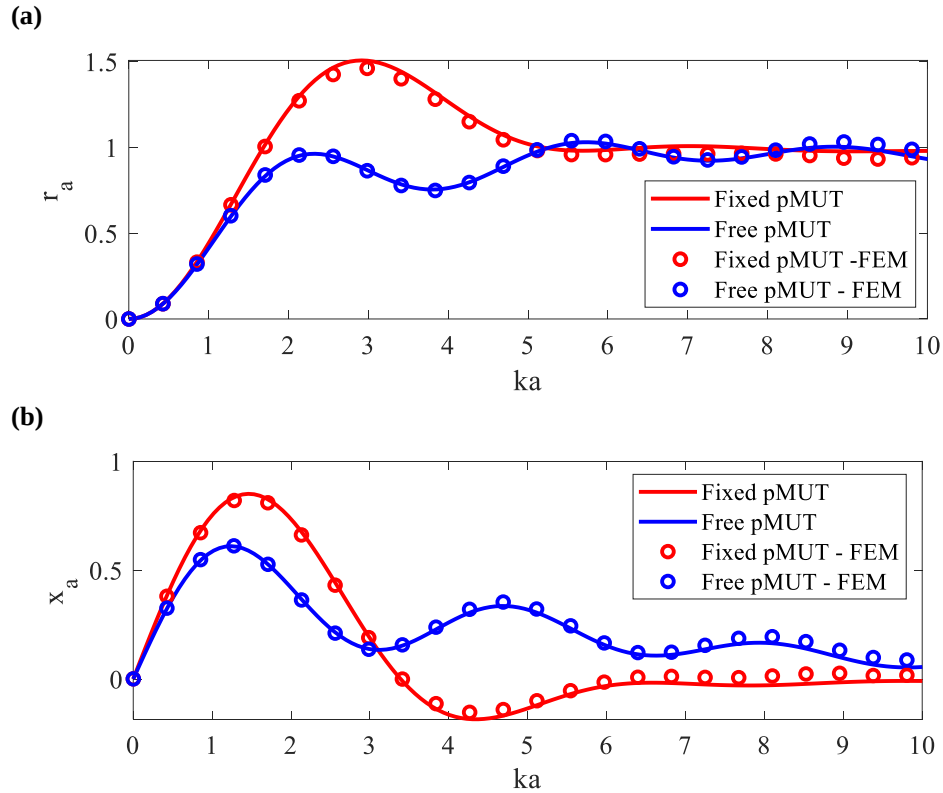


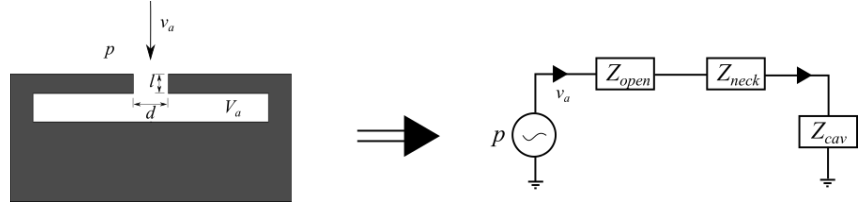
Figure 3.6: Comparison of the dimensionless resistance (a), r_a , and reactance (b), x_a , for a fixed pMUT (red line), free pMUT (blue color), and FEM simulations (symbols) as a function of wave number multiplied by the radius of the circular diaphragm (ka).

The behavior of both cases is similar in low ($<<1$) and high ($>>1$) ka regions. In the middle region, the resistive component is larger in fixed pMUT case; which is desirable. However, the reactive term is also larger in fixed pMUT case in the same region, which is undesired. As described earlier, ultrasonic transducers work in the low $ka(<<1)$ values, and both free and fixed boundary behaves similarly in this region.

3.3.4 Acoustic Domain: Bottom Surface

The bottom surface of the free plate is connected to an acoustic cavity which has a neck, a venting opening, and a cavity volume as shown in Figure 3.1. The acoustic cavity has the same medium as the one in the front side of the diaphragm. This is known as Helmholtz resonator which in general driven by external sound field shown in Figure 3.7a. However, the difference in this work is that Helmholtz resonator is driven by the volume source as shown in Figure 3.7b. This specific case of Helmholtz resonator excitation is also used in exhaust mufflers and bass-reflex loudspeakers [72].

(a)



(b)

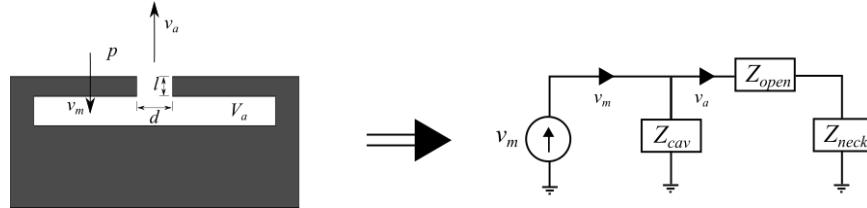


Figure 3.7: (a) A general Helmholtz resonator driven by the external sound field and the corresponding analog network describing its behavior. (b) A Helmholtz resonator driven by the volume source in its cavity and the corresponding analog network describing its behavior

In Figure 3.7, d is venting opening diameter; l is the neck length; p is sound field pressure; V_a is cavity volume; v_m is the speed of sound source in the cavity; Z_{open} is the radiation impedance at the opening; Z_{neck} is the acoustic impedance in the neck; and Z_{cav} is the acoustic impedance in the cavity.

In order to get the acoustic impedance at the bottom surface of the free pMUT, the Helmholtz resonator lumped elements, Z_{neck} , Z_{open} , and Z_{cav} , are connected in the analog network shown in Figure 3.7b. The lumped elements for a traditional Helmholtz resonator can be found in [48, 72, 73]. In order to have a linear behavior in Helmholtz resonator, it is assumed that the dimensions of the Helmholtz resonator are much smaller than the wavelength and the volume of the annular ring neck is much smaller than the cavity volume. An important note is that the neck type in this work is an annular ring. The radiation impedance from an annular ring can be calculated by using [74, 75]. However, in this work, this annular ring is transformed into a simpler geometry that is a disk. The acoustic surface areas are the same for the annular ring and equivalent disk, and the equivalent radius is:

$$a_{cv} = \sqrt{a_v(2a_v + a)} \quad (3.46)$$

The acoustic impedance is given as:

$$z_{a,b} = R_{a,b} + jX_{a,b} = \frac{\rho_m c_m}{\pi a_{cv}^2} (r_{a,b} + jx_{a,b}) \quad (3.47)$$

Where ρ_m is medium density; c_m is speed of sound in the medium; a_{cv} is the equivalent opening radius; and r_a , x_a are dimensionless resistive and reactive terms, respectively. The r_a and x_a functions can be simplified in the small ka region where most of the ultrasonic transducers operate at [48]:

$$r_{a,b} \approx \frac{(ka_{cv})^2}{2} \quad \text{and} \quad x_{a,b} \approx \frac{8ka_{cv}}{3\pi} \quad (3.48)$$

The real part of the radiation impedance behaves as a resistive term which is called $b_{ac,a}$, and the imaginary part behaves as a reactive term which is shown $m_{ac,a}$. In other words, the cavity opening also radiates waves and creates added mass on top of the mass inside the neck. Then:

$$b_{ac,a} = \frac{\rho_m c_m}{\pi a_{cv}^2} r_{a,b} \quad \text{and} \quad Z_{open} = b_{ac,a} \quad (3.49)$$

and

$$m_{ac,a} = \frac{\rho_m h_{total}}{\pi a_{cv}^2} + \frac{\rho_m c_m}{\pi a_{cv}^2} \frac{x_{a,b}}{\omega} \quad \text{and} \quad Z_{neck} = j\omega m_{ac,a} \quad (3.50)$$

The last impedance element in Figure 3.7b is the acoustic stiffness due to the cavity. The compressed pressure generated in the cavity returns this volume to its original position. This behavior is similar to a spring and its spring constant is given by [73]:

$$k_{ac,a} = \frac{\rho_m c_m^2}{V_{ac}} \quad \text{and} \quad Z_{cav} = \frac{\rho_m c_m^2}{j\omega V_{ac}} \quad (3.51)$$

One should note that, a in each subscript in the lumped element denotes the acoustic domain. In the mechanical domain, all parameters are multiplied by A_{eff}^2 .

3.4 Equivalent Circuit Model

As explained above, there are three energy domains and couplings in the transduction path: electrical, mechanical, and acoustical. In addition to being computational friendly, electrical circuit models enable designer to see the energy and power flow not only in the electrical domain but also in all physical domains [53].

Each physical domain and corresponding variables are shown in Figure 3.8a. In the electrical domain, an electrical voltage V_{in} is the effort variable and defines the potential difference between electrodes and current, i , which is the flow variable, and C_0 is the clamped capacitance also called as feedthrough capacitance. Electrical and mechanical

domains are coupled with an ideal transformer having the turn-ratio of η . The transformer turn-ratio is defined as the ratio of effort variables in the coupled domains:

$$\eta = \frac{F_e}{V_{in}} \quad (3.52)$$

In the mechanical domain, the generated piezoelectric force, F_e , is the effort variable and the velocity of the vibrating mass, v_m , is the flow variable. The symbols, m_m , k_m , and b_m are equivalent mechanical mass, stiffness and damping, respectively. Similarly, mechanical and acoustical domains are coupled with another ideal transformer having the turn-ratio of A_{eff} . This is also a well-known relation between the force and pressure.

$$A_{eff} = \frac{F_{surf}}{P_{surf}} \quad (3.53)$$

The effective surface area, A_{eff} , is given by:

$$A_{eff} = 0.645A_{total} \quad (3.54)$$

which corresponds to an effective radius, a_c , of:

$$a_c = 0.803a \quad (3.55)$$

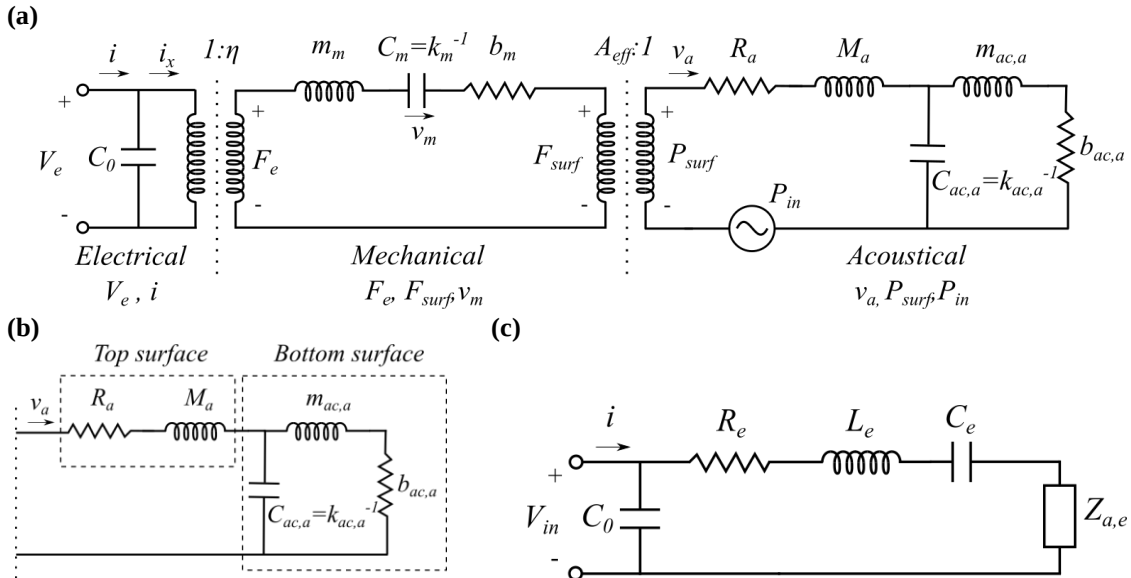


Figure 3.8: (a) Most general form of the equivalent circuit for the outer edge free pMUT showing physical domains. Each domain and corresponding effort and flow variables are given with ideal transformers. (b) Equivalent circuit of the total acoustic domain only, z_a , including front side impedance and Helmholtz resonator driven by the volume source in its cavity. (c) Butterworth–Van dyke (BVD) representation of the most general equivalent circuit together with acoustic domain impedance shown separately.

In the acoustical domain, the surface pressure, P_{surf} is the effort variable and the volumetric velocity, v_a is the flow variable. Table 3.2 gives the comparison of each lumped

parameters for the free and fixed boundary. The variable, z_a , represents the total acoustic radiation impedance of the vibrating surface in the acoustic domain and it is given by:

$$z_a = z_{a,f} + z_{a,b} = R_a + sM_a + \left(\frac{1}{sm_{ac,a} + b_{ac,a}} + \frac{s}{k_{ac,a}} \right)^{-1} \quad (3.56)$$

where s is the frequency parameter equals to $j\omega$ and:

$$m_{ac,a} = \frac{m_{ac,m}}{A_{eff}^2} \quad b_{ac,a} = \frac{b_{ac,m}}{A_{eff}^2} \quad k_{ac,a} = \frac{k_{ac,m}}{A_{eff}^2} \quad (3.57)$$

One primary comparison method of different designs is the electromechanical coupling coefficient which is expressed as the equivalent circuit parameters as [71]:

$$k_{eff}^2 = \frac{\eta^2 C_m}{\eta^2 C_m + C_0} \approx \frac{\eta^2 C_m}{C_0} \quad (3.58)$$

A large feedthrough capacitance is not desired since it stores the energy in the electrical domain. It is observed that the feedthrough capacitance in the free boundary case is 2.23x bigger, the mechanical capacitance of free boundary case is 2.74x bigger, and the turn-ratio of the free boundary case is 3.7x larger than those of the fixed boundary case. Overall, free boundary case has 16.8x higher electromechanical coefficient is 16.8x that that of the fixed boundary case.

Table 3.2: Comparison values of lumped element parameters in free and fixed boundary cases

Parameter	Free	Fixed
Clamped capacitance*, C_0	$\frac{\epsilon^S_{33}\pi a^2}{h_p}(1 - k_{31}^2)$	$0.449 \frac{\epsilon^S_{33}\pi a^2}{h_p}(1 - k_{31}^2)$
Turns Ratio, η	$2\pi(3.5)e_{31,f}Z_p$	$2\pi(0.947)e_{31,f}Z_p$
Mass, m_m	$0.481\pi a^2 \Lambda$	$0.184\pi a^2 \Lambda$
Stiffness, k_m	$7.001\pi \frac{D}{a^2}$	$19.226\pi \frac{D}{a^2}$
Damping**, b_m	Neglected	Neglected
Effective Surface Area, A_{eff}	$0.645A_{tot}$	$0.313A_{tot}$
Acoustic radius***, a_c	$0.803a$	$0.559a$
Radiation Impedance****, z_a	Given in (3.56)	$\frac{\rho_m c_m}{\pi a^2}(r_a + jx_a)$

* : Electrode coverage ratio is taken as unity (1) and 0.67 for free and fixed boundary case, respectively.

** : Mechanical damping is small, assumption: $\text{Re}(Z_a) \gg b_m$ [47]

*** : The given mode shapes in Table 2.1 are normalized such that the maximum displacement over the surface are is unity.

**** : The values of r_a and x_a can be obtained either integration of the given integral in equation (2.74).

3.5 Finite Element Model (FEM) Simulations

Figure 3.9 shows the geometries, physical domains, boundary conditions for the free pMUT case. The FEM simulations are conducted by using COMSOL Multiphysics v5.3. The pMUT is modeled as a 2D axisymmetric structure as the mode-of-interest is also axisymmetric. The structural domain composes of two layers including the piezoelectric and elastic layer. The piezoelectric layer is sandwiched between two electrodes. The electrodes are defined as the surface without any thickness because their relative thin thickness as compared to other dimensions. The mechanical domain is coupled with the electrical domain via piezoelectric effect multiphysics. The mechanical domain is coupled with the acoustic domain via acoustic-structure boundaries on the front and back side. Since the analytical model assumes an infinitely large domain, that is no reflections in the medium, a perfectly matched layer (PML) covering the acoustic domain is added. The simulations are done in the frequency domain. The PML layer effectively attenuates the incoming waves so that the reflected wave has no impact on the results. The material properties, geometric values, domain specific constants, and applied electric voltage are given in Table 2.5.

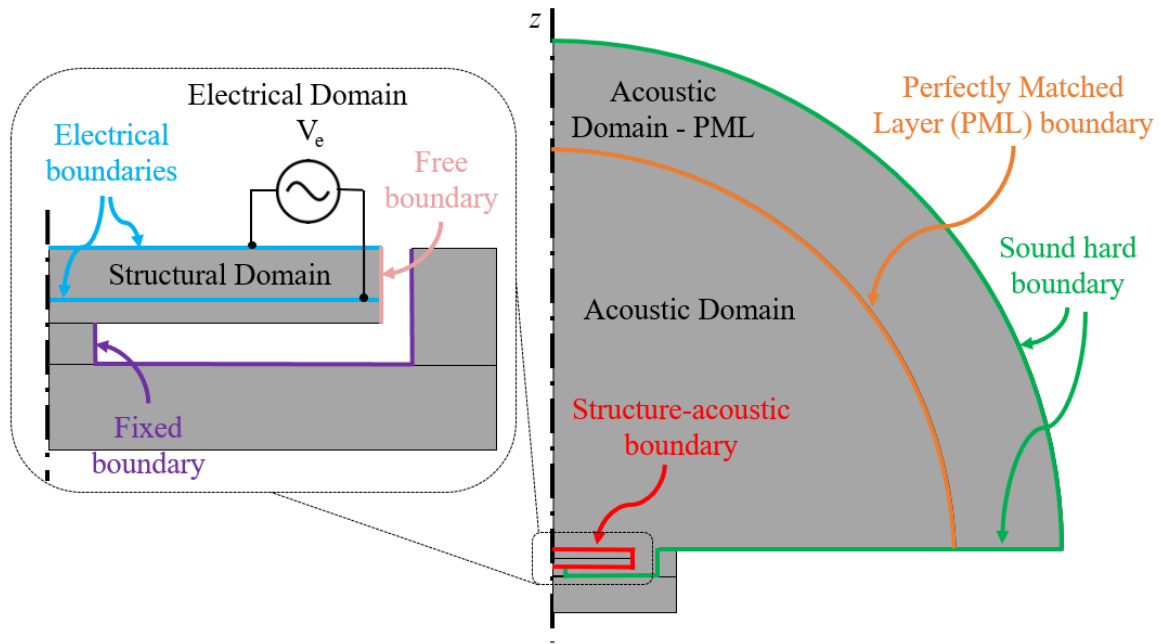


Figure 3.9: Geometry, physical domains, and boundary conditions in the 2D with axisymmetric axis for FEM simulations. The physical domains are structural domain, electrical domain, and acoustic domain. The structural domain has free boundary at the outer edge with the top structural layer of a piezoelectric structure sandwiched between two electrodes. The outer most boundary is sound hard boundary, and there is PML layer to mimic infinitely large domain.

Same mesh convergence study as in Chapter 2 is used here as well as the mesh sizes. As a result, ten elements per wavelength mesh size is selected to have errors of less than 1%. The simulation results are compared with the analytical method in the next section.

3.6 Comparison of Results

The reduced electrical equivalent circuit, also known as Butterworth–Van dyke (BVD) representation, of the pMUT is given in Figure 3.8c. The terms, R_e , L_e , and C_e represent the electrical resistance, inductance, and capacitance, respectively; and $Z_{a,e}$ represents the acoustical impedance transformed into the electrical domain. For the surface average velocity and displacement calculations, mechanical domain is used, where R_m , L_m , and C_m are the mechanical resistance, inductance, and capacitance, respectively; and $Z_{a,m}$ represents the acoustical impedance transformed into the mechanical domain.

In all results, the specific and simplest case of pMUT is used which is the single electrode and unimorph structure in the air medium. The piezoelectric material is selected to be AlN which is one of the most commonly used piezoelectric material [54]. In order to ease the complexities, the elastic layer is also selected to be AlN. All material properties, dimensions and medium properties are given in Table 3.3.

Table 3.3: Summary of the dimensions and material properties for comparison study

Parameter	Value
AlN Modulus of elasticity [55], Y	348 GPa
AlN Poisson's Ratio [55], ν	0.3
AlN Density [40, 55, 56], ρ	3300 kg/m ³
AlN Dielectric constant [55], ϵ_r	10.26
Piezoelectric coefficient [40, 56, 57], $e_{31,f}$	-1.02 C/m ²
Layer thickness, h_{top} and h_{bot}	1 μ m
Radius, a	200 μ m
Opening length, a_v	8 μ m
Neck length, $h_{top} + h_{bot}$	2 μ m
Cavity depth, h_c	540 μ m
Air density [58], ρ_m	1.23 kg/m ³
Speed of sound in air [58], c_m	343 m/s

The equivalent input impedance is obtained from the equivalent circuit given in Figure 3.8b as follow:

$$Z_{in} = \frac{V_{in}}{i} = \left[sC_0 + \frac{\eta^2 s}{s^2 m_m + sA_{eff}^2 Z_a + k_m} \right]^{-1} \quad (3.59)$$

where s is the frequency parameter equals to $j\omega$. The input impedance comparison between free and fixed boundary pMUTs with the same diameter is shown in Figure 3.10. Although the clamped capacitance is bigger in free boundary case, the total input impedance is smaller due to the mechanical domain.

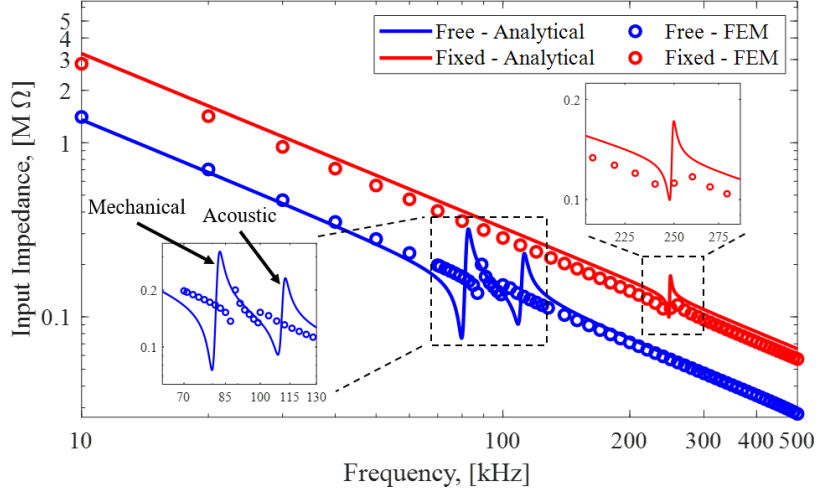


Figure 3.10: Electrical input impedance comparison for free and fixed boundary cases in analytical and FEM solutions. Zoomed-in image shows the region around the resonance of the diaphragm and acoustic resonance underneath the diaphragm.

The velocity and displacement of the diaphragm are obtained using the equivalent circuits transformed into the mechanical domain. The velocity and displacements are:

$$v_m = \frac{F_e}{Z_{\text{mechanical}}} = \frac{\eta V_{in} s}{s^2 m_m + s A_{\text{eff}}^2 z_a + k_m} \quad (3.60)$$

$$u_m = \frac{\partial w_m}{\partial t} = j\omega w_m$$

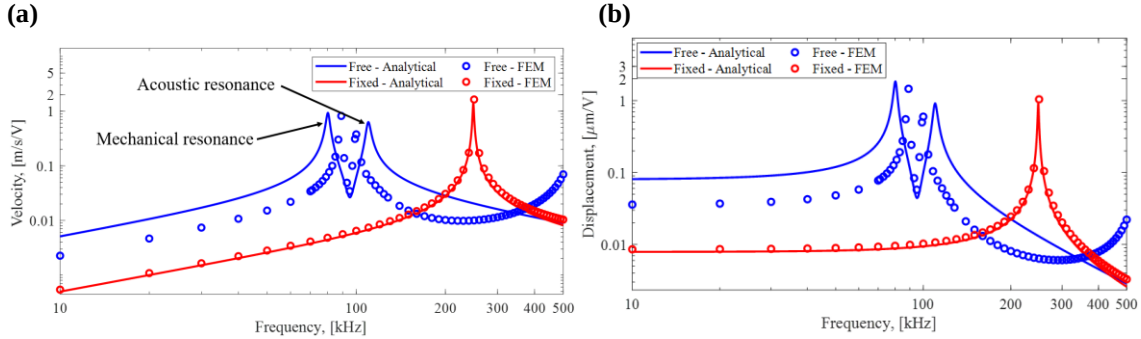


Figure 3.11: (a) Velocity and (b) displacement comparisons for free and fixed boundary cases in analytical and FEM solutions.

Figure 3.11a and Figure 3.11b show the velocity and displacement outputs versus the frequency, respectively, between free and fixed boundary cases in analytical and FEM simulations. In the free boundary case, there are two peaks in the velocity and displacement frequency responses. The first peak is the mechanical resonance of the free boundary structure, and the second one is due to the matched acoustic resonance. The combined response, which is one of the prominent advantages of free boundary, will have a larger bandwidth. In both velocity and displacement figures, the analytical model captures the FEM behavior; however, the off-resonance region deviates. This is as expected since all equivalent parameters are derived by using the fundamental mode shape of the structure. In higher frequencies, FEM results include the higher order effects.

The surface sound pressure level (SPL) comparison between free and fixed boundary cases in analytical and FEM simulations is shown in Figure 3.12.

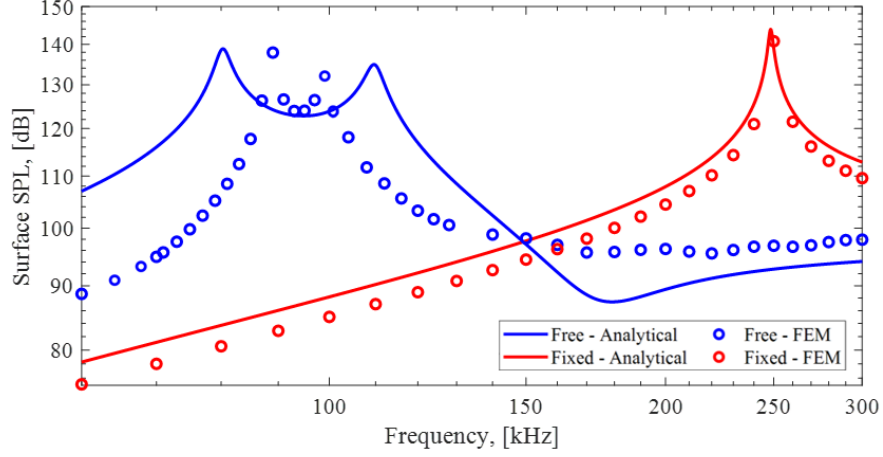


Figure 3.12: Surface sound pressure level (SPL) comparison for free and fixed boundary cases in analytical and FEM solutions.

The mechanical damping factors including thermomechanical damping, anchor loss, etc. are neglected ($b_m = 0$), assuming the dominating damping (B_m) comes from the acoustic radiation impedance on the surface due to the air medium. The damping factor is simplified [40] and the transducer bandwidth (BW) and quality factor (Q) are given as:

$$BW = \frac{1}{2\pi} \frac{B_m}{m_m} = \frac{1}{2\pi} \frac{\text{Re}(Z_{a,m})}{m_m}$$

$$Q = \frac{\sqrt{m_m k_m}}{B_m} = \frac{\sqrt{m_m k_m}}{\text{Re}(Z_{a,m})} \quad (3.61)$$

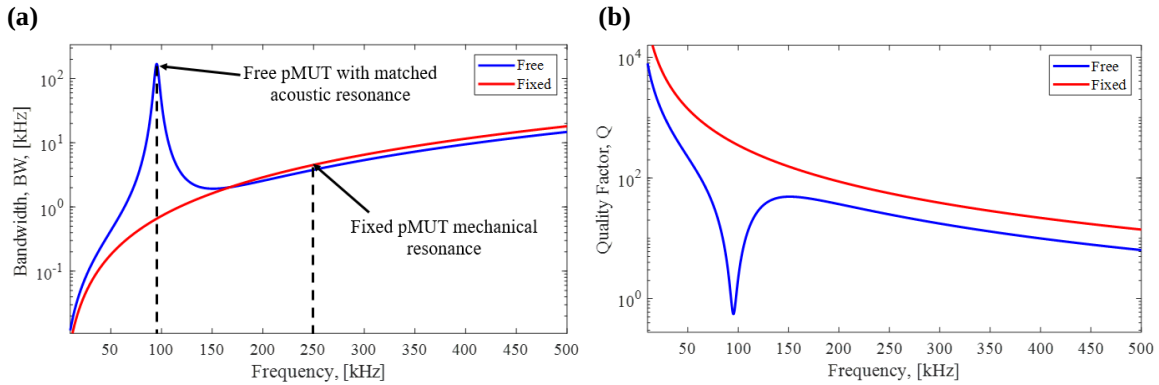


Figure 3.13: (a) Bandwidth, and (b) quality factor comparisons for free and fixed boundary cases in analytical and FEM solutions. The peak in the free boundary bandwidth is achieved by the mode matching of mechanical and acoustic resonances.

Bandwidth and quality factor comparisons between the free and fixed boundary cases in analytical and FEM solutions are given in Figure 3.13a and Figure 3.13b, respectively. The bandwidth enhancement changes due to the resonant frequency of the mechanical structure. Specific to the given diameter, the enhancement is about two-order of magnitude. In addition, the quality factor can be decreased, which is highly desired in acoustic

transmission. As described in [16], the bandwidth enhancement is more than one order of magnitude in the MHz frequency range.

Lastly, the on-axis pressure is calculated by setting $\varphi = 0^\circ$ in the pressure calculation equation and the Rayleigh's integral [1, 48-51] with the sectorial distance, R , as:

$$p(r', \theta', z) = \frac{i\rho_m c_m k}{2\pi} \iint_A \left(\frac{\partial w(r, \theta, t)}{\partial t} \right) \frac{e^{-ikR}}{R} dA \quad (3.62)$$

$$R = \sqrt{R_1^2 + r^2}$$

Figure 3.14 shows the on-axis pressure as a distance from the surface for the free and fixed boundary cases with corresponding resonance frequencies. The free boundary has slightly less pressure output due to the fact that its velocity is smaller due to the low resonant frequency. The high bandwidth, i.e. small quality factor, makes the vibration amplitude smaller. However, in the far field region, the pressure output for both cases are close to each other.

The Helmholtz resonator underneath the diaphragm can be designed using the cavity volume, V_{ac} , neck length, $h_{total} = h_1 + h_2$, and cavity opening, a_{cv} . While the cavity volume and the neck length determine the acoustic resonance frequency, the cavity opening has a strong effect on the bandwidth adjustment. Moreover, the neck length is determined by the mechanical resonance requirement and it has a small room for adjustments in the structural design. The effect of the vent opening dimension on the bandwidth enhancement is given in Figure 3.15 together with the free pMUT case without any acoustic cavity with the fixed volume and neck parameters given in Table 3.3.

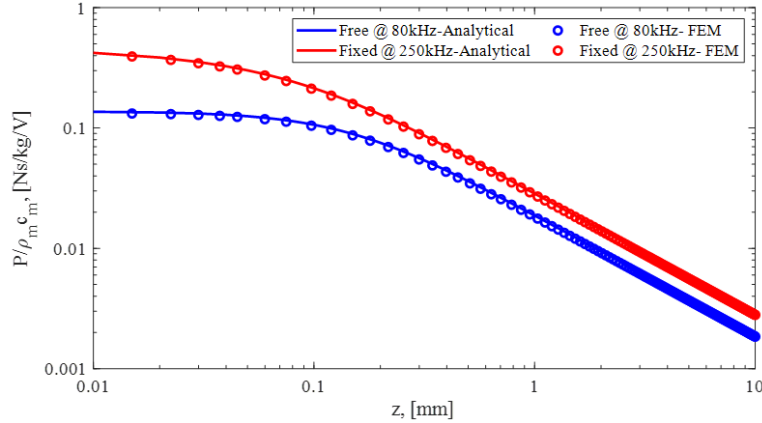


Figure 3.14: On-axis pressure comparison at a distance z from the diaphragm surface at 80 kHz in the free boundary case and 200 kHz in the fixed boundary case with FEM results.

As seen in Figure 3.15a and Figure 3.15b, the acoustic resonance frequency can be matched with the mechanical resonance in order to have the broader bandwidth. In addition, the mass added due to the acoustic radiation also shifts the mechanical resonance to lower frequencies as the cavity opening increases. In Figure 3.15c, the bandwidth of the free pMUT with and without the acoustic cavity for different cavity opening, a_{cv} , is shown.

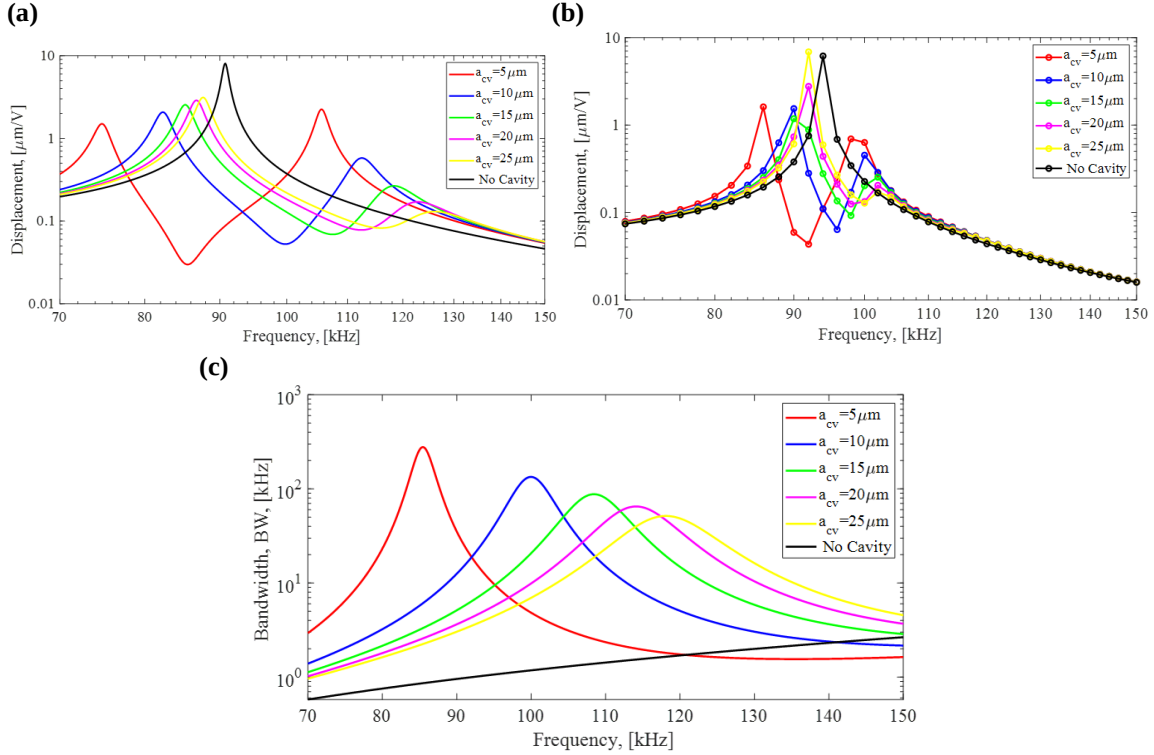


Figure 3.15: The displacement versus frequency responses with different sizes of cavity opening, a_{cv} , for (a) analytical and (b) FEM results. (c) Bandwidth of the free pMUT with different cavity opening, a_{cv} , showing the mode matching has resulted in prominent bandwidth enhancements.

It is seen that the bandwidth can be increased by two orders of magnitude compared to the pMUT without the cavity. This result indicates that the acoustic cavity enables transducers with broader bandwidths.

3.7 Conclusions

A novel pMUT with free boundary condition is introduced for the first time and compared with conventional pMUTs with the fixed boundary. The structure has a diaphragm with free boundary at the outer edge sitting on a central post. Due to the increased mechanical capacitance by the free edge and enhanced strain field distribution, the electromechanical coupling coefficient is more than 16x higher than that of a traditional clamped boundary pMUT. The acoustic cavity underneath the free boundary structure is utilized as an acoustic resonator, i.e. Helmholtz resonator. By matching the frequencies of mechanical and acoustical resonances, the bandwidth of the pMUT is increased. As a case study, the bandwidth of the free boundary pMUT is more than two orders of magnitude higher than that of a fixed boundary pMUT. In addition, the internal stresses created during the film deposition process are expected to be released with the released free edges.

Chapter 4

Ultrasound-Induced Airborne Haptics via pMUTs

In this chapter, a non-contact haptic device is presented using a dual electrode, bimorph, pinned-like boundary pMUT array. The radiated ultrasound from the pMUTs is modulated to get a low frequency radiation force that can be detected by the mechanoreceptors inside the human skin. The design and the detail fabrication of the pMUT array is explained. The device characterization experiment results are introduced. The effect of the modulation parameters has been demonstrated. Finally, the results of average sensation time and an experiment to write letters on the human skin using Morse code is presented.

4.1 Introduction

Recent advances in augmented and virtual reality (AR/VR) have introduced the needs of high intensity, low power, and small footprint haptic devices to increase the immersive experiences such as the sensation of touch in addition to visual and sound effects. To meet these needs, there has been great amounts of works in different transduction mechanisms for haptics such as capacitive, piezoelectric, magnetic, piezo resistive, optical etc. [76]. In the event of contact, the force and displacement information could be detected by the mechanoreceptors distributed in the dermis tissue [77-79]. The mechanoreceptors are differentiated as Meissner's Corpuscles, Merkel Disks, Ruffini Organs, and Pacinian Corpuscles based on their location and the speed of adaption to static stimuli [80]. The spatiotemporal sensitivities of the human skin depend on the density of mechanoreceptors. As such, the haptics sensation can be achieved by the stimulation of neurons directly [81-85] or forces on the skin [86-90]. The haptic force on the human skin can be created using the acoustic radiation force which is a non-contact type force and has the advantage of not being affected by the biological variance of the human skin surfaces. Haptic perception of unfocused ultrasound at the fingertip has the sensitivity of 0.4-3 mN at the low frequency range of 100-1000Hz [91]. For mobile devices, entertainment, and AR/VR applications, micromachined transducers are better for sound sources since they meet the requirements of high intensity, low power, and small footprint [13]. Here, piezoelectric micromachined transducers (pMUT) have advantages in features such as zero bias voltage, high displacement capability, ease of fabrication, and linear transduction over its capacitive counterparts such as capacitive piezoelectric micromachined transducers (cMUT) [92].

Ultrasound-induced sense of touch has the prominent advantages: (1) it is free from contact to skin, i.e., the skin is freely exposed to air for breath and the skin surface properties do not affect the performance, (2) a single transducer array can be used to generate different temporal actuations by amplitude modulation, (3) spatial resolution of the haptic feedback is better since it is determined by the frequency of the ultrasound, i.e., focal point dimensions, and (4) multimodal stimulation of human skin could be achieved as such surface texture, pressure, temperature, etc. The contact free ultrasound induced interface eliminates the issues of contact-type haptic devices due to the skin surface property effects, direct touching to the surface (not air contact), sweating, and others.

4.2 Mechanoreceptors in Human Skin and Ultrasonic Haptics

The ‘sense of touch’ in humans is achieved by two main processes: cutaneous and kinesthetic. These two process is characterized by the location of sensory input on the body. The cutaneous sense is achieved by the sensory inputs from the receptors embedded in the skin. On the other hand, kinesthetic sense is achieved by the sensory inputs from the receptors in muscles and joints [79]. The haptic system can use the sensor fusion of both cutaneous and kinesthetic systems.

For an adult human, there are 241 and 58 mechanoreceptors per centimeter area in the fingertips and palm areas [93]. In addition to the seventeen thousand mechanoreceptors which are responsible for pressure and vibration sensing; there is also thermos receptors responsible for temperature sensing, and free nerve endings (nociceptors) responsible for pain and damage sensing in the human skin [80, 94-96] which are summarized in Table 4.1.

Table 4.1: Types of the mechanoreceptors inside the human skin and their location, class, response type and temporal resolution properties.

Receptors	Location	Responds to	Class*	Freq. (Hz)
Hair Follicle Ending	Hairy skin areas	Hair displacement	-	-
Ruffini Corpuscle	Dermis of skin	Pressure and shear forces	SA	100-500
Pacinian Corpuscle	Deep layers of dermis	Vibratory stimulus	FA	40-800
Meissner’s Corpuscle	Dermis of skin	Motion and vibration	FA	10-200
Merkel Disk	Epidermis of skin	Texture, sensitive to edges	SA	0.4-100
Free nerve endings	Throughout the skin	Mechanical, thermal, or noxious	SA	40-800

* SA: Slowly adapting, FA: Fast adapting

The properties of the tactile sensing for human hand are spatial resolution, force sensitivity, dynamic range, linearity, hysteresis, temporal resolution, and robustness. In general, the human fingertip has the spatial resolution of about 1-2 mm [97]. However, two stimuli on the fingertip having a distance lower than 2 mm can be distinguished, if they are

stimulated at different times. This is called as pattern or texture discrimination, which is more sensitive. With a given average dimension of the human fingertip and 1-2 mm spatial resolution, the total area may have about 10x15 sensing elements. The force sensitivity of human mechanoreceptors depends on the frequency of the force; however, it has been shown that the minimum force that can be detected is around 0.3 mN [91]. Table 4.2 shows some examples for the force outputs of ultrasonic devices on airborne ultrasonic haptics. In another study, the sensitivity range is 0.01-10 N which means a dynamic range of 1000:1 [98]. The temporal resolution is given in Table 4.1 and it shows that humans can sense vibratory stimulus from a few Hz up to 800 Hz [99].

Today, numerous sensing systems have been developed for human-machine interfaces to sense signals such as physiological parameters relevant to human health conditions, including electro-cardio signal, pulse wave, blood pressure, and body motion [97]. On the other hand, the progress of wearable mechanical stimulators [100] remains very challenging. For example, traditional actuating systems have used large linear resonators, eccentric rotating masses, and voice coils to generate mechanical stimulations. As a result, these systems are bulky and often stationary with very limited applications. There are several “wearable” prototype stimulators in various research stages such as tactile devices on the fingertips to recreate interaction forces, but these systems are still bulky. Several soft and thin actuators have been reported to address the size issue based on piezoelectric polymers to make flexible, thin, and light-weight actuators [101, 102]. However, they require very high driving voltage (hundreds of volts) and they don’t have good spatial resolution for multi-modal stimulations to mimic complex sensations. Due to these limitations, there has been great effort to have airborne haptics [103].

Table 4.2: Types of the mechanoreceptors inside the human skin and their location, class, respond types and temporal resolution properties [94, 95].

Description of the device	Output Force @ a Distance	Total Number of Transducers	Spatial Resolution*
12-channel annular array [103, 104]	8mN @ 250 mm	91	20mm
4 focused annular arrays [105]	47mN @ 250 mm	364	20mm
18x18 ultrasound array [106, 107]	16mN @ 200 mm	3324	20mm
AUTD unit system [108, 109]	73mN @ 600 mm	2241	20mm
Aerial input/output system [110]	12mN @ 300 mm	384	3.4mm
Tactile visio-acoustic screen [111]	16mN @ 240 mm	249	10mm

* Spatial resolution equals to the focal point diameter

4.3 Concept and Operating Principle

An array of 12x12 pMUTs is used to emit unfocused ultrasonic signals as shown in Figure 4.1a by modulating the high operation frequency to a lower frequency range using

the pulse width modulation (PWM) scheme to induce the sensations on human mechanoreceptors remotely. Figure 4.1b illustrates the cross-sectional schematic of the structure with a deformed diaphragm. The diaphragm is $415\ \mu\text{m}$ in radius and is composed of two $1\ \mu\text{m}$ -thick piezoelectric active layers (the dual-electrode bimorph structure [54]) sandwiched by three 150nm -thick Molybdenum electrode layers (Figure 4.1b). The inner 70% of the diaphragm area composes the inner electrode and the rest composes the outer electrode. One of the practical solution to have high diaphragm deflections is to adjust the physical boundary condition of the diaphragm [33] such that the mechanical stiffness of the diaphragm is decreased and the saddle point inside the strain field is pushed towards the boundary. In this study a pinned boundary [23] is utilized by etching away the outer boundary of the upper piezoelectric layer.

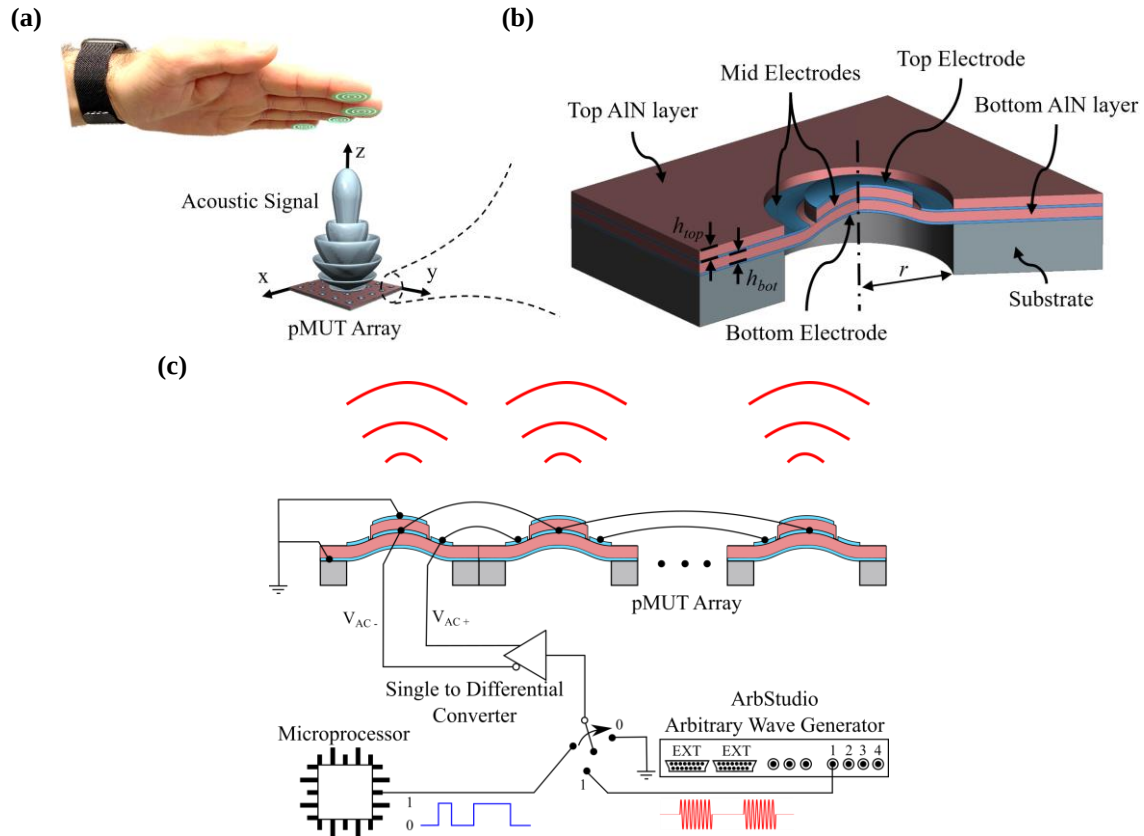


Figure 4.1: (a) Illustration of the haptic perception of a sound generated by a bimorph pMUT array. (b) Cross-sectional schematic view of the (deformed) pinned boundary, dual electrode, bimorph pMUT. (c) Operational principle of pMUT array with frequency modulation so that mechanoreceptors can detect the ultrasonic radiation and microprocessor to have pattern on the skin.

The operational principle is illustrated in Figure 4.1c. The transducers have high resonant frequency which is much larger than the mechanoreceptors' dynamic range. Therefore, this high frequency is down modulated into a lower region 100-300 Hz by the pulse width modulation (PWM). The modulated signal turned on and off in a timely manner which is controlled by a preprogrammed microprocessor. The pMUT array has the double electrode design and the inner and outer electrodes are driven with a 180° phase difference.

4.4 Device Fabrication

Polycrystalline Aluminum nitride (AlN) is deposited using the reactive sputtering process. The quality of polycrystalline AlN layers is measure using the X-ray diffraction (XRD) method (Figure 4.2a). The Full-Width-Half-Max (FWHM) of the rocking curve is an indicator of the polycrystalline orientation and a small FWHM is desirable for good crystal orientation [86]. In the work, it is measured as 1.6° (Figure 4.2b). Fabrication begins with the deposition of the AlN/Mo/AlN/Mo stack (Figure 4.3a) and follows by the patterning of the Mo layer as the middle electrode (Figure 4.3b). The thickness of the first AlN layer which also serves as the backside etch stop is around 200 nm. The first sputtered Mo layer which has the thickness of 150 nm is the bottom electrode. On top of the bottom electrode, a $1\mu\text{m}$ -thick AlN layer is sputtered as the first piezoelectric layer. The second 150 nm-thick Mo layer is deposited as the middle electrode. Another stack of material, AlN/Mo, is sputtered and the Mo layer is patterned to define the top electrodes (Figure 4.3c). The thicknesses of the second AlN and top Mo layers are $1\mu\text{m}$ and 150 nm, respectively. The upper AlN layer is etched to have the pinned boundary (Figure 4.3d). Patterning and etching of the AlN layers are done to open the contact pads on the bottom electrode (Figure 4.3e). Finally, a back side DRIE etch of the Si substrate is used to release the diaphragm [60] (Figure 4.3f). The detail fabrication process is given in Appendix D.

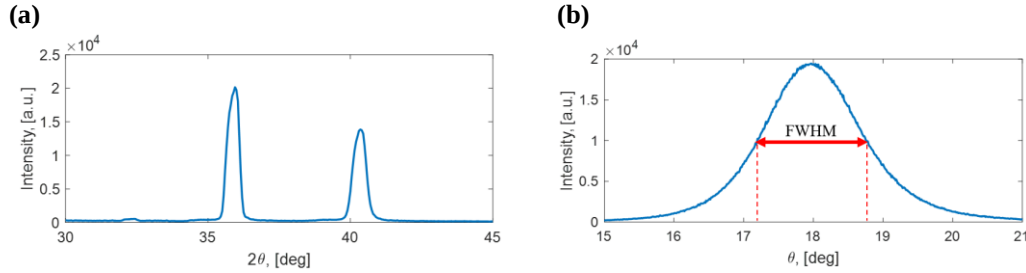


Figure 4.2: X-ray Diffraction measurements (XRD) of deposited AlN thin films. (a) Locked couple scan, (b) Rocking curve scan

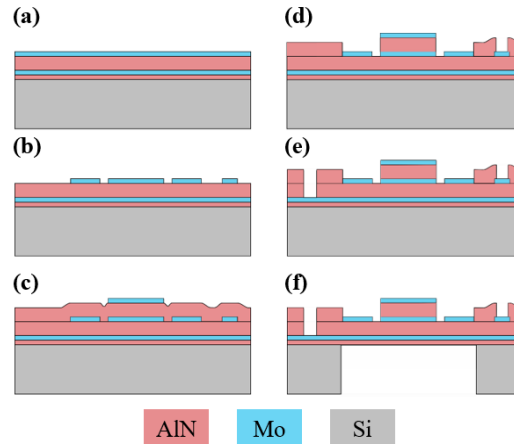


Figure 4.3: Fabrication flow of bimorph, dual electrode, pinned-clamped boundary pMUTs. (a) Deposition of Mo/AlN/Mo/AlN. (b) Patterning of mid-Mo layer via dry etch to define inner and outer electrodes. (c) Second Mo-AlN stack deposition and top Mo electrode patterning. (d) AlN etching for mid via and pinned boundary top layer. (e) AlN etching for bottom via. (f) Backside DRIE Si etching to define the diaphragm.

4.5 Device Characterization

Figure 4.4a shows an optical image of the fabricated pMUT array with 12×12 elements. The cross-section SEM image of the array and diaphragm is given in Figure 4.4b and Figure 4.4c, with two active AlN layers, three Mo electrodes (top, mid, and bottom), and the AlN seed layer at the bottom which is also the etch stop layer

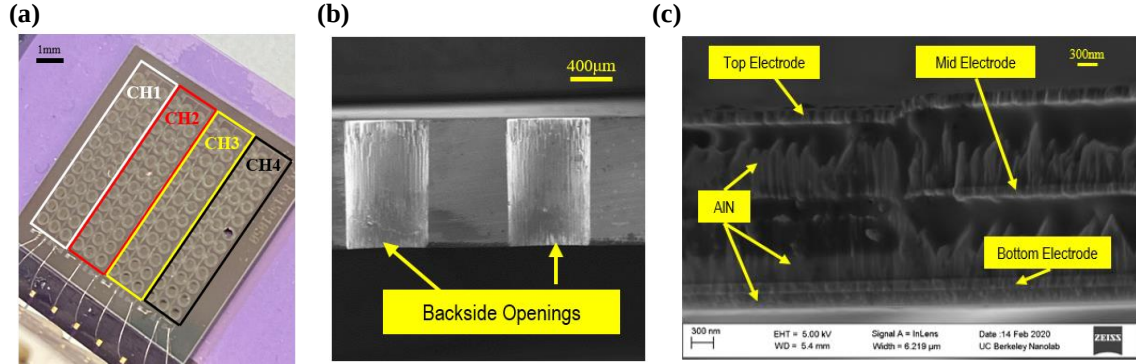


Figure 4.4: (a) An optical image of the fabricated 12×12 pMUT array. (b) A SEM image of the cross-sectional view showing the back side DRIE results. (c) A close-up SEM image of the cross-sectional view showing the two active AlN layers, the three Mo electrode layers, and the bottom stack of Mo/AlN, which also serves as the etch-stop layer for the DRIE process.

Due to the internal stress and stress gradient of the deposited thin films, the fabricated diaphragms of $415 \mu\text{m}$ in radius have bended after the backside etching and releasing process. This results a higher resonance frequency in air than the designed value. In Figure 4.5, the Digital Holographic Microscopy (DHM) measurements of the diaphragm surface profile without any excitation is shown. The intensity and phase graphs are given in Figure 4.5a and Figure 4.5b, respectively. Three different elements in the array are measured with the surface profile using the section view shown in Figure 4.5a, and results are given in Figure 4.5c. On average, the mid-point deflection is around $4.5 \mu\text{m}$ at zero excitation voltage.

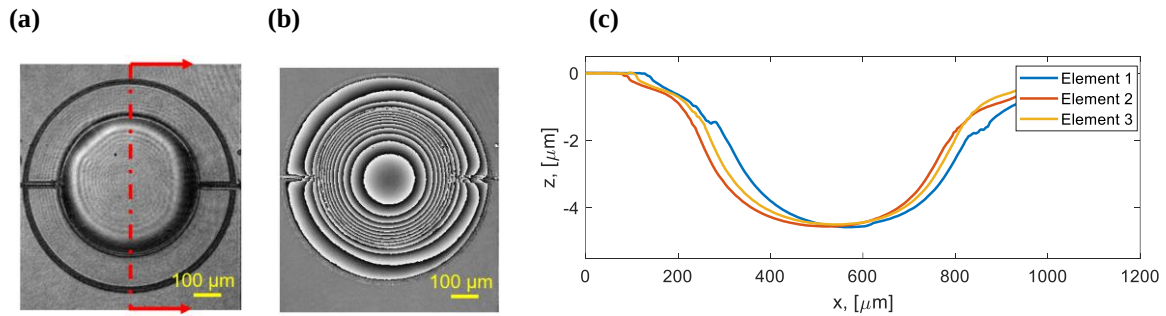


Figure 4.5: (a) Light intensity, and (b) phase images in DHM measurements. (c) The deflection of diaphragms due to internal stress and stress gradient after the backside releasing process under zero excitation voltage.

The array composes of 144 elements and the frequency of one of these elements is measured with Laser Doppler Vibrometer (LDV). The LDV experimental setup is given in Figure 4.6a, and the frequency response of an element is given in Figure 4.6b. For the combined response of the 144 elements, the impedance of the array is measured (Figure 4.6d). The natural frequency of the array is 109.4 kHz . In addition, the responses of the

array at the resonance under different amplitudes of excitation voltage are measured as shown in Figure 4.6c. It is found that the array has the linear behavior up to 6 V_p of the excitation voltage. It exhibits nonlinear behavior as the velocity does not linearly increase when the excitation voltage increases above 6 V_p. This is known as the nonlinear stiffening of the structure, and it is a well-known phenomenon.

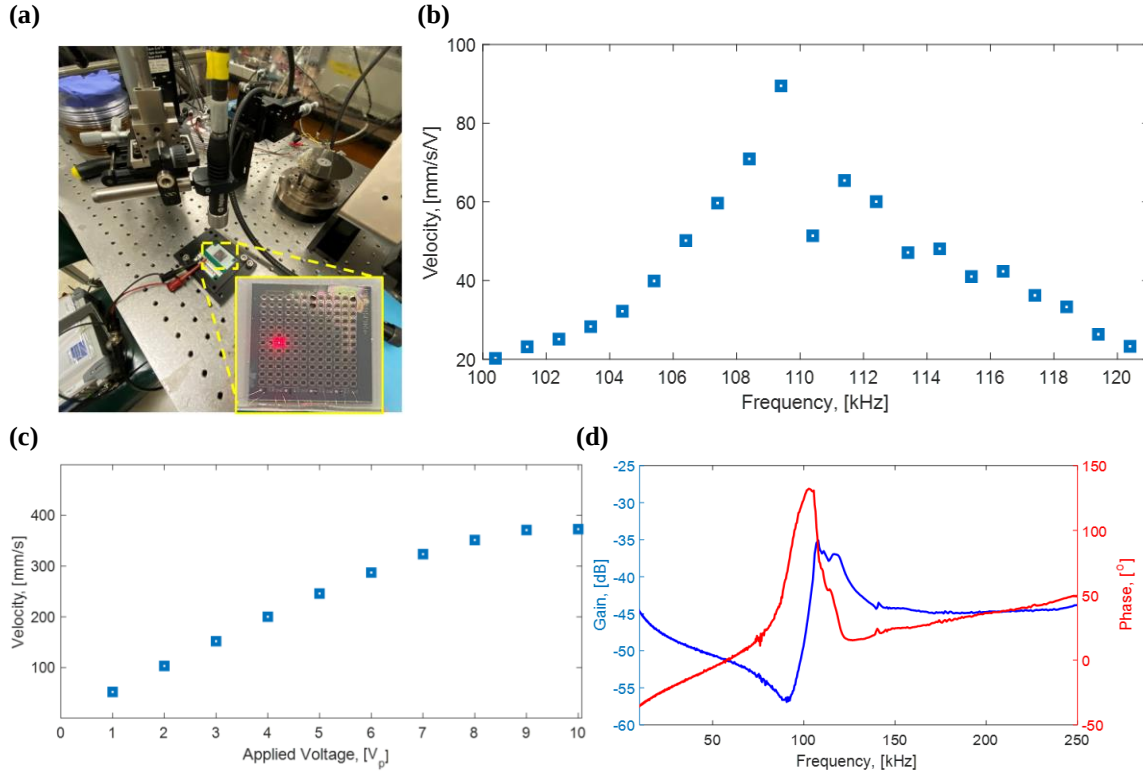


Figure 4.6: (a) LDV experimental setup. (b) Frequency response of a single element by means of velocity measurement. (c) The velocity vs applied voltage amplitude plot showing the start of the nonlinear behavior region. (d) Frequency response of a whole array by means of impedance measurement.

Figure 4.7a shows the optical image of a 2x2 pMUT array for the pressure output comparison. The pressure outputs of 1, 2, 4, and 8 elements are given in Figure 4.7b together with analytical calculations. It is important to note that the analytical model does not account for nonlinear effects; however, there is amplitude stiffening effect in the experiments after a certain applied voltage amplitude.

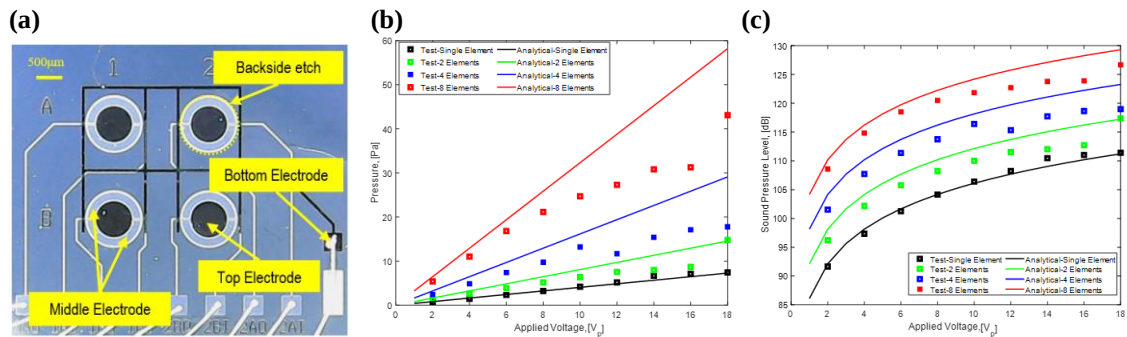
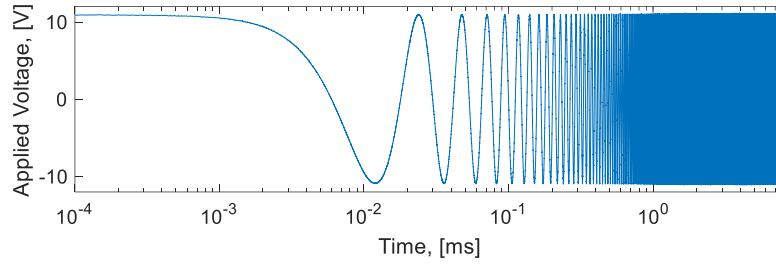


Figure 4.7: (a) Optical image of a 2x2 pMUT array for pressure output comparisons. (b) Pressure outputs of one, two, four, and eight pMUT elements 15 mm away, and (c) their corresponding sound pressure levels.

The device has been characterized in the ambient air environment by applying 8 ms of chirping signals at the frequency range of 40-200 kHz with 12V_p amplitude (Figure 4.8a). The pressure outputs of the 12×12 transducer array at its natural frequency is 950 Pa 15 mm away as shown in Figure 4.8b. While the prototype diaphragms have slightly different natural frequencies due to fabrication variations, the array has the maximum pressure output at 109.4 kHz, which is used for the rest of the test.

(a)



(b)

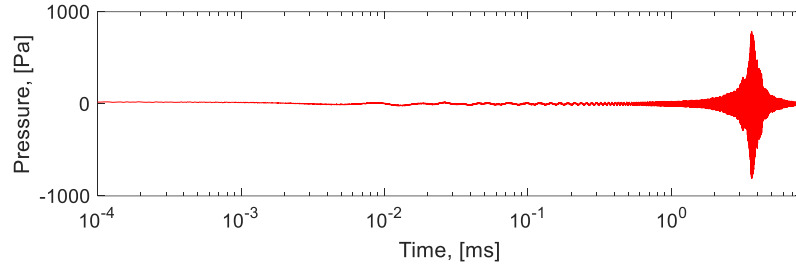


Figure 4.8: (a) Applied 8ms of chirping signals at the frequency range of 40-200 kHz with 12V_p amplitude. (b) The captured ultrasonic pressure.

The pressure at a distance from the source is given as:

$$P_{Rx} = G_{ch} P_{Tx} \quad (4.1)$$

where P_{Rx} is the pressure at receiving end; P_{Tx} is the pressure at the radiating source, and G_{ch} is the channel gain (attenuation) at the events of propagating, reflection, and etc. For a time-of-flight measurement, the channel gain is given by:

$$G_{ch} = G_{path} G_{target} \quad (4.2)$$

where G_{path} is the propagation path gain due to the media absorption as the conversion of vibrational energy into heat and spreading; and G_{target} is the reflecting surface gain with two parts: the reflection/transmission coefficients due to the acoustic impedance mismatch and the geometric gain. The path gain is composed of two parts: spreading and absorption which are given by [112]:

$$G_{path} = \underbrace{\frac{a_c}{2R}}_{\text{Spreading}} \underbrace{10^{-\alpha R}}_{\text{Absorption}} \quad (4.3)$$

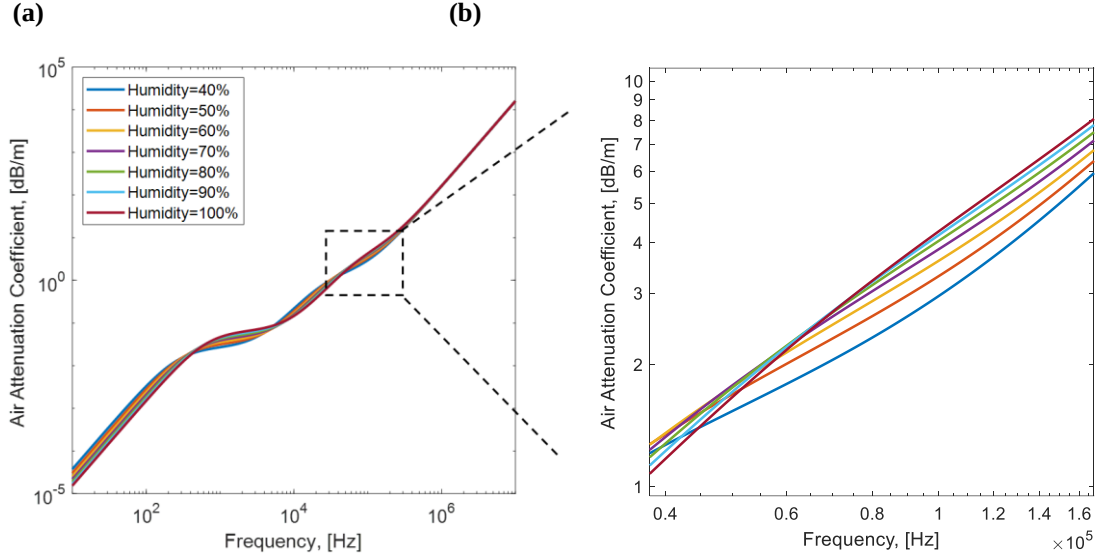


Figure 4.9: Air attenuation (absorption) coefficient with respect to frequency for: (a) a wide frequency range, (b) the frequencies in this chapter.

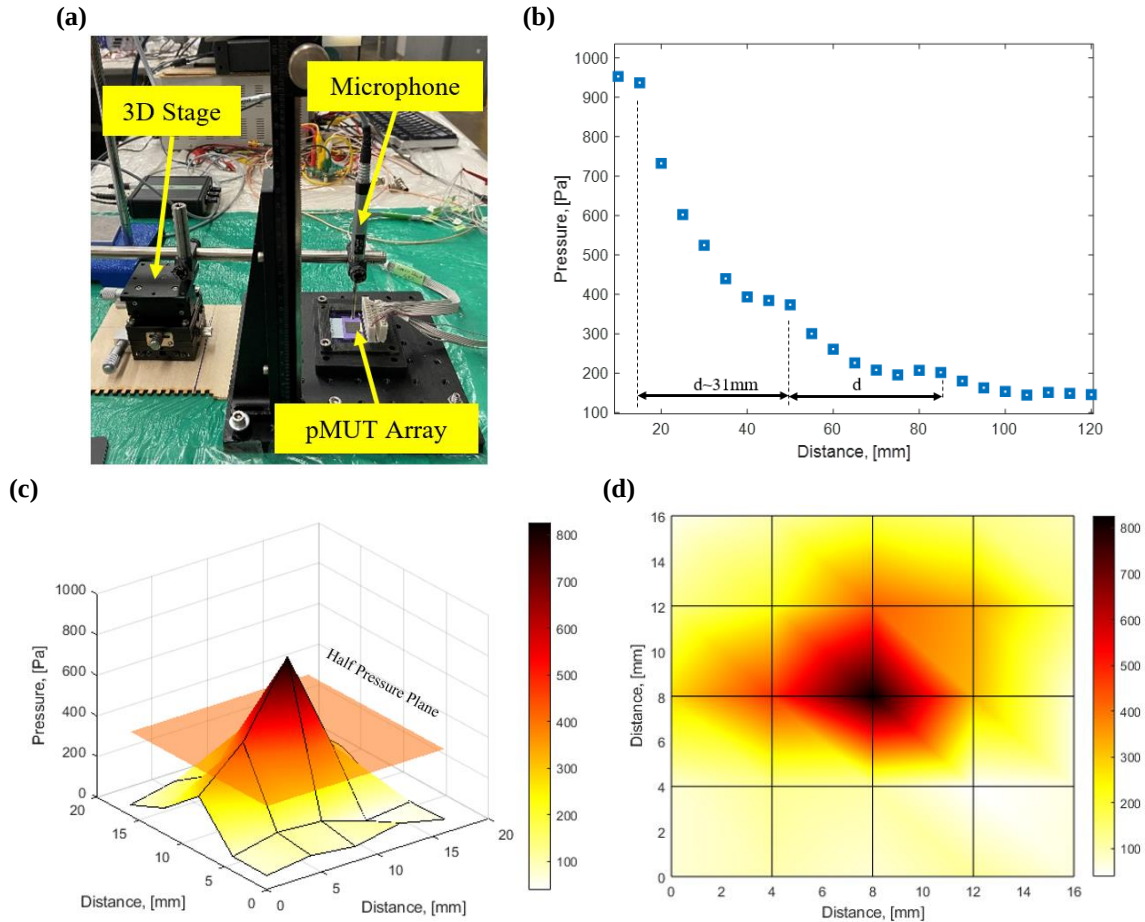


Figure 4.10: (a) Pressure experimental setup showing device under test, microphone, and 3D stage used for placing the microphone at any point in the air medium. (b) Pressure measurement versus axial distance. The wavelength of the ultrasound is around 31 mm which is clearly identified on the pressure vs. axial distance graph. (c) Pressure measurement versus planar distance together with half pressure plane. (d) Isopleth map of the planar pressure measurement for the focal point dimension measurement.

where a_c is the equivalent radius; α is the air attenuation coefficient; and R is the distance. The attenuation coefficient is a function of temperature, humidity, and the elements in the medium [48, 113-115] is given in Figure 4.9a with zoomed-in detailed version in Figure 4.9b for the frequency range in this work.

Experiments have been held in the air environment with an array composed of 12×12 pMUT elements. The device characterization has been studied using Bruel & Kjaer 4136 microphone located 15 mm away from the array surface with the 2669L preamplifier. The setup has a 3D stage in order to place the microphone at any point in the medium to characterize the total acoustic field generated by the pMUT array as shown in Figure 4.10a. The pMUT array is driven with 12 V_p amplitude at 109.4 kHz. The pressure with respect to the perpendicular distance from the array surface is shown in Figure 4.10b. At the 15 mm axial distance, the pressure output is around 950 Pa. The axial distance is fixed and the microphone is swept in the planar distance for the planar pressure measurement as given in Figure 4.10c with a plane showing half of the maximum pressure. Lastly, the isopleth map of the planar pressure measurement is given in Figure 4.10d in order to identify the natural focus diameter of the acoustic field. The focal dimensions of the 12×12 array is around 7 mm x 7 mm.

4.6 Frequency Modulation and Its Effects on Pressure Output

The natural frequency of the pMUT array (Figure 4.11a) is much larger than the maximum sensing frequency of the mechanoreceptors in human skin. Therefore, this high frequency is down modulated to a lower frequency using the pulse width modulation (PWM) as shown in Figure 4.11b. The PWM frequency is 100 Hz and duty cycle is 50%. Multiplication of these two generates the modulated excitation signal which is illustrated in Figure 4.11c. It is important to verify that the generated sound field also modulated with this low frequency. The distance between the pMUT array and the microphone is 15 mm as it has been determined from the maximum pressure output measurements in the device characterization process. The excitation and read-out circuitry are shown in Figure 4.12c. The pMUT array is driven using ArbStudio Arbitrary Wave Generator (AWG) with a square signal having an amplitude of 12 V_p at the natural frequency of the transverse fundamental mode of the pMUT diaphragms at 109.4 kHz with the PWM modulation having the frequency of 100 Hz and duty cycle of 50%. The microphone is placed 15 mm away from the surface of the array. The output of the microphone is fed into a PicoScope and to a computer. The applied signal and normalized pressure are shown in Figure 4.12 and Figure 4.12b. As seen, the measured pressure follows the modulated applied signal with small ramp up and ramp down lags. Therefore, it is concluded that the pressure modulation is achieved for sensing by the mechanoreceptors in human skin.

The effects of modulation parameters such as frequency and duty cycle on the output pressure are analyzed. In Figure 4.13a, the normalized pressure output as a function of duty cycle is shown for a fixed modulation frequency (100 Hz). It is seen that the maximum average pressure output is achieved at 50% duty cycle. On the other hand, Figure 4.13b shows the result of pressure output experiment for different modulation frequencies with a

given duty cycle, 50%. Lastly, Figure 4.13c summarizes all duty cycles between 10-90% and modulation frequencies between 20-500 Hz. Although mechanoreceptors in human skin are most sensitive to 200 Hz, the modulation frequency in this work is selected as 100 Hz in order to have a high average pressure output. It is noted that the instantaneous maximum pressure output does not change with respect to the modulation frequency and duty cycle. However, the pressure takes some time to build up and Figure 4.13d shows how the pressure builds up with different duty cycles.

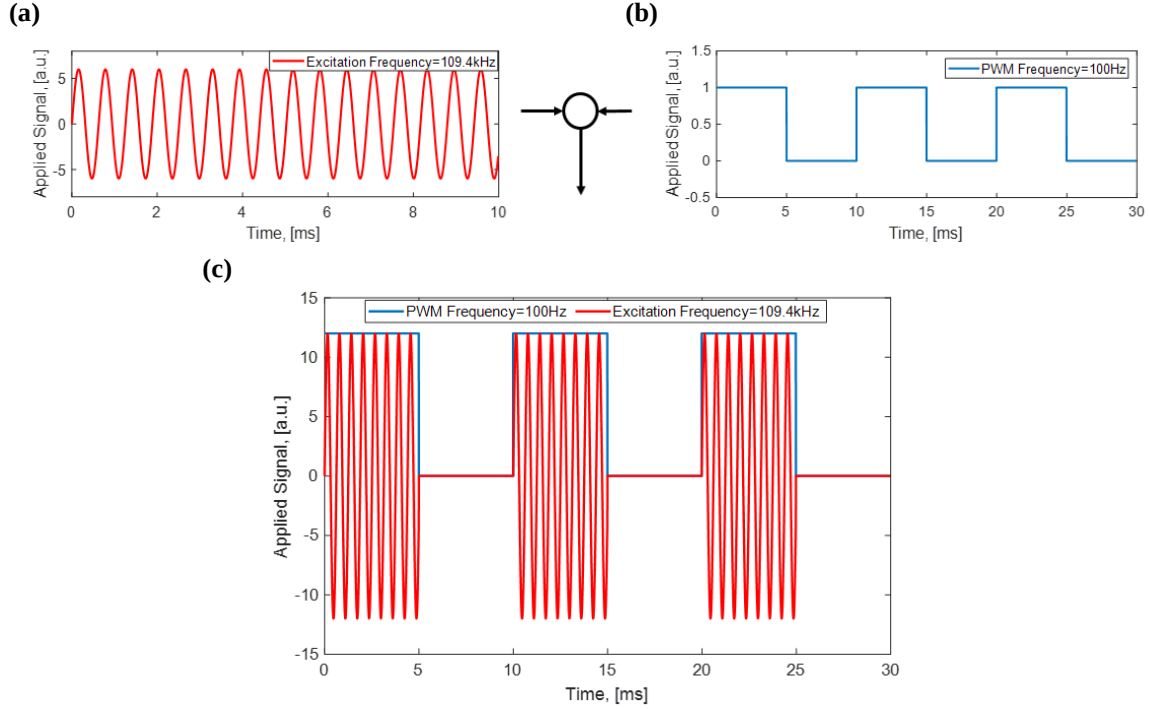


Figure 4.11: (a) Excitation signals of the original high frequency pMUT at 109.4 kHz. (b) Pulse width modulation signal with 100Hz frequency and 50% duty cycle. (c) Frequency modulated excitation signal.

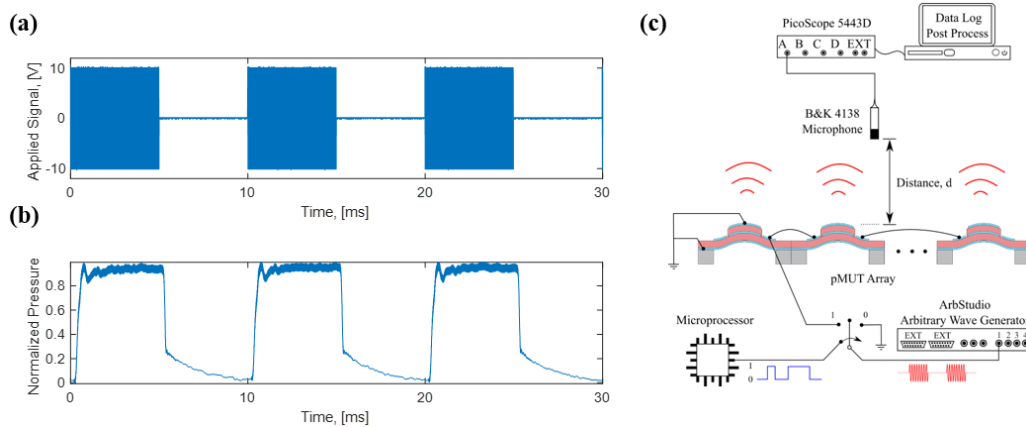


Figure 4.12: (a) Modulated applied signal; (b) read out pressure; (c) the excitation and read out circuitry.

Lastly, the effect of modulation on the maximum pressure output in the axial distance is shown in Figure 4.14a for different modulation frequencies. It is seen that the maximum instantaneous pressure is slightly different in each modulation frequency due to fact that it takes some time as explained above. In Figure 4.14.c, the effect of modulation on the

maximum pressure output in the planar direction is given. Overall, the formed focal point has similar dimensions; therefore, it is assumed that the beam dimensions are not affected by the PWM modulation frequency.

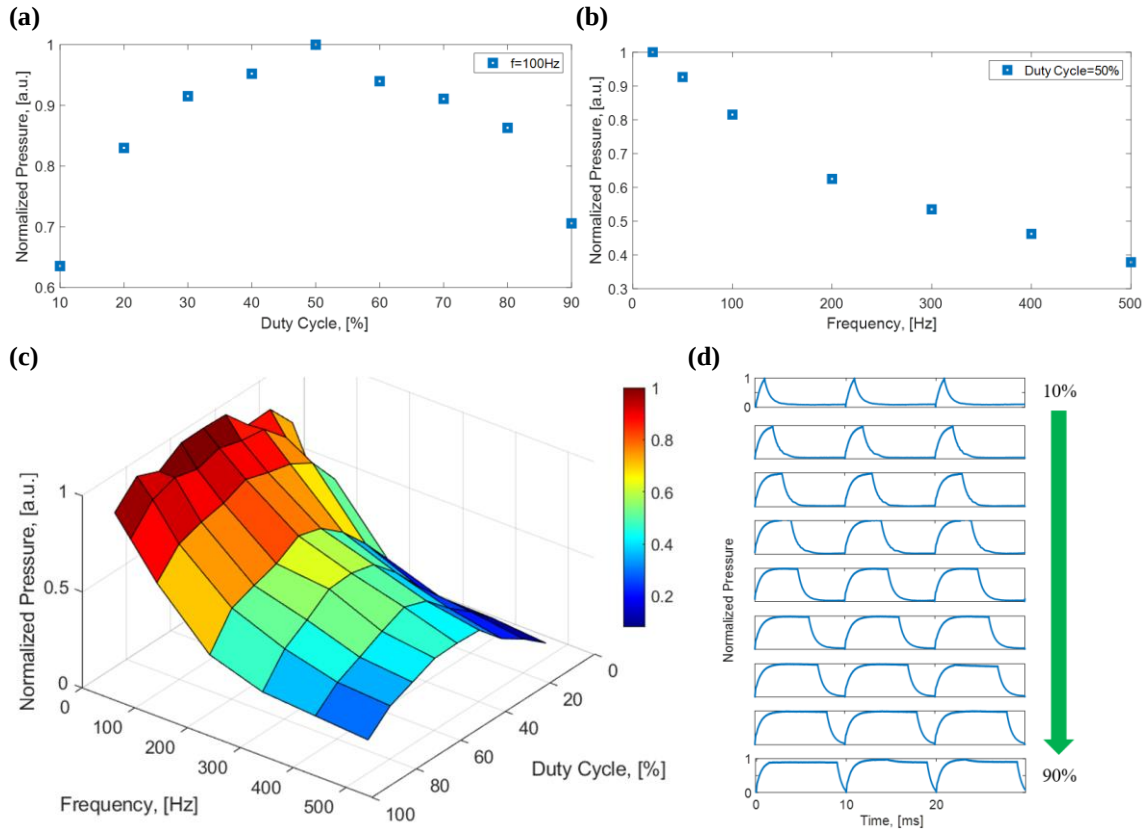


Figure 4.13: (a) Normalized pressure vs duty cycle with the PWM with a given modulation frequency of 100 Hz. (b) Normalized pressure vs modulation frequency with a given duty cycle of 50%. (c) Normalized pressure vs duty cycle between 10-90% and modulation frequency between 20-500 Hz. (d) The normalized pressure for duty cycles between 10-90% showing the buildup time.

The optimal case for haptic experiment is selected as 15 mm away from the pMUT array with a PWM modulation frequency of 100 Hz and the duty cycle of 50%. These settings result in a natural focal point region of the array transducers as 7 mm x 7 mm, which is sufficiently small for human fingers. In Figure 4.14d, the pressure output of the 12 x 12 array pMUT shown in 2D to verify that the natural focus point of the array is at around 15 mm distance away from the surface of the array.

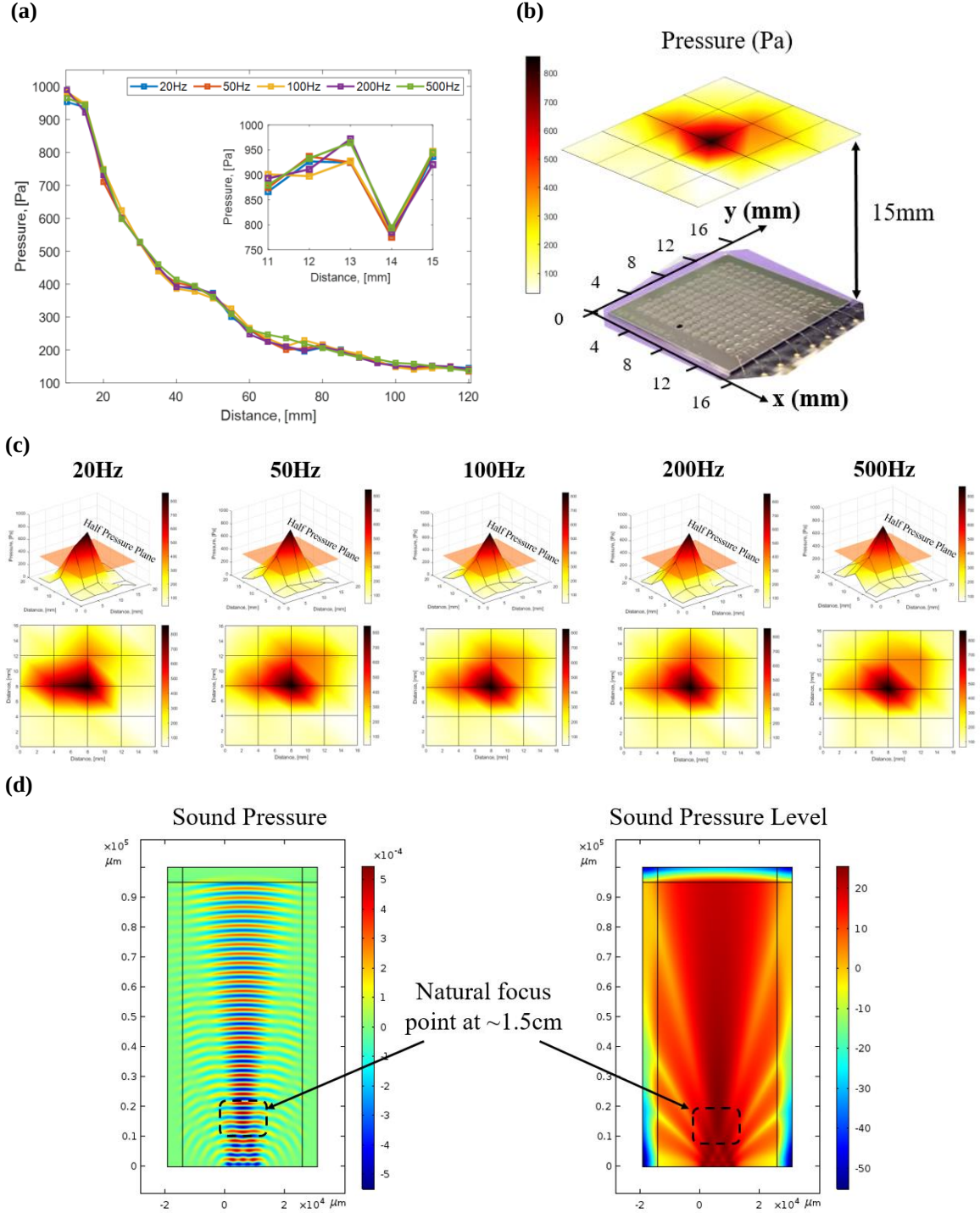


Figure 4.14: (a) Pressure measurement vs axial distance from the transducer surface with different modulation frequencies. Zoomed-in region is the distance between 11-15mm away from the surface. (b) Illustration of distance and modulation settings used in haptics experiments. The Distance is 15 mm away from the array surface and the modulation frequency is 100 Hz with the duty cycle if 50% to result a 7 mm x 7 mm focal point region. (c) Isopleth map of the planar pressure measurement for the focal point 15 mm away from the array surface for different modulation frequencies between 20-500 Hz. (d) FEM simulation of a 12 x 12 array in order to verify the distance of the natural focus point of the array.

4.7 Haptics Performance

The haptic sensation is generated by the acoustic radiation pressure. Assuming a plane wave, the acoustic radiation pressure is given as [110, 116-119]:

$$P_r = \alpha \frac{I}{c} = \alpha \frac{p^2}{\rho c^2} \quad (4.4)$$

where I is the sound intensity of the ultrasound; c is the speed of sound in the medium; ρ is the medium density; p is the acoustic pressure; and α is a constant ranging from 1 to 2 depending on the reflection coefficient R of the surface [120]:

$$\alpha = 1 + R^2 \quad (4.5)$$

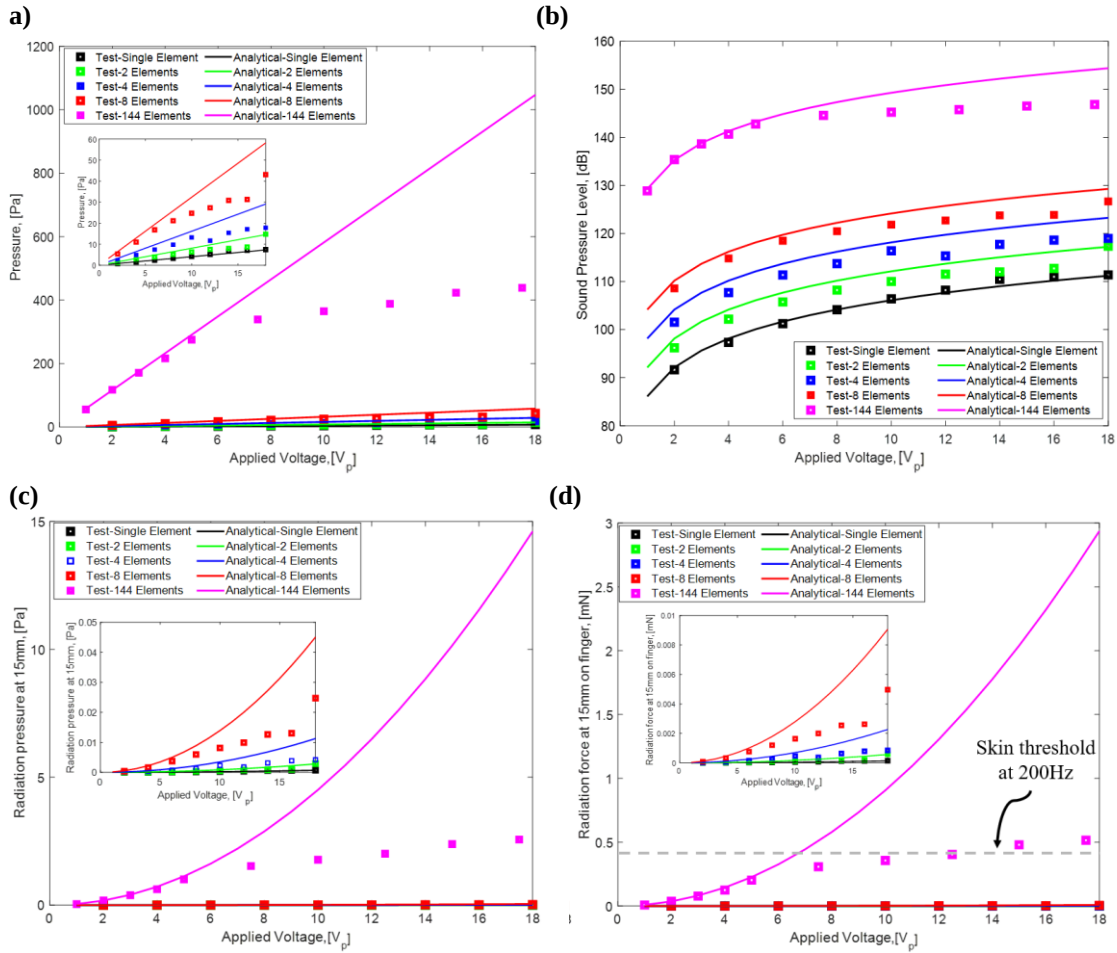


Figure 4.15: (a) Acoustic pressure, (b) Sound pressure level, (c) Acoustic radiation pressure, and (d) Acoustic radiation force on area equal to the average finger surface area 15mm away from the transducer array.

The reflection coefficient is defined by the acoustic impedances of the two media, Z_1 and Z_2 . Assuming the incident sound wave is in the first medium and reflected at the interface between first and second medium, the reflection coefficient is given as [48, 50]:

$$R = \frac{Z_1 - Z_2}{Z_1 + Z_2} \quad (4.6)$$

Finally, the acoustic radiation force is obtained as [121-125]:

$$F_r = A_{finger} P_r \quad (4.7)$$

where A_{finger} is the average finger surface area. The finger average radius is taken as $r_{finger} = 8\text{mm}$. The obtained acoustic pressure, acoustic radiation pressures and forces for 1, 2, 4, 8, and 144-element array are given in Figure 4.15. The sound pressure and sound pressure levels are given in Figure 4.15a and Figure 4.15b, respectively. They are converted into acoustic radiation pressure and acoustic radiation force using the equations (4.4) and (4.7), respectively. Figure 4.15c shows the acoustic radiation pressure and Figure 4.15d shows acoustic radiation force together with the human skin threshold at 200 Hz. Note that the applied acoustic radiation force is slightly larger than the threshold.

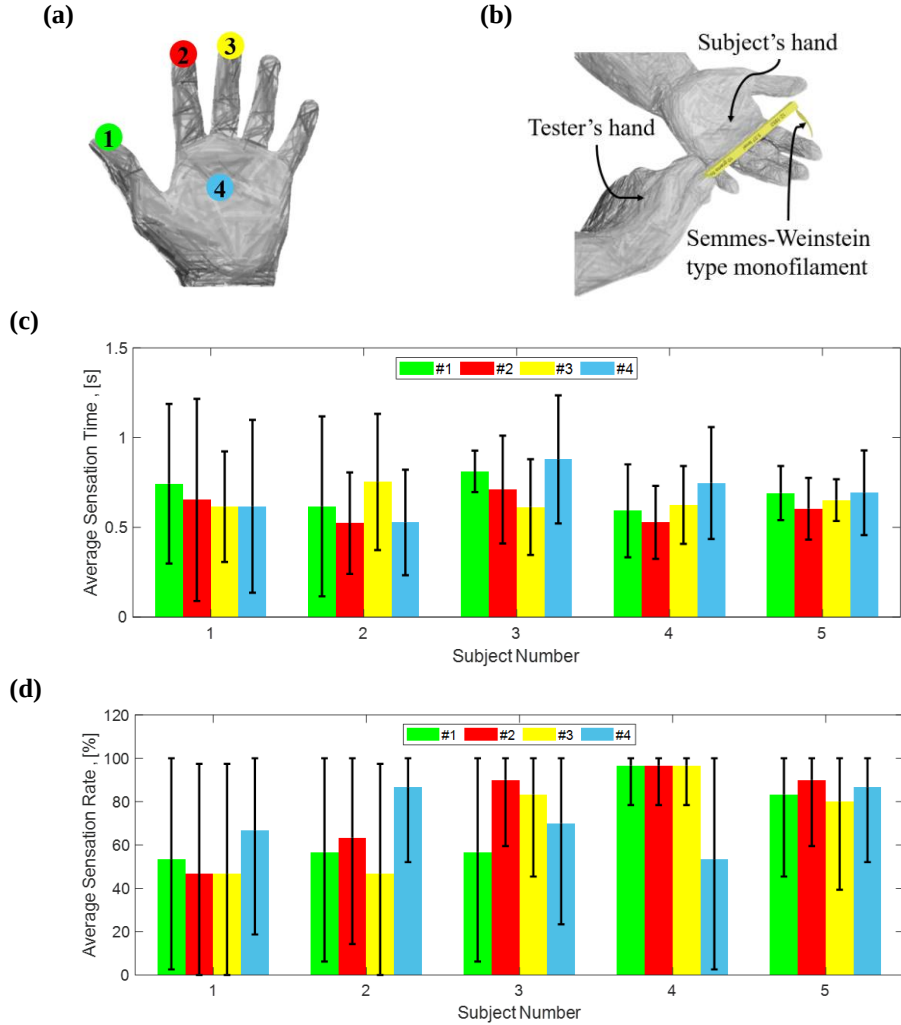


Figure 4.16: (a) Measured points on the volunteer's hand. (b) Non-invasive evaluation of cutaneous sensation level of each subject with Semmes-Weinstein type monofilament having a force rating of 0.687mN. (c) Average sensation time of five different volunteers. (d) Correct identification rate of each points by 5 subjects.

The sound pressure is converted into acoustic radiation pressure and acoustic radiation force using the equations (4.4) and (4.7), respectively. Figure 4.15.c shows the acoustic radiation pressure and Figure 4.15.d shows acoustic radiation force together with the human skin threshold at 200 Hz. Note that the applied acoustic radiation force is slightly larger than the threshold at $12V_p$ excitation voltage.

Before the ultrasonic haptic sensations experiment, the force sensitivity of each subject of both genders having the age range of 20-30 years old is determined by using the Semmes-Weinstein type monofilaments with force rating of 0.07 grams which equals to 0.687 mN. This value is selected as close as possible to the threshold value given in the literature [91]. During the sensitivity test, subjects are asked to close their eyes in order to isolate the haptic and visual sensation. There is no auditory input in this test; hence, subjects did not use ear plugs. The Figure 4.16a shows the measured points in volunteer's hand and Figure 4.16b shows the illustration of cutaneous sensation level with monofilaments. Each point on the subject's hand is tested 10 times for each subject. The average sensation time to 0.687 mN is given in Figure 4.16c with the same color code in Figure 4.16a to identify the locations. The average sensation time is 0.66 seconds with a standard deviation of 0.32 seconds. The overall correct identification rate is 72.5% with a standard deviation of 42.69% as shown Figure 4.16d.

The ultrasonic-induced haptic perception experiments have been conducted by testing the sensations of 5 different volunteers of both genders having the age range of 20-30 years old. These volunteers are asked to close their eyes and wear ear plugs in order to isolate them from possible environmental visual and sound disturbances. Their hands are placed inside an isolation chamber as shown in Figure 4.17a.

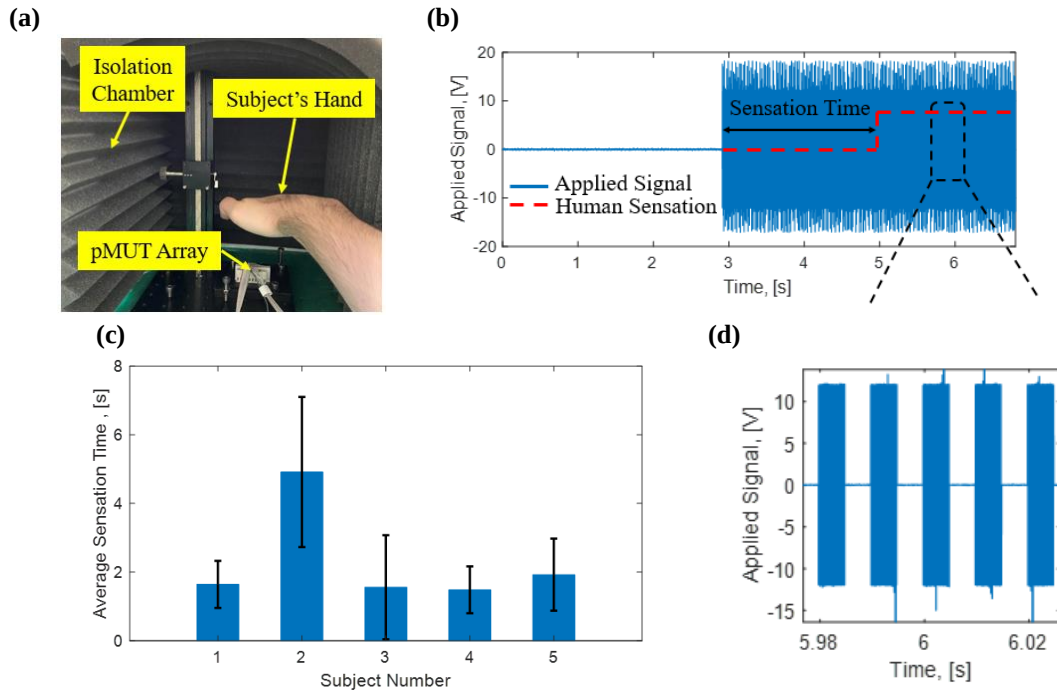


Figure 4.17: (a) Ultrasound-induced haptics experimental setup showing the relative placement of pMUT array and subject's hand inside an isolation chamber. (b) Applied signals (blue) and human sensation (red) showing a time delayed response. (c) Average sensation time of five different volunteers. (d) Zoomed-in applied signal showing the 100 Hz modulation with 12 V_p.

High frequency ultrasonic waves have been modulated to 100 Hz patterns by PWM with 50% duty cycle to stimulate mechanoreceptors in human skins as shown in Figure 4.17b and Figure 4.17d. The time for the sensation feeling has been measured with five volunteers. It is found that there is a time delay between the applied radiation pressure and human sensation as shown in Figure 4.17b. The average sensation time from 10 measurements per volunteer as well as the corresponding standard deviation are shown in Figure 4.17c as 2.3 seconds and 1.22 seconds, respectively.

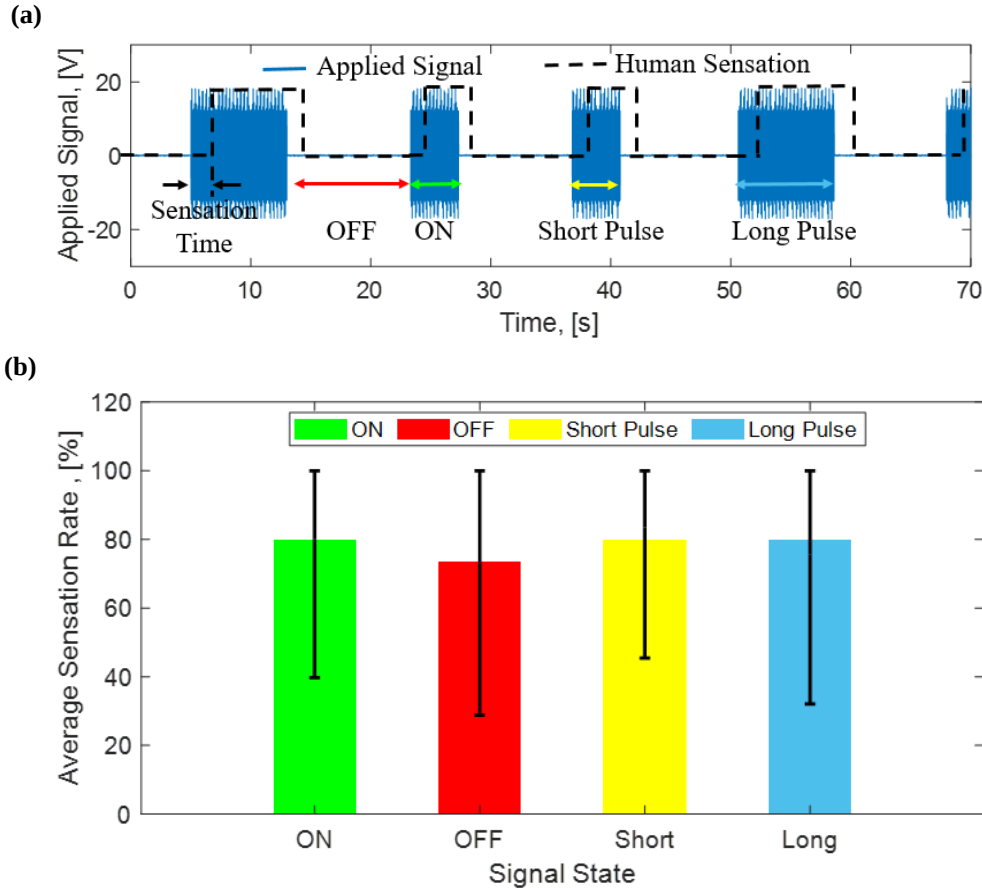


Figure 4.18: (a) Four types of switching experiments; switching on, switching off, short pulse, and long pulse. The human sensation time delay is included on the applied signals. (b) The correct identification rate of these four events by the five subject.

The modulated signals are switched between ON and OFF with a preprogrammed microprocessor shown in Figure 4.1c. There are four types in experiments: switching on, switching off, short pulse, and long pulse as shown in Figure 4.18a. A long pulse duration is defined as 8 seconds and a short pulse duration is defined as 4 seconds. The correct identification rate of these four events by the five subject is given in Figure 4.18b. Overall the correct identification rate is about 80%. Another outcome is that switching off is slightly difficult to be recognized as compared to other three events. The applied acoustic pressure is just above the skin threshold at 200 Hz and this threshold is in fact larger at 100 Hz. Therefore, turning signal off is difficult to be sensed by the subject given that the human mechanoreceptors adapt themselves to constant force amplitude and becoming less responsive.

Volunteers have also been tested with the transmission of the Morse code letters by means of long and short radiation pressure durations in Figure 4.19a. Three arbitrary letters, “C”, “A”, and “L”, are selected. Corresponding to these Morse code letter, the applied signal is given in Figure 4.19b. Similar to the switching experiment, a long pulse duration and short pulse are defined as 8 seconds and 4 seconds, respectively. The time period between the pulses is also 8 seconds. The average correct sensation rate of 5 volunteers is given in Figure 4.19c for each Morse character and the average correct sensing is 76%. It is observed that the correct sensation rate decreases towards the end of the experiment due to the fatigue of sensation for long testing durations.

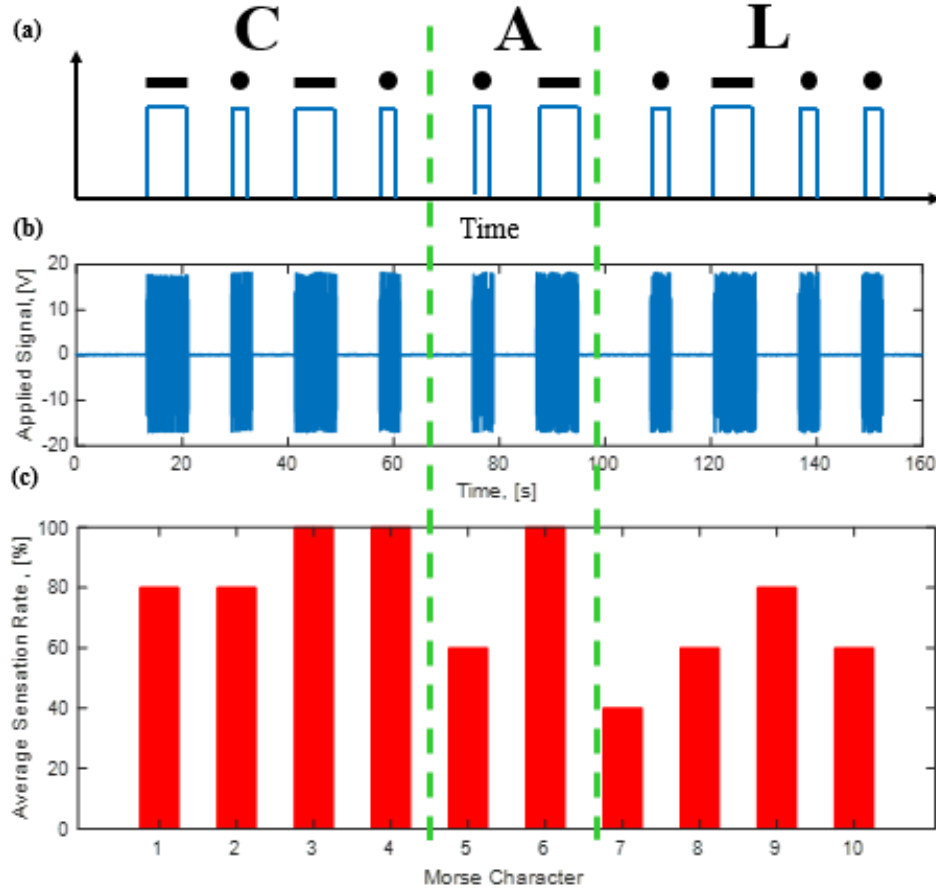


Figure 4.19: (a) The Morse code letter for “C”, ”A”, ”L” as dash and dots. (b) Applied signals to write “C”, ”A”, ”L” with the Morse code with short and long pulses. (c) Correct sensation results in terms of percentage of 5 volunteers.

4.8 Conclusions

In this chapter, an ultrasound-induced haptic interface device by using the pMUT array is demonstrated for the first time. By emitting the ultrasonic waves at the natural frequency of 109.4 kHz, a local maximum acoustic pressure at 950 Pa has been measured at 15 mm distance away from the transducer array. The pulse width modulation schemes have been utilized to convert the high frequency signals to a low frequency pattern at 100 Hz with 50% duty cycle in the haptic perception experiments under an applied amplitude is 12 V_p.

Corresponding radiation force is calculated as 0.4 mN which is reported as human skin threshold. It is found that the average sensation delay time of five different volunteer is 2.3 seconds and this is likely coming for the acoustic heating effect for the delay in response. The Morse code transmission experiment has been designed and conducted and results show an average correct identification rate of 76%. These results imply that the proposed concept has a promising potential to be used as high intensity, low power, and small footprint haptic perception interface for various practical fields, such as in medical, entertainment, and AR/VR applications.

Chapter 5

Effect of Reflector Curvature on the Pulse-Echo Method

In this chapter, the effect of structural curvature is measured in both amplitude and time-of-flight schemes. The sound pressure experiences an attenuation due to diffraction when it hits a curved structure whose dimensions are comparable to the wavelength of the ultrasonic wave. The characterization of this attenuation is important for applications needing high signal-to-noise ratios such as the blood pressure measurement. The diameter of the blood arteries and veins in the human body varies from tens of microns to millimeters. One practical issue in the detection of these non-flat surfaces is the ultrasonic detection attenuations in measurements. Furthermore, by using multiple receivers with known separation distances, it is possible to measure the time delays of each received signal. A non-contact radius-of-curvature measurement can be extracted from these time delay echo data as the speed of sound is a constant in the medium.

The effects of surface curvature by the pulse-echo sensing method in ultrasound is investigated with an analytical model, numerical simulations, and experimental validations. As a case study, the blood pressure measurement is selected with artificial tubes to emulate arteries and veins in the human body. The noncontact radius-of-curvature (RoC) measurement of a curved surface using ultrasonic transducers is also presented and the detailed fabrication flow of the pMUT array is explained. The proof-of-concept tests are held in the air environment using 3D printed curved surfaces with known RoCs.

5.1 Attenuation of Curved Structures in pMUT Measurements

This part characterizes the effects of the curved structure with the pulse-echo sensing method in the ultrasound detections by using theories, simulations and experimental validations. Results show good agreements with the sound attenuation due to the curved surfaces with the radius-of-curvature between 1-6 mm and frequency of 1-10 MHz.

5.1.1 Introduction

The advances in microfabrication have enabled micromachined ultrasonic transducers in various applications, including both sensors [126] and actuators [86]. Device performance is generally characterized by the acoustic power output in terms of the

actuation function. On the other hand, the sensing functions have achieved increased signal-to-noise ratio by advanced mechanical designs, materials [16, 54, 67], and minimized losses. As such, it is important to understand the losses in the medium due to medium absorptions [48], interface between two media, geometry [13], ... etc.. Small-size pMUTs with the array formation of many elements provide the feasibility of high spatial resolution detections in application such as blood vessels as illustrated in Figure 2.3a. However, one practical issue is the impact of the non-flat surfaces with possible detection attenuations.

5.1.2 Concept and Operating Principle

In general, the distance detection in ultrasonic transducers utilizes the pulse and echo method which is the time difference between the emitting ultrasound (T_x) and received echo signals (R_x) from an object. The blood vessel system is utilized as an example as shown in Figure 5.1a with various blood vessel dimensions [127].

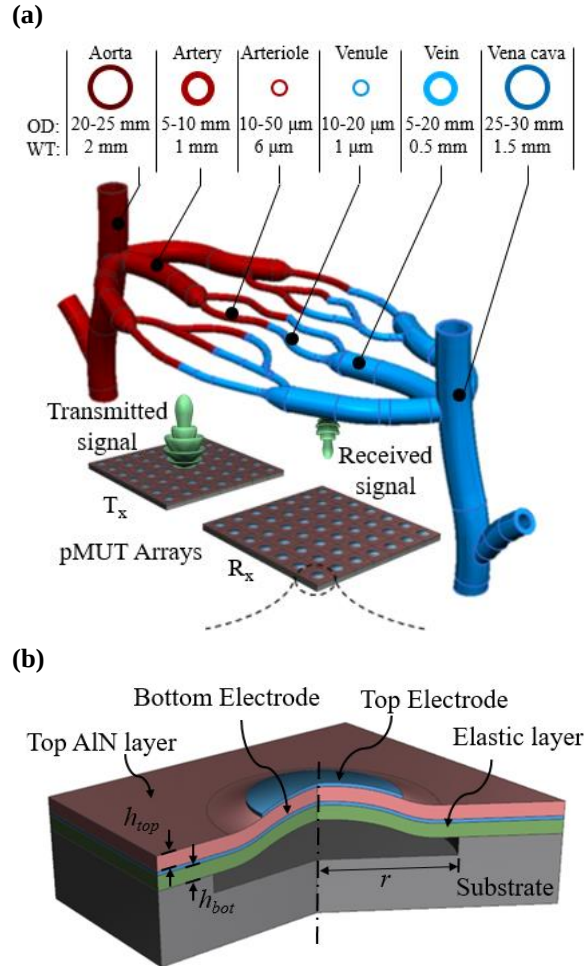


Figure 5.1: (a) Conceptual illustration of the ultrasonic blood pressure measurement using the pulse-echo method using transmitting, T_x , and receiving, R_x , transducer arrays for blood vessels, including arteries (red) and veins (blue). Outer Diameter (OD) and Wall Thickness (WT) of various blood vessels are given at the top. (b) Schematic view of the (deformed) unimorph, single electrode pMUT.

The distance between the plane of the pMUT array and the blood vessel surface is measured using the time-of-flight (ToF) or the Doppler shift schemes assuming the speed of sound in the medium is known. In this work, the transmitter and receiver array are composed of the unimorph single electrode piezoelectric micromachined ultrasonic transducers (Figure 5.1b).

5.1.3 Analytical Model and Simulations

As the sound wave propagates in a medium, it sees pressure attenuation in different forms such as medium absorptions [48], interface between two media, geometry [13], etc. The ratio of pressures at two points, the transmitting and receiving point in this case, is defined as the channel gain [56], G_{ch} , which is given by:

$$\frac{P_{Rx}}{P_{Tx}} = G_{ch} = G_{path} G_{target} \quad (5.1)$$

where P_{Rx} and P_{Tx} are the pressures at the receiver and transmitter positions, respectively and G_{path} is the propagation path gain which is due to the media absorption as the conversion of vibrational energy into heat. The degree of attenuation (gain<1) is defined by the attenuation coefficient, α , in Neper/m as a function of frequency [115]. On the other hand, $G_{target} (= G_{ac} G_{curv})$ is the reflecting surface gain with two parts; the reflection/transmission coefficient due to the acoustic impedance mismatch, G_{ac} , and the geometric gain which is the curvature effect specific to this work, G_{curv} .

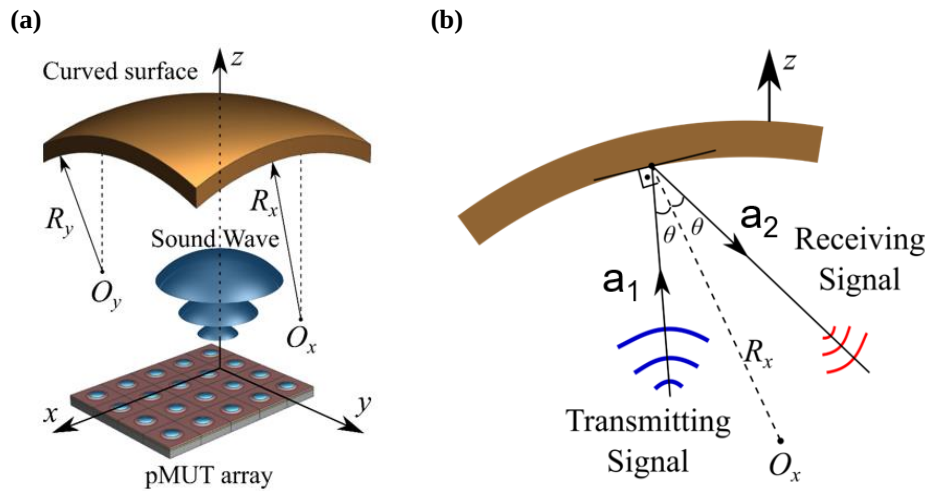


Figure 5.2: (a) Conceptual 3D illustration of sound wave generated by a sensor array placed on a surface with the radius of curvature R_x and R_y in the xz -plane and yz -plane, respectively. (b) Two-dimensional (xz -plane) illustration of the reflection on curved surface. Incident and reflected waves have the same angle, θ , with respect to the surface normal vector; a_1 and a_2 are the distances of transmitter and receiver to the reflection point on the curved surface.

If the reflective surface has the cylindrical-shape, it can be defined as a single curvature since its curvature is only in one plane. Similarly, a spherical surface can be defined to have curvatures in two planes (Figure 5.2a). For an infinitely long cylindrical surface, the attenuation due to the single curvature is given by [128]:

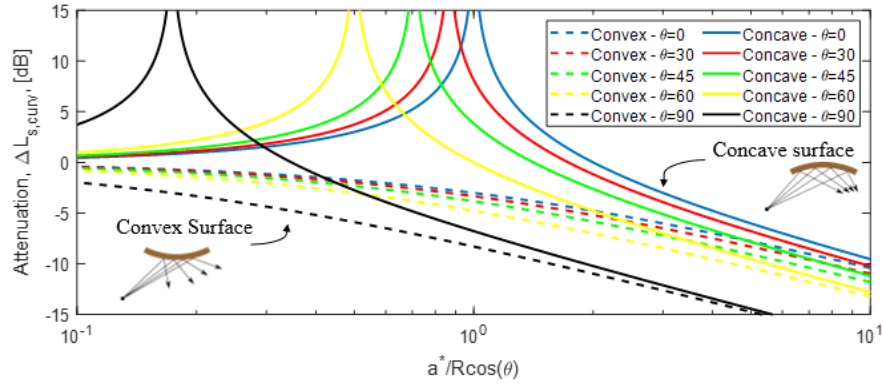
$$\Delta L_{s,curv} = -10 \log_{10} \left| 1 \pm \frac{a^*}{R \cos(\theta)} \right| \quad (5.2)$$

where R is the radius of the reflecting surface; θ is the angle between the sound propagation direction and the normal of the reflecting surface as shown in Figure 5.2b; and a^* is the effective distance as:

$$a^* = \frac{2a_1 a_2}{a_1 + a_2} \quad (5.3)$$

where a_1 and a_2 are the distances to the reflecting surface from transmitting and receiving points, respectively. The general single curvature attenuation with different reflections angles for concave and convex geometries is given in Figure 5.3.a.

(a)



(b)

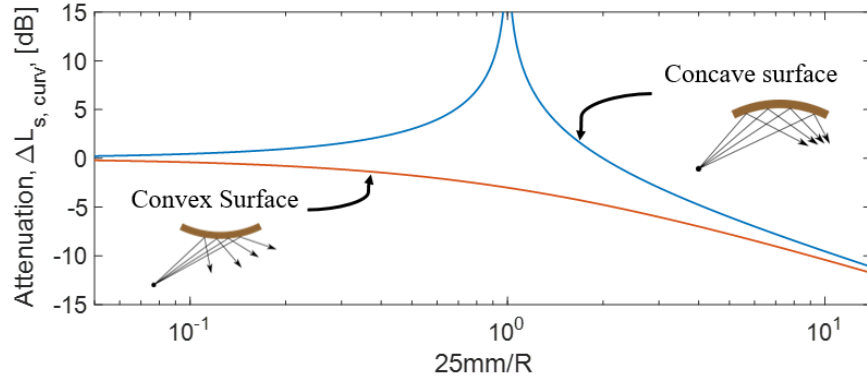


Figure 5.3: (a) The general single curvature attenuation with different reflections angles for concave and convex geometries. (b) Curvature attenuation specific to this work, that is ($a_1=a_2=a^*=d=25\text{mm}$, $\theta=0^\circ$)

In this work, the distance between the transducers arrays and a cylindrical surface is the same ($a_1 \approx a_2$), hence $a^* = a_1$. Also, the reflecting angle, θ , is zero. The analytical solution to the attenuation due to the single curvature is given in Figure 5.3b, with concave and convex curvatures. For the convex shape, the radius, R , is positive, and it is negative for the concave shapes. An important note is that if the transmitter and receiver are not in the plane perpendicular to the cylinder axis, the projected distances a_1 and a_2 , and angle,

θ , should be used. Furthermore, if the reflecting surface is spherical with the double curvature, the attenuation term in equation (5.2) should be doubled [128]

The MATLAB kWave toolbox is used in the simulation to define the elastic wave propagation in the homogenous medium [129]. The physical layouts of the transducers, reflecting surfaces and acoustic properties of the medium are shown in Figure 5.4a. In this work, human blood vessels are used as the demonstration example. Spatial and temporal values used in simulations are given in Table 5.1. The element (grid) size of 22 μm and a minimum of fourteen elements per wavelength are used. In all simulation cases, the transducers are driven with three cycles of a sine wave. A Perfectly Matched Layer, PML, encloses the domain is used to eliminate the effect of boundary reflections. In Figure 5.4b, snapshots of the simulated transmitted and received ultrasonic wave from a curved surface in the time domain are shown.

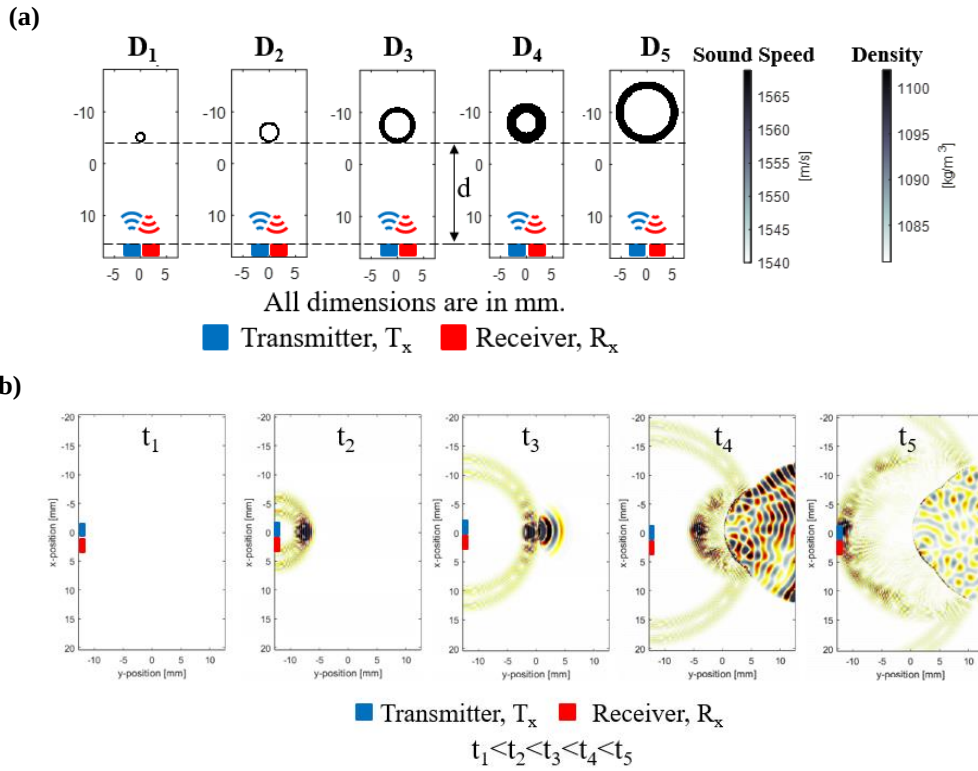


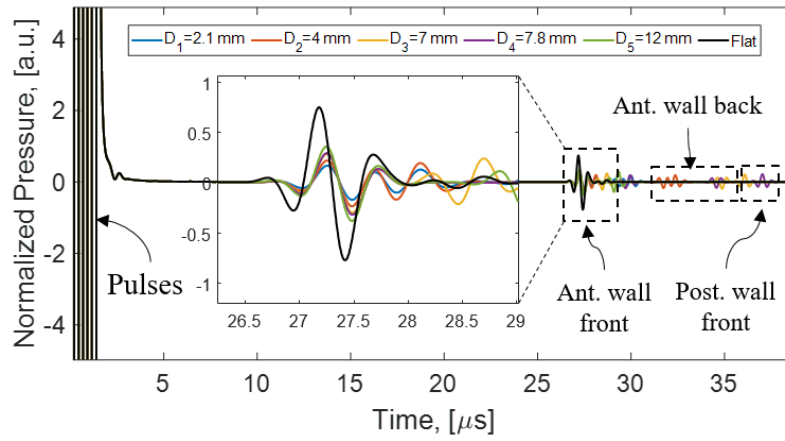
Figure 5.4: (a) The geometric layouts and acoustic properties of the homogenous medium used in the simulations. All dimensions are in mm. D_i is the outer wall diameter and i is from 1 to 5; d ($a^*=25$ mm) is the distance. (b) Snapshots of the simulated transmitted and received ultrasonic wave from a curved surface in the time domain. t_i is the i^{th} time and $t_i < t_{i+1}$

Table 5.1: The spatial and temporal parameters used in curvature attenuation simulations in MATLAB kWave toolbox.

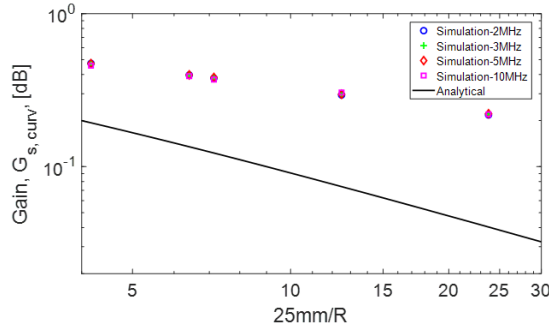
	D_1	D_2	D_3	D_4	D_5
Outer Diameter, [mm]	2.1	4	7	7.8	12
Wall Thickness, [mm]	0.5	1	1	2	1.5
Frequency, [MHz]	2,3,5, and 10				

Figure 5.5a shows the ToF simulation results by using a 2 MHz ultrasound system. The echoes from the anterior front wall, anterior back wall and posterior front wall are shown. The distances between the transducer and blood vessels in all cases are 25 mm and the echoes from all blood vessels have the same ToF. The sound attenuation from the curved surface is given in Figure 5.5b and Figure 5.5c as the gain and dB loss, respectively. The attenuation becomes more significant as the blood vessel diameter decreases. This is expected and the most significant attenuation is around 7 dB for the blood vessel with an outer diameter of 2.1 mm. The significance of this attenuation can be better realized if it is compared with the total attenuation. For a flat surface at the same settings, the channel attenuation is 72.75 dB. As such, the curvature attenuation has an effect of about 10%. As a last remark, the frequency in the given region does not affect the results since the geometric acoustic assumption hold for these wavelengths [130].

(a)



(b)



(c)

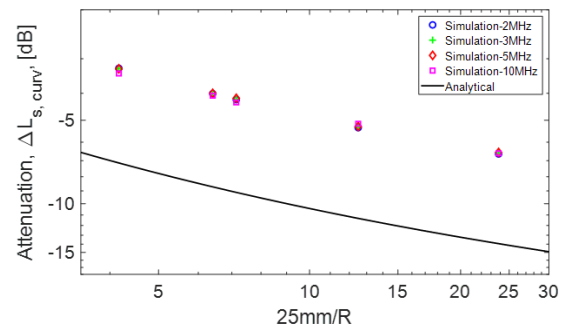


Figure 5.5: (a) Time-of-flight simulation at 2 MHz showing pulse and echo signals from the anterior front wall, anterior back wall, and posterior front wall. Effect of the surface curvature as: (b) gain, and (c) attenuation in dB.

5.1.4 Device Characterization

Optical and confocal microscope images of the pMUT array are shown in Figure 5.6a and Figure 5.6b, respectively. The array is fabricated on a cavity-SOI (silicon-on-insulator) wafer with a stack of Molybdenum-Aluminum Nitride-Molybdenum (Mo-AlN-Mo). Details of the fabrication and device characterization are given in [131]. Each array is 3 mm × 3 mm with a total of 39 × 39 pMUT elements. Each element is made of 1 μm-thick AlN piezoelectric layer and 5 μm-thick Si elastic layer with a diaphragm of 30 μm in radius.

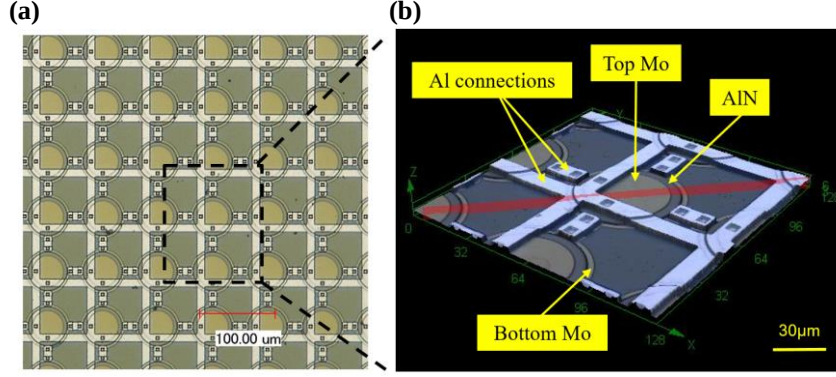


Figure 5.6: (a) The optical, and (b) confocal microscope image of the fabricated pMUT arrays having 39×39 elements.

The impedance measurements of the transmitter and receiver array in air are given in Figure 5.7a and Figure 5.7b, respectively. The transmitter array has anti-resonance and resonance frequencies of 16.04 MHz and 15.84 MHz, respectively resulting in an electromechanical coupling coefficient, k_{eff}^2 , of 2.48%. The receiver array, on the other hand, has anti-resonance and resonance frequencies of 15.73 MHz and 15.55 MHz, respectively resulting an electromechanical coupling coefficient, k_{eff}^2 , of 2.28%.

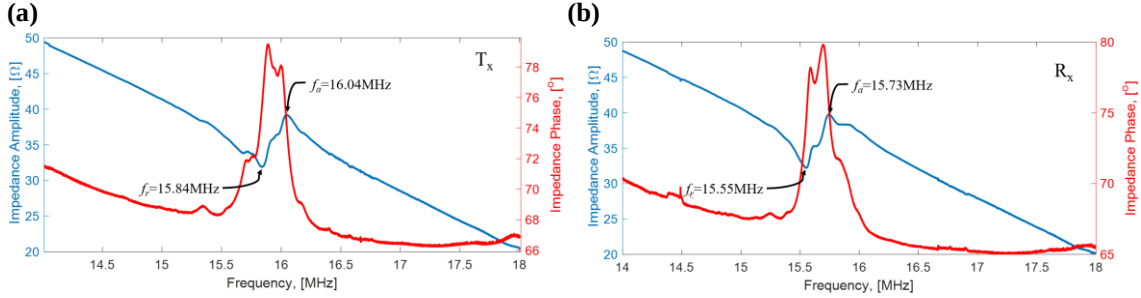


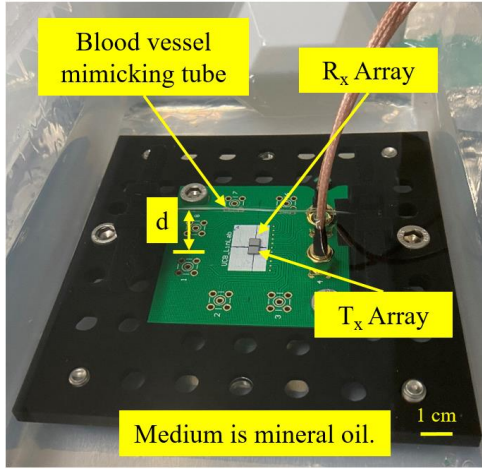
Figure 5.7: Impedance measurement results of: (a) Tx and (b) Rx arrays in air showing resonance and anti-resonance peaks for effective electromechanical coupling coefficient calculations.

5.1.5 Curvature Attenuation Experiments

All experiments are held in mineral oil to emulate the acoustic property of human tissue. The transmitting and receiving array are placed at a distance, d (25 mm in this test), away from the tube which has the acoustic properties similar to the blood vessel as seen in Figure 5.8a. The spatial and temporal sweeping parameters are given in Table 5.1. Tx is driven using the ArbStudio Arbitrary Wave Generator (AWG) with three sine wave signals having an amplitude of 12 V at the given frequencies. The received acoustic signal is transduced as charge output by the Rx array, and converted to voltage by a Trans-Impedance Amplifier (THS4631, TI). The voltage signal is passed through a low pass (LP) filter with a cut off frequency at 10% higher than that of the excitation frequency to suppress high frequency parasitic signals (Figure 5.8b).

In Figure 5.9a, ToF measurements from the anterior front wall at 2 MHz ultrasound signals are shown. The comparison of the analytical model, simulations, and experimental results for the structure with a curvature is given as gain and attenuation in Figure 5.9b and Figure 5.9c, respectively.

(a)



(b)

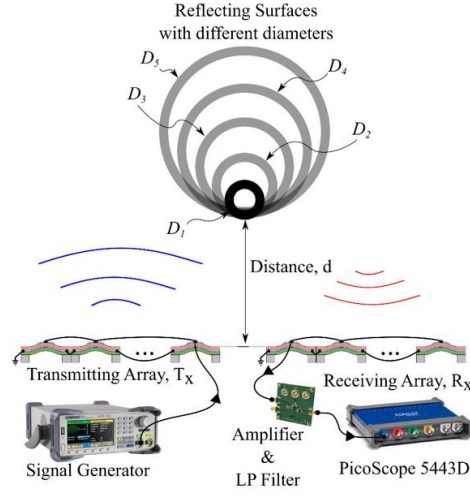
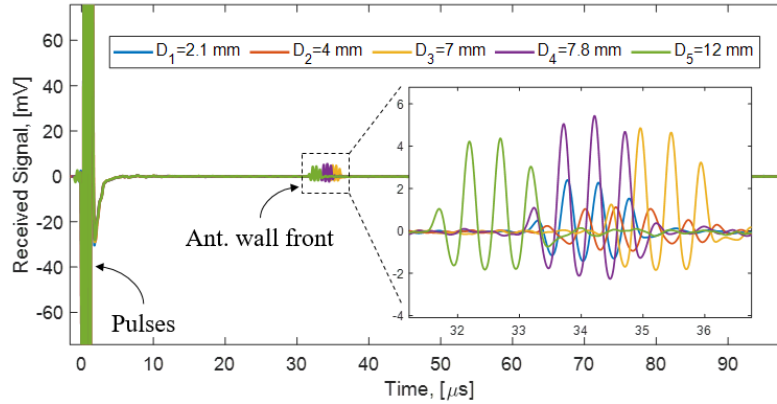


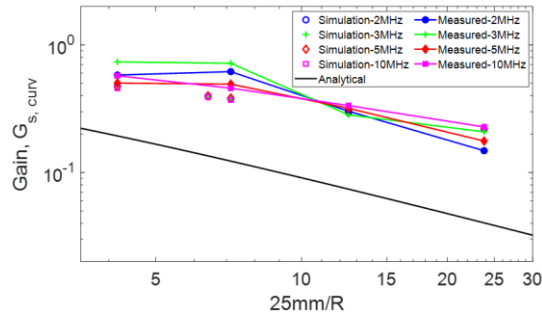
Figure 5.8: (a) Experimental setup in water with transmitting and receiving array placed across a tube with a known radius of curvature at distance d . (b) Experimental setup showing the pMUT arrays and the shapes of the reflecting surface and the excitation/readout circuitry.

The simulation and experimental results align well for the blood vessels dimension measurements (similar dimensions to arteries and veins) and the measured attenuation is about 7 dB. It is found that the frequency of the ultrasound signals has minimal effects on the attenuation magnitude, which shows the geometric acoustic assumption is still valid in those frequencies used in human tissue and blood.

(a)



(b)



(c)

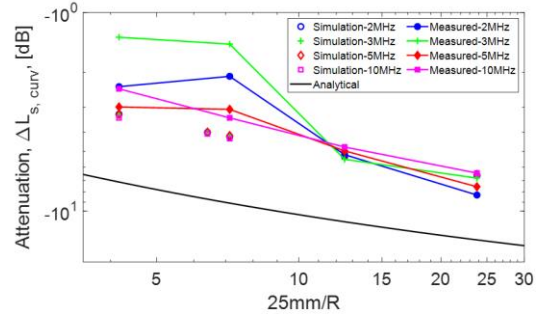


Figure 5.9: (a) ToF measurement of a 2 MHz ultrasonic transducer showing the pulse-echo signals from anterior front wall of the tube placed 25 mm away. Effect of the surface curvature as: (b) gain, and (c) attenuation in dB.

5.2 Radius of Curvature Measurement Using pMUTs

In the second part, the noncontact radius of curvature measurement of a surface using piezoelectric micromachined ultrasonic transducers is presented. The detailed fabrication flow of the pMUT array is explained. The proof-of-concept experiments are held in the air environment using 3D printed curved surfaces with known RoCs.

5.2.1 Introduction

Radius of curvature (RoC) of a structural surface is an important in many fields such as room acoustics design [132] and optical lenses [133]. There are two main RoC measurement techniques: contact and non-contact methods. Various non-contact measurement methods have been used widely due to two reasons: (1) contact methods have the risk of damaging the surface, and (2) contact methods have relatively low accuracy [134]. The commonly used non-contact measurement methods are the auto collimating microscope [135] and the interferometer [136]. Interferometer is often used due its accuracy, although it requires rather expensive instruments and well-adjusted environmental parameters with one major constraint that light cannot penetrate the opaque medium.

For the past decade, ultrasonic transducers have been used in many applications such as finger print sensors and gesture recognitions as well as medical imaging [49]. Compared to bulk transducers, micromachined ultrasonic transducers have the advantages of batch fabrication, higher bandwidth, better impedance match in air, high frequency range, small size, small form factor, and CMOS compatible fabrication [92]. Among all transduction mechanisms for micromachined transducers, capacitive [9] and piezoelectric [12] mechanisms have been utilized widely. Piezoelectric transduction has prominent futures such as no need to have a bias voltage, higher displacement for flexural mode transducers, ease of fabrication without the well-controlled thin gap as compared to those of ultrasound devices based on the capacitive transduction.

5.2.2 Concept and Operating Principle

The proposed method utilizes the pulse-echo technique, also known as time-of-flight (ToF) method, and it is based on the geometrical acoustics in which the behavior of the sound wave is described with geometrical rules as in geometrical optics. Geometrical acoustic is valid when the frequency of sound wave is high; i.e., dimensions of the reflecting surface are much higher than the wavelength of the ultrasonic wave [130]. In this study, the minimum dimension of the reflecting surface is 10x greater than that of the wavelength. Hence, the sound propagation is represented by rays which are the small portions of spherical wave fronts.

The conceptual illustration of the RoC measurement using pMUT array is given in Figure 5.10a. The pMUT array sits on the xy-plane, across a curved surface which has two curvatures on the xz-plane and yz-plane with RoC of R_x and R_y , respectively. As shown in the figure, acoustic rays can reflect according to Snell's law as the incoming and reflecting

rays have the same angle with respect to the surface normal vector, assuming no diffraction, bending, or interference [48].

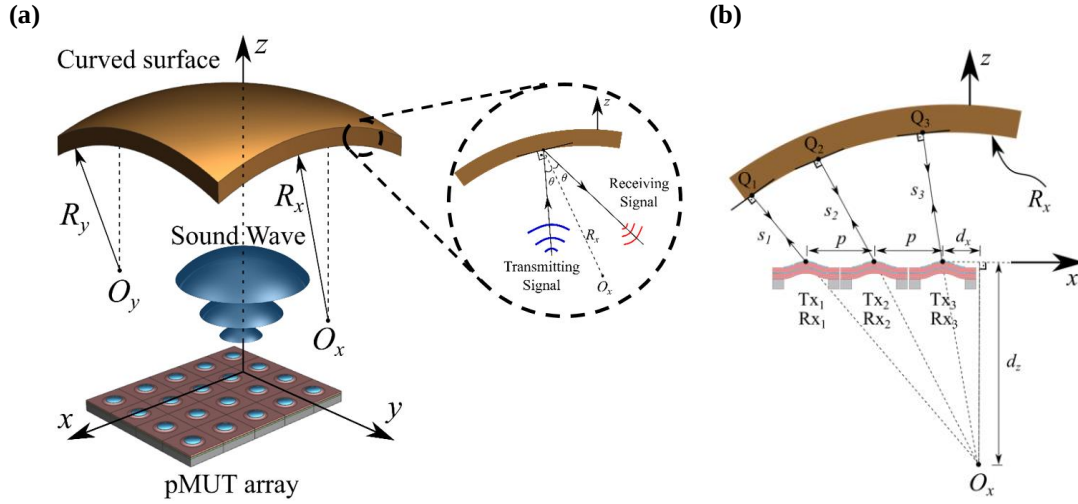


Figure 5.10: (a) Conceptual illustration of radius of curvature (RoC) measurement using a pMUT array. Curved surface across the pMUT array has two curvatures on the xz -plane and yz -plane with RoC of R_x and R_y and an incident angle, θ , with respect to the surface normal vector, respectively. (b) The analysis is visualized in a two-dimensional space, xz -plane. Three different pMUT channels with an array pitch, p , are utilized as transmitter and receiver in order to measure ToF for their own echoes.

Each channel in the x and y direction is used separately to measure R_x and R_y , respectively. Sequentially excited channels are utilized as both transmitter and receiver. The distance between the surface and each channel is extracted from the corresponding ToF measurements. Since a circle in a plane is defined with minimum three points, at least three measurements are required to measure the RoC in one plane.

5.2.3 Mathematical Model

A minimum of three different sound wave sources having a known distance in between them on the same plane are required. If the incoming wave front is normal to the surface tangent, it reflects back in the shortest distance with the highest amplitude to the original emitter and passes through the center for a curved surface, see in Figure 5.10b. Using geometrical relations, the radius can be extracted in terms of ToF and the distance between channels, i.e. array pitch, of the three acoustic waves.

The analysis is illustrated in a two-dimensional plane, xz -plane, in Figure 5.10b. In the figure, p is the distance between consecutive transducers, i.e. pitch; R_x is the RoC in the xz -plane; s_i is the minimum distance between the reflecting surface and i^{th} transducer and it is given by:

$$s_i = \frac{1}{2} t_i c \quad i = 1, 2, 3 \quad (5.4)$$

where c is the speed of sound in the medium and t_i is the ToF for i^{th} transducer. The radius of curvature in the given plane can be extracted as:

$$R_x = \frac{(2t_2^2 - t_1^2 - t_3^2)c^2 + 8p^2}{(8t_2 - 4t_1 - 4t_3)c} \quad (5.5)$$

which depends only on the ToF measurements and the pitch. The same procedure applies in the yz-plane as well.

5.2.4 Device Fabrication

The fabrication details are described in Chapter 4 and also given in Appendix D. This structure is bimorph, dual electrode, clamped design with 208 μm radius diaphragm. In Figure 5.11a-f, the cross sectional image of the fabrication is shown.

In Figure 5.11g, an optical image of the fabricated pMUT array is given. In the detailed image, a single pMUT device is shown with the top electrode, mid inner electrode and mid outer electrode, and the backside release mark. Lastly, cross-section SEM image of the diaphragm is given in Figure 5.11h, showing two active AlN layers, three Mo electrodes (top, mid, and bottom), and the AlN seed layer which is also the etch stop layer.

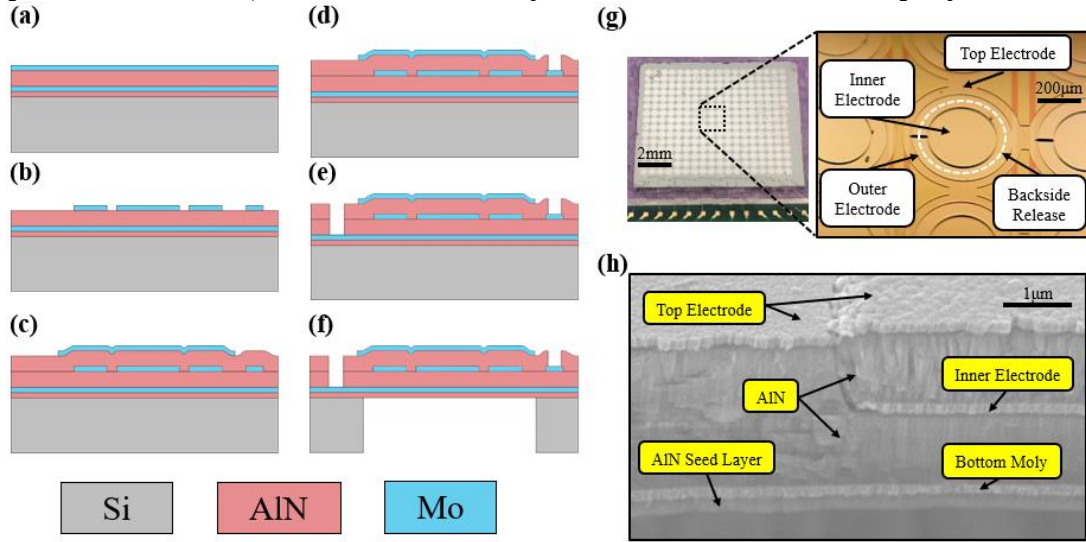


Figure 5.11: (a-f) Cross section of the diaphragm for the fabrication steps explained in Chapter 4 and Appendix D. (g) Optical image of the fabricated pMUT array - details shown in the zoomed-in image of a device with inner, outer and top electrodes. (h) SEM image of the diaphragm cross-sectional view showing the two layers of AlN, Mo electrodes and the bottom stack of Mo/AlN used as the seed layer.

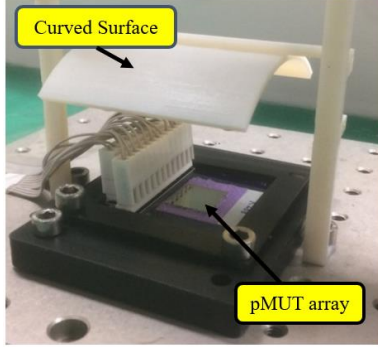
5.2.5 Experimental Setup

Experiments were held in the air environment with 3D printed curved surfaces placed 5 cm away from the pMUT array (Figure 5.12a). The pMUT array is composed of 3 channels in the x direction. Each channel composes of 4x20 pMUTs.

The read-out-layout is given in Figure 5.12b. Each channel is driven using ArbStudio Arbitrary Wave Generator (AWG) with a sine signal having an amplitude of 12Vpp and the natural frequency of the transverse fundamental mode of the diaphragms. The incoming sound wave is converted into a current signal via piezoelectric transduction by the fabricated devices. Then, the generated current is converted to voltage output using a

transimpedance amplifier (THS4631, TI). Before logging the data using the PicoScope5443 oscilloscope, voltage signal is passed through a low pass (LP) filter with a cut of frequency of 10% higher than the natural frequency of the diaphragms in order to suppress the high frequency parasitic noises.

(a)



(b)

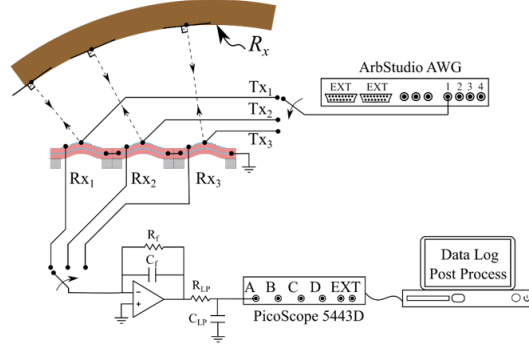


Figure 5.12: Experimental setup and read-out layout. (a) Fabricated pMUT array is placed at the bottom of a curved surface in air. (b) Three channels are used as both transmitting (Tx) and receiving (Rx) transducers. Output current of each Rx channel is converted into voltage using a transimpedance amplifier (TIA). Filtered signals are logged to be post processed.

5.2.6 Results and Discussions

Device characterization results are given in Figure 5.13. AlN pMUTs with 416 μm in diameter and 2 μm in thickness have the anti-resonance and resonance frequencies of 466.47 kHz and 462.83 kHz, respectively. The extracted electromechanical coupling factor, k_{eff}^2 , from the impedance measurement is 1.58%, using the given equation below [55]:

$$k_{eff}^2 = \frac{f_a^2 - f_r^2}{f_r^2} \quad (5.6)$$

where f_a and f_r the anti-resonance and resonance frequencies, respectively. The impedance measurement corresponds to the impedance of a channel which has 80 (4x20) pMUTs.

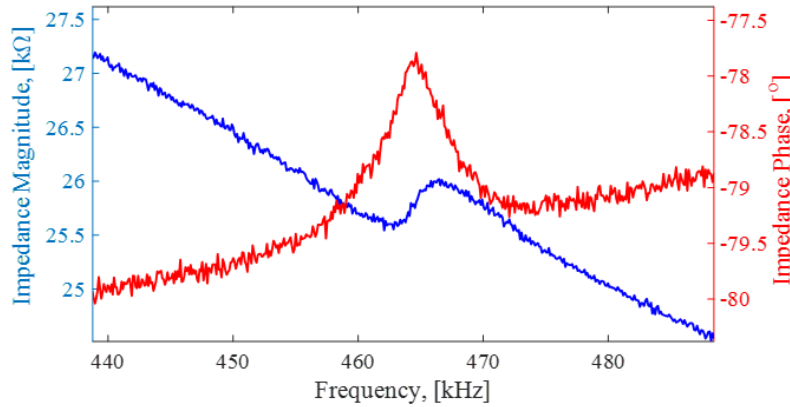


Figure 5.13: Impedance measurement result for a single 416 μm in diameter AlN pMUT channel showing the anti-resonant and resonant frequencies which are 466.47 kHz and 462.83 kHz, respectively. The extracted k_{eff}^2 is 1.58%.

To illustrate the ToF difference as explained in the method section, pulse-echo measurement of the three channels for a surface with RoC of 100 mm is shown in Figure 5.14a. Using the geometrical relation in equation (5.5), the RoC of the curvature is extracted. The same experiment is repeated for the surfaces with different RoCs between 20 mm and 2000 mm as shown in Figure 5.14b. Up to RoC of 200 mm, results align well with fabricated surfaces.

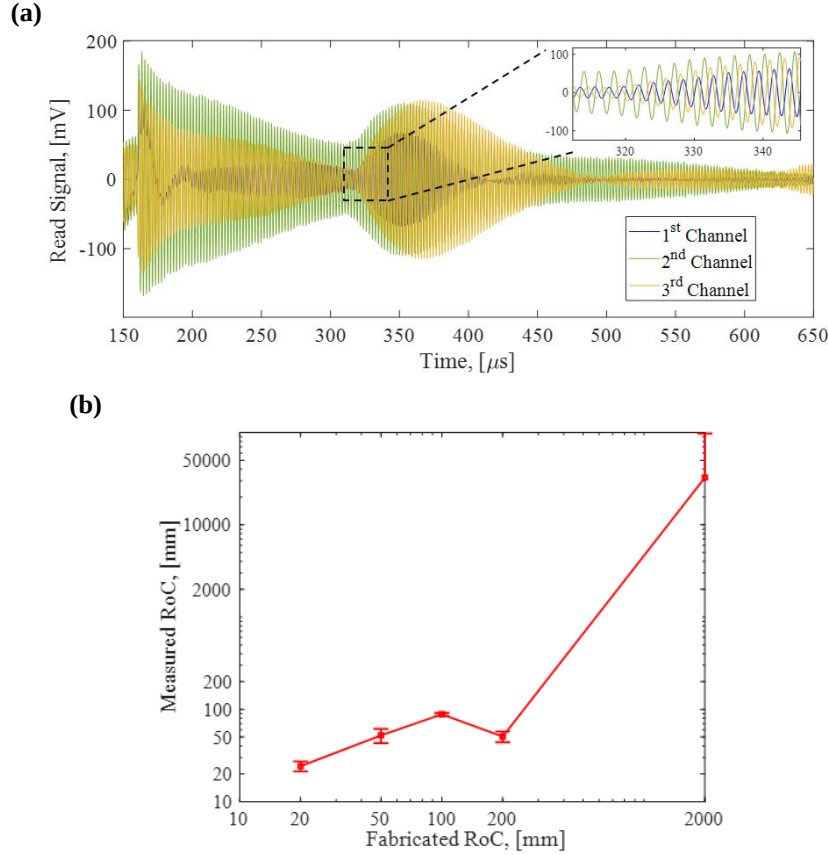


Figure 5.14: (a) Pulse-Echo measurements of three channels placed across a curved surface with RoC = 100 mm. Time of flights for each channel (zoomed-in image) are slightly different due to the curvature of the surface. (b) RoC measurement using pMUTs with 3D printed surfaces of different curvatures.

The wavelength of the emitted acoustic wave determines the minimum feature size that can be resolved in acoustic applications. In general, it is required to have smaller wavelengths. However, as the wavelength gets smaller, frequency gets higher which could be undesirable due to the acoustic impedance of the medium. In this study, we used an acoustic signal having the frequency of 460 kHz which corresponds to a wavelength of about 745 μ m. The operation frequency can also be designed for specific operations such as in liquid environment.

One important parameter is the axial resolution of the transducers, which is strongly related with the bandwidth of the transducers. As the bandwidth increases, the axial resolution can increase as the resolvable distance of two objects in the axial directions reduces. From the RoC measurements in Figure 5.14b, it is found that the bandwidth of the pMUTs is not high enough to resolve RoCs higher than 200 mm while the transducers can be redesigned to overcome that limitation.

5.3 Conclusions

The amplitude attenuation of the curved reflecting surface based on the pulse-echo method in ultrasonic measurements has been studied. The wave propagation in the homogeneous medium have been simulated with an element (grid) size of $22\text{ }\mu\text{m}$ by a minimum of fourteen elements per wavelength. The pMUT arrays are $3\text{ mm} \times 3\text{ mm}$ in size for a total of 39×39 elements made of diaphragms with $1\text{ }\mu\text{m}$ -thick AlN piezoelectric layer and $5\text{ }\mu\text{m}$ -thick Si elastic layer. By emitting the ultrasound wave with 12V in amplitude and three sine wave cycles at frequencies of 2-10 MHz, the attenuations on the surface with a radius of curvature of 1-6 mm have been measured in the mineral oil environment. The echo signals from a surface 25mm away are captured by a receiver array placed next to transmitter. For the artificial blood vessels dimensions similar to arteries and veins, the measured attenuation is 7 dB which is in a good agreement with simulations. These results indicate that the signal to noise ratio has a room of $\sim 10\text{ dB}$ for improvements by selecting appropriate reflecting surface in applications such as blood pressure measurements.

A scheme to measure the radius of curvature using a PMUT array is demonstrated. Experiments are conducted in air using 3D printed curved surfaces having RoCs of 20-2000 mm and placed 5 cm away the pMUT array. The extracted RoC values have a good agreement with the fabricated samples up to 200 mm in RoC. The resolutions can be further enhanced by optimizing the axial resolution of the transducers. These results imply that the proposed concept can be used to measure surface curvature inside an opaque media where optical counterparts cannot be used such as beams and columns inside the buildings, and implants inside the human bodies.

Chapter 6

Improved Ring-down Time and Axial Resolution of pMUTs via a Phase-shift Excitation Scheme

In this chapter, a 180° phase-shift excitation scheme is proposed in order to reduce the ring-down period of the ultrasonic transducers with better axial resolutions. The method is explained and experimented with both single and an array of pMUTs. The results show reduced ring-down period and improved axial resolution.

6.1 Introduction

While ultrasound applications in underwater and non-destructive testing have been known for a long time, novel applications have been introduced recently using micro-scale ultrasonic devices, such as in air range finding [137], fingerprint sensing [138], medical treatment and monitoring [126], and others [92]. Among the transduction mechanisms for micromachined transducers, capacitive [12] and piezoelectric [9] mechanisms have been utilized widely and devices based on piezoelectric transduction have prominent futures as: no need to have a bias voltage, possible large displacements from the flexural mode transducers, and the ease of fabrication process without a well-controlled thin gap [13].

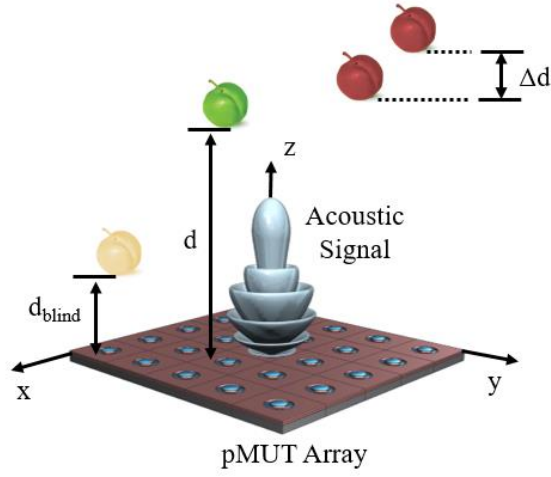
As shown in most of the ultrasonic transducers, pMUTs are used as both transmitting and receiving element in the time-of-flight (ToF) mode. Therefore, the excitation time and the ring-down time is added to the ToF, which causes a blind detection zone [139]. Various methods have been proposed to control this transient response, i.e., ring-down of the vibrating structures, including a changed waveform [139], binary encoding [140], active vibration control [141], and parallel resistors [142].

6.2 Concept and Operating Principle

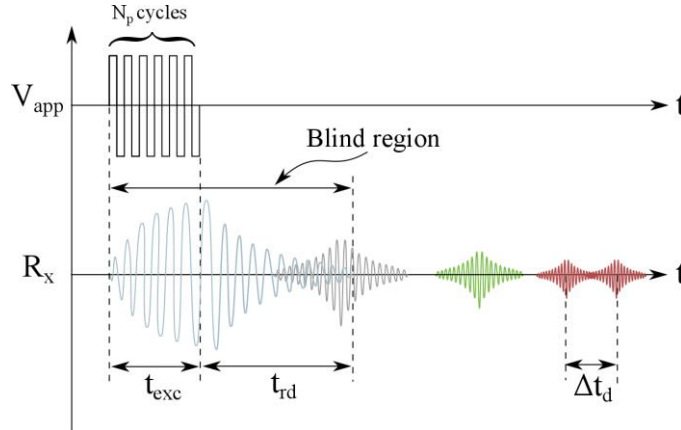
For range-finding applications by ultrasonic devices, a series (several pulses) of unfocused ultrasonic signals are emitted and received as shown in Figure 6.1a with two key regions affecting the system performances: the ring down region with a distance of d_{blind} , and the axial resolution with a distance of Δd . Traditionally, a number of excitation cycles are applied to produce ultrasound in the medium. Figure 6.1b shows the traditional excitation and receiving echo signals corresponding to the measurement setup in Figure 6.1a. The colored regions in the receiving signals match the same-colored objects. In the receiving side, the sum of the excitation and ring down determine the length of the blind distance. In practice, a low-quality factor is preferred for ultrasonic transducers for high

radiations as explained in Chapter 2. Otherwise, high quality factor can result in a long ring down period for possible cross-talks between emitting and receiving signals and leads to unresolvable objects.

(a)



(b)



(c)

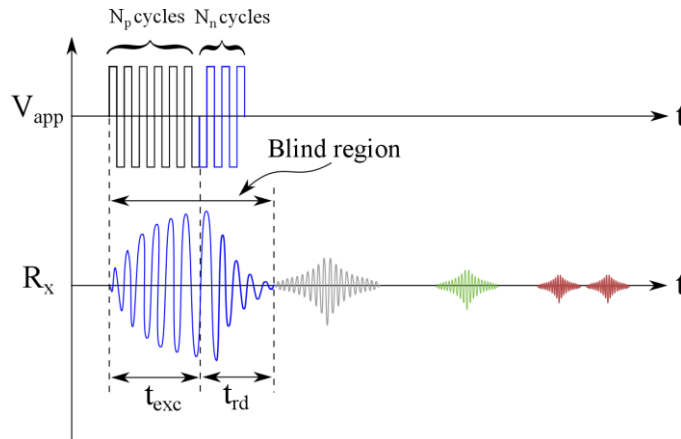


Figure 6.1: Illustrations and definitions for the proposed scheme: (a) a typical measurement setup with a distant, d , to the object; the distance of the blind region, d_{blind} , and the axial resolution, Δd , by a pMUT array; (b) traditional pulse excitations and receiving signals; (c) the proposed phase-shift cancellation scheme to reduce the blind region and increase the axial resolution.

Figure 6.1c illustrates the proposed ultrasound radiating pattern using the phase-shift cancellation scheme. After emitting N_p cycles (black color) of ultrasonic waves, N_n cycles (blue color) of signals with 180° phase-shift difference to the original ones are emitted. This results in the counter-reverse actuation of the excited pMUT diaphragm to quickly stop the vibration of the diaphragm and reduce the ring down period.

6.3 Fabrication of Devices

The fabrication details are described in Chapter 4 and also given in Appendix D. This structure is bimorph, dual electrode, pinned-clamped design with $415\ \mu\text{m}$ radius diaphragm. In Figure 6.2a-f, the cross sectional image of the fabrication is shown. In Figure 6.2g, an optical image of the fabricated pMUT array consisting of 4×4 elements is shown. Lastly, the cross-section SEM image is given in Figure 6.2h shows two active AlN layers, three Mo electrodes (top, mid, and bottom), and the AlN seed layer at the bottom which is also the etch stop layer.

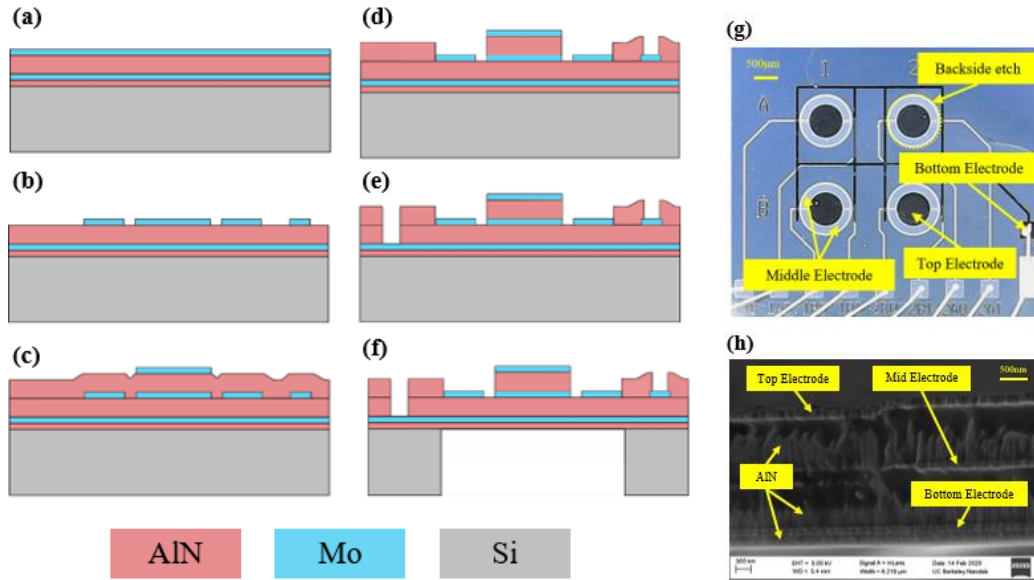


Figure 6.2: (a-f) Fabrication process flow of the transducers (see Chapter 4 and Appendix D for details). (g) An optical image of the fabricated pMUT array. (h) SEM image of the cross-sectional view showing the two active AlN layers, three Mo electrode layers and the bottom stack of Mo/AlN etch stop layer.

6.4 Experimental Setup

Experiments were conducted for both single transducer elements and an array of 80 elements. In these experiments, the number of positive excitation cycle, N_p , has been kept as ten cycles and the number of 180° phase-shift cycles, N_n , has been applied between 1-10. In Figure 6.3a, the pulse-echo experimental setup shows the transducer array and the reflecting surface. The fabricated pMUTs are driven using the ArbStudio Arbitrary Wave Generator (AWG) with ten positive cycles and one to ten 180° phase-shift cycles of the sine signal having an amplitude of 12Vpp at the natural frequency of the transverse fundamental mode of pMUT diaphragms at 160 kHz. The excitation and readout circuitry are in Figure 6.3b.

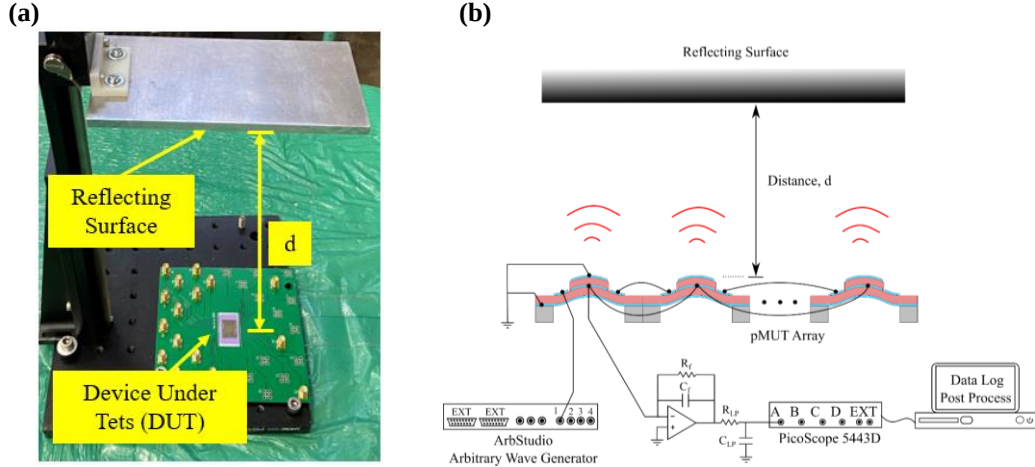


Figure 6.3: (a) Pulse-echo experimental setup showing the transducer array and reflecting surface. (b) Excitation and readout circuitry.

6.5 Experimental Results

The frequency response experiments are done in air and pMUT transducers with the dual electrode design have the middle electrode composed of the separate inner and outer electrodes. The frequency response was measured as a mean of transmission between the inner and outer electrodes. As shown in Figure 6.4, the operating frequency is 160 kHz, which is higher than the predicted value probably due to thin film stresses created during the fabrication process.

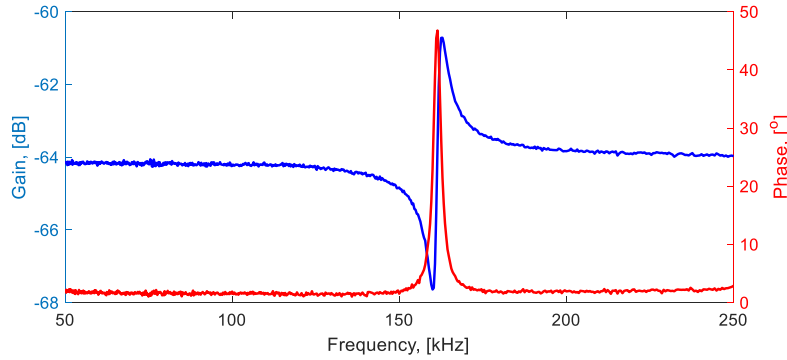


Figure 6.4: transmission between inner and outer electrodes.

In Figure 6.5a, the ring down period of a single pMUT element after ten excitation cycles and different number of the 180° phase-shift cycles have been tested. As the number of 180° phase-shift cycles increase, the ring down envelope gets smaller before it gets larger. There is an optimum number of 180° phase-shift cycles to have the minimum ring down. This is because the 180° phase-shift cycles counteract to the positive displacement. However, this scheme can become over-reacting to excite the diaphragms at its own phase and frequency. In the tests, the optimum number of 180° phase-shift cycles corresponding to the ten positive cycles is found as six.

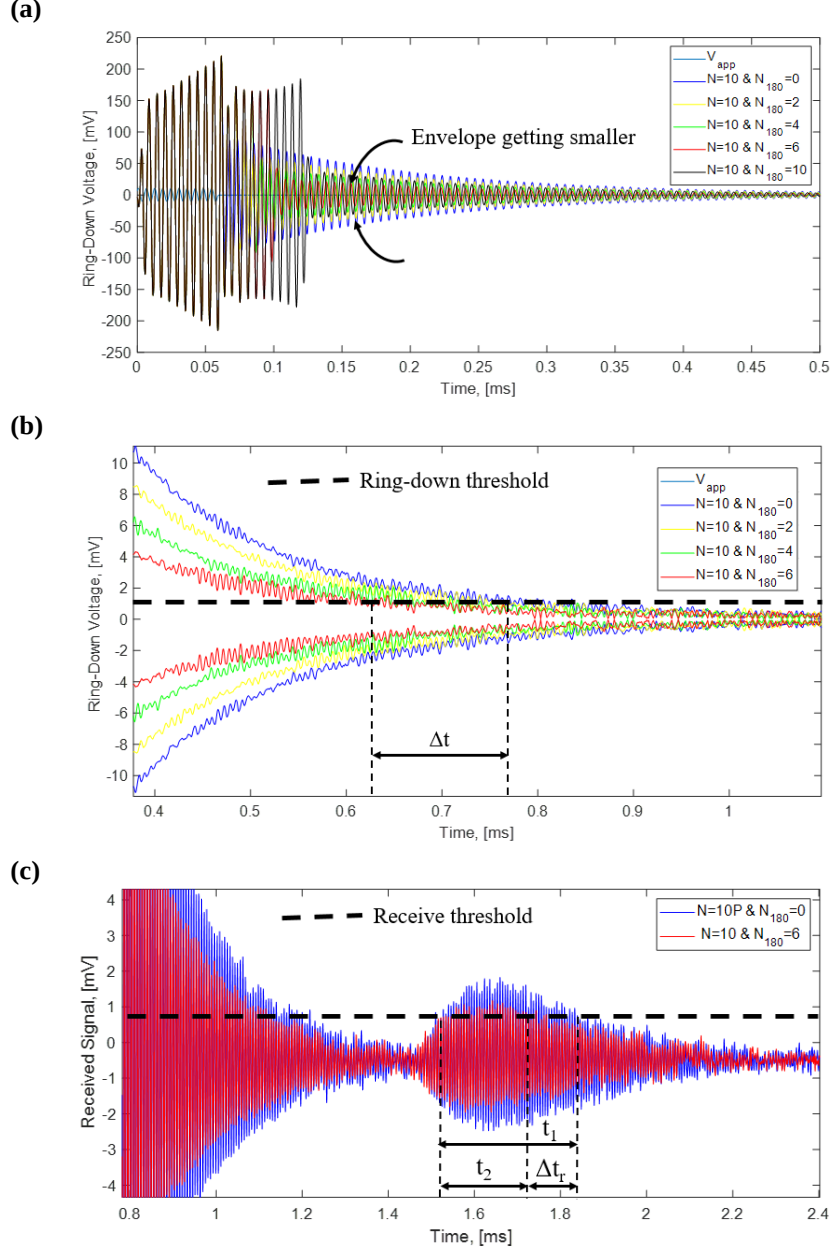


Figure 6.5: (a) Ring down period of a single pMUT element with ten excitation cycles and different number of 180° phase-shift cycles. (b) The envelope of the ring down period for different number of 180° phase-shift cycles. (c) Pulse echo measurement of a signal element 25 cm away for $N_n=0$ and $N_n=6$ cycles.

In Figure 6.5b, the envelope of the ring down period is given for different number of 180° phase-shift cycles. The Δt on the figure represents the largest time reduction in ring down period for a constant ring down threshold and it is measured as 0.102 ms. For a constant speed of sound in air, 343 m/s, Δt corresponds to a distance of 3.5 cm that means the blind region is shortened by 1.75 cm. Without any 180° phase-shift cycles, the blind region is 13.91 cm for this threshold. As such, the overall gain in the blind region is around 13%. Moreover, the axial resolution also benefits from the reduction of the ring down period.

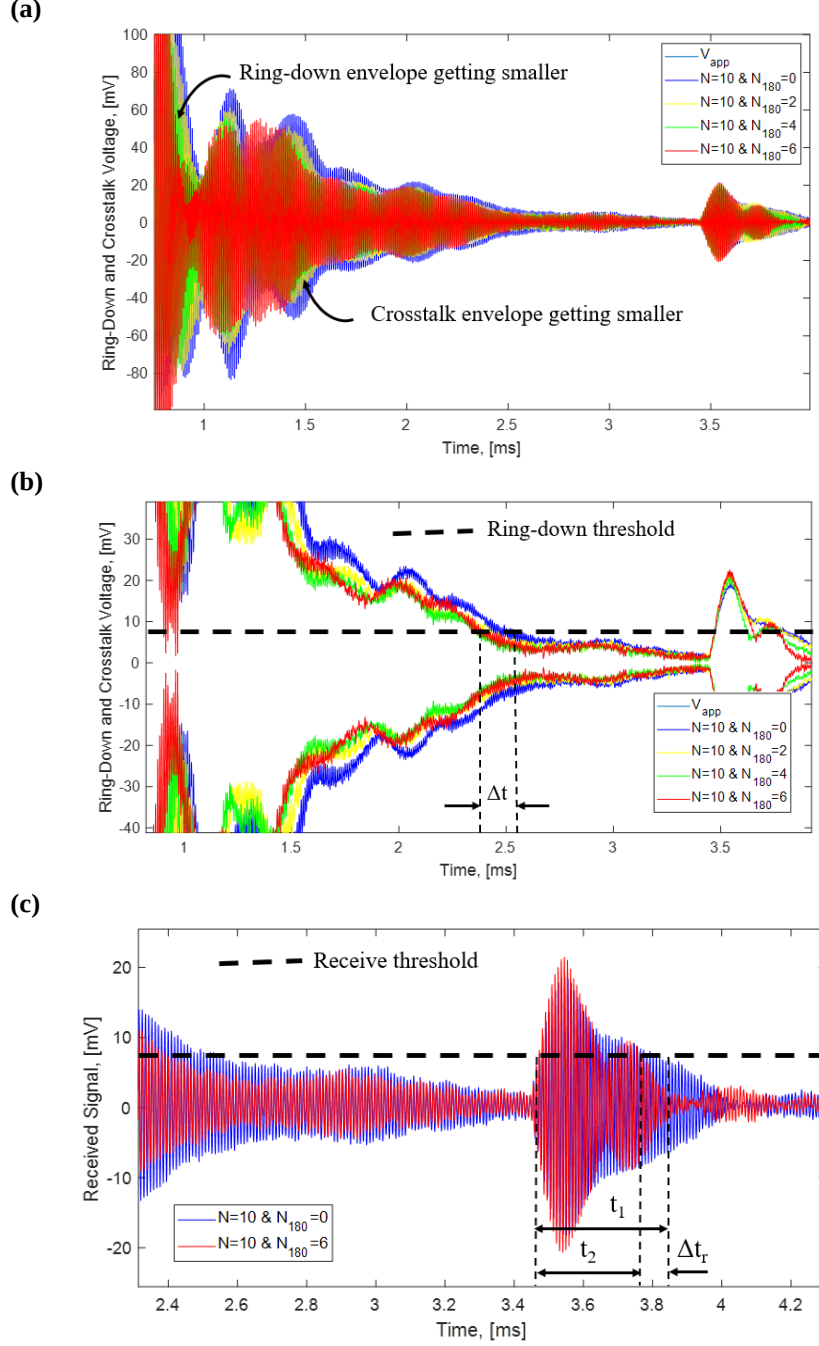


Figure 6.6: (a) Ring down period of a 80-element pMUT array with six excitation cycles and different number of 180° phase-shift excitation cycles. (b) The envelope of the ring down period for different number of 180° phase-shift cycles. (c) Pulse echo measurements of an 80-element pMUT array at 60 cm in distance for $N_n=0$ and $N_n=6$ cycles.

In Figure 6.5c, pulse-echo measurements of a signal element are given at a distance of 25 cm with zero and six 180° phase-shift cycles. The pulse width of the $N_p=10 \text{ \& } N_n=0$ excitation, t_1 , is 0.3 ms which corresponds to axial resolution of 5.15 cm. For the case of $N_p=10 \text{ \& } N_n=6$, the pulse width, t_2 , is 0.21 ms which corresponds to an axial resolution of 3.6 cm. As such, the overall gain in the pulse width is around 30% which is also the gain in the axial resolution.

In general, MUTs are not used as a single element as they are often utilized as an array. In Figure 6.6a, the ring down periods of an 80-element pMUT array with ten positive and different number of 180° phase-shift cycles are shown. In general, the ring down period of the array structure is similar to that of the single element. As the number of the 180° phase-shift cycles increases, the ring down envelope and the period become smaller. The optimum number of 180° phase-shift cycles corresponding to the ten positive cycles is found as six. In Figure 6.6b, the envelope of the ring down period is recorded for different number of 180° phase-shift cycles. For a given ring down threshold, Δt is measured as 0.191 ms which occurs for the case of applying six 180° phase-shift cycles. It corresponds to a distance of 6.65 cm that means the blind region is shortened by 3.3 cm. Hence the overall gain in the reduction of the blind region is 7.5%. This is smaller than the gain in that of the single element test which can be attributed to the crosstalk that is already counteracting to the ring down vibration for the array structure. In Figure 6.6c, the pulse echo measurement of the array is tested at a distance of 60 cm for zero and six 180° phase-shift cycles. For the $N_p=10$ & $N_n=0$ case, the pulse width (t_1) is 0.39 ms which corresponds to an axial resolution of 6.75 cm. However, for the $N_p=10$ & $N_n=6$ case, the pulse width (t_2) is 0.30 ms which corresponds to an axial resolution of 5.25 cm. The overall axial resolution gain is around 25%.

6.6 Conclusions

A 180° phase-shift excitation scheme is proposed to reduce the ring down period of the operations of ultrasonic transducers with better axial resolutions. The fabricated devices of an 80-element array with circular diaphragms of 415 μm in radius are fabricated by a CMOS compatible micromachining process to result a resonant frequency of 160 kHz. For a single element and an array structure, the ring down period is reduced by applying ten excitation cycles followed by six additional 180° phase-shift excitation cycles. By shrinking the ring down period, the blind region is shortened by 13% and 7.5% for the single element and 80-element array, respectively. In addition, the axial resolution of the system is increased by 30% for the single element and 25% for the array structure. These results suggest that the proposed excitation pattern has a promising potential to be used in the pulse echo applications of ultrasonic systems.

Chapter 7

Conclusions and Future Works

This dissertation presents the latest research results on piezoelectric micromachined ultrasonic transducers (pMUTs), including a two-term model in the equivalent lumped element circuit analysis of traditional clamped pMUTs, a free boundary structure to result in an ultra-high performing transducers, a ultrasound-induced airborne haptics application via pinned-clamped pMUTs, the structure curvature measurement with clamped pMUTs, and an excitation scheme for a better axial resolution and smaller blind zone region. The two-term mode shape approach improves the modelling results with an intuitive and hierarchical design procedure. The introduction of free boundary pMUTs with a mode matched acoustic resonance increases the bandwidth and the electromechanical coupling coefficient of the whole system. In the area of demonstration examples, the ultrasound-induced airborne haptics could provide non-contact, multimodal sensations with good spatial and temporal resolution for potential gaming and AR/VR applications. The structure curvature measurements in a non-contact mechanism by pMUTs can provide information for structural health as well as medical application such as blood pressure measurement by the curvature attenuation. Finally, the phase shift excitation enhances the axial resolutions and blind zone length limitations as compared with those of traditional clamped pMUTs.

7.1 Conclusions

Chapter 2 presented an improved, two-term mode shape model for the circular-shape pMUT. The small signal electro-mechano-acoustic equivalent model can analyze many parameters such as individual equivalent circuit parameters, input impedance, frequency, bandwidth, quality factor, directivity, and the on-axis pressure in the near and far fields. The proposed model has been compared with the literature with exact solution and finite element method simulations. The errors in all these parameters are below 0.5% with respect to those in the exact solution. In addition, the reductions of error in these parameters are 5x to three orders of magnitude as compared to those in the literature. It is found that the correct model of the deformation shape is critical for the model accuracy. The approximate energy-based method can be utilized to obtain accurate mode shapes and parameters for different shapes and types of ultrasonic transducers, such as rectangle and ring geometries, dual electrode, and bimorph structures.

In Chapter 3, a pMUT with the free boundary condition is introduced and compared with those of conventional designs with the clamped boundary. The free edge mode shape has been derived analytically and an assumed mode is proposed to ease the computational

complexity in the analyses. An equivalent circuit model is constructed with derived lumped parameters. The structure has a diaphragm with the free boundary at the outer edge sitting on a central post. Due to the increased mechanical capacitance and enhanced strain field distribution, the electromechanical coupling coefficient is more than 16x higher than that of a traditional clamped boundary pMUT. Furthermore, the acoustic cavity underneath the free boundary structure is utilized as an acoustic resonator to increase the coupling and bandwidth. As a case study, the bandwidth of the free boundary pMUT is more than two orders of magnitude higher than that of a fixed boundary pMUT. The free edge is also helpful to release the internal stresses created during the film deposition process.

An ultrasound-induced airborne haptic interface device by using the pMUT array is demonstrated as described in Chapter 4. By emitting the ultrasonic waves at the natural frequency of 109.4 kHz, a local maximum acoustic pressure at 950 Pa is measured 15 mm away from the transducer array. The pulse width modulation scheme is utilized to convert the high frequency signals to a low frequency pattern at 100 Hz with 50% duty cycle in the haptic perception experiments under an applied amplitude of 12 V_p. The radiation force is calculated as 0.4 mN which is reported as the minimum force required to stimulate human skin and it is found that the average sensation delay time of five different volunteer is 2.3 seconds. Two types of demonstration experiments are designed and conducted: switching on/off and Morse code transmission to achieve the average identification accuracy rates of 80% and 76%, respectively. These results imply that the proposed concept has a promising potential to be used as high intensity, low power, and small footprint haptic perception interfaces in various practical fields, such as in human machine interface, medical, entertainment, and AR/VR applications.

In Chapter 5, structures with curved surfaces are studied for two properties: (1) the signal attenuation magnitude due to the curved surface; and (2) the radius-of-curvature measurement by using the time of flight scheme. A pMUT array is 3 mm × 3 mm in size for a total of 39 × 39 elements made of diaphragms with 1 μm-thick AlN piezoelectric layer and 5 μm-thick Si elastic layer. By emitting the ultrasound wave with 12 V_p in amplitude and three sine wave cycles at frequencies of 2-10 MHz, the attenuations on curved surfaces with the radius of curvatures of 1-6 mm have been measured in the mineral oil environment. The echo signals from a surface 25 mm away are captured by a receiver array placed next to transmitter. For the blood vessels dimensions similar to arteries and veins, the measured attenuation is 7 dB which is in a good agreement with simulations. These results indicate that the signal to noise ratio has a room of ~10 dB for improvements by selecting appropriate reflecting surface in applications such as blood pressure measurement. In the radius of curvature measurements using a PMUT array, experiments are conducted in air using 3D printed surfaces having RoCs of 20-2000 mm placed 5 cm away the pMUT array. The extracted RoC values have a good agreement with the fabricated samples up to 200 mm in RoC. The resolutions can be further enhanced by optimizing the axial resolution of the transducers. These results imply that the proposed concept can be used to measure surface curvature inside an opaque media where optical counterparts cannot be used such as beams and columns inside buildings, and implants inside human bodies.

In Chapter 6, a 180° phase-shift excitation scheme is proposed to reduce the ring down

period for the operation of ultrasonic transducers with higher axial resolution. The fabricated devices have an 80-element array with circular diaphragms of 415 μm in radius by a CMOS compatible micromachining process to result in a resonant frequency of 160 kHz. By shrinking the ring down period, the blind region is shortened by 13% and 7.5% for the single element and 80-element array, respectively. In addition, the axial resolution of the system is increased by 30% and 25% for the single element and the array structure, respectively. These results suggest that the proposed excitation pattern has a promising potential to be used in the pulse echo applications of ultrasonic systems.

7.2 Future Works

The need for high performance ultrasonic transducers is an important topic for small-form factor hand-held systems toward low power consumption and high outputs for applications such as AR/VR, internet-of-things (IoT), and more. The novel free boundary design presented in this work illustrates an important approach for high performing micromachined ultrasonic transducers of ultra-large bandwidth and high electromechanical coupling coefficients. The analysis as demonstrated in this dissertation with the free boundary at the outer edge pMUT is the first step., while the fabrication process will be challenging such as the electrical connection through the central supporting post. On the other hand, the proposed free inner boundary design could be easier to fabricate, since the electrical connections can be done through the clamped outer edge. Assuming the silicon wafer is used as the substrate, the gas phase XeF_2 isotropic etching is recommended as it has high selectivity over AlN piezoelectric and Mo metal layers.

Fully immersive human machine interface requires haptic feedback other than visual and audial sensations. Haptic sensations in human is achieved by mechanoreceptors in the skins for pressure, texture, thermal, stretch stimulations. To achieve ultrasound-induced airborne haptics with multimodal excitation capability will be very challenging. The initial results presented here are promising, while exploring the multimodal stimulations with texture and thermal stimuli is another exciting research path to follow. In order to achieve this, three paths are proposed: the integration of pMUT arrays with different frequencies to realize different spatial resolutions, pMUT array pressure output enhancement for thermal stimuli, and beamforming and steering for variety of patterns.

Another interesting research direction is the closed loop control of the pMUT ring-down issue. In this work, an open loop control strategy is followed with a good degree of enhancement in axial resolution and blind zone length. A better way to minimize the ring-down time is to use a closed loop control strategy with one of the electrodes, such as the outer electrode as the sensor, and the inner electrode as the actuator. Monolithic integration of MEMS and CMOS can enable a minimum ring-down and best axial resolution without sacrificing the array size and power consumption in the feedback control.

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Appendix

A Strength of Materials Relations

A.1 Polar & Cartesian coordinates transformation

For axisymmetric structure like disks, it is more convenient to use polar coordinates instead of Cartesian coordinates (Figure A.1.1).

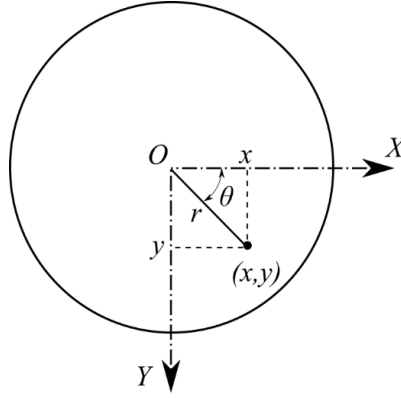


Figure A.1.1: Polar & Cartesian coordinates showing r , θ , x , and y axes.

The transformation relations are;

$$\left. \begin{array}{l} x = r \cos(\theta) \\ y = r \sin(\theta) \end{array} \right\} \Rightarrow \begin{array}{l} r^2 = x^2 + y^2 \\ \theta = \tan^{-1}\left(\frac{y}{x}\right) \end{array} \quad (\text{A.1.1})$$

The differential relations are;

$$\begin{aligned} \frac{\partial r}{\partial x} &= \cos(\theta) = \frac{x}{r} \\ \frac{\partial r}{\partial y} &= \sin(\theta) = \frac{y}{r} \\ \frac{\partial \theta}{\partial x} &= -\frac{y}{r^2} = -\frac{\sin(\theta)}{r} \\ \frac{\partial \theta}{\partial y} &= \frac{x}{r^2} = \frac{\cos(\theta)}{r} \end{aligned} \quad (\text{A.1.2})$$

The transverse displacement is w and slopes are given by [32, 143, 144]:

$$\begin{aligned}\frac{\partial w}{\partial x} &= \frac{\partial w}{\partial r} \frac{\partial r}{\partial x} + \frac{\partial w}{\partial \theta} \frac{\partial \theta}{\partial x} = \frac{\partial w}{\partial r} \cos(\theta) - \frac{1}{r} \frac{\partial w}{\partial \theta} \sin(\theta) \\ \frac{\partial w}{\partial y} &= \frac{\partial w}{\partial r} \frac{\partial r}{\partial y} + \frac{\partial w}{\partial \theta} \frac{\partial \theta}{\partial y} = \frac{\partial w}{\partial r} \sin(\theta) + \frac{1}{r} \frac{\partial w}{\partial \theta} \cos(\theta)\end{aligned}\quad (\text{A.1.3})$$

Similarly, second derivatives are found as:

$$\begin{aligned}\frac{\partial^2 w}{\partial x^2} &= \frac{\partial}{\partial x} \left(\frac{\partial w}{\partial x} \right) = \frac{\partial^2 w}{\partial r^2} \cos^2(\theta) - \frac{\partial^2 w}{\partial \theta \partial r} \frac{\sin(2\theta)}{r} + \frac{\partial w}{\partial r} \frac{\sin^2(\theta)}{r} + \frac{\partial w}{\partial \theta} \frac{\sin(2\theta)}{r^2} + \frac{\partial^2 w}{\partial \theta^2} \frac{\sin^2(\theta)}{r^2} \\ \frac{\partial^2 w}{\partial y^2} &= \frac{\partial}{\partial y} \left(\frac{\partial w}{\partial y} \right) = \frac{\partial^2 w}{\partial r^2} \sin^2(\theta) + \frac{\partial^2 w}{\partial r \partial \theta} \frac{\sin(2\theta)}{r} + \frac{\partial w}{\partial r} \frac{\cos^2(\theta)}{r} - \frac{\partial w}{\partial \theta} \frac{\sin(2\theta)}{r^2} + \frac{\partial^2 w}{\partial \theta^2} \frac{\cos^2(\theta)}{r^2} \\ \frac{\partial^2 w}{\partial x \partial y} &= \frac{\partial}{\partial x} \left(\frac{\partial w}{\partial y} \right) = \frac{\partial^2 w}{\partial r^2} \frac{\sin(2\theta)}{2} + \frac{\partial^2 w}{\partial \theta \partial r} \frac{\cos(2\theta)}{r} - \frac{\partial w}{\partial r} \frac{\sin(2\theta)}{2r} - \frac{\partial w}{\partial \theta} \frac{\cos(2\theta)}{r^2} + \frac{\partial^2 w}{\partial \theta^2} \frac{\sin(2\theta)}{2r^2}\end{aligned}\quad (\text{A.1.4})$$

Then the Laplacian operator is [32, 143, 144]:

$$\nabla^2 w = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \quad (\text{A.1.5})$$

and the biharmonic operator is: [32, 143, 144]:

$$\nabla^4 w = \frac{\partial^4 w}{\partial r^4} + \frac{2}{r} \frac{\partial^3 w}{\partial r^3} - \frac{1}{r^2} \frac{\partial^2 w}{\partial r^2} + \frac{1}{r^3} \frac{\partial w}{\partial r} + \frac{2}{r^2} \frac{\partial^4 w}{\partial r^2 \partial \theta^2} - \frac{2}{r^3} \frac{\partial^3 w}{\partial \theta^2 \partial r} + \frac{4}{r^4} \frac{\partial^2 w}{\partial \theta^2} + \frac{1}{r^4} \frac{\partial^4 w}{\partial \theta^4} \quad (\text{A.1.6})$$

A.2 Strain – Displacement Relations

Assuming the initial displacements are zero, strain - displacement relations which can be found in [32] are used. The bending deformation of a differential element is:

$$u = -z \frac{\partial w}{\partial r}, \quad v = -z \frac{\partial w}{r \partial \theta} \quad (\text{A.2.1})$$

For small deformation, linear strain - displacement relations are [32, 143, 144]:

$$\begin{aligned}\varepsilon_{rr} &= \frac{\partial u}{\partial r} = -z \frac{\partial^2 w}{\partial r^2} \\ \varepsilon_{\theta\theta} &= \frac{1}{r} \frac{\partial v}{\partial \theta} + \frac{u}{r} = -z \left(\frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) \\ \varepsilon_{r\theta} &= \frac{1}{r} \frac{\partial u}{\partial \theta} + r \frac{\partial}{\partial r} \left(\frac{v}{r} \right) = -z \left(\frac{2}{r} \frac{\partial^2 w}{\partial r \partial \theta} - \frac{2}{r} \frac{\partial w}{\partial \theta} \right)\end{aligned}\quad (\text{A.2.2})$$

The state of the stress in the plane is assumed to be the plane stress. Stress - strain relations are defined by the well-known relation, Hooke's Law as [143]:

$$\begin{aligned}\sigma_{rr} &= \frac{Y_r}{1 - \nu_r \nu_\theta} \varepsilon_{rr} + \frac{\nu_r Y_\theta}{1 - \nu_r \nu_\theta} \varepsilon_{\theta\theta} \\ \sigma_{\theta\theta} &= \frac{Y_\theta}{1 - \nu_r \nu_\theta} \varepsilon_{\theta\theta} + \frac{\nu_\theta Y_r}{1 - \nu_r \nu_\theta} \varepsilon_{rr} \\ \sigma_{r\theta} &= Y_{r\theta} \varepsilon_{r\theta}\end{aligned}\quad (\text{A.2.3})$$

where subscripts r and θ correspond to radial and circumferential (tangential) direction, respectively; Y is the Young's modulus, and ν is the Poisson's ratio. For the analysis done in this work, the properties are assumed to be isotropic for in-plane directions (axisymmetric structure). Then:

$$\begin{aligned}\sigma_{rr} &= \frac{Y}{1-\nu^2} \varepsilon_{rr} + \frac{\nu Y}{1-\nu^2} \varepsilon_{\theta\theta} \\ \sigma_{\theta\theta} &= \frac{Y}{1-\nu^2} \varepsilon_{\theta\theta} + \frac{\nu Y}{1-\nu^2} \varepsilon_{rr} \\ \sigma_{r\theta} &= \frac{Y}{2(1+\nu)} \varepsilon_{r\theta}\end{aligned}\tag{A.2.4}$$

A.3 Derivation of Force and Moment Resultants

As stated in the Kirchoff-Love assumptions, the normal stress in the transverse direction (σ_{zz}) is neglected for a thin plate subjected to bending forces. In addition, there will be no resulting forces in the r and θ direction, since the flexural deformation of the plate is considered [143].

$$\int_{-h/2}^{h/2} \sigma_{rr} dz = 0, \quad \int_{-h/2}^{h/2} \sigma_{\theta\theta} dz = 0\tag{A.3.1}$$

These in-plane stresses create bending moment around mid-surface; however, net forces at the mid-surface due to these stresses are zero. It should be noted that, the normal (σ_{rr} and $\sigma_{\theta\theta}$) and transverse shear stresses (σ_{rz} and $\sigma_{\theta z}$) are assumed to vary linearly along the thickness linearly and parabolically, respectively. The flexural rigidity is defined by:

$$D = \frac{Y}{1-\nu^2} \int_{-h/2}^{h/2} z^2 dz = \frac{Yh^3}{12(1-\nu^2)}\tag{A.3.2}$$

The force and moment resultants (Figure 2.4.d) with stress-strain and strain-displacement relations are calculated as [30, 31, 36, 37]:

$$\begin{aligned}M_r &= \int_{-h/2}^{h/2} \sigma_{rr} z dz = -D \left[\frac{\partial^2 w}{\partial r^2} + \nu \left(\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) \right] \\ M_\theta &= \int_{-h/2}^{h/2} \sigma_{\theta\theta} z dz = -D \left[\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} + \nu \frac{\partial^2 w}{\partial r^2} \right] \\ M_{r\theta} &= \int_{-h/2}^{h/2} \sigma_{r\theta} z dz = -(1-\nu) D \left(\frac{1}{r} \frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2} \frac{\partial w}{\partial \theta} \right) = M_{\theta r} \\ Q_r &= \int_{-h/2}^{h/2} \sigma_{rz} dz = -D \left[\frac{\partial^3 w}{\partial r^3} + \frac{1}{r} \frac{\partial^2 w}{\partial r^2} + \frac{1}{r^2} \left(\frac{\partial^3 w}{\partial r \partial \theta^2} - \frac{\partial w}{\partial r} \right) - \frac{2}{r^3} \frac{\partial^2 w}{\partial \theta^2} \right] \\ Q_\theta &= \int_{-h/2}^{h/2} \sigma_{\theta z} dz = -D \left[\frac{1}{r} \frac{\partial^3 w}{\partial \theta \partial r^2} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta \partial r} + \frac{1}{r^3} \frac{\partial^3 w}{\partial \theta^3} \right]\end{aligned}\tag{A.3.3}$$

Since the free body diagram (FBD) of the plate given as the force and moment resultant, one should note that $f(r, \theta, t)$ is the intensity of the externally distributed force. Dynamic equilibrium equations are written to derive EOM using the infinitesimal incremental approach. Writing force balance equation in the z direction results as [30, 31]:

$$\left(Q_r + \frac{\partial Q_r}{\partial r} dr\right) r d\theta + \left(Q_\theta + \frac{\partial Q_\theta}{\partial \theta} d\theta\right) dr + f(r, \theta, t) r dr d\theta - Q_r r d\theta - Q_\theta dr = -\rho h r dr d\theta \frac{\partial^2 w}{\partial t^2} \quad (\text{A.3.4})$$

$$\frac{\partial Q_r}{\partial r} + \frac{1}{r} \frac{\partial Q_\theta}{\partial \theta} + f(r, \theta, t) = -\rho h \frac{\partial^2 w}{\partial t^2}$$

where ρ and h are the density and thickness of the material, respectively. Equilibrium of moments in the r direction is given as [30, 31]:

$$Q_\theta = \frac{\partial M_{r\theta}}{\partial r} + \frac{1}{r} \frac{\partial M_\theta}{\partial \theta} + \frac{2}{r} M_{r\theta} \quad (\text{A.3.5})$$

Similarly, equilibrium of moments in the θ direction is found as [30, 31]:

$$Q_r = \frac{\partial M_r}{\partial r} + \frac{1}{r} \frac{\partial M_{r\theta}}{\partial \theta} + \frac{M_r - M_\theta}{r} \quad (\text{A.3.6})$$

Then, most general form of EOM of a transversely vibrating thin plate is found from the force resultant equilibrium equation.

$$\begin{aligned} & -D \left\{ \frac{\partial}{\partial r} \left[\frac{\partial^3 w}{\partial r^3} + \frac{1}{r} \frac{\partial^2 w}{\partial r^2} + \frac{1}{r^2} \left(\frac{\partial^3 w}{\partial r \partial \theta^2} - \frac{\partial w}{\partial r} \right) - \frac{2}{r^3} \frac{\partial^2 w}{\partial \theta^2} \right] \right\} \\ & -D \frac{1}{r} \left\{ \frac{\partial}{\partial \theta} \left[\frac{1}{r} \frac{\partial^3 w}{\partial \theta \partial r^2} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta \partial r} + \frac{1}{r^3} \frac{\partial^3 w}{\partial \theta^3} \right] \right\} + f(r, \theta, t) = -\rho h \frac{\partial^2 w}{\partial t^2} \end{aligned} \quad (\text{A.3.7})$$

which is simplified as:

$$D \nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = f(r, \theta, t) \quad (\text{A.3.8})$$

B Rayleigh-Ritz Solution Steps

B.1 Energy Based Approximate Solution for Clamped Edge

The most general form of potential and kinetic energy expressions are derived in polar coordinates as follow [36]:

$$V = \frac{D}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right)^2 - 2(1-\nu) \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) + 2(1-\nu) \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial w}{\partial \theta} \right)^2 \right] r d\theta dr \quad (\text{B.1.1})$$

$$T = \frac{\rho h}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial w}{\partial t} \right)^2 r dr d\theta \quad (\text{B.1.2})$$

All parameters are explained in Chapter 2. The deflection of the plate is symmetrical about the center and the deflection pattern is a function of only r . In addition to the axial symmetry, some of the terms have vanished due to fixed boundary conditions [36]. The potential energy expression in the polar coordinates is reduced to:

$$V = \frac{D}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right)^2 r dr d\theta \quad (\text{B.1.3})$$

With the given assumed mode shape in Chapter 2:

$$W_0 = c_1 \left[1 - \left(\frac{r}{a} \right)^2 \right]^2 + c_2 \left[1 - \left(\frac{r}{a} \right)^2 \right]^3 \quad (\text{B.1.4})$$

There are two trial functions included and the first two mode shapes and corresponding mode shapes will be obtained. The maximum energy equations are:

$$V_{\max} = \frac{D\pi}{a^2} \left(\frac{32}{3} c_1^2 + 16c_1c_2 + \frac{48}{5} c_2^2 \right) \quad (\text{B.1.5})$$

$$T_{\max}^* = a^2 \rho h \pi \left(\frac{1}{10} c_1^2 + \frac{1}{6} c_1c_2 + \frac{1}{14} c_2^2 \right) \quad (\text{B.1.6})$$

The partial derivatives of the maximum potential and kinetic energies with respect to Ritz coefficients:

$$\begin{aligned}
\frac{\partial V_{\max}}{\partial c_1} &= \frac{D\pi}{a^2} \left(\frac{64}{3}c_1 + 16c_2 \right) \\
\frac{\partial V_{\max}}{\partial c_2} &= \frac{D\pi}{a^2} \left(16c_1 + \frac{96}{5}c_2 \right) \\
\frac{\partial T_{\max}}{\partial c_1} &= a^2 h \rho \pi \left(\frac{1}{5}c_1 + \frac{1}{6}c_2 \right) \\
\frac{\partial T_{\max}}{\partial c_2} &= a^2 h \rho \pi \left(\frac{1}{6}c_1 + \frac{1}{7}c_2 \right)
\end{aligned} \tag{B.1.7}$$

Eigen value problem (EVP) can be written as:

$$\frac{D\pi}{a^2} \begin{bmatrix} \frac{64}{3} & 16 \\ 16 & \frac{96}{5} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} - \omega^2 a^2 h \rho \pi \begin{bmatrix} \frac{1}{5} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{7} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{B.1.8}$$

Then the stiffness and mass matrices, and Rayleigh's quotient are found as:

$$\begin{aligned}
[K] &= \begin{bmatrix} \frac{64}{3} & 16 \\ 16 & \frac{96}{5} \end{bmatrix} \\
[M] &= \begin{bmatrix} \frac{1}{5} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{7} \end{bmatrix} \\
\lambda &= \frac{\rho h a^4}{D} \omega^2
\end{aligned} \tag{B.1.9}$$

Solving the EVP yields:

$$\lambda = \begin{bmatrix} 104.3877 \\ 1854.01 \end{bmatrix} \tag{B.1.10}$$

Then the natural frequencies of the vibrations are:

$$\omega_1 = \frac{10.217}{a^2} \sqrt{\frac{D}{\rho h}} \quad \text{and} \quad \omega_2 = \frac{43.0582}{a^2} \sqrt{\frac{D}{\rho h}} \tag{B.1.11}$$

and corresponding eigenvectors are:

$$c^{(1)} = \begin{bmatrix} 3.067 \\ 1 \end{bmatrix} \quad \text{and} \quad c^{(2)} = \begin{bmatrix} -0.8384 \\ 1 \end{bmatrix} \tag{B.1.12}$$

B.2 Energy Based Approximate Solution for Free Edge

The most general form of potential and kinetic energy expressions are derived in polar coordinates as follow [36]:

$$V = \frac{D}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right)^2 - 2(1-\nu) \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial r} + \frac{1}{r^2} \frac{\partial^2 w}{\partial \theta^2} \right) + 2(1-\nu) \left[\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial w}{\partial \theta} \right)^2 \right] r d\theta dr \quad (\text{B.2.1})$$

$$T = \frac{\rho h}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial w}{\partial t} \right)^2 r dr d\theta \quad (\text{B.2.2})$$

All parameters are explained in Chapter 2. The deflection of the plate is symmetrical about the center and the deflection pattern is a function of only r . In addition to the axial symmetry, some of the terms have vanished due to fixed boundary conditions [36]. The potential energy expression in polar coordinates is reduced to:

$$V = \frac{D}{2} \int_0^{2\pi} \int_0^a \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right)^2 - 2(1-\nu) \frac{\partial^2 w}{\partial r^2} \left(\frac{1}{r} \frac{\partial w}{\partial r} \right) r d\theta dr \quad (\text{B.2.3})$$

With the given assumed mode shape in Chapter 2:

$$W_0 = c_1 \left(\frac{r}{a} \right)^3 + c_2 \left(\frac{r}{a} \right)^2 \ln \left(\frac{r}{a} \right) \quad (\text{B.2.4})$$

There are two trial functions included and the first two mode shape and corresponding mode shapes will be obtained. The maximum energy equations are:

$$V_{\max} = \frac{D\pi}{a^2} (13.95c_1^2 + 11.8c_1c_2 + 3.3c_2^2) \quad (\text{B.2.5})$$

$$T_{\max}^* = a^2 \rho h \pi \left(\frac{1}{8} c_1^2 - \frac{2}{49} c_1c_2 + \frac{1}{108} c_2^2 \right) \quad (\text{B.2.6})$$

The partial derivatives of the maximum potential and kinetic energies with respect to Ritz coefficients:

$$\begin{aligned} \frac{\partial V_{\max}}{\partial c_1} &= \frac{D\pi}{a^2} (27.9c_1 + 11.8c_2) \\ \frac{\partial V_{\max}}{\partial c_2} &= \frac{D\pi}{a^2} (11.8c_1 + 6.6c_2) \\ \frac{\partial T_{\max}}{\partial c_1} &= a^2 h \rho \pi \left(\frac{1}{4} c_1 - \frac{2}{49} c_2 \right) \\ \frac{\partial T_{\max}}{\partial c_2} &= a^2 h \rho \pi \left(-\frac{2}{49} c_1 + \frac{1}{54} c_2 \right) \end{aligned} \quad (\text{B.2.7})$$

Eigen value problem (EVP) can be written as:

$$\frac{D\pi}{a^2} \begin{bmatrix} 27.9 & 11.8 \\ 11.8 & 6.6 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} - \omega^2 a^2 h \rho \pi \begin{bmatrix} \frac{1}{4} & -\frac{2}{49} \\ -\frac{2}{49} & \frac{1}{54} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (\text{B.2.8})$$

Then the stiffness and mass matrices, and Rayleigh's quotient are found as:

$$\begin{aligned} [K] &= \begin{bmatrix} 27.9 & 11.8 \\ 11.8 & 6.6 \end{bmatrix} \\ [M] &= \begin{bmatrix} \frac{1}{4} & -\frac{2}{49} \\ -\frac{2}{49} & \frac{1}{54} \end{bmatrix} \\ \lambda &= \frac{\rho h a^4}{D} \omega^2 \end{aligned} \quad (\text{B.2.9})$$

Solving the EVP yields:

$$\lambda = \begin{bmatrix} 14.5457 \\ 1041.56 \end{bmatrix} \quad (\text{B.2.10})$$

Then the natural frequencies of the vibrations are:

$$\omega_1 = \frac{3.814}{a^2} \sqrt{\frac{D}{\rho h}} \quad \text{and} \quad \omega_2 = \frac{32.273}{a^2} \sqrt{\frac{D}{\rho h}} \quad (\text{B.2.11})$$

and corresponding eigenvectors are:

$$c^{(1)} = \begin{bmatrix} -0.51 \\ 1 \end{bmatrix} \quad \text{and} \quad c^{(2)} = \begin{bmatrix} 0.2336 \\ 1 \end{bmatrix} \quad (\text{B.2.12})$$

C Approximate Bessel Functions

The Bessel functions may have undetermined values which makes undetermined displacement in the plate. This is not physically possible; therefore, approximations of zeroth and first order Bessel functions are used in general [70]. The ascending series for Bessel functions are:

$$J_0(ka) = 1 - \frac{(ka)^2}{4} + \dots \quad J_1(ka) = \frac{ka}{2} - \frac{(ka)^3}{16} + \dots \quad (\text{C.1.1})$$

$$I_0(ka) = 1 + \frac{(ka)^2}{4} + \dots \quad I_1(ka) = \frac{ka}{2} + \frac{(ka)^3}{16} + \dots \quad (\text{C.1.2})$$

$$Y_0(ka) = J_0(ka) \log(ka) + \frac{(ka)^2}{4} \dots \quad (\text{C.1.3})$$

$$Y_1(ka) = J_1(ka) \log(ka) - \frac{1}{ka} - \frac{(ka)}{4} + (ka)^2 \dots$$

$$K_0(ka) = -I_0(ka) [\log(ka) - E] + \frac{(ka)^2}{4} \dots \quad (\text{C.1.4})$$

$$K_1(ka) = -I_1(ka) [\log(ka) - E] - \frac{1}{ka} - \frac{(ka)}{4} + \dots$$

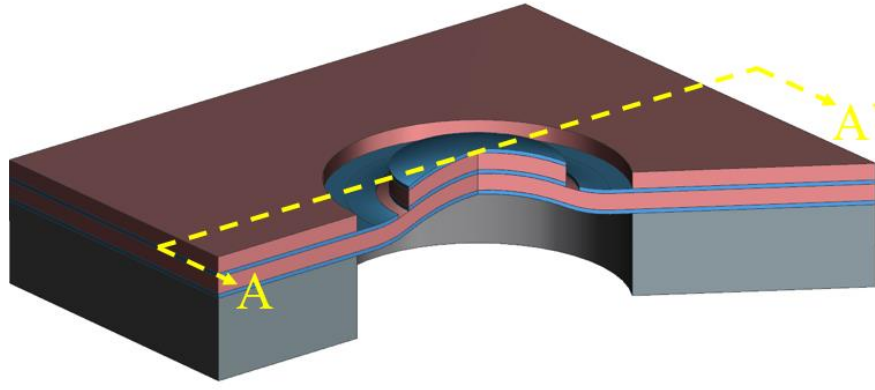
where E is given as:

$$E = \log(2) - \gamma \approx 0.11593 \quad (\text{C.1.5})$$

and γ is the Euler's constant.

D Bimorph, Dual Electrode, Clamped-Pinned Boundary pMUT Process Traveler

The 3D illustration of the bimorph, dual electrode, pinned-clamped pMUT is given below with the section cut AA`. This section view is used in fabrication process traveler. The fabrication has been done in University of California, Berkeley Marvell Nanofabrication laboratory (NanoLab) clean room. The tool names and recipes correspond the NanoLab tools as of the year 2022. The test processes are not included; however, it is strongly recommended to do a test run for every step before the device run.



0. NanoLab Tools

Table D.1: Required tools in UC Berkeley Marvell Nanofabrication Laboratory (NanoLab)

Cleaning	Deposition	Lithography	Etchers	Packaging	Metrology
msink6	aln2	picotrack1	lam7	disco	flexus
msink8	oxford2	picotrack2	sts2	westbond	nanospec
msink16		scgcoat6	sts-oxide	vacoven2	xrd
msink18		svgdev6	matrix-etch	hotplate	probe-eb8
msink1		ksaligner		msink5	ircam
		oven		headway2	

1. Starting wafers

p-type, Si, prime grade, 6" wafers.

Details/Comments: For labeling the wafers, use wafer scribe. The bottom left part of the major flat is scribed with letters and numbers which has round shapes like S, O, 3, 6, 8. The purpose is to make sure that no scribed line is aligned with the crystal directions of Si wafer. These scribed regions create stress concentration points which may be critical on QDR (quick dump rinse) and SRD (spin rinse drier).

2. Metrology: Initial wafer bow and stress measurement.

NanoLab Tool: flexus

Details/Comments: Single direction measurement is enough.

3. Cleaning: Wafer cleaning

NanoLab Tool: msink8

Steps: 10 min Piranha at 120°C + 4 cycle QDR + 1min HF at room temperature + 4 cycle QDR + SDR

Details/Comments: If the wafers are new, cleaning on msink8 only is enough, otherwise same process is suggested to follow on msink6.



4. Deposition: Deposition of bottom stack AlN/Mo/AlN

NanoLab Tool: aln2

Recipe: In this tool, recipe is known as sequence: zshao_aln_b_0926

Details/Comments: Do not forget to do an etching before the deposition to clean the wafer surface further. During deposition Ar and N₂ gases are used. In the standard recipe, the Ar/N₂ ratio is 1/3. However, it should be tuned according to desired the internal stress and FWHM. This sequence includes the AlN seed layer, bottom Mo, bottom AlN, and mid Mo layers.



5. Metrology: Film thickness

NanoLab Tool: nanospec

Recipe: AlN on Moly

Details/Comments: Check the goodness of fit, it should be much smaller than unity. Otherwise your measurement is not reliable.

6. Metrology: Film stress

NanoLab Tool: flexus

Details/Comments: Use the wafer ID assigned in the initial measurement. Both the film stress and stress gradient is critical in pMUT fabrication. It is possible to measure film stress which we want it to be as small as possible. However, there is no way of measuring stress gradient before releasing.

7. Metrology: XRay diffraction

NanoLab Tool: xrd

Details/Comments: Measure both Locked Couple and Rocking curves after optimizing the signal. The Full Width Half Max (FWHM) is defined on the Rocking Curve.

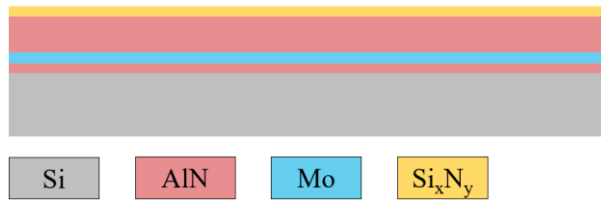
8. Deposition: Nitride barrier

NanoLab Tool: oxford2

Recipe: Standard 3min PECVD nitride recipe

Goal: 65 nm thickness

Details/Comments: Nitride deposition adds 20-30MPa Tensile stress. This barrier layer is used to insulate the bottom and mid electrodes. During the AlN deposition, there might be small gaps reaching from bottom surface to top surface due to contamination or grain boundaries resulting a short circuit between mid Mo and bottom Mo.



9. Metrology: Nitride barrier thickness

NanoLab Tool: nanospec

Details/Comments: There is no recipe for nitride layer thickness over AlN. Therefore, a test deposition should be done to determine deposition rate. 3 min deposition gives ~65 nm.

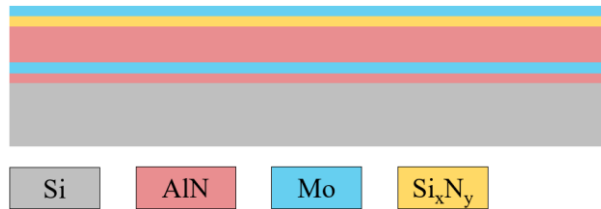
10. Deposition: Mid Mo

NanoLab Tool: aln2

Recipe: zshao_bi_Mo_45s

Goal: 150 nm thickness

Details/Comments: There is no recipe for Mo layer thickness over nitride. Therefore, the thickness will be measured after etching.



11. Lithography: Photoresist (PR) coating

NanoLab Tool: svgcoat6

Recipe: 01_OCG2um_Bake60s90C

Details/Comments: NanoLab currently does not have OCG PR line, it is replaced by MiR series. MiR701 PR is alternative with 2 μ m thickness.

12. Lithography: Exposure

NanoLab Tool: ksaligner

Details/Comments: The lamb test should be done before the exposure in order to determine the correct exposure time for sufficient exposure dose.

13. Lithography: PR develop

NanoLab Tool: svgdev6

Recipe: 03_noPEB_OCG25s

14. Lithography: Hardbake

NanoLab Tool: oven

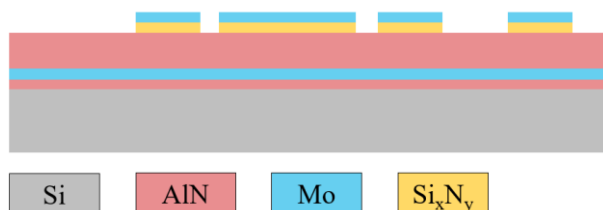
Recipe: 1 hour at 120°C

15. Etch: Mid electrode etch

NanoLab Tool: matrix-etch

Recipe: Ben_SF6_R.rcp

Details/Comments: The recipe is repeated as 60 s + 60 s + 60 s + 30 s. The etch is observed visually first to determine whether Mo etched or not. If longer duration is done at a single time, the etch rate will be different, so tune the time accordingly. The same recipe etches both Mo and nitride layers.



16. Lithography: PR strip

NanoLab Tool: msink1

Recipe: 10 min 1165 soaks at 80°C + 4 cycles QDR + SDR

Details/Comments: The PR stripper (1165) also etches the AlN. Although AlN etch rate is not high, care should be taken if longer 1165 soaks are required.

17. Metrology: Resistance measurement

NanoLab Tool: probe-eb8

Details/Comments: During the mask design, the resistive test structure has been added both to measure expected resistances and to check if closely placed wires opened. Check test structures at different locations to ensure all are etched.

18. Metrology: Mo thickness

NanoLab Tool: alphastep

Details/Comments: Expected Mo thickness is ~150 nm

19. Cleaning: Before the top AlN deposition

NanoLab Tool: msink1

Recipe: 10 min SVC-14 at 80°C + 4 cycles QDR + SDR

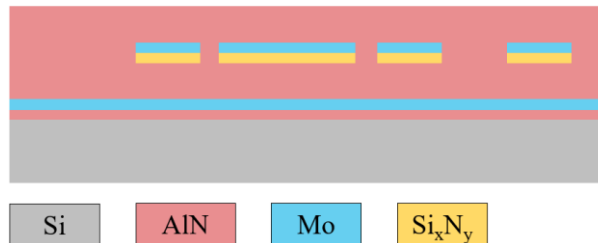
Details/Comments: After the cleaning, bake at 120°C for 30 min to dehydrate. It is better to check resistances after this step.

20. Deposition: Deposition of top stack AlN/Mo

NanoLab Tool: aln2

Recipe: In this tool, recipe is known as sequence: zshao_aln_t_1006

Details/Comments: Do not forget to do an etching before the deposition to clean the wafer surface further. During deposition Ar and N₂ gases are used. In the standard recipe, the Ar/N₂ ratio is 1/3. However, it should be tuned according to desired the internal stress and FWHM. This sequence includes the AlN seed layer, bottom Mo, bottom AlN, and mid Mo layers.



21. Metrology: Film stress

NanoLab Tool: flexus

Details/Comments: Use the wafer ID assigned in the initial measurement. Both the film stress and stress gradient is critical in pMUT fabrication. It is possible to measure film stress which we want it to be as small as possible. However, there is no way of measuring stress gradient before releasing.

22. Metrology: XRay diffraction

NanoLab Tool: xrd

Details/Comments: Measure both Locked Couple and Rocking curves after optimizing the signal. The Full Width Half Max (FWHM) is defined on the Rocking Curve.

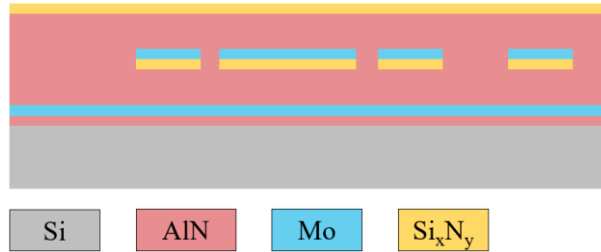
23. Deposition: Nitride barrier

NanoLab Tool: oxford2

Recipe: Standard 3 min PECVD nitride recipe

Goal: 65 nm thickness

Details/Comments: Nitride deposition adds 20-30MPa Tensile stress. This barrier layer is used to insulate the bottom and mid electrodes. During the AlN deposition, there might be small gaps reaching from bottom surface to top surface due to contamination or grain boundaries resulting a short circuit between mid Mo and bottom Mo.



24. Metrology: Nitride barrier thickness

NanoLab Tool: nanospec

Details/Comments: There is no recipe for nitride layer thickness over AlN. Therefore, a test deposition should be done to determine deposition rate. 3 min deposition gives ~65 nm.

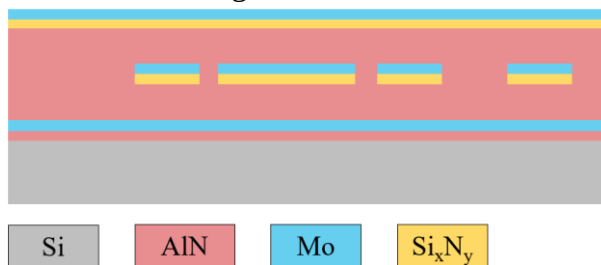
25. Deposition: Top Mo

NanoLab Tool: aln2

Recipe: zshao_bi_Mo_45s

Goal: 150 nm thickness

Details/Comments: There is no recipe for Mo layer thickness over nitride. Therefore, the thickness will be measured after etching.



26. Lithography: Photoresist (PR) coating

NanoLab Tool: svgcoat6

Recipe: 01_OCG2um_Bake60s90C

Details/Comments: NanoLab currently does not have OCG PR line, it is replaced by MiR series. MiR701 PR is alternative with 2 μm thickness.

27. Lithography: Exposure

NanoLab Tool: ksaligner

Details/Comments: The lamb test should be done before the exposure in order to determine the correct exposure time for sufficient exposure dose.

28. Lithography: PR develop

NanoLab Tool: svgdev6

Recipe: 03_noPEB_OCG25s

29. Lithography: Hardbake

NanoLab Tool: oven

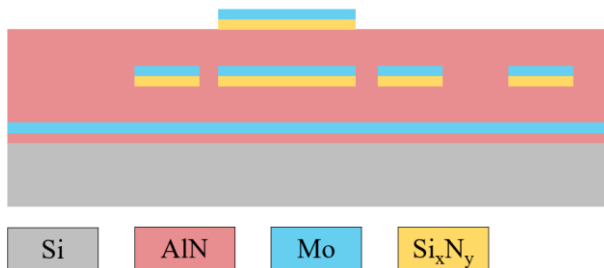
Recipe: 1 hour at 120°C

30. Etch: Top electrode etch

NanoLab Tool: matrix-etch

Recipe: Ben_SF6_R.rcp

Details/Comments: The recipe is repeated as 60 s + 60 s + 60 s + 30 s. The etch is observed visually first to determine whether Mo etched or not. If longer duration is done at a single time, the etch rate will be different, so tune the time accordingly. The same recipe etches both Mo and nitride layers.



31. Lithography: PR strip

NanoLab Tool: msink1

Recipe: 10 min 1165 soaks at 80°C + 4 cycles QDR + SDR

Details/Comments: The PR stripper (1165) also etches the AlN. Although AlN etch rate is not high, care should be taken if longer 1165 soaks are required.

32. Metrology: Resistance measurement

NanoLab Tool: probe-eb8

Details/Comments: During the mask design, the resistive test structure has been added both to measure expected resistances and to check if closely placed wires opened. Check test structures at different locations to ensure all are etched.

33. Metrology: Mo thickness

NanoLab Tool: alphastep

Details/Comments: Expected Mo thickness is ~150 nm

34. Lithography: Photoresist (PR) coating

NanoLab Tool: svgcoat6

Recipe: 01_OCG2um_Bake60s90C

Details/Comments: This time 2 layers of PR deposited with the same recipe due to selectivity issue in the etching. In total the PR thickness is 4μm.

35. Lithography: Exposure

NanoLab Tool: ksaligner

Details/Comments: The lamb test should be done before the exposure in order to determine the correct exposure time for sufficient exposure dose.

36. Lithography: PR develop

NanoLab Tool: svgdev6

Recipe: 03_noPEB_OCG25s

37. Lithography: Hardbake

NanoLab Tool: oven

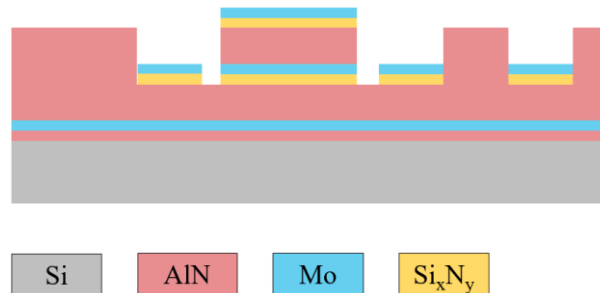
Recipe: 2 hours at 120°C

38. Etch: Mid via etch

NanoLab Tool: lam7

Recipe: spala_4120_Al

Details/Comments: The recipe is 3 min with etch rate of 7 nm/s. However, after the dry etch it is difficult to tell if the etch is finished. One way to get better idea is to check the resistance of the mid electrodes (couple of ohms). Another way is measure the thickness after PR strip; however, if the etch is not finished, then lithography should be repeated which is not convenient.



39. Lithography: PR strip

NanoLab Tool: msink1

Recipe: 10 min 1165 soaks at 80°C + 4 cycles QDR + SDR

Details/Comments: The PR stripper (1165) also etches the AlN. Although AlN etch rate is not high, care should be taken if longer 1165 soaks are required.

40. Metrology: Resistance measurement

NanoLab Tool: probe-eb8

Details/Comments: During the mask design, the resistive test structure has been added both to measure expected resistances and to check if closely placed wires opened. Check test structures at different locations to ensure all are etched.

41. Metrology: Mo thickness

NanoLab Tool: alphastep

Details/Comments: Expected Mo thickness is ~150 nm.

42. Lithography: Photoresist (PR) coating

NanoLab Tool: svgcoat6

Recipe: 01_OCG2um_Bake60s90C

Details/Comments: This time 2 layers of PR deposited with the same recipe due to selectivity issue in the etching. In total the PR thickness is 4 μm .

43. Lithography: Exposure

NanoLab Tool: ksaligner

Details/Comments: The lamb test should be done before the exposure in order to determine the correct exposure time for sufficient exposure dose. 15 s of exposure with 40 μm gap soft contact.

43. Lithography: PR develop

NanoLab Tool: svgdev6

Recipe: 03_noPEB_OCG25s

44. Lithography: Hardbake

NanoLab Tool: oven

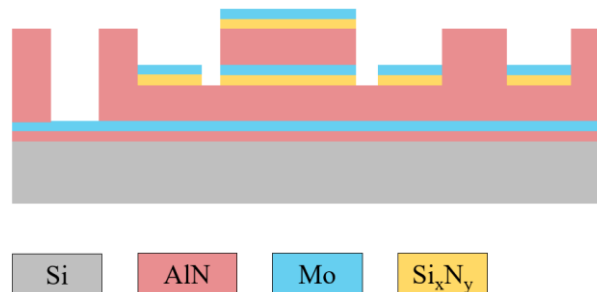
Recipe: 2 hours at 120°C

45. Etch: Bottom via etch

NanoLab Tool: lam7

Recipe: spala_4120_Al

Details/Comments: The recipe is 3 min with etch rate of 7 nm/s. However, after the dry etch it is difficult to tell if the etch is finished. One way to get better idea is to check the resistance of the mid electrodes (couple of ohms). Another way is measure the thickness after PR strip; however, if the etch is not finished, then lithography should be repeated which is not convenient.



46. Lithography: PR strip

NanoLab Tool: msink1

Recipe: 10 min 1165 soaks at 80°C + 4 cycles QDR + SDR

Details/Comments: The PR stripper (1165) also etches the AlN. Although AlN etch rate is not high, care should be taken if longer 1165 soaks are required.

47. Metrology: Resistance measurement

NanoLab Tool: probe-eb8

Details/Comments: During the mask design, the resistive test structure has been added both to measure expected resistances and to check if closely placed wires opened. Check test structures at different locations to ensure all are etched.

48. Deposition: SiO₂ deposition of handle wafer for encapsulation

NanoLab Tool: oxford2

Recipe: zsox-200n

Zsox-15u

Goal: 4 μ m SiO₂ deposition on handle wafer

Details/Comments: In NanoLab, it is required to use a handle wafer for through wafer etch which will be done for backside DRIE. The handle wafer and the device wafer will be bonded.



49. Deposition: SiO₂ deposition of device wafer backside and front side

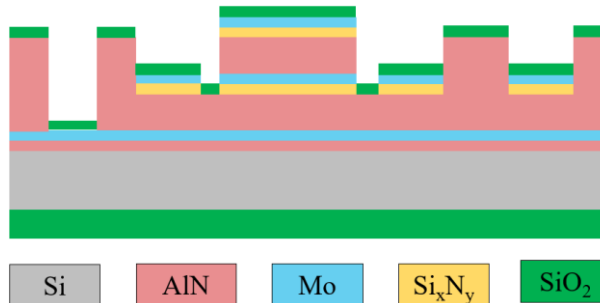
NanoLab Tool: oxford2

Recipe: Front side: zsox-300n

Back side: zsox-15u

Goal: 4 μ m SiO₂ deposition on device wafer backside

Details/Comments: The front side layer (~ 300 nm) is for protection of the devices: whereas, the back side thick layer (2.5 μ m) is hard mask for the backside DRIE.



50. Lithography: Photoresist (PR) coating

NanoLab Tool: picotrack1

Recipe: T1_AZ4620_8.0um, NoTEBR

Details/Comments: The backside etch is DRIE which requires long etching time for thorough wafer etch. Therefore, a thick PR used.

51. Lithography: Exposure

NanoLab Tool: ksaligner

Details/Comments: The lamb test should be done before the exposure in order to determine the correct exposure time for sufficient exposure dose. Backside mask exposure with 28 s with 90 μ m gap alignment.

52. Lithography: PR develop

NanoLab Tool: picotrack2

Recipe: T2_noPEB_MF26A2*60s_2Spray

Details/Comments: After the development, check the critical designs under microscope to see development was successful.

53. Lithography: Hardbake

NanoLab Tool: oven

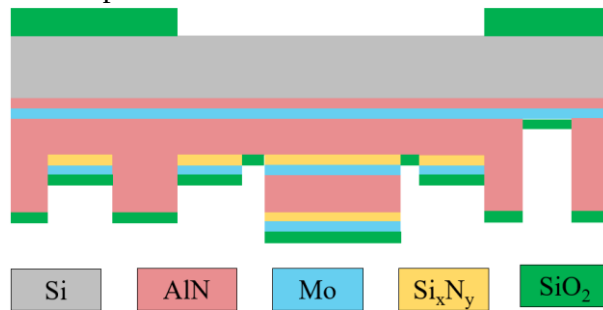
Recipe: 8 hours at 120°C

54. Etch: SiO₂ etch of device wafer back side

NanoLab Tool: sts-oxide

Recipe: beovino_no1

Details/Comments: This recipe is 5 min.

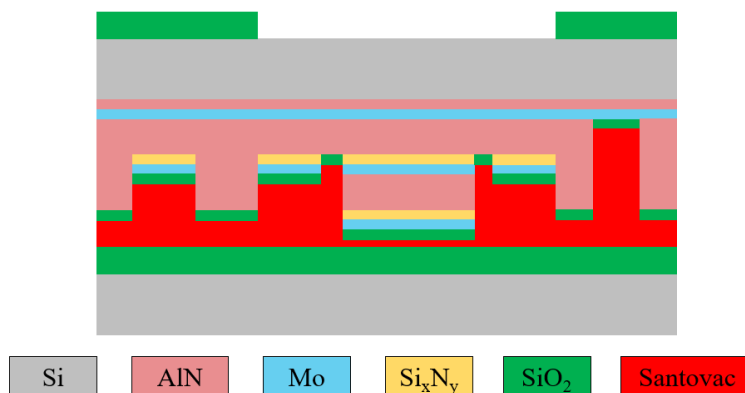


55. Wafer bonding: Bonding of device and handle wafers

NanoLab Tool: vacoven2

Recipe: ~6 drops of Santovac, Temp. set 4.5 for 60 min and 7 for 90 min

Details/Comments: Handle wafer bonding is manual process. It might require couple of dry runs in order to have Santovac fill all the surface area between two wafers. First place the handle wafer on the chuck with SiO₂ layer on the top. Pieces of the technicloth are placed at the outer edge so that excess Santovac will be on the technicloth. Otherwise, the handle wafer might be bonded to the chuck. Then pure the Santovac (~ 6 drops) on the handle wafer. Bend the shape memory alloys, SMA, (3 of them) on to the handle wafer. These SMA will return to its original position at elevated temperatures enabling the device and handle wafers bonding. Place the device wafer as SiO₂ layer on the top and AlN/Mo stacks on the bottom on top of the handle wafer. Then put them all together into the vacuum oven for 60 min at the set value 4.5 + 90 min at the set value 7.



55. Metrology: Check the Santovac bonding

NanoLab Tool: ircam

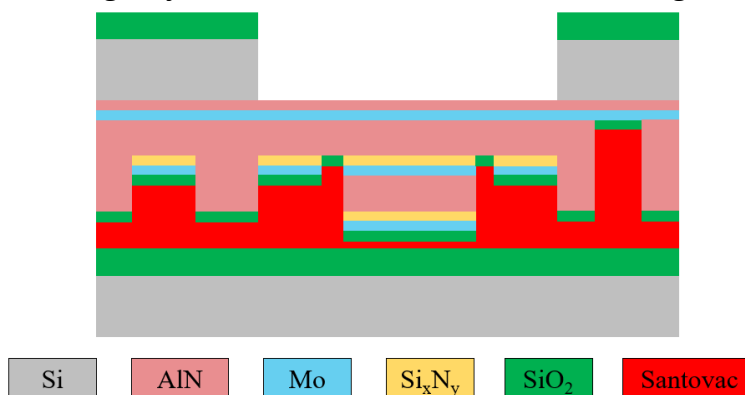
Details/Comments: The material stack here is transparent to IR wavelength. Check this bonded stack under ircam to see whether the Santovac covered all the surface area or not.

56. Etch: Si DRIE

NanoLab Tool: sts2

Recipe: beovino_HF_5s (600 cycles)
 beovino_DEEP_LF (200 cycles)
 beovino_DEEP_LF (100 cycles)
 beovino_DEEP_LF (25 cycles)
 beovino_DEEP_LF (30 cycles)

Details/Comments: After each etch step, it is checked under white light microscope to see the AlN seed layer which has blueish color. The required number of cycles to finish the etch depends on aspect ratio of the etch. That means as the radius of diaphragms gets smaller, it will take longer cycles to etch all the Si. Therefore, tuning must be done.



57. Debonding: Debonding after the DRIE

NanoLab Tool: hotplate

Details/Comments: Place the stack as the handle wafer is down. At 60°C, the Santovac becomes liquid in 5 min. Since the Santovac thickness is small, the bond is quite strong

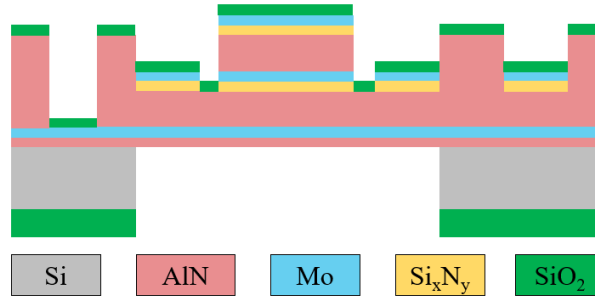
against the normal forces; however, it is very weak for shear forces. Slightly apply shear force on the device wafer, then it will slide over the handle wafer.

58. Cleaning: Removal of Santovac residue

NanoLab Tool: msink1

Recipe: 3 x (1 min Acetone + 1 min IPA + 1 min DI water)

Details/Comments: This is not soak; it should be sprayed on the wafer. Care must be taken, because diaphragms are defined at this point and they are fragile (only ~2 μm thick diaphragms). Spray these liquids at the very top of the wafer.



59. Dicing: PR coating

NanoLab Tool: msink5

Details/Comments: Coat the top surface of the wafer with spray-on photoresist. This is done before the dicing because, the PR will protect the diaphragms from the silicon ships created during mechanical sawing; i.e. dicing. IF you cannot obtain spray-on PR, then tape your device wafer onto another handle wafer with Capton tapes at 4-5 points at the edge and use spinner to coat PR. For spin coating, 1 min 500 rpm recipe should be used.

60. Dicing: Mechanical sawing

NanoLab Tool: disco

Recipe: spala_single

Details/Comments: The sawing parameters are: 2.0 mm/s feed speed, 18000 rpm rotational speed, water flows are 0.3:0.3:0.3.

61. Cleaning and Releasing: Wet etching AlN residues and the removal of PR and SiO₂ layers

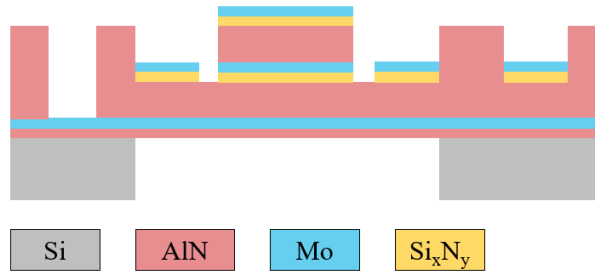
NanoLab Tool: msink18

Recipe – Cleaning: 1 min HF + 1 min DI Water + 1 min Acetone + 1 min IPA + 1 min DI Water at room temperature

Recipe – Release: H₃PO₄ and DI Water (3:1) at 70°C for 5 min

Details/Comments: The cleaning is done to remove the protective PR and any contamination from the dicing process. HF is for the SiO₂ layer removal. Rest is general cleaning steps. The releasing step is needed if there is any AlN layer remained after mid via and bottom via etches. The byproducts during these dry etching steps may prevent AlN

etched all the way, sometimes the last ~100 nm AlN remains. This residue increases the resistance in electrical path and also makes difficult to have wire bonds. Therefore, a last wet etching step of AlN may be needed.



62. Wire bonding: Electrical connections to the electrodes

NanoLab Tool: westbond

Recipe: Ultrasound power of 350 W for bottom via and 320 W for mod via and top electrode

Details/Comments: The powers of wire bonding should be tuned based on need.