

UC Merced

Proceedings of the Annual Meeting of the Cognitive Science Society

Title

A Quantitative Symbolic Approach to Individual Human Reasoning

Permalink

<https://escholarship.org/uc/item/5rv6124b>

Journal

Proceedings of the Annual Meeting of the Cognitive Science Society, 44(44)

Authors

Dietz, Emmanuelle
Fichte, Johannes Klaus
Hamiti, Florim

Publication Date

2022

Peer reviewed

A Quantitative Symbolic Approach to Individual Human Reasoning¹

Emmanuelle Dietz (emmanuelle.dietz@airbus.com)

Airbus Central Research & Technology, Hamburg, Germany

Johannes K. Fichte (johannes.fichte@tuwien.ac.at)

TU Wien, Vienna, Austria

Florim Hamiti (florim.hamiti@fit.fraunhofer.de)

Abstract

Cognitive theories for reasoning are about understanding how humans come to conclusions from a set of premises. Starting from hypothetical thoughts, we are interested which are the implications behind basic everyday language and how do we reason with them. A widely studied topic is whether cognitive theories can account for typical reasoning tasks and be confirmed by own empirical experiments. This paper takes a different view and we do not propose a theory, but instead take findings from the literature and show how these, formalized as cognitive principles within a logical framework, can establish a quantitative notion of reasoning, which we call plausibility. For this purpose, we employ techniques from non-monotonic reasoning and computer science, namely, a solving paradigm called answer set programming (ASP). Finally, we can fruitfully use plausibility reasoning in ASP to test the effects of an existing experiment and explain different majority responses.

Keywords: Answer Set Programming, Human Reasoning, Model Quantification, Individual Reasoning, Deduction, Abduction, Suppression Task, Non-monotonic Reasoning

Introduction

Usually, the adequacy of cognitive reasoning theories is assessed with respect to typical reasoning tasks, e.g., (Byrne, 1989; Wason, 1968) and own experiments. The aim is to understand how, from a hypothetical thought, humans reason and make conclusions. For example, given conditionals such as “if A, then B” together with a set of given premises, we can ask what humans conclude from this information. The adequacy of a cognitive theory is assessed by how well it can account the human data. Over the decades, many theories have been proposed (Johnson-Laird, 1983; Rips, 1994; Polk & Newell, 1995; Chater & Oaksford, 1999; Stenning & van Lambalgen, 2008; Hölldobler & Kencana Ramli, 2009). Here, we briefly discuss two dominant theories. The Probability Heuristics Model (PHM) is a cognitive theory where the environment is described by prior probabilities and updates are done according to Bayes’ theorem (Chater & Oaksford, 1999). PHM does not suggest how probabilities are computed, i.e. no implemented algorithm exists (López-Astorga, Ragni, & Johnson-Laird, 2021). The (Mental) Model Theory (Johnson-Laird, 1983) assumes that humans reason by constructing and manipulating mental models, which illustrate the possibilities of how the world is perceived by the reasoner (Khemlani & Johnson-Laird, 2013). The model theory with naive probabilities (Johnson-Laird, Legrenzi, Girotto, Legrenzi, & Caverni,

1999) provides a simple algorithm without using Bayes’ theorem, for computing subjective probabilities and was further extended (Khemlani, Lotstein, & Johnson-Laird, 2015; Khemlani & Johnson-Laird, 2016). These two theories seem to have conflicting viewpoints (Oaksford & Chater, 2020; Knauff & Gazzo Castañeda, 2021; Oaksford, 2021; Over, 2021) and so far, there is no agreement on whether an integration is possible (López-Astorga et al., 2021; Over, 2021).

So far, a widely accepted framework for cognitive reasoning does not exist. Even though there might be some agreement on the metrics for a good theory, e.g., generalizability (Thomson, Lebiere, Anderson, & Staszewski, 2015), simplicity, and predictive accuracy (Taatgen & Anderson, 2010), theories are not always formalized by their inventors and thus not applicable to tasks straightaway. When effort is done to make them testable and accessible to others, e.g., (Khemlani & Johnson-Laird, 2012), the theory might be ambiguously understood and not adequately modeled, e.g., (Baratgin et al., 2015). Quite an example of Newell’s observation—even though scientists make excellent research, they *never seem in the experimental literature to put the results of all the experiments together*, which obstructs progress (Newell, 1973). Among others, Newell suggested developing complete processing models, and a computer system that can perform all mental tasks. On the architectural level, a standard model of the mind was proposed, that depicts the *best consensus given the community’s understanding of the mind* (Laird, Lebiere, & Rosenbloom, 2017).

An additional challenge for cognitive reasoning is *to identify the relevant problems* that a model should account for (Ragni, 2020). Therefore, Ragni (2020) suggested establishing generally accepted benchmarks, similar to the PRECORE Challenge (Ragni, Riesterer, & Khemlani, 2019) for human reasoning tasks. The evaluation of this challenge was done with the benchmarking tool Cognitive COmputation for Behavioral Reasoning Analysis (CCOBRA) framework (Riesterer & Shadownox, 2021).

In other disciplines such as mathematics and computer science, annual problem challenges, such as the famous DIMACS challenges (Johnson, McGeoch, Grigoriadis, Monma, & Tarjan, 1990), SAT (Balyo et al., 2021), and ASP competitions (Gebser, Maratea, & Ricca, 2020), provided a community-building tool and contributed to tremendous progress in actual problem solving. On the side, these efforts result in common (intermediary) languages. At the same

¹Authors are stated in alphabetical order.

time, the outcome of the challenges defines the empirical upper bounds of the state-of-the-art model’s performance and determines the performance of new theories (Riesterer, Brand, & Ragni, 2020). Here, we do not propose a cognitive theory but formalize widely accepted findings as task-independent cognitive principles within one framework. These principles require that assumptions have to be general enough to be understood in various contexts. At the same time, they call for an unambiguous formalization that can immediately be instantiated to a specific context. As *logical reasoning is [...] considered one of the most fundamental cognitive activities* (Woleński, 2016), a logical formalization of higher-level cognitive assumptions might be suitable, even though not classical logic: The formalization and reasoning with *default assumptions*, which are facts that are true in the majority of contexts but not always, require *non-monotonic logic* (Reiter, 1978, 1980). A widely used modeling and problem solving paradigm in AI and computer science that implements non-monotonic reasoning is *answer set programming (ASP)* (Heule & Schaub, 2015; Gebser, Kaminski, Kaufmann, & Schaub, 2012). The solutions of the program *answer sets* or *stable models*, can then be understood as possible models for that program.

We use the well-established ASP paradigm to model human reasoning principles, and employ ASP for quantitative reasoning by defining a notion of plausibility that relates the number of models under assumptions of interest to the total number of models. Thereby, we obtain a framework that allows for implementing cognitive principles. In our work, we anti-disciplinary combine findings from cognitive science, the non-monotonic logic community in computer science, and a method of model quantification. However, it is important to emphasize that ASP is not a theory of cognitive reasoning.

Contributions. Our main contributions are as follows:

1. We show that existing cognitive principles can be well represented as rules in ASP following a natural semantics. Each program is a set of these rules and yields possible models.
2. We instantiate these general principles to a well-known task from empirical experiments into simple programs that do plausibility reasoning with ASP.
3. We illustrate, how we can turn potentially multiple models of an ASP program into a quantitative approach to reasoning (plausibility) to test the effects of existing experiments and explain different majority responses.

Preliminaries

Answer Set Programming (ASP). ASP is a popular declarative modeling and problem solving framework in computer science and artificial intelligence with roots in non-monotonic logic (Brewka, Eiter, & Truszczyński, 2011; Gebser, Kaufmann, & Schaub, 2012). In ASP, one states problems using propositional atoms, meaning, that an atom a can either be true or false. A program consists of rules, that state conclusions about atoms. Solutions to the program are called answer

sets (or stable models). A rule of the form $a \leftarrow b, \text{not } c$, intuitively, states that we can conclude a if b is true unless we have evidence that c is true. By default, in ASP, we assume that an atom a is false unless we can conclude it. This “in dubio pro reo”-like approach is known as closed-world assumption (CWA) (Reiter, 1980, 1978). Take the following example.

Example 1 (False by Default). *Consider the following conditional sentence. “If it is weekend (w), then she will go to the beach (b)”. However, we know that “She will not go to the beach (b), if it is cloudy (c).” We can rephrase this as follows: “If it is weekend (w), then she will go to the beach (b) unless it is cloudy (c).” This can be modeled as program $P_1 = \{b \leftarrow w, \text{not } c.\}$. \perp*

What are the (intended) models of the programs? Here we are interested in the answer sets (or stable models) of the programs, but we will not provide their formal definitions and rather explain the intuition by the next examples. The interested reader is referred to an extended version (Dietz, Fichte, & Hamiti, 2022) or introductory literature (Gebser, Kaminski, et al., 2012).

Example 2 (Answer Set). *The only answer set of P_1 is \emptyset , since we neither have evidence for cloudy, nor weekend, nor beach. If we know that “it is weekend”, we take program $P_2 = \{b \leftarrow w, \text{not } c. w.\}$. In P_2 , we have evidence for weekend by w , but no evidence for cloudy. From this knowledge, we can conclude beach from the rule in P_1 . The only answer set of P_2 is $\{w, b\}$. In contrast, $P_2 = \{b \leftarrow w, \text{not } c. w. c.\}$ has only the answer set $\{w, c\}$. We cannot conclude b , as we have evidence for c and the rule $b \leftarrow w, \text{not } c$ contains c as an exception to draw the conclusion. \perp*

In ASP, we can also make explicit choices to set an atom to true or not, which we illustrate in the following example.

Example 3 (Choices). *Take program P_1 from Example 1. If we know that it could either be weekend or not weekend, we add a choice rule to our program. A choice rule states that any combination of atoms inside the set are true, including none.² We obtain $P_3 = \{b \leftarrow w, \text{not } c. \{w\}.\}$. Then, P_3 has two answer sets $\{w, b\}$ and $\{\}$. \perp*

Next, we illustrate how adding rules can effect conclusions.

Example 4. *Consider program $P_4 = \underbrace{\{b \leftarrow w, \text{not } c.\}}_{r_1} \underbrace{\{w\}}_{r_2} \underbrace{\{s, c\}}_{r_3} \underbrace{. c \leftarrow \text{not } s}_{r_4}$ where r_1*

corresponds to the conditional in Ex. 1, r_2 to the choice in Ex. 3, and s means “it is sunny”. Choice r_3 states that either s or c are true, both are true, or none is true. We have six answer sets, namely, $\text{AS}(P_4) = \{\{s\}, \{c\}, \{c, s\}, \{b, s, w\}, \{c, s, w\}, \{c, w\}\}$. The set $X = \{b, w\}$ is not an answer set of P_4 , since we conclude cloudy (c) from rule r_4 if we have no evidence for sunny (s). \perp

Below, we also use variables in the programs, which provides us with a more expressive and compact way of representation. We omit formal details for space reasons, but again

²Sometimes we also write $n\{a_1; a_2; \dots; a_k\}m$, meaning that we chose at least n atoms and at most m atoms in the choice.

give an example. By $\{p(X).r(X) \leftarrow p(X)\}$ for $X \in \{a,b\}$, where a and b are constants, we mean $\{p(a).p(b).r(a) \leftarrow p(a).r(b) \leftarrow p(b)\}$. We assume that there is always at least one constant in P and $AS(P)$ is the set of all answer sets of P .

Quantitative Reasoning in ASP. Traditionally, when modeling in logic, one considers simple decision questions, i.e., yes-no questions (Copi, Cohen, & Rodych, 2019). In terms of ASP this would simply mean asking whether a given program has an answer set, i.e., $AS(P) \neq \emptyset$. Beyond, we find questions such as credulous and skeptical reasoning. Where *credulous* or *skeptical reasoning* asks whether an atom a is contained in *at least one* and *all* answer sets, respectively. We are also interested in computing the *plausibility* for a set Q of rules relating to the number of answer sets under the assumption of the total number of answer sets.

Definition 1 (Plausibility). Let P be a program and Q be a set atoms, called questions. Then, plausibility of P under Q is defined $\mathbb{P}[P, Q] := \frac{|AS(P \cup P_Q)|}{\max(1, |AS(P)|)}$ where P_Q consists of integrity rules that ask whether atoms in Q can be made true, i.e., $P_Q := \{\leftarrow \text{not } a \mid a \in Q\}$.

Later, when representing questions in ASP programs, we assume that Q is given by rules of the form “question(a).” for every $a \in Q$. When computing $|AS(P \cup Q)|$, we replace each question(a) as above by “ $\leftarrow \text{not } a$.” By using ASP, we describe the system and the outcome using rules within ASP. Answer sets represent the outcomes of our system. The question for plausibility still relates to inference and is done in terms of counting answer sets. Using modern implementations that solve answer set programs, we can obtain the plausibility by listing all answer sets and computing the relation. The problem is of high computational complexity and has only recently received more attention with the rise of more efficient solving techniques in the propositional setting that avoid enumeration (Lagniez & Marquis, 2017; Sharma, Roy, Soos, & Meel, 2019; Fichte, Hecher, & Hamiti, 2021; Fichte, Hecher, Thier, & Woltran, 2021; Fichte, Hecher, & Roland, 2021) or in the ASP setting (Kabir et al., 2022; Fichte, Gaggl, & Rusovac, 2022; Nadeem, Fichte, & Hecher, 2022).

Example 5. Consider P_4 from Ex. 4. For *skeptical reasoning* for s or c in P_4 , the outcome is no. Whereas for *credulous reasoning*, the outcome is yes. When considering the plausibility of P_4 under s being true, meaning $P_Q = \{\leftarrow \text{not } s\}$, we can see that $|AS(P_4 \cup P_Q)| = |\{\{s\}, \{c, s\}, \{b, s, w\}, \{c, s, w\}\}| = 4$ and $|AS(P_4)| = 6$, which yields $\mathbb{P}[P_4, Q] = 4/6$. \perp

Below, we illustrate counting and Bayesian views as well as differences to our notion. We follow a popular example by McElreath (2020b, 2020a). Recall that Bayes-Price theorem is used to compute the probability of an event, based on prior knowledge of conditions that might be related to the event. While it might seem quite plain, one can just list potential combinations and count possible ways instead.

Example 6. Assume that we have a bag of four marbles, which could be blue (b) or white (w). We are not aware of how

many of each is in the bag. From the four marbles, the cases (i) $wwww$; (ii) $bwww$; (iii) $bbww$; (iv) $bbbw$; and (v) $bbbb$ are possible. To obtain more detailed information about the content, we can take one marble remember its color and put it back. Assume that after repeating times, we observe bwb . To estimate Bayesian plausibility, we can count how many ways are to produce each of the Cases (i-v) assuming the seen data. In more detail, 0 ways for $wwww$, 3 ways for $bwww$, 8 ways for $bbww$, 9 ways for $bbbw$, and 0 ways for $bbbb$. In total 20 possible ways. Plausibility talks about an observation in relation to all possible ways. Here, $0/20 = 0$ for $wwww$, $3/20 = 0.15$ for $bwww$, $8/20 = 0.40$ for $bbww$, $9/20 = 0.45$ for $bbbw$, and $0/20 = 0$ for $bbbb$. Our framework allows to express this in our notion of plausibility. Therefore, we can model the 5 cases that can be produced and their resulting ways of producing the data. Then, ask for the number of solutions that can be produced in total and the one under the assumption say Case (ii) $bwww$. We provide a detailed ASP program in an extended version (Dietz et al., 2022). While our framework allows to express such questions we are more general and by plausibility in ASP, we express the relation of count under assumption and total count of possible answer sets. \perp

The existing probabilistic approaches to human reasoning differ from our proposal as those probabilities are either understood as subjective and are not derived from the quantification over models, e.g. (Chater & Oaksford, 1999) or attach the probabilities to different types of inferences, e.g. (Kleiter, 2018). We also use a slightly different approach than Johnson-Laird (1999) by considering the relationship on counting the values in the truth table that evaluate to true, but according to the answer set semantics.

Cognitive Principles in ASP

We employ accepted findings from the literature and formalize them as rules, called *cognitive principles*, within one framework and explain their effects. As a baseline, we consider the principles presented in the literature (Dietz & Kakas, 2020, 2021). These principles are task-independent and can be any assumption that humans seem to make regardless of whether they are valid in classical logic. ASP will be the framework in which we formalize them. Before we proceed with the rules and their representation in ASP, let us clarify that we do not present a new cognitive theory.

Presuppositions Grice’s (1975) conversational implicatures are about additional interpretations of the sentences we hear, not necessarily related to the content. For instance, we usually communicate according to the cooperation principle. Thus, when the experimenter (or someone we trust) states “ a is true or a is false”, we assume that this is true. In ASP, a fact $\text{prem}(a)$ is represented as either $\text{prem}(a)$ or $\text{nprem}(a)$, respectively (FACT principle).³ Yet, both cannot be true at the

³Throughout the paper, the negation of a statement $a(X)$ is represented with an auxiliary statement $na(X)$, i.e., having the same name as the statement, preceded by an ‘n’.

same time (CONSISTENCY principle).

Grice's maxim of relevance implies that everything that is said, seems to be relevant, suggesting that humans might generate hypotheses from the context (HYPOTHESIS principle). We account for this principle by establishing context-dependent hypotheses for each statement a that we are made aware of by adding $\text{hyp}(a)$.

$$\{\text{prem}(X); \text{nprem}(X)\} 1 \leftarrow \text{hyp}(X). \quad (\text{HYPOTHESIS})$$

The 1 denotes that at most one statement can be true ensuring the CONSISTENCY principle.

Types of Conditionals Conditions in conditionals can be of different types, such as necessary or sufficient (Byrne, Espino, & Santamaría, 1999; Byrne, 2005). Consider the two conditional sentences *If she meets with a friend, then she will go to the play* and *If she has enough money, then she will go to the play*. We assume that *she meets with a friend* is a sufficient condition whereas *she has enough money* is not sufficient but a necessary condition for *she will go to the play*. Assume that *she meets a friend*. Together with the above HYPOTHESIS principle and given the second conditional, humans might generate the hypothesis that *she does not have enough money* which functions as a **disabling condition** (Cummins, Lubart, Alknsnis, & Rist, 1991) to the modus ponens conclusion that *she will go to a play*. The following rule states that concl follows if condition is asserted to be true (modus ponens):

$$\text{concl} \leftarrow \text{prem}(X), \text{sufficient}(X). \quad (\text{SUFFICIENT})$$

The following rule states that nconcl follows if condition is false (denial of the consequent).

$$\text{nconcl} \leftarrow \text{nprem}(X), \text{necessary}(X). \quad (\text{NECESSARY})$$

Let us observe that *she does not meet a friend*. If this condition is also necessary, according to the NECESSARY principle we might conclude that *she will not go to the play*. Consider now additionally that *if she has free tickets, she will go to the play*. The hypothesis that *she has free tickets* functions as an **alternative cause** (Cummins et al., 1991) to the condition *she meets a friend*, for the conclusion *she will go to the play*. The following rule captures this idea:

$$\text{nconcl} \leftarrow \text{nprem}(X_1), \dots, \text{nprem}(X_n) \quad (\text{ALL SUFFICIENT})$$

where $\text{nprem}(X_1), \dots, \text{nprem}(X_n)$ is the conjunction of all X_i , $1 \leq i \leq n$, for which there exists a rule of the following form: $\text{concl} \leftarrow \text{prem}(X_i), \text{sufficient}(X_i)$. This rule states that nconcl follows when *all* its sufficient conditions are false.

Maxim of Inference to the best explanation Even though not valid in classical logic, humans have the ability to reason from observations to explanations, called abduction (Peirce, 1903). As reported by Kelley (1973); Sloman (1994), contrastive (or alternative) explanations might increase or decrease their plausibility, depending on the context.

In ASP, abduction can be implemented as cautious (or skeptical) abduction (Kakas, Kowalski, & Toni, 1993). Given a program P and an observation O , is \mathcal{E} an explanation for O ? This question can be answered in a two-step procedure: (i) Generate models of P , in which O holds: $\leftarrow O$. (EXPLAIN principle) (ii) Select models in which \mathcal{E} holds: $\text{question}(\mathcal{E})$. (CAUTIOUS principle)

We additionally require explanations to be minimal (MINIMAL principle): Given P , \mathcal{E} is a minimal explanation of O if and only if there is no other explanation \mathcal{E}' for O such that $\mathcal{E}' \subset \mathcal{E}$. In the sequel, O is either concl or nconcl and \mathcal{E} is either prem or nprem . Given that *if prem then concl*, the derivation from concl to prem corresponds to the (classical logically) invalid affirmation of the consequent, whereas the derivation from nconcl to nprem corresponds to the valid modus tollens.

Individual Reasoners Humans differ in their reasoning, c.f., (Khemlani & Johnson-Laird, 2016). We represent these differences as choice rules, surrounded by $\{\dots\}$, which can contain one or more variables. For instance, models in which $\text{hyp}(a)$ is true, false, or unknown, can be generated through the choice rule " $\{\text{hyp}(a)\}$ " (INDIVIDUALS principle).

Application to Human Reasoning

We discuss the application of cognitive principles within ASP by means of a typical reasoning task. The suppression task (Byrne, 1989) consists of two parts, where participants were divided into three groups and were asked whether they could derive conclusions given variations of a set of premises. First, we present the formalization in ASP guided by cognitive principles. In Part I, reasoning is done deductively, and, in Part II, it is done abductively. In contrast to other logic programming approaches (Stenning & van Lambalgen, 2008; Dietz, Hölldobler, & Ragni, 2012), we apply quantitative reasoning to the computed models which allows us to account for the majority's differences in the experimental results.

Part I: Search for conclusions Group I was given the following two premises: *If she has an essay to finish, then she will study late in the library. She has an essay to finish. (essay)* The participants were asked what of the following answer possibilities follows assuming that the above premises were true: *She will study late in the library, She will not study late in the library. or She may or may not study late in the library.* 96% of the participants in this group concluded that *She will study late in the library (library)*. Group II of participants additionally received the following premise: *If she has a textbook to finish, then she will study late in the library*, which yields to the same result: 96%⁴ of the participants in this group concluded that *She will study late in the library*. Group III of participants instead additionally received the following premise: *If the library is open then she will study late in the library*. In this

⁴We refer to the percentages from Byrne (1989). Table 2 also shows the percentages from Dieussaert (2000).

Case	All Groups	Group I	Group II	Group III
All cases	P_{basic}	$\{\text{necessary}(e)\}$		$\text{necessary}(e)$
<i>essay</i>	$\text{question}(\text{concl})$	$\text{prem}(e)$	$\text{prem}(e), \{\text{hyp}(t)\}$	$\{\text{hyp}(o); \text{prem}(e)\}$
<i>not essay</i>	$\text{question}(\text{nconcl}), \text{nprem}(e)$		$\{\text{hyp}(t)\}$	$\{\text{hyp}(o)\}$
<i>library</i>	$\{\text{concl}\}, \leftarrow \text{not concl}, \text{question}(\text{prem}(e))$	$\text{hyp}(e)$	$\{\text{hyp}(e); \text{hyp}(t)\}1$	$\text{hyp}(o), \text{hyp}(e)$
<i>not library</i>	$\{\text{nconcl}\}, \leftarrow \text{concl}, \text{question}(\text{nprem}(e))$	$\text{hyp}(e)$	$\text{hyp}(e), \text{hyp}(t)$	$\{\text{hyp}(o); \text{hyp}(e)\}1$

Table 1: Summary of the rules applied by case and group. The second row shows the rules by group that applied to all cases.

case, only 38% concluded that *She will study late in the library*. Even though the conclusion was logically valid in all three groups (modus ponens), a *suppression effect* in Group III could be observed (Byrne, 1989). This effect very well demonstrates the non-monotonic nature of human reasoning.

If instead, *She does not have an essay to finish* was given as a fact, only 4% of Group II concluded *She will not study late in the library*, whereas for Group I and Group III, it was 46% and 63%, respectively. Here, the conclusion was not valid (affirmation of the consequent), and the suppression effect could be observed in Group II. This case nicely shows that the suppression effect occurs independent on whether the conclusion is valid.

Motivated by the cognitive principles, the following rules, denoted by P_{basic} , are part of all cases and groups:

$\leftarrow \text{concl}, \text{nconcl}.$	(CONSISTENCY)
$\text{concl} \leftarrow \text{prem}(X), \text{sufficient}(X).$	(SUFFICIENT)
$\text{nconcl} \leftarrow \text{nprem}(X), \text{necessary}(X).$	(NECESSARY)
$\{\text{prem}(X); \text{nprem}(X)\}1 \leftarrow \text{hyp}(X).$	(HYPOTHESIS)
$\text{nconcl} \leftarrow \text{nprem}(e), \dots, \text{nprem}(t)$	(ALL SUFFICIENT)
$\text{sufficient}(e).$	(SUFFICIENT)
$\text{sufficient}(t).$	(SUFFICIENT)
$\text{necessary}(o).$	(NECESSARY)

concl and nconcl here refer to *She will study late in the library* and *She will not study late in the library*, respectively. The last three rules state that e (*She has an essay to finish*) and t (*She has a textbook to read*) are sufficient for concl , whereas o (*The library is open*) is necessary for concl . Table 1 shows all the programs for all the cases and groups.

Consider the first three rows in Table 1. For different groups of participants different underlying principles are assumed: By the FACT principle for case *essay* and *not essay* we assume $\text{prem}(e)$ and $\text{nprem}(e)$, respectively. Similar to the answer possibilities that were given to the participants, in ASP we ask the program whether an answer follows by $\text{question}(\text{concl})$ or $\text{question}(\text{nconcl})$. The different groups are made aware of different contexts, which is represented by the HYPOTHESIS principle: The program for Group II can build the hypothesis $\text{hyp}(t)$, whereas Group III can build the hypothesis $\text{hyp}(o)$. To account for different participants (INDIVIDUALS principle), choice rules (rules surrounded by $\{\dots\}$) are used: Consider $\{\text{necessary}(e)\}$ in Group I for all cases: It allows the

generation of models in which $\text{necessary}(e)$ is true, false, or unknown. Choice rules enable us to deal with conditions that might result in conflicting conclusions. Consider $\{\text{hyp}(o), \text{prem}(e)\}$ (in case *essay*, Group III). Assume $\text{hyp}(o)$: Because $\text{necessary}(o) \in P_{\text{basic}}$, by the NECESSARY principle nconcl follows. If we assume $\text{prem}(e)$, as $\text{sufficient}(e) \in P_{\text{basic}}$, by the SUFFICIENT principle, concl follows.

Part II: Search for Explanations The second part of the experiment was similar, except that the given facts were different. In the first case, participants were asked what follows, given the fact that *She will study late in the library* (*library*). For Group I and III, 71% and 54% derived the non-valid (affirmation of the consequent) conclusion that *She has an essay to finish*, whereas the suppression effect occurred for Group II, with only 13%. In the second case, they were asked what follows, given the fact that *She will not study late in the library* (*not library*). Here, 92% and 96% of participants in Group I and II derived the (logically valid) modus tollens conclusion *She does not have an essay to finish*, whereas the suppression effect occurred for Group III, with only 33%.

Following the EXPLAIN principle, participants might have understood the given fact as an observation and searched for explanations. $\leftarrow \text{not concl}$ generates all models in which concl holds and $\text{question}(\text{prem}(e))$ selects the models in which $\text{prem}(e)$ holds; similar for nconcl . These models are explanations for the given observation.

To account for different participants, we specify choice rules: $\{\text{concl}\}$ allowing to generate models in which concl is either false, true, or unknown. The cases in which concl is simply assumed to hold, represents participants who possibly did not search for explanations or generated other explanations based on their background knowledge. Similar for nconcl .

Consider the special cases of choice rules for the generation of explanations in row 5 and 6 in Table 1: For Group II, case *library*, $\{\text{hyp}(e); \text{hyp}(t)\}1$ excludes the cases where both $\text{hyp}(e)$ and $\text{hyp}(t)$ are true. As both e and t are *sufficient conditions* for *library*, it is enough to assume either $\text{prem}(e)$ or $\text{prem}(t)$ to hold as an explanation for *library*. For Group III, case *not library*, $\{\text{hyp}(e); \text{hyp}(o)\}1$ excludes the cases where both $\text{hyp}(e)$ and $\text{hyp}(o)$ are true. As both e and o are *necessary conditions* for *library*, it is enough to assume either $\text{nprem}(e)$ or $\text{nprem}(o)$ to hold as explanation for *not library*.

Note that for both groups this rule is not relevant for the other cases. In Group II, both $\text{nprem}(e)$ and $\text{nprem}(t)$ need to

hold to be an explanation for *not library* whereas in Group III, both $\text{prem}(e)$ and $\text{prem}(o)$ need to hold to be an explanation for *library*. These choice rules motivated by the MINIMAL principle are case-specific. Computing minimal explanations is expensive in general (Eiter & Gottlob, 1993).

From Counting Models to Plausibility Table 2 shows the number of generated models according to the given programs in Table 1 including the plausibility for *library*, *not library*, *essay*, and *not essay*, respectively. The plausibility is computed via ASP as described in the preliminaries. For each group (column 2) the number of all models for the program (column 4) and the number of all models that satisfied the question (column 3) are depicted. Columns 5 to 7 show the computed plausibility of quantitative ASP, the experimental results in the literature (Byrne, 1989) and (Dieussaert et al., 2000), respectively. ASP does not only model well the suppression effect in all four cases but also accommodates for the difference between high percentages (Group I and II for the cases *essay* and *not library*) and significant percentages (Group I and III for case *not essay* and *library*). Interestingly, whenever a suppression effect occurs in a group, ASP also generates more models compared to the other groups. This seems to agree with the assumption that inferences which leads to multiple models should be more difficult than the ones on a single model (López-Astorga et al., 2021).

Discussion and Outlook

To the best of our knowledge, no approach for human reasoning has considered quantitative model counting. Additionally, we provide an online accessible formalization of the task such that the results can be replicated.

We believe that a good model needs to account for the assumptions of various theories, for individual reasoners, and can rigorously be applied to benchmarks. An approach, which is guided by general cognitive principles, can account for individuals, is rigorously applicable as shown in other domains, and, as motivated by established theories, likely accounts for a variety of tasks.

The computed plausibility in this paper is solely based on the number of models, ignoring the quality of the respective models. However, some models might be easier to be considered by humans than other models (Knauff, Rauh, Schlieder, & Strube, 1998; Ragni, Fangmeier, Webber, & Knauff, 2006), meaning that they are not equiprobable. An additional preference relation, either on the rule level or on the model level, can easily be implemented in ASP (Brewka, Delgrande, Romero, & Schaub, 2015), could account for these differences or weighted counting (Sang, Beame, & Kautz, 2005).

Conclusion and Future Work

In this work, we showed how model human reasoning principles can be formalized within answer set programming (ASP),

³The programs, models and the results can be found online: <https://github.com/eadietz/bst2asp>

Cases	Group	models				
		Question	total	ASP	B.	D. ⁺
<i>essay</i>	I	2	2	100	96	88
	II	4	4	100	96	93
	III	3	7	43	38	60
↪ concluded <i>She will study late in the library</i>						
<i>not essay</i>	I	1	2	50	46	49
	II	1	4	25	4	22
	III	5	8	63	63	49
↪ concluded <i>She will not study late in the library</i>						
<i>library</i>	I	2	5	40	71	53
	II	1	7	14	13	16
	III	2	4	50	54	55
↪ concluded <i>She has an essay to finish</i>						
<i>not library</i>	I	1	1	100	92	69
	II	1	1	100	96	69
	III	1	2	50	33	44
↪ concluded <i>She does not have an essay to finish</i>						

Table 2: The results in ASP compared to the experimental results in (Byrne, 1989; Dieussaert et al., 2000), abbr. by B and D⁺, resp. The first two columns refer to cases and groups. Columns 3 and 4 refer to the number of models that satisfy the question and all models of the program. The highlighted rows show in which group the suppression effect occurred.³

which is a popular modeling problem, and reasoning framework in artificial intelligence (AI). By counting answer sets, we establish a notion of quantitative reasoning in terms of plausibility and account for different majority responses in cognitive reasoning. While the constructed models were guided by cognitive principles, we clearly do not believe that human reasoning works similarly as ASP computation. Instead, ASP helps to represent principles.

Putting our results into the light of Newell’s considerations on progress within the cognitive community (Newell, 1973), our work might be seen as yet another framework for mental modeling. However, we use well-established techniques from AI for representing cognitive principles and making small steps to converge. Thereby, we incorporate existing approaches and open ASP to the cognitive theory community.

In addition, we aim to investigate whether preferences over answer sets or weighted counting could allow for more detailed modeling of cognitive principles. Furthermore, inspired by our idea of employing existing techniques from AI, and as already mentioned in the introduction, the cognitive science community could discuss and design an event establishing benchmarks for human reasoning tasks as suggested in (Ragni, 2020) explaining different majority responses using one or many existing frameworks.

Acknowledgements

Work has partially been carried out while the second author was visiting the Simons Institute at UC Berkeley. The authors gratefully acknowledge the valuable feedback by the anonymous reviewers.

References

- Balyo, T., Froleys, N., Heule, M. J. H., Iser, M., Järvisalo, M., & Suda, M. (Eds.). (2021). *Proceedings of sat competition 2021*.
- Baratgin, J., Douven, I., Evans, J., Oaksford, M., Over, D., & Politzer, G. (2015). The new paradigm and mental models. *Trends in Cognitive Sciences*, 19(10), 547–548.
- Brewka, G., Delgrande, J. P., Romero, J., & Schaub, T. (2015). asprin: Customizing answer set preferences without a headache. In B. Bonet & S. Koenig (Eds.), *Proceedings of the twenty-ninth AAAI conference on artificial intelligence, january 25-30, 2015, austin, texas, USA* (pp. 1467–1474). AAAI Press.
- Brewka, G., Eiter, T., & Truszczyński, M. (2011). Answer set programming at a glance. *Communications of the ACM*, 54(12), 92–103. doi: 10.1145/2043174.2043195
- Byrne, R. M. J. (1989). Suppressing valid inferences with conditionals. *Journal of Memory and Language*, 31, 61–83.
- Byrne, R. M. J. (2005). *The rational imagination: How people create alternatives to reality*. MIT press.
- Byrne, R. M. J., Espino, O., & Santamaría, C. (1999). Counterexamples and the suppression of inferences. *Journal of Memory and Language*, 40(3), 347–373.
- Chater, N., & Oaksford, M. (1999). The probability heuristics model of syllogistic reasoning. *Cognitive Psychology*, 38, 191–258.
- Copi, I. M., Cohen, C., & Rodych, V. (2019). *Introduction to logic* (15th edition ed.). Taylor & Francis Ltd.
- Cummins, D. D., Lubart, T., Alksnis, O., & Rist, R. (1991). Conditional reasoning and causation. *Memory & cognition*, 19(3), 274–282.
- Dietz, E., Fichte, J. K., & Hamiti, F. (2022). A quantitative symbolic approach to individual human reasoning. *CoRR*. Retrieved from <https://arxiv.org/abs/2205.05030> doi: 10.48550/ARXIV.2205.05030
- Dietz, E., Hölldobler, S., & Ragni, M. (2012). A computational logic approach to the suppression task. In N. Miyake, D. Peebles, & R. P. Cooper (Eds.), *In proc. of 34th conf. of Cognitive Science Society* (pp. 1500–1505). Cognitive Science Society.
- Dietz, E., & Kakas, A. C. (2020). Cognitive argumentation and the suppression task. *CoRR, abs/2002.10149*. Retrieved from <https://arxiv.org/abs/2002.10149>
- Dietz, E., & Kakas, A. C. (2021). Cognitive argumentation and the selection task. In *Proceedings of the annual meeting of the Cognitive Science Society*, 43 (pp. 1588–1594). Cognitive Science Society.
- Dieussaert, K., Schaeken, W., Schroyens, W., & D’Ydewalle, G. (2000). Strategies during complex conditional inferences. *Thinking & Reasoning*, 6(2), 125–161.
- Eiter, T., & Gottlob, G. (1993). The complexity of logic-based abduction. In P. Enjalbert, A. Finkel, & K. W. Wagner (Eds.), *Stacs 93* (pp. 70–79). Berlin, Heidelberg: Springer Berlin Heidelberg.
- Fichte, J. K., Gaggl, S. A., & Rusovac, D. (2022). Rushing and strolling among answer sets – navigation made easy. In *AAAI’2022*.
- Fichte, J. K., Hecher, M., & Hamiti, F. (2021, December). The model counting competition 2020. *ACM J. Experimental Algorithmics*, 26(13). doi: 10.1145/3459080
- Fichte, J. K., Hecher, M., & Roland, V. (2021). Parallel model counting with cuda: Algorithm engineering for efficient hardware utilization. In *CP’21*.
- Fichte, J. K., Hecher, M., Thier, P., & Woltran, S. (2021). Exploiting Database Management Systems and Treewidth for Counting. *Theory Pract. Log. Program.*, 1–30.
- Gebser, M., Kaminski, R., Kaufmann, B., & Schaub, T. (2012). *Answer set solving in practice*. Morgan & Claypool. doi: 10.2200/S00457ED1V01Y201211AIM019
- Gebser, M., Kaufmann, B., & Schaub, T. (2012). Conflict-driven answer set solving: From theory to practice. *Artificial Intelligence*, 187–188. doi: 10.1016/j.artint.2012.04.001
- Gebser, M., Maratea, M., & Ricca, F. (2020). The seventh answer set programming competition: Design and results. *Theory and Practice of Logic Programming*, 20(2), 176–204.
- Grice, H. P. (1975). Logic and conversation. In P. Cole & J. L. Morgan (Eds.), *Syntax and semantics* (Vol. 3). New York: Academic Press.
- Heule, M., & Schaub, T. (2015). What’s hot in the SAT and ASP competitions. In B. Bonet & S. Koenig (Eds.), *Proceedings of the twenty-ninth AAAI conference on artificial intelligence, january 25-30, 2015, austin, texas, USA* (pp. 4322–4323). AAAI Press.
- Hölldobler, S., & Kencana Ramli, C. D. P. (2009). Logic programs under three-valued lukasiewicz semantics. In P. M. Hill & D. S. Warren (Eds.), *Logic programming, 25th international conference, ICLP 2009, pasadena, ca, usa, july 14-17, 2009. proceedings* (Vol. 5649, pp. 464–478). Springer.
- Johnson, D., McGeoch, C., Grigoriadis, M., Monma, C., & Tarjan, B. (1990). *DIMACS implementation challenges*. <http://dimacs.rutgers.edu/programs/challenge>.
- Johnson-Laird, P. N. (1983). *Mental models: towards a cognitive science of language, inference, and consciousness*. Cambridge, MA: Harvard University Press.
- Johnson-Laird, P. N. (1999). Deductive reasoning. *Annual Review of Psychology*, 50(1), 109–135. (PMID: 15012459) doi: 10.1146/annurev.psych.50.1.109
- Johnson-Laird, P. N., Legrenzi, P., Girotto, V., Legrenzi, M. S., & Caverni, J. P. (1999). Naive probability: a mental model theory of extensional reasoning. *Psychological review*, 106

- 1, 62-88.
- Kabir, M., Everardo, F., Shukla, A., Fichte, J. K., Hecher, M., & Meel, K. (2022). ApproxASP - a scalable approximate answer set counter. In *AAAI'2022*.
- Kakas, A. C., Kowalski, R. A., & Toni, F. (1993, 12). Abductive Logic Programming. *Journal of Logic and Computation*, 2(6), 719-770.
- Kelley, H. (1973). The processes of causal attribution. *American Psychologist*, 28(2), 107-128.
- Khemlani, S., & Johnson-Laird, P. N. (2012). Theories of the syllogism: A meta-analysis. *Psychological Bulletin*, 138(3), 427-457.
- Khemlani, S., & Johnson-Laird, P. N. (2016). How people differ in syllogistic reasoning. *38th Conference of the Cognitive Science Society*.
- Khemlani, S., & Johnson-Laird, P. N. (2013). The processes of inference. *Argument & Computation*, 4(1), 4-20.
- Khemlani, S., Lotstein, M., & Johnson-Laird, P. N. (2015). Naive probability: Model-based estimates of unique events. *Cognitive Science*, 39(6), 1216-1258. doi: 10.1111/cogs.12193
- Kleiter, G. D. (2018). Imprecise uncertain reasoning: A distributional approach. *Frontiers in Psychology*, 9. doi: 10.3389/fpsyg.2018.02051
- Knauff, M., & Gazzo Castañeda, L. E. (2021). When nomenclature matters: is the “new paradigm” really a new paradigm for the psychology of reasoning? *Thinking & Reasoning*, 0(0), 1-30.
- Knauff, M., Rauh, R., Schlieder, C., & Strube, G. (1998). Continuity effect and figural bias in spatial relational inference. In Erlbaum (Ed.), *Proceedings of the twentieth meeting of the cognitive science society*.
- Lagniez, J.-M., & Marquis, P. (2017). An improved decision-DDNF compiler. In C. Sierra (Ed.), *Proceedings of the 26th international joint conference on artificial intelligence (ijcai'17)* (p. 667-673). Melbourne, VIC, Australia: The AAAI Press.
- Laird, J. E., Lebiere, C., & Rosenbloom, P. S. (2017, December). A Standard Model of the Mind: Toward a Common Computational Framework across Artificial Intelligence, Cognitive Science, Neuroscience, and Robotics. *AI Magazine*, 38(4), 13.
- López-Astorga, M., Ragni, M., & Johnson-Laird, P. N. (2021). The probability of conditionals: A review. *Psychological Bulletin*.
- McElreath, R. (2020a). *Bayesian inference is just counting*. <https://speakerdeck.com/rmcelreath/bayesian-inference-is-just-counting?slide=11>. (Talk.)
- McElreath, R. (2020b). *Statistical rethinking*. Chapman and Hall/CRC. doi: 10.1201/9781315372495
- Nadeem, M. A., Fichte, J. K., & Hecher, M. (2022). Plausibility reasoning via projected answer set counting – a hybrid approach. In *IJCAI'22*. (To appear.)
- Newell, A. (1973). You can't play 20 questions with nature and win: Projective comments on the papers of this symposium. In *Visual information*. New York: Academic Press.
- Oaksford, M. (2021). Mental models, computational explanation and bayesian cognitive science: commentary on knauff and gazzo castañeda (2022). *Thinking & Reasoning*, 0(0), 1-12.
- Oaksford, M., & Chater, N. (2020). New paradigms in the psychology of reasoning. *Annual review of psychology*, 305-330.
- Over, D. (2021, 12). The new paradigm and massive modalization. *Thinking & Reasoning*, 1-7.
- Peirce, C. S. (1903). *Harvard Lectures on Pragmatism: Lecture VII. MS [R] 315*.
- Polk, T. A., & Newell, A. (1995). Deduction as verbal reasoning. *Psychological Review*, 102, 533-566.
- Ragni, M. (2020). Artificial intelligence and high-level cognition. In P. Marquis, O. Papini, & H. Prade (Eds.), *A guided tour of artificial intelligence research* (Vol. 3, pp. 457-486). Germany: Springer.
- Ragni, M., Fangmeier, T., Webber, L., & Knauff, M. (2006). Complexity in spatial reasoning. In Erlbaum (Ed.), *Proceedings of the 28th annual cognitive science conference*.
- Ragni, M., Riesterer, N., & Khemlani, S. (2019). Predicting individual human reasoning: The precore-challenge. In A. K. Goel, C. M. Seifert, & C. Freksa (Eds.), *Proceedings of the 41th annual meeting of the cognitive science society, cogsci 2019: Creativity + cognition + computation, montreal, canada, july 24-27, 2019* (pp. 9-10). cognitive-sciencesociety.org.
- Reiter, R. (1978). On closed world data bases. In H. Gallaire & J. Minker (Eds.), *Logic and data bases* (pp. 55-76). Springer.
- Reiter, R. (1980, April). A logic for default reasoning. *Artificial Intelligence*, 13, 81-132. doi: 10.1016/0004-3702(80)90014-4
- Riesterer, N., Brand, D., & Ragni, M. (2020). Predictive modeling of individual human cognition: Upper bounds and a new perspective on performance. *Topics in Cognitive Science*, 12(3), 960-974.
- Riesterer, N., & Shadownox. (2021, December). *Cognitive-computationlab/ccobra: Release v1.3.0*. Zenodo. (See also <https://github.com/CognitiveComputationLab/ccobra>.) doi: 10.5281/zenodo.5807735
- Rips, L. J. (1994). *The Psychology of Proof: Deductive Reasoning in Human Thinking*. The MIT Press.
- Sang, T., Beame, P., & Kautz, H. (2005). Performing bayesian inference by weighted model counting. In M. M. Veloso & S. Kambhampati (Eds.), *Proceedings of the 29th national conference on artificial intelligence (aaai'05)*. The AAAI Press.
- Sharma, S., Roy, S., Soos, M., & Meel, K. S. (2019). GANAK: A scalable probabilistic exact model counter. In S. Kraus (Ed.), *Proceedings of the 28th international joint conference on artificial intelligence (ijcai'19)* (pp. 1169-1176). Macao, China: IJCAI.

- Sloman, S. (1994). When explanations compete: the role of explanatory coherence on judgements of likelihood. *Cognition*, 52(1), 1 - 21.
- Stenning, K., & van Lambalgen, M. (2008). Human reasoning and cognitive science. In *Cambridge ma.* MIT Press.
- Taatgen, N., & Anderson, J. R. (2010). The past, present, and future of cognitive architectures. *Topics in Cognitive Science*, 2(4), 693-704.
- Thomson, R., Lebiere, C., Anderson, J. R., & Staszewski, J. (2015). A general instance-based learning framework for studying intuitive decision-making in a cognitive architecture. *Applied Research in Memory and Cognition*, 4(3), 180-190. (Modeling and Aiding Intuition in Organizational Decision Making)
- Wason, P. (1968). Reasoning about a rule. *Quarterly J. of Experimental Psychology*, 20(3), 273–281.
- Woleński, J. (2016). Logic in the light of cognitive science. *Studies in Logic, Grammar and Rhetoric*, 48(1), 87–101.