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## Title

Simple quantitative models for integrative planning frameworks
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## Author

Rausser, Gordon C.
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## 1. INTRODUCTION

Many agribusiness firms are decentralized along at least three major cross centers: procurement, production, and marketing. Unfortunately, in decision models that have been constructed for agribusiness firms, we typically find only one of the centers being emphasized. In part, this is a direct result of the training of staff analysts. Their training and perspective are often compartmentalized; they are either marketing analysts, production engineers or economists, and commodity-procurement specialists. The integration of these staff functions faces many of the same obstacles confronted in bringing researchers together from several disciplines, e.g., anthropology, psychology, sociology, economics, etc., to examine interdisciplinary problems. Such efforts result in progress which is generally slow and often hampered by jargon and miscommunication.

Similar, although not as formidable, obstacles arise when experts in operations research combine forces with experts in econometrics or in decision analysis. Based on my experience in managing such a group of experts in developing integrated procurement, production, and market-planning models while on the faculty at the Harvard School of Business Administration, I have come to the conclusion that the key success factor in such team efforts is simple formulations which explicitly recognize the inherent decentralized functional components of individual companies.

Along with the initial emphasis on simplicity, the modeling process must be problem rather than solution oriented. This point has been observed by numerous analysts and perhaps most forcefully by Crowder (1976). The assumptions underlying the various components of the model must be stated explicitly, and potential users must be able to easily extend model components
to examine the sensitivity of the results to alternative assumptions. This, of course, facilitates the value and potential use of the model formulation. Clearly, to enhance the use of the model for decision-making purposes, it must generate results that are timely. Finally, in constructing the initial simplified model representations, the bridge to more complex representation should be planned and formally organized. The analysts should travel over the bridge only when the expected insights and richness of results or, more concretely, additional accuracy outweighs the additional cost of complexity. ${ }^{1}$

The above focus led to the search for-and development of-a number of real world case studies at the Harvard Graduate School of Business Administration. In this paper, one of these case studies is developed. The case study is based on a major U. S. chocolate confectionary firm which imposed upon the researchers the requirement that the company remain anonymous. This requirement was imposed in large part because of the proprietory data that was made available to the research analysts. For this reason, the analysis will refer to a fictitious firm by the name of Mackintosh Chocolate Company.

In what follows, a group of simple quantitative models are developed for each of Mackintosh's three major cost centers: procurement, production, and marketing. In section 2, some background material on the company is provided along with a description of the procurement, production, and marketing and promotion functions. In section 3, simply quantitative models are specified and empiricized for each of these three functions. An attempt is made to illustrate how the models can be utilized to examine a number of decision problems that extend well beyond the initial specification.

## 2. MACKINTOSH CHOCOLATE COMPANY

Mackintosh Chocolate Company is one of the largest privately owned corporations in the United States. It began in 1910 as a converter and processor of cocoa beans to produce and market proprietary products such as candy bars and packaged chocolate powder and industrial products (bulk chocolate, liquor, and cocoa butter). Currently, it grinds about 100,000 tons of cocoa annually, and its major competitor grinds approximately 80,000 annually. Mackintosh Chocolate Company is predominantly in the proprietary products business with about 80 percent of its cocoa beans used for such products.

### 2.1. Cocoa Procurement

The major raw ingredients of milk chocolate are: cocoa beans, sugar, and milk. The prices of cocoa beans as well as their by-products (cocoa powder, cocoa butter, and cocoa liquor) are, in general, both higher and subject to wider fluctuation than those of the other ingredients. As a result, there has been a definite but gradual trend toward reducing the cocoa content of chocolate bars. In addition, various qualities of cocoa beans can be utilized to form blended milk chocolate bars. This feature, along with the possibilities of altering the mix of raw ingredients (cocoa, sugar, and milk) to form the ultimate products, allows cocoa manufacturing firms some flexibility in dealing with raw ingredient price fluctuations.

At Mackintosh, the Procurement Division has instituted a policy of forward contracting to secure the quality and quantity of the necessary raw ingredients. Over the past six months and continuing until July, 1977, the time of the case analysis, they have contracted for both sugar and milk at fixed
prices ( 16 cents per pound for sugar and 94 cents per pound of powdered milk). These contracts permit Mackintosh to select the time of delivery for the amounts requested by the Milk Chocolate (MC) Division. In the case of cocoa, due largely to unanticipated price fluctuations, a "differential" price purchasing policy was utilized.

For cocoa, the general practice followed by Mackintosh is to contract for cocoa beans with New York dealers who, in turn, purchase the raw beans directly from the producing countries. The contracts specify a fixed "differential" or a specific number of points ( 100 points is equal to 0.01 cents) above the price of a particular futures month. The major futures markets for cocoa beans are in London and New York; other minor markets are located in Paris and Amsterdam. MacKintosh uses only the New York market in its contracting with dealers. This market allows trading for futures contracts terminating in the months of March, May, July, September, and December.

In essence the dealer contracts allow Mackintosh to utilize the futures market as a pricing mechanism for its procurement of raw cocoa beans. The contracts provide for a transaction known as "buyer's call." They permit Mackintosh to assure availability of its raw cocoa needs and simultaneously determine the purchase price of these beans. The dealer insures the physical availability of particular varieties of cocoa beans and Mackintosh agrees to pay the dealer a specified number of points (currently 800 for Brazil beans) above the price of a particular New York futures contract. The premium or differential guarantees the dealer's costs and margins. Since the dealer's own profit is assured, he allows Mackintosh to select the day and time when it believes the price of the specific futures contract is most attractive. When Mackintosh so decides, it buys the specified number of futures contracts on the dealer's account, which fixes the purchase price. This price equals the
price on the purchased futures contracts plus the specified dealer contract price premium or differential. Hence, the cocoa procurement policy at Mackintosh is known as being "on differential."

The successful implementation of the above procurement policy requires that Mackintosh forecast the movements in cocoa futures prices. The Procurement Division, a cost center, is responsible for generating these forecasts. Until very recently, these forecasts were generated on the basis of the general trends along with judgmental assessments of experienced cocoa traders and Mackintosh's purchasing agents. These assessments were often conflicting due largely to the complexity of the cocoa commodity system and the alternative perceptions of those responsible for submitting price forecasts to the MC. Division.

### 2.2. Cocoa Commodity System

The cocoa commodity system is international in scope. This commodity can be grown only in tropical climates. A cocoa tree begins to bear on a commercial scale after 3 to 5 years, obtains its maximum yield after 10 to 15 years, and continues to produce until it is approximately 50 to 60 years old. Almost all of the world's cocoa is produced in the following countries: Ghana, Nigeria, Ivory Coast, Cameroon, and Brazil. Over much of the historical period, Ghana alone produces approximately 30 percent of the world's outputs.

While cocoa is produced only in certain tropical developing countries, it is consumed mainly in the industrial countries. Thus by far the greater part of cocoa enters international trade. The volume of world cocoa exports has grown at a long-term average rate of 2.4 percent a year over the last 25
years. The United States is the largest importer of cocoa, accounting for approximately 20 percent of the world's utilization. It is followed by the Federal Republic of Germany with about 13 percent, the Netherlands with approximately 10 percent, and the United Kingdom with about 8 percent. Since the late 1950s, the combined share of the Soviet Union and other Eastern European countries has doubled.

The crop year for cocoa runs from October of one year to September of the next. There are two harvesting periods for cocoa. The first is referred to as "main crop" and is harvested from October through March and accounts for approximately 80 percent of the total output. The second or "mid crop" is harvested from May through June. Once this commodity has been harvested and cleaned, it is referred to as cocoa nibs and may be ground into a mass called chocolate liquor. Consumption of cocoa in importing countries is measured in terms of such grindings.

Conventional wisdom seems to hold the view that cocoa prices fluctuate widely because demand is relatively inelastic with respect to price, and short-term supply is unstable due largely to weather conditions. Moreover, national marketing boards in many of the producing countries have been known sometimes to hold beans from the market and otherwise attempt to influence futures markets. The fixed investment features of cocoa production and the nonbearing period of new cocoa plantings also implies that supply takes at least three to five years to adjust to long-term increases in demand. Hence, in the short run, supply remains almost unaffected even under rather drastic price changes. Consequently, only a moderate surplus or deficit causes substantial price movements.

### 2.3. Production

The MC Division of Mackintosh has responsibility for the production of four products: milk chocolate, chocolate powder, cocoa butter, and cocoa liquor. Milk chocolate is sold from this division directly to wholesalers. Chocolate powder, however, is a by-product obtained in the production of milk chocolate. Cocoa butter and cocoa liquor are intermediate products which are utilized in the production of milk chocolate but which also can be sold to other divisions of MacKintosh. In addition, chocolate powder is transferred directly to the Powder Division for further processing and packaging. The operation of the MC Division is described by the process-flow diagram in Figure 1.

In the MC Division, processing of raw cocoa beans consists of a number of stages starting with cleaning and roasting, followed by grinding, pressing, and mixing of cocoa intermediate products with sugar and milk power to obtain milk chocolate. Currently, only Ghana and Brazil origin beans were utilized in the division's blended products.

The cleaning and roasting stage begins with a machine which is an elaborate filter designed to separate foreign matter which has crept into cocoa bags. Air currents are used to blow away materials lighter than beans; another current lifts the beans away from heavier unwanted fragments. The machine then screens the beans into batches of different sizes. This sorting prepares the beans for uniform roasting. Within the MC Division, the cleaning and roasting stage is completely integrated and the total monthly capacity in terms of raw Ghana cocoa beans is 3 million pounds. (Due to impurities in raw Brazil beans, 1 pound of these beans requires 1.1 units of available cleaning and roasting capacity.)

FIGURE 1
Cocoa Processing in MC Division


In the grinding stage, cocoa beans of different varieties are first blended to maintain minimal quality standards. After blending, the mixture is milled between pairs of carborundum grind stones or in multiroll steel refiners. Milling generates heat which liquefies the fat content of the bean. The grinding. process continues until a dark-brown liquor mass is formed. The total monthly grinding capacity in the MC Division of Mackintosh is, again in terms of raw Ghana cocoa beans, also 3 million pounds. (Raw Brazil beans also require 1 unit of grinding capacity per pound.) The liquor output from grinding is 18 percent less in weight than the input of roasted beans.

After the grinding process is completed, the liquor moves in two distinct directions. One direction is to the cocoa press, which separates liquor into its two basic constituents--butter and powder (obtained in equal amounts). The second is to the mixing stage which produces milk chocolate. This product is essentially liquor combined with cocoa butter along with sugar and milk powder to increase its palatability.

The press stage utilizes a hydraulic ram, working through pistons, which compresses pods of liquor, squeezing out predetermined amounts of cocoa butter, and leaving behind a hard press cake of cocoa solids. This cake is cooled when discharged, crushed between rollers, and then pulverized to yield chocolate powder. In the MC Division, the total pressing capacity is 2.4 mil lion pounds of liquor.

Finally, butter, which was squeezed out of the liquor in the cocoa-press stage, is mixed with liquor which is not pressed, along with prescribed amounts of powdered sugar and milk, to obtain the final product--milk chocolate. After sufficient mixing and conditioning in a "tempering kettle"
to enhance shelf life, the milk chocolate is molded and packaged. In the past, as well as over the foreseeable future, the monthly capacity of this last stage of mixing and packaging far exceeds any conceivable level of milk chocolate sales.

Due to the desire of the division to maintain a stable labor force, the cost of labor within the division was essentially fixed. Furthermore, on a monthly basis, the overhead cost associated with packaging was also fixed.

### 2.4. Marketing and Promotion

Mackintosh markets a number of products but its principal product is a milk chocolate bar, whose brand name was Mack's are sold to wholesalers in cases containing one gross (144) chocolate bars, at a price of $\$ 18.72$ a case, representing a discount of 35 percent from the retail price of 20 cents per $1 \frac{1}{4}$-ounce chocolate bar. The wholesalers distributed the cases directly to large supermarket chains and indirectly via jobbers to small food stores, drug stores, and other outlets which sold candy, chewing gum, and similar products over the counter.

During the period of the case analysis, a proposal on a promotional campaign for the month of March was under examination. During the past three years, Mack's had established a pattern of launcing two such campaigns a year, one in the spring and one in the fall, although the precise month in which each campaign took place tended to vary from year to year. Each of these campaigns lasted for a month, during which wholesales were offered the product at a discount of 5 percent from the regular wholesale price, or $\$ 17.784$ per case.

In evaluating the promotional campaign, a number of practical issues arose. Do wholesaler's promotions simply provide an opportunity for wholesalers to stock at a low price The marketing staff argued that, to a
certain extent, this does in fact happen. In a month following one of Mack's promotions, a drop in orders usually occurred as wholesalers allowed their inventories to return to normal levels. However, the surge in orders during the month in which promotions took place was generally rather substantial. This was due in part to wholesalers filling their warehouses; in part to taking market share away from Mack's major substitutable product, Scrumptious; and, finally, in part to wholesalers expanding their overall demand for chocolate bars. The additional incentive provided by the price discount motivated wholesalers to move the Mack product out of their warehouses and onto supermarket shelves. The additional shelf space generated sales which neither the Mack nor the Scrumptious product would otherwise have received.

Scrumptious bars were produced by the American Confectionary Company-a large producer of candies and other food items. Scrumptious and Mack each had about 50 percent of the U. S. market for domestic milk chocolate bars. The competition for shelf space and, indirectly, for consumers by both Mackintosh and the American Confectionary Company was one of the major reasons that the marketing staff felt it would not be wise simply to reduce permanently the price on Mack's by 5 percent. Some years ago, MacKintosh had pursued such a strategy and found itself in a price war with American Confectionary. In effect, whenever MacIntosh announced a promotion, American Confectionary would follow with a promotion of its own for the same month. When this happened, both Mackintosh and American Confectionary suffered. As a result, over the last few years a pattern developed in which American Confectionary promoted Scrumptious in one of the summer months and in one of the winter months while Mackintosh promoted in the spring and in the fall. For example, during the previous year the Mackintosh promotion occurred in September; and American Confectionary conducted its promotion in November.

Another question that arose is whether the traditional 5 percent discount was optimal. Here, the marketing staff argued that more than a 5 percent discount would require an increase in sales larger than the offset to the decrease in unit contribution which they did not expect could be achieved. A smaller discount, on the other hand, would not be sufficient, they argued, to motivate wholesalers to move the MacKintosh product onto supermarket shelves.

Another issue related to the length of the promotion. The historical practice has been to promote for only one month. The marketing staff argued that a longer promotion would probably be viewed by American Confectionary as an attempt to achieve a permanent price cut which would be interpreted by them as an invitation to retaliate. On the other hand, they argued that a promotion lasting less than a month would be a mess logistically. There would not be sufficient time to process wholesalers' orders and to deliver the Mackintosh product to their warehouses.

Finally, the timing of promotion vis-a-vis the American Confectionary product was seriously questioned. For example, would some improvement in returns be obtained if Mackintosh announced a promotion in the month immediately following the announcement of a promotion for Scrumptious or vice versa. On this issue, the marketing staff argued that Mackintosh would suffer if it began a promotion in a month when the wholesalers were trying to clear their warehouses and eliminate an overstock position. The strategy of alternating promotions with those for Scrumptious was based on assumptions about the likely response of American Confectionary. It was, of course, clear that, when American Confectionary promoted and MacKintosh did not promote, Scrumptious took some of the market share away from Mack. However, when both American Confectionary and Mackintosh promoted at the same time, both companies benefited from an expanded market.

## 3. QUANTITATIVE MODELS

For each of the functions--procurement, production, and marketing--some simple models were constructed to analyze issues raised in section 2. An attempt was made to develop the models as separable, operational decisionmaking tools which could be easily combined for an integrative analysis. This desire led to a block-recursive specification. In the following subsections, each of the blocks is reported and their links are analyzed.

### 3.1. Procurement Mode1

The practice at Mackintosh was to develop judgmental price forecasts of cocoa by a committee composed of procurement specialists. The principal information used in forming the judgmental forecasts included:

1. The weekly main crop purchases by the market boards of Ghana and Nigeria.
2. A series of world production estimates issued by the Foreign Agricultural Service of the U. S. Department of Agriculture and private organizations such as Gill and Duffus of London.
3. Quarterly grind reports issued by the Association of Cocoa and Chocolate Manufacturers.
4. The New York Cocoa Exchange reports on stocks of cocoa available for delivery as well as futures prices, daily open interest, and volume figures.
5. The Census Bureau Monthly Reports which give poundage and dollar sales of chocolate items in the United States.
6. The grind and production forecasts available for the upcoming crop year generated principally by Gill and Duffus and the Foreign Agricultural Service.
7. Quarterly futures price forecasts (for the quarter preceding the terminal point on a particular futures contract market) obtained from a private consulting firm.

The process of combining this information into a judgmental assessment of likely movements in futures prices seemed to Mackintosh management to be fraught with human error. They were particularly concerned about the Committee's inability to take into account feedback effects between production, price, grind, and world inventory levels of raw cocoa beans. It often seemed that the desires, wishes, and hopes of various Committee members significantly biased the price forecast downward. Hence, it was decided to develop a consistent price forecasting model.

A number of alternative frameworks exist for forecasting the most important uncertain quantities, namely, production, grind, price, and inventory carryover. These frameworks vary from formulations which concentrated on the production effects of weather within various regions of Ghana to formulations which decomposed the major consuming regions of the world into developed countries, developing countries, and centrally planned countries. At one end of the spectrum, studies reported in industry journals argued that the major, if not the only, distinguishable factor for price movements was the "carryover ratio." The carryover ratio, computed as the "world stocks at the end of the crop year" to the "world grind during the crop year," was viewed as providing a measure of the near term need to ration and/or encourage inventory holding and as an excellent proxy for the market assessment of the longer term supply-demand balance. At the other end, a number of academic journals reported formulations which specified production, grind, and inventoryregression equations and equilibrium conditions for generating price forecasts. There was some consistency among these studies. They all seemed to suggest that the elasticity demand for cocoa beans is about -.25 , that is to say a 100 percent rise in price leads to about a 25 percent decrease in consumption.

On the basis of these studies (data reported in Table 1), it was decided to operate with a forecasting model containing equations for world production, world grind, world carryover stocks, and New York futures prices. At the outset, in the hopes of enhancing its use, the model formulation was kept reasonably simple and understandable yet sufficiently accurate in forecasting quarterly futures prices. The estimated equations obtained for quarterly production, grind, and inventory along with the price equation (see Table 2) and the resulting price forecast for the first quarter of 1977 are reported in Table 3. These forecasts were compared to those generated by industry sources, especially the Foreign Agricultural Service and Gill and Duffus production forecasts (see Table 4), the Gill and Duffus annual grind forecast (see Table 4), and the private consulting firm quarterly price forecast (see Table 5).

The focus of the procurement decision problem is on the four estimated equations appearing in Table 2. These four equations attempt to represent the interactions among production, consumption, inventories, and prices as well as the lagged feedback effects among these four basic variables. The model is basically stepwise recursive; if the additive error or disturbance terms of each equation are unrelated, conventional ordinary least-squares estimation procedures will provide desirable estimates. If this condition is not satisfied, then Zellner's (1962) seemingly unrelated or some instrumental variable (or simultaneous equation estimation procedure) can be employed to estimate the unknown effects. The estimated parameters and their associated standard errors reported in Table 2 are obtained from an iterated instrumental variable estimation procedure developed by Brundy and Jorgenson (1971) and others.

TABLE 1
Quarterly Data for Cocoa Price Forecasting Model＊

| lear | Marter | Cojereno：s Variables |  |  |  | Exogensus Var：ables |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\gamma$ | I | G | P | $\mathrm{X}_{1}$ |  | ${ }_{3}$ | $\mathrm{X}_{4}$ | ${ }_{5}$ | $\mathrm{i}_{6}$ | $\mathrm{X}_{7, t-13}$ | $\mathrm{i}_{x, t-s}$ | $X_{y}$ |
| 1965 | 4 | 7414.59 | 11713.3 | 3715. | 14.24 | 0 | 0 | 0 | 0 | 1 | 181 | 2790.47 | 2547．27 | 6102.1 |
| 1906 | 1 | 2553.75 | 10799.1 | 3468. | 16.05 | 0 | 0 | 1 | 0 | 0 | 183 | 2761．43 | 2266.6 | 5861． 20 |
|  | 2 | 749.35 | 8121.4 | 3－27． | 29．5j | 0 | 0 | 0 | 0 | 0 | 188 | 25：7．3i | 2：3：0\％ | 555j． 9 9 |
|  | 3 | 1415.49 | 6235.89 | 3293. | 20.55 | 0 | 0 | 0 | 1 | 0 | 181 | 2835. | 2431．s3 | 5744.43 |
|  | 4 | 8435.27 | 10977.2 | 3697. | 24.21 | 0 | 0 | 0 | 0 | 1 | 192 | 2890.44 | 2232.14 | 62：2．73 |
| 1967 | 1 | 2409.89 | 9941.05 | 3446. | 20.46 | 0 | 0 | 1 | 0 | 0 | 187 | 2949.26 | 1944.67 | 6493.59 |
|  | 2 | \＄20．95 | 7349.99 | 3412. | 25.85 | 0 | 0 | 0 | 0 | 0 | 192 | 3453.23 | 1422.7 | 7115.01 |
|  | 3 | 1；04．12 | － 5776.11 | 3278. | 23.22 | 0 | 0 | 0 | 1 | 0 | 183 | 4013.09 | 1489．8 | 7710.54 |
|  | 4 | 8353.72 | 10437.8 | 3722. | 23.77 | 0 | 0 | 0 | 0 | 1 | 200 | 3831．84 | 1540．03 | $7650 . 〔 7$ |
| 1968 | 1 | 2052.69 | 9540.51 | 3549. | 27.05 | 0 | 0 | 1 | 0 | 0 | 198 | $4 \geq 00.2$ | 2003.81 | 7：23．69 |
| － | 2 | 7i5．65 | 6353.16 | 3433. | 25.23 | 0 | 0 | 0 | 0 | 0 | 203 | 4005.85 | 2035．4？ | $745 \%$ |
|  | 3 | 1593.91 | 5110.07 | 3362. | 24.93 | 0 | 0 | 0 | 1 | 0 | 198 | $3 ? 60.85$ | 2491．1 | 7427. |
|  | 4 | 7549.02 | 8903.09 | 3756. | 25.09 | 0 | 0 | 0 | 0 | 1 | 215 | 3778.43 | 2084．91 | 7366.1 |
| 1969 | 1 | 1874．57 | 7338.67 | 3439. | 39.94 | 0 | 0 | 1 | 0 | 0 | 215 | 3759.65 | 2629.25 | 6250.75 |
|  | $2$ | $957.3$ | $4973.96$ | $3322$ | $34.42$ | 0 | 0 | 0 | 0 | 0 | $221$ | $3779.26$ | $2352.36$ | $6502.39$ |
|  | 3 | 1910.41 | 3639.38 | 3185. | 33.75 | 0 | 0 | 0 | 1 | 0 | 212 | 3241． 55 | 2430.55 | $6484.04$ |
|  | 4 | 8805.55 | 8920.44 | 3584． 5 | 33.47 | 0 | 0 | 0 | 0 | 1 | 227 | 3044.93 | 2753.76 | 6391.66 |
| 1970 | 1 | 2241.04 | 7830.98 | 3336.5 | 36.81 | 0 | 0 | 1 | 0 | 0 | 224 | 3201． 50 | 2574．2： | 6559.81 |
| ． | 2 | 1012.2 | 5476.67 | 3365.5 | 25.24 | 0 | 0 | 0 | 0 | 0 | 228 | 3155.65 | 2525.35 | 7514.56 |
|  | 3 | 2136.53 | $4363.8$ | 3249.5 | 20.93 | 0 | 0 | 0 | 1 | 0 | 215 | 3013.23 | 2564．22 | 7722.71 |
|  | 4 | 8851.23 | 9628.02 | 36.17 ． | 28.56 | 0 | 0 | 0 | 0 | 1 | 227 | $30 \leq 0.7$ | 4020.09 | 7200.59 |
| 1971 | 1 | 2565.44 | 8808.46 | 3485. | 23.81 | 0 | 0 | 1 | 0 | 0 | 225 | 3024.44 | 3：78．42 | 7373.83 |
|  | 2 | 995.05 | 6333.5 | 3471． | 19.96 | 0 | 0 | 0 | 0 | 0 | 234 | 2937.85 | 3：30．26 | 75ご．0\％ |
| $4$ | 3 | 228？．68 | 5169.18 | 3447 ． | 18.13 | 0 | 0 | 0 | 1 | 0 | 222 | 2896．78 | 3290.12 | $7551.45$ |
| $\geq$ | 4 | 9315．32 | 11029.0 | 3985． 5 | 25.76 | 1 | 0 | 0 | 0 | 1 | 237 | 2925．4 4 | $508.03$ | $7314.54$ |
| 3972 | 1 | 302，．65 | 10190.1 | 3850.5 | 17.98 | 1 | 0 | 1 | 0 | 0 | 237 | 2907.75 | 232？．12 | $7356 . \bar{i} 6$ |
|  | 2 | 671.75 | $730 \prime .4$ | 3767.5 | 19.80 | 1 | 0 | 0 | 0 | 0 | 246 | 2：53．6 | 217 2．06 | 7173．56 |
|  | 3 | 1888.46 | 5472.36 | 3716.5 | 21.45 | 1 | 0 | 0 | 1 | 0 | 236 | 2786.86 | 287̇．15 | 6920．：5 |
|  | 4 | 0539.73 | 10745．1 | $431 \therefore$ ． | 24.15 | 0 | 0 | 0 | 0 | 1 | 259 | 27\％5．13 | $2 \therefore \therefore \therefore 5$ | 6t．．5．${ }^{\text {c }}$ |
| 1973 | $1$ | $2093.6$ | 8836.69 | 4005. | 25.67 | 0 | 0 | 1 | 0 | 0 | 262 | 323：．35 | 202－． 29 | 6962．5S |
|  | 2 | 701.86 | 5622.55 | 3916. | 24.02 | 0 | 0 | 0 | 0 | 0 | 271 | 3356． 35 | ： 55.5 | 6632． |
|  | 3 | 1446.85 | 3432.42 | 3630. | 36.34 | 0 | 0 | 0 | 1 | 0 | 250 | 38－̇． 6 | 2206.25 | 5442．53 |
|  | 4. | 8032．53 | 7521.95 | 4003. | 40.7 | 0 | 1 | 0 | 0 | 1 | 278 | 3825．69 | ：785．61 | 4723.64 |
| 1974 | $1$ | 3313.22 | $\bigcirc 6754.17$ | 4081. | $35.38$ | 0 | 1 | 1 | 0 | 0 | 269 | 3679.97 |  | 7447.77 |
|  | 2 | 975.48 | 4158.64 | 3571. | 38.40 | 0 | 1 | 0 | 0 | 0 | 275 | 2840.51 | 2284． 2 | 7945．92 |
|  | 3 | 1078．s\％ | 2726．5 | 335！． | 50．5．2 | 0 | 1 | 0 | 1 | 0 | 257 | 2463.96 | $\therefore-3$ ．-6 | ，455．94 |
|  | 4 | 9105.03 | 8061.72 | 3769.5 | 42.96 | 0 | 0 | 0 | 0 | 1 | 264 | 2730.49 | 2545．85 | 3：25．3？ |
| 1975 | 1 | 2561.05 | 7073.27 | 3649.5 | 41.46 | c | 0 | 1 | 0 | 0 | 244 | 3252.73 | 2446.59 | 5555．47 |
|  | 2 | 1189.53 | 4676.6 | 3589.3 | 36.34 | c | 0 | 0 | 0 | 0 | 248 | 3061.41 | $3559.8$ | 9351．： |
|  | 3 | 2335.35 | 3429.65 | 3579.3 | 24.96 | 0 | 0 | 0 | 1 | 0 | 241 | 3118．8： | $4211.96$ | $9623.97$ |
|  | 4 | 9148.05 | 8776.7 | 3801. | 29.96 | 0 | 0 | 0 | 0 | 1 | 262 | 3497.64 | 3350.17 | 9235.37 |
| 1976 | 1 | 2811.74 | 7687.44 | 3901. | 29.78 | 0 | 0 | 1 | 0 | 0 | 264 | 3489．80 | 3597． 85 | 7663.62 |
|  | 2 | 1051．53 | 5047.96 | 3691. | 34.01 | 0 | 0 | 0 | 0 | 0 | 275 | 3505．95 | 456，$\therefore 5$ | 7251．04 |
|  | 3 | 1930.6 | 3237.55 | 3741. | $\therefore \therefore .57$ | 0 | 0 | 0 | 1 | 0 | 262 | 3832． | 402．9．9 | 6552.39 |

[^0]TABLE 1 continued.

## Definition of Variables

Endogenous (Variables Determined by the Model)
Y: Quarterly World Cocoa Production
G: Quarterly World Cocoa Grind
I: Quarterly World Cocoa Ending Inventory
P: Quarterly Average New York Cocoa Futures Price Quoted for Contracts Terminating in the Following Quarter (March, May, July, December), Deflated by a Wholesale Price Index.

Exogenous (Variables Determined Outside the Model)
$X_{1}$ : Durmy Variable for the 1971/72 Crop Year
$\mathrm{X}_{2}$ : Dummy Variable for the 1973/74 Crop Year
$X_{3}$ : Dummy Variable for the First Quarter
$X_{4}$ : Dumny Variable for the Third Quarter
$X_{5}$ : Dummy Variable for the Fourth Quarter
$X_{6}$ : Quarterly Index of Deflated Income for Major Cocoa Consuming Countries
$X_{7}$ : Quarterly Lagged Coffee Prices
$X_{8}$ : Quarterly Lagged Index of Producer Country Cocoa Prices
$\mathrm{X}_{9}$ : Quarterly Cocoa Manufacturers' Margins

TABLE 2

## Estimated Cocoa Price Forecasting Model*

Estimated Production Equation: $\quad\left(R^{2}=.987\right)$

$$
\begin{aligned}
& +\underset{(.021)}{.992}\left[\frac{Y_{t-8}+Y_{t-12}+Y_{t-16}+Y_{t-20}}{4}\right]+\underset{(895.35)}{1840.64 X_{1 t}-1225.29 X_{2 t}}
\end{aligned}
$$

Estimated Grind Equation: $\quad\left(\mathrm{R}^{2}=.907\right)$

$$
\begin{aligned}
\mathrm{G}_{\mathrm{t}}= & 1326.80+.051 \mathrm{X}_{9 t-5}-.043 \mathrm{P}_{\mathrm{t}}-.021 \mathrm{P}_{\mathrm{t}-1}-.049 \mathrm{P}_{\mathrm{t}-2}(.018)(.012)(.026) \\
& (306.2)(.016)(102) \\
& -.096 \mathrm{P}_{\mathrm{t}-3}+.051 I_{\mathrm{t}-4}+9.049 \mathrm{X}_{6 \mathrm{t}-4}+392.76 \mathrm{X}_{5 t} \\
& (.030)(.019)(.740)(120.81) \\
+ & 81.06 \mathrm{X}_{4 \mathrm{t}}+42.20 \mathrm{X}_{3 \mathrm{t}}-.094 \mathrm{P}_{\mathrm{t}} \mathrm{X}_{5 \mathrm{t}} \\
& (46.96) \quad(51.40)(.029)
\end{aligned}
$$

## Inventory Carryover Identity Equation:

$$
I_{t}=Y_{t}+I_{t-1}-G_{t}
$$

Estimated Price Equation: $\left(\mathrm{R}^{2}=.834\right)$

$$
\begin{aligned}
P_{t}= & 20.78-.002 I_{t-1}+.942 P_{t-1}-40.34 \\
& (8.18)(.102) \\
& \left.-\left(\frac{Y_{t-2}+Y_{t-3}+Y_{t-4}+Y_{t-5}+Y_{t-2}+Y_{t-3}+Y_{t-4}}{G_{t-2}+G_{t-3}+G_{t-4}+G_{t-5}}\right)\right]
\end{aligned}
$$

[^1]
## TABLE 3

First Quarter Price Forecast Probability Distribution for May, 1977, Contract*

```
DISTRIBUTION OF P PRINTOUT ? l
```

MEAN $=57.837$
$\operatorname{STDDEV}=7.633$
PRINTOUT ? 2

## FRACTILES

| .001 | .01 | .1 | .25 | .5 | .75 | .9 | .99 | .999 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 33.23 | 39.80 | 48.14 | 52.77 | 57.84 | 62.91 | 67.53 | 75.88 | 82.44 |

*Note that these forecasts are deflated by a Wholesale Price Index. To obtain the corresponding undeflated (nominal) values multiply by 2.732, the Wholesale Price Index for the first quarter 1977.

TABLE 4
Annual Production and Grind Forecasts (Thousand metric tons)

|  | Production Forecasts <br> (Foreign Agricultural <br> Service) | (Gill \& Duffus) | Grind Forecasts <br> Crop Year |  |
| :--- | :---: | :---: | :---: | :---: |
| $1969 / 70$ | 1,335 | 1,326 | (Gil1\& Duffus) |  |
| $1970 / 71$ | 1,387 | 1,455 | 1,307 |  |
| $1971 / 72$ | 1,532 | 1,651 | 1971 | 1,364 |
| $1972 / 73$ | 1,527 | 1,510 | 1972 | 1,486 |
| $1973 / 74$ | 1,461 | 1,469 | 1973 | 1,584 |
| $1974 / 75$ | 1,466 | 1,456 | 1974 | 1,494 |
| $1975 / 76$ | 1,568 | 1,521 | 1975 | 1,402 |
| $1976 / 77$ | 1,457 |  | 1976 | 1,500 |

TABLE 5

# Consulting Firm Quarterly Mean Price Forecasts for Contracts Terminating in the Following Quarter Deflated by a Wholesale Price Index 



For the production equation, over 98 percent of the variation associated with quarterly cocoa production is explained by lagged coffee prices, lagged producer prices, lagged average production, and the dummy variables for the crop years 1971-72 and 1973-74. As expected, a positive relationship is exhibited between lagged producer prices and quarterly production. The structure of the lag suggests that, after eight quarters, a significant supply response to price occurs. The rigidity of producer prices for the African countries during the crop year is one of the main reasons why shorter term supply response was not observed. The lagged average production variable (composed of the same quarter two, three, four, and five years ago) transformed production totals into a proxy for the existing tree stock of bearing age. The greater the tree stock, the larger the production capacity which is reflected in the sign of the estimated coefficient. Coffee prices lagged 13 periods representing a measure of the opportunity cost of allocating land to cocoa trees. Hence, in accordance with a priori reasoning, a negative relationship is revealed for this variable. The structure of the lag suggests that, approximately three years after an increase in coffee prices, cocoa production would decline.

Finally, the two qualitative variables represent the favorable weather conditions of 1971-72 and the unfavorable weather conditions of 1973-74. In using this equation for forecasting purposes, of course, the importance of forecasting weather conditions becomes obvious. These two dummy variables imply that, for the sample record, all years except the years 1971-72 and 1973-74 were normal (or randomly drawn from the same probability distribution for weather and thus reflected in the error term). If weather conditions can be reasonably approximated by three states, then we need only forecast the
state of weather. If the three states do not provide a reasonable approximation of weather conditions (and weather is distinguishable), then some judgmental assessment would have to be added to the effects of weather on quarterly production.

Approximately 90 percent of the variation associated with quarterly grind is explained by lag manufacturers' margins, current and lagged futures prices, a lagged index of income, and seasonable dummy variables. The observed effects of all these variables are in correspondence with a priori reasoning. Each of these explanatory variables is also highly significant. The positive relationship between quarterly grind and margins covers a period of five quarters. This lag is justified owing to the substantial period of time required before a major margin change is translated into price and/or weight changes which affect consumer demand and, ultimately, grind. The effects of futures prices on quarterly grind are distributed over time due largely to bottlenecks, inertia, uncertainty, and the like. A positive relationship for lagged inventory was obtained as expected. The structure of this lag variable implies that the quantity of cocoa beans available in the previous year affects the current quantity of cocoa beans ground. The rationale is that manufacturers make grind decisions approximately a year in advance and are largely influenced by the quantity of cocoa beans in the supply pipeline at that time. The lag for the positive income effects is approximated by four quarters. Finally, the seasonal dumm variables suggest that, on average (ceteris paribus), the smallest quantity of cocoa beans is ground during the second quarter and significantly greater quantities are ground during the fourth and first quarters. Finally, note that an interaction explanatory variable, $P_{t} X_{5 t}$, is included to allow for a slope to change in the effects of price on the fourth-quarter grind. This estimated slope change coefficient
and its associated standard error suggest that grind is more sensitive to price changes in the fourth quarter than in the remaining quarters.

The inventory equation simply closes the representation for the system and is formulated as an identity. Note that inventory carryover could have been specified as a behavioral equation, and an equilibrium condition (the equality of supply and demand) could have been used to generate the average futures price of the current quarter. Over the sample period, inventories generated by the model had a correlation coefficient with actual inventories of .982 .

Approximately 83 percent of the variation in the quarterly futures prices is explained by three explanatory variables: lagged inventory, lagged futures prices, and the first difference of the lagged (average) ratio of production to grind. Each of these explanatory variables is highly significant. As expected, an increase in inventory in the previous quarters has a depressing effect on futures prices. The effect of lagged futures prices also corresponds with a priori reasoning and is suggestive of a geometric lag in past inventories and past first difference (average) ratios of production to grind. The latter variable characterizes periods of inventory accumulation or depletion. Positive values of this variable suggest times of accumulation while negative values would indicate times of depletion. Hence, the expected effect of this variable on futures prices is negative. Note that this variable was averaged to eliminate pronounced seasonal effects.

An examination of the estimated errors for each of the above equations suggests no pronounced heteroscedasticity, distinguishability, or autocorrelation. Conventional examination-of-residuals procedures were employed, and the results suggest randomness.

In using the four equation models for price forecasting purposes, an obvious dichotomy arises for single versus multiperiod prediction. For a single period, all the explanatory variables in the price equation are lagged; and thus, for a single quarter forecast beyond the current period, actual data on lagged inventories, production, grind, and futures prices can be employed. For multiperiod forecasts, however, the following sequence must be followed:

1. Generate a production forecast from the production equation.
2. Generate a price forecast from the price equation.
3. Substitute the price forecast into the grind equation to generate a forecast of current grind.
4. Substitute the production forecast, the grind forecast, along with lagged inventory into the inventory carryover identity equation to obtain an ending inventory forecast.
5. Continue in this fashion until the terminal point of the forecast horizon.

The emphasis in assessing the validity of the price forecasting is on the correspondence of the estimated signed effects with a priori reasoning. The relationship in all cases is specified linearily; no strong arguments can be advanced for retarding, accelerating, or other nonlinear relationships and, thus, Occam's razor applies. Although the goodness-of-fit measures are very high for each of the four basic equations, there still is a fair degree of variability in the uncertain prices for the first quarter of 1977 (see Table 3). The standard deviation for the deflated futures price forecast is 7.63 cents. It should also be noted that this standard deviation is the lower bound estimate of the true standard deviation since the explanatory variables are stochastic.

With regard to the alternative forecasts of production and grind (see Table 4) and prices (see Table 5), the relevant issue is how these alternative forecasts can be combined to obtain improved accuracy. As shown in Bates and Granger (1969), Newbold and Granger (1974), and Falconer and Sivesind (1977), the "best" forecast should make use of all relevant information. The composite forecasting schemes suggested by this work are straightforward and based on the simple argument that alternative forecasts which offer nonequivalent partial explanations can be effectively combined in a composite forecast. This composite forecast can be obtained simply by regressing actual data beyond the sample period against forecasted values, again beyond the sample period. If each of the individual forecasts is unbiased, then the estimated coefficients associated with the individual forecasts will sum to unity. No restrictions are imposed upon the type of individual forecast utilized; it can be judgmental, based on regression analysis, based on an econometric model, or a Box-Jenkins type time series analysis.

Based on the individual contribution of each individual forecast toward reducing uncertainty, composite predictors transform the individual forecast into a single prediction which may be more accurate than any of the individual forecasts. The amount of improvement of the composite forecast over individual forecasts hinges on the information that each individual forecast can add to the total predicted power of the forecast. Thus, the essential question is not whether one forecast is superior to another but whether the forecast embodies complementary prediction information.

### 3.2. Production Planning Model

Given the size of Mackintosh and the nature of production planning, the product allocation decisions may be viewed as "small risk"; they are at most "narrow risk" decisions. Hence, the expected value of production costs is the relevant criterion for the production planning problem. As a result and emphasizing simplicity, a linear-programming formulation of the production planning problem was constructed (see Table 6). In this formulation, the following information is taken as given: a price of $\$ 2.10$ per pound of liquor which might be sold to other divisions of MacKintosh; a price of $\$ 2.25$ per pound at which any amount of chocolate powder produced could be sold; a price of $\$ 2.40$ per pound at which any amount of cocoa butter could be sold; and an amount of 2 million pounds of milk chocolate which will be sold directly to wholesalers during the month of February.

For the month of February, Ghana beans offered to various divisions would average 3.0 points per pound while Brazil beans would average 2.0 points per pound. ${ }^{2}$ The minimal quality requirement on raw cocoa beans used by the MC Division is 2.7. Moreover, for the month of February, the prices charged the MC Division would involve a 29 cents (or 2,900 points) differential for Ghana beans and an 8 cents (or 800 points) differential for Brazil beans. In accordance with the Procurement Division's pricing policy, this meant that MC would be charged a price equal to whatever the futures prices on the May, 1977, contract might happen to average over the first quarter of 1977 plus 29 cents for Ghana beans.

Applying the expectation operator to the objective function of production costs, including the uncertain procurement prices for G-Beans and B-Beans, we

TABLE 6

## Production Planning

UEJtCTIVES:

|  | G-BEANS | B-BEANS | L-PRFSS | L-MIX | L-SOLD | CHOC-POK | RUT-SOLD | BUT-MIX | SUGAR | MILK |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| COST | 1.870 | 1.660 | .0000 | .0000 | -2.100 | -2.250 | -2.400 | .0000 | .1600 | .9400 |

CONSTRAINTS:

|  | G-bEANS | B-BEANS | L-PRESS | L-MIX | L-SOLD | CHOC-POW | BUT-SOLD | BUT-MIX | SUGAR | MILK | RELATION | $\mathrm{R}-\mathrm{H}-\mathrm{S}$ | : |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| CAP-CRR | 1.000 | 1.100 | . 0000 | . 0000 | . 0000 | . 0000 | . 0000 | . 0000 | . 0000 | . 0000 | LE | 3000. | , |
| QU:LIITY | . 3000 | -. 7000 | . 0000 | . 0000 | . 0000 | . 0000 | . 0000 | . 0000 | . 0000 | . 0000 | GE | . 0000 | - |
| CAP-GHND | 1.000 | 1.000 | . 0000 | . 0000 | , 0000 | . 01000 | . 0000 | . 0000 | . 0000 | . 0000 | LE | 3000. |  |
| CRIMDING | .8900 | . 8200 | -1.000 | -1.000 | -1.000 | . 0000 | . 0000 | . $0000 \%$ | . 0000 | . 0000 | EQ | . 0000 |  |
| CAY-PRES | . 0000 | . 0000 | 1.000 | . 0000 | . 0000 | . 0000 | . 0000 | . 0000 | . 0000 | . 0000 | LE | 2400. | , |
| PRESS | . 0000 | . 0000 | 1.000 | . 0000 | . 0000 | $-1.000$ | -1.000 | -1.000 | . 0000 | . 0000 | EQ | . 0000 |  |
| BUT $=\mathrm{CHOC}$ | . 0000 | . 0000 | . 0000 | . 0000 | . 0000 | 1.000 | $-1.000$ | -1.000 | . 0000 | . 0000 | EQ | . 0000 | ; |
| SUG-MIX | . 0000 | . 0000 | . 0000 | . 5000 | . 0000 | . 0190 | . 0000 | . 5000 | -. 5000 | . 5000 | EQ | . 0000 |  |
| MILK-MIX | . 0000 | . 0000 | .0000 | . 1000 | . 0000 | . 0000 | . 0000 | .1000 | . 1000 | -. 9000 | EQ | . 0000 |  |
| FAI, -MAX | . 0000 | . 0000 | .0000 | -1.200 | .0000 | . $00: 10$ | . 0000 | 1.000 | . 0000 | . 0000 | LE | . 0000 |  |
| B/:-M1N | . 0000 | . 0000 | . 0000 | -. 8000 | . 0000 | . 0000 | . 0000 | 1.000 ? | . 0000 | . 0000 | GE | . 0000 | N |
| Mllarchoc | . 0000 | . 0000 | . 0000 | -. 000 . | . 0000 | . $0: 000$ | . 0000 | 1.000 | 1.000 | 1.000 | EO | 2000. |  |

TABLE 6 continued.
Definition of Variables (all variables in units of 1000 lbs.)

1. G-BEANS: Ghana beans purchased by the MC Division;
2. B-BEANS: Brazil beans purchased by the MC division;
3. L-PRESS: Cocoa liquor pressed;
4. L-MIX: Cocoa liquor sent to mixing and packaging;
5. L-SOLD: Cocoa liquor sold;
6. CHOC-POW: Chocolate powder produced;
7. BUT-SOLD: Cocoa butter sold;
8. BUT-MIX: Cocoa butter sent to mixing and packaging;
9. SUG: Powdered sugar purchased by the MC Division;
10: MILK: Powdered milk purchased by the MC Division.
Definition of Constraints
10. CAP-C\&R: Monthly cleaning-and-roasting capacity;
11. QUALITY: Minimal point score needed to achieve desired quality;
12. CAP-GRND: Monthly grinding capacity;
13. GRINDING: Defines relations between materials entering and leaving the grinding stage;
14. CAP-PRES: Monthly press capacity;
15. PRESS: Defines relations between materials entering and leavingthe press stage;
16. BUT=CHOC: Defines mix of cocoa butter and chocolate powder coming out of press stage;
17. SUG-MIX: Defines ratio of sugar to ingredients involved in mixing stage;
18. MILK-MIX: Defines ratio of milk to ingredients involved in mixing stage;
19. B/L-MAX: Specifies maximum butter-to-liquor ratio permitted in mixing stage;
20. B/L-MIN: Specifies minimum butter-to-liquor ratio permitted in mixing stage;
21. MILKCHOC: Milk-chocolate production required by marketing.
Objective Function
Net cost in thousands of dollars per month.
obtain the result appearing in Table 6. Note that the coefficient of G-Beans is the mean-price forecast (see Table 3) plus 29 cents and the coefficient of B-Beans is the same mean-price forecast plus 8 cents.

In analyzing the output of Table 6, net cost is measured in thousands of dollars per month and all variables are in units of 1,000 pounds. Thus, the reported shadow values are dollars per pound. To illustrate the use of these measures, consider (a) from output *3* (Table 7), the marginal cost of producing an additional pound of milk chocolate is 81.11733 ; and (b) the marginal cost of producing milk chocolate remains constant over a broad range, viz., .000015257 to 4133.7 . Hence, the cost of producing an additional 1,000 cases or 11,250 pounds is:

$$
(11,250)(\$ 1.11733)=\$ 12,570
$$

As indicated by output *5*, the optimal solution will remain unchanged as long as the price of chocolate powder is within the interval (2.0073, 2.400). Hence, an increase in the price of chocolate powder from, say, $\$ 2.25$ to $\$ 2.35$ will not alter the production plans. Note also that, for the full month, such a price increase would lower net cost by:

$$
971,953 \cdot(\$ 2.35-\$ 2.25)=\$ 97,195
$$

The results appearing in Table 7 can also be employed to evaluate simple investment decisions assuming that February represents a typical month over the life of the proposed investment. For example, a rather simple comparison of the two options--(1) purchase 200,000 pounds of additional cleaning and roasting capacity and (2) do not purchase additional capacity--can be examined. For the first option, we must compute the net present value of the

TABLE 7
Computer Output from Linear Programming Analysis of MC Production Planning

```
MAXIMIZE OR MINIMIZE? MIN
OPTIMAL SOLUTION FOUND.
    COST 1944.86
OUTPUT OPTION? E
ALL ITENS NOT LISTED IN SECTIONS 1 - 4 HAVE THE VALUE zERO.
*l* DECISION VARIARLES
    1. G-EEFNS 2038.83
    2. R-BEANS 873.786
    3. L-PRESS 1943.91
    4. L-MIX 444.444
    6. CHOC-POW 971.953
    7. RUM-SOLD 616.39?
    8. BUJT-MIX 355.556
    9. SLGAR \therefore i000.00
    10. MILR .......}200.00
*2* SLACK(+) AND SUPPLUS(-) IN CONSTRAINTS
    3. +CAP-GRND 87.3786
    5. +CAP-PRES 456.095
    10. +B/L-MAX 177.778
*3* SHADOW PRICES FOR CONSTRAIMS
    1. CAP-C&R -.966019E-01
    2. QUALITY . }20034
    4. GPINDING 2.32500
    6. PRESS 2.3250C
    7. SUT=CHCC .750000E-01
    8. SUG-NIX 2.19833
    9. MILK-MIX 1.41833
    11. E/L-NIN . 416666E-Cl
    12. MILKCHOC 1.11733
*4* REDUCED COSTS FOR DECICION VARIABIES
    5. L-SOLD . 225000
*5* RANGES ON COEFFICIENTS OF OBJECTIVE COST
            VARIARLE LOWER BOUND CURRENT VALUE UPPER ROUND
\begin{tabular}{llll} 
1. G-BEANS & 1.6224 & 1.8700 & 2.0121 \\
2. B-SEANS & \(-.74565 E+07\) & 1.6600 & 1.8664 \\
3. L-PRESS & \(-.75000 E-01\) & .00000 & .12134 \\
4. L-MIX & UNBOUNDED & .00000 & \(.75000 E-01\) \\
5. L-SOLD & -2.3250 & -2.1000 & UNROUNDED \\
6. CHOC-FOW & -2.4000 & -2.2500 & -2.0073 \\
7. RUT-SOLD & -7.4329 & -2.4000 & -2.2500 \\
8. BUT-MIX & \(-.75000 E-01\) & .00000 & UNROUNDED \\
9. SUGAR & UNEOUNDED & .16000 & \(.27962 E+C 7\) \\
10. MILK & UNROUNDED & .94000 & UNBOUNDED
\end{tabular}
*6* RANGES ON VAIUES OF RIGHT-HAND-SIDE. R-H-S
                CONSTRNT LOWER BOUND CUPRENT VALUE UPPER BOUND
        1. CAP-C&R
        2. QUALITY -1909.1 .00000 900.00
        1451.5
        3C00.0
        2. QUALITY -1909.1 .00000 900.00
        3. CAP-GFND 2912.6 3000.0 UNOOUNDED
        4. GRINDING -456.09 .00000 1232.8
        5. CAP-PFES 1943.9 2400.0 UNSOUNEED
        6. PRESS UNBOUNDED .000CO !232.B
        7. BUT=CHOC -1943.9 .00000 1232.8
        8. SUGMIX -800.00 .00CこO 853.47
        9. MILK-MIX -800.00 .00000 200.00
    10. B/L-MAX -177.78 .OCNOS UNEOUNDED
    11.'R/L-MIN -640.00 .0C000 145.45
    12. MILKCHOC .15259E-04 2000.0 4133.7
```

200,000 pounds of additional capacity. Assuming zero salvage value, this requires the following information:
(a) The investment cost (say, 8500,000 ).
(b) The life of the additional capacity.
(c) The monthly incremental labor and overhead costs associated with this investment.
(d) The company after-tax hurdle rate (monthly).
(e) The company tax rate and the computed monthly tax savings resulting from depreciation and investment tax credits less additional tax payments resulting from cost reductions (f).
(f) The monthly reductions in net costs resulting from the proposed investment.

This information would allow the net present value to be computed by discouting at rate (d) the sum $[(f)+(e)-(c)]$ over the period (b) and subtracting the investment cost (a). Unfortunately, the only information available is (a) and (f). The typical month's reduction in costs can be determined from Table 7 by noting that:
(1) An additional pound of cleaning and roasting capacity will reduce net costs by . 0966019 cents (see output *3*).
(2) The per-pound reduction in net costs remains constant for increases in cleaning and roasting capacity up to 90,000 pounds (see output *6*).
(3) For increases above 90,000 pounds, the relevant shadow value can be expected to fall to 0 due to the bottleneck represented by available grinding capacity (currently a slack of $87,378.6$ pounds).

The equivalence of the upper bound on the R-H-S for the C\&R constraint and the slack for the grinding capacity constraint can be demonstrated. The quality constraint and current prices quoted by procurement will result in 70 percent of total raw cocoa bean utilization being represented by Ghana beans and 30 percent by Brazil beans. In other words, in terms of quality constraint,

$$
.70 \cdot(3.0)+.30(2.0)=2.7
$$

and, for this constraint to remain tight,

$$
\Delta G-B e a n s=(.7 / .3) \Delta B-\text { Beans } .
$$

Furthermore, for 90,000 additional pounds of cleaning and roasting capacity:

$$
(\Delta G-\text { Beans })+1.1 \text { ( } \Delta B \text {-Beans })=90,000
$$

Solving these two equations for $\Delta G$-Beans and $\Delta B-B e a n s$, we have:

$$
\begin{aligned}
& \Delta G-\text { Beans }=61.1650 \\
& \Delta B-\text { Beans }=26.2136
\end{aligned}
$$

Total Increment: 87.3786.

This total increment is precisely the amount of slack available in grinding capacity.

The monthly amount of cost reductions (f) that should be discounted over the life of the investment is:

$$
90,000 \cdot(.0966019)=\$ 8,694 .
$$

For a zero hurdle rate, this monthly increment in "cash flow" implies a payback period on the 8500 k of 4.79 years. Hence, this potential expansion may indeed be attractive.

Disregarding the bottleneck represented by available grinding capacity, a less rigorous analysis would assert that the shadow price of . 0966019 applies for the first 90,000 pounds; and, for the next 510,000 pounds, the average shadow price is between 0 and some value less than .0966019. Thus, the monthly cost reduction is between

$$
90,000 \cdot \$ .0966019=\$ 8,694
$$

and

$$
600,000 \cdot \$ .0966019=\$ 57,961
$$

implying an undiscounted payback period of between 4.79 and .718 years. Whether such an expansion would appear attractive would depend on the net present value of the after-tax cash flows discounted at the appropriate hurdle rate.

Relaxing the assumption of February as a typical month is not likely to improve the situation unless a simultaneous expansion in grinding capacity is also admitted as a relevant alternative. Under these circumstances, probability distributions for the following uncertainties over the complete planning horizon would be needed:

1. Price for G-Beans.
2. Price for B-Beans.
3. Quality of G-Beans.
4. Quality of B-Beans.
5. Price of liquor.
6. Price of chocolate powder.
7. Price of chocolate butter.
8. Price of sugar.
9. Price of milk.
10. Demand for milk chocolate.

Assessment of probabilities on these uncertainties would involve the use of regression analysis and judgments. Furthermore, a simulation model could be suggested to evaluate the two options (expansion versus no expansion) under the criterion of expected net present value.

From the results in Table 7, various rental decisions can be evaluated. For example, suppose one of the cleaner/roasters, with a capacity of 120,000 pounds, is on its last legs. In addition, suppose a new machine is on order, but it will not arrive until March 1. It is clear that, if we knew that the 120,000 pound cleaner/roaster will fail on February 1, we would be prepared to rent up to 200,000 pounds (the 120,000 pounds we may lose plus the 90,000 pounds for which the shadow price remains constant) of additional capacity at a price less than or equal to .0966019 cents per pound. However, if the equipment does not fail until March 1, we would still be prepared to rent up to 90,000 pounds at less than or equal to the appropriate shadow value. Hence, the relevant rental amount of equipment ranges from 0 to 210,000 pounds.

To proceed with the decision analysis for this problem, a number of assumptions are required:
(1) The per unit rental rate is known with certainty or can be negotiated.
(2) The set-up and removal costs (e.g., transportation, installation, additional overhead, etc.) are known with certainty.
(3) There is no delivery delay.
(4) The rental decision must be made before the knowledge of the equipment failure, say, before February 1.
(5) Partial equipment failure is not relevant.
(6) Temporary repair of the equipment is too costly and thus can be excluded a priori.
(7) The quality of output from the rental capacity will be indistinguishable from current equipment output.
(8) The nonmonetary consequences of dealing with a firm renting the equipment to MacKintosh are unimportant.

Given the magnitude of the consequences on the rental decision relative to total net costs of the MC Division and the size of Mackintosh, the expected
monetary value is the appropriate criterion. Finally, the major uncertainty is the point at which the equipment fails between February 1 and March 1. Note for practical purposes that, since new equipment will be delivered on March 1, the probability of failure (or simply terminating utilization of equipment) by the end of February can be treated as one. Given an assessment of the probability distribution associated with this uncertainty, the decision diagram is as follows.


Amount of
Pental Caracity

Failure Time

To proceed with the endpoint computation, let $\mathrm{R}_{\mathrm{j}}$ denote the rental rate for rental capacity of $\mathbf{j} 1,000$ pounds. Let $S$ denote the set-up and removal costs (in thousands of dollars), assumed independent of the rental capacity. Given that the shadow value for increases in capacity above 90,000 pounds is 0 , the before-tax change in costs (relative to the optimal solution of Table 7) is given (in thousands of dollars) for each end position as:

$$
\begin{aligned}
& A=B: 90 \cdot .0966019-R_{210}-S \\
& C:-R_{120}-S \\
& D: 90 \cdot .0966019-R_{120}-S \\
& E:-30 \cdot .0966019-R_{90} \\
& F: 90 \cdot .0966019-R_{90}-S \\
& G:-120 \cdot .0966019 \\
& H: 0 .
\end{aligned}
$$

For failure at any time between February 1 and March 1 , the end-position value will be an appropriately weighted average of the corresponding February 1 and March 1 end-position values. Since incremental taxes will be proportional to before-tax change in costs, we car analyze the problem in terms of pretax dollars by folding the tree back using five-bracket medians for the probability distribution on failure time. Depending upon the actual situation, sensitivity analysis may be warranted for some or all of assumptions 1 through 8.

Shortly after the linear-programming model was constructed, the Procurement Division considered the purchase of Ivory Coast beans (I-Beans, 500,000 pounds) at a 2,100 point differential to the first quarter's average futures price on the May, 1977, contract. The average quality of these beans is 2.5, and they use the same capacity in cleaning, roasting, and grinding as Ghana beans. This alternative of purchasing I-Beans can be examined by augmenting the formulation appearing in Table 6 and solving again. This involves the addition of
(1) The term 1.79 (I-Beans) in the objective function.
(2) The constraint I-Beans $\leq 500$.
(3) A column in the tableau for I-Beans
1.0
-0.2
1.0
0.82
0.0
0.0
0.0
0.0
0.0
0.0
0.0
0.0
1.0

Obviously, if I-Beans appear in the solution to the revised problem as a decision variable with a positive value, then they should be purchased.

A definitive answer to the question of whether or not I-Beans should be purchased can be obtained by "pricing out" the value of I-Beans using the shadow prices in the output of Table 7. To do so, observe that the purchase of 1 pound of I-Beans results in
(1) A 1-pound increase in cleaning and roasting capacity costing . 0966 cents per pound.
(2) A .2-unit increase in quality, ${ }^{3}$ worth .2003 cents per unit.
(3) A 1-pound decrease in grinding capacity costs nothing since the constraint is not binding.
(4) A . 82 -pound increase in grinding output which is worth $\$ 2.325$ per pound (the average of the selling prices of additional chocolate powder and cocoa butter, produced in equal quantities, which can be sold).

Assuming April is otherwise the same as February, these changes result in a net value of

$$
-1 \cdot \$ .0966+.2 \cdot \$ .2003-1 \cdot \$ .0+.82 \cdot \$ 2.325=\$ 1.770
$$

which is .02 cents less than the cost of $\$ 1.79$. Thus, buying 1 pound of I-Beans would increase costs by .02 cents. Since the shadow prices can at best remain the same if the amount purchased increases, additional pounds of I-Beans purchased will increase costs by at least . 02 cents per pound and, therefore, no I-Beans should be purchased.

### 3.3. Marketing and Promotion Formulation

To investigate the issues raised in section 2.4 due to proprietary concerns, only limited data were made available. In particular, monthly data on the number of cases (in hundreds) ordered, retail prices, and whether a promotion had occurred were available for the period January, 1973, through January, 1977. A regression equation was specified with the number of cases ordered as the dependent variable (deseasonalized); and the independent variables consisted of six dummy variables, having to do with whether or not a promotion had occurred, and one variable having to do with the retail price of Mack's chocolate bars. In particular, all the dummy variables, $x_{2}$ through $x_{7}$, were set equal to 0 in each month with the following exceptions:
$x_{2}=1$ if Mack's were promoted this month and Scrumptious were not, $x_{3}=1$ if Mack's were promoted last month and Scrumptious were not, $x_{4}=1$ if Scrumptious were promoted this month and Mack's were not, $x_{5}=1$ if Scrumptious were promoted last month and Mack's were not,
$x_{6}=1$ if both Mack's and Scrumptious were promoted this month, and $x_{7}=1$ if both Mack's and Scrumptious were promoted last month.

The price-related variable, $x_{8}$, reflected the per ounce retail price in constant cents (constant is defined in terms of the food component of the consumer price index). In fact, it was company policy to price Mack's so that the cost per ounce kept reasonable pace with the food component of the consumer price index. However, because prices could be changed in increments of no less than 5 cents and weight could be changed in increments of no less than 1/8 ounce, the price per ounce--measured in constant dollars or cents--would vary between changes in size or price of a Mack's bar. MacK's current price of 20 cents for a $1 \frac{1}{4}$-ounce bar, or 16 cents per ounce, was matched by Scrumptious and was well within the range where the price and size would remain the same. In the past, either the price or the weight had been changed whenever the retail price, measured in current (i.e., January, 1977) cents, fell below 14.5 cents per ounce. Similar retail pricing policies had been followed by Scrumptious and other candy bar manufacturers, and MacKintosh management believed that they could not unilaterally deviate from such a policy. All the values of variable $x_{8}$ have been adjusted to January, 1977, dollars. The estimated equation is reported in Table 8. ${ }^{4}$ A trend term was not included simply because per capita consumption of chocolate is decreasing at about the same rate as population is increasing. Attempts to determine whether promotion effects persist beyond the following month revealed no noticeable relationship. This issue was investigated by lagging the promotion-related dummies one or more periods.

The estimated equation in Table 8, when combined with the production planning model of section 3.2 , can be used to evaluate promotion alternatives.

## TABLE 8

## Estimated Equation for MacK's Case Orders



For example, assuming that (a) neither Mackintosh nor ACC promotes in February, (b) the monthly seasonal index is 1.1 in March and 1.20 in April, and (c) ACC does not run a promotion in either March or April, the expected contribution to Mackintosh of running a promotion in March can be determined. Because a promotion in any one month affects the number of cases ordered both in the month in question and in the succeeding month, the relevant horizon is two months.
"Contribution" is revenue less variable cost. Variable cost per pound of milk chocolate equals $\$ 1.117$ (the shadow price of the milk chocolate constraint, Table 6). Because there are 11.25 pounds per case, this amounts to $11.25 \cdot \$ 1.117=\$ 12.566$ per case. Notice that the range over which this shadow price applies is virtually 0 to $4,133,700$ pounds (Table 6) or from 0 to $4,133,700 / 11.25=367,400$ cases, which easily covers the range of cases ordered in March and April under the specified assumptions. ${ }^{5}$

Revenue per case is $\$ 18.72$, normally, and $\$ 17.784$ in any month during which there is a promotion. Contribution per case is, therefore, 818.72 $\$ 12.566=\$ 6.154$, normally; and $\$ 17.784-\$ 12.566=\$ 5.218$ during a promotion. Notice that these numbers apply to all cases ordered, not just the increment due to promotion. The expected cases ordered are computed in Table 9.

Expected contribution is the expected value of contribution per case times cases ordered; because this is a linear function of cases ordered, expected contribution is contribution per case times expected cases ordered. Thus, if Mackintosh promotes in March and ACC does not, expected contribution is: $\$ 1,378,000$ in March and $\$ 1,182,000$ in April or $\$ 2,560,000$ over two months. 6

TABLE 9

## Computation of Expected Cases Ordered



[^2]The above analysis is not very useful largely because it must assume a strategy on the part of ACC with respect to their Scrumptious product. Since data were not available on costs, sales, prices, etc., of Scrumptious, a first cut at determining the strategy of ACC was to assume that their costs and revenues were the same as MacKintosh's. Under the circumstances, this assumption seemed reasonable since ACC also had approximately 50 percent of the market.

Given the above simplification and no distinct seasonal pattern, the promotion alternatives facing MacKintosh can be analyzed as a two-person nonzero-sum game in which each "player" (Mackintosh and ACC) has two strategies, "promote this month" and "do not promote this month," and in which the "payoffs" are expected contributions to each player over the two-month horizon. The experience of the marketing staff outlined in section 3.4 ruled out the possibility of one company promoting one month after its competitor's promotion. ${ }^{7}$

Consider the four possible pairs of strategies: (a) Mackintosh and ACC both promote, (b) MacKintosh promotes and ACC does not, (c) ACC promotes and Mackintosh does not, and (d) neither promotes. Total contribution to Mackintosh for each of these pairs of strategies is computed in Table 10.

The payoff table is shown in Table 11 where the entry in the upper left of each cell is the expected contribution to Mackintosh and the entry in the lower right (computed by symmetry) is the expected contribution to ACC. There are two equilibria: Mackintosh promotes and ACC does not and ACC promotes and Mackintosh does not.

Short-run considerations might result in each firm trying to force the outcome to be the equilibrium point more favorable to it. That is, each firm

TABLE 10
Contribution to MacKintosh for Possible Pairs of Strategies


TABLE 11
Expected Contribution to MacKintosh and ACC (Thousand dollars)

will promote hoping that the other will respond in its own best interest which is not to promote. The result of all this will be the lowest contribution $(2,092)$ to both firms. But restriction to two promotions per year by each firm makes alternation clearly preferable since each can obtain each of the two equilibrium points half of the time. This will give rise to a contribution of $.5(2,238+2,146)=2,192$ to each. Such a strategy is clearly preferred to the promotion war $(2,092)$ or to both refraining from promotion $(2,184)$.

If there were a pronounced seasonal pattern, each firm would find it more profitable to promote in the high season since the effects of promotion on orders is multiplicative. Thus, there will remain an unresolved conflict of interest even after the firms reach an understanding with regard to alternative promotions. One way out might be to alternate not only within each year but also across years with each firm promoting in the high season one year and in the low season the following year.

## 4. CONCLUDING REMARKS

The principal purposes of the models constructed here along with other related case studies are to: (a) show how various methods can be integrated to achieve a more meaningful analysis; (b) construct models which allow potential users to easily extend various components to examine the sensitivity of results to alternative assumptions; (c) develop decision-making models which allow results to be generated on a timely basis; and (d) provide real world case studies which force students to recognize what additional information will have to be collected to complete an analysis of typical issues that often arise in agribusiness firms. Too often, decision science or managerial economic courses offered to graduate business students emphasize various solution procedures to generic problems rather than how various models that can be combined to achieve an integrative focus. Our experience in developing simple quantitative models strongly suggests that agribusiness firms can benefit significantly from the focus provided by such integration.

The first model in this case study concentrates on price forecasting and Mackintosh's procurement policies. The techniques employed are more advanced than most graduate students of business are taught, namely, the model formulation extends beyond multivariate regression analysis. The second simple model focuses on a production planning framework and is based on a linear programming formulation. The link between the first two models is the mean price forecasts which appear as coefficients in the objective function of linear programming formulation. The third model examines the timing of promotion decisions within the marketing division. The link between this model and the production planning framework is the shadow price on the production of milk chocolate--the product that is being promoted and marketed by Mackintosh. Two
alternative versions of the marketing promotion model are examined; the first in effect imposes standard competitive assumptions while the second emphasizes the game-theoretic aspects of MacKintosh's promotion timing.

## FOOTNOTES

${ }^{1}$ This trade-off in the explicitly investigated context of statistical analysis has been referred to as post-Bayesian analysis (Faden and Rausser, 1976).
${ }^{2}$ The Procurement Division has the responsibility of assessing the quality of raw cocoa utilized by Mackintosh. A numerical scale is used with a range from one to four.
${ }^{3}$ Determined from $(3.0 G+2.0 B+2.5 I) /(G+B+I)=\geq 2.7$, or $.3 G-.7 B \geq$ . 2 I instead of $.3 G-.7 B \geq 0$ as in the original problem.
${ }^{4}$ Note that $x_{1}=1$ for all observations and, thus, 2047.0 is the estimated intercept term.
${ }^{5}$ Notice that the total cost of the optimal solution, or $\$ 1,944,860$, is not simply the product of the shadow price on milk chocolate times the requirement of $2,000,000$ pounds of milk chocolate: there is a fixed element of "cost," actually negative, which would be incurred even if milk chocolate requirements were set equal to zero and which represents the difference between the costs of raw materials and the revenues generated by sales of by-products. Thus, it is not strictly correct to compute contribution as (1) total revenues less (2) the cost of the optimal solution, or $\$ 1,944,860$, less (3) $\$ 1.117$ times the difference between pounds ordered and 2,000,000 pounds.
${ }^{6}$ Note that, if neither Mackintosh nor ACC promotes in March or April, expected contribution is: $\$ 1,201,000$ in March and $\$ 1,311,000$ in April or a total of $\$ 2,512,000$ over two months. Hence, Mackintosh's promotion in March is better than neither promoting by $\$ 2,560,000-\$ 2,512,000=\$ 48,000$.
${ }^{7}$ Notice that expressing payoffs in terms of number of cases ordered fails to capture the fact that, although a promotion increases unit demand during
the month in which the promotion is held, demand generates less contribution per case in a promotion month than in a nonpromotion month. Also, it is incorrect to express payoffs in terms of revenue--it is not true that revenue generated is proportional to contribution--nor is it correct to express payoffs as the contribution on the difference between cases ordered as a result of a promotion and cases which would have been ordered if there had been no promotion: the per case contribution applies to all cases ordered.


[^0]:    ＊t－i refers to the current quarter for $1=0$ ，the previous quarter for $1-1$ ，two quarters ago for $1=2$ ，ard so on．

[^1]:    *The UNC's are reported below the estimated b's in parentheses.

[^2]:    *issintes no chansi in food component of consumer-nrice index, ounces per bar, or rulail price per bar in March or April.

