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Publication Date
1959-07-16

UCRL-8836

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BERKELEY, CALIFORNIA

UCRL-8836

UNIVERSITY OF CALIFORNIA

Lawrence Radiation Laboratory
Berkeley, California

Contract No. W-7405-eng-48

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Printed for the U. S. Atomic Energy Commission

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ABSTRACT

A simple, approximate method of incorporating the effects of a nuclear spin-orbit potential in the wave function has been developed. When used with the appropriate optical-model wave function for the central potential, it permits closed-form solutions for polarizations and cross sections that are about as simple to obtain as the Born-approximation solutions. Its application is discussed in two types of problems: the elastic scattering of nucleons by nuclei, and the "direct" interaction type of nuclear rearrangement collision. In this latter problem it is found that the elastic scattering of initial- and final-state particles from the nuclear spin-orbit potential is primarily responsible for the observed polarization effects.

* Work done under the auspices of the U.S. Atomic Energy Commission.

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1. INTRODUCTION

In recent years the theoretical description of both the elastic and the inelastic scattering of nucleons by nuclei has achieved considerable success by the use of the optical model[1] and the direct-interaction picture,[2] respectively. The aim of an earlier paper[3] (hereafter designated as I) was to combine these two models to obtain a simple description of nuclear rearrangement collisions. To accomplish this, the direct-interaction picture was modified by representing the initial- and final-state particles by elastic scattering states rather than by Born-approximation plane wave states.

In order to avoid the usual calculational difficulties of such problems, a major goal of I was to derive closed-form wave functions to represent these elastic scattering states. By means of a high-energy WKB approximation, simple forms were obtained for the wave functions that describe the scattering from the nuclear central potential. The rather important effects of including elastic scattering were shown for a particular inelastic process--the deuteron pickup reaction.

The purpose of this present note is to briefly describe the results of an analysis, similar to that of I, that incorporates the nuclear spin-orbit potential. An approximate elastic wave function that includes the effects of both the central and spin-orbit potentials is given in Section 2. A simple example that shows its utility is given in Section 3, where the polarization due to the elastic scattering of nucleons by nuclei is calculated with this wave function, and is compared with the more exact computer results. Finally in Section 4, the approximate wave functions are used to obtain the polarization of particles produced by an inelastic nuclear process--the deuteron pickup reaction considered in I.

2. THE SPIN-ORBIT WAVE FUNCTION

We consider the nonrelativistic Schroedinger equation for a nucleon of mass m and energy E_0 interacting elastically with a nucleus. Equation (18) of I can be generalized to read

$$\left(- \frac{\hbar^2}{2m} \nabla_r^2 + \mathcal{V}_c(\vec{r}) + \mathcal{U}_c(\vec{r})\, \vec{\sigma}\cdot\vec{\ell} - E_0 \right) \psi(\vec{r}, \lambda) = 0 , \tag{1}$$

where $\mathcal{V}_c$ and $\mathcal{U}_c\, \vec{\sigma}\cdot\vec{\ell}$ are the central and spin-orbit potentials, respectively, $\vec{\sigma}$ is the nucleon spin operator, and λ the nucleon spin variable. We define the form of the two potentials to be[4]

$$\mathcal{U}_c(\vec{r}) = \frac{\mu a^2}{\hbar} \frac{1}{r} \frac{\partial}{\partial r} \rho(\vec{r}) = \gamma \frac{\rho'(\vec{r})}{r} , \tag{2}$$

$$\mathcal{V}_c(\vec{r}) = \beta\, \rho(\vec{r}) , \tag{3}$$

where $\rho(\vec{r})$ is the nuclear density distribution. An approximate solution of Eq. (1) can now be obtained by considering a simplified two-dimensional solution similar to the one-dimensional WKB approximation used in I, but where the spin-orbit potential here is a function of the coordinate perpendicular to the direction of motion of the incoming nucleon. Although such a two-dimensional model is a rather crude approximation, we can test the validity of the solution by examining the extent to which it satisfies Eq. (1).

The three-dimensional form of this approximate solution turns out to be

$$\psi(\vec{r}) \cong [1 + \alpha\, f(\vec{r})\rho(r)\vec{\sigma}\cdot\vec{\ell}\,]\psi_o(\vec{r}) \,, \tag{4}$$

where α is a constant and $\psi_o(\vec{r})$ is the WKB solution for the central potential alone [See Eqs. (23), (24), and (25) of I]. The function $f(\vec{r})$ approaches unity at low energies and, provided the angular momentum $L = \hbar k R_o$ is not too large, depends only weakly on $\vec{r}$ at higher energies. We shall assume that at the energies used in the applications below, we may set $f(\vec{r}) = 1$. Now the form of Eq. (4) is made physically reasonable by considering an argument due to Fermi;[5] upon entering the nucleus, the particle feels two potentials: the central potential, which acts everywhere inside the nucleus and which changes the local wave number of the particle in accordance with the approximate WKB solution, and the spin-orbit potential, which acts only at the nuclear surface. Incoming particles feel the effect of this latter potential only while traversing the nuclear boundaries. A mathematical description of this physical effect is provided by the use of the density function $\rho(\vec{r})$ in Eq. (4).

To test the validity of Eq. (4) and find the value of α, we substitute the wave function of Eq. (4) into Eq. (1) and make the following assumptions:

(a) $\rho(r)$ falls from its maximum value to zero in a distance 2ϵ, where $\epsilon \ll R_o$, the radius of the density distribution,

(b) $\rho'(r) = \rho''(r) = 0$ for $r < R_o - \epsilon$,

(c) the average value of ρ'' from $R_o - \epsilon$ to $R_o + \epsilon$ is zero.

(d) $f(\vec{r}) = 1$, and

(e) $\left(-\frac{\hbar^2\nabla^2}{2m} + \mathcal{V}_c(\vec{r}) - E_o\right)\psi_o(\vec{r}) = 0$ where $\psi_o(\vec{r})$ is given by Eq. (24) of I.

We then obtain an approximate solution, provided

$$\alpha = \frac{2m}{\hbar} \times \frac{\gamma}{4} = \frac{m\mu a^2}{2\hbar^3} . \tag{5}$$

Using this value of α together with $\psi_o(\vec{r})$ given in I, we obtain our approximate analytic wave function for the case of a square-well potential:

$$\psi(\vec{r}) = \left(1 + \frac{m\mu a^2}{2\hbar^3} \vec{\sigma}\cdot\vec{\ell}\right) \exp\left\{i(n-1)k_o \sqrt{R_o^2 - r^2 \sin^2\theta} + i\, n\, \vec{k}_o\cdot\vec{r}\right\} \tag{6}$$

for $r < R_o$, and

$$\psi(\vec{r}) = \exp\{ i\, \vec{k}_o\cdot\vec{r} \} \tag{7}$$

for $r > R_o$. The index of refraction, n, of Eq. (6) is defined in I:

$$n^2 = 1 - \frac{\mathcal{V}_c}{E_o} . \tag{8}$$

3. ELASTIC SCATTERING POLARIZATION

The polarization of nucleons scattered elastically from nuclei has been investigated by a number of authors,[6] and the more recent theoretical results have usually been in good agreement with experiments. In this section we describe the results of using the wave function of Eq. (6) to calculate the polarization. We are not primarily interested in an exact calculation, but merely want to show the effects of the spin-orbit term. Therefore we shall keep the analysis simple and use only the <u>zero-order</u> term in the central potential, i.e., we let $n = 1$ in Eq. (6), but include the <u>higher-order</u> effects in the spin-orbit potential.

The general form of the matrix element for a spin-$\frac{1}{2}$ particle and a spin-zero nucleus is

$$f(\theta) = A(\theta) + B(\theta)\vec{\sigma}\cdot\hat{n} , \tag{9}$$

where $\hat{n}$ is a unit vector perpendicular to the plane of scattering. In our approximation, with $n = 1$, the scattering amplitude is

$$f(\theta) \sim \int_0^{R_o} d\vec{r}\, e^{-i\vec{k}_f\cdot\vec{r}} (\mathcal{V}_c + \mathcal{U}_c\, \vec{\sigma}\cdot\vec{\ell})\{1 + \alpha\, \vec{\sigma}\cdot\vec{\ell}\}\, e^{i\vec{k}_o\cdot\vec{r}} , \tag{10}$$

where $\vec{k}_o$ and $\vec{k}_f$ are the initial and final nucleon momenta, respectively, and $|\vec{k}_o| = |\vec{k}_f|$. The only difference between Eq. (10) and the usual Born-approximation analysis[7] is the addition of the term involving α, which yields results of higher order than the Born approximation only for the spin-orbit term. To show the effect of this term, we first set $\alpha = 0$ to obtain the Born-approximation amplitudes:

$$A(\theta) \sim 4\pi R_o^3 \mathcal{V}_c \frac{j_1(\Delta k R_o)}{\Delta k R_o} , \tag{11}$$

and

$$B(\theta) \sim i\,\mu\, a^2 k^2 \sin\theta \;\; 4\pi R_o^3 \frac{j_1(\Delta k R_o)}{\Delta k R_o} , \tag{12}$$

where $\Delta\vec{k} = \vec{k}_o - \vec{k}_f$ is the momentum transfer. The polarization is defined by the relation

$$P = \frac{2\,\mathrm{Re}(AB^*)}{|A|^2 + |B|^2} . \tag{13}$$

The Born approximation gives the correct polarization for small angles,[8] but does not predict the observed diffraction effects near the first zero of the spherical Bessel function, since this is a common factor to $A(\theta)$ and $B(\theta)$ and therefore cancels in Eq. (13). (See Fig. 1.) However, if we go beyond the Born approximation and use the higher-order spin-orbit term $(\alpha \neq 0)$, $B(\theta)$ will include a contribution in addition to that given in Eq. (12), of the form

$$\alpha \mathcal{V}_c \int_0^{R_o} d\vec{r} \; e^{-i\vec{k}_f\cdot\vec{r}} \; \vec{\sigma}\cdot\vec{\ell} \; e^{i\vec{k}_o\cdot\vec{r}} = -i\left(\frac{m \mathcal{V}_c}{2\hbar^2}\right)k^2 \sin\theta \;\; 4\pi R_o^3 \frac{j_2(\Delta k R_o)}{(\Delta k R_o)^2} \; . \tag{14}$$

By adding the contribution of Eq. (14) to $B(\theta)$ in Eq. (12), we obtain an expression for the polarization that exhibits diffraction effects; the angle-dependent Bessel function no longer cancels, and the polarization shows a remarkable similarity to the exact calculation. (See Fig. 1.)

We may also calculate $f(\theta)$ more exactly by using the complete wave function $\psi_o(\vec{r})$, of Eq. (6), instead of a plane wave. This calculation gives spherical Bessel functions, $j_1(\Delta k' R_o)$ and $j_2(\Delta k' R_o)$, where the argument, $\Delta k' R_o = | \vec{k}_f - n\vec{k}_o | R_o$, is complex due to the imaginary part of the index of refraction.[9] The results are in good agreement with the exact WKB calculation for the polarization in high-energy scattering problems.

It should be emphasized again that in our first example we have investigated the higher-order terms in the spin-orbit potential alone, and not the central potential. Other analyses that yield simple results for polarization effects[10] have proceeded from the opposite point of view, i.e., they include the higher-order terms in the central potential only and neglect them in the spin-orbit potential. However, as the calculation above suggests, the higher-order spin-orbit terms are not negligible. In fact, at least in the high-energy WKB approximation used here, it can be shown that the contributions from higher-order terms in each potential are of the same order of magnitude.

Although the approximate methods described above are able to yield adequate results for the scattering of nucleons in an energy region where the kinetic energy is much larger than the potential, they cannot be expected to be as generally useful as the computer calculations in which the Schroedinger equation is solved exactly. However, it is interesting that the main qualitative features of the scattering and polarization can be obtained from the very simple Born-approximation type of calculation, and that a detailed analysis is not necessary to afford at least a physical understanding of the experimental results.

4. POLARIZATION FROM INELASTIC "DIRECT" PROCESSES

We have seen the usefulness of our optical-model wave functions in the calculation and physical interpretation of elastic nuclear scattering processes. A more important application, however, is found in the description of high-energy inelastic nuclear processes for which even computer calculations become exceedingly cumbersome. The description of a rearrangement collision which proceeds via a "direct" interaction can be improved considerably over the Born approximation by the inclusion of elastic scattering effects. We require only a formal expression for the transition matrix in which the initial and final scattering states can be approximated by elastic scattering states.

For our example here, we consider the deuteron pickup reaction in which a high-energy proton enters a nucleus, picks up a neutron, and emerges as a final-state deuteron. We wish to include in our treatment the elastic nuclear scatterings of the initial-state proton and of the final-state deuteron. The formalism for use in an optical-model approximation has been derived in I, and the calculation of the differential cross section (excluding spin-orbit scattering effects) can also be found there.

In considering the polarization of final-state deuterons, we include only the effects of the nuclear spin-orbit potential on the initial and final scattering states, and we neglect the spin effects of the two-body potential through which the "direct" pickup process occurs, and also the spin effects of the nuclear potential in which the picked up neutron is initially bound. These latter effects are neglected since they lead to polarizations[11] considerably smaller than those predicted by our spin-orbit scattering effects. For a complete treatment these terms should be included, but our purpose here is to show that the spin-orbit scattering is the major factor in producing polarized deuterons.

It can be shown from invariance arguments[12] that, for two spin-$\frac{1}{2}$ particles and a spin-zero nucleus, the most general form of the transition matrix, M, is

$$M = A(\theta) + B(\theta)\vec{\sigma}_p \cdot \hat{n} + C(\theta)\vec{\sigma}_n \cdot \hat{n} + \cdots , \tag{15}$$

where $\hat{n}$ is a unit vector perpendicular to the plane defined by the momenta of the incoming proton and outgoing deuteron, and $\vec{\sigma}_p$ and $\vec{\sigma}_n$ are the spin vectors for the proton and neutron respectively. In this application higher-order terms in $\vec{\sigma}_p$ and $\vec{\sigma}_n$ are small and can be neglected.

We calculate M by replacing the optical-model wave functions ψ_0 in Eq. (36) of I by the ψ given in Eq. (6) above. The spin function for the outgoing deuteron is assumed to be characterized by the neutron spin function, in accordance with the formalism derived by Francis and Watson[13] and discussed at some length in I. The details of the calculation are not given here, but the method is straightforward and is exactly the same as that used in I, i.e., the functions in the integrands are approximated by Gaussians, which allows closed-form expressions for the integrals over momentum space.

Once we have found A, B, and C of Eq. (15), we can use the matrix notation of Wolfenstein and Ashkin[12] to evaluate the polarization. Specific applications of this formalism to deuteron problems have been investigated by Lakin[14] and Stapp.[15] However, since the tensor terms in the deuteron formalism turn out to be small for the pickup reaction, it is simpler for the purposes here to consider just the formalism for two spin-$\frac{1}{2}$ particles, and at the end to project out the states that do not give deuterons.

Following Wolfenstein,[7] we write the differential cross section for the deuterons resulting from the scattering of initially polarized protons:

$$I = \frac{1}{4} \mathrm{Tr}\, [T(MM^{\dagger})] + \frac{1}{4} P\, \vec{\mu}\cdot \mathrm{Tr}\, [T(M\vec{\sigma}_p M^{\dagger})] \quad , \qquad (16)$$

where $P\vec{\mu}$ gives the amount and direction of the initial proton polarization, and the operator T projects out the singlet spin state of the n-p system and leaves only triplet deuteron states:

$$T = \frac{3 + \vec{\sigma}_p \cdot \vec{\sigma}_n}{4} \quad . \qquad (17)$$

For the specific case we wish to compare with experiment, we assume that the initial proton polarization is in a direction normal to the plane of scattering. Then Eq. (16) becomes

$$I = \frac{3}{4}(|A|^2 + |B|^2 + |C|^2) + \frac{1}{2}\mathrm{Re}(BC^*) + P\left\{\mathrm{Re}[A(\frac{3}{2}B^* + \frac{1}{2}C^*)]\right\}\cos\phi \quad , \tag{18}$$

and the left-right asymmetry is

$$\epsilon = \frac{\mathrm{Re}\,[A(\frac{3}{2}B^* + \frac{1}{2}C^*)]}{\frac{3}{4}(|A|^2 + |B|^2 + |C|^2) + \frac{1}{2}\mathrm{Re}(BC^*)} \tag{19}$$

A comparison of the results obtained by using Eq. (19) with recent experiments from the bombardment of C^{12} by 145-Mev protons[16] is given in Fig. 2. The values for the optical-model parameters for the central potential are the same as those tabulated in I, and the spin-orbit parameter chosen is $\mu a^2 = 5 \times 10^{-26}$ Mev-cm^2, which is the value used in Reference 4.

It can be seen that the main features of the data are predicted by our theory. Particularly, it is clear that the major contribution to the polarization must come from the spin-orbit scatterings, since the results obtained by considering just the effect of the initial state of the bound neutron[11] lead to a maximum asymmetry of 1/3 , which is much too small to account for the experimental results.

Because of the simplicity of the wave functions used, the analysis here allows a qualitative explanation of the form of the observed polarization, as it also did for the elastic-scattering case of Section 3. The increasing positive slope for small angles can be understood from the fact that the effects of the optical potential (spin-orbit elastic scattering) become increasingly important as the angle increases away from the forward direction. A maximum in the polarization is reached, however, since for very large angles, inelastic initial- and final-state scatterings (which are not considered in the theory here) contribute to the cross section.

ACKNOWLEDGMENTS

The author is deeply indebted to Professor Kenneth M. Watson for his support and many helpful suggestions during the course of this work. Particular thanks are also due Dr. Henry Stapp for many stimulating discussions.

FIGURES

Figure 1: The polarization from the elastic scattering of 300-Mev protons on aluminum. The solid curve is the Born approximation and the dashed curve shows the results of the simplified calculation in the text. The nuclear radius was chosen as $1.3\ A^{1/3} \times 10^{-13}$ cm. Coulomb effects have been neglected in both these curves. The dot-and-dash curve shows the results of the exact computer calculation of Bjorklund et al. (Reference 6).

Figure 2: The asymmetry of outgoing deuterons from the pickup reaction on carbon. The smooth curve gives the results of the calculation in Section 4.

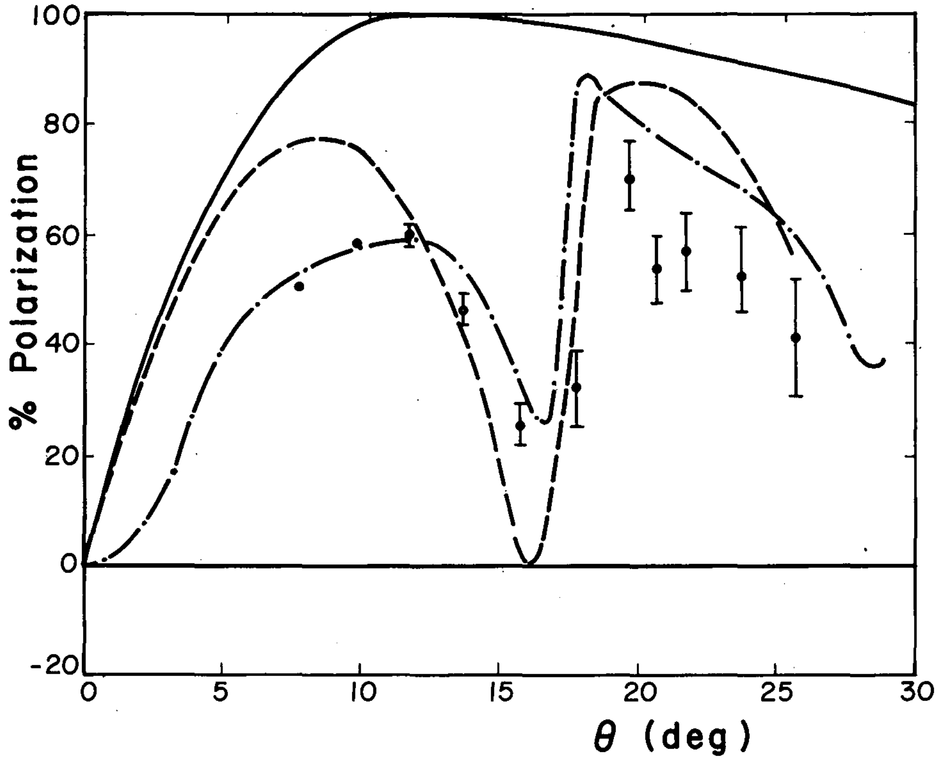

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Fig. 1.

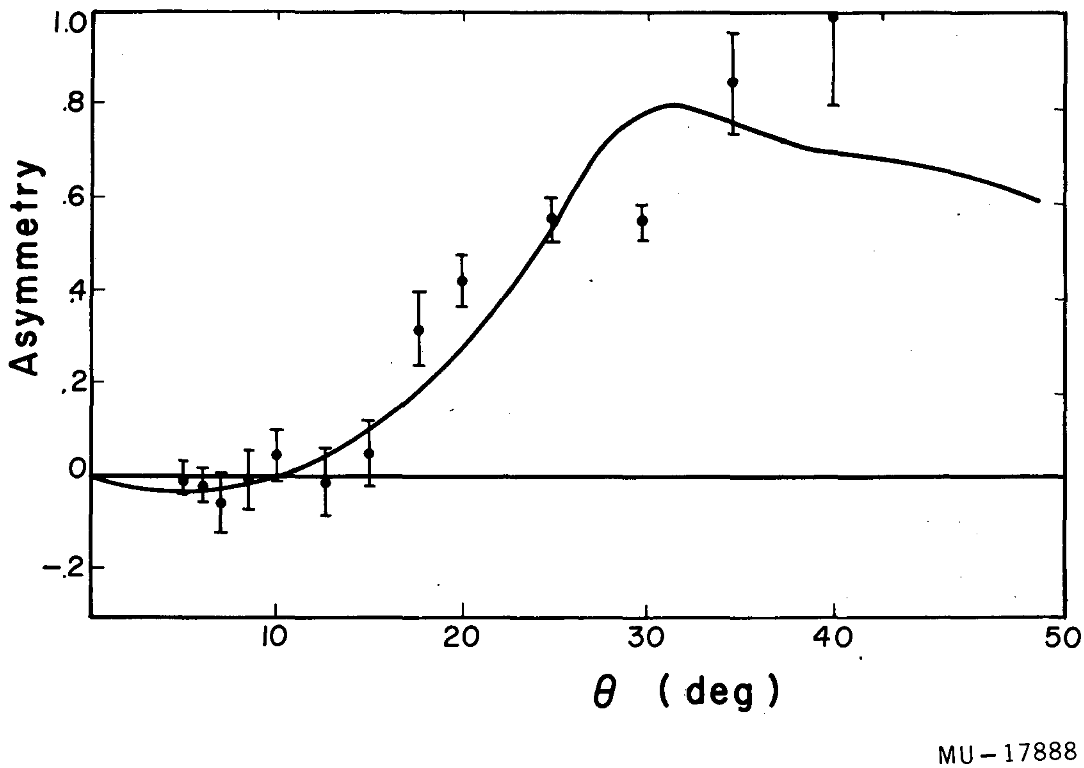

Fig. 2.

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