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Author

Agnew, Duncan Carr

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An Improbable Observation of the Diurnal Core Resonance

Duncan Carr Agnew

To the memory of John Wahr

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Abstract The resonance associated with the ellipticity of the core-mantle boundary is usually measured with observations of either the Earth's nutations, or of tidal gravity, strain, or tilt. But, improbably, it can also be seen in a dataset collected and processed with older and simpler technologies: the harmonic constants for the ocean tides. One effect of the resonance is to decrease the ratio of the amplitude of the P_1 constituent to the amplitude of the K_1 constituent to 0.96 of the ratio in the equilibrium tidal potential. The compilation of ocean-tide harmonic constants prepared by the International Hydrographic Bureau between 1930 and 1980 shows considerable scatter in this ratio; however, if problematic stations and regions are removed, this

Institute of Geophysics and Planetary Physics

Scripps Institution of Oceanography

University of California San Diego

La Jolla, California, USA

Tel.: +1-534-2590

E-mail: dagnew@ucsd.edu

1 dataset clearly shows a decreased ratio. While these data apply only a weak con-
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3 straint to the frequency of the resonance, they also show that the effect could have
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5 been observed long before it actually was.
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8 9 10 **1 Introduction**

11
12 Using nutations and tides to determine properties of the Earth goes back to the nine-
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14 teenth century (Brush, 1979; Kushner, 1990), but this approach was overtaken by
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16 seismological methods early in the twentieth. Even the early researches identified a
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18 mode of oscillation in which the interior fluid, acting as a solid, precessed because
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20 of pressure forces on the ellipsoidal solid-fluid boundary. The first models of this
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22 with realistic properties for the core and mantle were those of Jeffreys and Vicente
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24 (1957a,b) and Molodensky (1961), both of whom showed that this mode of oscilla-
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26 tion, now called either the Free Core Nutation (FCN) or Nearly Diurnal Free Wobble
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28 (NDFW) would result in a resonance in the response of the Earth to tidal forces that
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30 produce a net torque, as the diurnal tides do. The relevant theory was further devel-
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32 oped using both normal-mode theory for the rotational modes (Wahr, 1981b,a) and
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34 semi-analytic models (Sasao et al, 1980; Dehant et al, 1993; Mathews et al, 1995;
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36 Mathews, 2001; Dehant and Mathews, 2015); these have included other kinds of
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38 core-mantle coupling as well as improved Earth models.
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42 The first observation of this resonance was claimed by Melchior (1966), who
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44 used tilt data; subsequent measurements of it using Earth tides have focused more
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46 on gravity (Cummins and Wahr, 1993; Sato et al, 1994; Ducarme et al, 2007; Rosat
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48 et al, 2009) and strain (Polzer et al, 1996; Amoruso et al, 2012). The strongest ob-
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1 servational constraints come from observations of the Earth's nutations using high-
2 precision VLBI astrometry; these led Gwinn et al (1986) to the finding that the fre-
3 quency of the resonance differed from that expected for a core-mantle boundary with
4 the ellipticity for a hydrostatic Earth model. Subsequent measurements (Herring et al,
5 2002; Koot et al, 2008; Rosat and Lambert, 2009; Chao and Hsieh, 2015) have come
6 in tandem with improvements in models: paradoxically, we learn about the core-
7 mantle boundary by measurements of very distant quasars.

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18 Wahr and Sasao (1981) showed that the core resonance should also be present in
19 ocean-tide data, a more complicated case than other tidal measurements because the
20 physics has to include not just the Earth's response to external forcing but also to the
21 loading from the tide itself: the loading, just like the external forcing, is affected by
22 the NDFW resonance. Wahr and Sasao (1981) used gravity-tide data to argue that this
23 effect was present, but the only systematic search for it has been by Ooe and Tamura
24 (1985), using tide-gauge data from Japan. Ray (2017) has recently pointed out that the
25 resonance in ocean tides needs to be included in the "inference" of tidal constituents,
26 notably of P_1 from K_1 ; as evidence for this, a high-quality set of open-ocean pressure
27 data (Ray, 2013) shows the resonance effect quite clearly.

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40 Given this, we can ask if the resonance could also be seen in the tidal data col-
41 lected along coastlines. Somewhat improbably, the answer is yes, although the preci-
42 sion is too low to contribute new information about the NDFW. It is startling to realize
43 that this effect, usually sought using VLBI and superconducting gravimeters, can be
44 observed using a sensor no more complicated than a float, recording with a pencil
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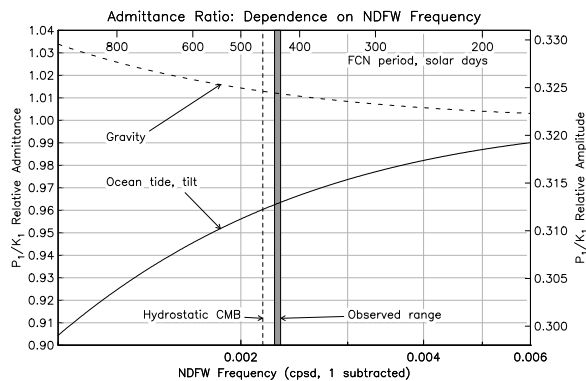


Fig. 1 The solid line shows the relative size of P_1 and K_1 in the ocean tide (or tilt) as a function of the core resonance frequency f_c , in cycles per sidereal day (cpsd). The dashed line shows the same for tidal gravity. The left axis shows relative size as admittance (normalizing by the relative amplitude of the constituents in the tidal potential), and the right axis shows this as the ratio of measured constituent amplitudes.

writing on rolls of paper, and with data processing done mostly by human computers with (sometimes) mechanical calculators.

Section 2 of this paper reviews the theory for the resonance in ocean-tide data, while Section 3 describes the sources of ocean-tide data, and Section 4 describes the results, which rely on there being large amounts of data to average.

2 Effect of the NDFW on Ocean Tides

As Wahr and Sasao (1981) point out, the full equation for the ocean-tide resonance is complicated by the need to include loading. The resonance effect is confined to the spherical harmonic of degree two and order one, but this will perturb the total tide, couple into all spherical harmonics, and so needs to be computed from the tidal model itself, just as with the self-attraction and loading effects that have to be included in the tidal equations (Ray, 1998). Wahr and Sasao (1981) give the full solution for

1 an equilibrium ocean tide for an ocean-covered Earth; for a more realistic situation
 2
 3 they extend this to the following approximate expression, giving the response $R(f)$
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 5 relative to a reference frequency f_0 :
 6

$$7 \quad R(f, f_0) = \frac{\gamma(f)}{\gamma(f_0)} \frac{1}{1 + K(\gamma'(f) - \gamma'(f_0))/\gamma(f_0)} \quad (1)$$

8
 9 where γ and γ' are the Love-number combinations relating the equilibrium tide to the
 10
 11 tidal potential and to a tidal load, in both cases for a spherical harmonic of degree
 12
 13 two and order one: $\gamma = 1 + k - h$ and $\gamma' = 1 + k' - h'$, with h , k , h' , and k' being the
 14
 15 Love numbers and load Love numbers. The constant K is
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$$19 \quad K = \frac{3\rho t_{21}(f_0)}{5\rho_E g \phi(f_0)} \quad (2)$$

20
 21 where ρ and ρ_E are the mean density of ocean water (1.035) and the solid Earth (5.51)
 22
 23 respectively; $g\phi(f_0)$ is the height of the driving potential for the constituent at f_0 ; and
 24
 25 $t_{21}(f_0)$ is the (2,1) spherical harmonic coefficient for a tidal model at frequency f_0 ,
 26
 27 which is complex-valued. The approximation consists of assuming that t_{21} is the same
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 29 for another tide at frequency f as it is for the tide at f_0 .
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 35 The expression for the frequency-dependent γ can be derived from those for the
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 37 Love numbers h and k in Pétit and Luzum (2010). These expressions include the
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 39 resonances for the Chandler wobble and the Inner Core Nutation; ignoring the latter
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 41 and treating the former as a constant term, we obtain
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$$43 \quad \gamma(f) = 0.7021 - \frac{8.98 \times 10^{-5}}{f - f_N} \quad (3)$$

44
 45 where f_N is the frequency of the NDFW, and both it and f are expressed in cycles
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 47 per solar day (cpd; hereafter “day” means solar day unless otherwise specified). The
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1 NDFW frequency in an earth-fixed frame is related to its period in a space-fixed
 2 frame, T_N , by $T_N = (f_N - 1.0027379)^{-1}$; for a hydrostatic earth $T_N = 460$ days, but
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 4 both tidal and nutation data give values of about 430 days.
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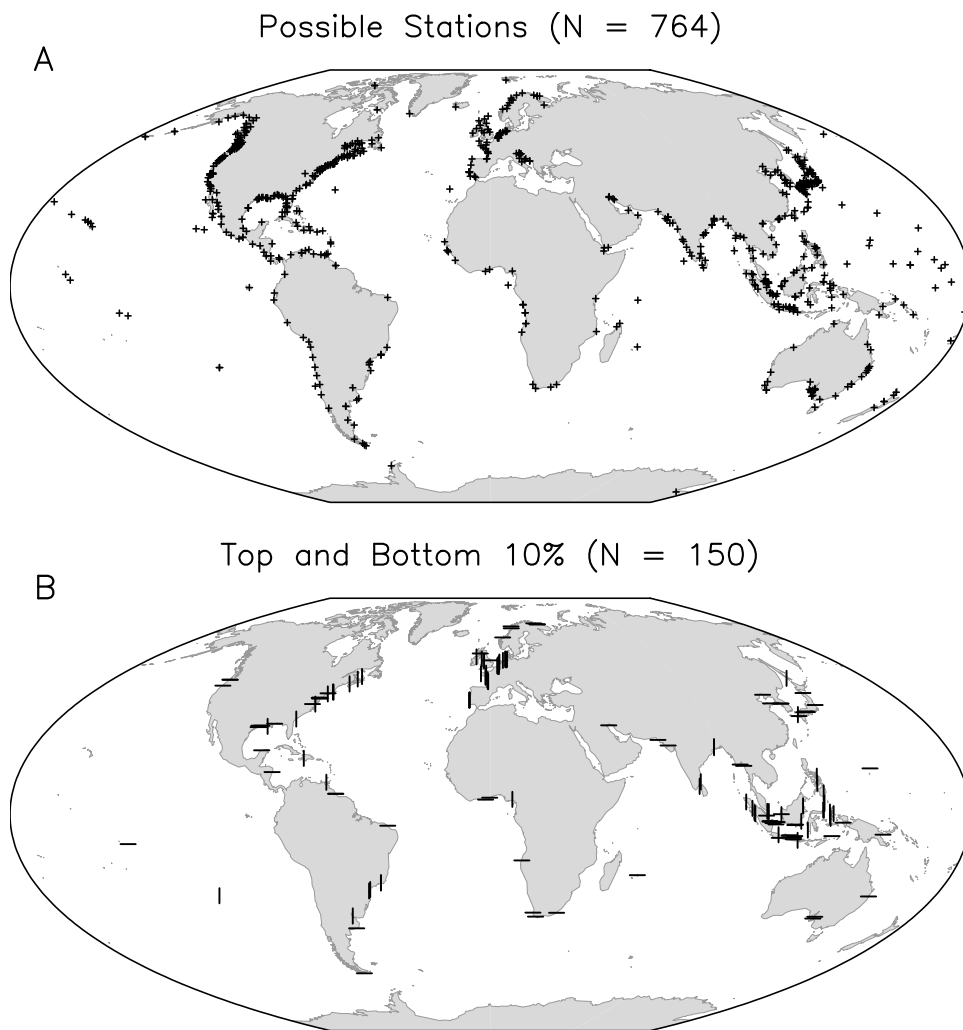
7 A similar expression applies to the combination for load Love numbers:
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$$10 \quad \gamma(f) = 1.6877 - \frac{1.39 \times 10^{-4}}{f - f_N} \quad (4)$$

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 12
 13 Spherical-harmonic expansions of modern tide models give values of $gt_{21}(f)/\phi(f)$
 14 which are all close to a complex number with amplitude 0.31 and phase 132° ; putting
 15
 16 this into equation (1) shows that the amplitude of K is 0.034. Using the K_1 frequency
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 18 for f_0 and the P_1 for f , the total loading term in equation (1) is approximately $1.4 \times$
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 20 10^{-3} . We therefore ignore it and take $R(f, f_0) = \gamma(f)/\gamma(f_0)$. Figure 1 shows R as a
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 22 function of f_N , to make the point that, over the range of plausible values for f_N , the
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 24 variation in R is small; for observed values of T_N it is 0.964.
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31 **3 Ocean-Tide Data**

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 34 My analysis uses the amplitudes of the P_1 and K_1 harmonic constituents for ocean
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 36 tides, measured by coastal tide gauges. I first give some history to help readers under-
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 38 stand how, why, and when this information became available – and, more recently,
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 40 ceased to be so. A key point is that, interesting as the ocean tides are as a scientific
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 42 problem, and important as sea-level change is, most tidal measurements have been
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 44 (and are) collected and processed to predict future tides for the benefit of maritime
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 46 trade, and to define the (legally important) boundary between land and sea. So there
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 48 has been much more data collected than there would have been for scientific research.
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36 **Fig. 2** Panel A (top) shows distribution of “winnowed” stations from the IHO Tidal Constituent Data Bank.
37 These have an analysis interval longer than 354 days, K_1 amplitude of at least 5 cm, diurnal nonlinear tides
38 (MP or SO) no more than 20% of the P_1 tide, and a phase difference between the P_1 and K_1 constituents
39 less than 15° . Panel B (bottom) shows the stations from this dataset for which the P_1/K_1 amplitude ratio
40 is in the bottom or top 10% of the distribution of observed ratios (vertical lines for top, horizontal for
41 bottom). Equal-area projection (McBryde-Thomas flat-polar quartic).
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1 But the commercial value of predicted tides means that, like other economically (and
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3 military) valuable information, tidal parameters may not be freely available.
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6 This is well illustrated by tide prediction in the early nineteenth century. Very
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8 simple methods (Cartwright, 1999) were widely available, but better methods of tide
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10 prediction for active ports such as Liverpool and London were kept secret by those
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12 who profited by supplying them to almanac publishers (Rossiter, 1972; Woodworth,
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14 2002). This changed (Hughes, 2006; Reidy, 2008) in the 1830's when Lubbock and
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16 Dessiou developed non-harmonic methods. This decade also saw the invention of
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18 self-recording tide gauges, which provided the first continuous record of the tides, as
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20 opposed to times and heights of high and low water. Such continuous records were in
21
22 turn crucial to two new techniques: the harmonic method of tidal analysis developed
23
24 by Thomson, Roberts, and Darwin between 1867 and 1883 (Darwin, 1883) and the
25
26 Tide Predicting Machine invented by Thomson and Roberts in the 1870's (Anony-
27
28 mous, 1926b; Cartwright, 1999; Woodworth, 2016). Together these could predict the
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30 tides more accurately for more diverse tidal regimes than the nonharmonic method,
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32 using much less data. The harmonic method also meant that the tidal behavior of any
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34 location could be described by a relatively small collection of numbers, namely the
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36 harmonic constants. Publishing these numbers made it relatively easy to predict the
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38 tides throughout the world, with great benefit to mariners (Hughes and Wall, 2007).
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42 Thus it is unsurprising that the International Hydrographic Bureau (IHB), founded
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44 in 1921 to promote standardization and the exchange of hydrographic information
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46 (Bermejo, 1997) included tidal matters – perhaps also because one of its Directors, J.
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48 H. Pfaff, worked in this area (Pfaff, 1926, 1927). In 1924 the IHB found that many hy-
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1 drographic offices did not use harmonic analysis and prediction. To encourage such
2 use, in 1926 the IHB published tables (Anonymous, 1926a) for harmonic-constant
3 calculation, including a list giving constants for over a thousand locations. The 1926
4 International Hydrographic Conference resolved that the IHB should collect and pub-
5 lish even more constants (Ritchie, 1980), bringing them up to date annually. This was
6 done by creating IHB Special Publication 26, in which groups of constants were pub-
7 lished as separate fascicles, produced as different sources contributed them. The first
8 fascicle was published in 1931, and by 1940 constants were available for 1464 loca-
9 tions, along with (usually) the time and duration of data analyzed, and information
10 on the datums used.

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23 Publication and distribution of harmonic constants in fascicles continued until
24 1968, when the IHB stopped the practice, though it continued to compile information,
25 available as individual photocopies on request (Anonymous, 1976). To make this in-
26 formation more available, the 1972 International Hydrographic Conference resolved
27 that it should be put into machine-readable form. This was done by the Canadian
28 Hydrographic Service, which created the International Hydrographic Organization
29 (IHO) Tidal Data Bank. (The IHB had been renamed as the IHO in 1970 (Bermejo,
30 1997), though the IHB designation remained for the central office in Monaco). This
31 data bank was updated as additional harmonic constants were added, and a magnetic
32 tape of it was distributed to each national office belonging to the IHO; this tape could
33 be purchased, though at a high price.

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46 Unfortunately, it soon became apparent that the data could be used in ways that
47 were both unexpected and, for some, unwelcome. With the advent of personal com-
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puters, a market developed for tide-prediction software, the developers of which relied on the IHO Data Bank for the harmonic-constant values – though these were regarded by the IHO as belonging to the agencies contributing them. For a time the managers of the Data Bank separated values that could be made generally available from those for which this was not allowed without consulting the original contributor, but this became a significant burden, while at the same time new harmonic constants were rarely contributed. In 2000 the IHO therefore requested its members to vote on continuing to operate the Data Bank (Anonymous, 2000); the outcome was that it was terminated, making the harmonic constants unavailable except to those who already had copies. In addition, legal threats by some of the contributing entities discouraged commercial use of this information by third parties.

The harmonic constants in the IHO Data Bank are almost all for coastal sites, especially those in locations of maritime activity. At many locations the tidal observations were made for a month or less, sometimes using continuously-recording gauges and sometimes using tide poles. The number of tidal constituents varies from the four largest tides to up to 50 in areas with large nonlinear effects. And, given that many of the data come from sites not directly open to the ocean, such nonlinear behavior is not uncommon. The data were collected at different times from the 1850's to the 1980's; a wide range of methods (most using hand computation, and few using least squares) were used to estimate the harmonic constants from the actual sea-level measurements. All of this means that the quality of these constants is much more variable, and much less representative of the global tides, than those obtained by

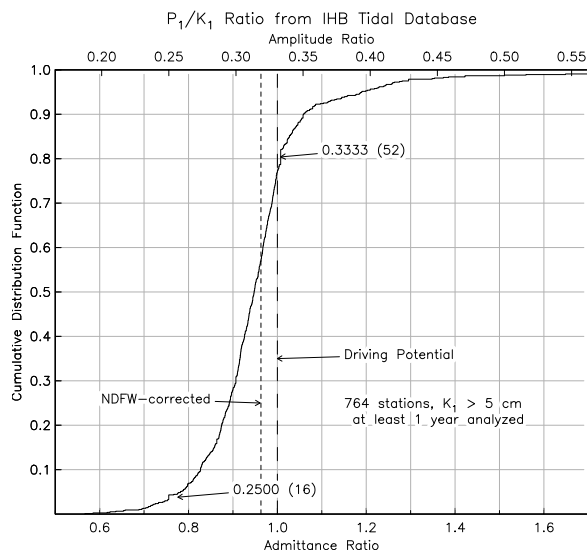


Fig. 3 Empirical cumulative distribution for the P_1/K_1 admittance and amplitude ratios for all stations shown in Figure 2A, along with the expected ratio for an earth with and without a core resonance.

Ponchaut et al (2001), not to mention those estimated from open-ocean pressure data by Ray (2013, 2017).

Nevertheless, I shall show that the NDFW resonance can be clearly seen in this collection of values, as a ratio of P_1 to K_1 amplitudes that is much closer to that described in Section 2 than the ratio in the driving potential. This can only be done by combining some data selection with substantial averaging: it is not that many sites show the appropriate ratio, but that their average is close to it.

4 Results for Different Data Sets

To get the best results from the rather heterogeneous dataset just described, it is important to remove, or winnow out, locations for which the harmonic constants might be questionable. To some extent this can be done on an *a priori* basis by looking at

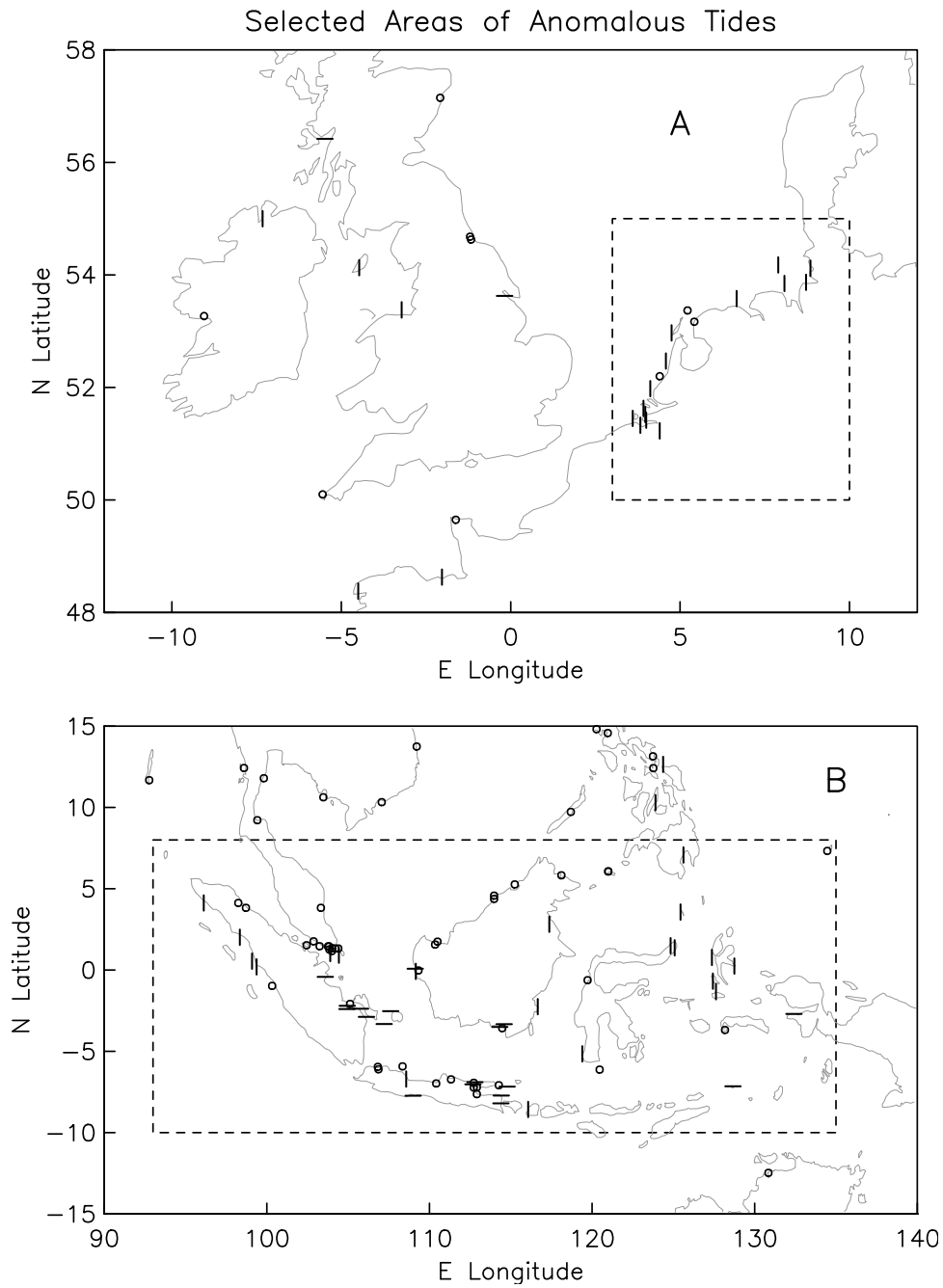


Fig. 4 Detail maps of two regions from Figure 2B that show a high concentration of extreme P_1/K_1 amplitude ratios. The dashed lines show the regions removed from the list.

1 other criteria than the actual ratio of P_1 to K_1 – though an initial look at this ratio was
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3 useful in flagging possible gross errors in the database. Appendix A describes these
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5 relatively few problems in more detail.
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8 It is not possible to determine the P_1 and K_1 tides reliably if the record length
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10 is less than six months. Looking at the distribution of durations in the database, it
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12 was apparent that there were not many stations with durations longer than 0.5 yr and
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14 shorter than 0.96 yr (355 days, a popular length for classical harmonic analysis). Of
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16 the 4090 stations available, 2974 had a duration that was unknown or less than 0.96
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18 yr, leaving 1116 to be considered further. Of these, 26 lacked P_1 or K_1 amplitudes,
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20 and 111 had K_1 amplitudes less than 0.05 m. This amplitude cutoff is the same used
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22 by Ray (2017), though it might be reasonable to make it larger in view of the greater
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24 noise level for many coastal locations.
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27 As noted above, many stations in the IHO Data Bank are in locations more likely
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29 to be affected by nonlinearity. In order to ameliorate this problem, no station was
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31 included if the P_1 amplitude was less than five times that of the larger of two nonlinear
32
33 tides in the diurnal band, MP_1 (frequency 0.9350 cpd) and SO_1 (frequency 1.0705
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35 cpd): these are well enough separated from other constituents to be reliably measured.
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37 This criterion removed 133 stations.
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40 Clearly, if there is a local resonance in the ocean with a frequency close to those
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42 of the P_1 and K_1 tides, the ratio between them could be more affected by this than
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44 by the difference in the driving potential. Such a resonance might also be expected
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46 to create a difference in phases; for this reason any station for which the P_1 and K_1
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48 phases differed by more than 15° was eliminated. There were 80 such stations.
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1 This winnowing left 766 stations; as described in Appendix A one was eliminated
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3 as having an outlying value for the P_1/K_1 ratio, and not agreeing with nearby ocean
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5 models derived from satellite ranging. Of the remaining 765, 30% had been put into
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7 the data bank before 1941, and another 30% between 1949 and 1968, suggesting
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9 that close to 60% were derived using hand computation. The oldest sea-level data
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11 used are from Cat Island, Mississippi, which were collected in 1848 and 1849; the
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13 oldest harmonic constants are probably for Hilbre Island, near Liverpool, computed
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15 by Roberts in the 1870's (Baird and Darwin, 1885). The newest data and constants
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17 come from the late 1970's.
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21 For each station, I computed the P_1/K_1 ratio and converted this to relative admit-
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23 tance by multiplying by $0.36883/0.12205$, the ratio of K_1 to P_1 given in the tables of
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25 Cartwright and Edden (1973) for the mid 20th century, the epoch most appropriate to
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27 when most of these amplitudes were estimated.
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30 The resulting 765 values of R have a mean value of 0.960 and a median of 0.948,
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32 with the extremes being 0.567 and 2.418. Figure 3 shows the distribution of ratios;
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34 this is presented as a cumulative distribution function. Such a presentation, by avoid-
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36 ing the binning needed for a histogram, can show features not otherwise visible: in
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38 this case, an excess of values with P_1/K_1 equal to one-third. The original values are
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40 in centimeters but only given to one decimal place; a Monte Carlo simulation using
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42 resampled values of K_1 , a range of ratios, and rounding the P_1 amplitude to one dec-
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44 imal, shows the same effect at about the same level: so these are simply the effect
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46 of computation, rather than evidence for P_1 having been inferred from the potential
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48 values (which gives a ratio close to this).
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1 Figure 2A shows the distribution of the winnowed station set. This shows, what is
2 common in all sea-level measurements, a high density around Japan, North America,
3 and northwest Europe – along with certain gaps caused by geopolitical issues, such
4 as on the coasts of the former Soviet Union.
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10 Figure 2B shows the locations of the stations with the lowest 10% and highest
11 10% of values for R , the question being whether there are particular regions that
12 might be pruned from the dataset to reduce outliers. Two regions stand out for a
13 relatively high proportion of extreme values for R and a plausible reason why this
14 might be locally distorted. The first is along the Belgian, Dutch, and German coasts
15 of the North Sea, where almost all the stations show high values of R , some extremely
16 high (Antwerp, on the Scheldt, has $R = 2.3$). The other region is Indonesia, where
17 complex bathymetry and coastlines create a great many short-wavelength variations
18 in the tides. Also, many of harmonic constants in this region come from data collected
19 in the 1890's – and, as reference to the original printed fascicles has shown, quite
20 often with tide poles rather than tide gauges.
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34 Figure 4 shows these two regions with the areas within which stations were
35 pruned from the list: it is important to realize that all stations in these areas were
36 removed, not just those with extreme values. This leaves 670 values of R , with a
37 mean value of 0.943, and a median value of 0.946. The extremes are 0.618 (Ogusi,
38 in Kyushu, Japan) and 1.295 (Bristol, Rhode Island, USA). It is perhaps worth not-
39 ing that the first of these two uses data from 1915 and the second from 1890, while
40 both are in regions somewhat separate from the open ocean: Omura and Narragansett
41 Bays, respectively.
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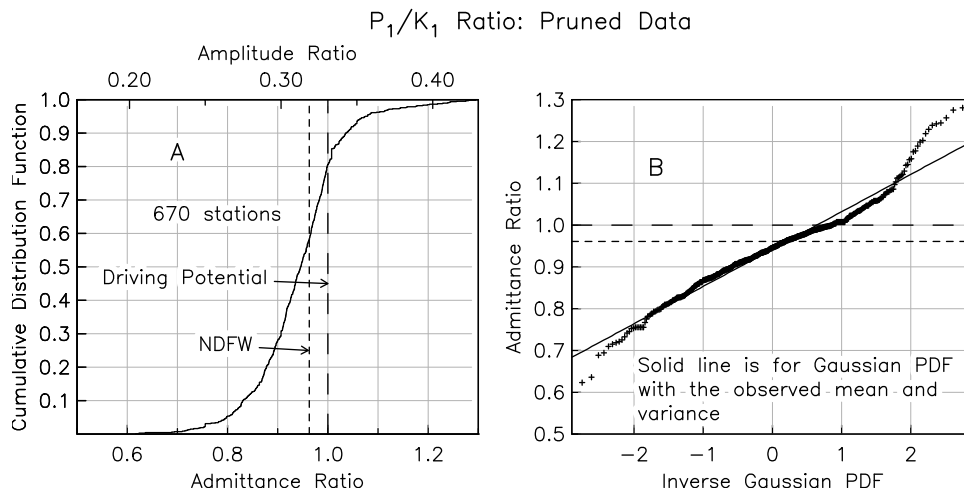


Fig. 5 Probability distribution for for the P_1/K_1 amplitude ratio for all stations shown in Figure 2A, omitting those from the regions shown in Figure 4. This distribution is shown on the left (A) as an empirical cumulative distribution and on the right (B) as a probability plot. The sloping line in panel (B) corresponds to a normal distribution with the same mean and variance as these data.

Figure 5 shows the cumulative distribution function for these pruned data, along with a probability plot. The latter shows that the data, while still slightly heavy-tailed, are close to Gaussian. The average value, mean or median, for R is robustly determined as 0.94. The nominal standard deviation if the data are further pruned by removing the tails beyond 2σ is 0.06, though it is not clear how meaningful this value is in terms of actual uncertainty. This value is about 2% below the value of R expected from theory and (much more accurate) observations (Figure 1).

5 Conclusions

The main result of this paper is a simple one: ocean-tide data, even from such a heterogeneous source as the IHO Tidal Data Bank, clearly show the presence of the

1 NDFW resonance in the ratio of the P_1 to the K_1 tides. This adds to the evidence
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3
4 educed by Ray (2017) that, when inferring the P_1 tide, the ratio should be the ratio of
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6 the constituents in the driving potential, modified by the NDFW admittance.
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8 Two questions remain, one scientific and the other historical and hypothetical.
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10 The scientific question is what, if any, conclusions we can draw from the difference
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12 between the value of $R = 0.94$ found here, and the NDFW value of $R = 0.964$. Ray
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14 (2017) finds a value of $R = 0.961$ from open-ocean measurements, and his histogram
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16 of values obtained is peaked for P_1/K_1 amplitude ratios from 0.315 to 0.320, while
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18 the R found here gives a ratio of 0.311. One plausible explanation lies in the geo-
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20 graphic distribution of stations show in Figure 2, which does not sample the coast-
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22 line, much less the ocean, in anything approaching an even distribution. It may be
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24 that the discrepancy between coastal and open-ocean values of R could be explained
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26 by large-scale variations in the admittance for the ocean tide, perhaps in terms of the
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28 diurnal modes determined by Skiba et al (2013). But this is well beyond the scope
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30 of this paper, and probably beyond, as well, what can realistically be extracted from
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32 these data.
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36 The historical and hypothetical question is, could the ocean-tide data, properly
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38 examined, have provided observational evidence for the NDFW prior to the tilt ob-
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40 servations of Melchior (1966)? The answer is, yes, these data could have provided
41
42 such evidence. Taking the “pruned” dataset described in Section 4 and limiting it to
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44 harmonic constants sent to the IHB before 1940 gives a median value of $R = 0.94$,
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46 based on 164 stations. If we extend the cutoff date to 1960, the same dataset gives
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48 a median value of $R = 0.93$ based on 327 stations. But anyone working in this field
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1 could be forgiven for not attempting such an estimate – it remains startling that this
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3 can be done, and could fairly have been judged to be improbable.
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6 7 8 **A Corrections to the IHO Data Bank** 9

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11 The following modifications were made to the values in the IHO Tidal Data Bank prior to the processing
12 described here. The tidal constants were originally published as separate sheets bound into fascicles: sheets
13 1-1967 are each for individual stations, though only sheets 1-1180 were published. Groups of stations,
14 usually with fewer constituents, were published on sheets 2000-2347 and 3000-3055.
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18 Sheet 167 (Bass Harbour, Malaysia). The data bank value for P_1 is 0.5 cm; reference to the original
19 published sheet shows that this should be 5.5 cm. (This location was not actually part of the winnowed
20 data because of the ratio of P_1 to the nonlinear tide SO_1 .)
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24 Sheet 169 (Sydney, Australia). The data bank value for P_1 is 0.5 cm; reference to the original published
25 sheet shows that this should be 4.7 cm.
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28 Sheet 670 (Stockton, California). The data bank value for P_1 is 2.0 cm; the original published sheet
29 shows a value of 1.999 cm, but this sheet also gives the amplitude in feet (the original units), and this
30 amplitude corresponds to 19.99 cm, so in this case there is a typographical error on the sheet.
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32
33 Sheet 1445 (Yeosu, Korea). The data bank gives two values for K_2 and none for K_1 ; looking at the
34 phase of other diurnal tides it is clear that the first K_2 value should be assigned to K_1 .
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36
37 Sheet 1780 (Nagapatnam, India). The data bank value for K_1 is 0.5 cm and for P_1 is 22.3 cm. I have
38 instead used the values given in Darwin (1888): 6.8 cm for K_1 and 2.2 cm for P_1 .
39

40
41 Sheet 2313 (Santander, Spain). The data bank and published sheet both give P_1 an amplitude of 9.0
42 cm, larger than K_1 (6.4 cm). The values for K_1 match a number of global models (EOT11A, FES2004,
43 TPXO7.2ATLAS, GOT4P7), which is to be expected since this is a harbor open to the ocean. But these
44 models all give values around 2 to 3 cm for P_1 . I have therefore rejected this station.
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48 **Acknowledgements** I thank Bernie Zetler for making NOAA's copy of the IHO Data Bank tape available
49 to me in 1981, and Walter Zürn and Richard Ray for comments on an early draft of this paper. Spherical-
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1 harmonic expansions of modern tide models are from Richard Ray at <http://bowie.gsfc.nasa.gov/ggfc/tides/harmonics.html>;
2
3 and his recent paper on tidal inference stimulated me to write this one.
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