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Authors
Sarnecka, Barbara W
Goldman, Meghan C
Slusser, Emily B

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How Counting Leads to Children's First Representations of Exact, Large Numbers

Barbara W. Sarnecka and Meghan C. Goldman

University of California, Irvine

Emily B. Slusser

Wesleyan University

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Abstract

Young children initially learn to 'count' without understanding either what counting means, or what numerical quantities the individual number words pick out. Over a period of many months, children assign progressively more sophisticated meanings to the number words—linking them to discrete objects, to quantification, to numerosity, and so on. Eventually, children come to understand the logic of counting. Along with this knowledge comes an implicit understanding of the successor function, as well as of the principle of equinumerosity, or exact equality between sets. Thus, when children arrive at a mature understanding of counting, they have (for the first time in their lives) a way of mentally representing exact, large numbers.

Keywords: bootstrapping, cardinality, children, counting, development, early childhood, equinumerosity, integers, language, natural numbers, numbers, preschool, successor function.

How Counting Leads to Children's First Representations of Exact, Large Numbers

Counting is the first overtly numerical activity that most children do. However, young children's counting behavior is often difficult to interpret, because what they *do* is not a perfect indicator of what they *know*. Of course this is true for all behavior—but in the case of counting, the gap between doing it right and knowing what it means has been of particular interest to developmental scientists.

The gap is important because, over the past several decades, children's counting behavior has been the main evidence used to argue that children have (or don't have) natural-number concepts. (By natural-number concepts, we mean the mental representation of exact numerical quantities such as six or seven, not explicit ideas about entities called 'natural numbers.') Interest in these topics dates back to at least the 1970s, when a few researchers (e.g., Schaeffer et al. 1974; Gelman and Gallistel 1978) shifted their focus away from Piagetian conservation-of-number tasks, and started to look instead at the number-concept knowledge demonstrated by younger children, especially in their counting behavior.

As these researchers observed, correct counting follows several rules. For example, you must recite the counting list in the same order every time; you must point to one and only one object with each number word you say; you must point to all the objects without skipping or double-counting any of them; and—crucially—you must understand that the last number word you say tells the number of objects in the whole set. Note that all but the last rule is about something you must *do*. The last rule, which Gelman and Gallistel (1978) called the *cardinal principle* and Schaeffer et al. (1974) called the *cardinality rule,* is about something you must *know*. It's easy to measure what children do, but hard to measure what they know. So naturally, this is the rule that scientists have been arguing about ever since.

Gelman and Gallistel (1978) originally proposed that children have an innate or early-developing knowledge of how to count, and that this is itself evidence that children have innate or early-developing number concepts. These claims inspired many researchers in the 1980s and 1990s, to try to figure out which came first: Procedural knowledge of counting (knowing that you have to say the counting list in the same order every time, that you have to point to one object for every number word, etc.) vs. conceptual knowledge of counting—especially the cardinal principle. Although space precludes a complete review of that literature here, the general finding was that there is a gap between procedural and conceptual knowledge, with children acquiring the procedural knowledge first. In other words, children learn to count (and they appear to be doing everything right) long before they understand that counting reveals the cardinal number of items in the set (Baroody and Price 1983; Briars and Siegler 1984; Frye et al. 1989; Fuson 1988; Miller et al. 1995; Slaughter et al. 2011; Wagner and Walters 1982).

We will take time here to describe just one task that demonstrates this gap, because the task re-appears throughout this chapter. This task is called 'Give-N' or 'Give-A-Number.' In this task, the child is given a set of objects (e.g., a bowl of 15 small plastic apples) and is asked to give a certain number of the objects to a puppet. For example, the child might be asked to ''Give *two* apples to Kermit." The oft-replicated result is that, while many two- to four-year-olds can count perfectly well (e.g., they count a row of 10 apples without error), these same children are unable to give the right number of objects in the Give-N task. Instead of using counting to generate the right number of items, they just grab a handful. Even when they are explicitly told to count (e.g., "Can you count and make sure you gave Kermit N apples?" or, "But Kermit wanted N apples—can you fix it so there are N?") they still don't use the results of their counting to solve the problem (Le Corre et al. 2006).

Studies using the Give-N task have shown that children move through a predictable series of performance levels, often called number-knower levels (e.g., Condry and Spelke 2008; Le Corre and Carey 2007; Le Corre et al. 2006; Lee and Sarnecka 2010, 2011; Negen and Sarnecka in press; Sarnecka and Gelman 2004; Sarnecka and Lee 2009; Slusser and Sarnecka 2011; Wynn 1990). These number-knower levels are found not only in child speakers of English, but also in Japanese (Sarnecka et al. 2007), Mandarin Chinese (Le Corre et al. 2003; Li et al. 2003) and Russian (Sarnecka et al. 2007).

The number-knower levels are as follows. At the earliest (i.e., the 'pre-number-knower') level, the child makes no distinctions among the meanings of different number words. On the Give-N task, pre-number knowers might always give one object, or might always give a handful, but the number given is unrelated to the number requested. At the next level (called the 'one-knower' level), the child knows that "one" means 1. On the Give-N task, this child gives exactly 1 object when asked for one, and gives 2 or more objects when asked for any other number. After this comes the 'two-knower' level, when the child knows that "two" means 2. Two-knowers give 1 object when asked for ''one," and 2 objects when asked for ''two," but they don't reliably produce the right answers for any higher number words. The two-knower level is followed by a 'three-knower' and then a 'four-knower' level.

Mathieu Le Corre coined the term 'subset-knowers' to describe children at the one-, two-, three- and four-knower levels, because even though they can typically count to ten or higher, they only know the exact meanings of a subset of the words in their counting list. Subset-knowers are distinct from both pre-number knowers (who don't yet know the exact meanings of any number words), and 'cardinal-principle-knowers' (abbreviated 'CP-knowers') who know the exact meanings of all the number words, as high as they can count. More precisely, CP-knowers understand that the set size associated with any number word is generated by counting up to that number word from "one," and adding one object to the set for every word in the counting list. In this way, counting links each number word to an exact set size.

This pattern of development (where procedural skill precedes conceptual understanding) is interesting because it suggests a special kind of learning, called conceptual-role bootstrapping (Carey 2009; see also Block 1986; Quine 1960). Conceptual-role bootstrapping is a process wherein the learner (here, a young child) first learns a placeholder structure—a set of symbols that are structured (i.e., they have some fixed relation to each other). In this case, the set of symbols are the counting words. The list is structured because it has an order: it starts with *one*, followed by *two*, then *three*, etc. But the words are not initially defined in terms of other knowledge the child has. In other words, at the beginning, the child learns to recite the counting list without knowing what each word means.

Under Carey's conceptual-role bootstrapping account, the process of learning what the words mean *is* the process of acquiring natural-number concepts. This is where natural-number concepts come from. And if this proposal is correct, then we should be able to find behavioral evidence for it. Specifically, there should be in an intermediate state of knowledge where children have acquired the symbol structure (i.e., memorized the counting list) but don't yet understand what each number word means. The children in this intermediate state are the subset-knowers.

Remember that what separates subset-knowers from CP-knowers is that only CP-knowers understand Gelman and Gallistel's (1978) *cardinal principle*, which is much more than a rule about how to count – it is the principle that makes the meaning of any cardinal number word a function of that word's ordinal position in the counting list. In other words, the cardinal principle guarantees that for any counting list, in any language, the sixth word in the list must mean 6, the twentieth word must mean 20, and the thousandth word must mean 1,000. To understand the cardinal principle is to understand how counting represents number.

An Aside: Why Don't We Find any Five-Knowers?

People sometimes ask why the knower-levels only go up to 'four-knower.' Why aren't there any five-knowers, six-knowers, or seven-knowers? Couldn't there be a child who knows what the numbers *one* through *seven* mean, but doesn't know what *eight* means, and still doesn't understand the cardinal principle? According to the bootstrapping account, the answer is no. The reason is that only the meanings of the smallest number words (1-4) can be perceived accurately without counting. Humans have innate, nonverbal cognitive systems that allow them to represent small, exact numbers (up to 3 or 4) as well as large, approximate numbers (see Feigenson et al. 2004 for review), but these nonverbal systems cannot distinguish, for example, 7 from 8 items. Thus, the largest exact number that children can recognize directly seems to be 4. Numbers higher than 4 must be represented through counting.

Studies of Number-Knower Levels and Number-Concept Development

If this account of where exact-number concepts come from is right, then subset-knowers present a wonderful opportunity to study number-concept construction in progress. The meanings that subset-knowers assign to number words should be partial and incomplete—including some, but not all aspects of the exact-number concepts that adults and older children have. Describing this process, and figuring out what meanings subset-knowers assign to higher numbers and when, is the general motivation for the studies discussed below.

Number-Knower Levels Versus Counting or Estimation

First, it is necessary to confirm that number-knower levels are a real phenomenon. As explained above, the knower-levels framework describes a pattern where each child either succeeds up to a given number (4 or below), and then fails at all the higher numbers (in which case the child is a subset-knower), or uses counting and succeeds at the higher numbers as well (in which case the child is a CP-knower). But this description is idealized. In real life, even subset-knowers sometimes grab (by lucky accident) 6 items when asked for "six"; and CP-knowers sometimes make counting errors, and so end up giving the wrong number of items even though they understand how counting works. Also, subset-knowers sometimes do count the objects, even though don't know how counting solves the problem. So how can we be sure that the 'number-knower levels' framework is actually the right way to think about all this?

An alternative way of thinking about these data would be to assume that children do have exact-number concepts from the beginning. So when they count, we might assume that they do know what all of the number words mean, and that they use either estimation or counting to do the Give-N task. In either case (estimation or counting), smaller sets are easier to produce than larger ones, so something like the knower-levels pattern might emerge. That is, each child would still perform correctly up to a given number, and fail at higher numbers. When children give the right answer (e.g., they give 4 items when asked for "four"), it's difficult to know what strategy they used: estimation, counting or simple recognition as in the knower-levels account. But when they give the wrong answer for higher number words, we can learn something from their mistakes. In particular, both estimation and counting produce different patterns of errors than simple guessing—and simple guessing on the higher numbers is what the knower-levels account predicts.

In a recent meta-analysis (Sarnecka and Lee 2009), we looked at Give-N data from 280 children, ages 24 to 48 months (about 4,500 Give-N trials in all), and found that most wrong answers were simply guesses, not counting or estimation errors. We could see this in several ways: First, the mean of the errors was unrelated to the number asked for. If children were either estimating or counting, then errors that fell close to the target should be more common than errors that fell far away from it. (E.g., if you are trying to estimate or count 10 items, you are more likely to produce 9 or 11 by mistake than 5 or 15.) However, in the data we reviewed, the wrong guesses produced by subset-knowers were unrelated to the number word asked for, indicating that they really didn't know what number they were trying to produce.

Second, children's errors were asymmetrical and lower-bounded by the numbers they knew. Both counting and estimation should produce symmetrical patterns of error, because people are equally likely to undercount/underestimate as to overcount/overestimate. However, in the data we reviewed, children's errors were asymmetrical, in exactly the way predicted by the knower-levels account.

Take for example, children who correctly gave 1, 2 and 3 items when asked (these are three-knowers in the bootstrapping account). These children did *not* give 1, 2 or 3 items when asked for "four" or any higher number. If they were asked for "four", they were much more likely to give 5 items (as a wrong answer) than 3 items, even though these errors are both the same distance from the correct answer of 4. If children were counting or estimating, this asymmetry wouldn't make sense. Counting and estimation errors are just as likely to produce too few items as too many. Only the knower-levels framework explains why errors would be asymmetrical: Three-knowers actually know what "one," "two," and "three" mean, and they know that the number-word meanings are exclusive (Wynn 1990, 1992), so they restrict their guesses about the meaning of "four" to higher set sizes.

The analysis verified other aspects of the knower-levels account as well: children learned the number-word meanings in order (i.e., there was no evidence of any child learning the meaning of a higher number word before a lower one), and once they figured out the cardinal principle of counting, they generalized this principle to the rest of their count list (i.e., there was no evidence of any child knowing what some, but not all of the higher number words meant). Overall, these analyses strongly supported the number-knower levels/bootstrapping account of number-concept development.

Age Ranges for Each Knower-Level

An obvious question to about the number-knower levels is: How old are the children at each level? The answer depends on the individual child, and differs dramatically for children in different socioeconomic environments. Most studies to date have involved children from relatively privileged socioeconomic backgrounds. Aggregate results from many studies with high-SES children show that these children typically reach an understanding of cardinality sometime between 34 and 51 months old (see Figure 1). That is, any time from a few months before their third birthday to a few months after their fourth birthday. (Because these data are cross-sectional, the fact that many CP-knowers were above age 51 months is not informative; we don't know how long those children were CP-knowers before they were tested. To estimate the range of ages at which children in this population typically become CP-knowers, we looked at the youngest CP-knower and the oldest subset-knower, excluding outliers.)

These data were collected from children attending private preschools in the relatively affluent and educated communities of Ann Arbor, Michigan; Cambridge, Massachusetts; Irvine, California; and Taipei, Taiwan. There is evidence that children from less privileged backgrounds typically come to understand cardinality significantly later, by up to a year or even more (Gunderson and Levine 2011; Jordan and Levine 2009; Klibanoff et al. 2006; Levine et al.

2011a, b).

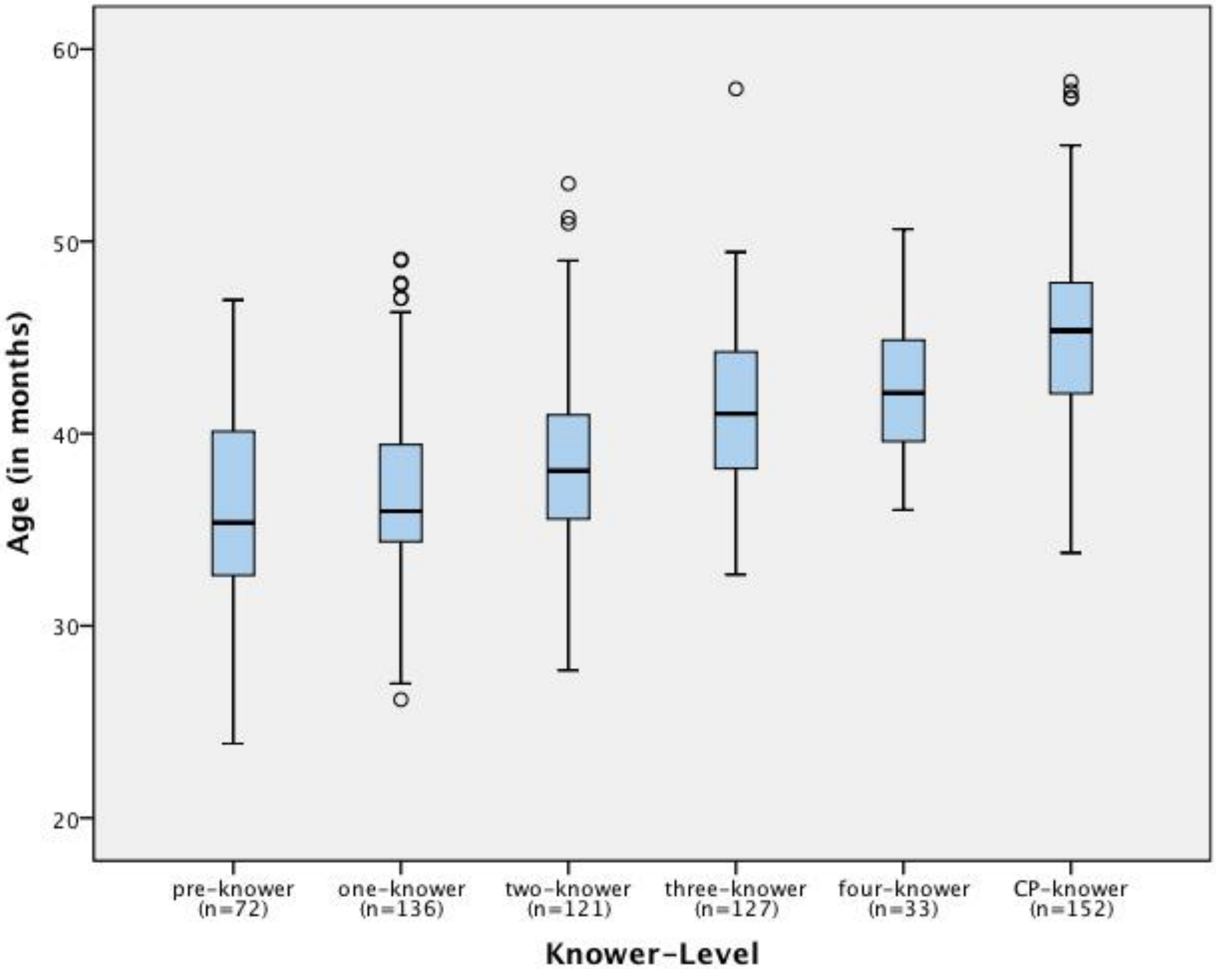

Figure 1. Aggregate data from 641 two- to four-year-old children tested on the Give-N task. Data are taken from Negen and Sarnecka (2009), Negen and Sarnecka (in press), Sarnecka and Carey (2008), Sarnecka and Gelman (2004), Sarnecka et al. (2007), Sarnecka and Lee (2009), Slusser et al. (in revision, under review), Slusser and Sarnecka (2011).

Perhaps the most important point to take away from the data in Figure 1 is not the absolute age for each knower level (as these ages differ depending on the child's socio-economic circumstances), but the wide individual variation, shown by how much the knower-level groups overlap in age. Although there is a correlation between number-knower level and age, any three-year-old in this population could plausibly fall into any of the performance levels, from preknower to CP-knower.

Relation of Number-Knower Level to Vocabulary Development

A basic empirical question about number-knower levels is how they relate to the child's general vocabulary. According to the bootstrapping account, language plays an important role in number-concept development because (1) the initial placeholder structure (i.e., the list of counting words) is a part of language, and (2) the processes used to construct the number concepts (i.e., to figure out the meanings of these symbols) likely subserve word learning in other domains as well. Thus, it is reasonable to expect that controlling for age, children who know more word meanings in general will also know more number-word meanings. (Note that this is not a direct test of the bootstrapping account itself—it is something that should be true if number-concept development is tied in any way to language.)

To answer this question, we gave 59 children, ages 30 to 60 months, the Give-N task, as well as tests of expressive and receptive vocabulary (Negen and Sarnecka in press). As predicted, strong correlations were found between number-knower level and vocabulary (see Figure 2), providing indirect evidence for theories of number-concept development (including the bootstrapping theory) that assign an important role to language.

Within-Child, Cross-Linguistic Consistency of Number-Knower Levels

A more specific prediction of the bootstrapping account is that if a child speaks two languages (e.g., Spanish and English), her number-knower level should be the same (or nearly the same) in both of them. Here's why: Subset-knowers are in an intermediate state of knowledge. They have memorized a counting list (i.e., they have acquired a placeholder structure), but they haven't yet learned what each word means. Because the counting list is memorized so long before the word meanings are learned, bilingual children have ample time to memorize the counting lists in both of their languages before they learn the exact meanings of the words in either language. So when a bilingual child learns that (e.g.) "three" means 3, it should be a short step to realizing that *tres* (the third word in the Spanish counting list) also means 3. If the Spanish and English placeholder structures are both known, and are waiting to be filled in with meaning, then the newly-acquired concept '3' can take its place in both of them.

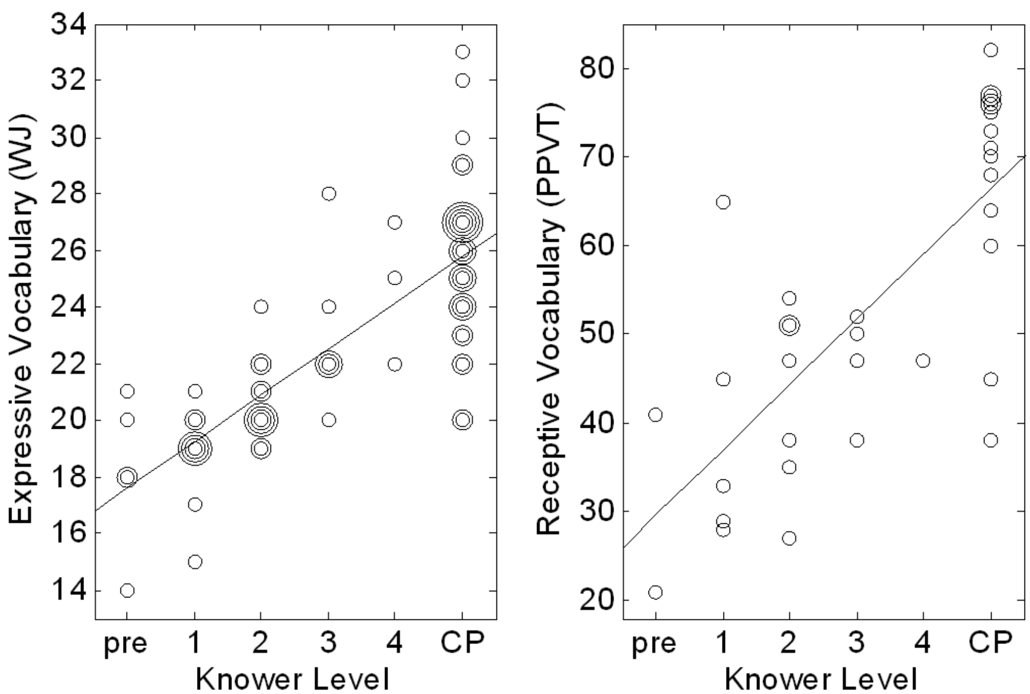

Figure 2. Relations among expressive vocabulary, receptive vocabulary, and knower-level variables. Concentric markers are used when data points overlap. Correlations were significant at $p < .01$, even when partialed for age (Negen and Sarnecka in press).

Thus, the prediction of the bootstrapping account is that children should know the same (or nearly the same) set of number-word meanings in both of their languages. To test whether this was true, we looked at data from 65 bilingual children, ages 41 to 66 months (Goldman and Sarnecka 2011). These children used English at their Head Start program, which most of them started at age 4, and either a mixture of Spanish and English or only Spanish at home. Results are illustrated in Figure 3. In general, the data conform to the bootstrapping prediction—that is, even though there was wide individual variation in how much time the children spent in Spanish-language versus English-language environments, most children showed the same or similar knower-levels in both Spanish and English. In Figure 3, this is illustrated by the fact that the cells on the diagonal (which correspond to having the same knower-level in both languages) had the highest number of children, and children who did not fall on the diagonal usually fell close to it (indicating that their knower-levels were similar, but not the same in both languages). Data from Korean/English and Mandarin/English bilingual preschoolers from higher-income households showed the same pattern (Sarnecka et al. 2011).

Importantly for the bootstrapping account, this within-child, cross-language consistency is more true for numbers than for other kinds of words. We looked at bilingual children's knowledge (in both their languages) of three types of words: Numbers, colors (*red*, *yellow*, etc.) and common nouns for animals and vehicles (*tiger*, *bus*, etc.). Children's number-word knowledge was significantly more correlated (across their two languages) than their color-word or common-noun knowledge (Sarnecka et al. 2011), again confirming the predictions of the bootstrapping account.

Describing the Number Concepts Under Construction

According to the bootstrapping account, subset-knowers should have a partial and incomplete understanding of what the number words mean—these are the number concepts still under construction. So, many of our studies are aimed at figuring out exactly what subset-knowers understand about numbers (or operationally, what meanings they assign to number words) along the way.

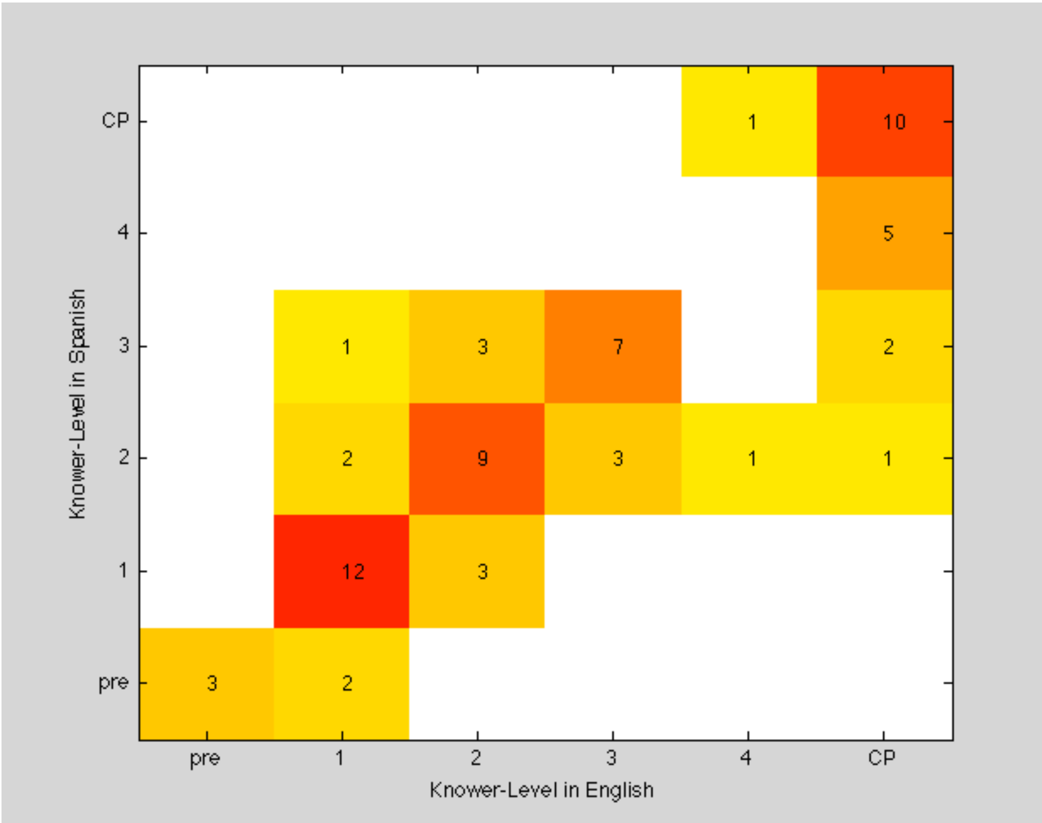

Figure 3. Heat map of data from 65 bilingual children, ages 41 to 66 months, tested on the Give-N task in each of their languages separately. Numbers in each cell indicate the number of children with that combination of English and Spanish knower-levels. Cells on the diagonal represent children with the same knower-level in English and Spanish. For all participants, the overall correlation between their knower-levels in English and Spanish was highly significant at $p < .0001$ (Goldman and Sarnecka 2011).

Number is About Discrete Things, Not Continuous Substances

Part of understanding numbers is knowing that number is a property of sets—and that sets are comprised of discrete individuals. We reasoned that this aspect of number knowledge might plausibly be acquired during the bootstrapping process, and that it might be understood by subset-knowers before they acquire the cardinal principle. In other words, discreteness might be a piece of the number-concept ‘puzzle’ that children discover relatively early.

Language may provide a clue to this aspect of numbers (Bloom & Wynn, 1997), because number words quantify over count nouns (e.g., *five blocks*) but not over mass nouns (e.g., **five water* is ungrammatical). To find out whether subset-knowers understand this about number words, we tested 170 children, ages 30 to 54 months, on the Blocks and Water task (Slusser et al. in revision). This task asked whether young children—even those who cannot yet produce or identify a set of exactly five or six objects—already know that *five* and *six* refer to discontinuous quantities (i.e., to sets of discrete individuals).

In the first experiment, children were presented with two empty cups. The experimenter then placed five (or six) objects in one cup and five (or six) scoops of a continuous substance in the other cup. Four trials asked children about a number word (e.g., "Which cup has five?"). The other four trials asked about a quantifier (e.g., "Which cup has more?"). For half of the trials, the cup with discrete objects was full; for the other half, the cup with the continuous substance was full (see Figure 4).

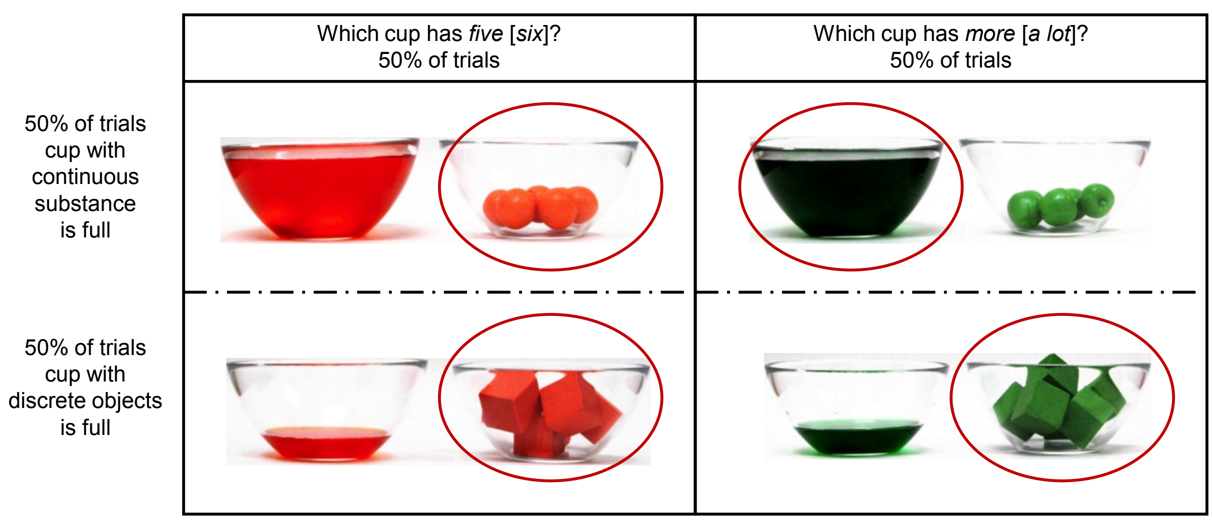

Figure 4. Blocks and Water task design. The correct answer for each trial type is circled.

Results showed that, while children correctly chose the full cup when asked which cup has *more* or *a lot*, only children at the three-knower level or higher consistently chose the blocks over the water as examples of *five* and *six*. One- and two-knowers, on the other hand, were as likely to extend *five* or *six* to continuous substances as to sets of discrete objects.

A second experiment used a slightly modified version of the Blocks and Water task, and asked children about low number words (*one* and *two*) as well as high number words (*five* and *six*). Results from this experiment confirmed that even one- and two-knowers understand that *one* and *two* refer to sets of discrete objects. What's more, in this experiment one- and two-knowers chose the cup with discrete objects when asked about *five* and *six*, but only if they were asked about the low numbers first. This finding suggests that even one- and two-knowers have at least a fragile grasp of the fact that *five* and *six* (and perhaps number words as a class) are about discontinuous, rather than continuous quanitities. And by the time children have learned the meanings of the first three number words, this knowledge is quite robust.

Connecting number words to discrete quantification is only one step in natural-number-concept construction. Another thing that children must understand is what types of changes to a set are relevant to number (or operationally, to number words).

Some Kinds of Actions Affect Number; Others Don't

To test whether children understand that only changing the numerosity of a set will also change its number word, we gave 54 children, ages 34 to 49 months, the Transform Sets task (Sarnecka and Gelman 2004). In this task, children were shown a set of objects labeled with a number word (e.g., "I'm putting *six* buttons in this box."). Then some action was performed on the set (shaking the whole box, rotating the whole box, adding an object or removing an object), and the children were asked, "Now how many buttons? Is it five, or six?" Both subset- and CP-knowers judged that the original number word should still apply on trials where the box had been shaken or rotated, but that the number word should change on trials where an item had been added or removed from the set. Like the results from the blocks-and-water task described above, this result also shows how subset-knowers' understanding of higher numbers (literally, of higher number-word meanings) includes some, but not all aspects of the full concept. A three-knower, for example, seems to understand that "six" is a word that pertains to sets of discrete items (rather than continuous substances), and that "six" will no longer apply to the set if items are added or removed. But that three-knower still doesn't know exactly how many items "six" is, or how counting is related to the exact meaning of the word.

Number Words are About Numerosity, Not Total Area or Other Properties of a Set

The Transform-Sets task showed that children recognize some actions (i.e., adding or removing items) as changing the number of items in a set, and other actions (i.e., shaking or rotating the whole box) as not changing the number of items. But the Transform-Sets task did not distinguish number from the broader dimension of quantity, and there is a difference between the two. An undergraduate assigned to write a 15-page paper is tempted to use a large font size to fill up the space; a researcher writing a 15-page grant proposal looks for ever smaller and narrower fonts to cram in more information. At the end, the essays will each occupy the same quantity of space (15 pages) and may even use the same quantity of ink, but the number of words will be very different.

Even very young infants can detect changes in both numerosity and continuous spatial extent (such as summed surface area or contour length, e.g., Clearfield and Mix 1999; Feigenson et al. 2002). In the Transform-Sets task described above, the number of items in the box changed, but so did the quantity of stuff. After all, six plastic apples minus one plastic apple equals, not only fewer apples, but also less plastic. Thus, the fact that subset-knowers succeed on the Transform-Sets task doesn't necessarily prove that they know number words are about *numerosity per se*—they might just know that number words are about *quantity*.

To answer this question, we tested 116 children, ages 30 to 48 months, on the Match-to-Sample task (Slusser and Sarnecka 2011). For this task, the experimenter showed the child a sample picture while saying, for example, "This picture has *four* turtles." The experimenter then placed two more pictures on the table, saying, "Find another picture with *four* turtles." One picture had the same number of items as the sample. The other had a different number of items (either half or twice as many), but matched the sample in either total summed area, or total summed contour length of the items. (See Figure 5.) The experiment also included control trials, where children were asked to match pictures according to mood (happy or sad) or color. For example, "This picture has *happy* turtles. Find another picture with *happy* turtles."

Results from these experiments showed that children subset knowers generally fail to extend number words (*four, five, eight* and *ten*) based on numerosity. (These children did just fine on the mood and color trials, so we know that they understood the task.) CP-knowers, on the other hand, succeeded robustly. We did not allow children to count the items, but even without counting, the CP-knowers understood that two sets of the same numerosity should be labeled by the same number word, whereas sets of clearly different numerosities (they were always different by a ratio of 1:2) must have different number words.

As in the Blocks and Water task described above, there was also evidence for progressively better performance across the subset-knower levels, suggesting that children gradually identify numerosity as the dimension of experience that number words refer to. Again, this result supports the 'bootstrapping' account, which posits step-by-step construction of natural-number concepts.

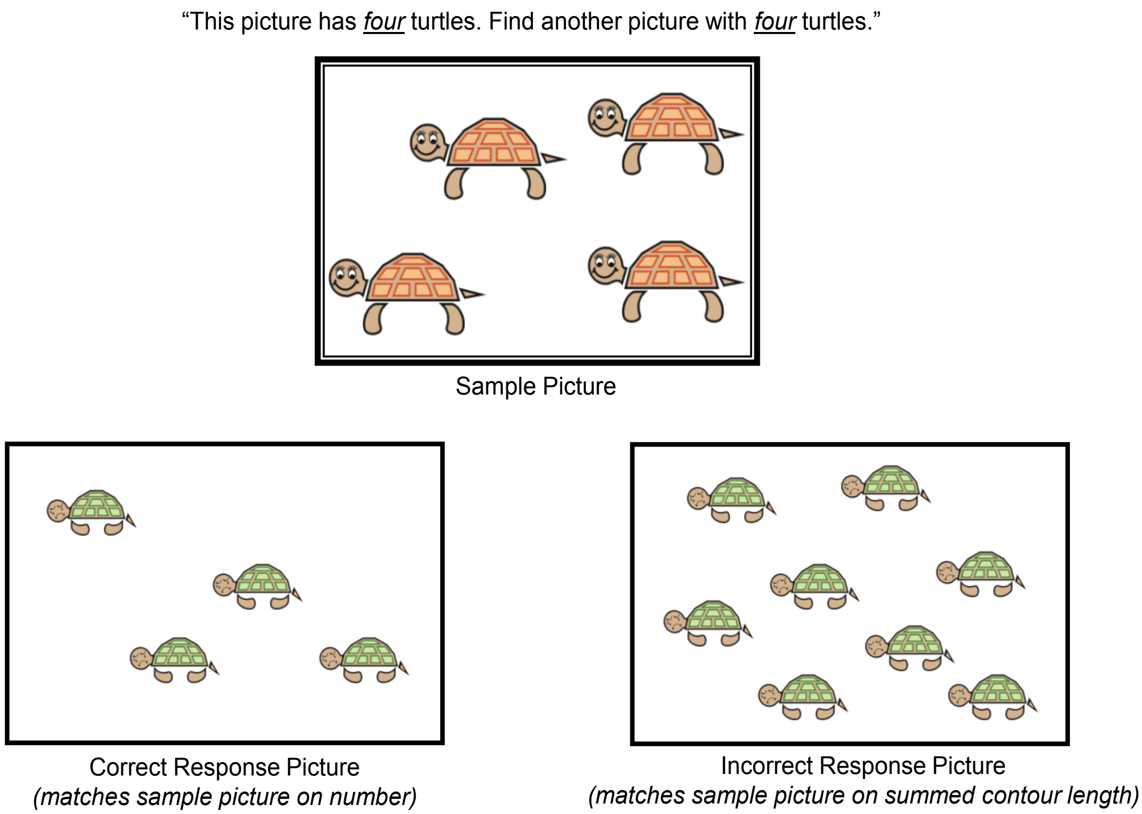

Figure 5. Example of a Match-to-Sample trial where the correct response picture matches the sample picture on number, and the other (incorrect) response picture matches on summed contour length (neither response picture matches the sample picture on color or mood).

Cardinality and the Successor Function

If the bootstrapping story is correct, then the shift from subset-knower to CP-knower is more profound than it appears. It is not just about learning a counting rule; it is also about understanding how counting instantiates the successor function. The successor function is the rule that generates each natural number by adding 1 to the number before it. Counting represents the successor function because represents the process of adding one to a set, over and over again.

To understand this is to understand the cardinal meaning of every word in your counting list. It is to understand what the numbers are.

To explore children's understanding of the successor function, we tested 73 children, ages 24 to 48 months, on a battery of tasks (Sarnecka and Carey 2008). All of these children could count to ten and could correctly count ten objects in a line. In other words, they had mastered the procedural aspects of counting. However, tests of successor-function knowledge told a different story.

In one task, children were shown two plates, each with 6 items (e.g., toy apples), and were told, "This plate has six, and that plate has six. And now I'll move one." Then the experimenter picked up an item from one plate and moved it to the other plate. Next came the test question: the experimenter said to the child, "Now there's a plate with *five*, and a plate with *seven*. Which plate has *five*?" (On half of the trials, the child was asked, "Which plate has *seven*?") Children were not allowed to count the items. Only four-knowers and CP-knowers succeeded on this task. One-, two-, and three-knowers performed at chance. These results suggest that only children who understand cardinality (or those who are on the verge of understanding it) realize that moving forward in the count list means adding items to a set, whereas moving backward means subtracting items. This is an essential part of understanding how counting embodies the successor function.

In another task, children were shown a box with (e.g.) five toy apples inside, and were told, "There are five apples in this box." Then the experimenter added either one or two more apples and asked, "Now how many? Is it six or seven?" Again, children were not allowed to count the items. On this task, only CP-knowers succeeded. That is, only CP-knowers understood that adding 1 item to the set meant moving forward one word in the counting list, whereas adding

2 items meant moving forward two words. This may be the final piece of the puzzle—the last thing that children figure out as they come to understand how counting embodies the successor function, which is effectively to understand what numbers are.

Cardinality and Equinumerosity

Another profound difference between the knowledge of subset-knowers and CP-knowers is that only CP-knowers appear to robustly understand *equinumerosity* (also called *exact equality*)—the notion that any set of N can be put into one-to-one correspondence with any other set of N. For example, any set of ten things can be matched up one-to-one with any other set of ten things: If you have ten flowers and ten vases, you will have exactly one flower for each vase.

We (Sarnecka and Gelman 2004; Sarnecka and Wright in press) tested children's understanding of equinumerosity, by presenting preschoolers with a scenario in which two stuffed animals (a frog and a lion) were each given "snacks" (laminated cards with pictures of food). On half of the trials, the snacks were equal (e.g., Frog and Lion each got 6 peaches); the other half of the trials featured unequal snacks (e.g., Frog got 5 muffins, Lion got 6). The pictures were designed to line up clearly, so that the one-to-one correspondence (or lack thereof) between the sets was visually obvious.

At the beginning of each trial, the child was asked whether the animals' snacks were 'just the same.' If children did not correctly identify the sets as 'the same' or 'not the same,' the experimenter corrected the error and drew the child's attention to the sameness or difference between the sets.

For the test question, the experimenter told the child the number of items in one snack, and then asked about the other (e.g., "Frog has *six* peaches. Do you think Lion has *five* or *six*?"). Children were not allowed to count the items.

Only CP-knowers succeeded robustly on this task. In other words, CP-knowers knew (without counting) that if Frog had six, and the sets were the same, then Lion must also have six. On the other hand, if Frog had six and the sets were not the same, then Lion must have some other number. This understanding of *equinumerosity* as an aspect of number-word meanings is not a rule about counting, but it understood by children who understand the cardinal principle of counting. This provides convergent evidence that the cardinal-principle induction (i.e., the conceptual achievement of figuring out the cardinal principle) is not just an insight about counting—it is a deeper insight about what numbers are.

Conclusions

The studies described here flesh out Carey's (2009) bootstrapping story of where natural-number concepts come from. They do not appear to be innate—rather, they are constructed. Specifically, they are constructed, piece by piece, as meanings for a set of initially meaningless placeholder symbols—the counting words. Children initially learn to say these words in order, and to point to objects while saying them, much as one might point to objects while chanting *eenie, meenie, minie, mo.* But gradually, over a period of months or years, the number words are assigned progressively more sophisticated meanings. Children figure out that these words have something to do with discrete objects, with quantification, with numerosity, and eventually that the counting list represents numerosities in increasing order, with each word in the list indicating the addition of one item to the set. This one-to-one correspondence (between words in the counting list and items in the set) is then generalizable to relations among sets—leading to the knowledge that any set of six can be put into one-to-one correspondence with all and only other sets of six.

Thus, cardinality seems to be the marker of a profound conceptual achievement, involving an implicit understanding of the successor function and of equinumerosity, as well as of how counting works. In this way, the process of learning to count—which starts when the child memorizes a few words of the counting list, and continues to the point of understanding the cardinal principle—is actually the process by which children develop their first mental representations of exact, large numbers.

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