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Upstream flood pattern recognition based on downstream events

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Abstract Reverse stream flood routing determines the upstream hydrograph in a stream reach given the downstream hydrograph. The Muskingum model of flood routing involves parameters that govern the routed hydrograph. These parameters are herein estimated using simulation methods coupled with optimization tools to achieve optimized parameters. Different simulation methods are shown to perform unequally in the estimation of nonlinear Muskingum parameters. This paper presents two simulation methods for nonlinear Muskingum reverse flood routing: (1) Euler equations and (2) Runge-Kutta 4th order equations. Moreover, the generalized reduced gradient (GRG) is used as the optimization tool that minimized the sum of the squared deviations (*SSQ*) between observed and routed inflows in a benchmark flood routing problem. Results show the Runge-Kutta 4th order equations yield better routed hydrographs with smaller *SSQ* than obtained in previous research and with the first simulation method (Euler equations).

Keywords Reverse stream flow routing · Nonlinear Muskingum model · Euler equations · Runge-Kutta equations, generalized reduced gradient

Introduction

Flood routing calculates outflow hydrographs along a river given inflow hydrographs. In contrast, reverse flood routing determines the inflow hydrograph given the outflow hydrograph. Das (2009) cited applications of reverse flood routing (Das 2009). Those applications enable enacting emergency rehabilitation projects after the passage of large floods. An understanding about flood damage in the upstream reaches of the river can be gained by implementing reverse routing. Human activities, such as commercial open sand and gravel mining on river beds, construction of flow retarding structures and bridges, and the like, may alter the local flow characteristics in a reach of the river. Reverse flood routing can assist in determining the upstream inflow hydrograph whenever a desired downstream hydrograph is specified.

Generally, hydraulic and hydrologic approaches are applied for flood routing through water bodies. Hydraulic approaches rely on the equations of conservation of energy or momentum in streams that are applied to flood routing using geometric and hydraulic data. One well-known hydraulic model is the river analysis system

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(RAS) (Hydrologic Engineering Center 2010). Hydrologic flood routing, on the other hand, is simple in comparison and relies on limited parameters coupled with linear and nonlinear equations based on the continuity equation to route flood hydrographs (Orouji et al. 2013). McCarthy (1938) introduced and applied the Muskingum routing method in the Ohio River.

The nonlinear Muskingum flood routing model parameters can be found by trial and error, although this may take a large number of trials and become computationally burdensome. Thus, use of an optimization method coupled with a hydrography simulation model is recommended to find optimally simulated hydrographs (Fallah-Mehdipour et al. 2011). The Muskingum model applies a continuity equation that has a first-order differential structure and is solved by forward and backward methods in forward routing and reverse routing, respectively. Tung (1985) applied Euler and Runge-Kutta 4th order equations as the simulation methods in applications of the Muskingum model. Other mathematically based optimization tools, such as segmented least squares (S-LSQ), nonlinear least squares (N-LSQ), Broyden–Fletcher–Goldfarb–Shannon (BFGS), Lagrange multiplier (LM), nelder-mead simplex (NMS), and GRG were applied respectively by Gill (1978), Yoon and Padmanabhan (1993), Geem (2006), Das (2004, 2009), and Hamedi et al. (2014).

Pattern search (PS) (Tung 1985), genetic algorithm (GA) (Mohan 1997), harmony search (HS) (Kim et al. 2001), particle swarm optimization (PSO) (Chu and Chang 2009), parameter-setting-free HS (PSF-HS) (Geem 2011), differential evolution (DE) (Xu et al. 2012), simulated annealing (SA), and shuffled frog leaping algorithm (SFLA) (Orouji et al. 2013) were evolutionary algorithms applied to Muskingum forward routing.

The forward Muskingum method is applied to calculate the outflow hydrograph from the inflow hydrograph and a set of Muskingum model parameters. Ideally, the same set of parameters would apply to reverse routing, whereby an inflow hydrograph is calculated from an outflow hydrograph. In actuality, inflow and outflow hydrograph data are not ideal. Therefore, one set of Muskingum model parameter values may not be applicable for forward and reverse routing of flood hydrographs (Das 2009). Das (2009) applied LM to find optimal parameters of a reverse routing Muskingum model minimizing the sum of squares of the normalized difference between observed and estimated inflows subjected to the satisfaction of the routing equation. The results indicated parameter values for three types of Muskingum channel storage models and corresponding inflow hydrographs fit the observed inflows satisfactorily. LM was shown capable of estimating optimal parameters for use in reverse routing, although it does so through a time-consuming process employing the derivations of different Muskingum parameters (K, X, and m), inflow (I), and Lagrange multiplier (λ) simultaneously. Thus, application of less computationally burdensome optimization methods could decrease processing time in reverse flood routing.

This paper couples Euler and Runge-Kutta 4th order differential methods with GRG as the optimization tool in reverse flood routing of a benchmark problem. This paper's results contrast the capability of the Euler method with that of the Runge-Kutta 4th order method in hydrograph simulation. Moreover, the performance of LM and GRG are compared to determine optimal nonlinear Muskingum parameters in the optimization step for parameter estimation.

Methodology

The optimal nonlinear Muskingum model for reverse flood routing herein presented applies the Euler and Runge-Kutta 4th order equations as the simulation methods and GRG as the optimization tool.

Hydrograph simulation methods

McCarthy (1938) stated the continuity and storage equations used in the Muskingum routing model, as follows:

Continuity:

$$\frac{dS_t}{dt} = I_t - O_t \tag{1}$$

Storage:

$$S_t = K[XI_t + (1-X)O_t] \tag{2}$$

in which, S_t, I_t, and O_t= storage, inflow, and outflow at time t, respectively; K= storage-time constant for a river reach; which has a value reasonably close to the travel time of flow through the river reach; and X= weighting factor usually ranging between 0 and 0.5 for reservoir storage, and between 0 and 0.3 for stream channels

(Mohan 1997). There are two nonlinear forms of the Muskingum model, which were applied by Chow (1959) and Gill (1978), as follows:

$$S_t = K[\mathrm{XI}_t + (1-X)O_t]^m \tag{3}$$

$$S_t = K\left[\mathrm{XI}_t^m + (1-X)O_t^m\right] \tag{4}$$

in which, m= exponent that presents nonlinearity between storage and weighted flow.

Reverse flood routing proceeds backwards in time, starting with the last of N simulation steps. The following steps are applied to route hydrograph using the Euler equations:

Step (1). Initial values are assumed for the parameters K, X, and m.

Step (2). The storage volume at last time step, S_N, is calculated with Eq. (5). It should be noted that the estimated input flow ($\hat{I}_N$) is the same as the observed output flow (O_N) in the last routing time step (step N), $\hat{I}_N = O_N$:

$$S_N = K\left[X\hat{I}_N + (1-X)O_N\right]^m \quad j = N \tag{5}$$

Step (3). $\frac{\Delta S_j}{\Delta t}$ is the rate of storage volume variation, determined as follows:

$$\frac{\Delta S_j}{\Delta t} = \left(\frac{1}{X}\right)\left(\frac{S_j}{K}\right)^{1/m} - \left(\frac{1}{X}\right)O_j \quad j = N, N\text{-}1, \ldots, 1 \tag{6}$$

Step (4). Storage volume is calculated with Eq. (7):

$$S_j = S_{j+1} - \Delta t\left(\frac{\Delta S_{j+1}}{\Delta t}\right) \quad j = N\text{-}1, N\text{-}2, \ldots, 0 \tag{7}$$

Step (5). Input flow is estimated with Eq. (8):

$$\hat{I}_j = \left(\frac{1}{X}\right)\left(\frac{S_j}{K}\right)^{1/m} - \left(\frac{1-X}{X}\right)O_j \quad j = N\text{-}1, N\text{-}2, \ldots, 0 \tag{8}$$

Step (6). Steps (3) to (5) are repeated for all N time steps.

The Runge-Kutta 4th order equations are widely applied in numerical simulation because of their high accuracy. The following steps are applied with the Runge-Kutta 4th order equations:

Step (1). Initial values are assumed for K, X, and m.

Step (2). Storage volume at last time step, S_N, is calculated with Eq. (5) where $\hat{I}_N = O_N$.

Step (3). $\frac{\Delta S_j}{\Delta t}$ is calculated with:

$$K_{j1} = \left(\frac{1}{X}\right)\left(\frac{S_j}{K}\right)^{1/m} - \left(\frac{1}{X}\right)O_j \quad j = N, N\text{-}1, \ldots, 1 \tag{9}$$

$$K_{j2} = \left(\frac{1}{X}\right)\left(\frac{S_j + 0.5K_{j1}\Delta t}{K}\right)^{1/m} - \left(\frac{1}{X}\right)\left(\frac{O_j + O_{j-1}}{2}\right) \quad j = N, N\text{-}1, \ldots, 1 \tag{10}$$

$$K_{j3} = \left(\frac{1}{X}\right)\left(\frac{S_j + 0.5K_{j2}\Delta t}{K}\right)^{1/m} - \left(\frac{1}{X}\right)\left(\frac{O_j + O_{j-1}}{2}\right) \quad j = N, N\text{-}1, \ldots, 1 \tag{11}$$

$$K_{j4} = \left(\frac{1}{X}\right)\left(\frac{S_j + K_{j3}\Delta t}{K}\right)^{1/m} - \left(\frac{1}{X}\right)O_{j-1} \quad j = N, N\text{-}1, \ldots, 1 \tag{12}$$

$$\frac{\Delta S_j}{\Delta t} = {}^{1/6}\left(K_{j1} + 2K_{j2} + 2K_{j3} + K_{j4}\right) j = N, N\text{-}1, \ldots, 1 \tag{13}$$

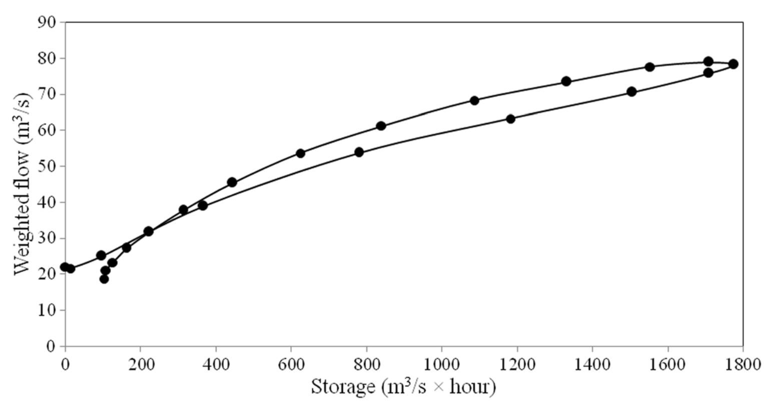

Fig. 1 Nonlinear relation between storage and weighted flow

Table 1 Calculated input hydrograph using the Euler equations

j	Time (hour)	O_j (m^3/s)	J_j (m^3/s)	S_j (m^3/s)	$\frac{\Delta S_j}{\Delta t}$ (m^3/s)	$\hat{I}_j$ (m^3/s)	$(I_j-\hat{I}_j)^2$ (m^3/s)
0	0	22	22	24.12	–	24.12	4.51
1	6	21	23	77.34	− 8.06	12.94	101.26
2	12	21	35	200.86	20.59	41.59	43.38
3	18	26	71	483.59	47.12	73.12	4.50
4	24	34	103	850.36	61.13	95.13	61.97
5	30	44	111	1224.40	62.34	106.34	21.70
6	36	55	109	1540.67	52.71	107.71	1.66
7	42	66	100	1748.17	34.58	100.58	0.34
8	48	75	86	1834.95	14.46	89.46	12.00
9	54	82	71	1788.05	− 7.82	74.18	10.13
10	60	85	59	1636.79	− 25.21	59.79	0.62
11	66	84	47	1420.03	− 36.13	47.87	0.76
12	72	80	39	1164.11	− 42.65	37.35	2.73
13	78	73	32	903.95	− 43.36	29.64	5.57
14	84	64	28	655.62	− 39.72	24.28	13.84
15	90	54	24	466.87	− 33.13	20.87	9.77
16	96	44	22	326.68	− 23.36	20.64	1.86
17	102	36	21	235.67	− 15.17	20.83	0.03
18	108	30	20	159.96	− 12.62	17.38	6.86
19	114	25	19	134.01	− 4.33	20.67	2.80
20	120	22	19	88.20	− 7.63	14.37	21.47
21	126	19	18	82.20	− 1.00	18.00	0.00
Sum	–	–	–	–	–	–	327.78

in which, K_{j1}= rate of storage volume variation at jth time step calculated using K, K_{j2}= rate of storage volume variation between j-1th and jth time step calculated using K_{j1}, K_{j3}= rate of storage volume variation between j-1th and jth time step calculated with K_{j2}, K_{j4}= rate of storage volume variation at jth time step calculated using K_{j3}.

Step (4). Storage volume is calculated with Eq. (7).
Step (5). Input flow is estimated with Eq. (8)
Step (6). Steps (3) to (5) are repeatedly calculated for all N time steps.

The minimization of the SSQ is the objective function use to estimate the Muskingum parameters K, X, and m.

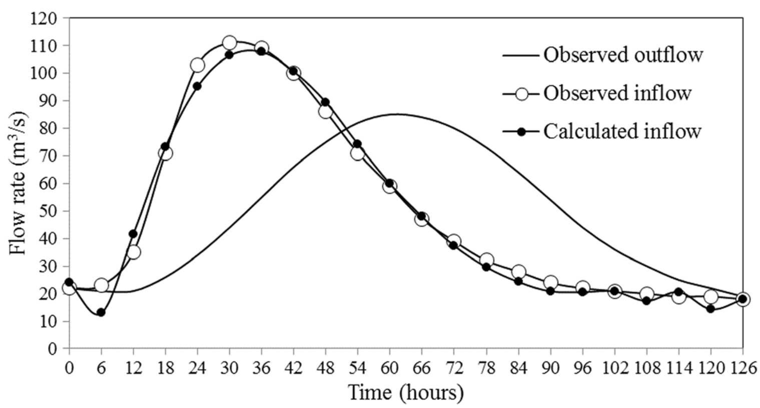

Fig. 2 Observed and calculated hydrographs with Euler equations

Table 2 Calculated input hydrograph using Runge-Kutta 4th-order equations

j	Time (hour)	O_j (m^3/s)	I_j (m^3/s)	S_j (m^3)	K_{j1} (m^3/s)	K_{j2} (m^3/s)	K_{j3} (m^3/s)	K_{j4} (m^3/s)	$\hat{I}_j$ (m^3/s)	$(I_j-\hat{I}_j)^2$ (m^3/s)
0	0	22	22	290.3	–	–	–	–	23.1	1.08
1	6	21	23	232.4	− 4.2	− 8.0	− 9.9	− 17.9	16.8	6.22
2	12	21	35	405.4	19.9	27.1	29.5	40.0	40.9	5.90
3	18	26	71	825.6	46.0	66.5	71.6	98.0	72.0	0.97
4	24	34	103	1373.7	61.2	87.7	93.0	125.4	95.2	7.77
5	30	44	111	1947.2	63.6	92.2	97.1	131.3	107.6	3.41
6	36	55	109	2459.9	54.3	82.1	86.4	119.3	109.3	0.28
7	42	66	100	2833.6	35.6	60.1	63.7	92.6	101.6	1.56
8	48	75	86	3037.2	14.3	32.1	34.7	55.7	89.3	3.33
9	54	82	71	3053.4	− 9.3	1.6	3.2	16.0	72.7	1.74
10	60	85	59	2895.4	− 27.5	− 26.5	− 26.3	− 25.1	57.5	1.52
11	66	84	47	2610.7	− 38.6	− 46.5	− 47.8	− 57.5	45.4	1.62
12	72	80	39	2244.1	− 44.6	− 59.2	− 61.7	− 80.2	35.4	3.58
13	78	73	32	1838.9	− 44.0	− 64.5	− 68.5	− .2	29.0	3.03
14	84	64	28	1439.6	− 38.7	− 62.5	− 67.7	− 100.2	25.3	2.68
15	90	54	24	1080.1	− 30.3	− 55.0	− 61.2	− 96.8	23.7	0.29
16	96	44	22	729.4	− 19.7	− 42.6	− 49.2	− 84.2	24.3	2.28
17	102	36	21	587.1	− 11.8	−29.5	− 35.3	− 64.0	24.2	3.20
18	108	30	20	434.9	− 7.9	−21.2	−26.2	− 49.5	22.1	2.14
19	114	25	19	334.6	− 3.2	− 13.2	− 17.4	− 35.9	21.8	2.81
20	120	22	19	265.3	− 2.6	− 9.0	− 12.0	24.7	19.4	0.40
21	126	19	18	209.5	− 1.0	− 6.7	− 9.7	− 21.9	18	0.00
Sum	–	–	–	–	–	–	–	–	–	226.50

Moreover, Eq. (15) is imposed as a constraint to avoid negative input flows:

$$\text{Min SSQ} = \sum_{j=1}^{N} \left(I_j\text{-}\hat{I}_j\right)^2 \quad j = 0, 1, 2\ldots, N \tag{14}$$

$$\frac{1}{m} > \frac{I_j\left[(1-X)O_j\right]}{I_j\left(\frac{S_j}{K}\right)} \quad j = 0, 1, 2\ldots, N \tag{15}$$

in which, I_j= observed input flow at jth time step.

Optimization tool

The GRG is applied for finding the best Muskingum parameters. GRG is a reduced gradient or gradient projection method that extends algorithms suitable for handling linear constraints to cope with nonlinear constraints. The general form of the optimization problem herein considered is as follows:

$$\text{Minimize } f(x) \tag{16}$$

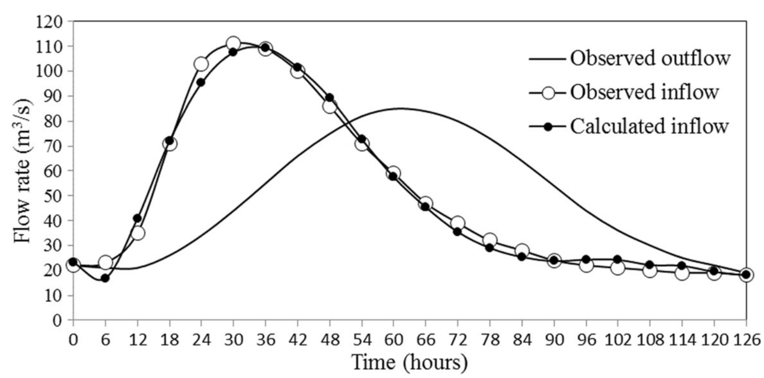

Fig. 3 Observed and calculated hydrographs with Runge-Kutta 4th order equations

Table 3 Comparison of optimal *SSQ* and Muskingum parameters by LM and GRG

Method	K	X	m	SSQ
LM	0.852	0.292	2.263	329.56
Euler equations-GRG	0.162	0.358	2.129	327.78
Runge-Kutta 4th-order equations-GRG	0.916	0.287	1.855	226.50

$$g_j(x) \leq 0 \quad j = 1, 2, \ldots, M \tag{17}$$

$$x_i^L \leq x_i \leq x_i^U \quad i = 1, 2, \ldots, L \tag{18}$$

in which, $f(x)$, $g(x)$, and $h(x)$= continuously differentiable real valued functions, x= decision variable vector, M = number of constraints, L = number of decision variables [herein $L = 3$ (K, X, m)], and x_i^L and x_i^U = lower and upper allowable values for ith decision variable, respectively.

The GRG transforms inequality constraints to equality constraints using nonnegative slack variables which are added to the constraints.

The GRC relies on the a first-order Taylor's expansion for minimizing a linearized objective subject to linearized constraints at a current feasible point x. More information about the GRG is available in Martin (2013).

Results and discussion

The performance of the methods stated in the previous section relies on a benchmark problem (Wilson (1974)). Figure 1 depicts a nonlinear relation between S and $[XI + (1 - X)O]$ with 6 time intervals ($\Delta t = 6$) and 21

Table 4 Observed and calculated input hydrographs

j	Time	Observed data: m^3/s		Calculated inflow: m^3/s		
	(hour)	O_j	I_j	LM	Euler equations-GRG	Runge-Kutta 4th-order equations-GRG
0	0	22	22	20.4	24.1	23.1
1	6	21	23	21.2	13.0	16.8
2	12	21	35	39.2	41.6	40.9
3	18	26	71	66.0	73.1	72.0
4	24	34	103	89.5	95.1	95.2
5	30	44	111	103.5	106.3	107.6
6	36	55	109	106.8	107.7	109.3
7	42	66	100	1006	100.6	101.6
8	48	75	86	89.4	89.5	89.3
9	54	82	71	73.7	74.2	72.7
10	60	85	59	59.1	59.8	57.5
11	66	84	47	47.3	47.9	45.4
12	72	80	39	37.3	37.4	35.4
13	78	73	32	30.4	29.6	29.0
14	84	64	28	26.1	24.3	25.3
15	90	54	24	23.7	20.9	23.7
16	96	44	22	23.5	20.6	24.3
17	102	36	21	22.7	20.8	24.2
18	108	30	20	20.7	17.4	22.1
19	114	25	19	20.2	20.7	21.8
20	120	22	19	17.5	14.4	19.4
21	126	19	18	18.0	18.0	18.0

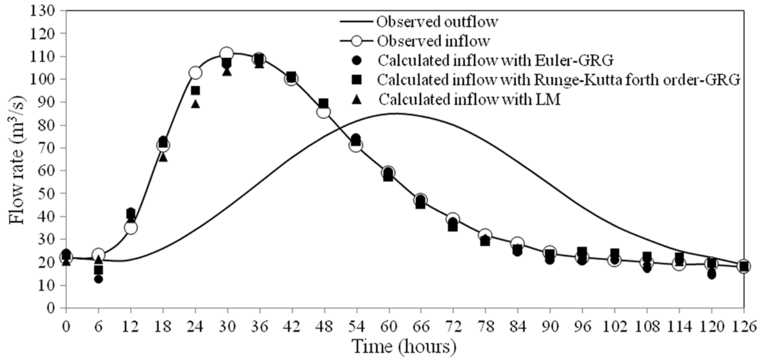

Fig. 4 Observed and calculated input hydrograph with several methods

time steps ($N = 21$). This problem was applied to exemplify reverse flood routing by Das (2009).

Table 1 presents results from the Euler equations reported by Wilson (1974). Figure 2 illustrates observed input and output hydrographs and calculated inflow hydrograph. The optimal *SSQ* equals 327.78 with optimal parameters $m = 2.129$, $X = 0.358$, and $K = 0.162$.

Table 2 presents optimal parameters from the Runge-Kutta 4th order equations. The minimum (best) value of objective function is $SSQ = 226.5$, $m = 1.855$, and $X = 0.287$, and $K = 0.916$. Figure 3 shows observed and optimal calculated hydrographs. Clearly, the Runge-Kutta 4th order equations yielded smaller (better) *SSQ* than the Euler equations.

Das (2009) determined the input routed hydrograph for the benchmark problem by Wilson (1974) using LM as the optimization tool. In this paper, the GRG was applied for optimization and results are listed in Table 3.

It is seen in Table 3 that the obtained *SSQ* using the Runge-Kutta 4th order equations-GRG method is 30 and 31% smaller (better) than those calculated with the Euler equations-GRG and LM methods, respectively. Moreover, the GRG performed better than the LM. Table 4 and Fig. 4 show observed and calculated input hydrographs calculated with the different methods.

The sensitivity of *SSQ* objective function to increasing nonlinear Muskingum model parameters using Euler and Runge-Kutta 4th order equations was analyzed and results are listed in Table 5. It is evident in Table 5 that a given parameter value was raised by 1%, while the other two parameters were kept unchanged at their optimal values. The results listed in Table 5 demonstrate that the parameters from the simulation method based on Runge-Kutta 4th order equation and GRG are more sensitive than those from the simulation method based

Table 5 Sensitivity analysis of objective function with respect to increasing nonlinear Muskingum model parameters

Method		Parameters of the nonlinear Muskingum model			Percent variation of *SSQ*
		K	*X*	*m*	
Euler equations	Optimal values	0.162	0.358	2.129	–
	Sensitivity analysis	0.164	0.358	2.129	0.01
		0.162	0.362	2.129	0.00
		0.162	0.358	2.150	0.61
Runge-Kutta 4th order equations	Optimal values	0.916	0.287	1.855	–
	Sensitivity analysis	0.925	0.287	1.855	0.02
		0.916	0.290	1.855	0.01
		0.916	0.287	1.874	1.66

on the Euler equations with GRC. Moreover, *m* was the most sensitive parameter, as compared to *K* and *X*.

Concluding remarks

This paper applied the Euler and Runge-Kutta 4th order equations to reverse-simulate flood hydrographs. These two methods were coupled with GRG as the optimization tool. Our results established the Runge-Kutta 4th order equations exhibited better performance in this benchmark reverse flood routing problem than the Euler equations. Moreover, the GRG outperformed the LM as an optimization tool in the reverse flood routing example.

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