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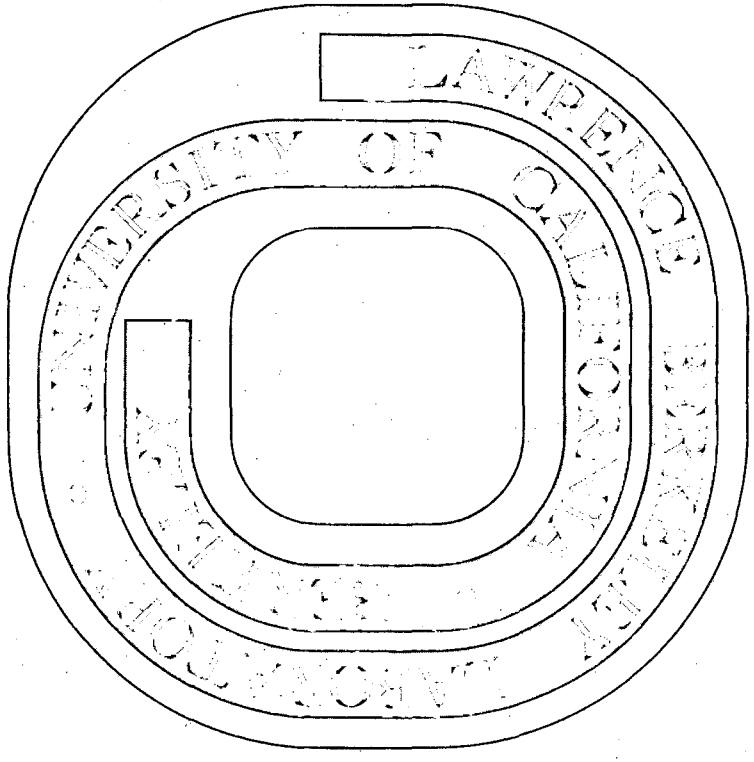

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ABSTRACT

We show that, within the framework of the dual resonance models, it is possible to find a scale transformation on the Chan variables that generates the Bjorken scaling law in the deep inelastic lepton-hadronic scatterings and the Feynman scaling law in the hadronic inclusive reactions. Applying the similar scale transformations to the exclusive production processes, we show that the invariant amplitude, for both the lepton-hadronic and the pure hadronic exclusive reactions, are not scaled, but the nonscaled parts are factorizable from the scaled parts. We then conclude that the hadronic cross sections at fixed multiplicities as well as the lepton-hadronic structure functions at fixed multiplicities, are not scaled in general.

I. INTRODUCTION

Two famous scaling laws, one suggested by Bjorken in the deep-inelastic lepton-hadronic reactions and the other conjectured by Feynman in the hadronic inclusive reactions, have been widely discussed by many authors. This paper aims at the explanations of the origins of these two scaling laws within the framework of the dual resonance model (and the parton dual resonance model). We give a unified treatment of the Bjorken's scaling law and the Feynman's scaling law, and show that they in fact stem from similar origins.

The crucial keys to the explanations are contained in the correct choices of the scale transformations,[1] which are uniquely determined by the physically required discontinuities. By the physically required discontinuities, we mean the discontinuities across the missing mass square variables, to the strict exclusions of all extra pieces of imaginary parts contributed from all other channels. This strict exclusion uniquely fixes the scale transformations, which gives rise to the scaling behaviors.

In Sec. II, we use the dual resonance amplitude to derive the generalized Feynman's scaling law, and use the parton dual resonance model[2] to derive the generalized Bjorken scaling law. They result in explicit formulas. In Sec. III, we apply the same scale transformations to the exclusive production processes, and show that the invariant amplitudes are not scaled, but the nonscaled parts are universally known. As consequences of these nonscaling behaviors, it is argued that the cross sections at fixed multiplicities and the structure functions at fixed multiplicities, are not scaled in general. In Sec. IV, we conclude the possible multiplicity distributions in the

deep inelastic lepton-hadronic collisions as well as in the hadron-hadronic reactions.

II. THE ORIGINS OF THE SCALING LAWS IN THE INCLUSIVE REACTIONS

A. The Generalized Feynman's Scaling Law

Consider the generalized hadronic inclusive reaction

$$h_1 + h_2 \rightarrow h_3 + h_4 + \cdots + h_n + \text{anything}. \qquad (1)$$

Since we use the optical theorem to construct a model for the cross section, we approximate the "anything" (= missing masses) by the resonances, and take the absolute square of the amplitude. Hence we get a 2n-point function (with the important $i\epsilon$-prescription however). We then take the imaginary part in the missing mass square variable $s_{1n} \equiv (k_1 + k_2 + \cdots + k_n)^2$ so that the cross section is given by the formula (Fig. 1):

$$s_{12}\, d\sigma \Big/ \left(\prod_{i=3}^{n} d^3k_i/k_{0i} \right) \approx \frac{G}{\pi} \operatorname{Im} s_{1n}\, \bar{B}_{2n}(k_1, k_2, \cdots, k_n;\ -k_n, \cdots, -k_2, -k_1), \qquad (2)$$

where

$$\begin{aligned} s_{12} &\equiv (k_1 + k_2)^2, \quad \text{(incident energy)}, \\ s_{1n} &\equiv (k_1 + \cdots + k_n)^2, \quad \text{(missing mass square)}, \end{aligned} \qquad (3)$$

and the standard 2n-point function is

$$\bar{B}_{2n} = \int_0^1 \prod_{i=1}^{n-2} \frac{dx_i\, dy_i\, dz}{(1 - x_i)(1 - y_i)(1 - z)} \prod_{i=1}^{n-2} (x_i y_i)^{-\alpha_{1,i+1}(s_{1i+1})-1}$$

$$\times\ z^{-\alpha_{1,n}(s_{1n})-1} (1 - x_1 \cdots x_{n-2} z y_{n-2} \cdots y_1)^{-\alpha_{1,2n}(0)}$$

$$\times \prod_{2 \le i \le j \le 2n-1}{}' U_{ij}^{-\alpha_{ij}(s_{ij})}, \qquad (4)$$

with

$$k_i \equiv -k_{2n+1-i}, \tag{5a}$$

$$s_{ij} \equiv (k_i + k_{i+1} + \cdots + k_j)^2, \quad s_{ij} = s_{2n+1-j,2n+1-i}, \tag{5b}$$

$$U_{ij} = \frac{(1 - x_{i-1}\cdots x_{j-2})(1 - x_{i-2}\cdots x_{j-1})}{(1 - x_{i-2}\cdots x_{j-2})(1 - x_{i-1}\cdots x_{j-1})}, \quad 2 \le i < j \le n,$$

$$= \frac{(1 - y_{2n-1-i}\cdots y_{2n-j})(1 - y_{2n-i}\cdots y_{2n-1-j})}{(1 - y_{2n-i}\cdots y_{2n-j})(1 - y_{2n-1-i}\cdots y_{2n-1-j})},$$

$$n + 1 \le i < j \le 2n - 1, \tag{5c}$$

$$= \frac{(1 - x_{i-1}\cdots x_{n-2} z y_{n-2}\cdots y_{2n-j})(1 - x_{i-2}\cdots z \cdots y_{2n-1-j})}{(1 - x_{i-2}\cdots x_{n-2} z y_{n-2}\cdots y_{2n-j})(1 - x_{i-1}\cdots z \cdots y_{2n-1-j})},$$

$$2 \le i \le n,$$

$$n + 1 \le j \le 2n-1,$$

$$x_0 = y_0 \equiv 0, \qquad x_{n-1} = y_{n-1} \equiv z. \tag{5d}$$

The symbol Π' means that we delete the channel $\alpha_{2,2n-1} = \alpha_{1,2n}$. The $i\epsilon$-prescription states that all the invariant variables belonging to the right-hand side of Fig. 1, must be analytically continued from their cuts in opposite directions to those belonging to the left-hand side.

We are interested in the generalized Feynman scaling limit, defined by

incident energy: $s_{12} \to +\infty$

momenta transfer: s_{ij} = fixed, $2 \le i < j \le n$, (6)

scaling variables: $\omega_i \equiv \dfrac{-t_{1i}}{s_{12}}$ = fixed > 0,

$$t_{1i} \equiv (k_1 + k_i)^2 < 0, \quad i = 3,\cdots n.$$

Under this limit, we see that $s_{1i} \equiv (k_1 + k_2 + \cdots + k_i)^2 \approx$ $\approx s_{12}(1 - \omega_3 - \omega_4 \cdots - \omega_i)$, which approach to $+\infty$ in the same order as s_{12}, if $\omega_3 + \omega_4 + \cdots + \omega_i < 1$. The single particle inclusive reaction $h_1 + h_2 \to h_3 +$ anything, is the case $n = 3$, so the Feynman's scaling variable is $x \equiv 2p_{\parallel}^{c}/(s)^{\frac{1}{2}}$.

To take the scaling limit (6) in Eq. (4), we can first consider the limits $s_{12}, s_{13}, \cdots s_{1n} \to -\infty$, where the 2n-point function is convergent. In this region all inequalities $\omega_i < 0$ and $\omega_3 + \omega_4 + \cdots + \omega_i < 1$ are satisfied automatically. We then make a scale transformation, after which, we can rotate $s_{12}, s_{13}, \cdots s_{1n}$ to $+\infty$, and take the imaginary part in s_{1n}. Because $s_{12}, s_{13}, \cdots s_{1n-1}$ approach to $+\infty$ at the same time as s_{1n} does, the extra pieces of imaginary parts contributed from $s_{12}, s_{13}, \cdots s_{1n-1}$ are going to be mixed up with the physically required discontinuity in s_{1n}. It is the necessity of disentangling the unwanted extra pieces of imaginary parts in $s_{12}, \cdots s_{1n-1}$ that forces us to choose the scale transformation in Eq. (9).

We first write Eq. (3) as

$$\bar{B}_{2n} = \int_0^\infty \prod_{i=1}^{n-2} \frac{d(\ell n \frac{1}{x_i})\, d(\ell n \frac{1}{y_i})\, d(\ell n \frac{1}{z})}{(1 - x_i)(1 - y_i)(1 - z)} \prod_{i=1}^{n-1} (x_i y_i)^{-\alpha_{1,i+1}(0)}$$

$$\times\ z^{-\alpha_{1,n}(0)} (1 - \prod_{i=1}^{n-2} x_i y_i z)^{-\alpha_{1,2n}(0)}$$

$$\times \prod_{2\le i<j\le 2n-1}{}' U_{ij}^{-\alpha_{ij}(s_{ij})} \exp\left\{ s_{1n}\left[\ell n \frac{1}{z} + \sum_{i=1}^{n-2} \left(\frac{s_{1i+1}}{s_{1n}}\right) \ell n \left(\frac{1}{x_i y_i}\right)\right]\right\} \tag{7}$$

As $s_{12}, s_{13}, \cdots s_{1n} \to -\infty$, the important region that contributes to the $(2n - 3)$ dimensional integral is when $\ell n \frac{1}{x_i}$, $\ell n \frac{1}{y_i}$, $i = 1,\cdots n-2$ and $\ell n \frac{1}{z}$ are small. We then make the scale transformation[3]

$$\begin{aligned} \ell n \frac{1}{x_i} &= \kappa_i \rho\, \alpha_i, \\ \ell n \frac{1}{y_i} &= \kappa_i \rho\, \alpha_i', \qquad \kappa_i > 0, \\ \ell n \frac{1}{z} &= \rho \left[1 - \sum_{i=1}^{n-2} (\alpha_i + \alpha_i') \right], \end{aligned} \tag{8}$$

with

$$1 - \sum_{i=1}^{n-2} (\alpha_i + \alpha_i') \ge 0, \tag{9}$$

$$\kappa_i \equiv \frac{s_{1n}}{s_{1i+1}} \approx \frac{1 - \omega_3 - \cdots - \omega_n}{1 - \omega_3 - \cdots - \omega_{i+1}}, \quad i = 1,2,\cdots,n-2.$$

We further expand $1 - x_i \approx \rho\, \kappa_i\, \alpha_i$, $1 - y_i \approx \rho\, \kappa_i\, \alpha_i'$, and $1 - z \approx \rho[1 - \sum(\alpha_i + \alpha_i')]$ in Eq. (7). Then we find the ρ integral is

$$\int_0^\infty d\rho\, \rho^{-1-\alpha_{1,2n}(0)} \exp(s_{1n}\rho) = \Gamma\left(-\alpha_{1,2n}(0)\right) (s_{1n})^{\alpha_{1,2n}(0)}. \tag{10}$$

Equation (10) has a cut if $s_{1n} > 0$, and this is precisely the physical cut corresponding to the imaginary part which we required.[4] Therefore, we see that because of the choice of the scale transformation Eq. (8), which is scaled in the sense of ratios of s_{1i}, that we have successfully isolated the physical discontinuity from the other unwanted cuts.

We now can analytically continue s_{1i}, $i = 2,\cdots,n-1$, to $+\infty \pm i\epsilon$ so that the variables s_{1i}, $i = 2,\cdots n-1$, in the right-hand side channels (in Fig. 1) approach $+\infty + i\epsilon$, but those in the left-hand side channels approach $+\infty - i\epsilon$, while s_{1n} approaches $+\infty \pm i\epsilon$. We then take imaginary part of the amplitude. From Eq. (10), the imaginary part is

$$\frac{1}{\Gamma\left(\alpha_{1,2n}(0) + 1\right)} (s_{1n})^{\alpha_{1,2n}(0)}, \quad (\text{for } s_{1n} \to +\infty). \tag{11}$$

Hence from Eqs. (7), (8), (10), (11), and (2), we get[5]

$$s_{12}\, d\sigma \Big/ \prod_{i=3}^{n} d^3 k_i \Big/ k_{0i} \approx \frac{G}{\pi\, \Gamma\left(\alpha_{1,2n}(0) + 1\right)} (s_{1n})^{\alpha_{1,2n}(0)}$$

$$\times \int_0^{Z\ge 0} \prod_{i=1}^{n-2} \left(\frac{d\alpha_i\, d\alpha_i'}{\alpha_i \alpha_i'}\right) \frac{[Z + \sum \kappa_i (\alpha_i + \alpha_i')]^{-\alpha_{1,2n}(0)}}{Z}$$

$$\times \prod_{2\le i\le n-1} \left[\frac{(Y_{i-1})(Y_{i-2} + Z)}{(Y_{i-2})(Y_{i-1} + Z)} (\alpha \leftrightarrow \alpha') \right]^{-\alpha(s_{in})}$$

Equation (12) continued

$$\times \prod_{2\le i\le n-1} \left[\frac{(Y_{i-1}+Z+Y'_j)(Y_{i-2}+Z+Y'_{j-1})}{(Y_{i-2}+Z+Y'_j)(Y_{i-1}+Z+Y'_{j-1})}\ (\alpha \leftrightarrow \alpha')\right]^{-\alpha(k_i)}$$

$$\times \prod_{2\le i<j\le n-1} \left[\frac{(\kappa_{i-1}\alpha_{i-1}+\cdots+\kappa_{j-2}\alpha_{j-2})(\kappa_{i-2}\alpha_{i-2}+\cdots+\kappa_{j-1}\alpha_{j-1})}{(\kappa_{i-2}\alpha_{i-2}+\cdots+\kappa_{j-2}\alpha_{j-2})(\kappa_{i-1}\alpha_{i-1}+\cdots+\kappa_{j-1}\alpha_{j-1})}\ (\alpha \leftrightarrow \alpha')\right.$$

$$\left. \times \frac{(Y_{i-1}+Z+Y'_j)(Y_{i-2}+Z+Y'_{j-1})}{(Y_{i-2}+Z+Y'_j)(Y_{i-1}+Z+Y'_{j-1})}\ (\alpha \leftrightarrow \alpha')\right]^{-\alpha(s_{ij})}$$

$$\times \prod_{3\le j\le n}$$

$$\times \left\{ \frac{\left[Z+\sum_{i=j-2}^{n-2}\kappa_i(\alpha_i+\alpha'_i)\right]\left[Z+\sum_{i=j-1}^{n-2}\kappa_i(\alpha_i+\alpha'_i)\right]}{\left[Z+\kappa_{j-2}\alpha_{j-2}+\sum_{i=j-1}^{n-2}\kappa_i(\alpha_i+\alpha'_i)\right]\left[Z+\kappa_{j-2}\alpha'_{j-2}+\sum_{i=j-1}^{n-2}\kappa_i(\alpha_i+\alpha'_i)\right]} \right\}^{-\alpha_{j,2n+1-j}(0)}, \tag{12}$$

where $\alpha_{j,2n+1-j}(0)$ have the quantum number of the vacuum,

$$Z \equiv 1 - \sum_{i=1}^{n-2}(\alpha_i+\alpha'_i) \equiv \kappa_{n-1}\alpha'_{n-1} \equiv \kappa_{n-1}\alpha_{n-1},$$

$$\kappa_0\alpha_0 + \text{anything} \equiv \kappa_0\alpha'_0 + \text{anything} \equiv 1, \tag{13}$$

$$Y_i \equiv \kappa_i\alpha_i + \kappa_{i+1}\alpha_{i+1} + \cdots + \kappa_{n-2}\alpha_{n-2},$$

$$Y'_i \equiv \kappa_{n-2}\alpha'_{n-2} + \kappa_{n-3}\alpha'_{n-3} + \cdots + \kappa_i\alpha'_i.$$

We immediately observe that, if $\alpha_{1,2n}(0) \equiv 1$, then (a) Eq. (12) approaches a limitation distribution, (b) the cross section is scaled

as a function of $\kappa_1,\cdots\kappa_{n-2}$ and the momenta transfers s_{ij}, $2 \le i < j \le n$.

Let us briefly rederive the formula for the single particle case[5] $h_1 + h_2 \to h_3 + \text{anything}$. Taking $n = 3$ in Eq. (12), we get

$$s_{12}d\sigma/d^3k_3/k_{03} \approx \frac{G}{\pi\Gamma(\alpha_{16}(0)+1)}(s_{13})^{\alpha_{16}(0)}\int_0^1 d\alpha_1\int_0^{1-\alpha_1} d\alpha'_1$$

$$\times \frac{[1-\alpha_1-\alpha'_1+\kappa_1(\alpha_1+\alpha'_1)]^{-\alpha_{16}(0)}}{\alpha_1\alpha'_1(1-\alpha_1-\alpha'_1)}$$

$$\times \left[\frac{(\kappa_1\alpha_1)(\kappa_1\alpha'_1)}{(\kappa_1\alpha_1+1-\alpha_1-\alpha'_1)(\kappa_1\alpha'_1+1-\alpha_1-\alpha'_1)}\right]^{-\alpha(s_{23})}$$

$$\times \left[\frac{(\kappa_1\alpha_1+1-\alpha_1-\alpha'_1)(\kappa_1\alpha'_1+1-\alpha_1-\alpha'_1)}{(\kappa_1\alpha_1+\kappa_1\alpha'_1+1-\alpha_1-\alpha'_1)^2}\right]^{-\alpha(k_2)}$$

$$\times \left[\frac{(1-\alpha_1-\alpha'_1)(1-\alpha_1-\alpha'_1+\kappa_1(\alpha_1+\alpha'_1))}{(\kappa_1\alpha_1+1-\alpha_1-\alpha'_1)(\kappa_1\alpha'_1+1-\alpha_1-\alpha'_1)}\right]^{-\alpha_{34}(0)}, \tag{14}$$

with

$$\kappa_1 \equiv \frac{s_{13}}{s_{12}} \approx 1-\omega_3 = 1-x > 0, \tag{15}$$

$$s_{23} \approx \frac{(1-x)^2}{x}M^2 - \frac{1}{x}p_\perp^2.$$

We can directly read off the predictions ($\alpha_{16} \equiv 1$):

$$\text{(a)}\quad d\sigma/d^3k_3/k_{03} \approx g(\kappa_1, s_{23}) = f(x, p_\perp^2), \quad \text{(limit fragmentation)},$$

$$\text{(b)}\quad x \to 1, \quad d\sigma/d^3k_3/k_{03} \approx (1-x)^{-2\alpha(s_{23})+1} f(s_{23}), \quad \text{(triple-reggeon limit)}, \tag{16}$$

$$\text{(c)}\quad x \to 0, \quad d\sigma/d^3k_3/k_{03} \underset{p_\perp^2 \gg M^2}{\approx} c\, x^{-\alpha_{34}+1} p_\perp^{-5} \exp(-4p_\perp^2), \quad \text{(pionization limit)}.$$

Hence the representation, Eq. (14), for the single particle inclusive reaction is completely in agreement with previous work.[5]

One can similarly study the two-particle case, $n = 4$. Other workers have already studied this case, we will not elaborate here.

B. The Generalized Bjorken Scaling Law

Consider the reactions

$$\begin{aligned} \ell + h_n &\to \ell + h_3 + h_4 + \cdots + h_{n-1} + \text{anything}, \\ \ell + \bar{\ell} &\to h_3 + h_4 + \cdots + h_{n-1} + \bar{h}_n + \text{anything}. \end{aligned} \tag{17}$$

Isolate the strong interaction parts, we have

$$\begin{aligned} \text{"}\gamma\text{"} + h_n &\to h_3 + h_4 + \cdots + h_{n-1} + \text{anything}, \\ \text{"}\gamma\text{"} &\to h_3 + h_4 + \cdots + h_{n-1} + \bar{h}_n + \text{anything}. \end{aligned} \tag{18}$$

We use the parton dual resonance model[2] to obtain a formula for the generalized virtual forward Compton scattering

$$T_{\mu\nu}^{(i)} = \int d^4k_2\, d^4k_{2n-1} \times \frac{\kappa_{\mu\nu}^{(i)}\, \bar{B}_{2n}(q-k_2, k_2, k_3, \cdots k_n; -k_n, \cdots -k_3, -k_{2n-1}, -q-k_{2n-1})}{(k_2^2-m^2)[(k_2-q)^2-m^2](k_{2n-1}^2-m^2)[(q+k_{2n-1})^2-m^2]}. \tag{19}$$

After performing the two-loop momenta integrals, we get[2,6]

$$T_{\mu\nu}^{(i)} = \int_0^\infty da_2\, da_{2n-1}\, da_1\, da_{2n} \prod_{i=1}^{n-2} d(\ell n \tfrac{1}{x_i})\, d(\ell n \tfrac{1}{y_i})\, d(\ell n \tfrac{1}{z})$$

$$\times\ \kappa_{\mu\nu}^{(i)} \frac{\pi^4}{C^2}\{G\} \exp(-J)$$

$$\times\ \exp\left\{q^2\left[\ell n \frac{1}{x_1 y_1} + \sum_{i=2}^{n-2}\left(\frac{s_{i+1}}{q^2}\right)\ell n\left(\frac{1}{x_i y_i}\right) + s_n\, \ell n \frac{1}{z} + (a_1+a_{2n})F(\omega's)\right]\right\}, \tag{20}$$

where $\kappa_{\mu\nu}^{(i)}$, C, $\{G\}$, J, and $F(\omega's)$ have explicit forms. Here we only mention that J is a function of the momenta transfers $s_{ij} \equiv (k_i + k_{i+1} + \cdots + k_j)^2$, $3 \le i < j \le n$, and $F(\omega's)$ is a function of the scaling variables $\omega_i \equiv (2k_i \cdot q)/(-q^2)$, $i = 3, \cdots, n$. Also in Eq. (20), $s_i \equiv (q + k_3 + \cdots + k_i)^2$, $s_n = (q + k_3 + \cdots + k_n)^2$ = missing mass square, and the incident energy is $s \equiv (q + k_n)^2$; (for $q^2 < 0$).

We are interested in the limit $q^2 \to -\infty$, $s_i \to -\infty$, $i = 3, \cdots, n$. In this limit, all $\omega_i < 0$, and so $F(\omega's)$ is positive definite, hence all terms in the last exponent of Eq. (20) are negative definite.

To choose the correct scale transformation, we observe that

(a) the parton is nonobservable, therefore we must avoid the imaginary part contributed from $F(\omega's)$,

(b) the physically correct discontinuity should be taken across s_n only, hence we need to transform away all undesired pieces contributed from q^2, s_i, $3 \le i \le n-1$.

Thus, we have to perform the scale transformation

$$
\begin{aligned}
a_1' &= a_1\, F(\omega's)\ , \\
a_{2n}' &= a_{2n}\, F(\omega's), \\
\ell n\,\frac{1}{x_1} &= \rho\,\beta_1, \quad \ell n\,\frac{1}{y_1} = \rho\,\beta_1', \\
\ell n\,\frac{1}{x_i} &= \left(\frac{q^2}{s_{i+1}}\right)\rho\,\beta_i, \quad \ell n\,\frac{1}{y_i} = \left(\frac{q^2}{s_{i+1}}\right)\rho\,\beta_i', \quad i = 2,\cdots,n-2, \\
\ell n\,\frac{1}{z} &= \rho\,\beta_{n-1}, \\
a_1' &= \rho\,\beta_n, \\
a_{2n}' &= \rho(1 - \beta_1 - \beta_1' - \cdots - \beta_n), \quad 1 - \beta_1 - \beta_1' - \cdots - \beta_n \geq 0.
\end{aligned}
\tag{21}
$$

We then expand everything else in terms of ρ and β_i's. The ρ integral is

$$
\int_0^\infty d\rho\ \rho^{1-\alpha_{1,2n}(0)}\ \exp\left\{q^2\left[1 - \left(1 - \frac{s_n}{q^2}\right)\beta_{n-1}\right]\right\}
$$

$$
= \Gamma(2 - \alpha_{1,2n}(0))\left\{\frac{1}{q^2\left[1 - \left(1 - \frac{s_n}{q^2}\right)\beta_{n-1}\right]}\right\}^{2-\alpha_{1,2n}(0)} \cdot \tag{22}
$$

Equation (22) is analytic in s_n if $s_n < 0$, since then $1 - (1 - \frac{s_n}{q^2})\beta_{n-1} > 0$. It has a cut in s_n if $1 < (1 - \frac{s_n}{q^2})\beta_{n-1}$, i.e., if $s_n >$ threshold. To get the scale invariant result, we set[7] $\alpha_{1,2n}(0) \equiv 1$. So, we see that, apart from a factor $(q^2)^{-1}$ in Eq. (22) and $\ell n|q^2|$ in C, our result now will be functions of s_i/q^2 and the

momenta transfers s_{ij}, $3 \leq i < j \leq n$,—which is the generalized Bjorken scaling law.

Now we keep q^2, s_3, s_4,$\cdots$ s_{n-1} fixed at $-\infty$ but analytically continue s_n to $+\infty \pm i\epsilon$. Taking its imarginary part is then trivial; we simply put $\beta_{n-1} = (1 - \frac{s_n}{q^2})^{-1}$ in Eq. (20)[after the scale transformation (21), of course], together with the Θ-function constraint $\Theta[1 - \beta_1 - \beta_1' - \cdots - \beta_{n-2}' - (1 - \frac{s_n}{q^2})^{-1}]$. We can further absorb[2,6] the Θ-function constraint by making the change of variables such that the range of integrations of β_i is unchanged

$$
\begin{aligned}
\beta_i &= \left[1 - \left(1 - \frac{s_n}{q^2}\right)^{-1}\right]\alpha_i, \\
\beta_i' &= \left[1 - \left(1 - \frac{s_n}{q^2}\right)^{-1}\right]\alpha_i', \quad i = 1,2,\cdots n-2.
\end{aligned}
\tag{23}
$$

We further define

$$
\tau_{ij} \equiv \frac{2k_i\cdot k_j}{-q^2}\ , \qquad 3 \leq i < j \leq n,
$$

$$
y_1 \equiv \frac{s_n}{-q^2} \approx \omega_3 + \omega_4 + \cdots + \omega_n + \sum_{3\leq i<j\leq n} \tau_{ij} - 1, \tag{24a}
$$

$$
y_i = \frac{s_n}{-s_{i+1}} \approx \frac{y_1}{\left(1 - \omega_3 - \cdots - \omega_{i+1} - \sum_{3\leq \ell<j\leq i+1} \tau_{\ell j}\right)}\ ,
$$

$$
i = 2,\cdots n - 2,
$$

and

$$
\begin{aligned}
[(k_\ell + k_j)\cdot q]W_2^{(i)}(\ell j) &\to 2M\, F_2^{(i)}(\ell j), \quad \ell,j = 3,\cdots n, \\
W_1^{(i)} &\to F_1^{(i)}, \qquad i = 1,2,
\end{aligned}
\tag{24b}
$$

with

$$W_{\mu\nu}^{(i)} = \frac{1}{2}\sum_{\ell,j=3}^{n}\left[\left(k_\ell - \frac{k_\ell\cdot q}{q^2}q\right)_\mu\left(k_j - \frac{k_j\cdot q}{q^2}q\right)_\nu + \left(k_\ell - \frac{k_\ell\cdot q}{q^2}q\right)_\nu\left(k_j - \frac{k_j\cdot q}{q^2}q\right)_\mu\right]\frac{W_2^{(i)(\ell j)}}{M^2} - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)W_1^{(i)}$$

$$= \mathrm{Im}\; s_n\, T_{\mu\nu}^{(i)} \,. \qquad (25)$$

We finally arrive at the generalized Bjorken scaling results for all structure functions $(k,\ell = 3,\cdots,n)$

$$\left\{\begin{array}{l} F_1^{(1)} \\ F_1^{(2)} \\ F_2^{(1)}(k\ell) = F_2^{(2)}(k\ell) \end{array}\right\} = \frac{\pi^\ell}{\ell n|q^2|}\int_0^{1-\sum(\alpha_i+\alpha'_i)}\prod_{i=1}^{n-2}\left(\frac{d\alpha_i\, d\alpha'_i}{\alpha_i\alpha'_i}\right)$$

$$\times \int_d^\infty d\alpha\left(1-\frac{d}{\alpha}\right)\left\{\begin{array}{l} \frac{2}{\alpha q^2} \\ 1 \\ \frac{2M}{\alpha^2}(\omega_k+\omega_\ell)\,\ell n\, Z_k\;\ell n\, Z_\ell \end{array}\right\}$$

$$\times \frac{1}{\left\{1-\frac{1}{\alpha}\left(\sum_{i=3}^{n}\omega_i\;\ell n\, Z_i\right)\right\}^2}\exp\left[-m^2(\alpha-d) - \frac{1}{\alpha}\left(\sum_{i=3}^{n}\omega_i\;\ell n\, Z_i\right)^2\right]$$

Equation (26a) continued

Equation (26a) continued

$$\times\left\{\frac{[1-\sum(\alpha_i+\alpha'_i)]y_i}{[1+\sum y_i(\alpha_i+\alpha'_i)]}\left[\frac{\left(1+\sum y_i(\alpha_i+\alpha'_i)\right)^2}{\alpha_1 y_1\alpha'_1 y_1}\right]^{\alpha_{23}}\right.$$

$$\times\left[\frac{\left(1+y_1\alpha_1+\sum_{i=2}^{n-2}y_i(\alpha_i+\alpha'_i)\right)\left(1+y_1\alpha'_1+\sum_{i=2}^{n-2}y_i(\alpha_i+\alpha'_i)\right)}{\alpha_1 y_1\alpha'_1 y_1}\right]^{-\alpha_0}$$

$$\times\prod_{i=4}^{n}\left[\frac{(y_1\alpha_1+\cdots+y_{i-2}\alpha_{i-2})(Y_1+1+Y'_i)}{(y_1\alpha_1+\cdots+y_{i-1}\alpha_{i-1})(Y_1+1+Y'_{i-1})}(\alpha\longleftrightarrow\alpha')\right]^{-\alpha(s_{3i})}$$

$$\times\prod_{3\le i<j\le n}\left[\frac{(y_{i-1}\alpha_{i-1}+\cdots+y_{j-2}\alpha_{j-2})(y_{i-2}\alpha_{i-2}+\cdots+y_{j-1}\alpha_{j-1})}{(y_{i-2}\alpha_{i-2}+\cdots+y_{j-2}\alpha_{j-2})(y_{i-1}\alpha_{i-1}+\cdots+y_{j-1}\alpha_{j-1})}\right.$$

$$\left.\left.\times(\alpha\longleftrightarrow\alpha')\frac{(Y_{i-1}+1+Y'_j)(Y_{i-2}+1+Y'_{j-1})}{(Y_{i-2}+1+Y'_j)(Y_{i-1}+1+Y'_{j-1})}(\alpha\longleftrightarrow\alpha')\right]^{-\alpha(s_{ij})}\right.$$

$$\left.\times\prod_{i=1}^{n-2}\left[\frac{\left(1+\sum_{j=i+1}^{n-2}y_j(\alpha_j+\alpha'_j)\right)\left(1+\sum_{j=i}^{n-2}y_j(\alpha_j+\alpha'_j)\right)}{\left(1+y_i\alpha_i+\sum_{j=i+1}^{n-2}y_j(\alpha_j+\alpha'_j)\right)\left(1+y_i\alpha'_i+\sum_{j=i+1}^{n-2}y_j(\alpha_j+\alpha'_j)\right)}\right]^{-\alpha_t(0)}\right\} \qquad (26a)$$

where

$$y_{n-1}\,\alpha_{n-1} \equiv y_{n-1}\,\alpha'_{n-1} \equiv 1$$

$$Y_i \equiv y_i\alpha_i + y_{i+1}\alpha_{i+1} + \cdots + y_{n-2}\alpha_{n-2}, \quad i \le n-2,$$

$$Y'_i \equiv y_{n-2}\alpha'_{n-2} + y_{n-3}\alpha'_{n-3} + \cdots + y_i\alpha'_i, \quad i \le n-3,$$

$$1 + Y'_i \equiv 1, \quad \text{if } i > n-2,$$

Equation (26b) continued

Equation (26b) continued

$$d \equiv \ell n \left[\frac{1 + \sum y_i(\alpha_i + \alpha_i')}{\alpha_1 y_1 \alpha_1' y_1} \right], \tag{26b}$$

$$Z_i = \frac{(Y_1 + 1 + Y_{i-1}')(Y_1' + 1 + Y_{i-1})}{(y_1\alpha_1 + \cdots + y_{i-2}\alpha_{i-2})(y_1'\alpha_1' + \cdots + y_{i-2}\alpha_{i-2}')}$$

and

$$\sum \equiv \sum_{i=1}^{n-2} .$$

Equation (26) is true in the generalized Bjorken limits $q^2 \to \pm +\infty$, $s_i \to \mp \infty$, $i = 3,\cdots n-1$; $s_n, s \to +\infty$, but s_{ij} = fixed, $3 \le i < j \le n$. It holds for both[2,6] the lepton-hadronic and the colliding beam reactions, Eq. (17).

III. APPLICATIONS TO THE EXCLUSIVE PRODUCTION PROCESSES

We apply the idea of scale transformations, developed in the previous section, to the hadronic exclusive processes as well as to the lepton-hadronic exclusive processes.

A. Hadronic Exclusive Reactions

Consider the reaction

$$h_1 + h_2 \to h_3 + \cdots + h_{n+1}. \tag{27}$$

[We can regard h_{n+1} is the ground state of the excited leg in Eq. (11), i.e., the "anything." Then the exclusive process Eq. (27) is a particular case of the inclusive process Eq. (1), with the missing mass is fixed at the ground state.]

The invariant amplitude is the standard $n+1$-point function (Fig. 3)

$$B_{n+1} = \int_0^1 \prod_{i=1}^{n-2} \frac{dx_i}{(1 - x_i)} \prod_{i=1}^{n-2} x_i^{-\alpha(s_{1,i+1})-1}$$

$$\times (1 - x_1 \cdots x_{n-2})^{-\alpha_{1,n+1}(k_1 + k_{n+1})} \prod_{2 \le i < j \le n-1}{}' U_{ij}^{-\alpha_{ij}(s_{ij})}. \tag{28}$$

The symbol $\prod'$ means that we delete the channel $\alpha_{2,n} = \alpha_{1,n+1}$.

We take the limit $s_{1,i} \to -\infty$, $i = 2,\cdots,n-1$, but keep s_{ij} = fixed, $2 \le i < j \le n$, and y_i = fixed > 0, with

$$y_i \equiv \frac{s_{12}}{s_{1,i+1}} \approx \frac{1}{1 - \omega_3 - \omega_4 - \cdots - \omega_{i+1}}, \quad i = 2,3,\cdots n-1. \tag{29}$$

As suggested from Eq. (8), we make the scale transformation

$$\ell n \frac{1}{x_i} = y_i \rho \beta_i, \quad y_i > 0, \quad i = 2,\cdots,n-2,$$
$$\ell n \frac{1}{x_1} = \rho(1 - \beta_2 - \beta_3 - \cdots - \beta_{n-2}), \tag{30}$$
$$1 - \beta_2 - \beta_3 - \cdots - \beta_{n-2} \geq 0.$$

Substituting Eq. (30) in Eq. (28), and doing the ρ integral, we get

$$B_{n+1} \sim (s_{12})^{\alpha_{1,n+1}(k_1+k_{n+1})} \int_0^{1-\beta_2-\cdots-\beta_{n-2}\geq 0} \prod_{i=2}^{n-2}\left(\frac{d\beta_i}{\beta_i}\right)$$
$$\times \left[(1 - \beta_2 - \cdots - \beta_{n-2}) + \sum_{i=2}^{n-2} y_i\beta_i\right]^{-\alpha_{1,n+1}(k_1+k_{n+1})}$$
$$\times \prod_{2\leq i<j\leq n-1}\left[\frac{(y_{i-1}\beta_{i-1}+\cdots+y_{j-2}\beta_{j-2})(y_{i-2}\beta_{i-2}+\cdots+y_{j-1}\beta_{j-1})}{(y_{i-2}\beta_{i-2}+\cdots+y_{j-2}\beta_{j-2})(y_{i-1}\beta_{i-1}+\cdots+y_{j-1}\beta_{j-1})}\right]^{-\alpha_{ij}(s_{ij})}, \tag{31}$$

with

$$y_1 \equiv 1,$$
$$\beta_1 \equiv 1 - \beta_2 - \beta_3 - \cdots - \beta_{n-2}. \tag{32}$$

We then define Eq. (30) to be held for $s_{1i} \to +\infty$, $i = 2,\cdots n-1$. Equation (31) shows that the invariant amplitude for n-1 particle production has the expected regge behavior, which is not scaled, but which is factorized from the remaining scaled part.

Since the invariant amplitude is not scaled, the cross section at fixed multiplicity n-m, given by

$$\sigma^{(n-m)} \sim \int d\phi_{n-m}\left|\sum_{\text{perm.}} B_{n+1}\right|^2 = s_{12}^{2\alpha_{1,n+1}(0)-2} F_{(n-m)}(\ell n^{-1} s_{12}, \omega_i, s_{ij}), \tag{33}$$

where (m-1) is the number of detected particles and ϕ_{n-m} is the n-m body phase space integral, will not be scaled in general. But the sum over all multiplicities $\sum_{n-m=2}^{\infty} \sigma^{(n)}$ should be scaled, as proved in Eq. (12).

B. The Lepton-Hadronic Exclusive Reactions

Consider the reactions

$$\text{"}\gamma\text{"} + h_3 \to h_4 + \cdots + h_{n+1},$$
$$\text{"}\gamma\text{"} \to \bar{h}_3 + h_4 + \cdots + h_{n+1}. \tag{34}$$

We adopt the idea[2] that a heavy virtual photon behaves like a parton-antiparton pair in its participation of the strong interaction. We then write down the invariant amplitude for producing (n-2) particles in the parton dual resonance model:

$$T^{(n-2)} = \int d^4k_2 \frac{B_{n+1}(q - k_2, k_2, k_3, \cdots, k_{n+1})}{(k_2^2 - m^2)[(k_2 - q)^2 - m^2]}. \tag{35}$$

We define the kinematic variables

incident energy: $s = (q + k_3)^2$,

momenta transfers: $s_{ij} = (k_i + k_{i+1} + \cdots + k_j)^2$,

$$3 \leq i < j \leq n,$$

Equation (36) continued

Equation (36) continued

$$\text{scaling variables:} \quad \omega_i = \frac{2k_i\cdot q}{-q^2}, \quad i = 3,\cdots,n+1,$$

$$\tau_{ij} = \frac{2k_i\cdot k_j}{-q^2}, \quad 3 \le i < j \le n+1. \tag{36}$$

One relation among ω_i's is

$$\omega_3 + \omega_4 + \cdots + \omega_n = 1. \tag{37}$$

In Eq. (33), we have neglected the spin complications of the virtual photon and the two partons. They are irrelevant in the following discussions. (They certainly can be correctly taken into account, as in Ref. 2.)

We now do the loop momentum integration over d^4k_2, and get

$$T^{(n-2)} = \int_0^\infty da_1\, da_2 \prod_{i=1}^{n-2} \frac{d(\ell n\, \frac{1}{x_i})}{(1 - x_i)}$$

$$\times \left\{(1 - x_1)^{-\alpha_{23}+\alpha_0} \prod_{i=1}^{n-3}\left(\frac{1 - x_1\cdots x_i}{1 - x_1\cdots x_{i+1}}\right)^{-\alpha(s_{3,i+2})}\right.$$

$$\left.\times\ (1 - x_1\cdots x_{n-2})^{-\alpha(s_{3n})} \prod_{3\le i<j\le n} U_{ij}^{-\alpha(s_{ij})}\right\}$$

$$\times\ \frac{\pi^2}{C^2} \exp\left\{q^2\left[\sum_{i=1}^{n-2}\left(\frac{s_{1,i+1}}{q^2}\right)\ell n\left(\frac{1}{x_i}\right) + \frac{a_1}{C}F\right]\right\}$$

$$\times\ \exp\left\{-m^2(a_1 + a_2) - \frac{1}{C}\left[\sum_{i=3}^{n} k_i\ \ell n(1 - x_1x_2\cdots x_{i-2})^{-1}\right]^2\right\}, \tag{38}$$

where

$$s_{1,i} \equiv (q + k_3 + \cdots + k_i)^2, \quad s_{12} = q^2,$$

$$\alpha_0 + s_{ij} = \alpha(s_{ij}), \quad \alpha_0 + k_i^2 = 0, \tag{39}$$

and

$$C = a_1 + a_2 + \ell n(1 - x_1)^{-1},$$

$$F = \left[a_2 + \ell n\left(\frac{1 - x_1\cdots x_{n-2}}{1 - x_1}\right) - \sum_{i=3}^{n-1} \omega_i\ \ell n\left(\frac{1 - x_1\cdots x_{n-2}}{1 - x_1\cdots x_{i-2}}\right)\right]. \tag{40}$$

We take the limits $q^2 \to -\infty$, $s_{1,i} \to -\infty$, hence all $\omega_i < 0$, and F is positive definite. We then make the scale transformation, as suggested from Eq. (21):

$$a_1' = a_1 \frac{F}{C},$$

$$\ell n\, \frac{1}{x_i} = y_i\, \rho\, \beta_i, \qquad y_i > 0, \qquad i = 1,2,\cdots n-2, \tag{41}$$

$$a_1' = \rho(1 - \beta_1 - \cdots - \beta_{n-2}),$$

with

$$y_i \equiv \left(\frac{q^2}{s_{1,i+1}}\right) \approx \frac{1}{1 - \omega_3 - \omega_4 \cdots - \omega_{i+1}}, \quad y_1 \equiv 1.$$

Expanding everything else in terms of ρ and β_i, we find for the ρ integral

$$\int_0^\infty d\rho\ \rho^{-\alpha_{23}} \exp(q^2\rho) = \Gamma(1 - \alpha_{23})\left(\frac{1}{q^2}\right)^{-\alpha_{23}+1}. \tag{42}$$

The remarkable feature is that the ρ integral is independent of s_{3n}. This results from a delicated cancellation between the loop integration and the original $n+1$-point function. In the light-cone language, it means that the singularities of the operator expansions near the light cone are c numbers.[8]

Substituting Eqs. (41), (42) in Eq. (38), we then find

$$T^{(n-2)} \sim \left(\frac{1}{q^2}\right)^{-\alpha_{23}+1} \frac{\pi^2\,\Gamma(1-\alpha_{23})}{\ell n|q^2|} \int_0^{1-\beta_1\cdots-\beta_{n-2}\geq 0} \prod_{i=1}^{n-2}\left(\frac{d\beta_i}{\beta_i}\right)$$

$$\times \int_0^\infty da_2\, \frac{\exp(-m^2 a_2)}{[F]} \left\{ \beta_1^{\alpha_0-\alpha_{23}} (y_1\beta_1 + \cdots + y_{n-2}\beta_{n-2})^{-\alpha(s_{3n})} \right.$$

$$\times \prod_{i=1}^{n-3}\left(\frac{y_1\beta_1 + \cdots + y_i\beta_i}{y_1\beta_1 + \cdots + y_{i+1}\beta_{i+1}}\right)^{-\alpha(s_{3,i+2})}$$

$$\times \prod_{i=1}^{n-2} (y_1\beta_1 + \cdots + y_i\beta_i)^{2k_{i+2}\cdot(k_3+k_4+\cdots+k_n)}$$

$$\left. \times \prod_{3\leq i<j\leq n} \left[\frac{(y_{i-1}\beta_{i-1}+\cdots+y_{j-2}\beta_{j-2})(y_{i-2}\beta_{i-2}+\cdots+y_{j-1}\beta_{j-1})}{(y_{i-2}\beta_{i-2}+\cdots+y_{j-2}\beta_{j-2})(y_{i-1}\beta_{i-1}+\cdots+y_{j-1}\beta_{j-1})}\right]^{-\alpha(s_{ij})} \right\}, \tag{43}$$

where

$$F = a_2 + \ell n\left(\frac{y_1\beta_1+\cdots+y_{n-2}\beta_{n-2}}{y_1\beta_1}\right) - \sum_{i=3}^{n-1} \omega_i\, \ell n\left(\frac{y_1\beta_1+\cdots+y_{n-2}\beta_{n-2}}{y_1\beta_1+\cdots+y_{i-2}\beta_{i-2}}\right), \tag{44}$$

$$\text{anything} + y_{n-1}\beta_{n-1} \equiv 1.$$

We then define Eq. (39) to be true for $q^2 \to +\infty$, and $s \to +\infty$, i.e., it holds for both the lepton-hadronic and the colliding beam exclusive reactions.

It should be pointed out that the q^2 dependence in Eq. (43), apart from the factor $\ell n^{-1}|q^2|$, is the asymptotic form factors[2] in the parton dual resonance model:

$$G(q^2) \underset{|q^2|\to\infty}{\sim} C\left(\frac{1}{q^2}\right)^{-\alpha_{23}+1}. \tag{45}$$

Thus the exclusive invariant amplitude is asymptotically proportional to the universally asymptotic form factors.

Again, since the invariant amplitude is not scaled, the structure function at fixed multiplicity $(n-2)$, given by

$$\nu W_2^{(n-1)} \sim q^2 \int d\phi_{n-2} \left| \sum_{\text{perm.}} T^{(n-2)} \right|^2, \tag{46}$$

is not scaled in general. But the overall structure function

$\nu W_2 = \sum_{n=2}^{\infty} \nu W_2^{(n)}$ does scale.

The lepton-hadronic exclusive experiment is being carried out at Cornell. Let us therefore study a little detail for the case $n = 4$,

$$\begin{aligned} &"\gamma" + h_3 \to h_4 + h_5, \\ &"\gamma" \to \bar{h}_3 + h_4 + h_5. \end{aligned} \tag{47}$$

The invariant amplitude, from Eq. (43), is

$$T^{(2)} \sim \frac{G(q^2)}{\ell n|q^2|}\,\pi^2\,\Gamma(1-\alpha_{23})\int_0^1 d\beta_1\int_0^{1-\beta_1} d\beta_2\int_0^\infty da$$

$$\times\ \frac{\exp(-m^2 a)}{\left[a + (1-\omega_3)\,\ell n\left(\frac{\beta_1 + y_2\beta_2}{\beta_1}\right)\right]}$$

$$\times\ \left\{\frac{\beta_1^{\alpha_0-\alpha_{23}}}{\beta_1\beta_2}\,(y_2\beta_2)^{-\alpha_0}\left[\frac{\beta_1(\beta_1 + y_2\beta_2)}{y_2\beta_2}\right]^t\right\}\ , \qquad (48)$$

with $t \equiv (k_3 + k_4)^2 = s_{34}$, $y_2 = \frac{q^2}{s_3} = \frac{1}{1-\omega_3}$. In obtaining Eq. (48), we have used the bootstrap conditions $\alpha_0 + k_i^2 = 0$, $i = 3,4,5$.

Let us now list the predictions from Eq. (48):

$$\text{(a)}\quad T^{(2)} \underset{\omega_3\to+\infty}{\sim} \frac{G(q^2)}{\ell n|q^2|}\,\omega_3^{\alpha(t)}\,f(t), \qquad \text{(the regge limit)}$$

$$\text{(b)}\quad T^{(2)} \underset{\omega_3\to 1^{\pm}}{\sim} G(q^2)(\omega_3 - 1)^{\alpha_0}\,f'(t), \qquad \text{(the threshold behaviors)} \qquad (49)$$

$$\text{(c)}\quad T^{(2)} \underset{\omega_3\to 0}{\sim} \frac{G(q^2)}{\ell n|q^2|}\,f''(t), \qquad \text{(the pionization limit for the colliding beam reaction).}$$

The structure function $\nu W_2^{(2)}$ for multiplicity two can be calculated; we get

$$\nu W_2^{(2)} \sim q^2\int d\phi_2|T^{(2)}|^2 \sim q^2\,G^2(q^2)\,\frac{1}{\ell n^2|q^2|}\,f^{(2)}(\omega). \qquad (50)$$

We see that, apart from the factor $\ell n^{-2}|q^2|$, the q^2 dependence factor $q^2\,G^2(q^2)$ is identical to that given by Bloom and Gilman for the threshold behavior in the electroproduction. The result, Eq. (50), is consistent with Roy's[9] wee parton calculation.

From Eqs. (50), (46), and (44), it is clear that, in general, each individual $\nu W_2^{(n)}$ at fixed multiplicity n, falls off in q^2 [of the order of $q^2\,G^2(q^2)$], but because more and more channels contributed as $s \equiv (q + k_3)^2$ (the incident energy) increases, we still get a sclaing result for the overall νW_2. This may be regarded as the physical origin of the Bjorken's scaling law.

IV. CONCLUSIONS

In this paper we have fully discussed the scaling laws in the electromagnetic interaction and in the strong interaction. We have shown that by the correct choices of the scale transformations, the two scaling laws can be naturally explained, and that they in fact stem from similar origins. We have further shown that the exclusive reactions do not scale in general, but the nonscaled parts may be found from the scaled parts.

From the behavior of the unscaled parts, we can indirectly infer the average multiplicity distributions. If we assume that the sum of all exclusive processes should produce the inclusive results, i.e.,

$$\sigma_{inc} = \sum_{n=2}^{\infty} \sigma^{(n)} ,$$
$$\nu W_2 = \sum_{n=2}^{\infty} \nu W_2^{(n)} , \qquad (51)$$

then, because the left-hand sides of Eq. (51) are scaled, the terms on the right-hand sides summations must combine in very delicate ways, so that they produce the scaling results. However, we know that the nonscaled parts of each individual term on the right-hand sides are smooth (one is regge-behaved, the other is proportional to the square of the form factor), therefore the summation over all n must produce smooth behaviors to cancel with the nonscaled parts. This implies that the average multiplicity distribution $\bar{n}$ must be function of s or $-q^2$, but not ω. Thus we can roughly conclude that our work favors a

multiplicity distribution which depends on $\log s$, rather than $\log \omega$. (If we only consider the planar diagrams in the n dimensional phase space integrals.)

The problems of spin, internal symmetry, and ghosts do not play any fundamental roles in this work. What is essential, in the scaling limits, is the regge behaviors in various channels, together with a (factorizable) pomeron pole of intercept $\alpha_0 \equiv 1$.

Finally, we mention that we have neglected the permutation of external legs, and we have not considered the nonplanar loop contributions to the inclusive reactions.

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FOOTNOTES AND REFERENCES

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1. Eden, Landshoff, Olive, and Polkinghorne, The Analytic S-matrix (Cambridge University Press, England, 1966), p. 131-146.

2. L. p. Yu, A Simple Parton Dual Resonance Model for Electroproductic and Lepton-Pair Annihilation Processes With the Incorporation of Final-State Interaction, Lawrence Radiation Laboratory Report UCRL-20610, March, 1971.

3. Let us point out right now that because κ_i are the ratios of s_{1i}'s, the scale transformation produces the scaling invariant results.

4. There are also imaginary parts that come from κ_i, but these pieces are cancelled by the $i\epsilon$-prescription, which demand the remaining integral to be real.

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FIGURE CAPTIONS

Fig. 1. The 2n-point function model for the hadronic inclusive reaction.

Fig. 2. The 2n-point function parton dual resonance model for the lepton-hadronic and colliding beam inclusive reactions.

Fig. 3. The n + 1-point function model for the hadronic exclusive reaction.

Fig. 4. The n + 1-point parton dual resonance model for the lepton-hadronic and the colliding beam exclusive reactions.

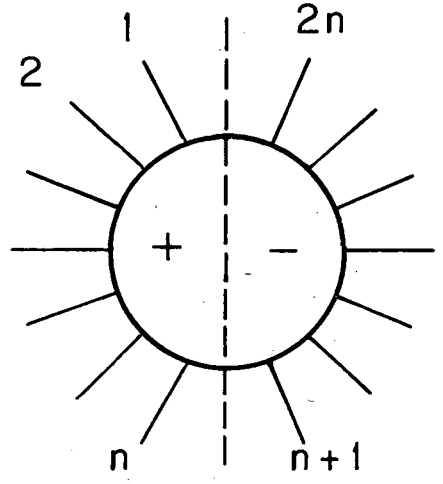

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Fig. 1

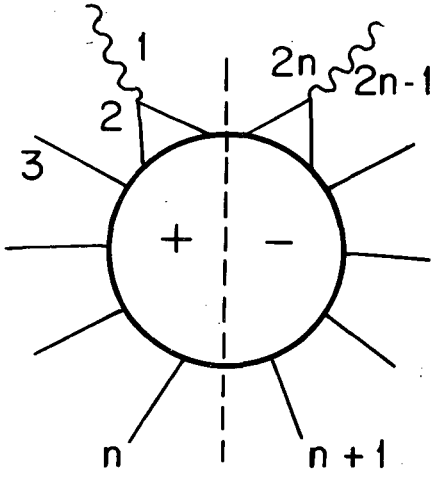

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Fig. 2

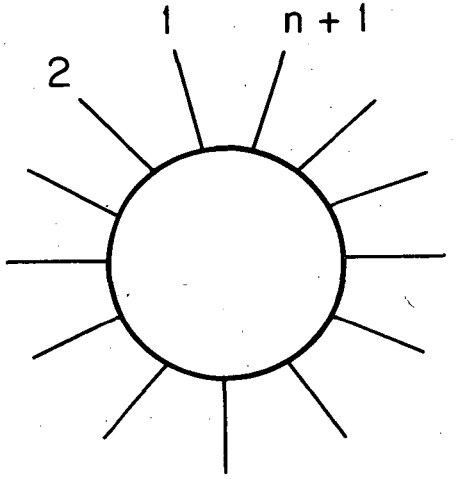

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Fig. 3

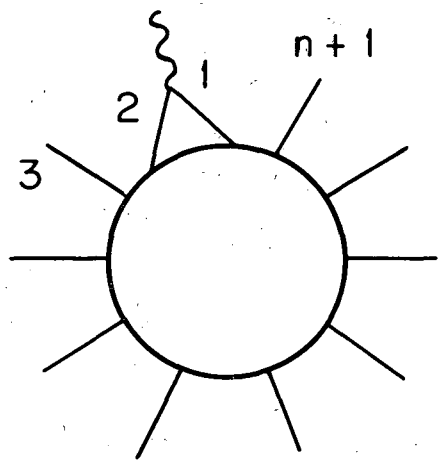

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Fig. 4

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