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Publication Date

1998-07-01



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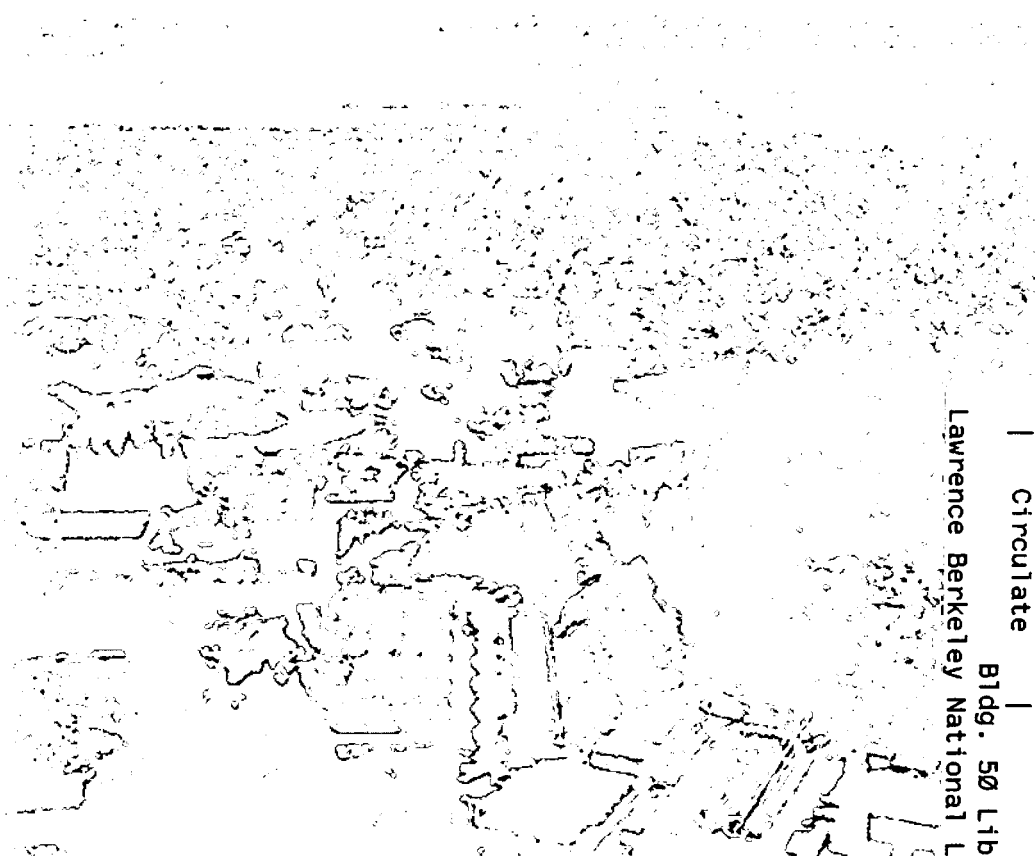
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July 1998

Submitted to
Physical Review D



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**The Final-State Interaction in the Two-Body Nonleptonic
Decay of a Heavy Particle**

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The final-state interaction in the two-body nonleptonic decay of a heavy particle

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Abstract

We attempt to understand the final-state interaction in the two-body nonleptonic decay of a heavy particle for which many multibody ($N \geq 3$) decay channels are also open. No matter how many multibody channels couple to the two-body channels, the analyticity of the S-matrix relates the phase and the magnitude of the two-body decay amplitude through a dispersion relation. In general, however, the phase cannot be determined by strong interactions alone. The dispersion relation requires on a general ground that the final-state interaction phases be small for the two-body decay amplitudes when the initial particle is very heavy. We then analyze the final-state interaction phases in terms of the s -channel eigenstates of the S-matrix and obtain semiquantitative results applicable to the B decay with a random S-matrix hypothesis. We use the high-energy scattering data and the dual resonance model as a guide to the relevant aspects of strong interaction dynamics at long and intermediate distances.

PACS numbers: 11.55.Fv, 11.80.Gw, 12.40.Nn, 13.20.Fc, 13.20.He

I. INTRODUCTION

The final-state interaction in the nonleptonic weak decay is difficult to estimate when a large number of multibody channels are open. While the short-distance final-state interaction is small [1] and its computation is noncontroversial, we have little understanding, theoretically or experimentally, of the long-distance final-state interactions.

The long-distance final-state interaction phases were computed from the high-energy Regge exchange amplitudes in the elastic rescattering approximation [2,3]. The experimental data appeared in favor of large final-state interaction phases at least for the D decay [5]. It was asserted that the measured phases of the two-body D decay amplitudes can be reproduced in the elastic approximation to the final hadron interactions [6]. However, it is fairly obvious from an analysis of the partial-wave unitarity with the diffractive scattering [4] that the elastic approximation cannot be justified at high mass scales, for instance, in the B decay where the two-body final states can couple to a very large number of multibody final states. Even when we are interested in the final-state interaction phases of the two-body channels alone, we cannot determine them without knowing the coupling of the two-body channels to the multibody channels. The Regge amplitudes alone do not provide all necessary pieces of information. Actually, strong interactions and CP-conserving weak interactions are entangled in the decay phases. In this paper we shall make a modest attempt toward understanding of the inelastic final-state interactions. Because of the limitation in the numerical computation of the long-distance effects, we are able to present our results only in a semiquantitative way.

We present two approaches here. The first one uses the analyticity of the S-matrix. For the two-body decay, the phase and the magnitude of amplitude are tightly related to each other by a dispersion relation no matter how many multibody channels couple to two-body channels. The same phase-amplitude relation does not hold for the multibody decay. So far no theorist has ever attempted to study the correlation between the phase and the magnitude from this aspect. No dynamical assumption is introduced in this approach.

The origin of difficulty in the final-state interaction at high mass scales is in that so many channels are open and communicate with each other. In the second approach we analyze the decay in terms of the s -channel eigenstates of the S-matrix and treat a large number of open eigenchannels statistically by introducing a randomness hypothesis [7]. While we give up much of numerical predictability in this approach, we are still able to see general trends in the final-state interaction at high mass scales. Both approaches lead us to conclude that the long-distance final-state interaction phases should be small for two-body heavy hadron decay. Though our conclusion favoring small final-state interactions may be in line of some of the existing literature, our method and picture are completely orthogonal to them.

In Section II, after a brief review of the analyticity of the decay amplitude into general N -body channels, we derive for the two-body decay a dispersion relation which relates the phase and the amplitude through the Omnès-Mushkelishvili integral [8]. Using this dispersion relation we separate from the physical decay amplitude the final-state interactions below an arbitrarily chosen timelike energy scale. We see in this form that a final-state interaction phase of any origin cannot persist to very high energies.

In Section III, we shall study the final-state interaction phases from the viewpoint of the eigenphase shifts of the strong-interaction S-matrix. We write the hadron scattering

amplitude in terms of the eigenphase shifts, and classify the eigenchannels into the resonant and nonresonant ones according to the dual resonance model. Then we express the final-state interaction in terms of the eigenphase shifts. We study the high-energy behaviors of the eigenphase shifts from the scattering data.

In Section IV we introduce the dynamical postulate that the composition of eigenchannels is statistically random when very many of them exist in degeneracy. We make a quantitative estimate of the decay amplitude phases within the limitation of the method.

Finally in Section V, we apply our findings to the actual D and B decays. For the D decay, the elastic approximation combined with the Regge asymptotic behavior is not allowed in determining the final-state interaction phases. The random-phase method is probably a poor approximation. The observed large phase difference between different isospin channels can be accommodated but not predicted. The random phase approximation has the best chance in the B decay where the final hadron multiplicity is high. All two-body decay amplitudes are dominantly real up to CP violations in the B decay. If the color suppression exists prior to final-state interaction corrections, it should be preserved even in the presence of final-state interactions. Interestingly, the same high-energy behavior of the eigenphase shifts that makes the elastic scattering amplitudes purely imaginary leads to the almost real two-body decay amplitudes.

II. BASIC PROPERTIES OF THE INELASTIC DECAY AMPLITUDES

A. Analyticity

We consider the weak decay

$$H \rightarrow \text{hadrons} \quad (1)$$

where H is a heavy particle such as the D and B mesons. The final state is generally a multiparticle state of N hadrons. Going off the H mass shell, we call the H mass squared as the variable s and examine the analytic property of the invariant decay amplitude $M(s)$ in the complex s -plane.

The analytic property of the S-matrix elements was extensively studied decades ago [9]. We obtained many rules of computation by examining the Feynman diagrams though a rigorous proof without referring to the diagrams was given only in the limited cases. To avoid inessential complications, we consider the case where H and the final hadrons are all spinless. We shall use the *in-out* formalism [10] to simplify our notations. The invariant amplitude for the decay into the N -hadron state f is defined by

$$\langle f^{\text{out}}(p_i) | H_w | H(P) \rangle = M^+(p_i), \quad \left(\sum_i p_i = P \right) \quad (2)$$

where the one-particle states are normalized as $\langle \mathbf{p} | \mathbf{p}' \rangle = (2\pi)^3 \delta(\mathbf{p} - \mathbf{p}')$. $M^+(p_i)$ is actually the function of all possible Lorentz invariants made of p_i and P . Aside from $s (= P^2)$, there are $(N+1)(N-2)/2$ independent invariants, which we may choose as

$$s_{ij} = (p_i + p_j)^2 \quad (i > j (\neq N-1)), \quad (3)$$

since there is one linear dependency relation,

$$s = 2 \sum_{i>j} (p_i \cdot p_j) + \sum_i m_i^2. \quad (4)$$

We may include short-distance strong interactions in H_w by using the QCD-corrected effective Hamiltonian. Though we shall treat H_w as a local operator, the analytic property to be discussed below does not depend on locality of H_w . What is important is that H_w transfers no energy-momentum.

To study analyticity, we introduce the auxiliary (unphysical) amplitude,

$$\langle f^{in}(p_i) | H_w | H(P) \rangle = M^-(p_i). \quad (5)$$

We can obtain $M^-(p_i)$ by imposing the incoming boundary condition on the N hadrons in Eq.(2). Diagrammatically, it amounts to flipping the sign of $i\epsilon$ in all Feynman propagators. If H_w is time-reversal invariant, it holds that

$$\langle f^{out}(p_i) | H_w | H(P) \rangle = \langle H(P') | H_w | f(p'_i)^{in} \rangle, \quad (6)$$

where p'_i and P' are obtained from p_i and P by reversing the signs of their space components. Eq.(6) reads

$$M^+(p_i) = M^-(p'_i)^*, \quad (7)$$

where the asterisk denotes a complex conjugate. The reversal of the signs of the space components of p_i does not change the Lorentz invariants, s_{ij} and s . Since both $M^+(p_i)$ and $M^-(p'_i)$ are real below all thresholds and therefore coincide with each other there, Eq.(7) means that they actually represent values of a single analytic function on two different Riemann sheets. In terms of the Lorentz invariants, Eq.(7) can be written as

$$M(s + i\epsilon, s_{ij} + i\epsilon) = M(s - i\epsilon, s_{ij} - i\epsilon)^*. \quad (8)$$

When H_w is not T-invariant, it is convenient to work with each short-distance-corrected weak Hamiltonian individually after separating out a T-violating phase; $H_w = \overline{H}_w e^{i\delta_{CP}}$. Then Eq.(8) is valid for \overline{H}_w . We shall use the words "T-violation" and "CP-violation" as equivalent, assuming the CPT invariance.

One problem about the multibody decay amplitude is that there are so many variables; three independent variables even for the three-body decay, one more than in the Mandelstam representation for two-body scattering. More a serious obstacle is that the real analyticity relation Eq.(8) holds only when we go across all cuts in s and s_{ij} from $+i\epsilon$ to $-i\epsilon$ simultaneously. We are able to write a dispersion relation in one of the variables keeping the others fixed, for instance, in variable s keeping $s_{ij} ((ij) \neq (N, N-1))$ fixed to $s_{ij} + i\epsilon$. Then $M(s - i\epsilon, s_{ij} + i\epsilon)$, which is not simply related to the physical amplitude, enters the dispersion integral. In the case of a three-body final state, for instance, such an amplitude is the complex conjugate of the unphysical amplitude $M(s + i\epsilon, s_{ij} - i\epsilon)$ in which the particle pairs (1,2) and (1,3) interact with the wrong sign phases $-\delta_{12}$ and $-\delta_{13}$, respectively, while the particle pair (2,3) interacts with the right sign phase δ_{23} . Only for the two-body decay amplitudes does Eq.(8) give the simple real analyticity relation:

$$M(s + i\epsilon) = M(s - i\epsilon)^*, \quad (9)$$

so that the discontinuity in the complex s -plane is directly related to a physical process. For the decays of $N \geq 3$, we can write only multivariable dispersion relations as an extension of the Mandelstam representation. We see little chance of extracting a useful information out of them.

B. Dispersion relation

The standard dispersion relation relates the real part of $M(s)$ to the imaginary part. To relate the phase to the magnitude, we write the dispersion relation for logarithm of $M(s)$. Notice that $\ln M(s)$ has the same analytic property as $M(s)$ except for cuts due to zeros of $M(s)$. Choosing the contour of the Cauchy integral as usual, we can write the dispersion relation in the once-subtracted form:

$$\ln M(s) - \ln M(0) = \frac{s}{\pi} \int_{s_0}^{\infty} \frac{\delta(s')}{s'(s' - s)} ds', \quad (10)$$

where s_0 is the lowest threshold of final states. The phase $\delta(s)$ should be normalized to zero at $s = s_0$ in this representation to keep $\ln M(s)$ finite at $s = s_0$. When Eq.(10) is exponentiated, it is the representation of Mushkelishvili which was first applied by Omnès to a study of the electromagnetic form factors. Hereafter we shall refer to this exponentiated dispersion relation as the Omnès-Mushkelishvili representation [8]. The application was limited to the energy region where the two-body scattering is elastic. Contrary to some misconception [3], however, the same dispersion relation can be derived for the two-body decay even in the presence of inelastic channels. We emphasize this point since otherwise the representation will be of no use in the heavy hadron decay. The key observation here is that once the phase along the cut is known, a real analytic function is unique.

Actually there is one uncertainty that cannot be fixed by the analyticity. It arises from possible zeros of $M(s)$. If $M(s)$ has zeros at s_i ($i = 1, 2, 3, \dots$), they generate logarithmic singularities for $\ln M(s)$ and contribute to the dispersion integral. When such zeros are included, the amplitude $M(s)$ is expressed as

$$M(s) = P(s) \exp\left(\frac{s}{\pi} \int_{s_0}^{\infty} \frac{\delta(s')}{s'(s' - s)} ds'\right), \quad (11)$$

where $P(s) = M(0)\prod_i(1 - s/s_i)$. We need some physical argument to determine the polynomial $P(s)$. In the days of the bootstrap theory we used to resort to a certain philosophy of determinism: there must not be a free parameter which we cannot control on. We do not think that we can argue along the same line in the context of QCD. In our case we avoid this problem as follows.

Let us write Eq.(11) back in the form

$$\ln M(s) = \ln P(s) + \lim_{\Lambda^2 \rightarrow \infty} \frac{1}{\pi} \left(\int_{s_0}^{\Lambda^2} \frac{\delta(s')}{s' - s} ds' - \int_{s_0}^{\Lambda^2} \frac{\delta(s')}{s'} ds' \right) \quad (12)$$

and then define $M(s; m^{*2})$ by

$$\ln M(s; m^{*2}) = \ln P(s) + \lim_{\Lambda^2 \rightarrow \infty} \frac{1}{\pi} \left(\int_{m^{*2}}^{\Lambda^2} \frac{\delta(s')}{s' - s} ds' - \int_{s_0}^{\Lambda^2} \frac{\delta(s')}{s'} ds' \right). \quad (13)$$

$M(s; m^{*2})$ is the unphysical decay amplitude in which the final-state interactions of the energy range from the threshold to $\sqrt{s} = m^*$ has been removed. In terms of this amplitude, the physical decay matrix element $M(s)$ is expressed as

$$M(s) = M(s; m^{*2}) \exp \left(\frac{1}{\pi} \int_{s_0}^{m^{*2}} \frac{\delta(s')}{s' - s} ds' \right). \quad (14)$$

In this form all strong-interaction corrections below $\sqrt{s} = m^*$ are explicitly factored out in the exponent. The correction factor includes both short- and long-distance corrections except for the interaction responsible for formation of hadrons. Concerning the hadron formation interaction, we encounter a fundamental issue in the final-state interaction theory. Both the final-state interactions and the formation of hadrons result from the same QCD force. Nonetheless, in order to formulate the final-state interaction theory, we must separate the hadron formation forces from the long-distance interactions between hadrons.¹ The representation above provides one way to do so. While $M(s; m^{*2})$ cannot be measured in experiment, it is the closest to what theorists have been calculating for decay matrix elements by various methods without final-state interactions. The appropriate choice of a value for m^* in Eq.(14) is

$$m_H \leq m^* \leq m_W. \quad (15)$$

A remark is in order on the two-body final state. The dispersion relation in Eq.(14) holds for any two-body final state. In the elastic energy region, we normally choose an isospin or an SU(3) eigenstate for it. The reason is that such a state is an eigenchannel of the strong interaction S-matrix, and therefore that the phase is identified with the strong interaction phase by Watson's theorem [11]. At high energies where two-body states couple to multibody final states, however, two-body final states are no longer eigenstates of the S-matrix, no matter which isospin states we may choose. Then it happens that the net phase $\delta(s)$ of the two-body decay amplitude depends not only on strong interactions but also on weak interactions, even when one computes amplitudes due to a single effective weak Hamiltonian. Though this was already pointed out [4] in the past, it is worth emphasizing since it is the origin of all complications when rescattering is inelastic. We shall see the point more clearly in Section III. From this viewpoint, two-body isospin eigenstates in the inelastic region are just as bad as the I_3 eigenstates of indefinite isospin in the elastic region. When inelastic channels are open, writing for instance the decay amplitudes for $B^0 \rightarrow D^- \pi^+$, $\rightarrow \bar{D}^0 \pi^0$ in the $I = 1/2$ and $3/2$ amplitudes contributes to very little to solving the problem.

¹Recall that the hadrons were considered as elementary when the phase theorem was proved [11].

C. High-energy behavior of the phase

Imagine that $\delta(s)$ approaches the asymptotic value $\delta(\infty)$ at some value m well below $\sqrt{s} = m_H$.²

$$\delta(s) \simeq \delta(\infty) \quad (m \leq \sqrt{s} \leq m_W). \quad (16)$$

The exponent of Eq.(14) can be estimated for such $\delta(s)$ as

$$\begin{aligned} \frac{1}{\pi} \int_{s_0}^{m^{*2}} \frac{\delta(s')}{s' - m_H^2 - i\epsilon} ds' &= \frac{1}{\pi} \left(\int_{m^2}^{m^{*2}} + \int_{s_0}^{m^2} \right) \frac{\delta(s')}{s' - m_H^2} ds', \\ &= \frac{2\delta(\infty)}{\pi} \ln \left(\frac{m^*}{m_H} \right) + i\pi + O \left(\frac{m^2}{m_H^2} \right). \end{aligned} \quad (17)$$

The contribution of $O(m^2/m_H^2)$ from the region below $\sqrt{s} = m$ is negligibly small if $m_H^2 \gg m^2$. The dominant contribution comes from the asymptotic energy region. When exponentiated, this integral generates an enhancement or suppression factor of $(m^*/m_H)^{2\delta(\infty)/\pi}$ for $M(s)$. If, for instance, the phase reaches $\delta(\infty) = \pm\pi/2$, the strong interaction correction would alter the amplitude by factor 16 for $m_H = m_B$ and $m^* = m_W$. It means an enhancement or a suppression of factor ~ 250 in rate. There is no evidence for such a huge dynamical enhancement or suppression when we compare the observed two-body decay rates with the theoretical estimates in the D and B decays. The so-called color suppression observed in the B decay should be attributed not to a severe dynamical suppression by strong interactions but to lack of strong-interaction corrections, since the suppression exists prior to final-state interactions.

The obvious alternative to the asymptotic behavior of Eq.(16) is

$$\delta(s) \rightarrow 0 \quad (m \leq \sqrt{s} \leq m_W). \quad (18)$$

The enhancement or suppression is milder in this case.

The fractional power $(m^*/m_H)^{2\delta(\infty)/\pi}$ does not appear in the conventional calculation of the decay matrix elements. The short-distance QCD corrections enter in fractional powers of $(\ln m^*/\ln m_H)$. If we wish to reproduce the short-distance correction factor of the renormalization group with our final-state interaction integral, we would choose such that $\delta(s)$ approach zero asymptotically as

$$\delta(s) \rightarrow \bar{\delta}/\ln s, \quad (m \leq \sqrt{s} \leq m_W) \quad (19)$$

where $\bar{\delta} = \gamma_w/b_0$, γ_w is a constant determined by the anomalous dimension of H_w , and b_0 is from the running QCD coupling. The asymptotic behavior of Eq.(19) leads to

²Since there is no large characteristic energy scale of strong interactions, it is reasonable to assume that $M(s)$ approaches its asymptopia as early as hadron scattering amplitudes do. If $\delta(s)$ should keep oscillating, $\ln M(s)$ would behave like $e^{|\delta|}$ along some direction in the complex s -plane, which would prevent us from writing the dispersion relation to start with.

$(\ln m^*/\ln m_H)^{2\bar{\delta}/\pi}$ for the amplitude. Since our correction factor includes both short- and long-distance effects, Eq.(19) should represent only the portion of the asymptotic phase that is attributed to the short-distance interactions. We must study the long-distance effects by some other means.

The main result of our dispersion relation is that the phase of the two-body decay amplitude of a heavy hadron should be zero or a small value for a large initial mass ($s \rightarrow \infty$). It should certainly not be $\pm 90^\circ$. It is worth noting that, unlike the phase, the magnitude of the amplitude can be subject to substantial final-state interaction corrections since it picks up the effects of all energies.

III. EIGENPHASE SHIFTS

In this Section we study the decay phases from the viewpoint of the eigenphases of the S-matrix of high-energy scattering. We need to know about the composition of the eigenchannels and their high-energy asymptotic behavior. We shall use the dual resonance model as our guide since it is the most successful model that incorporates the relevant aspects of the quark model spectroscopy and long-distance hadron scattering.

The partial-wave S-matrix of strong-interaction is diagonalized in terms of the eigenchannels carrying the quantum numbers of hadron H . Labeling the eigenchannels by $|a\rangle, |b\rangle, \dots$ we can express the S-matrix elements as

$$\begin{aligned} S_{ab} &= \langle b^{out} | a^{in} \rangle \\ &= \delta_{ab} e^{2i\delta_a(s)}. \end{aligned} \quad (20)$$

Without loss of generality we normalize the eigenphase shifts to zero at their respective thresholds. Experimentally observed are the hadronic states with each particle carrying definite momentum. We project those states onto the J^P eigenstates and denote them by h . By completeness of eigenchannels, h can be expanded as

$$|h\rangle = \sum_a O_{ha} |a\rangle, \quad (21)$$

where we can choose O_{ha} to be an orthogonal matrix as a consequence of T-invariance for strong interactions.

The partial-wave amplitude $a^J(s)$ for elastic scattering is expanded in the eigenphase shifts as

$$a^J(s) = \sum_a O_{ha}^2 e^{i\delta_a} \sin \delta_a, \quad (22)$$

or in the real and imaginary parts,

$$\begin{aligned} \text{Re} a^J(s) &= \sum_a O_{ha}^2 \cos \delta_a \sin \delta_a, \\ \text{Im} a^J(s) &= \sum_a O_{ha}^2 \sin^2 \delta_a. \end{aligned} \quad (23)$$

Both O_{ha} and δ_a are s -dependent. Strictly speaking, once multibody states are included, we must label the eigenchannels with continuous parameters. Therefore the discrete summation

in Eqs.(21) and (24) is symbolic. The state density per unit energy in a volume characteristic of strong interactions ($\sim m_\pi^{-3}$) may substitute as the effective number of states for multibody channels.

Actually the dual resonance model [12] of the late 1960's answers to how many states exist at energy \sqrt{s} . In this model, the number of states was counted to be [13]

$$n_0 \sim \frac{1}{(\alpha's)^{(d+1)/2}} \exp(\sqrt{s}/m_0), \quad (24)$$

where α' is the Regge slope ($\simeq 1\text{GeV}^{-2}$), d is the space-time dimension, and $m_0 = (3/2\alpha'd)^{1/2}/\pi$, which takes a value $\sim 200\text{MeV}$ for $d = 4$. This state density n_0 contains the states of all angular momentum $J \leq (\alpha's)^{1/2}$ at mass \sqrt{s} for the interval of $\sim 1\text{GeV}^{-2}(\approx \alpha')$. While the highest J state has no degeneracy, degeneracy of states rapidly increases with descending J . It also includes the states of negative norm on the daughter Regge trajectories in the case of 4-dimensional space-time. Nonetheless the state density of Eq.(24), particularly the exponential dependence, gives an order-of-magnitude estimate for the number of the $J=0$ states in the dual resonance model. Hagedorn [14] introduced a statistical model of hadrons with quite a different motivation. It is amusing that his model led to essentially the same state density with a very close value ($\sim 160\text{ MeV}$) for m_0 but with a slightly difference power of s in front. In Hagedorn's model, the closeness of m_0 to the pion mass was explained by the fact that every time energy increases by m_0 , one more pion evaporates and causes the exponential growth of the state density.

The phenomenological success of the dual resonance model [15] confirmed that the s -channel resonances are dual to the non-Pomeron Regge exchanges while the Pomeron is dual to the nonresonant continuum in the s -channel [16]. The dual resonance model at tree-level incorporates only the non-Pomeron trajectories. The Pomeron term was included by adding the two-body nonresonant intermediate states. At high energies, the resonant states are so broad in width that they are not recognized as resonances but make up the smooth Regge asymptotic behavior of non-Pomeron exchange. In the yet higher energy region, the diffractive scattering dominates so that the s -channel states consist almost entirely of nonresonant states. Henceforth we shall call the s -channel states dual to the Pomeron and to the non-Pomerons as the nonresonant and the resonant channels, respectively, even though no resonance peak appears in the resonant channels at high energies. We shall also use the words, the diffractive and nondiffractive channels, for them. In the dual resonance model, the effective number of the nonresonant channels is even larger than n_0 of the resonant channels in the high-energy limit.

The distinction between the Pomeron and the non-Pomeron trajectories is best described by the quark diagram [17]. (See Figure 1.) The non-Pomeron exchange in the boson-boson scattering is described by a pair of quark-antiquark in the intermediate state (Fig.1a) which represents a tower of resonances in the s -channel and at the same time the Regge trajectory exchanges in the t -channel. In contrast, the Pomeron exchange corresponds to the "disconnected" quark diagram of the $q\bar{q}q\bar{q}$ four-quark intermediate states in the s -channel as shown in Fig.1b. In the context of QCD one pair of $q\bar{q}$ exchanges relatively soft gluons with the other pair of $q\bar{q}$ in the Pomeron exchange [18]. This quark diagram not only ensures the exchange degeneracy for a pair of non-Pomeron trajectories with opposite signatures, as observed in scattering experiment [16], but also explains the absence of the mesons with exotic quantum numbers such as $I \geq 3/2$ in meson spectroscopy.

A. Final-state interactions in eigenphase shifts

The T-invariance relation in Section II can be easily extended to the decay amplitude into the eigenchannels. Using the completeness condition $\sum_a |a^{out}\rangle\langle a^{out}| = 1$ and $\langle a^{out}|a^{in}\rangle = e^{2i\delta_a}$, we obtain

$$\langle a^{out}|H_w|H\rangle = e^{2i\delta_a(s)}\langle a^{out}|H_w|H\rangle^*, \quad (25)$$

where $s = m_H^2$. Accordingly the decay amplitude $M_a(s)$ into the eigenchannel a carries the eigenphase $\delta_a(s)$. Inserting the complete set of eigenchannels and using the expansion of $|h\rangle$, we obtain

$$\langle h^{out}|H_w|H\rangle = \sum_a O_{ha}\langle a^{out}|H_w|H\rangle. \quad (26)$$

The decay amplitude $M_h(s)$ for $H \rightarrow h$ is now expressed in the terms of the eigenchannel decay amplitudes $M_a(s)$:

$$M_h(s) = \sum_a O_{ha}M_a(s). \quad (27)$$

Separating the phase $\delta_a(s)$ from $M_a(s)$ as

$$M_a(s) \equiv \bar{M}_a(s)e^{i\delta_a(s)}, \quad (28)$$

we put Eq.(27) in the form

$$M_h(s) = \sum_a O_{ha}\bar{M}_a(s)e^{i\delta_a(s)}. \quad (29)$$

Comparing Eq.(29) with Eq.(22), we clearly see that the net phase of $M_h(s)$ has little to do with that of $a^J(s)$. The phase of $M_h(s)$ agrees with that of $a^J(s)$, barring an accident, only when there is only one eigenchannel so that the elastic unitarity holds,

$$|1 + 2ia^J(s)| = 1. \quad (30)$$

It was pointed out that the partial-wave projection of the diffraction amplitude for the $\pi\pi$ scattering at the B mass is far short of the unitarity limit [4]; $|1 + 2ia^J(s)| \simeq 0.6$. For the D decay, which occurs near the resonance region or a little above it, the nonleading Regge terms cannot be ignored in two-body scattering. We shall see in Section V that even after adding the non-Pomeron terms the elastic unitarity does not hold at the D mass.

The amplitude $\bar{M}_a(s)$ defined in Eq.(28) still contains strong-interaction effects. To define the eigenchannel decay amplitude free of final-state interactions, we must separate out not only the phase correction but also the magnitude correction $\Delta_a(s)$ by $\bar{M}_a(s) = \bar{M}_{0a}(s)\Delta_a(s)$. If all inelastic channels are approximated as two-body or quasi-two-body states, $\Delta_a(s)$ can be written in the Omnès-Mushkelishvili representation:

$$\Delta_a(s) = \exp\left(\frac{\mathcal{P}}{\pi} \int_{s_{0a}}^{m_W^2} \frac{\delta_a(s')}{s' - s} ds'\right), \quad (31)$$

where \mathcal{P} stands for the principal value integral. One obvious property of $\Delta_a(s)$ is that it is positive definite. Whether $\Delta_a(m_H^2)$ gives an enhancement (> 1) or a suppression (< 1) depends on the sign of $\delta_a(s)$ for all values of s from the threshold s_{0a} to m_W^2 , not just the on-shell value $\delta(m_H^2)$. In the nonresonant channels, we shall argue later that δ_a is small in magnitude and the sign of δ_a can be easily flipped as energy changes when a large number of channels mix. If so, a correlation of the magnitude correction $\Delta_a(m_H^2)$ with the on-shell phase value $\delta_a(m_H^2)$ is tenuous, if any. With the magnitude enhancement factor written out, Eq.(29) turns into

$$M_h(s) = \sum_a O_{ha} \overline{M}_{0a}(s) \Delta_a(s) e^{i\delta_a(s)}. \quad (32)$$

For the resonant channels, one can compute the Feynman diagram for the decay process $H \rightarrow R(\text{resonance}) \rightarrow h(\text{two-body})$ through a resonance R . The decay amplitude for the resonant eigenchannel r takes the form,

$$M_r(s) = f_{HR} \frac{\sqrt{m_R \Gamma_r}}{m_R^2 - s - im_R \Gamma_{tot}}, \quad (33)$$

where f_{HR} is the H - R pole transition strength and $\Gamma_{tot,r}$ are the total and partial decay widths of R . Note that $\Gamma_{tot} = O(m_H)$. In the quark model f_{HR} is the overlap of the wave functions of H and R . Since the overlap does not increase with s , $M_r(s)$ decreases like $1/s$ or faster as $s \rightarrow \infty$. If we express the resonant channel contributions in the form of Eq.(32), it means $\overline{M}_{0r}(s) \Delta_r(s) \rightarrow 1/s$ or faster for each resonant channel r .

B. Strong interaction scattering

Elastic scattering provides useful pieces of information about the eigenphase shifts. Experiment shows that the imaginary part dominates over the real part for the invariant amplitude $T(s, t)$ of elastic scattering at high energies. Theoretically, the dominance of $\text{Im}T(s, t)$ is a general consequence of analyticity and crossing symmetry, not specific to the Regge theory, when the total cross section approaches a constant up to powers of $\log s$. The amplitude of the flat Pomeron trajectory,

$$T(s, t) = is\sigma_{tot} e^{bt}, \quad (34)$$

gives a reasonably good description of the diffractive scattering in the whole high-energy region relevant to us.³ Eq.(34) leads to the elastic cross section,⁴

³At energies above $\sqrt{s} \approx 100$ GeV, the total cross sections actually rise very slowly with energy. One fit to pp -collisions gives $\sigma_{tot}(s) \simeq \beta_P(s/s_0)^{0.08} + \beta_{\rho-f}(s/s_0)^{-0.56}$ [21]. This s -dependence requires that the forward scattering amplitude contains a real part: $\text{Re}T(s, 0)/\text{Im}T(s, 0) \rightarrow \tan(\pi\epsilon/2)$, where $\epsilon = 0.08$. The forward scattering amplitude contains a real part by about 10% even at extremely high energies. However such high energies have no direct relevance to the final-state interaction of the B decay.

⁴We ignore all hadron masses as compared with \sqrt{s} throughout this section.

$$\sigma_{el} = \sigma_{tot}^2/32\pi b, \quad (35)$$

and to the partial-wave amplitude

$$a^J(s) = i\sigma_{tot}/16\pi b, \quad (J \ll (s/s_0)^{1/2}). \quad (36)$$

When we parametrize $T(s, t)$ by Eq.(34), the partial-wave elasticity $\sigma_{el}^J/\sigma_{tot}^J (= \text{Im}a^J(s))$ for $J \ll (s/s_0)^{1/2}$ is equal to twice the total elasticity, $(1/\sigma_{tot}) \int (d\sigma_{el}/dt) dt$. The values for σ_{tot} and b can be extracted from the experimental data on pp , πp , and Kp scattering. The factorization of the Regge residues⁵ relates the high-energy total cross sections by

$$\sigma_{tot}^{MM'} = \sigma_{tot}^{Mp} \sigma_{tot}^{M'p} / \sigma_{tot}^{pp}, \quad (37)$$

where M and M' stand for mesons. With $\sigma_{tot}^{pp} = 37\text{mb}$, $\sigma_{tot}^{\pi p} = 21\text{mb}$, and $\sigma_{tot}^{Kp} = 17\text{mb}$ for the diffractive contribution of σ_{tot} at $\sqrt{s} = 2 \sim 8 \text{ GeV}$ [19], we obtain

$$\sigma_{tot}^{\pi\pi} = 12\text{mb} \quad \sigma_{tot}^{K\pi} = 10\text{mb}. \quad (38)$$

The numerical values are roughly in line with the empirical law of the quark number counting for the total cross sections: $\sigma_{tot}^{MM'} : \sigma_{tot}^{Mp} : \sigma_{tot}^{pp} = 2^2 : 2 \times 3 : 3^2$. The diffraction width parameter b obeys the inequality $b_{pp} > b_{\pi p} > b_{Kp}$. In one analysis [20]

$$b_{pp} \simeq 5\text{GeV}^{-2}, \quad b_{\pi p} \simeq 4.3\text{GeV}^{-2}, \quad b_{Kp} \simeq 3.2\text{GeV}^{-2}. \quad (39)$$

The parameter b is related to the effective target size of colliding hadrons in elastic scattering. The inequality $b_{pp} > b_{\pi p}$ indicates that the proton is a little more spread than the pion. The electromagnetic form factors show the same trend: $(r_p^2/6)^{1/2} \simeq \sqrt{2}m_\rho^{-1}$ and $(r_\pi^2/6)^{1/2} \simeq m_\rho^{-1}$. The relation $b_{Kp} < b_{\pi p}$ may be interpreted as a result of the less intense interaction of the s-quark with the u/d-quarks, which reduces the effective size of K as well as σ_{tot}^{Kp} . This line of argument leads us to the b -parameters for $\pi\pi$ and $K\pi$ scattering somewhere around

$$b_{\pi\pi} \simeq 3.7\text{GeV}^{-2}, \quad b_{K\pi} \simeq 2.8\text{GeV}^{-2}. \quad (40)$$

If we extrapolate this reasoning to the $D\pi$ scattering and ignore the c-quark interaction with the u/d-quarks, we are led with $\sigma_{tot}^{D\pi} = \sigma_{tot}^{\pi\pi}/2$ and $b_{D\pi} < b_{K\pi}$ to

$$\sigma_{tot}^{D\pi} = 6\text{mb} \quad b_{D\pi} \simeq 2.4\text{GeV}^{-2}. \quad (41)$$

When we substitute these values of parameters in the partial-wave projection of Eq.(36), we obtain

$$\text{Im}a^J(s) \simeq \begin{cases} 0.16 & (\pi\pi) \\ 0.17 & (K\pi) \\ 0.12 & (D\pi) \end{cases} \quad (42)$$

⁵The factorization can be proved only for relatively simple J-plane singularities. It is an assumption for the more general Pomeron.

The value of $\text{Im}a^J(s)$ is even smaller for $\psi\pi$ and ψK . The precise values of the right-hand side of Eq.(42) are not important in the following. It is no surprise that the elastic unitarity is not satisfied for any of the above processes:

$$|1 + 2ia^J(s)| = 0.66 \sim 0.76. \quad (43)$$

Note that $|1 + 2ia^J(s)| = 0.5$ is the limit of the completely absorptive black target. When $a^J(s)$ is purely imaginary, the value of $|1 + 2ia^J(s)|$ does not tell the whole story. The partial-wave inelasticity reveals more of the scattering mechanism. The numbers in Eq.(42) give the partial-wave inelasticity ($=1 - \text{Im}a^J$) in the range of

$$\sigma_{inel}^J / \sigma_{tot}^J = 0.83 \sim 0.88. \quad (44)$$

It should be pointed out here that the s -wave phase shift is dominated by long-distance physics. The largest contribution to $a^J(s)$ for small J comes from the region of the momentum transfer $-1/b < t \leq 0$. The contribution from the perturbative QCD region of large $|t|$ is small. It is $\sim 1/|t|^{1/2}$ that determines the effective distance of interactions. It is true, however, that the s -wave contains a larger share of short-distance physics than high partial-waves.

Comparing Eq.(42) with $\text{Im}a^J$ of Eq.(22), we find that the average or typical eigenphase shift of the diffractive channels should be in the range of

$$\sin^2 \delta_d \simeq 1/8 \sim 1/6, \quad (45)$$

where the subscript d for the phase shift stands for "diffractive". The smallness of $\text{Re}a^J / \text{Im}a^J$ for the diffractive scattering suggests that δ_d ($d = 1, 2, \dots$) spreads over positive and negative values (modulo $n\pi$) in an approximately symmetric distribution with respect to $\delta_d = 0$. For such a distribution a large cancellation occurs among different eigenchannels in $\text{Re}a^J$ ($\propto \cos \delta_d \sin \delta_d$) while every term adds up in $\text{Im}a^J$ ($\propto \sin^2 \delta_d$). What about the $n\pi$ ambiguity for δ_d ? Since the diffractive channels are nonresonant, δ_d starts at 0 and does not go over $\pi/2$. Because of Wigner's theorem [22] on the causality constraint on phase shifts, it is not very likely for δ_d to turn clockwise and go over $-\pi/2$. Therefore δ_d 's stay in $-\pi/2 < \delta_d < \pi/2$, spreading symmetrically with respect to $\delta_d = 0$ over the range roughly

$$-\pi/8 < \delta_d < \pi/8. \quad (46)$$

Here again, the precise values of the upper and lower bounds are unimportant to us.

Even in the decay into a diffractive multibody channel a resonance can appear in a subchannel giving rise to a large phase when the subchannel invariant mass coincides with the resonance mass. For instance, $H \rightarrow R\pi$ (nonresonant) $\rightarrow K\bar{K}\pi$ at $m_{K\bar{K}} = m_R$. If R is a sharp resonance of $K\bar{K}$, we treat the process as a two-body decay into $R\pi$. If not, the final-state is a three-body channel. Though the decay amplitude has a large phase at $m_{K\bar{K}} = m_R$, this phase is washed out after the $m_{K\bar{K}}$ is integrated over with \sqrt{s} fixed to m_H .

Let us turn to the nondiffractive channels. Their contribution to σ_{tot} falls off like $1/s^{1/2}$ or faster relative to that of the diffractive channels. To obtain the subdominant terms in s , we add the nonleading Regge contributions. One may wonder about a possible isolated s -channel non-Regge singularity which may contribute only to a single angular momentum.

Especially relevant is the fixed singularity $a^J \propto \delta_{J0}/J$. This singularity generates a constant term in $T(s, t)$ for all values of s and t . Since hadrons are composites of the scale Λ_{QCD} , there must not be such a hard interaction.

According to the Regge duality, the nondiffractive and diffractive portions of the amplitude are separately dual to the resonant and nonresonant states, respectively, in the s -channel. Therefore the nondiffractive or resonant contribution $a_r^J(s)$ to the partial-wave amplitude can be expressed as

$$a_r^J(s) = \sum_r O_{hr}^2 e^{i\delta_r} \sin \delta_r, \quad (47)$$

where the subscript r stands for "resonant". In these channels, the eigenphase shift δ_r turns around counter clockwise slowly passing $\pi/2$ at the resonance and approaches π asymptotically. Comparison of the asymptotic energy dependences of the diffractive and nondiffractive amplitudes gives us

$$\frac{\sum_r O_{hr}^2 \sin^2 \delta_r(s)}{\sum_d O_{hd}^2 \sin^2 \delta_d(s)} = (\bar{\beta}_{\rho-f}/\bar{\beta}_P) s^{-1/2}, \quad (48)$$

where $\bar{\beta}_{P, \rho-f}$ are the properly normalized Regge residues at $t = 0$ of the Pomeron and the ρ - f_2 trajectory, and s is in the unit of 1 GeV^2 . We can extract the Regge residues from the energy dependence of total cross sections. For $\pi^+ p$ ($\pi^- p$) scattering [19], for instance,

$$\bar{\beta}_{\rho-f}/\bar{\beta}_P \simeq 0.75(1.18). \quad (49)$$

We can relate the left-hand side of Eq.(48) to the numbers of the resonant and nonresonant eigenchannels. By replacing $\sin^2 \delta_r$ by unity for the resonant channels and substituting the average value for $\sin^2 \delta_d$, we obtain

$$\frac{\sum_r O_{hr}^2 \sin^2 \delta_r(s)}{\sum_d O_{hd}^2 \sin^2 \delta_d(s)} \simeq \frac{n_r}{(n - n_r) \langle \sin^2 \delta_d \rangle}. \quad (50)$$

With Eqs.(48) and (49), we obtain

$$n_r \approx n \langle \sin^2 \delta_d \rangle / s^{1/2}, \quad (51)$$

which agrees with our expectation $n_r \ll n$.

IV. RANDOM S-MATRIX APPROXIMATION

When a very large number of eigenchannels are present, studying individual channels is impractical. To study the spacings and widths for hundreds of the densely populated resonances in complex nuclei, nuclear physicists introduced a certain randomness hypothesis in the multichannel S-matrix. The work was started by Wigner [23], pursued by many [25], and brought into a mathematical sophistication by Dyson [26]. It succeeded in reproducing various features of those resonances [25]. A similarity of the nuclear resonances to the multitude of the hadron channels in our problem suggests us to study physics of the uncontrollably many eigenchannels with the randomness hypothesis.

The randomness of the channel mixing O_{ha} can arise in our problem when a very large number of eigenchannels exist in degeneracy with the two-body final-states. Since even a small coupling strongly mixes a pair of degenerate states, any two-body state in the highly inelastic energy region is a linear combination of many eigenchannels. The expansion coefficients of a given state into eigenchannels, namely O_{ha} , is sensitive to the strength of channel couplings, but many features of physics, for instance δ_a , should not be sensitive to small variations of channel couplings. It is therefore reasonable to postulate that quantities of our interest can be computed by replacing products of O_{ha} with their statistical averages over the phase space of the $O(n)$ rotations of O_{ha} . After the $O(n)$ average has been taken for products of O_{ha} , we are left with δ_a . In our problem unlike the nuclear resonances, we cannot postulate a complete randomness for the distribution of δ_a . If we assumed that both δ_a and O_{ha} are completely random or chaotic, Eq.(24) would lead us to $\langle O_{ha} O_{kb} \rangle = \delta_{ab} \delta_{hk} / n$, $\langle \sin^2 \delta_a \rangle = 1/2$, and $\langle \cos \delta_a \sin \delta_a \rangle = 0$. Consequently

$$\text{Re}a^J = 0, \quad \text{Im}a^J = 0.5. \quad (52)$$

This corresponds to the scattering from a black disc that gives the 50% elasticity due to the shadow scattering in disagreement with Eq.(42). In the actual high-energy scattering, σ_{el}/σ_{tot} is considerably less than 50%. It means that a hadron target behaves like an opaque disc at high energies. To describe such a target, we must postulate randomness only within the restricted range as specified in Eq.(46).⁶

A. Dominance of the real part for decay amplitudes

We start with Eq.(32) and isolate all eigenchannel dependences by expressing $M_a(s) = \overline{M}_{0a} \Delta_a e^{i\delta_a}$ back in terms of the decay amplitudes of the hadron basis k which are free of the final-state interaction (of $\sqrt{s} \leq m_W$). We denote such decay amplitudes by $\overline{M}_{0k}(s)$. Writing the nonresonant and resonant channels separately, we have

$$\overline{M}_{0k}(s) = \sum_d O_{kd} \overline{M}_{0d}(s) + \sum_r O_{kr} \overline{M}_{0r}(s). \quad (53)$$

Substituting the inverted relation of this in Eq.(32), we obtain

$$M_h(s) = \sum_{d,k} O_{hd} O_{kd} \overline{M}_{0k}(s) \Delta_d(s) e^{i\delta_d(s)} + \sum_{r,k} O_{hr} O_{kr} \overline{M}_{0k}(s) \Delta_r(s) e^{i\delta_r(s)}. \quad (54)$$

Using $\langle O_{ha} O_{kb} \rangle = \delta_{hk} \delta_{ab} / n$, we obtain the random phase values for the real and imaginary parts of $M_h(s)$:

$$M_h(s) = \overline{M}_{0h}(s) (\langle \Delta \cos \delta \rangle + i \langle \Delta \sin \delta \rangle), \quad (55)$$

⁶We previously studied mainly the magnitude of the squared decay amplitude in this statistical model [7]. Here we focus on the decay phase by refining some of the previous postulates. In Ref [7], the magnitude factor $\Delta_a(s)$ was not separated but ignored for simplicity.

where

$$\langle \Delta \cos \delta \rangle = \frac{1}{n} \left(\sum_d \Delta_d(s) \cos \delta_d(s) + \sum_r \Delta_r(s) \cos \delta_r(s) \right), \quad (56)$$

and $\cos \delta \rightarrow \sin \delta$ for $\langle \Delta \sin \delta \rangle$.

Let us leave out the resonant eigenchannels for the moment. Then, first of all, the phase of $M_h(s)$ approaches a common limit in Eq.(55) independent of isospins or charge states of h , say, $K^- \pi^+$ and $\bar{K}^0 \pi^0$. The common limit does not depend on the effective weak interaction H_w . This marked simplicity is valid only in the random phase limit. Considering that $\text{Im}a^J$ takes roughly the same value for all meson-meson scatterings (cf. Eq.(42)), we expect the common decay phase is not very sensitive to the final hadrons.

We have pointed out earlier that in the quasi-two-body approximation to the inelastic channels, $\Delta_a(s)$ is positive definite and uncorrelated with the on-shell $\delta_a(s)$. If this is the case, the terms of different eigenchannels cancel each other in $\text{Im}M_h(s)$ because of the random signs of $\sin \delta_a$:⁷

$$\text{Im}M_h(s) = \sum_{\delta_d > 0} \Delta_d(s) \sin \delta_d - \sum_{\delta_{d'} < 0} \Delta_{d'}(s) |\sin \delta_{d'}|. \quad (57)$$

In contrast, all eigenchannels add up in the real part since $\cos \delta_d > 0$ in the restricted range of Eq.(46). It is similar to the situation in the elastic scattering amplitude of Eq.(24), but this time the cancellation occurs in the imaginary part instead of the real part. Because of the deviation of $\Delta_d(s)$ from unity due to enhancement and suppression, the cancellation may not be as good as in the scattering. We can set with Eqs.(55) and (56) a loose upper bound on the imaginary-to-real ratio for $M_h(s)$. In terms of $\tan \delta_h = \text{Im}M_h(s)/\text{Re}M_h(s)$, it is given by⁸

$$|\tan \delta_h| < |\langle \sin \delta \rangle| / \langle \cos \delta \rangle. \quad (58)$$

The right-hand side is less than ~ 0.4 according to Eq.(45). This number would be realized when no cancellation occurs in Eq.(57). Since $\Delta_d(s)$ and the sign of the on-shell $\delta_d(s)$ are only tenuously correlated, we expect in reality a fairly high degree of cancellation between the terms of $\delta_d > 0$ and $\delta_d < 0$. Therefore the actual value of the imaginary-to-real ratio is most likely much smaller than 0.4. By pushing our approximation further, let us set Δ_d to a common number⁹ and $\cos \delta_d \simeq 1$. Then we obtain

$$\tan \delta_h \simeq \frac{\text{Re}a^J}{\text{Im}a^J} \langle \sin^2 \delta \rangle. \quad (59)$$

⁷The possibility of many phases averaging out to a small net decay phase was mentioned earlier by Wolfenstein [24] to the author.

⁸In Section II, δ_h was simply written as $\delta(s)$.

⁹If the sign of the on-shell δ_d is not correlated with $\Delta_d > 1$ or < 1 , setting Δ_d to a number independent is justifiable.

The right-hand side is zero for the purely diffractive amplitude. Even if we use the parametrization $\sigma_{tot} \sim s^{0.08}$ fitted to much higher energies which requires a nonnegligible real part for $T(s, 0)$, the right-hand side is $\simeq 0.04\pi \times \langle \sin^2 \delta \rangle \simeq 2 \times 10^{-2}$.

Next we look into the contribution of the resonant channels to the decay phase. It falls like $s^{\alpha_\rho - f - 1} \simeq s^{-1/2}$ relative to the diffractive contribution. In addition to the $s^{-1/2}$ suppression, the chiral structure of the weak interaction suppresses the transition from the initial state H of $J^P = 0^-$ to the resonant channels. Let us examine this suppression.

Keeping in mind that in the dual resonance model the resonant channels are made of two-quark states in the quark diagram (Fig.1a), we examine the transition of $H(= \bar{q}Q)$ to the resonant state $R(= \bar{q}q')$.¹⁰ The bare weak Hamiltonian which causes the transition is the four-quark operator,

$$H_w = 4(G/\sqrt{2})V_{qq'}^*V_{qQ}(\bar{q}'_L q_L)(\bar{q}_L Q_L), \quad (60)$$

where we have suppressed the Dirac structure of the quark bilinears. The short-distance QCD corrections to H_w induce the effective interactions such as $(\bar{q}'_L(\lambda_a/2)q_L)(\bar{q}_L(\lambda_a/2)Q_L)$ and the penguin interaction $(\bar{q}_R(\lambda_a/2)q_R)(\bar{q}'_L(\lambda_a/2)Q_L)$. The weak transition $H \rightarrow R$ is the so-called annihilation or exchange process. The matrix elements of the annihilation and exchange processes are suppressed by the decay constant $f_H(= O(f_\pi))$ of the meson H and, for the non-penguin interactions, also by the factor $(m_{q'} + m_q)$ of chirality mismatch on the side of R . The suppression due to the first one, f_H/m_H , together with the energy dependence suppression $s^{-1/2}$ or n_r/n is severe enough to make the phase difference contribution of the resonant channels negligibly small. It should be reminded that the *resonances* in the resonant channels are so broad ($\Gamma_H = O(m_H)$) that no sharp resonance enhancement arises. The suppression of f_H/m_H occurs for the transition $H \rightarrow R$ through the penguin interaction too.

In short, the dominant decay process is through the spectator diagrams which lead to $q\bar{q}q\bar{q}$ not $q\bar{q}$ in the final state. The decay $H \rightarrow q\bar{q}q\bar{q}$ followed by a pair annihilation into $q\bar{q}$ is no other than the annihilation or exchange process, as we can see by drawing the diagram. The rescattering of $q\bar{q}q\bar{q}$ without a $q\bar{q}$ annihilation is a diffractive process not a resonant one in the sense of the dual resonance model. Therefore, we can conclude that the decay phase due to the resonant channels are negligibly small. We estimate a typical magnitude of the phase differences of this origin as

$$\Delta\delta \approx (f_H/m_H)(1\text{GeV}/s^{1/2}) \quad (61)$$

for the decay modes where the spectator process dominates. When the chiral mismatch occurs for R , a decay matrix element is suppressed by the additional factor of $m_{q'}/m_H$.

B. Phase difference between amplitudes

What we can measure in experiment is not the absolute phases but the phase differences. Though the phase of $\langle M_h(s) \rangle$ is independent of the charge or isospin states of h ,

¹⁰The following argument is valid also for $H \rightarrow R(= \bar{q}''q')$ with minor modifications.

the fluctuation around the average value can be isospin dependent and therefore generate phase differences. The physical origin of this type of phase differences can be explained as follows: A pair of decay amplitudes have different compositions of diffractive eigenchannels. Therefore the interference between eigenchannels sums up to different net phases for the amplitudes. The phases of this origin are washed out in the random limit. The interference between the diffractive and nondiffractive eigenchannels is another possible source of the relative phases.

We first examine the relative phase due to the fluctuations. The fluctuation is the standard deviation from the random phase limit of $\langle M_h(s) \rangle$. Averaging out the product of four O_{ha} in the phase space of $O(n)$ group¹¹, we obtain for $n \gg n_r$

$$\begin{aligned} |\Delta \text{Im} M_h(s)|^2 &\equiv \langle (\text{Im} M_h(s) - \langle \text{Im} M_h(s) \rangle)^2 \rangle, \\ &= \langle |\overline{M}_0|^2 \rangle (\langle \Delta^2 \sin^2 \delta \rangle - \langle \Delta \sin \delta \rangle^2) + O\left(\frac{1}{n} |M_h(s)|^2\right), \\ |\Delta \text{Re} M_h(s)|^2 &= (\sin \delta \rightarrow \cos \delta), \end{aligned} \quad (62)$$

where the brackets denote the averages such as

$$\langle |\overline{M}_0|^2 \rangle = \frac{1}{n} \sum_d |\overline{M}_{0d}|^2. \quad (63)$$

The sum $\sum_{a=(d,r)} |\overline{M}_{0a}|^2 / 2m_H$ is the total rate for the decays induced by H_w without strong interaction corrections. Since the decay amplitude is dominantly real, the phase difference is more sensitive to $\Delta \text{Im} M_h(s)$ than to $\Delta \text{Re} M_h(s)$. The relative magnitude of the standard deviation $\Delta M_h(s)$ to the average $|M_h(s)|$ depends on the channel number n . If n is nearly as large as the dual resonance model indicates, or even a small fraction of it, $|\Delta M_h(s)|$ is negligibly small and no relative phase arises from fluctuations. In this case, the two-body channel couples to an enormous number of inelastic nonresonant channels though its couplings to individual channels are very weak accordingly. The fluctuations almost even out when this happens. On the other hand, if n is so small that $1/n$ is comparable to the branching fraction to the channel h , $\Delta \text{Im} M_h(s)$ can be close to $|\text{Im} M_h(s)|$ itself. When $|M_h(s)|$ is small by an accidental cancellation of high degree among different eigenchannels, the fluctuation can be large in proportion. To ensure that the random phase is a good approximation, therefore, we should apply its predictions only to the two-body modes having relatively large branching fractions.

The phase differences arising from an interference with the resonant eigenchannels are suppressed by the chiral structure of the weak interaction operators and decrease like $s^{-1/2}$ with energy. Their contribution is of the order of $\Delta\delta$ as given in Eq.(61) in general. However, our argument fails in the modes where the spectator process is forbidden or highly suppressed. In such processes the diffractive eigenchannels dual to the Pomeron are missing or highly suppressed so that large phases and phase differences may potentially arise. It is unfortunate that the decay branching is generally very small for them.

¹¹It is straightforward to obtain $\langle O_{ka} O_{la} O_{mb} O_{nb} \rangle = (\delta_{kl} \delta_{mn} + \delta_{km} \delta_{ln} + \delta_{kn} \delta_{lm}) / n(n+2)$ for $a = b$ and $= [((n+1)\delta_{kl} \delta_{mn} - \delta_{kn} \delta_{lm} - \delta_{km} \delta_{ln})] / (n-1)n(n+2)$ for $a \neq b$ [7].

V. THE D AND B DECAYS

We shall look into the specific cases of the D and B mesons with a few critical remarks on some of the recent attempts to compute the final-state interaction phases of the two-body decays of D and B .

A. The D meson

The most prominent two-body decay of the D meson is the $\bar{K}\pi$ modes. From the observed decay rates of $D^+ \rightarrow \bar{K}^0 \pi^+$ and $D^0 \rightarrow \pi^+ K^-, \bar{K}^0 \pi^0$, the phase and magnitude of the decay amplitude were determined for $D \rightarrow \pi \bar{K}$ of definite isospin. They discovered a large relative phase between the $I=1/2$ and $3/2$ decay amplitudes [5];

$$\delta_{3/2}(m_D^2) - \delta_{1/2}(m_D^2) = (96 \pm 13)^\circ \quad (64)$$

with the amplitude ratio

$$|M_{3/2}/M_{1/2}| = 0.27 \pm 0.03. \quad (65)$$

Since the D meson mass is too low for the Pomeron to dominate in the $K\pi$ scattering, our argument in the preceding Section does not apply with a good accuracy. The phases of the s -wave $K\pi$ elastic scattering amplitudes $a_I(s)$ at energy m_D was computed with the Regge theory and identified with the phases of the decay amplitudes for the $K\pi$ isospin eigenstates.

The result of calculation in Ref. [6] happens to be in a good agreement with the observed phase difference, Eq.(64), within the uncertainties in the values of the Regge parameters. We argue however that such an agreement is fortuitous. In order to identify the scattering phases with the decay phases, the scattering must be elastic. The authors of Ref [6] computed only the phase difference of the $K\pi$ amplitudes, not the magnitudes of them. If they had done so, they would have obtained

$$\begin{aligned} a_{1/2}(m_D^2) &= 0.08 + 0.21i \\ a_{3/2}(m_D^2) &= -0.35 + 0.18i. \end{aligned} \quad (66)$$

They do not satisfy the elastic unitarity: $|1 + 2ia_{1/2}| = 0.60$ and $|1 + 2ia_{3/2}| = 0.95$. The $I = 1/2$ amplitude is deep inside the Argand diagram while the $I = 3/2$ amplitude nearly satisfies the elastic unitarity. Actually, if one insists that $K\pi$ scattering be elastic at $\sqrt{s} = m_D$, one can make $a_{3/2}$ unitary in a model-independent way. Since the $I = 3/2$ channel is exotic, *i.e.*, contains no s -channel resonances dual to the non-Pomeron trajectories, the imaginary part of $a_{3/2}$ in Eq.(67) comes entirely from the Pomeron. The non-Pomeron trajectories can contribute only to the real part in the case of $a_{3/2}$. Therefore one should make up for the unitarity violation of $a_{3/2}$ by adjusting its real part:

$$a_{3/2}(m_D^2) = -0.38 + 0.18i, \quad (67)$$

which gives

$$\delta_{3/2}(m_D^2) = 155^\circ. \quad (68)$$

Unlike $a_{3/2}$ the unitarity deficit of $a_{1/2}$ is very large. To recover the elastic unitarity for $a_{1/2}$, the Pomeron daughters and the exchange-degenerate pairs of trajectories such as the daughters of ρ - f_2 and of K^* - K_2 must contribute just as much as or more than the parent trajectories. If so, the Regge expansion itself would be unjustified at this energy.

In the $I = 1/2$ channel there is an obvious candidate for a cause of inelasticity, that is, $\bar{K}\eta$ channel which should enter according to the flavor SU(3) symmetry. Then the $(\bar{K}\pi)_{I=1/2}$ involves at least two eigenchannels, the $I = 1/2$ channels of the 8 and 27 representations of SU(3). In addition, there are multibody channels. The D meson decays into $\bar{K}\pi\pi\pi$ of both $I = 1/2$ and $3/2$ including the modes such as $\bar{K}^*\rho$ and $\bar{K}a_1$. The combined branching fraction of all $\bar{K}\pi\pi\pi$ modes is even larger than that of $\bar{K}\pi$. Many $J^P = 0^+$ states are contained in $\bar{K}\pi\pi\pi$, with which $\bar{K}\pi$ mixes in general. It is not surprising if many eigenchannels are open for both $I = 1/2$ and $3/2$ already at the energy of the D meson

Is there any chance to identify the $D \rightarrow \bar{K}\pi$ decay phase with the $\bar{K}\pi$ scattering phase all the way up to high energies despite the presence of inelastic channels? We do not think so. In Eq.(67) the real part of a_I is positive for $I = 1/2$ and negative for $I = 3/2$ at the D meson mass. If it happens that $a_{1/2}$ and $a_{3/2}$ approach purely imaginary numbers at high energies, $\delta_{1/2}(s)$ would turn counter clockwise to $\pi/2$ and $\delta_{3/2}(s)$ would turn clockwise to $-\pi/2$. The phase-amplitude dispersion of Section II then requires that $M_{1/2}(D \rightarrow K\pi)$ is enhanced over $M_{3/2}(D \rightarrow K\pi)$ by factor $(m^*/m_D)^2$, where m^* is the highest energy up to which one is willing to make the identification of the decay phase with the scattering phase. This relative enhancement factor is far too large unless m^* is chosen to be comparable to m_D .

To summarize, the D-meson mass is not low enough to allow identification of the final-interaction phase with the elastic scattering phase. Yet it is not high enough for our random S-matrix approximation to work with a good accuracy. The large phase difference obtained by experimentalists can be accommodated but not predicted.

B. The B decay

The meson-meson scattering at energy $\sqrt{s} = m_B$ is most likely asymptotic for $\pi\pi$, $K\pi$, $\bar{D}\pi$ or even for $\bar{D}D_s$. The resonances exist only at much lower energies, which means that the contribution of the nondiffractive scattering, *i.e.*, the non-Pomeron exchanges, to the asymptotic scattering amplitude is negligible. For instance, if we make an estimate for the elastic $K\pi$ scattering at energy m_B using the Regge parametrization of Ref [6], the diffractive contribution gives

$$|1 + 2ia_I| = 0.64 \quad (I = 1/2, 3/2). \quad (69)$$

The nondiffractive contributions modify it only slightly into $|1 + 2ia_{1/2}| = 0.63$ and $|1 + 2ia_{3/2}| = 0.65$. The two-body scattering is clearly in the asymptotic energy region. The s -wave inelasticity for $K\pi$ is

$$\sigma_{inel}/\sigma_{tot} = 0.83 \quad (70)$$

from our estimate in Eq.(42). There is no chance for the elastic approximation to work at this energy. The fact that the largest branching fractions observed in the B decay are at

the level of 1% is a clear evidence for the presence of very many open channels. The B decay is the process where our results of Section IV should apply best. We shall restate our predictions for the B decay below.

First of all, the two-body B decay amplitudes should be dominantly real up to CP violation phases. According to our very crude estimate made in Eq.(59), the decay phases should be 7° or smaller. The relative phases should be even smaller. As has already been pointed out, this is in a sharp contrast to the conclusion of the elastic approximation which predicts that the phase differences are small but the phases themselves are close to 90° . Though some of the existing literature reach the same or similar conclusions, our s -channel picture for the origin of the almost real decay amplitudes is quite different from and orthogonal to them. Our phase-amplitude dispersion relation corroborates the smallness of the decay phases. Up to the short-distance QCD corrections, the magnitude of the final-state interaction phase is independent of isospins in the first-order approximation. This small and common final-state interaction phase is the reason why the color suppression appears to hold well in the B -decay.

As for a phase difference between a pair of decay amplitudes, the portion of the relative phase that arises from the first-order correction to the random phase limit is not calculable, but small when the channel number n is large. The typical branching fraction for the two-body modes $B \rightarrow \bar{D}M, \bar{D}^*M \dots$, where M is π, ρ, \dots , is somewhere between 0.1% and 1% while the inclusive nonleptonic branching due to $b \rightarrow c\bar{u}d$ should be close to 70%. Therefore the effective number of open channels n with this set of quantum numbers is a hundred or more. For the decay modes such as $B \rightarrow D^- \rho^+$ that have relatively large branching fractions, the randomness approximation gives reliable predictions while the phases of the suppressed decay modes such as $B^0 \rightarrow \bar{D}^0 \rho^0$ cannot be reliably predicted in this approximation. In other words, the phase of the $B^0 \rightarrow \bar{D}^0 \rho^0$ amplitude can be large. The reason is that the two isospin amplitudes $(\bar{D}\rho)_{1/2,3/2}$ of $B^0 \rightarrow \bar{D}^0 \rho^0$ nearly cancel each other in the $B^0 \rightarrow \bar{D}^0 \rho^0$ combination, thus enhancing the phase fluctuation contributions.

The relative phase which arises from the interference with the subdominant nondiffractive eigenchannels is negligibly small since the contribution of the nondiffractive channels is suppressed by f_B/m_B . According to our estimate in Eq.(61), the phase difference of this origin is expected to be

$$\Delta\delta \sim 1/300. \quad (71)$$

The phase difference due to the fluctuations is more important in comparison. We are unable to estimate the relative importance between the fluctuations and the short-distance effects to the phase differences. Notable exceptions are the decay modes for which the annihilation or exchange process dominates. For example, the decay $B^0 \rightarrow K^+ K^-$ for which no spectator diagram can be drawn. In this case the decay proceeds only through $B \rightarrow R \rightarrow K^+ K^-$ in the picture of the dual resonance model. There are no diffractive eigenchannels in the s -channel for this mode. Therefore the $B^0 \rightarrow K^+ K^-$ amplitude can have a large phase unlike the amplitude for the spectator-dominated mode $B^0 \rightarrow K^0 \bar{K}^0$. The rescattering from $K^0 \bar{K}^0$ to the $K^+ K^-$ final state can occur through a $q\bar{q}$ pair exchange, namely, the non-Pomeron exchange. By stretching the corresponding quark diagram out, however, we find that the this rescattering decay process is actually due to one of the annihilation diagrams. Unfortunately the branching fraction is too small for such interesting decay modes.

VI. CONCLUDING REMARKS

A reliable computation of the long-distance final-state interaction phases is nearly an impossible task. The elastic rescattering approximation is probably not viable even for the D decay. It is out of question for the B decay. Nonetheless we are able to extract a few relevant pieces of information with the phase-magnitude dispersion relation and with the eigenphase analysis. The conclusion from the dispersion relation is rigorous while the argument based on the eigenphase shifts resorts to the random channel-mixing postulate and the dual resonance model. The complexity of long-distance strong interactions appears formidable if we try to go any step further along this line. It may be possible to reach essentially the same qualitative conclusions with more an intuitive qualitative reasoning [27]. We have tried in a way different from anybody else, maybe in a hard way. We hope that our approach sheds a light on some aspects of the problem which have so far not been appreciated.

ACKNOWLEDGMENTS

This work was supported in part by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, Division of High Energy Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098 and in part by the National Science Foundation under Grant PHY-95-14797.

REFERENCES

- [1] M. Bander, D. Silverman, and A. Soni, Phys. Rev. Lett. **43**, 242 (1979): J. Gérard and W.-S. Hou, Phys. Rev. Lett. **62**, 855 (1989); Phys. Rev. D **43**, 2909 (1991): H. Simma, G. Eilam, and D. Wyler, Nucl. Phys. **B352**, 367 (1991): R. Fleisher, Z. Phys. C **58**, 438 (1993); **62**, 81 (1994): N. G. Deshpande and X.-G. He, Phys. Lett. B **336**, 471 (1994): G. Kramer, W.F. Palmer, and H. Simma, Nucl. Phys. **B428**, 77 (1994): Z. f. Phys. C **66**, 429 (1995).
- [2] H. Zhang, Phys. Lett. B **356**, 107 (1995): B. Blok and I. Halperin, Phys. Lett. B **385**, 324 (1996): B. Blok, M. Gronau, and J.L. Rosner, Phys. Rev. Lett. **78**, 3999 (1997): G. Nardulli and T.N. Pham, Phys. Lett. B **391**, 165 (1997).
- [3] A.N. Kamal and C.W. Luo, Phys. Lett. B **398**, 151 (1997).
- [4] J.F. Donoghue, E. Golowich, A.A. Petrov, and J.M. Soares, Phys. Rev. Lett, **77**, 2178 (1996).
- [5] M. Bishai et al.(CLEO Collaboration), Phys. Rev. Lett. **78**, 3261(1997).
- [6] D. Delépine, J.-M. Gérard, J. Pestieau, and J. Weyers, Univ. Cath. de Louvain preprint, UCL-IPT-98-01; hep-ph/9802361.
- [7] R.N. Cahn and M. Suzuki, Lawrence Berkeley National Laboratory preprint LBNL-40626; UCB-PTH-97/40; hep-ph 9708208.
- [8] N.I. Mushkelishvili, *Singular Integral Equations* (Noordhoff, Gronigen, 1953), p.204: R. Omnes, Nuovo Cim. **8**, 316 (1958).
- [9] R.J. Eden P.V. Landshoff, D. Olive, and J.C. Polkinghorne, *The Analytic S-Matrix* (Cambridge University Press, Cambridge, 1965) and references quoted therein.
- [10] H. Lehmann, K. Symanzik, and W. Zimmermann, Nuovo Cim. **1**, 42 (1955).
- [11] K.M. Watson, Phys. Rev. **88**, 1163 (1952).
- [12] G. Veneziano, Nuovo Cim. **57A**, 190 (1968).
- [13] S. Fubini and G. Veneziano, Nuovo Cim. **64A**, 811 (1969): K. Bardakci and S. Mandelstam, Phys. Rev. **184**, 1640 (1969): J.H. Schwarz, Phys. Rep. **8C**, 269 (1973).
- [14] R. Hagedorn, Nuovo Cim. **56A**, 1027 (1968).
- [15] C. Lovelace, Phys. Lett. B **28**, 264 (1968): J.L. Petersen, Phys. Rep. **2C**, 157 (1971).
- [16] P.D.B. Collins and E.J. Squires, *Regge Poles in Particle Physics* (Springer-Verlag, New York, 1968) and references therein.
- [17] H. Harari, Phys. Rev. Lett. **22**, 5692 (1969): J. Rosner, Phys. Rev. Lett. **22**, 689 (1969).
- [18] F.E. Low, Phys. Rev. D **12**, 163 (1975): S. Nussinov, Phys. Rev. Lett. **34**, 1286 (1975).
- [19] V. Barger and R.J.N. Phillips, Nucl. Phys. **B32**, 93 (1971).
- [20] G.Y. Chow and J. Rix, Phys. Rev. **184**, 1714 (1969).
- [21] A. Donnachie and P.V. Landshoff, Phys. Lett. B **296**, 227 (1992).
- [22] E.P. Wigner, Phys. Rev. **98**, 145 (1955): R.G. Newton, *Scattering Theory of Waves and Particles* (McGraw-Hill Inc., New York, 1966) p.314.
- [23] E.P. Wigner, Ann. Math. **53**, 36 (1951); **62**, 548 (1955); **65**, 203 (1957); **67**, 325 (1958): Phys. Rev. **98**, 145 (1955): L. Landau and Ya. Smorodinsky, *Lectures on the Theory of the Atomic Nuclei* (Consultants Bureau, Inc., New York, 1958), p.55.
- [24] L. Wolfenstein, a private communication.
- [25] R.G. Thomas and C.E. Porter, Phys. Rev. **104**, 483 (1956): I.I. Gurevich and M.I. Pevsner, Nucl.Phys. **2**, 575 (1957): S. Bloomberg and C.E. Porter, Phys. Rev. **110**, 786 (1958): N. Rosenzweig, Phys. Rev. Lett. **1**, 101 (1958): C.E. Porter and N. Rosenzweig,

- Phys. Rev. **120**, 1698 (1960): M. Mehta, Nucl. Phys. **18**, 395 (1960): M. Mehta and M. Gaudin, Nucl. Phys. **18**, 420 (1960): M. Gaudin, Nucl. Phys. **25**, 447 (1961).
- [26] F.J. Dyson, J. Math. Phys. **3**, 140, 157, and 166 (1962).
- [27] J.D. Bjorken, Nucl. Phys. **B** (Proc. Suppl.) **11**, 325 (1989).

FIGURES

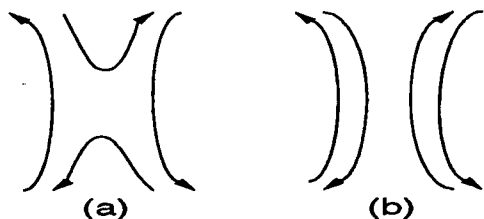


FIG. 1. Duality in the quark diagrams for the elastic boson-boson scattering. (a) The resonant channels dual to the non-Pomeron Regge trajectories. Being made of a $q\bar{q}$ pair, the intermediate resonant states cannot have the exotic (non- $q\bar{q}$) quantum numbers. (b) The nonresonant channels dual to the Pomeron consist of $q\bar{q}q\bar{q}$. In the Pomeron exchange process, one boson ($q\bar{q}$) exchanges gluons with the other boson ($q\bar{q}$). Since gluons carry no flavors, the Pomeron is necessarily a flavor singlet. The s -channel intermediate states consist of a pair of hadrons or hadron resonances which does not resonate.

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