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EWPD Constraints on Flavor Symmetric Vector Fields

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Abstract

Electroweak precision data constraints on flavor symmetric vector fields are determined. The flavor multiplets of spin one that we examine are the complete set of fields that couple to quark bi-linears at tree level while not initially breaking the quark global flavor symmetry group. Flavor safe vector masses proximate to, and in some cases below, the electroweak symmetry breaking scale are found to be allowed. Many of these fields provide a flavor safe mechanism to explain the $t\bar{t}$ forward backward anomaly, and can simultaneously significantly raise the allowed values of the Standard Model Higgs mass consistent with electroweak precision data.

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I. INTRODUCTION

The global flavor symmetry group of the Standard Model (SM) is only broken by the Yukawa matrices. As a result, the SM predicts a non-trivial and definite pattern in flavor changing decays and meson mixing observables; there are no flavor changing neutral currents (FCNC's) at tree level, and flavor changing charged currents follow the pattern of the CKM matrix. The predicted pattern of flavor changing observables in the SM is consistent with what is observed in precise measurements of $\text{Br}(b \rightarrow s + \gamma)$, $K^0 - \bar{K}^0$, $B^0 - \bar{B}^0$ mixing, etc. The lack of any apparent pattern of statistically significant deviations in these observables¹ places strong constraints on New Physics (NP) at the TeV scale invoked to resolve the hierarchy problem of the electroweak symmetry breaking (EWSB) scale, v , in the SM.

One way to attempt to reconcile these facts with NP that appears at the \sim TeV scale is to assume that the NP has a non-generic flavor structure. A popular assumption that can frequently accommodate this tension is to assume that NP has exactly the dominant flavor breaking pattern that has been experimentally established, *i.e.*, that NP that couples to quarks has the same flavor breaking structure as in the SM. This is known as the principle of Minimal Flavor Violation (MFV), which states that there is a unique source of flavor symmetry breaking [5–7] at scales measurable in flavor physics observables. Another, closely related assumption is that a TeV scale new physics sector is completely flavor symmetric with respect to the quark flavor symmetry group $G_F = \text{U}(3)_Q \times \text{U}(3)_U \times \text{U}(3)_D$. Breaking of this flavor symmetry is not completely absent because the quark masses already break G_F through Yukawa interactions

$$\mathcal{L}_Y = Y_U \bar{u}_R H^T i \sigma_2 Q_L - Y_D \bar{d}_R H^\dagger Q_L + \text{h.c.} \quad (1)$$

and this breaking will appear in loop corrections to the couplings of the NP to quark bilinears. This flavor breaking however will follow the SM pattern and be of an MFV

¹ Note that some sets of measurements do show interesting patterns of deviations from the SM, in particular the measurement of the CP violating phase in B_s mixing at the Tevatron through dimuon final states and $B \rightarrow J/\psi\phi$; see [1, 2] for related discussions. The recently reported LHC_b measurement [3] of $B \rightarrow J/\psi\phi$ does not support a NP interpretation of this data, while recent updates to $D\phi$ measurements of the same decay [4] are still able to accommodate the large phase required by the dimuon anomaly.

form.

Further flavor breaking can be MFV like or deviate from the pattern expected in MFV. The top quark is the only quark with an $\mathcal{O}(1)$ Yukawa coupling, y_t . This large coupling generally forces phenomenologically viable $\sim v$ scale NP flavor breaking effects $\propto y_t$ to be aligned with the SM flavor breaking. This can be taken to mean that flavor breaking involving the third generation ($G_F \rightarrow H_F$ where $H_F = U(2)_Q \times U(2)_U \times U(2)_D \times U(1)^3$) should be aligned with the SM. Versions of this scenario are referred to as next to minimal flavor violation in the literature [8].

The scalar and vector fields that can couple to the SM at dimension four respecting SM gauge symmetry and not breaking G_F have recently been classified and studied in some detail [9–11]. In this paper, we examine oblique electroweak precision data (EWPD) corrections due to the flavor symmetric vector field multiplets of this form in much more detail. We seek to perform as general an analysis of EWPD as possible on phenomenologically interesting flavor symmetric vector fields in this paper, and consequently only consider the (large) $G_F \rightarrow H_F$ flavor breaking in the mass spectrum. Sub-dominant flavor breaking could be MFV-like — where all flavor breaking comes with SM Yukawa insertions — or could deviate from the SM pattern of flavor breaking. In the later case, potentially important flavor changing observable based constraints could exist. For oblique EWPD constraints, only the leading flavor breaking we include is expected to be relevant. In fact, we will show that the breaking of G_F is not directly related to the breaking of custodial symmetry $SU_C(2)$ to the approximation we work to, and thus we can neglect flavor breaking in general when considering oblique EWPD constraints. We find that vector masses $m_v \sim v$ are consistent with EWPD in most of the allowed representations, and in certain cases in large regions of parameter space. The allowed vector representations [11] are shown in Table 1.

Note that interest in flavor symmetric fields has been increased recently due to the ability of a number of these representations to explain the CDF and DØ measurements [12, 13] of an anomalously large $t\bar{t}$ forward backward asymmetry, $A_{FB}^{t\bar{t}}$. Flavor symmetric fields can in principle explain this anomaly in a flavor safe manner [14]. In addition, some fields of this form can potentially explain both the $t\bar{t}$ anomaly and same sign dimuon

anomaly reported in [15] by $D\bar{O}$ at the same time [11].²

EWPD constrains the vector fields we study through effectively constraining the higher dimensional operators that are present in the theory when the NP is integrated out. Oblique EWPD constraints can generally be studied by using the STU parameters [17–19] which assumes that the masses of the new states are $\sim \text{TeV}$. When considering masses comparable to (or lower than) the EWSB scale, the STUWVX formalism of [20] which does not expand in v^2/m_{NP}^2 is preferred and can be more constraining [21]. The latter formalism reduces to the STU parameters when large NP scales are present and we will use the STUWVX formalism of [20] in this paper, as we are interesting in studying how light the vector multiplet masses can be.

² However, it should be noted that recent LHC results on the $t\bar{t}$ invariant mass spectrum [16] and the measurement of time dependent $B \rightarrow J/\psi\phi$ have cast some doubt on the NP interpretation of Tevatron anomalies in $A_{FB}^{t\bar{t}}$ or the dimuon anomaly [3].

II. ELECTROWEAK SECTOR OF MFV VECTOR LAGRANGIANS

Table I lists the G_F symmetric representations that couple to quark bilinears without Yukawa suppression, while preserving SM gauge invariance.³ In this section, we construct the gauge sector of these Lagrangians⁴ for cases V-XI and determine the contribution of these fields to the self energies of the SM gauge bosons. As massive flavor symmetric vectors are effective fields in a non-renormalizable extension of the SM, we also consider the contribution of higher-dimension operators, which are suppressed by the cutoff scale, Λ , of the effective theory [23]. These operators are necessary to obtain finite oblique corrections in some cases. One naively expects the scaling $\Lambda \sim 4\pi m_v$, but the separation of these scales can be smaller. Indeed, this might be expected to be the case due to the relatively large number of degrees of freedom in the vector multiplets. As a non-perturbative study is clearly beyond the scope of this initial work, we assume that the operators suppressed by higher powers of Λ that we do not retain are sufficiently suppressed.

Case	$SU(3)_c$	$SU_L(2)$	$U(1)_Y$	$SU(3)_{U_R} \times SU(3)_{D_R} \times SU(3)_{Q_L}$	couples to
$I_{s,o}$	1,8	1	0	(1,1,1)	$\bar{d}_R \gamma^\mu d_R$
$II_{s,o}$	1,8	1	0	(1,1,1)	$\bar{u}_R \gamma^\mu u_R$
$III_{s,o}$	1,8	1	0	(1,1,1)	$\bar{Q}_L \gamma^\mu Q_L$
$IV_{s,o}$	1,8	3	0	(1,1,1)	$\bar{Q}_L \gamma^\mu Q_L$
$V_{s,o}$	1,8	1	0	(1,8,1)	$\bar{d}_R \gamma^\mu d_R$
$VI_{s,o}$	1,8	1	0	(8,1,1)	$\bar{u}_R \gamma^\mu u_R$
$VII_{s,o}$	1,8	1	-1	($\bar{3}$,3,1)	$\bar{d}_R \gamma^\mu u_R$
$VIII_{s,o}$	1,8	1	0	(1,1,8)	$\bar{Q}_L \gamma^\mu Q_L$
$IX_{s,o}$	1,8	3	0	(1,1,8)	$\bar{Q}_L \gamma^\mu Q_L$
$X_{\bar{3},6}$	$\bar{3},6$	2	-1/6	(1,3,3)	$\bar{d}_R \gamma^\mu Q_R$
$XI_{\bar{3},6}$	$\bar{3},6$	2	5/6	(3,1,3)	$\bar{u}_R \gamma^\mu Q_R$

TABLE I: G_F symmetric vector representations from Ref.[11].

³ Cases I-IV are flavor singlets and are already discussed extensively in the literature. See Ref. [22] for a recent discussion.

⁴ See [11] for the Yukawa sectors of these models and related phenomenological constraints.

A. Cases V, VI, and VIII

These cases are not charged under the electroweak gauge group. The vector fields are parameterized using Gell-Mann matrices for the color, τ^A , and the flavor, t^B , representations. The Lagrangians in the mass eigenstate basis for cases $C = \{V, VI, VIII\}$ are

$$\begin{aligned}\mathcal{L}_C &= (1 + \delta_{a,o}) \left(-\mathcal{L}_C^{\text{kin}} + \mathcal{L}_C^{\text{mass}} \right) + \mathcal{L}_C^{\text{int}} + \mathcal{L}_C^{\text{Yuk}} + \text{h.c.}, \\ \mathcal{L}_C^{\text{kin}} &= \frac{1}{2} \text{Tr} \left(V^{a;\mu\nu} V_{\mu\nu}^{a;\dagger} \right) + \frac{\kappa}{2} \text{Tr} \left(V_{\mu\nu}^s \Delta_C \right) B^{\mu\nu}, \\ \mathcal{L}_C^{\text{mass}} &= \left(m^2 + \lambda H^\dagger H \right) \text{Tr} \left(V^{a;\mu} V_\mu^{a;\dagger} + \zeta_1 V_\mu^a \Delta_C V_\mu^{a;\dagger} + \zeta_2 \Delta_C V_\mu^a \Delta_C V_\mu^{a;\dagger} + \dots \right), \\ \mathcal{L}_C^{\text{int}} &= -\beta \text{Tr} \left(V_\mu^s \Delta_C \right) H^\dagger D^\mu H,\end{aligned}\tag{2}$$

where $a = o, s$ stands for the octet and singlet sub-cases and $V_{\mu\nu}^s = \partial_\mu V_\nu^s - \partial_\nu V_\mu^s$. The trace is over flavor space. In the color octet case, a covariant derivative is needed and the trace extends to color space. Note that the flavor symmetry ensures that the flavor basis of the vector fields that couple through the operators shown is the same as the flavor basis obtained after rotating to the quark mass field basis in \mathcal{L}^{Yuk} , *i.e.*, that no further flavor violation is present due to a misalignment of the flavor eigenbases.

We have included explicit insertions of Yukawa matrices in $\mathcal{L}_C^{\text{mass}}$ as an illustrative example. A series of flavor breaking insertions are also possible on the other terms in the Lagrangian inside the trace. Using the formalism of MFV, the insertions are parameterized by a series in powers of $\Delta_C = \{Y_d Y_d^\dagger, Y_u Y_u^\dagger, Y_u^\dagger Y_u\}$ for $C = \{V, VI, VIII\}$. For flavor breaking insertions in the kinetic terms, we can always re-diagonalize the fields with a finite renormalization and we neglect the resulting mass splittings, implicitly absorbing these splittings into the leading order mass definition for each flavor. The insertion of Yukawa matrices into the mass terms causes mass splittings among the different flavors. We retain the leading flavor breaking due to the top or bottom Yukawa leading to the mass spectrum

$$m_{1,2,3}^2 = m^2 + \frac{\lambda}{2} v^2, \quad m_{4,5,6,7}^2 = m_1^2 \left(1 + \frac{\zeta_1}{2} y^2 \right), \quad m_8^2 = m_1^2 \left(1 + \frac{2\zeta_1}{3} y^2 + \frac{2\zeta_2}{3} y^4 \right).\tag{3}$$

Here we have used the conventions $\langle H \rangle = v/\sqrt{2}$ and $\text{Tr}(\tau^a \tau^b) = \text{Tr}(t^a t^b) = \delta^{ab}/2$.

$\mathcal{L}_C^{\text{int}}$ arises only in the color singlet case, and is suppressed by an insertion of Δ_C . This operator leads to tree level mixing of the SM gauge bosons with the new vector

multiplet suppressed by the appropriate Yukawa matrices. When only third generation Yukawa matrices are retained, only the 8 flavor component mixes. For simplicity, we only consider a single mixing with Z , A and V , treating $\beta, \kappa \ll 1$ rather than sum the geometric series that results if all insertions of Δ_C are unsuppressed.⁵ We treat this mixing as a perturbation. The fields are transformed to a new field basis \tilde{V}_8, \tilde{B} with diagonalized kinetic terms in the presence of the kinetic mixing between the V^8 colour singlet vector and the B field. The required transformation on the field basis is

$$\begin{pmatrix} V_8^\alpha \\ B^\alpha \end{pmatrix} = \begin{pmatrix} 1 & \frac{2\kappa y^2}{\sqrt{3}} \frac{m_B^2}{m_B^2 - m_8^2} \\ \frac{2\kappa y^2}{\sqrt{3}} \frac{m_8^2}{m_8^2 - m_B^2} & 1 \end{pmatrix} \begin{pmatrix} \tilde{V}_8^\alpha \\ \tilde{B}^\alpha \end{pmatrix} \quad (4)$$

This transformation leaves the bare tree level mass terms of the B field (after EW symmetry breaking), m_B , and the tree level mass of the vector field V_8 unchanged. We neglect the interaction terms in \mathcal{L}^{Yuk} , that are discussed in Ref. [11], assuming the direct coupling to the light quarks is small enough that tree level vector exchanges can be neglected, and an oblique EWPD analysis is appropriate. Dijet constraints on the coupling of these fields to light quarks at LHC generically constrain this coupling to be $\sim \mathcal{O}(0.1)$ which is consistent with this assumption.⁶ Consistency of this analysis also requires we neglect the tree level exchanges $\mathcal{O}(g^2 \kappa^2)$ from the field redefinition of the B field in the covariant derivative in the quark kinetic terms. We also neglect $\mathcal{O}(\beta \kappa)$ contributions in the Lagrangian after the field redefinitions — these effects can be removed by a higher order re-diagonalization of the mass and kinetic operators. With these assumptions, the color singlet vector fields give contributions to the self-energies of the SM gauge bosons,

$$\Pi_{ZA}(p^2) = 0, \quad \Pi_{AA}(p^2) = 0, \quad \Pi_{ZZ}(p^2) = \frac{|\beta|^2 v^2 m_Z^2 y^4}{12(p^2 - m_8^2)}. \quad (5)$$

Here y is the appropriate Yukawa coupling. Only terms proportional to $g_{\mu\nu}$ are shown.

⁵ One can always reinterpret this parameter to correspond to a series of insertions $\beta \Delta_C + \beta' (\Delta_C)^2 + \dots$.

⁶ However, the coupling of these fields that involve the top quark could be far larger due to flavor splitting effects, allowing these fields to still explain the $A_{FB}^{t\bar{t}}$ anomaly while this EWPD analysis is appropriate; see [11] for a more detailed discussion.

B. Case VII

Case VII is a weak singlet, but has non-zero hypercharge, $Y = -1$. We expand the fields in terms of the color Gell-Mann matrices, τ^A . $V_\mu^\dagger \neq V_\mu$ because the V_μ fields are in the $(\bar{3}, 3, 1)$ representation of G_F . The fields are flavor bi-fundamentals, $V^\mu = (V^\mu)^i_j$, where i and j are the indices of the $(\bar{3}, 1, 1)$ and $(1, 3, 1)$ representations respectively. The Lagrangians in the mass eigenstate basis are

$$\begin{aligned} \mathcal{L}_{VII} &= (1 + \delta_{a,o}) (-\mathcal{L}_{VII}^{\text{kin}} + \mathcal{L}_{VII}^{\text{mass}}) + \mathcal{L}_{VII}^{\text{int}} + \mathcal{L}_{VII}^{\text{c.t.}} + \mathcal{L}_{VII}^{\text{Yuk}} + \text{h.c.}, \\ \mathcal{L}_{VII}^{\text{kin}} &= \text{Tr} (V^{a;\mu\nu} V_{\mu\nu}^{a;\dagger}) + i g_1 \xi B^{\mu\nu} \text{Tr} ([V_\mu^a, V_\nu^{a;\dagger}]), \\ \mathcal{L}_{VII}^{\text{mass}} &= (m^2 + \lambda H^\dagger H) \text{Tr} (V^{a;\mu} V_\mu^{a;\dagger} + \zeta_1 V^{a;\mu} Y_u Y_u^\dagger V_\mu^{a;\dagger} + \zeta_2 V^{a;\mu} Y_u Y_u^\dagger V_\mu^{a;\dagger} Y_d Y_d^\dagger + \dots), \\ \mathcal{L}_{VII}^{\text{int}} &= -\beta \text{Tr} [(V_\mu^s \Delta_{VII})^\dagger] (D^\mu H)^\dagger \tilde{H}, \end{aligned} \quad (6)$$

where $V_{\mu\nu}^s = D_\mu V_\nu^s - D_\nu V_\mu^s$, $D_\mu = \partial_\mu - i g_1 Y B_\mu$, $\Delta_{VII} = Y_u Y_d^\dagger$, and $\tilde{H} = i\sigma_2 H^*$ (and as before a color gauge field term is included in the covariant derivative in the case of color octet vector). The mass splittings for case VII are as follows

$$m_{11,21,12,22}^2 = m^2 + \frac{\lambda}{2} v^2, \quad m_{13,31,23,32}^2 = m_{11}^2 \left(1 + \frac{\zeta_1}{2} y_t^2 \right), \quad (7)$$

$$m_{33}^2 = m_{11}^2 \left(1 + \frac{\zeta_1}{2} y_t^2 + \frac{\zeta_2}{2} y_b^2 y_t^2 \right). \quad (8)$$

$\mathcal{L}^{\text{c.t.}}$ contains all the terms needed to make this theory finite, which includes higher-dimensional operators as the theory is non-renormalizable. See Section II E for details on these operators. The vector field multiplet contributions to the self-energies of the SM gauge bosons are,

$$\begin{aligned} \Pi_{ZZ}(p^2) &= D(R_C) s_w^2 g_1^2 \sum_{f=1}^9 [f + \xi g + \xi^2 h] (p^2, m_f^2), \\ \Pi_{ZA}(p^2) &= -D(R_C) s_w c_w g_1^2 \sum_{f=1}^9 [f + \xi g + \xi^2 h] (p^2, m_f^2), \\ \Pi_{AA}(p^2) &= D(R_C) c_w^2 g_1^2 \sum_{f=1}^9 [f + \xi g + \xi^2 h] (p^2, m_f^2), \\ \Pi_{WW}(p^2) &= \delta_{a,s} \frac{y_t^2 y_b^2 |\beta|^2 v^2 m_W^2}{8(p^2 - m_{33}^2)} \end{aligned} \quad (9)$$

where $D(R_C)$ is the dimension of the color representation. Here s_w, c_w are the sine and cosine of the weak mixing angle with convention $e = g_1/c_w$, and g_1 is the hypercharge

coupling. The form factors f, g, h are defined in the Appendix A.

C. Case IX

Case IX is a weak triplet that has zero hypercharge. In addition to being parameterized by τ^A and t^B for color and flavor, the fields are also parameterized using the Pauli matrices, $V_\mu = \sigma_i V_\mu^i$, for the $SU_L(2)$ representation. We suppress the color singlet, octet label on the field in this section for clarity⁷ while including the $SU_L(2)$ index, the Lagrangians are

$$\begin{aligned}
\mathcal{L}_{IX} &= (1 + \delta_{a,o}) (-\mathcal{L}_{IX}^{\text{kin}} + \mathcal{L}_{IX}^{\text{mass}}) + \mathcal{L}_{IX}^{\text{int}} + \mathcal{L}_{IX}^{\text{c.t.}} + \mathcal{L}_{IX}^{\text{yuk}} + \text{h.c.}, \quad (10) \\
\mathcal{L}_{IX}^{\text{kin}} &= \frac{1}{2} \text{Tr} (V_i^{\mu\nu} V_{\mu\nu}^{i;\dagger}) + \frac{\kappa}{2} \text{Tr} (V_{\mu\nu}^i \Delta_{IX}) W_i^{\mu\nu} - g_2 \xi \epsilon_{ijk} W^{i;\mu\nu} \text{Tr} (V_\mu^j V_\nu^{k\dagger}), \\
\mathcal{L}_{IX}^{\text{mass}} &= (m^2 + \lambda H^\dagger H) \text{Tr} (V_i^\mu V_\mu^{i;\dagger} + \zeta_1 V_i^\mu \Delta_{IX} V_\mu^{i\dagger} + \zeta_2 \Delta_{IX} V_i^\mu \Delta_{IX} V_\mu^{i;\dagger} + \dots), \\
\mathcal{L}_{IX}^{\text{int}} &= -\beta \text{Tr} (V_\mu^i \Delta_{IX}) H^\dagger \sigma_i D^\mu H,
\end{aligned}$$

where $V_{\mu\nu}^i = (D_\mu V_\nu)^i - (D_\nu V_\mu)^i$, $D_\mu^{ij} = \partial_\mu \delta^{ij} - g_2 \epsilon^{ijk} W_{k\mu}$, and $\Delta_{IX} = Y_u^\dagger Y_u$. The mass splittings are the same as in Case VIII. In this section we neglect the effects of the kinetic mixing operator considering the case $\kappa \ll \beta, g_2 \xi, g_2$.⁸

The contributions to the self-energies of the SM gauge bosons are then

$$\begin{aligned}
\Pi_{ZZ}(p^2) &= 2 D(R_C) c_w^2 g_2^2 \sum_{f=1}^8 [f + \xi g + \xi^2 h] (p^2, m_f^2) + \delta_{a,s} \frac{|\beta|^2 y_t^4 v^2 m_Z^2}{12(p^2 - m_8^2)}, \quad (11) \\
\Pi_{ZA}(p^2) &= 2 D(R_C) s_w c_w g_2^2 \sum_{f=1}^8 [f + \xi g + \xi^2 h] (p^2, m_f^2), \\
\Pi_{AA}(p^2) &= 2 D(R_C) s_w^2 g_2^2 \sum_{f=1}^8 [f + \xi g + \xi^2 h] (p^2, m_f^2), \\
\Pi_{WW}(p^2) &= 2 D(R_C) g_2^2 \sum_{f=1}^8 [f + \xi g + \xi^2 h] (p^2, m_f^2) + \delta_{a,s} \frac{|\beta|^2 y_t^4 v^2 m_W^2}{3(p^2 - m_8^2)}.
\end{aligned}$$

⁷ The κ, β operators are only for the singlet case as before.

⁸ The effect of the kinetic mixing with a nonabelian field has recently been studied in [24, 25] for example and our results can be directly extended to include kinetic mixing. Many of the effects of kinetic mixing can be absorbed into a redefinition of the remaining unknown parameters of this model once a diagonalization of the kinetic terms is undertaken. The transformation to canonical kinetic terms is exactly of the form given in Section II A for each isospin state when the kinetic mixing is not neglected.

D. Cases X and XI

These fields are $SU_L(2)$ doublets, flavor bi-fundamentals, and are in either the color sextet or anti-triplet representations, $a = 6, \bar{3}$. The only difference between the two cases is they have different hypercharges, $Y_C = \{-1/6, 5/6\}$ for $C = \{X, XI\}$. The Lagrangians are

$$\begin{aligned} \mathcal{L}_C &= -\mathcal{L}_C^{\text{kin}} + \mathcal{L}_C^{\text{mass}} + \mathcal{L}_C^{\text{c.t.}} + \mathcal{L}_C^{\text{yuk}} + \text{h.c.}, \\ \mathcal{L}_C^{\text{kin}} &= \frac{1}{2} V^{a;\mu\nu} V_{\mu\nu}^{a;\dagger} + i g_1 \xi_1 B^{\mu\nu} V_\mu^a V_\nu^{a;\dagger} + i g_2 \xi_2 V_\mu^a W^{\mu\nu} V_\nu^{a;\dagger}, \\ \mathcal{L}_C^{\text{mass}} &= (m^2 + \lambda_1 H^\dagger H) (V^{a;\mu} V_\mu^{a;\dagger} + \zeta_1 V^{a;\mu} \Delta_C V_\mu^{a;\dagger} + \dots) \\ &\quad + \lambda_2 (H^{\alpha\dagger} V_\alpha^{a;\mu} V_\mu^{a;\beta\dagger} H_\beta + \dots) + \lambda_3 (\tilde{H}^{\alpha\dagger} V_\alpha^{a;\mu} V_\mu^{a;\beta\dagger} \tilde{H}_\beta + \dots), \end{aligned} \quad (12)$$

where $V_{\mu\nu}^6 = D_\mu V_\nu^6 - D_\nu V_\mu^6$, $D_\mu = \partial_\mu - i g_3 \tau_6^A A_\mu^A - i g_2 \sigma^i W_\mu^i - i g_1 Y B_\mu$, and $\Delta_C = Y_u Y_u^\dagger$ in both cases. In the last line of $\mathcal{L}_C^{\text{mass}}$ the weak indices, α and β , are explicit. We did not explicitly write down in the Lagrangian the flavor breaking insertions in the additional Higgs terms that appear in the mass splittings below. In these cases there is a mass splitting in the electroweak doublet

$$m_{Q_+}^2 = m^2 + (\lambda_1 + \lambda_2) \frac{v^2}{2}, \quad m_{Q_-}^2 = m^2 + (\lambda_1 + \lambda_3) \frac{v^2}{2} \quad (13)$$

where $Q_\pm = Y \pm 1/2$. Note that if there were no mass splitting between the weak states then there would be no contribution to $\Pi_{W^3 B}$ because the interaction has the form $\text{Tr}(V_\mu^\dagger \sigma^3 V^\mu)$. In addition there is the usual mass splitting in flavor space

$$m_{11,12,21,22}^2 = m_{Q_\pm}^2, \quad m_{13,23,32,31,33}^2 = m_{Q_\pm}^2 (1 + \zeta y_t^2) \quad (14)$$

The contribution of the vector fields to the self-energies of the SM gauge bosons are written in the electroweak basis in this case to reduce clutter,

$$\begin{aligned} \Pi_{W^3 W^3}(p^2) &= D(R_C) \frac{g_2^2}{4} \sum_{f,L} [f + \xi_2 g + \xi_2^2 h] (p^2, m_{f,L}^2), \\ \Pi_{W^3 B}(p^2) &= D(R_C) \frac{g_2 g_1 Y}{2} \sum_{f,L} (-1)^{L-1} \left[f + \frac{1}{2} (\xi_1 + \xi_2) g + \xi_1 \xi_2 h \right] (p^2, m_{f,L}^2), \\ \Pi_{BB}(p^2) &= D(R_C) g_1^2 Y^2 \sum_{f,L} [f + \xi_1 g + \xi_1^2 h] (p^2, m_{f,L}^2), \\ \Pi_{W^1 W^1}(p^2) &= D(R_C) \frac{g_2^2}{2} \sum_f [f + \xi_2 g + \xi_2^2 h] (p^2, m_{f,Q_+}^2, m_{f,Q_-}^2), \end{aligned} \quad (15)$$

where the sums over flavor f and weak L states run from 1 to 9 and 2 respectively. For contributions from Π_{W^3B} , it is the difference of weak states rather than the sum that contributes. The factor of $(-1)^{L-1}$ accounts for this. For Π_{WW} , there is no sum over weak state because both particles of the weak doublet need to be in the loop to conserve electric charge. Note that the mass splitting among weak states causes Π_{WW} to be a function of both masses.

The S parameter is negative in certain regions of parameter space for cases X and XI. For example, with $\xi_1 = \xi_2 = 0$ and $m_{\pm}^2 \gg m_Z^2$, $S \propto Y_C \ln(m_{\pm}^2/m_Z^2)$.

E. Counterterms and Higher-Dimensional Operators

The vectors are effective fields, and the Lagrangians contain an infinite number of non-renormalizable operators. At low scales, p , compared to the cutoff scale of the effective theory, Λ , the contribution of these terms to self-energies is suppressed by powers of $(p/\Lambda)^n$. The ratio of scales can be set by the external momentum p^2/Λ^2 or can be set by the ratio of other invariants m_v^2/Λ^2 depending on the operator of interest. Because of this suppression we neglect contributions to the self-energies from almost all of these other operators. The reason why we did not neglect the contributions from all of the higher dimensional operators is explained in what follows.

Being psuedo-observable, the STUVWX parameters must be free of divergences and independent of the renormalization point μ . Contributions to the self-energies have two origins. Contributions of the first kind contain no powers of momentum in the numerator from internal propagators. As expected, the resulting STUVWX parameters are finite and μ -independent. These are the only terms that would be found in a renormalizable theory. There is no need to add a field renormalization term, such as $Z_B B^{\mu\nu} B_{\mu\nu}$, to the Lagrangian because its contribution to each of these parameters is identically zero.

The second type of contributions contain all of the other terms that come about from at least one propagator's $p_\mu p_\nu$ piece. In this case, contributions to the STUVWX parameters are divergent and require higher-dimensional operators as counter terms. $\mathcal{L}_C^{\text{c.t.}}$ contains all the counterterms necessary to absorb the divergences of the self-energies from the dimension-4 operators. Not all of the operators are needed as counterterms in each case.

With this understanding, the counterterms are

$$\begin{aligned}
\mathcal{L}_C^{\text{c.t.}} = & \frac{Z_1}{\Lambda^2} \partial_\rho B_{\mu\nu} \partial^\rho B^{\mu\nu} + \frac{Z_2}{\Lambda^4} \partial_\tau \partial_\rho B_{\mu\nu} \partial^\tau \partial^\rho B^{\mu\nu} + \frac{Z_3}{\Lambda^2} |H^\dagger D_\mu H|^2 \\
& + \frac{Z_4}{\Lambda^2} \text{Tr} (D_\rho W_{\mu\nu} D^\rho W^{\mu\nu}) + \frac{Z_5}{\Lambda^4} \text{Tr} (D_\tau D_\rho W_{\mu\nu} D^\tau D^\rho W^{\mu\nu}) \\
& + \frac{Z_6}{\Lambda^2} H^\dagger W_{\mu\nu} H B^{\mu\nu} + \frac{Z_7}{\Lambda^4} H^\dagger D_\rho W_{\mu\nu} H \partial^\rho B^{\mu\nu} + \frac{Z_8}{\Lambda^6} H^\dagger D_\tau D_\rho W_{\mu\nu} H \partial^\tau \partial^\rho B^{\mu\nu}
\end{aligned} \tag{16}$$

The divergences come in the form $2/\varepsilon - \gamma + \ln(4\pi\mu^2/m^2)$ (using dimensional-regularization in $D = 4 - \varepsilon$ dimensions) and we use the operators to cancel the divergences and μ dependence. In practice, we take $\mu = 1$ TeV for numerical evaluations and include in our fit a finite contribution from one higher-dimensional operator to absorb the μ -dependence when a divergence is canceled by a higher dimensional operator.

III. FIT TO ELECTROWEAK PRECISION DATA

Electroweak precision data provides strong constraints on the MFV vectors under consideration. A convenient subset of the full set of corrections are the oblique corrections. Purely NP contributions to the self-energies of the electroweak gauge bosons can be written in the following form $\Pi_{ab}^{\mu\nu}(k) = \Pi(k^2)_{ab} g^{\mu\nu} + f(k^2) k^\mu k^\nu$ where $ab = \{W^+W^-, ZZ, AA, ZA\}$. When the masses of the new states are heavy compared to the EWSB scale, the vacuum polarizations can be expanded in momentum $\Pi_{ab}(q^2) \approx \Pi_{ab}(0) + q^2 \Pi'_{ab}(0)$ and the STU parameters can be used. However, when one wishes to consider masses comparable to (or smaller than) the EWSB scale, this expansion in momentum becomes invalid. In that case, the STUWVX parameters (defined in [20]), which do not expand in v^2/m_{NP}^2 , can be used and can be more constraining. The theoretical predictions of EWPD of the 2008 PDG [26] was used to perform a modern fit to the STUVWX parameters in Ref. [21]. The results of the fit are given in Table II. The correlation coefficient matrix of the fit is given by

$$\rho = \begin{pmatrix} 1 & 0.60 & 0.38 & -0.57 & 0 & -0.86 \\ 0.60 & 1 & -0.49 & -0.95 & 0 & -0.13 \\ 0.38 & -0.49 & 1 & 0.46 & -0.01 & -0.76 \\ -0.57 & -0.95 & 0.46 & 1 & 0 & 0.13 \\ 0 & 0 & -0.01 & 0 & 1 & 0 \\ -0.86 & -0.13 & -0.76 & 0.13 & 0 & 1 \end{pmatrix}. \tag{17}$$

The least-squared estimators $\hat{\boldsymbol{\theta}}$ for the set of parameters $\boldsymbol{\theta}$ of a given model, is determined by the minimum of

$$\chi^2(\boldsymbol{\theta}) = (\mathbf{y} - \mathbf{F}(\boldsymbol{\theta}))^T V^{-1} (\mathbf{y} - \mathbf{F}(\boldsymbol{\theta})) \quad (18)$$

where \mathbf{y} is a vector of the best-fit values of the STUVWX parameters, V^{-1} is the inverse of the covariance matrix $V_{ij} = \sigma_i \rho_{ij} \sigma_j$, and $\mathbf{F}(\boldsymbol{\theta})$ is the corresponding vector of predicted values of the model. We determine the 1- and 2- σ confidence regions of allowed parameter space by requiring that the contribution of NP to $\chi^2(\boldsymbol{\theta})$ satisfy

$$\chi^2(\boldsymbol{\theta}) \leq \chi_{min}^2 + \Delta\chi^2 \quad (19)$$

where $\Delta\chi^2$ corresponds to the probability that the confidence region of parameter space determined with the cumulative distribution function and contains the best fit value of the parameters.

IV. RESULTS

In this section, constraints on flavor symmetric vectors are discussed on a case-by-case basis. Mass splittings among flavor states are generally assumed to be small and not relevant for these constraints. This follows from the fact that there are no loops with different flavor multiplet species to the approximation we work to, so the breaking of G_F is not linked to the breaking of custodial symmetry $SU_C(2)$. Note however that in certain

Oblique	$y \pm \sigma$
S	0.07 ± 0.41
T	-0.40 ± 0.28
U	0.65 ± 0.33
V	0.43 ± 0.29
W	3.0 ± 2.5
X	-0.17 ± 0.15

TABLE II: EWPD fit to STUVWX results of Ref. [21]. y is the best-fit value, and σ is the square root of the diagonal element of the determined covariance matrix.

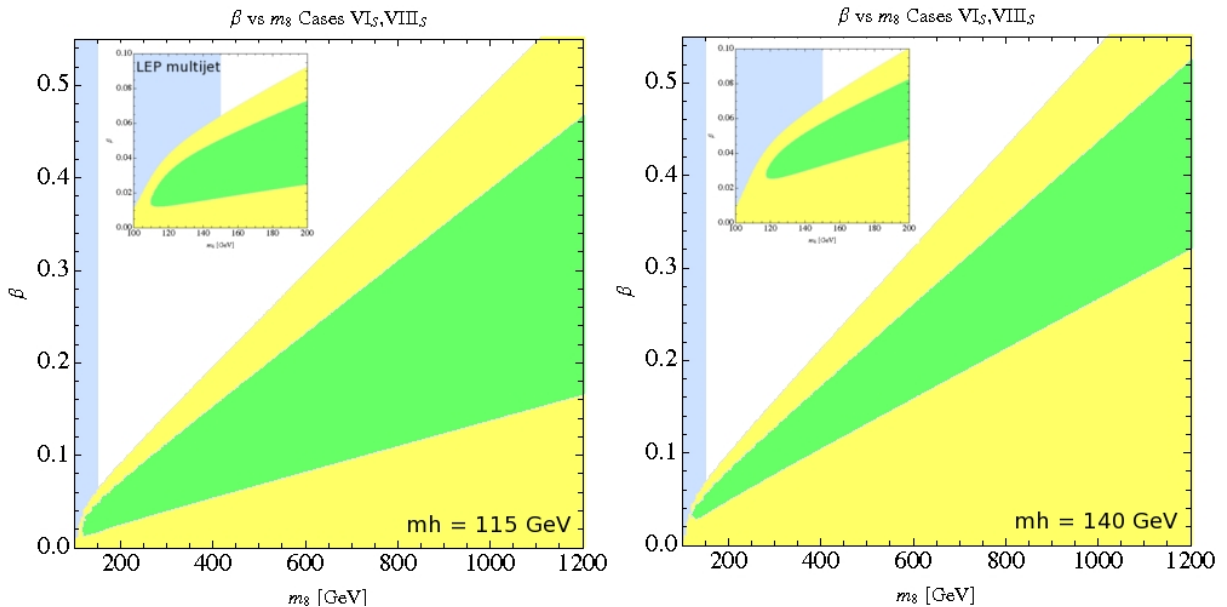


FIG. 1: In all of the plots, the green and yellow regions represent regions of parameter space that are allowed at 1- and 2- σ respectively. Also shown is the approximate bound from LEP due to multijet final states when the coupling to light quarks is not neglected. Note that these bounds are complementary in that the oblique analysis fails in this case.

cases, such as the constraint plots for Section A, what is plotted is the constraint space for β and m_8 . The other vectors in these multiplets are split in general from the mass m_8 with a splitting of $\mathcal{O}(\zeta_{1,2})$ and the appropriate Yukawa suppression.

A. Cases V, VI, and VIII

In all of these color octet cases, EWPD does not place limits on the parameters of the model in the approximation that we are working to — the relevant operator is forbidden. However, the absence of vector pair-production at LEP implies a kinematic bound for color singlet and octet vectors of at least $m_V \gtrsim 105$ GeV. When the direct coupling of the vectors to quarks is $\mathcal{O}(1)$, the experimental bound become stronger, $m_V \gtrsim 150$ GeV from anomalous multi-jet events at LEP [11]. We also include the latter bound in the figure. Dedicated collider searches can significantly raise these mass bounds.

The bounds from EWPD on case V_s are particularly weak as $y_b \ll 1$ and the other parameters in this model are unconstrained. In theories such as the large $\tan \beta$ limit of the MSSM where y_b becomes $\mathcal{O}(1)$, the constraints are similar to cases VI_s and $VIII_s$. In

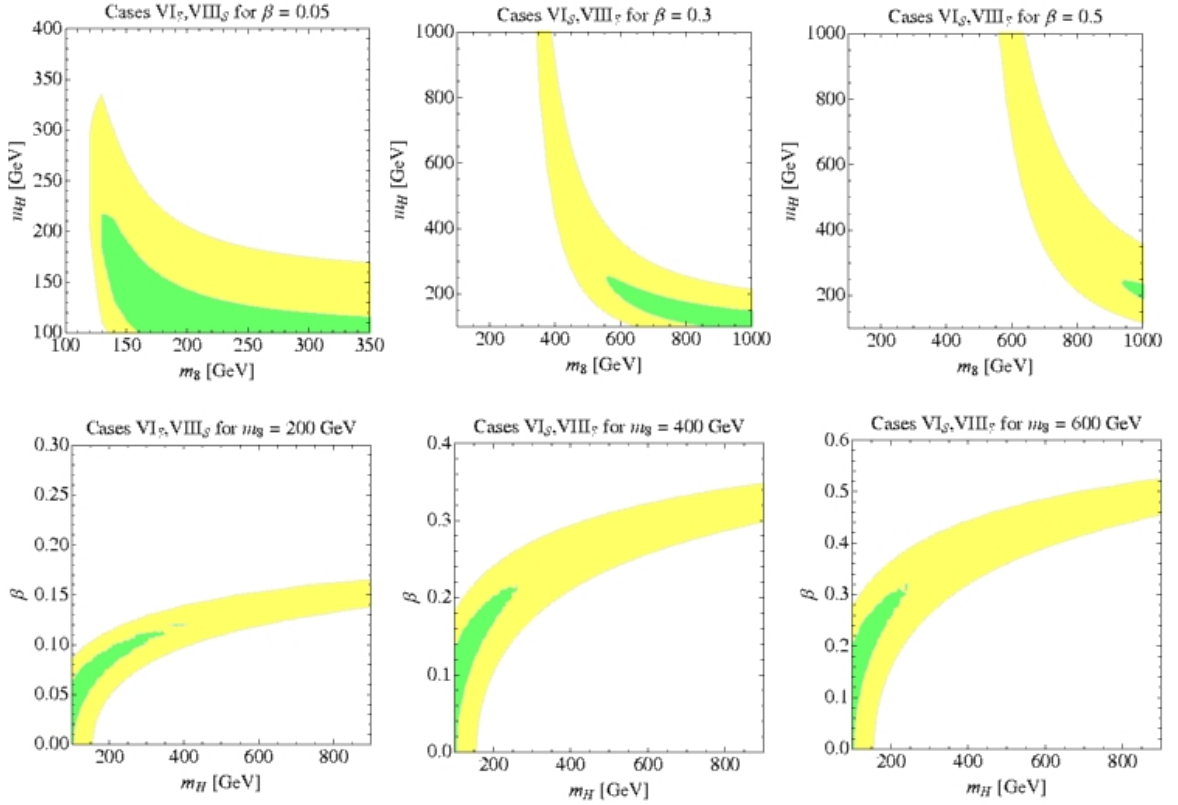


FIG. 2: Allowed confidence regions at 1- and 2- σ when the Higgs mass is raised through a $SU_C(2)$ violating operator - due to coupling to a flavor symmetric vector multiplet.

Fig. 1 the Higgs mass is fixed to $m_h = 115(140)$ GeV in the left (right) figure with its one loop contribution to the EW parameters floated from a reference value of $\hat{m}_h = 96$ GeV in the fit.

Operators such as $\text{Tr}(V_\mu^s \Delta_C) H^\dagger D^\mu H$ lead to a violation of $SU_C(2)$ and can act to raise the Higgs mass by giving a positive contribution to the T parameter. We illustrate the effect on the best fit value of the Higgs mass in Fig. 2. The entire light mass region of the Higgs in the SM may be excluded in the near future by CMS and/or ATLAS or a combination of the experimental data sets. Simple (flavor safe) mechanisms to raise the Higgs mass in the EWPD as demonstrated here would then be of greater interest. This mechanism also exists in several of the remaining cases when an operator of this form is allowed. We will generally fix the Higgs mass in what follows to reduce the parameter space.

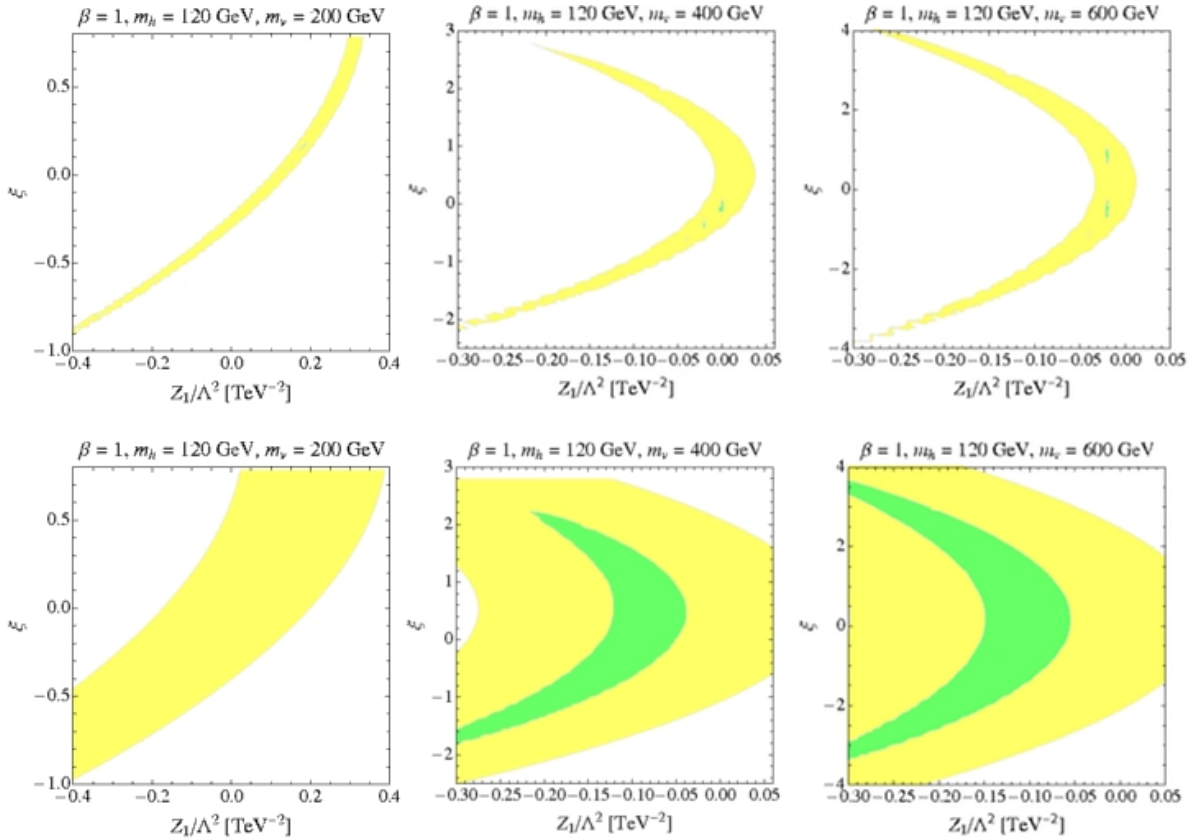


FIG. 3: Space of allowed ξ and Z_1/Λ^2 for cases VII_o (top) and VII_s (bottom). Here we have set $\zeta_{1,2} = 0$, neglecting flavour breaking. The regions shown are weakly dependent on variations in β , we have fixed $\beta = 1$ and $m_h = 120$ GeV. Left to right the masses are $m_v = 200, 400, 600$ GeV.

B. Case VII

Eliminating the μ -dependence from STUVWX determines a relationship between ξ and Z_1/Λ^2 as a function of the other (allowed) parameters in the model. For numerical purposes, we ignore the dimension-8 operator with coefficient Z_2 . Fig. 3 shows the relationships between ξ and Z_1/Λ^2 for various masses. The allowed regions in the multi-dimensional parameter space has a nontrivial dependence on the various parameters, as is further illustrated in Fig. 4. Generically the parameter space for VII_s is less constrained than VII_o. Both cases require a strong correlation between the finite part of the counter term and the remaining parameters to not be ruled out.

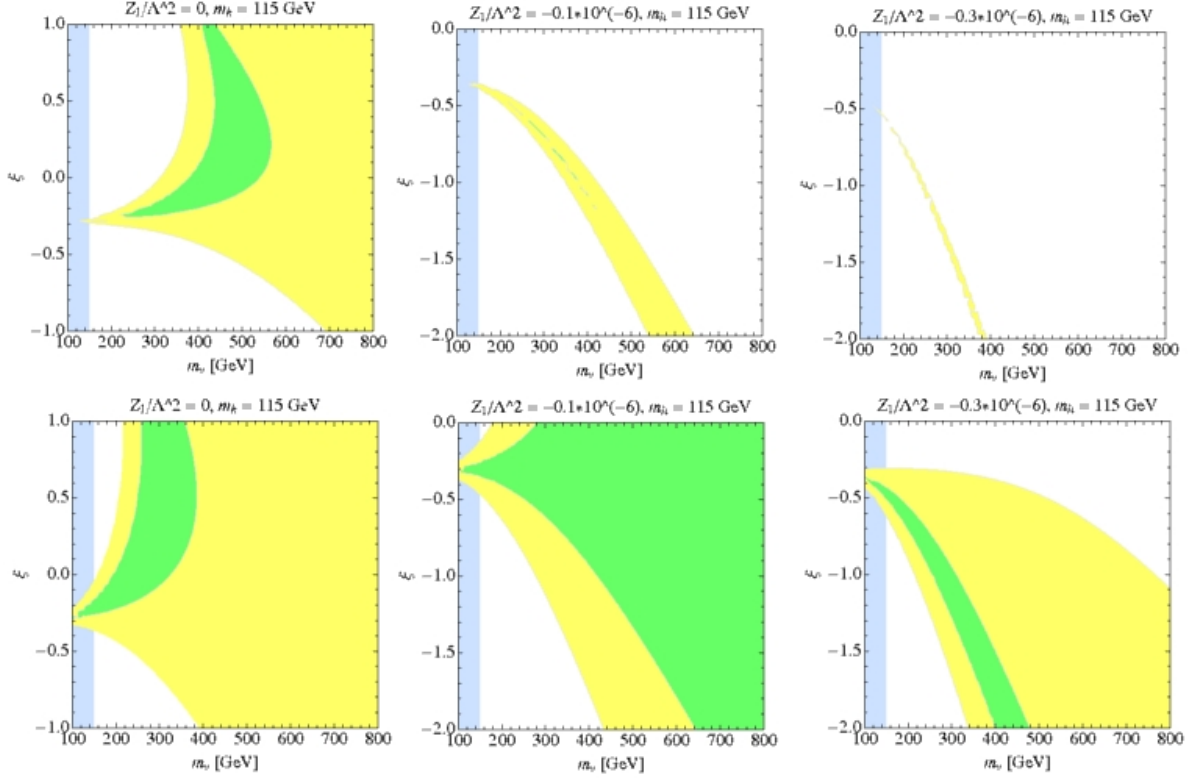


FIG. 4: Space of allowed ξ and m_V for cases VII_o (top) and VII_s (bottom). Here we neglecting flavor breaking and β as the allowed parameter space is weakly dependent on these parameters. We have set $m_h = 115$ GeV. Left to right the counterterm values are $Z_1/\Lambda^2 = (0, -0.1, -0.3) \times \text{TeV}^{-2}$.

C. Case IX

The operator proportional to β again leads to a relaxation of the Higgs mass bounds as in Cases VI_s, VIII_s. We set $\beta = 0$ and $m_h = 115$ GeV in what follows and examine the remaining parameter space. This case has the strongest constraints from oblique EWPD. A strong correlation is required between ξ , m and Z_4/Λ^2 for the allowed parameter space. This is illustrated in Fig. 5 for IX_s. We do not find viable parameter space for IX_o.

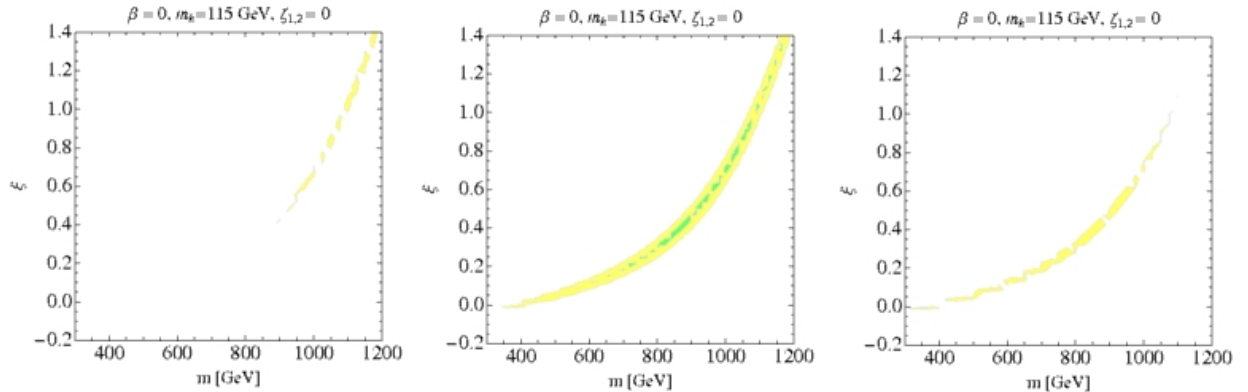


FIG. 5: Space of allowed ξ and m for case IX_s. Here we have set $\beta = 0$ and used $m_h = 115$ GeV. Left to right the counterterm is $Z_4/\Lambda^2 = (-3, 0, 3) \times 10^{-2} \text{ TeV}^{-2}$.

D. Cases X and XI

There is no operator proportional to $H^\dagger D^\mu H$ in these cases that directly violates $SU_C(2)$ and allows the Higgs mass to be raised. In cases X_{6,3} the allowed parameter space dependence on the parameter ξ_1 is trivial, not showing a significant correlation with Z/Λ^2 , ξ_2 or m_V . The correlation between ξ_2 and Z/Λ^2 in the allowed parameter space is shown in Fig. 6. The allowed masses in a joint fit with fixed $m_h = 115$ GeV is shown in Fig. 7 for model X₃. For comparison the required correlations between the parameters for case XI₃ are shown in Fig. 8.

V. CONCLUSIONS

In this paper we have examined the constraints that oblique EWPD places on flavor symmetric vector fields. We have examined these constraints on the vector field multiplets that transform under the flavor group and couple to quark bi-linears at tree level, while not initially breaking the quark global flavor symmetry group. These extensions to the SM are treated as effective fields in fitting to oblique EWPD, including appropriate counterterms to make the EWPD pseudo-observables finite and renormalization scale independent when required. We have found that large regions of parameter space exist where a joint fit to these fields with the Higgs allows a good fit, while the masses of the vector multiplets are

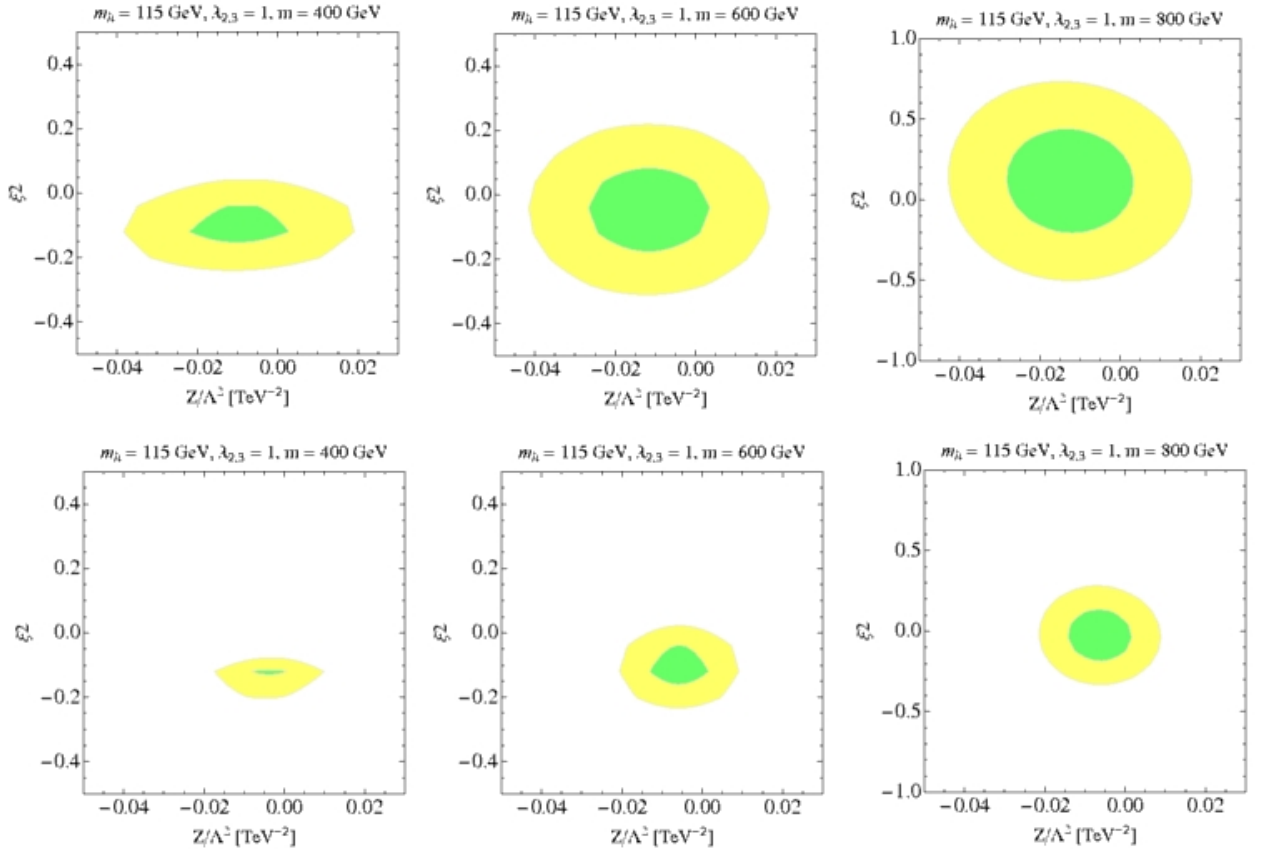


FIG. 6: Space of allowed ξ_2 and Z/Λ^2 for cases X_3 (top) and X_6 (bottom). Here we neglect flavour breaking and ξ_1 . Breaking of the mass degeneracy of the $SU_L(2)$ states is included, again the effect on the parameter space is negligible. Left to right the masses are $m_\nu = 400, 600, 800$ GeV.

$\sim v$. Vectors of this form can act to significantly relax the mass bound on the Higgs in a flavor safe manner, as we have demonstrated in detail for Cases VI_s , and $VIII_s$. Flavor safe mechanisms to raise the Higgs mass bound may be of greater interest if the entire light Higgs mass parameter space is excluded experimentally in the near future.

Conversely, it is interesting to note that large regions of parameter space exist in the models where the Higgs mass is in the light mass region with $m_h \sim 115$ GeV, joint fits to EWPD are improved over the SM alone, and field content is allowed that could possibly explain the $A_{FB}^{t\bar{t}}$ anomaly. Vector fields of the form we have considered are relatively unconstrained by indirect searches in flavor physics due to their flavor symmetry, and

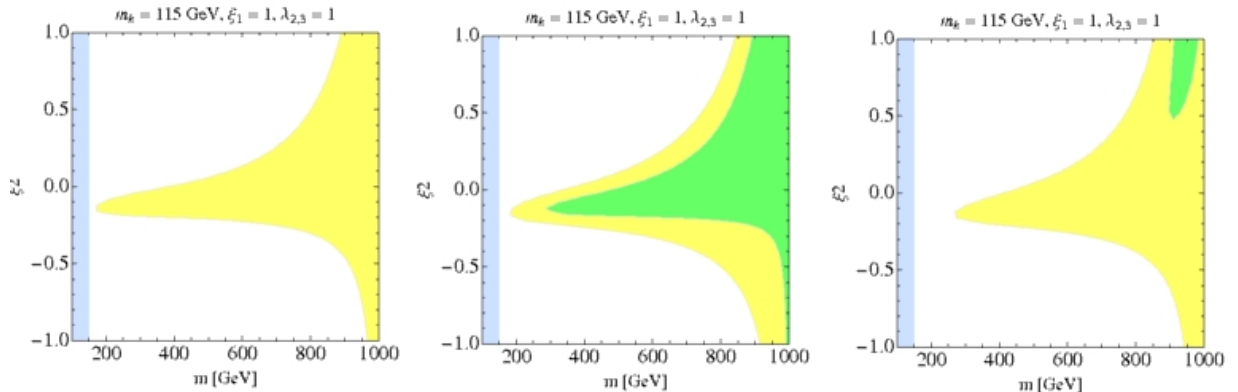


FIG. 7: Space of allowed ξ_2 and m for $X_{\bar{3}}$. Flavour breaking is neglected and we set $\xi_1 = 1$ and $\lambda_{2,3} = 1$ - the dependence on these parameters is negligible. We have also set $m_h = 115 \text{ GeV}$. Left to right $Z/\Lambda^2 = (0.01, -0.01, -0.03) \text{ TeV}^{-2}$. For case X_6 the parameter space with $Z/\Lambda^2 = -0.01 \text{ TeV}^{-2}$ is similar, while there is no allowed parameter space for the other values of Z/Λ^2 .

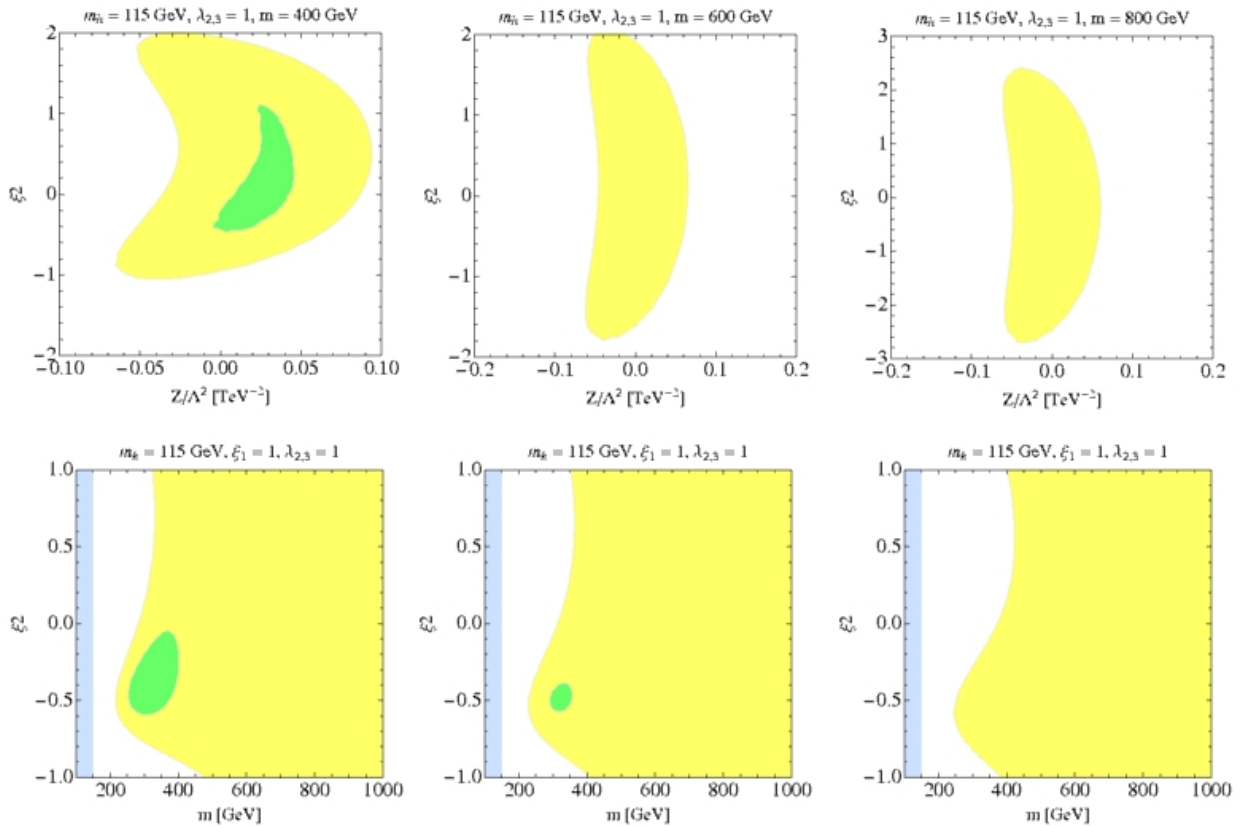


FIG. 8: Space of allowed ξ_2 and Z/Λ^2 for case $XI_{\bar{3}}$ (top), left to right the masses are $m_v = 400, 600, 800 \text{ GeV}$, other parameters same as above, Z/Λ^2 same as above left to right for $XI_{\bar{3}}$ (bottom row).

have been shown to be consistent with oblique EWP constraints. Further dedicated studies of the constraints on these flavour multiplets from non oblique precision EW observables, such as R_b , may provide stronger constraints on the allowed mass scales. Dedicated direct collider studies of flavor safe vector fields also have the potential to raise the mass bounds on these models, or discover models of this form at LHC.

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Appendix A: Self-Energy Form Factors

Throughout this work the following form factors appear:

$$\begin{aligned}
f(p^2, m^2, M^2) &= \frac{1}{576\pi^2 m^2 M^2} \left[3(2p^2(m^2 + M^2) - m^4 - 14m^2 M^2 - 9M^4) A_0(M^2) \right. \\
&\quad + 3(2p^2(m^2 + M^2) - 9m^4 - 14m^2 M^2 - M^4) A_0(m^2) \\
&\quad + 3(24m^2 M^2(m^2 + M^2) + p^2(3m^4 - 2m^2 M^2 + 3M^4) \\
&\quad \quad \left. - 2p^4(m^2 + M^2)) B_0(p^2, m^2, M^2) \right. \\
&\quad + 2(3(m^6 + 11m^4 M^2 + 11m^2 M^4 + M^6) - 4p^2(m^4 + m^2 M^2 + M^4) \\
&\quad \quad \left. + p^4(m^2 + M^2)) \right. \\
&\quad \left. + 3 \frac{(m^2 - M^2)^2}{p^2} (m^4 + 10m^2 M^2 + M^4) (B_0(0, m^2, M^2) - B_0(p^2, m^2, M^2)) \right], \\
g(p^2, m^2, M^2) &= \frac{m^2 + M^2}{32\pi^2 m^2 M^2} \left[(2p^2(m^2 + M^2) - p^4 - (m^2 - M^2)^2) B_0(p^2, m^2, M^2) \right. \\
&\quad \left. + (m^2 - M^2 + p^2) A_0(m^2) + (M^2 - m^2 + p^2) A_0(M^2) \right], \\
h(p^2, m^2, M^2) &= \frac{1}{576\pi^2 m^2 M^2} \left[6(m^2 - M^2)^2 (m^2 + M^2) (B_0(0, m^2, M^2) - B_0(p^2, m^2, M^2)) \right. \\
&\quad + 3p^2((3m^4 + 10m^2 M^2 + 3M^4) B_0(p^2, m^2, M^2) + (M^2 - 9m^2) A_0(m^2) \\
&\quad \quad \left. + (m^2 - 9M^2) A_0(M^2) - 2(m^2 + M^2)^2) \right. \\
&\quad \left. + p^4(3(A_0(m^2) + A_0(M^2)) + 8(m^2 + M^2)) - p^6(3B_0(p^2, m^2, M^2) + 2) \right].
\end{aligned}$$

The above expressions simplify when the particles in the loop have the same mass. We define $f(p^2, m^2) \equiv f(p^2, m^2, m^2)$ as the form factor when the masses are equal.

$$\begin{aligned}
f(p^2, m^2) &= \frac{1}{144\pi^2 m^2} \left[3 (12m^4 + m^2 p^2 - p^4) B_0(p^2, m^2, m^2) \right. \\
&\quad \left. + 6 (p^2 - 6m^2) A_0(m^2) + 36m^4 - 6m^2 p^2 + p^4 \right], \\
g(p^2, m^2) &= \frac{p^2}{16\pi^2 m^2} \left[(4m^2 - p^2) B_0(p^2, m^2, m^2) + 2A_0(m^2) \right], \\
h(p^2, m^2) &= \frac{p^2}{576\pi^2 m^4} \left[3 (16m^4 - p^4) B_0(p^2, m^2, m^2) + 6 (p^2 - 8m^2) A_0(m^2) \right. \\
&\quad \left. - 2 (12m^4 - 8m^2 p^2 + p^4) \right].
\end{aligned}$$

Notice that when the masses in the above form factors are equal, the form factor vanishes at zero-momentum. This ensures that gauge invariance, which requires $\Pi_{AA}(0) = \Pi_{ZA}(0) = 0$, is satisfied. It also ensures that the STUVWX parameters have the correct limiting forms. Consider S in model VII, for example: since the Taylor expansion of any form factor about $p^2 = 0$ starts at one derivative one sees from Eqs. 9 that the one derivative contribution to the S parameter vanishes. This is precisely as expected from the Peskin-Takeuchi definition of S , proportional to $\Pi'_{3B}(0)$: in model VII the vector boson couples to B but not to W^3 .

The one-loop form factors have been written in terms of Passarino-Veltman functions

$$\begin{aligned}
A_0(m^2) &= 16\pi^2 \mu^{4-D} \int \frac{d^D q}{i(2\pi)^D} \frac{1}{q^2 - m^2 + i\epsilon}, \\
B_0(p^2, m^2, M^2) &= 16\pi^2 \mu^{4-D} \int \frac{d^D q}{i(2\pi)^D} \frac{1}{[q^2 - m^2 + i\epsilon] [(q+p)^2 - M^2 + i\epsilon]}.
\end{aligned}$$

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