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The Analysis of Thick-Target Thick-Catcher Nuclear Recoil Experiments*

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ABSTRACT

In a "thick-target thick-catcher" experiment a relatively thick foil surrounded by foils of some other material is exposed to a beam of particles. The distribution of a reaction product in the three foils is measured. The results are analyzed by the standard two-step vector representation. The velocity of the recoiling nucleus is given by the sum of a forward velocity v (taken here to be constant) imparted in the first step and a velocity V (taken here to be isotropic) imparted in the second step. The range-velocity relation is taken to be: $\text{range} = K \times \text{velocity}^N$, where K and N are constants. The distribution of the recoil energy E in the second step, corresponding to the velocity V , is of the form

$$P(E) dE = (E-E_0) \exp[(E-E_0)/\tau] dE/\tau^2$$

where E_0 is the effective coulomb barrier and $\tau = (\langle E \rangle - E_0)/2$. The analysis is made in two parts, $V > v$ and $V < v$. Experimental results can be analyzed to obtain values of v , $\langle R \rangle$, $\langle V \rangle$, $\langle E \rangle$, etc.

NUCLEAR REACTIONS	Analysis of thick-target thick-catcher nuclear recoil experiments.	
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1.

Experiments on the interaction of high-energy particles with complex nuclei are of two general types: (1) measurements on the emitted and scattered nucleons and small aggregates of nucleons, like alpha particles, and (2) measurements on the large residual nuclei that remain at the end of the reaction. The study of the large residual nuclei usually involves the measurement of either their excitation functions or their recoil properties.

One of the simplest experiments to perform in order to determine the recoil properties of nuclei is the "thick-target thick-catcher" experiment (Figure 1).¹⁻³ The incident beam passes through a target foil and two catcher foils placed on either side of it. The product, which is formed in the target foil, may either recoil completely within the target foil or into one of the catcher foils. This technique requires that the product not be formed directly in the catcher foil. In a later publication we will describe a method that can be followed in case suitable catcher foils are not available.

The results of these experiments are usually analyzed by the standard two-step vector representation.¹⁻³ The velocity of the recoiling residual nucleus is taken to be the sum of a forward velocity v imparted in the first step to an intermediate nucleus and a velocity V (taken here to be isotropic) imparted in the second step to the residual nucleus (Figure 2). The values of v and V and related quantities are calculated in this analysis.

In the "thick-target thick-catcher" experiment the thicknesses of the target and catcher foils are larger than the longest recoil range. The quantities measured are the fractions F and B of the residual nuclei that recoil out of the target foil into the forward and backward directions, respectively. The thickness of the target foil is identified by W .

An additional parameter for the reaction can be obtained by orienting the foils in line with the beam so as to measure the fraction of nuclei that recoil out of the target foil in a perpendicular direction. This information has been interpreted by some workers in terms of a perpendicular component of the velocity imparted in the first step of the reaction and by others in terms of an anisotropy in the angular distribution of the velocity V from the second step. The discussion of this type of experiment will not be included in this report.⁴

2.

The analysis presented here has several features:

1. The value of v is taken to be constant. The rationale for this is presented below.
2. A distribution in the values of V is assumed. This distribution is expressed in a general form, which is appropriate for a wide variety of nuclear reactions (Equation (23)).
3. A general type of range-velocity relationship is assumed (Equation (1)).

The effect of various assumed distributions in v and V on the analysis of this type of experiment was examined by Porile.⁵ He found that the values of $\langle v \rangle$ and $\langle V \rangle$ obtained from his analysis were independent of the assumed distributions for most of the cases he studied. His analysis involved the Monte Carlo technique, whereas the one presented here is based on the integration of analytical functions.

THE ANALYSIS

The results of the recoil experiments depend on the range-energy relation of the recoiling nuclei. It is convenient to express this relation as

$$R = KV^N \quad (1)$$

where R is the range, and K and N are constants. This expression has been found to be valid for the distribution of recoil energies from the reactions studied so far. The constants K and N can be evaluated from tables of ranges of nuclei recoiling into various materials.^{6,7}

The thickness of material that a recoiling nucleus will traverse, projected on the forward direction, is

$$t = R \left(\frac{v}{V} + \cos\theta \right) \left(1 + \frac{v^2}{V^2} + 2\frac{v}{V}\cos\theta \right)^{(N-1)/2} \quad (2)$$

where R is the range of a nucleus with a velocity V in the target material.

For given values of v and V , the fraction of nuclei recoiling into the forward catcher is

$$F = \frac{1}{2W} \int_0^{K(v+V)^N} \int_0^\theta \sin\theta \, d\theta \, dt \quad (3)$$

The evaluation of Equation (3) differs for the two possible cases: $V > v$ and $V < v$. Each of these cases will now be considered.

3.

$$V > v$$

For this case Equation (3) becomes

$$F = \frac{1}{2W} \frac{v \sqrt{1 - \frac{v^2}{V^2}}}{V} t \cos \theta$$

$$= \frac{R}{2W(N+1)(N+3)(v^2/V^2)} \left[\left(1 - \frac{v^2}{V^2}\right)^{\frac{N+3}{2}} - \left\{1 - (N+2)\frac{v}{V}\right\} \left(1 + \frac{v}{V}\right)^{N+2} \right] \quad (4)$$

The value for B is obtained from Equation (4) by changing the sign of v . The ratio $\frac{v}{V}$ can be obtained from the experimental value of $\frac{F}{B}$, if $\frac{v}{V}$ is single-valued,

$$\frac{F}{B} = \frac{(1 - v^2/V^2)^{(N+3)/2} - [1 - (N+2)v/V] (1 + v/V)^{N+2}}{(1 - v^2/V^2)^{(N+3)/2} - [1 + (N+2)v/V] (1 - v/V)^{N+2}} \quad (5)$$

The value of R can now be obtained from Equation (4) and V from Equation (1), provided V itself is single-valued.

If $\frac{v}{V}$ is not single-valued, the exact expression (Equation (4)), can be expanded as a power series to obtain

$$F = \frac{R}{4W} \left[1 + a_1 \frac{v}{V} + a_2 \frac{v^2}{V^2} + a_3 \frac{v^3}{V^3} + a_4 \frac{v^4}{V^4} \right] \quad (6)$$

where

$$a_1 = 2(N+2)/3 \quad (7)$$

$$a_2 = (N+1)^2/4 \quad (8)$$

$$a_3 = N(N-1)(N+2)/15 \quad (9)$$

$$a_4 = (N^2-1)(N^2-N-3)/72 \quad (10)$$

Equation (6) is exact for $N = 1$ and $N = 3$ and nearly so for intermediate values of N . In the general case, where v and V have many values, this equation becomes

$$F = [\langle R \rangle + a_1 \langle R \frac{v}{V} \rangle + a_2 \langle R \frac{v^2}{V^2} \rangle + a_3 \langle R \frac{v^3}{V^3} \rangle + a_4 \langle R \frac{v^4}{V^4} \rangle] / 4W \quad (11)$$

Average quantities are indicated by $\langle \rangle$. The expression for B is obtained by changing the sign of v .

Adding F and B and making the approximation $\langle R \frac{v^n}{V^n} \rangle = \langle R \rangle \frac{v^n}{V^n}$,

we get

$$\langle R \rangle = 2W(F+B) / [1 + a_2 \frac{v^2}{V^2} + a_4 \frac{v^4}{V^4}] \quad (12)$$

With $\frac{v}{V}$ from Equation (5) we can calculate $\langle R \rangle$. The value of $\langle R \rangle$, obtained this way, is close to the correct value because the last two terms in the denominator of Equation (12) are usually small. For $N = 1$, $\langle R \rangle$ gives $\langle V \rangle$, according to Equation (1). For $N = 2$, $\langle V^2 \rangle = \frac{2}{m} \langle E \rangle$ is obtained, where m = mass of the recoiling nucleus, and E = kinetic energy imparted to it in the second step of the reaction.

The value of $\langle \frac{v}{V} \rangle$ cannot be obtained directly from either Equation (5) or Equation (11), because the various powers of this quantity are weighted by R in a thick-target experiment. This effect can cause an appreciable error, if disregarded, for many types of reaction. This quantity can be obtained by replacing the thick target foil in Figure 1 with one that is very thin with respect to the range R . For this "thin-target thick-catcher" experiment

$$\langle \frac{v}{V} \rangle = F - B \quad (13)$$

see reference 2.

It will be assumed that v is not a function of V . In this case $\langle \frac{v}{V} \rangle = \langle v \rangle \cdot \langle \frac{1}{V} \rangle$ and $\langle R \frac{v}{V} \rangle = \langle v \rangle \cdot \langle \frac{R}{V} \rangle$. Except for the small effect of the higher-order terms in Equation (11), $\langle v \rangle$ can be treated as a constant, denoted by v .

In order to complete the analysis of "thick-target thick-catcher" experiments, it is necessary to know the distribution of V . This information can be obtained from a "thin-target thin-catcher" experiment, e.g. a 2π differential range experiment. The foil arrangement, shown in Figure 1, is replaced by a thin target sandwiched between many thin catcher foils on both sides. The distribution of recoiling nuclei $A(t)$ is measured as a function of t the thickness of foils traversed.

5.

The results of this type of measurement on the reaction $^{27}\text{Al}(p, 3p_n)^{24}\text{Na}$ at a proton energy of 2.9 Gev are shown in Figure 3.⁸ The activity of ^{24}Na is seen to drop by increasingly large factors overall with increasing values of t . Similar results were obtained for this type of study on the reactions of protons with tantalum, gold, and bismuth to produce ^{149}Tb .⁹

The distribution in V can be obtained from these results by a procedure based on the treatment given in reference 9. There is shown that

$$A(t) = \text{constant} \int_t^\infty \frac{P(R)}{R} dR \quad (14)$$

where $P(R) dR$ is the distribution in R . The constant in this equation is different for the forward and backward catcher foils. We can approximate $A(t)$ by the following expression

$$A(t) = A_0 e^{-\text{constant} t^M} \quad (15)$$

where M is a constant giving the best fit for the activity distribution in the catcher foils.

This fitting method is shown in Figure 4 for the data from Figure 3. The values of M for the forward and backward cases are essentially the same, $M = 1.8$. In the ^{149}Tb work $A(t)$ was found to be exponential, implying $M = 1.0$.⁹

By combining Equations (14) and (15), we get the distribution in R

$$P(R) dR = \text{constant} R e^{-(CR)^M} d(CR)^M \quad (16)$$

where C is a constant. If we express Equation (1) as

$$R = k E^{N/2} \quad (17)$$

where $k = (2/m)^{N/2} K$, we find that the average value of E is

$$\langle E \rangle = \frac{\Gamma(1 + \frac{1}{M} + \frac{2}{MN})}{\Gamma(1 + \frac{1}{M})} (Ck)^{-2/N} \quad (18)$$

with Γ representing the gamma function.

We can express Equation (16) in terms of the energy parameter

$$x = (CR)^{2/N} = \frac{\Gamma(1 + \frac{1}{M} + \frac{2}{MN})}{\Gamma(1 + \frac{1}{M})} \frac{E}{\langle E \rangle} \quad (19)$$

to obtain

$$P(x) dx = \frac{MN}{2\Gamma(1 + \frac{1}{M})} x^{N(M+1)/2-1} e^{-x^{MN/2}} dx \quad (20)$$

In the ^{149}Tb work $M = 1.0$ and $N = 2.0$.⁹ For this case

$$P(E) dE = 4E e^{-2E/\langle E \rangle} dE/\langle E \rangle^2 \quad (21)$$

For ^{24}Na nuclei recoiling into aluminum or plastic material, $N = 1.28$.⁶

With $M = 1.8$ Equation (20) becomes

$$P(E) dE = 2.4 E^{0.8} e^{-1.5(E/\langle E \rangle)^{1.15}} dE/\langle E \rangle^{1.8} \quad (22)$$

The distributions given by Equations (21) and (22) are almost identical (Figure 5). The Maxwellian form, Equation (21), is thus valid for both cases.

In the general case the presence of a coulomb barrier must be included.¹⁰ The distribution used here is Equation (21), shifted by the effective coulomb barrier E_0 .

$$P(E) dE = (E - E_0) e^{-(E - E_0)/\tau} dE/\tau^2 \quad (23)$$

where

$$\tau = (\langle E \rangle - E_0)/2 \quad (24)$$

For ^{24}Na from aluminum and ^{149}Tb from tantalum, gold, and bismuth the 2π differential-range results are consistent with $E_0 = 0$. The recoil of these products is probably due to a succession of "evaporation" or similar events. Whenever a charge particle is emitted, there will be a minimum recoil "kick", because of the coulomb barrier. The succession of these events results in a smearing out, or "random walk", process with a decrease in the value of E_0 .

When a single recoil event dominates, as in fission, the value of E_0 will be large. A similar effect has been found for light nuclei that

7.

are produced by bombarding heavy elements with high-energy protons.¹⁰ In that study the value of E_0 was found to be approximately one-half of the coulomb barrier, calculated for a radius parameter of 1.44 F. The calculation reported here will be repeated for various values of E_0 to determine the sensitivity of the final results to its value.

The average value of quantities of the form E^g , where g is a constant, is

$$\langle E^g \rangle = \int_{E_L}^{\infty} E^g P(E) dE = \tau^g G(g, E_L) \quad (25)$$

where

$$G(g, E_L) = \left[\Gamma\left(2+g, \frac{E_L}{\tau}\right) - \frac{E_0}{\tau} \Gamma\left(1+g, \frac{E_L}{\tau}\right) \right] e^{E_0/\tau} \quad (26)$$

and

$$\Gamma(w, y) = \int_y^{\infty} x^{w-1} e^{-x} dx \quad (27)$$

Thus,

$$\langle R \rangle = k \langle E^{N/2} \rangle = k \tau^{N/2} G(N/2, E_L) \quad (28)$$

The lower limit for the integrals in Equations (25) and (28) is $E_L = E_0$.

In order to evaluate $\langle E^g \rangle$, the value of τ must be determined. This will be done by an iterative process. An initial value of τ can be obtained with Equation (28) by substituting $\langle R \rangle \approx 2W(F+B)$ from Equation (12) and by setting $E_L = E_0 = 0$ to get

$$\tau \approx \left[\frac{2W(F+B)}{k\Gamma(2+N/2)} \right]^{2/N} \quad (29)$$

Substituting $E_L = E_0$ in Equation (25), we get

$$\left\langle \frac{V}{V} \right\rangle = G(-0.5, E_0) Y \quad (30)$$

where

$$Y^2 = mv^2/2\tau \quad (31)$$

When $E_0 = 0$, $\left\langle \frac{V}{V} \right\rangle = \frac{\sqrt{\pi}}{2} Y$.

From Equations (1), (17), and (31) we get

$$R_V = KV^N = kT^{N/2} Y^N \quad (32)$$

The following quantities, restricted to $V > v$, are needed in the remaining part of the analysis.

$$\left| \frac{\langle R \frac{V^n}{V^n} \rangle}{\langle R \rangle} \right|_{V > v} = A_n Y^n \quad (33)$$

where

$$A_n = \frac{G(\frac{N-n}{2}, E_L)}{G(\frac{N}{2}, E_L)} \quad (34)$$

and the integer $n \leq 4$, see Equation (11). The value of E corresponding to $V = v$ is τY^2 from Equation (31). Therefore, the value of E_L is the larger of E_0 or τY^2 in Equation (34). For $N \leq 2$ when $E_0 > 0$ we drop terms containing A_4 because our computer subroutine for Equation (27) is restricted to values of $w > 0$. There are similar restrictions for other terms for $N < 1$.

From Equations (11) and (33) we get

$$Y = \frac{f}{a_1 A_1} \frac{F - B}{F + B} \quad (35)$$

where

$$f = \frac{1 + a_2 A_2 Y^2 + a_4 A_4 Y^4}{1 + \frac{a_3 A_3}{a_1 A_1} Y^2} \quad (36)$$

We set $f = 1$ and $A_1 = \Gamma(1.5 + N/2)/\Gamma(2 + N/2)$, its value at $E_L = E_0 = 0$, to obtain the initial value of Y . The value of $\langle R \rangle$ can now be evaluated from

$$\langle R \rangle = 2W(F + B)/[1 + a_2 A_2 Y^2 + a_4 Y^4] \quad (37)$$

which is based on Equations (11) and (33).

For $E_0 \geq \tau Y^2$, V is always greater than v and $E_L = E_0$. For this case the iterations with Equations (28) and (33) through (37) are repeated until τ and Y converge to constant values. For $E_0 < \tau Y^2$ some values of V will

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be smaller than v . This source of recoils in the forward direction will be estimated in the next section.

$$V < v$$

For this case the target can be divided into three regions (Figure 6). In region I all of the nuclei with $V < v$ recoil into the forward catcher. In region II some nuclei escape, and in region III all remain. The fraction of the nuclei with $V < v$ from region I is

$$F_I = \frac{K(v-V)^N}{W} \quad (38)$$

The fraction that are produced in region II and escape the target foil is

$$F_{II} = \int (1 - \cos\theta) dt / 2W \quad (39)$$

evaluated at the limits $\cos\theta = \pm 1$. The sum of Equations (38) and (39) gives F' , the fraction nuclei with $V < v$ recoiling into the forward catcher.

$$F' = R_V(v/V) \left[\frac{(1+\frac{V}{v})^{N+3} - (1-\frac{V}{v})^{N+3}}{N+3} + \frac{1 - (\frac{V}{v})^2}{N+1} \left\{ (1+\frac{V}{v})^{N+1} - (1-\frac{V}{v})^{N+1} \right\} \right] / 4W \quad (40)$$

See also reference 11.

It is convenient to expand Equation (40) as a power series

$$F' = R_V \left[1 + d_1 \frac{V^2}{v^2} + d_2 \frac{V^4}{v^4} + d_3 \frac{V^6}{v^6} \right] / W \quad (41)$$

where

$$d_1 = \frac{(N+2)(N-1)}{3!} \quad (42)$$

$$d_2 = \frac{(N+2)N(N-1)(N-3)}{5!} \quad (43)$$

$$d_3 = \frac{(N^2-4)N(N-1)(N-3)(N-5)}{7!} \quad (44)$$

Equation (41) is exact for integral values of N from 1 to 5 and nearly so for intermediate values. In the general case, where $\frac{V}{v}$ has many values, quantities of the form $\frac{V^{2n}}{v^{2n}}$ become

$$\left\langle \frac{V^{2n}}{v^{2n}} \right\rangle = \frac{D_n}{Y^{2n}} = \frac{G(n, E_0) - G(n, E_L)}{Y^{2n}} \quad (45)$$

restricted to values of $E_L = \tau Y^2$ greater than E_0 . Thus,

$$F' = R_V [1 + d_1 D_1 / Y^2 + d_2 D_2 / Y^4 + d_3 D_3 / Y^6] / W \quad (46)$$

with R_V given by Equation (32).

The number of nuclei with $V < v$ recoiling into the forward catcher, divided by the total number of nuclei, is

$$F_2 = F' P(V < v) \quad (47)$$

where $P(V < v)$, the fraction of nuclei with $V < v$, is given by

$$P(V < v) = 1 - (1 + Y^2 - E_O / \tau) e^{-(Y^2 - E_O / \tau)} \quad (48)$$

$$\approx \frac{1}{2} (Y^2 - E_O / \tau)^2 \quad (49)$$

For $E_O \geq \tau Y^2$, $P(V < v) = 0$.

The number of nuclei with $V > v$ recoiling into the forward direction, divided by the total number, is

$$F_1 = F - F_2 \quad (50)$$

More accurate values of Y and τ can now be obtained by replacing F by F_1 in Equations (35), (36), and (37) and by substituting the more accurate value of $\langle R \rangle$ from Equation (37) into Equation (28). The calculations in this and the preceding sections are repeated until the values of τ , Y , and F_2 converge to constant values.

RESULTS

The values of various quantities can be obtained from Equations (24), (25), and (31). Thus $\langle \frac{V}{V} \rangle$ is given by Equation (30) and $\langle E \rangle$ by Equation (24). Also

$$\langle V \rangle = \left(\frac{2\tau}{m} \right)^{\frac{1}{2}} G(0.5, E_O) \quad (51)$$

11.

When $E_0 = 0$, $\langle V \rangle = \frac{3\sqrt{\pi}}{4} \left(\frac{\langle E \rangle}{m} \right)^{\frac{1}{2}}$.

The value of $\frac{V}{V}$ from a "thick-target thick-catcher" experiment, evaluated by means of Equation (5), is approximately equal to $\frac{\langle Rv/V \rangle}{\langle R \rangle}$, see Equation (11). The value of $\langle \frac{V}{V} \rangle$ can be obtained by multiplying this value by a correction factor, based on Equations (30) and (33).

$$\text{C.F.} = \frac{\langle \frac{V}{V} \rangle}{\frac{\langle Rv/V \rangle}{\langle R \rangle}} = \frac{G(\frac{N}{2}, E_0) G(-0.5, E_0)}{G(\frac{N}{2} - 0.5, E_0)} \quad (52)$$

When $E_0 = 0$, $\text{C.F.} = \frac{\sqrt{\pi}}{2} \frac{\Gamma(2 + N/2)}{\Gamma(1.5 + N/2)}$.

Table I

N	C.F. for $E_0 = 0$
1.0	1.178
1.5	1.258
2.0	1.333
2.5	1.405
3.0	1.473

The value of C.F. approaches unity as the value of E_0 approaches $\langle E \rangle$. This method is less accurate for large values of $\langle \frac{V}{V} \rangle$, because of the neglect of higher-order terms in Equation (11). For these cases the complete analysis may be required.

Poskanzer, Cumming, and Wolfgang compared values of $\frac{V}{V}$ for the reaction $^{27}\text{Al}(p, 3pn)^{24}\text{Na}$ from thin-target and thick-target experiments and obtained a ratio of 1.2.⁸ This agrees with the calculated value of 1.22 for C.F. ($N=1.28$ from the Northcliffe-Schilling tables⁶.) A similar agreement is obtained for production of ^{149}Tb by proton bombardments on gold, see Table II.

Results of the "thick-target thick-catcher" experiments on gold¹² were analyzed by the method reported here and are given on the left side of Table II. The analysis of the same results, based on the assumption that $\frac{V}{V} = \text{constant}$, was made in reference 12 and is reproduced in the next three columns. The values for $\langle \frac{V}{V} \rangle = \text{C.F.} \frac{V}{V}$ are given in the next column, where $\text{C.F.} = 1.333$ from Table I for $N = 2$. The results of earlier experiments with a thin target surrounded by many thin catcher foils are given in the last three columns.⁹

The values of $\langle E \rangle$ and $\langle \frac{V}{V} \rangle$ were determined by an analysis that was similar to the one reported here with one difference: the effect of range straggling was included in the thin-target experiments, but not in the present analysis.

Table II
p + $^{197}\text{Au} \rightarrow ^{149}\text{Tb}$

E_p	This analysis $E_0 = 0. \quad R = .231 E^{1.00}$ (ref. 12)						Analysis of reference 12			C.F. $\frac{V}{V}$ $\parallel$ $\langle \frac{V}{V} \rangle$	Thin target, reference 9		
	$P(V < v)$	v	$\langle R \rangle$	$\langle V \rangle$	$\langle E \rangle$	$\langle \frac{V}{V} \rangle$	$\langle R \rangle$	$\langle E \rangle$	$\frac{V}{V}$		$\langle E \rangle$	$\langle \frac{V}{V} \rangle$	F-B
		$(\frac{\text{Mev}}{\text{AMU}})^{\frac{1}{2}}$	$\frac{\text{mg}}{\text{cm}^2}$	$(\frac{\text{Mev}}{\text{AMU}})^{\frac{1}{2}}$	Mev		$\frac{\text{mg}}{\text{cm}^2}$	Mev			Mev		
1	.059	.099	.86	.21	3.7	.55	.91	3.9	.40	.53	3.8	.59	.53
3	.014	.067	.87	.21	3.8	.37	.89	3.9	.28	.37	3.7	.48	.44
11.5	.003	.046	.90	.22	3.9	.25	.91	3.9	.19	.25			
300	.002	.038	.83	.21	3.6	.22	.84	3.6	.16	.21			
Average value			.87	.21	3.8		.89	3.8					

The analysis of reference 12 and the one given here result in similar values for $\langle R \rangle$, $\langle E \rangle$, and $\langle \frac{V}{V} \rangle$. The best agreement is obtained when the values of $\langle \frac{V}{V} \rangle$ and hence $P(V < v)$ are small. When $N \neq 2$, $\langle E \rangle$ can be calculated from Equations (24) and (28) with $\langle R \rangle$ from Equations (5) and (12), or more accurately from the complete analysis presented here.

The values of $\langle E \rangle$, which are based on recoil ranges in gold, agree with the values from thin-target measurements of recoil ranges in aluminum. This agreement indicates that the two types of experiment give consistent results and also provides a check on the range-energy tables.⁶ The range-energy relation for ^{149}Tb recoiling into aluminum, used in reference 9, was measured directly,^{11,13} whereas the ranges in gold were based on stopping-power measurements for other systems.⁶ The effect of range straggling was neglected in our analysis, because of these uncertainties.⁷

The thick-target values of $\langle \frac{V}{V} \rangle$ in Table II agree with the thin-target values, based on the value of F-B and Equation (13), within 20 percent or better. The calculated effect of range straggling is included in the value of $\langle \frac{V}{V} \rangle$ given in the next-to-last column.

13.

A comparison of the values of the relative momentum transfer $\frac{p}{p_{CN}}$ is given in Table III. The momentum of the intermediate nucleus, given by p , corresponds to the velocity v . The momentum, calculated for a compound-nucleus reaction, is p_{CN} . The value of this ratio does not depend on the mass of the intermediate nucleus.

Table III

Values of p/p_{CN} for the reaction $p + {}^{197}\text{Au} \rightarrow {}^{149}\text{Tb}$

E_p (Gev)	Thick target, this analysis	Thin target, reference 9
1	.35	.38
3	.106	.13
11.5	.023	
300	.001	

For many reactions that are studied by the recoil method the value of E_0 may not be known. Fortunately, most results of the analysis are only slightly affected by the value of E_0 , see Table IV (except for $\langle \frac{v}{V} \rangle$). An approximate value for E_0 is sufficient for most purposes. The relative values for $\langle v \rangle$, which are equal to the square root of the values for $\langle E \rangle$, are not given in the table.

The analysis of experimental results for reactions in which E_0 is large is given in Tables V to VIII. For proton bombardments of gold (Tables V and VI) the value of E_0 was taken to be 38 Mev for ${}^{24}\text{Na}$ and 40 Mev for ${}^{28}\text{Mg}$, based on the observation that the effective coulomb barrier is $\sim .45$ times the nominal barrier, calculated for a radius parameter of $1.44 F$.¹⁰ The emitting nucleus was assumed to be ${}^{180}_{73}\text{Ta}$. When this procedure was followed for the experiments with uranium targets (Tables VII and VIII), with ${}^{220}_{86}\text{Rn}$ as the emitting nucleus¹⁰, the values obtained for E_0 were 43 Mev for ${}^{24}\text{Na}$ and 47 Mev for ${}^{28}\text{Mg}$. Since these values were larger than nearly all calculated values of $\langle E \rangle$, smaller values were used.

Table IV

Relative values ($= 1.0$ at $E_0 = 0$) for contrived cases: $R = E^{N/2}$, $2W(F+B) = 12.0$

N	$E_0/\langle E \rangle$	v	$\langle v/V \rangle$	$\langle R \rangle$	$\langle E \rangle$
F/B = 1.5					
1.0	.4	1.000	.87	1.002	.918
	.8	1.000	.85	1.002	.888
2.0	.4	.959	.81	1.001	1.001
	.8	.942	.76	1.002	1.002
3.0	.4	.938	.77	1.001	1.067
	.8	.906	.69	1.001	1.104
F/B = 9.0					
1.0	.4	1.014	.84	1.066	1.047
	.8	.959	.78	1.041	.960
2.0	.4	.956	.80	1.035	1.035
	.8	.936	.74	1.051	1.051
3.0	.4	.936	.76	1.025	1.084
	.8	.902	.69	1.041	1.132

Table V*

E_p	v	$\frac{p}{p_{CN}}$	$\langle \frac{v}{V} \rangle$	$\langle R \rangle$	$\langle V \rangle$	$\langle E \rangle$
(Gev)	$(\frac{\text{Mev}}{\text{AMU}})^{\frac{1}{2}}$			$(\frac{\text{mg}}{\text{cm}^2})$	$(\frac{\text{Mev}}{\text{AMU}})^{\frac{1}{2}}$	Mev
.7	.195	.88	.098	14.8	1.89	48.5
3	.262	.41	.135	13.9	1.83	45.7
6	.197	.17	.105	13.0	1.77	42.6
11.5	.147	.072	.080	12.5	1.74	41.1
28	.105	.022	.059	11.7	1.68	38.4
200	.088	.003	.051	11.1	1.64	36.5
300	.090	.002	.050	11.7	1.68	38.5

*The analysis of data from Table II of reference 14.

$E_0 = 38$ Mev (except at 200 Gev, where $E_0 = 30$ Mev).

$R = .305 E^{1.00}$ (reference 7).

15.

Table VI*

E_p	v	$\frac{p}{p_{CN}}$	$\langle \frac{v}{V} \rangle$	$\langle R \rangle$	$\langle V \rangle$	$\langle E \rangle$
(Gev)	$(\frac{\text{Mev}}{\text{AMU}})^{\frac{1}{2}}$			$(\frac{\text{mg}}{\text{cm}^2})$	$(\frac{\text{Mev}}{\text{AMU}})^{\frac{1}{2}}$	Mev
.7	.186	.84	.088	17.9	2.03	65.2
3	.278	.44	.140	16.0	1.90	57.3
6	.184	.16	.098	14.3	1.77	49.8
11.5	.138	.067	.074	14.1	1.76	49.0
28	.098	.021	.055	13.1	1.69	45.2
200	.082	.002	.047	12.7	1.65	43.2
300	.082	.002	.045	13.2	1.69	45.3

*The analysis of data from Table II of reference 14.

$E_0 = 40 \text{ Mev.}$

$R = .515 E^{.85}$ (reference 7).

Table VII*

E_p	v	$\frac{p}{p_{CN}}$	$\langle \frac{v}{V} \rangle$	$\langle R \rangle$	$\langle V \rangle$	$\langle E \rangle$
(Gev)	$(\frac{\text{Mev}}{\text{AMU}})^{\frac{1}{2}}$			$(\frac{\text{mg}}{\text{cm}^2})$	$(\frac{\text{Mev}}{\text{AMU}})^{\frac{1}{2}}$	Mev
.7	.170	.93	.107	13.2	1.52	31.2
3	.277	.53	.154	15.6	1.70	39.1
6	.252	.27	.145	14.8	1.64	36.4
11.5	.173	.102	.103	14.1	1.58	34.0
300	.096	.002	.061	12.7	1.48	29.8

*The analysis of data from Table II of reference 14.

$E_0 = 32 \text{ Mev}$ (except at .7 and 300 Gev, where $E_0 = 24 \text{ Mev}$).

$R = 1.00 E^{.75}$ (reference 6).

Table VIII*

E_p	v	$\frac{p}{p_{CN}}$	$\langle \frac{v}{V} \rangle$	$\langle R \rangle$	$\langle V \rangle$	$\langle E \rangle$
(Gev)	$(\frac{\text{Mev}}{\text{AMU}})^{\frac{1}{2}}$			$(\frac{\text{mg}}{\text{cm}^2})$	$(\frac{\text{Mev}}{\text{AMU}})^{\frac{1}{2}}$	Mev
.7	.164	.89	.090	17.1	1.73	47.2
3	.247	.47	.133	17.5	1.76	48.8
6	.223	.24	.125	16.3	1.67	44.3
11.5	.152	.089	.088	15.5	1.62	41.4
300	.089	.002	.055	14.3	1.54	37.4

*The analysis of data from Table II of reference 14.

$E_0 = 40$ Mev (except at 300 Gev, where $E_0 = 30$ Mev).

$R = .95 E^{.75}$ (reference 6).

Some of the values in Tables V to VIII can be compared with energy spectra measured with (dE/dx) -E counters. The value of $\langle E \rangle$ given in Table V for the reaction $p + {}^{197}\text{Au} \rightarrow {}^{24}\text{Na}$ at a proton energy of 28 Gev is 38.4 Mev. This value is near the peak value of the energy distribution of all Na isotopes, measured by Rensberg and Perry at a laboratory angle of 90° .⁴ A similar comparison for the reaction $p + {}^{238}\text{U} \rightarrow {}^{24}\text{Na}$ (or $\rightarrow {}^{28}\text{Mg}$) at 4.9 Gev shows a large discrepancy. The values given here, ~ 37 Mev for ${}^{24}\text{Na}$ (Table VII) and ~ 46 Mev for ${}^{28}\text{Mg}$ (Table VIII), are appreciably smaller than the peak energy values of the two elements.¹⁵ Part of this discrepancy may be due to the fact that specific isotopes were measured in the "thick-target thick-catcher" experiments, whereas many isotopes of each element were measured together in the counter experiments.

Another possible source of error was the range-energy relations that are used in the calculations for Tables VII and VIII. The accuracy of the analysis presented here depends directly on the accuracy of these relations. The constants k and N in Equation (17) for reactions with gold (Tables V and VI) are based on stopping-power measurements of several light elements, including magnesium, in gold.⁷ These range-energy relations are, therefore, expected to be more accurate than those for uranium, which are based on stopping-powers in other elements.⁶

Three effects have been neglected in the analysis given here:

1. A possible perpendicular component of v . The neglect of this component has little effect on the quantities calculated in this report for $N \approx 1$.¹⁶
2. Anisotropy in the distribution of V . This effect is small for fission reactions.^{16,17}
3. Scattering in the target and catcher foils. The inclusion of this effect results in lower values of $\langle E \rangle$ and related quantities.^{9,18}

None of these effects was included here, because they would have complicated the analysis without any clear improvement in the results. They should be included at a later time when the details of the stopping process are better known.

Although the interpretation of the results given here in terms of the reaction mechanism is outside the scope of this report, it is interesting to note the large momentum transfer (p/p_{CN}) observed below 1 Gev for the production of ^{24}Na , ^{28}Mg , and ^{149}Tb from heavy elements (Tables V to VIII and reference 9).

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CAPTIONS

- Figure 1. Arrangements of foils in the usual type of "thick-target thick-catcher" recoil experiment. The center foil is the target. The catcher foils on either side are usually some material of low atomic weight, like aluminum or plastic.
- Figure 2. Two-step model.
- Figure 3. Results of the 2π differential-range experiment on the reaction $^{27}\text{Al}(p, 3p_n)^{24}\text{Na}$. The ordinate is the ^{24}Na activity in each catcher foil divided by the thickness of the foil. The catcher foils were Formvar plastic. This figure is from reference 8.
- Figure 4. Data of Figure 3 fitted to Equation (15).
- Figure 5. Solid line -- the distribution given by Equation (21).
Dashed line -- the distribution given by Equation (22).
- Figure 6. For $V < v$ target consists of three regions: I - all recoils escape into forward catcher. II - some recoils escape. III - none escape in forward direction. (This figure is drawn for the case where $N = 1$ in Equations (1) and (17)).

Figure 1.

Figure 2.

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Figure 3.

Figure 4.

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Figure 5.

Figure 6.

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