Lawrence Berkeley National Laboratory
Recent Work

Title
LOCALLY COUPLED EVOLUTION OF WAVE AND PARTICLE DISTRIBUTION IN GENERAL MAGNETOPLASMA GEOMETRY

Permalink
https://escholarship.org/uc/item/5v57h6xr

Authors
McDonald, S.W.
Grebogi, C.
Kaufman, A.N.

Publication Date
1985-05-01

eScholarship.org Powered by the California Digital Library
University of California

LBL-15829
Preprint c.2

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

RECEIVED
LAWRENCE
BERKELEY LABORATORY

JUL 10 1985

LIBRARY AND
DOCUMENTS SECTION

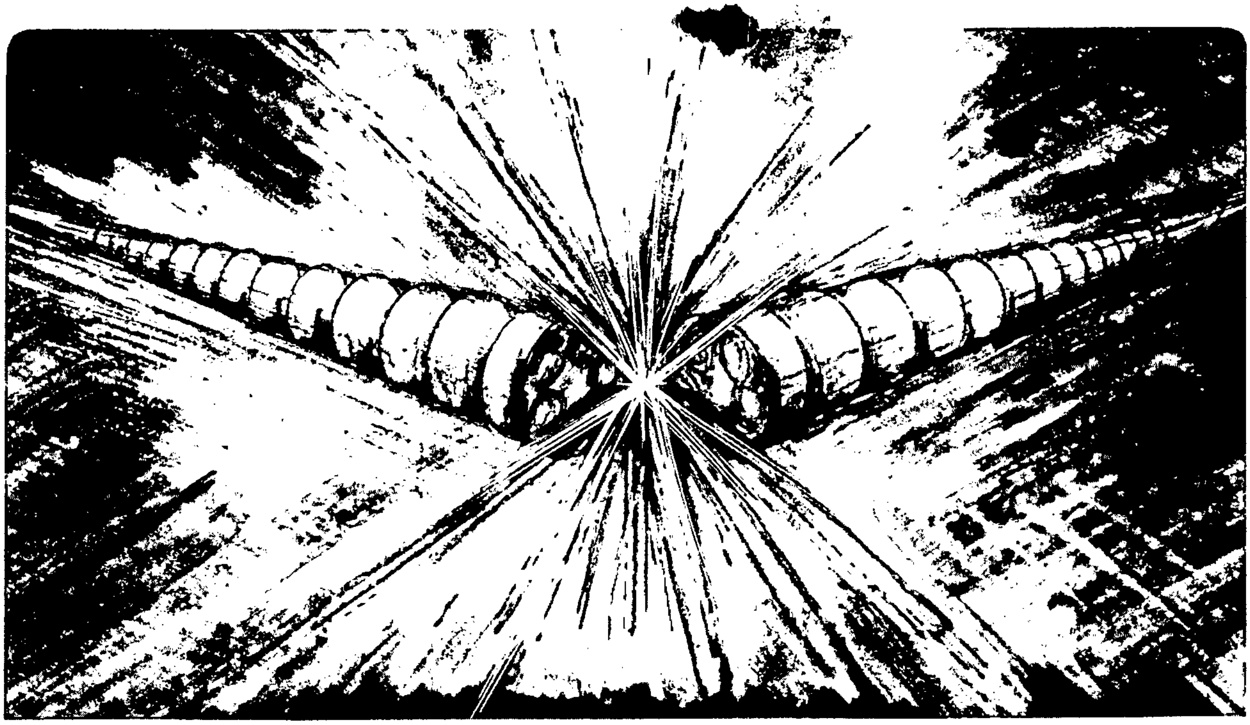
Accelerator & Fusion Research Division

Submitted to Physics Letters A

LOCALLY COUPLED EVOLUTION OF WAVE AND PARTICLE DISTRIBUTION IN GENERAL MAGNETOPLASMA GEOMETRY

S.W. McDonald, C. Grebogi, and A.N. Kaufman

May 1985



LBL-15829 c.2

Prepared for the U.S. Department of Energy under Contract DE-AC03-76SF00098

DISCLAIMER

This document was prepared as an account of work sponsored by the United States Government. While this document is believed to contain correct information, neither the United States Government nor any agency thereof, nor the Regents of the University of California, nor any of their employees, makes any warranty, express or implied, or assumes any legal responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by its trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof, or the Regents of the University of California. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof or the Regents of the University of California.

Submitted to Physics Letters A

LBL-15829

Locally Coupled Evolution of Wave and Particle Distribution in General Magnetoplasma Geometry*

Steven W. McDonald,+ Celso Grebogi,+ and Allan N. Kaufman

Lawrence Berkeley Laboratory
and
Physics Department
University of California
Berkeley, CA 94720

May 1985

Abstract

Self-consistent kinetic equations are obtained for the evolution of plasma-wave action density in ray phase space and oscillation-center density in guiding-center phase space. Both resonant and nonresonant interactions are included, in a strong, weakly-nonuniform magnetic field. Energy conservation and entropy production are the hallmarks of self-consistency.

* This work was supported by the U.S. Department of Energy under Contract No. DE-AC03-76SF00098.

\+ Present address: Laboratory for Plasma and Fusion Energy Studies, University of Maryland.

Many years ago Dewar [1] formulated the concept of the oscillation center in terms of a well-defined canonical tranformation of particle phase space. He derived an evolution equation for F(z), the oscillation-center phase-space density, which clearly separated the nonresonant effect of the waves, represented by the ponderomotive Hamiltonian K(z), and the quasilinear diffusion of F due to resonant interaction with the waves. He then proposed, on an intuitive basis, a self-consistent evolution equation for the (incoherent) wave-action density J(y) in ray phase space $y = (\underline{k},\underline{x})$, and obtained from these two equations an appropriate set of energy-momentum conservation laws.

Dewar's pioneering work pertained to an unmagnetized plasma; in the present paper we consider the analogous, but considerably more complex, problem for a weakly non-uniform magnetized plasma. We draw on a number of new techniques and understandings developed since, to obtain this generalization, as well as to include wave emission by discrete particles. We list these important new tools, which are essential for our derivation: (1) the Weyl symbol calculus leads rigorously to the wave kinetic equation for J(y) [2], allowing the inclusion of incoherent discrete-particle sources; (2) the guiding-center phase-space is represented in Littlejohn's symplectic form, in terms of Poisson brackets [3]; (3) the canonical Lie transform [4] is used to transform the linear wave-perturbation Hamiltonian to second order, thereby obtaining K(z) in terms of a Lie derivative; (4) the K-χ theorem [5] is used to relate the linear susceptibility $\chi(y)$, needed for the wave kinetic equation, to the Hamiltonian K(z).

With these powerful techniques, the derivation is reasonably straightforward. In this paper we present the results in detail, and refer to some extensions of this work.

To simplify the notation, we suppress the labels for particle species and wave branches, and set e=m=c=1. Our results for the oscillation centers are expressed in terms of the Poisson brackets on functions of the gyrophase θ, its conjugate the magnetic moment μ, parallel momentum p, and guiding-center position $\underline{X}$, denoted collectively as z. Since F(z) and K(z) are both (by construction) θ-independent, their Poisson bracket [3]

$$\{F,K\} = \underline{b}\cdot[\nabla K\times\nabla F + (\nabla F)(\partial K/\partial p)-(\nabla K)(\partial F/\partial p)]/B^* \qquad (1)$$

contains no μ-derivative. Here $\underline{b}(\underline{X})$ is the direction of the magnetic field $\underline{B}$, while $B^* = \underline{b}\cdot\underline{B}^*$, with $\underline{B}^* = \underline{B} + p\,\nabla\times\underline{b}$.

The oscillation-center Hamiltonian

$$K(z) = H_o(z) + K_2(z) \qquad (2)$$

consists of its "unperturbed" part [3]

$$H_o(z) = p^2/2 + e\phi(\underline{X}) + \mu B(\underline{X}) \qquad (3)$$

and the wave-induced ponderomotive part [6]

$$K_2(z) = \int d^6y\; J(y)\;\delta\omega(y)/\delta F(z). \qquad (4)$$

(Here $d^6y = d^3x\; d^3k/(2\pi)^3$.) The frequency $\omega(\underline{k},\underline{x})$ depends functionally on F(z), and thus yields K_2 directly.

To obtain this frequency, we first consider the local susceptibility, $\underset{\sim}{\chi}(\underline{k},\omega;\underline{x},\underline{t})$, which is the centered local Fourier transform [2] of the two-point susceptibility, obtained by Lie transform methods:

$$\underset{\sim}{\chi}\,(\underline{k},\omega;\underline{x},t) = -\omega_s^2(\underline{x},t)\underset{\sim}{\underline{I}}/\omega^2 + \Sigma_\ell \int d^6z\,(\omega - d_\ell K)^{-1}\,\underset{\sim}{\alpha}_\ell\; d_\ell F/\omega^2. \qquad (5)$$

(Here $\omega_s^2 = 4\pi n_s e_s^2/m_s$). This expression involves the characteristic Lie operator

$$d_\ell = \{\ell\theta + \underline{k}\cdot\underline{X},\;\cdot\;\} = \ell\partial/\partial\mu + \underline{k}\cdot(\underline{B}^*\partial/\partial p + \underline{b}\times\nabla)/B^* \qquad (6)$$

which arises when the interaction Hamiltonian is expressed as a Fourier series in gyrophase (exp $i\ell\theta$). In differential geometric language, it is the Hamiltonian vector field generated by exp $(i\ell\theta + i\underline{k}\cdot\underline{X})$. The corresponding Fourier coefficients

$$\underline{i}_\ell = (\dot{\underline{X}} + \ell\Omega(\underline{X})\underline{k}_\perp/k_\perp^2)\, J_\ell + (\underline{b}\times\underline{k}/k_\perp^2)\, 2i\Omega\mu\, (\partial J_\ell/\partial\mu) \tag{7}$$

form the coupling tensor

$$\underset{\sim}{\alpha}_\ell = \underline{i}_\ell \underline{i}_\ell^* \, \delta^3(\underline{x}-\underline{X}-\ell\underline{b}\times\underline{k}/k_\perp^2), \tag{8}$$

which appears in (5). (In (7), J_ℓ is the usual Bessel function, and $\Omega = eB/mc$ is the local gyrofrequency.) The resonance denominator of (5),

$$\omega - d_\ell K = \omega - \{\ell\theta + \underline{k}\cdot\underline{X},\ K\} = \omega - \ell\dot{\theta} - \underline{k}\cdot\dot{\underline{X}}, \tag{9}$$

thus includes the ponderomotive drifts and gyrofrequency shift. The numerator of (5), $d_\ell F$, involves all the gradients of F; the action of the same operator d_ℓ on both F and K is of course crucial for the conservation laws.

The susceptibility $\underset{\sim}{\chi}$ (to be precise, its Hermitian part) is combined with the vacuum local Maxwell dispersion tensor to form the dispersion tensor [2] $\underset{\sim}{D}(\underline{k},\omega;\underline{x},t)$. Its local eigenvalues $D(\underline{k},\omega;\underline{x},t)$ and eigenvectors $\hat{e}(\underline{k},\omega;\underline{x},t)$ then yield, by setting D=0, the frequency function $\omega(\underline{k},\underline{x};t)$ and the corresponding polarization $\hat{e}(\underline{k},\underline{x};t)$.

Since $D = \hat{e}^*\cdot\underset{\sim}{D}\cdot\hat{e}$, we can obtain $\delta\omega/\delta F$ by implicit functional differentiation:

$$\delta\omega(y)/\delta F(z) = [\delta^3(\underline{x}-\underline{X}) - d_\ell(\alpha_\ell(\omega - d_\ell K)^{-1})]/\omega^2\bar{D}, \tag{10}$$

where $\bar{D} = \partial D/\partial\omega$ and $\alpha_\ell = \hat{e}^*\cdot\underset{\sim}{\alpha}_\ell\cdot\hat{e}$. Substituting (10) into (4), and using the definition [2] of J:

$$\langle\underline{EE}\rangle\ (\underline{k},\omega;\underline{x},t) = 4\pi\hat{e}\hat{e}^*\,\delta(D)\,J, \tag{11}$$

we obtain an expression for K_2 in terms of $\langle\underline{EE}\rangle$, in agreement with what we can obtain directly [7] by Lie transforms.

The frequency function $\omega(\underline{k},\underline{x};t)$ is the ray Hamiltonian, and appears as the generator of the flow in ray phase space. The non-resonant propagation is given by

$$\{J(y), \omega(y)\} = (\partial J/\partial \underline{x})\cdot(\partial\omega/\partial\underline{k}) - (\partial J/\partial\underline{k})\cdot(\partial\omega/\partial\underline{x}) \qquad (12)$$

and preserves the action, as is well known. (Note that here {,} is a canonical bracket on ray phase space; which bracket is meant is always clear from the context.)

The full evolution, including linear resonant interaction, is found (by the methods of Ref. (2)) to be

$$\partial J/\partial t + \{J,\omega\} = \int d^6z\ \Sigma_\ell\ \Gamma_\ell, \qquad (13)$$

where the ℓth gyroresonant coupling is given by

$$\Gamma_\ell(y,z) = 2\pi\ \delta(\omega(y)-d_\ell K(z))\ \alpha_\ell\ [J(y)d_\ell F(z) + F(z)]/\overline{D}, \qquad (14)$$

and $d^6z = 2\pi B^* d^3X d\mu dp$ is the Liouville measure of guiding-center phase space. The term of (14) linear in J represents gyroresonant damping/growth, with the same Lie derivative $d_\ell F$ as in (5), as expected from Kramers-Kronig. The term independent of J represents incoherent emission by gyroresonant discrete particles.

The self-consistent evolution of F must preserve the total energy $\mathcal{H}(F,J) = \int d^6y\ J\ \omega + \int d^6z\ F\ H_0$. This determines the dissipative terms of its kinetic equation:

$$\partial F/\partial t + \{F,K\} = \int d^6y\ \Sigma_\ell d_\ell \Gamma_\ell. \qquad (15)$$

Inserting (14) into (15), we see that (15) is a Fokker-Planck equation of remarkably concise form. The term linear in J represents quasi-linear diffusion in $(\underline{X},\mu,p)$. It is easy to verify that the local diffusion matrix is symmetric. The term independent of J is the radiation reaction due to the emission process. This evolution equation can be obtained directly, of course, by generalizing Dewar's averaging method to deal with the non-canonical Poisson structure [8].

A crucial test for the self-consistency of these coupled evolution equations is the entropy theorem. Defining the entropy functional $S(J,F) = \int d^6y \ln J(y) - \int d^6z\, F(z) \ln F(z)$, we immediately obtain the desired monotonic increase of entropy:

$$dS/dt \sim \delta(\omega - d_\ell K)\ (J\, d_\ell F + F)^2. \tag{16}$$

Note that this dissipation is due only to gyroresonant interaction. Thus, as expected, this interaction produces a tendency toward a thermal equilibrium given by $J\, d_\ell F + F = 0$, or

$$J^{-1} = -\, d_\ell \ln F. \tag{17}$$

In the absence of other symmetries, the unique solution of (17) is the Gibbs distribution $F(z) \sim \exp[-\beta K(z)]$ and the Rayleigh-Jeans distribution $J(y)\omega(y) = \beta^{-1}$, where β is the effective temperature of the resonant particles and waves, and the resonance condition $\omega = d_\ell K$ of (16) has been used.

The set (13), (15) is incomplete, in that the evolution of the background fields $\underline{B},\phi$ is omitted. A proper treatment of them requires knowing the quasistatic polarization and magnetization associated with the wave distribution. This aspect has recently been investigated [9], using a Lorentz-covariant phase-space Lagrangian approach. The results will be reported in a future publication, together with the associated energy-momentum conservation laws.

These results were presented at the April 1982 Controlled Fusion Theory Conference at Santa Fe, NM. We are indebted to Dr. Gary Smith for checking our results, uncovering some sign errors, and discussing the application of our formalism to practical situations [10].

References

1. R. Dewar, Phys. Fluids 16 (1973) 1102.
2. S. McDonald and A. Kaufman, "The Weyl Representation for Electromagnetic Waves: The Wave Kinetic Equation," submitted to Phys. Rev. A; S. W. McDonald, Ph.D. thesis, Univ. of Cal. Berkeley, 1983, LBL-14837, Univ. Microfilms #8413506.
3. R. Littlejohn, J. Plasma Physics 29 (1983) 111.
4. J. Cary, Physics Reports 79 (1981) 129.
5. J. Cary and A. Kaufman, Phys. Fluids 24 (1981) 1238; A. Kaufman and D. Holm, Physics Letters 105A (1984) 277.
6. A. Kaufman, Physica Scripta T2/2 (1982) 517.
7. C. Grebogi, A. Kaufman, and R. Littlejohn, Phys. Rev. Lett. 43 (1979) 1668.
8. A. Kaufman, J. Wurtele, and S. Omohundro, in preparation.
9. B. Boghosian, P. Similon, and A. Kaufman, in preparation.
10. B. Boghosian and G. Smith, Bull. Am. Phys. Soc. (1982) 1116.

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

Reference to a company or product name does not imply approval or recommendation of the product by the University of California or the U.S. Department of Energy to the exclusion of others that may be suitable.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720