Resolving Impasses in Problem Solving: An Eye Movement Study

Günther Knoblich (knoblich@mpipf-muenchen.mpg.de)
Cognition and Action,
Max-Planck-Insitute for Psychological Research
Leopoldstr. 24, 80802 Munich, Germany

Stellan Ohlsson (stellan@uic.edu) and Gary Raney(geraney@uic.edu)
Department of Psychology
University of Illinois at Chicago
1007 W. Harrison St., Chicago, IL 60607

Abstract

Insight problems cause impasses because they deceive the problem solver into constructing an inappropriate initial representation. The main theoretical problem of explaining insight is to identify the cognitive processes by which impasses are resolved. In past work, we have hypothesized two such processes: constraint relaxation and chunk decomposition. In the study reported here, we derive detailed predictions about the structure of eye movements from these hypotheses. Eye movement data from a study of match stick algebra problems were consistent with the predictions. The results support the view that a key component of creative thinking is to overcome the processing imperatives of past experience.

Impasse Resolution

Some problems are difficult because they require the problem solver to find the right sequence of steps in a vast space of alternative step sequences (Newell & Simon, 1972). The main resources for solving such problems are search heuristics and memories of prior problem solving episodes, both of which help constrain the search. In contrast, other problems, traditionally called *insight problems*, quickly generate the impression that they are unsolvable (Ohlsson, 1984; Seifert, Meyer, Davidson, Patalano, & Yaniv, 1995). Instead of floundering in a myriad of possibilities, problem solvers cannot think of even a single useful step to take: they quickly find themselves at an *impasse*.

Insight problems cause impasses because they deceive the problem solver into constructing an incomplete or overly constrained problem space that does not include the solution (Kaplan & Simon, 1990; Ohlsson, 1984; Wickelgren, 1974). Impasses are encountered after an initial phase in which the procedures activated by the initial representation have been applied to the problem without success (Ohlsson, 1992). During impasses no further ideas about how to proceed come to mind. Feeling-of-knowing judgments show that problem solvers do not feel that they are approaching the solution until the moment immediately before it is attained (Metcalfe & Wiebe, 1987).

If the impasse persists, the problem solver will eventually have to give up and declare failure. To resolve the impasse, the problem solver must revise his or her initial representation of the problem. The new representation might change the problem space by activating previously dormant but task relevant knowledge (operators, procedures, rules, etc.) that allow problem solving to continue. Identifying the cognitive processes involved in resolving impasses is the main theoretical problem in explaining insight problem solving in particular and creative thinking in general.

In past work we have proposed two processes for impasse resolution. First, constraint relaxation changes the representation of the goal (Ohlsson, 1992). Isaak and Just (1995) have suggested that constraint relaxation was involved in many historically important technological innovations. Second, chunk decomposition changes the representation of the problem situation (Knoblich, Ohlsson, Haider, & Rhenius, submitted). The purpose of the study reported here was to develop and test the implications of these process hypotheses for eye movements during problem solving. Before reporting the study, we develop the process hypotheses in more detail vis-a-vis the particular task domain we used.

Process Hypotheses

Constraint Relaxation

The initial representation of the goal to be sought in solving a particular problem is biased by prior knowledge. Such knowledge generates expectation about, among other things, which aspects of the problem situation are invariant and which are variable. For instance, when confronted with an arithmetic problem like $10 + 2 = ?$, the problem solver knows that the arithmetic operation is an invariant aspect of the task; the only thing that can vary is the value of the answer. His or her representation of the goal will naturally constrain the solution to look like the initial state except that the result of the arithmetic operation is filled in.

When encountering an unfamiliar type of problem, it is less obvious which aspects of the initial problem situation should be thought of as invariant and which as variable. Consider the domain of *match stick algebra* (Knoblich et al., submitted). A match stick algebra problem consists of an incorrect arithmetic expression written with Roman numerals and with the operators "+" and "-" and the equal sign (see Figure 1); for brevity, we shall let the term "operator" include the equal sign from this point on. The numerals and operators are constructed out of match sticks. The task is to move exactly one stick so as to change the given expression into a true expression consisting of nothing but Roman numerals in the range of I to XIII and the arithmetic operators.

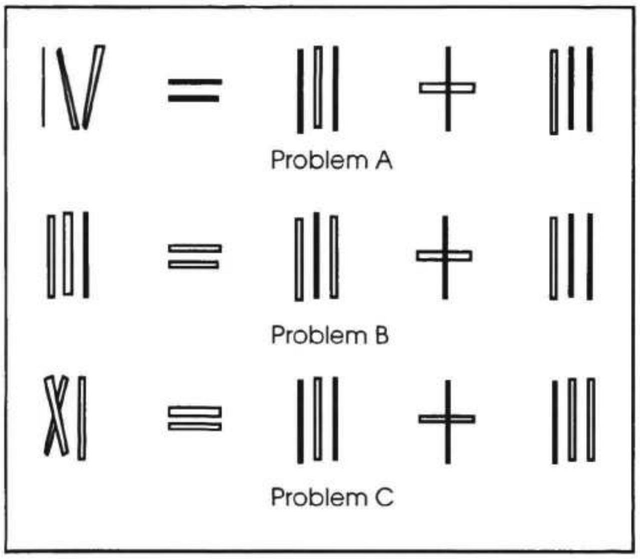

Figure 1. Match stick algebra problems.

Match stick algebra is unfamiliar to many people. However, their similarity to standard arithmetic makes it highly likely that these problems activate prior knowledge of arithmetic. This will bias the problem solver to view the arithmetic operators as constants and only the values as variable. In this representation, the only moves that will be considered are those that transform the values. Problem A in Figure 1 can be solved with a move of this type: pick up the vertical stick (I) in the left hand side of the expression and move it to the other side of the V, thus changing the IV to a VI and the expression as a whole to VI = III + III, a true arithmetic expression.

However, problem B cannot be solved as long as we only consider moves that change values. To find the solution to B, the operators have to be considered variable as well. To solve B, pick up the vertical stick in the plus sign to the right, rotate it 90 degrees and put it down again to make a second equal sign. This transforms the expression into III = III = III, a true arithmetic expression.

In short, constraint relaxation extends the space of possible solutions by replacing constants in the goal structure by variables in response to an impasse.

Chunk Decomposition

Familiarity with certain configurations of perceptual features establish those as patters (chunks) in memory. Configurations of perceptual features that match a chunk are automatically recognized as an instance of that chunk. When encoding a problem, perceptual chunks that have proven useful in past encounters with superficially similiar problems are automatically applied. In most situations, chunks faciliate problem solving (Chase & Simon, 1973; Ericsson & Lehmann, 1996).

Nevertheless, chunks that have proven useful in the past might hinder the solution to an unfamiliar problem. To find a solution, the components of a chunk might have to be separated from each other and reconfigured into a different patterns. We refer to this as *decomposing* the chunk.

The possibility of decomposition may be more or less obvious depending on characteristics of the chunks. To decompose a chunk like "IV" into "I" and "V" is easy, because those componens are themselves meaningful symbols (chunks) within the system of Roman numerals. We call this a *loose* chunk. To solve problem A (see Figure 1), it is sufficient to decompose that chunk.

However, other chunks consists of components that are not meaningful, at least not in the context of match sick algebra. To solve problem B (see Figure 1), the problem solver has to decompose the plus sign into one vertical and one horizontal stick. To solve problem C, the problem solver has to decompose the numeral X into two sticks, one that is slanted to the left and one that is slanted to the right. In neither case are the components of the chunks meaningful within the system of Roman numerals. We call these *tight* chunks.

In short, chunk decomposition splits chunks into their components in response to an impasse. This leads to a more fine grained problem representation which might activate dormant but relevant knowledge (operators, procedurs, rules, etc.) that can be applied to the problem and hence resolve the impasse and enable problem solving to resume.

Eye Movement Predictions

At the surface, the three problems in Figure 1 differ only in the numeral on the left-hand side. However, to solve problem B, the problem solver has to relax constraints on altering operators that do not need to be relaxed to solve problems A or C. To solve problems B and C, the problem solver needs to decompose tight chunks that do not need to be decomposed to solve problem A. Hence, we predicat that problems B and C are more difficult than problem A, in spite of their surface similarities, and that B is more difficult than C.

However, the probability of solution and the time to solution do neither allow us to determine how the problem solving process unfolds nor how succssful and unsuccessful problem solvers differ. Think aloud protocols provide a window onto problem solving (Ericsson & Simon, 1993), but problem solvers do not verbalize during impasses (Duncker, 1945) and verbalizing may interfer with the processes involved in representational change (Schooler, Ohlsson, & Brooks, 1993). To overcome these problems, we recorded the participants' eye movements.

How do the two processes of constraint relaxation and chunk decomposition influence the movements of the problem solver's eyes? The answer depends on the particular quantity or measure that we derive from the raw eye movement recordings.

Mean *fixation duration* is likely to be affected by impasses, because the duration of fixations indicates how long an information item is processed in working memory (Just & Carpenter, 1976). During impasses, problem solving activity ceases or at least slows down. Hence, processing speed should drop and mean fixation duration should increase. Because we expect impasses in problems B and C but not in problem A, mean fixation duration should be greater in the former two problems. Furthermore, because impasses only occur after the initial exploration of the problem space, mean fixation duration should increase as problem solving continues for problems B and C but not for problem A.

A second measure is the *proportion of fixation time* spent on different elements of the task. This measure indicates which elements were processed most extensively. Differences in the proportion of fixation time devoted to a particular element during successive intervals is a sign that the participants' attention shifted from one element to another during the problem solving process.

In problem B relaxing the constraint to keep operators constant should result in a shift of attention from the result and the operands (i.e., the values) to the operators. Because problem B can only be solved when this constraint is relaxed, this shift should be present or more pronounced in participants who solved problem B, and absent or less pronounced in participant who failed to solve that problem.

In problem C the process of decomposing the tight chunk "X" should result in a shift of attention to that chunk. Because problem C can only be solved when this chunk is decomposed, the shift should be present or more pronounced in participants who solved problem C, and absent or less pronounced in participants who failed to solve that problem.

Our final measure is the *proportion of fixation changes* that moves the point of fixation between a particular pair of problem elements. A high proportion of fixation changes between elements I and J indicates that these two elements were processed together.

Predictions about fixation changes can be derived for problem B. Relaxing the constraint to keep operators constant should allow the problem solver to consider moving sticks between operators as well as between operators and values. Hence, the proportion of fixation changes between operators should increase as problem solving continues. Because problem B can only be solved when such moves are considered, this increase should be present or more pronounced in participants who solved problem B, and absent or less pronounced in participans who did not.

Method

Twentyfour undergraduate students at the University of Illinois at Chicago participated in the study for course credit. All participants attempted the three problems in Figure 1. The problems were presented in random order on a computer screen. Eye movements were recorded concurrently. As soon as a participant announced that she or he had found a solution, the experimenter terminated the eye movement recording and the participant said the solution out aloud. If a problem was not solved within five minutes, the participant was told the solution and the next problem was presented.

Results

Frequency of Solution and Solution Time

Figure 2 shows the solution rate in terms of the percentage of problems solved (panel a), and the mean solution time for the successful solutions (panel b). Consistent with our predictions, problem B, which requires both the relaxation of the constraint to keep operators constant and the decomposition of a tight chunk, was solved least often, and the successful solutions required more time than for the other two problems. Problem C, which requires the decomposition of a tight chunk but not constraint relaxation, was solved more often than B but less often than A, and the mean solution time for the successful solutions was also between the corresponding measures for the other two problems. Problem A, which does not require either constraint relaxation or the decomposition of any tight chunk, was solved most often and fastest. These results are consistent with our hypotheses, but they only provide weak support because many other process hypotheses might predict the same outcome pattern. Stronger support requires more fine grained analyses of the participants' behavior.

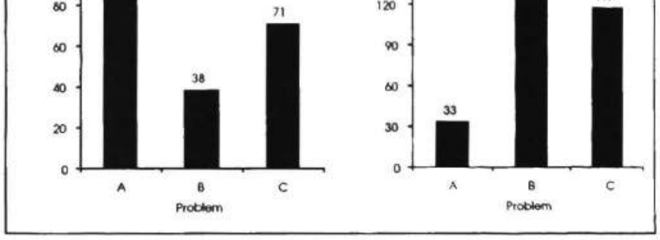

Figure 2. Solution rate (a) and mean solution time (b) for each of three match stick algebra problems.

Eye Movements

We divided each problem solving performance into three invervals, each interval representing one-third of the overall duration of the performance. (Intervals thus were of equal duration within each performance, but varied in duration across performances.) The three eye movement measures were calculated separately for each interval and participant and averaged across participants for each problem. We discuss each measure separatly.

Fixation duration. As we predicted, mean fixation duration was greater for problems B and C than for problem A; see Figure 3. Moreover, the fixation duration for problems B and C increased monotonically across the three intervals. In contrast, the fixation duration for problem A increased from interval 1 to interval 2, but did not increase from interval 2 to interval 3. These results support the hypotheses that the participants encountered more impasses on problems B and C, particularly towards the end of the problem solving attempt.

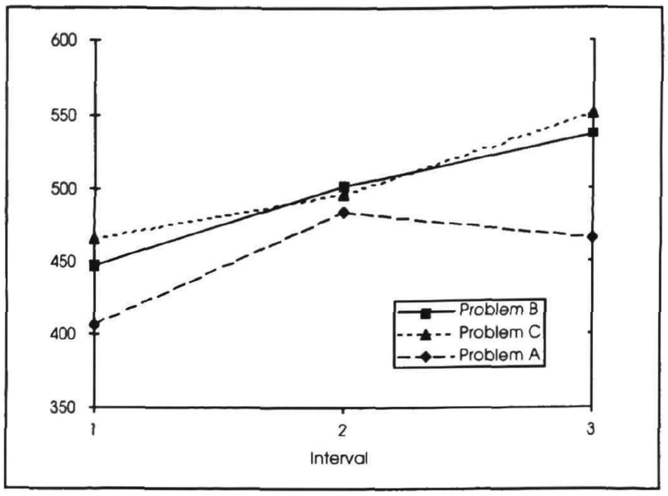

Figure 3. Mean fixation duration as a function of interval and problem

Fixation time. We computed the proportion of fixation time allocated to the different problem elements for each interval. Figure 4 shows the result for participants who solved and did not solve problem B. During the first interval there were no differences in attention allocation between those who solved the problem and those who did not. All participants payed more attention to the numerals in the equation than to the operators. This is consistent with the hypothesis that the initial goal representation constrains the problem space to moves that change values but not operators. However, the two groups differ in intervals 2 and 3. For participants who did not solve the problem, the initial pattern of attention allocation did not exhibit any consistent trend across the three invervals. For those who solved the problem, attention gradually migrated from the numerical values to the operators. This is consistent with the hypothesis that a key step in the solution is to relax the constraint that operators must be kept constant. Interestingly, the change in attention allocation is already present between the first and second intervals, i.e., at a time when problem solvers are still at an impasse and still think that the problem is unsolvable.

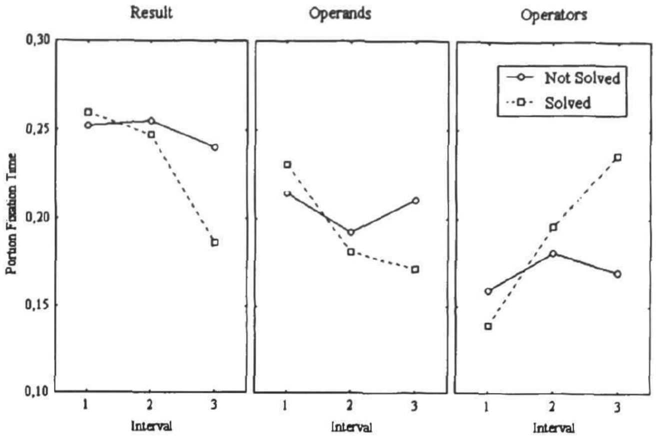

Figure 4. Mean proportion fixation time allocated to results, operands and operators for successful and unsuccessful solutions to problem B.

Figure 5 shows the corresponding results for problem C. During the first interval attention was allocated more to the result than to the operands and operators for both successful and unsuccessful problem solvers. For unsuccessful solvers, the pattern of attention allocation was virtually unchanged in intervals 2 and 3. For the successful solvers, on the other hand, the amount of fixation time allocated to the crucial component -- the numeral X -- increased monotonically across intervals.

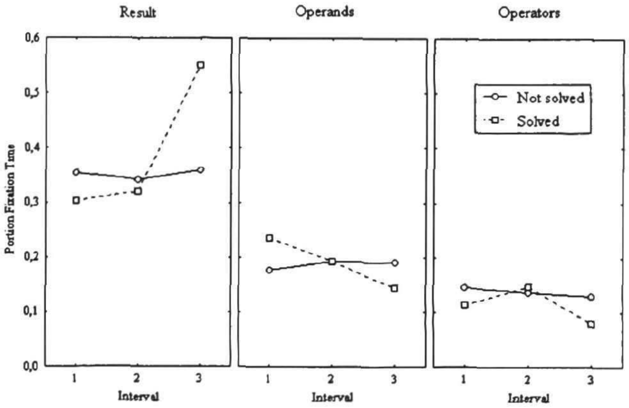

Figure 5. Mean proportion fixation time allocated to results, operands and operators for successful and unsuccessful solutions to problem C.

Moreover, the proportion fixation time allocated to the result almost doubled from interval 2 to interval 3 for the successful solutions. This result is consistent with the hypothesis that problem C can only be solved when the problem solver pays close attention to the tight chunk that need to be decomposed.

Fixation changes. We computed the proportion of fixation changes that moved the fixation point between operators for problem B. Figure 6 shows the results for participants who solved and did not solve problem B.

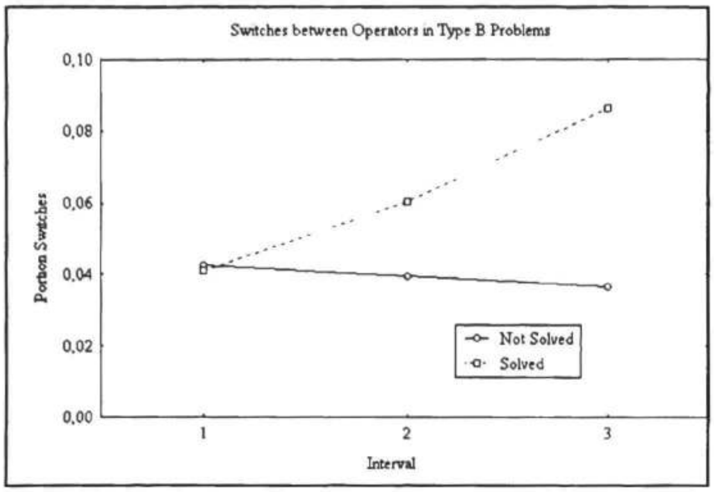

Figure 6. Proportion fixation changes that moved the fixation point between operators for each interval for successful and unsuccessful solutions to problem B.

Consistent with the constraint relaxation hypothesis, the proportion of fixation changes between operators was equal for both groups in the first interval and remained constant in intervals 2 and 3 for unsuccessful participants, but increased monotonically across the three intervals for successful participants. This result is consistent with the hypothesis that participants who solved the problem eventually related the two crucial operators while the unsuccessful participants did not.

Discussion

The fundamental characterisic of insight problem solving is the occurrence of an impasse, i.e., a state of mind in which the problem solver has run out of ideas about what to do next. The main theoretical problem of explaining insight is to specify the cognitive processes by which impasses are resolved (Ohlsson, 1984; Ohlsson, 1992). We propose that constraint relaxation extends the problem space by changing the status of certain problem elements from invariants to variables that can be manipulated, and that chunk decomposition extends the problem space by allowing features or components of the problem situation that are normally perceived as linked in a particular configuration to be separated and reconfigured. When these processes occur, previously unheeded possiblities suddenly come to mind and problem solving can continue.

These hypotheses generate detailed predictions about the expected pattern of eye movements on insight problems. In particular, they generate predictions about differences between superficially similar problems and between successful and unsuccessful problem solvers. The fact that our data were consistent with these predictions lend strong support to the hypotheses. Although there are alternative theories of insight problem solving (Simonton, 1988; Smith, Ward, & Finke, 1995; Sternberg & Davidson, 1995; Weisberg, 1986), they are too vague to generate detailed predictions about the temporal structure of problem solving behavior, about the differential difficulty of individual problems or about differences between problem solvers.

The constraint relaxation and chunk decomposition hypotheses are instances of a more general principle: Although human beings have to base their approach to each new problem or situation on past experience -- there is no other choice -- success vis-a-vis an unfamiliar problem might neverheless require that the mind overrides the computational imperatives of experience. Automatized encoding rules and habitual response patterns have to be suppressed in order for novel actions to come to mind. Although the particular processes involved might vary across task domains, we suggest that overcoming past experience is a fundamental component of creative thinking in general.

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