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Essays in Macroeconomics: Business Cycles, Monetary Policy, and Labor Market

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Essays in Macroeconomics: Business Cycles, Monetary Policy, and Labor Market

by

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Committee in charge:

Professor Yuriy Gorodnichenko, Chair
Professor David Romer
Assistant Professor Benjamin Schoefer

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Abstract

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In this dissertation, I study labor market dynamics and positive and normative analysis of monetary policy over the business cycle. The first chapter studies how monetary policy can be more inclusive to benefit people who are particularly vulnerable over the business cycle by developing a tractable New Keynesian model with two types of labor. The second chapter focuses on labor market dynamics during the early COVID-19 period. This chapter studies the effect of the Stay-at-Home (SAH) orders on the labor market outcomes. The third chapter focuses on the effects of monetary policy. This chapter tries to estimate the extent of the information channel of monetary policy.

In Chapter 1, "Good Jobs and Bad Jobs over the Business Cycle: Implications for Inclusive Monetary Policy," I study how monetary policy can be more inclusive and benefit people who are more vulnerable to economic fluctuations. To shed light on this question, I study heterogeneity ("types") in labor market arrangements and implications of this heterogeneity for welfare and optimal monetary policy. I document that the experiences of regular and irregular workers over the business cycle differ considerably. For example, the share of irregular workers in employment rises during recessions, suggesting that firms actively adjust labor composition over the business cycle. I develop a tractable New Keynesian model with regular and irregular labor types that reflect the cyclical nature of labor composition. I find that workers, who are marginally attached to either the regular or the irregular labor market, face larger volatilities in their consumption and disutility from labor supply and hence suffer larger welfare losses over the business cycle. I find that optimal monetary policy rule should react to employment dynamics in specific segments of the labor market than the overall stance of the labor market. When a central bank follows that rule, it benefits not only people who are more vulnerable to economic fluctuations but generate higher economy-wide welfare.

Chapter 2, "Unemployment Effects of Stay-at-Home Orders: Evidence from High Frequency Claims Data," is based on the joint work with Peter McCrory, Todd Messer, and

Preston Mui, which is forthcoming in *Review of Economics and Statistics*. We use the high-frequency, decentralized implementation of Stay-at-Home orders in the United States to disentangle the labor market effects of Stay-At-Home orders from the general economic disruption wrought by the COVID-19 pandemic. We find that each week of SAH exposure increased a state's weekly initial unemployment insurance (UI) claims by 1.9% of its employment level relative to other states. A back-of-the-envelope calculation implies that, of the 17 million UI claims between March 14 and April 4, only 4 million were attributable to SAH orders. We present a currency union model to provide conditions for mapping this estimate to aggregate employment losses.

Chapter 3, "Estimating the Effects of Central Bank Communications," is based on the joint work with Nicholas Sander. We estimate the extent to which expectations changes depend on explicit information given by central banks. We compare impulse responses to high-frequency monetary surprises during announcements when the Bank of England also releases a detailed inflation report to those where a simple press statement is released. We find that when a simple press statement is released policy has conventional signs: unemployment and inflation fall following a surprise tightening. However, when a detailed inflation report is released, surprise tightening raise unemployment and inflation suggesting the information effect can be controlled by central banks.

To my husband and my family

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Chapter 1

Composition Margin of Labor Adjustment over the Business Cycle: Implications for Monetary Policy

Introductory Comments

This chapter studies how monetary policy can be more inclusive and benefit people who are more exposed to economic fluctuations. To shed light on this question, I study heterogeneity in labor market arrangements and implications of this heterogeneity for labor market dynamics, welfare and optimal monetary policy. By developing a tractable New Keynesian model featuring two types of labor, regular and irregular labor types, I demonstrate that workers who frequently move between regular and irregular labor markets face the largest burden of economic fluctuations. I show that an interest rate rule which targets these contingent workers rather than the overall stance of the labor market generates higher aggregate welfare.

"With regard to the employment side of our mandate, our revised statement emphasizes that maximum employment is a broad-based and inclusive goal. This change reflects our appreciation for the benefits of a strong labor market, particularly for many in low- and moderate-income communities."

— Jerome Powell, at the *Jackson Hole Symposium* on August 27th, 2020.

1.1 Introduction

When the Federal Reserve Chair Jerome Powell unveiled the new framework for monetary policy last summer, he emphasized that maximum employment is a broad-based and inclusive goal for the Federal Reserve. He stressed that a strong labor market benefits particularly people from low- and moderate-income communities who are more vulnerable to economic fluctuations. The costs of business cycles are not evenly distributed in the economy. Some groups of people suffer more from economic fluctuations. Nevertheless, monetary policies have relied only on "aggregate" labor market variables, ignoring this heterogeneity in the incidence of business cycle costs. Can monetary policies be more inclusive? How should monetary policies be implemented to ease the burden of the vulnerable and yet to increase aggregate welfare? This chapter seeks to answer these questions by shedding new light on heterogeneity ("types") in labor market arrangements and the implications of this heterogeneity for welfare and optimal monetary policy.

To study the differential experience of different types, I first classify various types of labor arrangements into two broad types: permanent, full-time jobs ("regular types") and other types of jobs such as temporary jobs, informal jobs, part-time jobs, and so on ("irregular types").[1] Regular jobs incur higher productivity but difficult to create or destruct and irregular jobs incur lower productivity but easier to create or destruct. Using microdata from the Current Population Survey (CPS) in the United States and full-time and part-time jobs as proxies for regular and irregular jobs, I document four stylized facts.[2] First, the share of irregular jobs rises during recessions. Second, many workers move directly from regular to irregular jobs during economic downturns. These direct flows are not only highly countercyclical but also quantitatively large.[3] Third, the share of irregular jobs significantly rises during recessions, because many regular workers directly move to

[1]The latter group has received a number of names in the literature, e.g., marginally attached workers, temporary workers, informal jobs, part-time workers, etc. I will use "irregular workers" to highlight that this group is likely to be least protected from business cycle fluctuations.

[2]I validate the use of full-time and part-time jobs as proxies for regular and irregular jobs in Appendix A.1.1.

[3]Heterogeneous experiences of different types of labor over the business cycle are well documented in the literature. Particularly in the context of the United States, there are empirical studies examining the distinct feature of involuntary part-time workers from full-time workers during the Great Recessions (see, for example, Canon et al., 2014; Cajner et al., 2014; Warren, 2017; Lariau, 2018; Mukoyama, Shintani, and Teramoto, 2019; Borowczyk-Martins and Lalé, 2019).

irregular jobs, not because those out of the labor force enter the irregular labor market. Fourth, while the number of regular workers increases in response to a positive government spending shock, the number of irregular workers decreases. Similarly, the number of people who participate in the regular labor market significantly decreases in response to a contractionary monetary policy shock. The opposite is true for the number of those participating in the irregular labor market. This illustrates that regular and irregular jobs exhibit differential dynamics in response to shocks.

Informed by these facts from the data, I develop a tractable New Keynesian model following an approach of Galí (2011, 2020), and Christiano, Trabandt, and Walentin (2021), who reformulate the standard New Keynesian framework to incorporate unemployment. In particular, this chapter extends the modeling strategy of Christiano, Trabandt, and Walentin (2021). Importantly, the approach of Christiano, Trabandt, and Walentin (2021) deviates from the assumption of perfect consumption insurance against individual workers' labor market outcomes. This feature enables me to have a meaningful heterogeneity in consumption (and hence welfare) over the business cycle for individual workers who face different labor market risks.

Unlike Christiano, Trabandt, and Walentin (2021), my model features *two types* of labor. Therefore, it is possible to examine differential dynamics between the two labor markets and different income risks that different workers face. Specifically, I introduce regular and irregular types of labor into Christiano, Trabandt, and Walentin (2021). When firms decide to adjust the total labor input, firms either newly create or destruct each type of job or they transfer one type to the other via promotion and demotion. That is, they alter the composition of job types. Workers choose which labor market to participate in and what types of jobs to seek. Workers and firms make decisions over labor types every period, which leads to changes in the composition of labor types. I call this margin of labor adjustment "*the composition margin*" and examine its implications for welfare and monetary policy in a standard New Keynesian model with sticky prices.

After calibrating my model, I show that my framework can successfully replicate the empirical patterns. My model can generate differential dynamics of the regular and the irregular labor markets in response to aggregate shocks. For example, consistent with the results from the data, firms decrease the number of regular workers but increase the number of irregular workers in response to a contractionary monetary policy shock. This, in turn, generates a large increase in the share of irregular workers. I demonstrate that aggregate (un)employment dynamics mask considerable heterogeneity in outcomes for different labor types. The changes in the relative demand for the two types make some portion of workers move frequently between the two labor markets, increasing these workers' uncertainties in their labor market outcomes.

I use my model to explore heterogeneity in the costs of economic fluctuations borne by different workers. I show that workers, who are likely to move between the regular and irregular labor markets (contingent "regular" workers) or between the irregular labor market and not-in-the-labor-force (contingent "irregular" workers), pay substantially larger welfare costs over the business cycle. Workers who are marginally attached to either the regular or the irregular jobs encounter higher uncertainties over their labor market status. Hence, they face the risks of larger fluctuations in their disutility from supplying labor and consumption, which varies according to their labor market status with the imperfect consumption insurance. In addition, the levels of consumption per each labor market status themselves vary over the business cycles, which makes those workers experience the largest consumption volatility. Moreover, the share of contingent regular workers is larger than the share of contingent irregular workers, with the changes of the composition of worker types.

Can the monetary authority achieve higher *aggregate* welfare by stabilizing the income fluctuations of these contingent workers? I find that an alternative interest rate (Taylor) rule, which stabilizes the labor-type composition based on the size of each labor market, not only improves the welfare of contingent workers but also achieves higher economy-wide welfare. This alternative monetary policy rule stabilizing the composition of labor types can minimize the number of contingent workers and the movements of them between the two labor markets. As a result, it can stabilize the consumption volatility they experience over the business cycle. This alternative specification of a Taylor rule achieves higher aggregate welfare than the conventional specification with the overall unemployment gaps. These results suggest that the central bank can achieve higher aggregate welfare by targeting more vulnerable groups, who move across the two labor markets in response to the business cycle, rather than by focusing on the overall stance of the labor market.

Related Literature

This chapter studies optimal monetary policy when incorporating the heterogeneity in labor market arrangements. In this regard, the most closely related paper is Galí (2020). He studies monetary policy implications of introducing insider-outsider labor markets into a New Keynesian model embedding a theory of involuntary unemployment of Galí (2011). In contrast to Galí (2020), my approach features imperfect consumption insurance, and therefore the level of consumption is different according to labor market status. This distinct feature enables me to explore heterogeneity in the costs of economic fluctuations borne by different workers according to labor market status, and therefore to examine if monetary policy can be more inclusive.

With regular and irregular work arrangements, this chapter examines differential labor market dynamics between the two labor markets. In this regard, my work is closely related to the literature studying labor market dynamics with two types of labor (see, for example, Blanchard and Summers, 1986; Alonso-Borrego, Fernández-Villaverde, and Galdón-Sánchez, 2005; Mukoyama, Shintani, and Teramoto, 2019). Among them, the most closely related paper is Mukoyama, Shintani, and Teramoto (2019). They introduce the full-time and the part-time labor markets into monetary DSGE models and generate differential labor market dynamics between the two labor markets. However, I explore different mechanisms than theirs. While Mukoyama, Shintani, and Teramoto (2019) focus on the on-the-job search of part-timers for full-time jobs, my model focuses more on the firm side. In my model, firms' ability to transfer one type of labor to the other via promotion and/or demotion is the key to generate the opposite responses of the two labor market variables. Moreover, in contrast to Mukoyama, Shintani, and Teramoto (2019), my work explores the welfare and policy implications of incorporating the two types of labor.

Lastly, this chapter contributes to explore heterogeneity in the costs of economic fluctuations borne by different workers. To study the welfare implications of incorporating more than one type of labor, I calculate the welfare costs of eliminating business cycles in a fashion similar to Lucas (1987). Similar to the results in the earlier literature (see, for example, Mukoyama and Şahin, 2006; Krusell et al., 2009), I show that there is substantial heterogeneity in the cost of business cycles among workers with different characteristics. In particular, my model introduces a new group of workers who pay substantially larger costs of economic fluctuations: contingent regular workers who frequently move between the regular and the irregular labor markets and switch their job types. Because these workers face larger uncertainties regarding their labor market status, they experience larger fluctuations in their labor income over the business cycle.

The rest of the chapter is organized as follows. Section 1.2 documents the importance of the changes in compositions of labor types using the micro data from the Current Population Survey in the United States. Section 1.3 lays out the model with two types of labor and multiple ways for firms to adjust labor composition. Section 1.4 explains calibration of the model. Section 1.5 examines the labor market dynamics from the model. Section 1.6 investigates welfare implications of the findings in Section 5 and discuss monetary policy implications. The chapter concludes in Section 1.7.

1.2 Evidence: the Importance of the Composition Margin over the Business Cycle

This section documents basic macroeconomic facts about regular and irregular labor over the business cycle using micro data from the Current Population Survey (CPS) in the United States between January 1976 and December 2019. To that end, I group labor market outcomes into five states:[4] (i) employed full-time (e^{FT}), (ii) employed part-time (e^{PT}), (iii) unemployed full-time (u^{FT}), (iv) unemployed part-time (u^{PT}), and (v) not in the labor force (n), following the CPS distinction of full-time and part-time status. The CPS distinguishes full-time workers, who work over 35 hours or more per week, and part-time workers, who work less than 35 hours per week. For the unemployed, the full-time versus part-time status is determined by the sort of jobs they are mainly looking for. If they mainly seek full-time (part-time) jobs, they are classified as unemployed in full-time, u^{FT}

[4]For the summary statistics, refer to Table A.1 in Appendix A.1. See also A.1 for the details about the construction of the series.

(unemployed in part-time, u^{PT}). I then examine if the composition of the two types significantly changes in response to aggregate shocks, and if the two labor markets exhibit differential dynamics to those shocks.

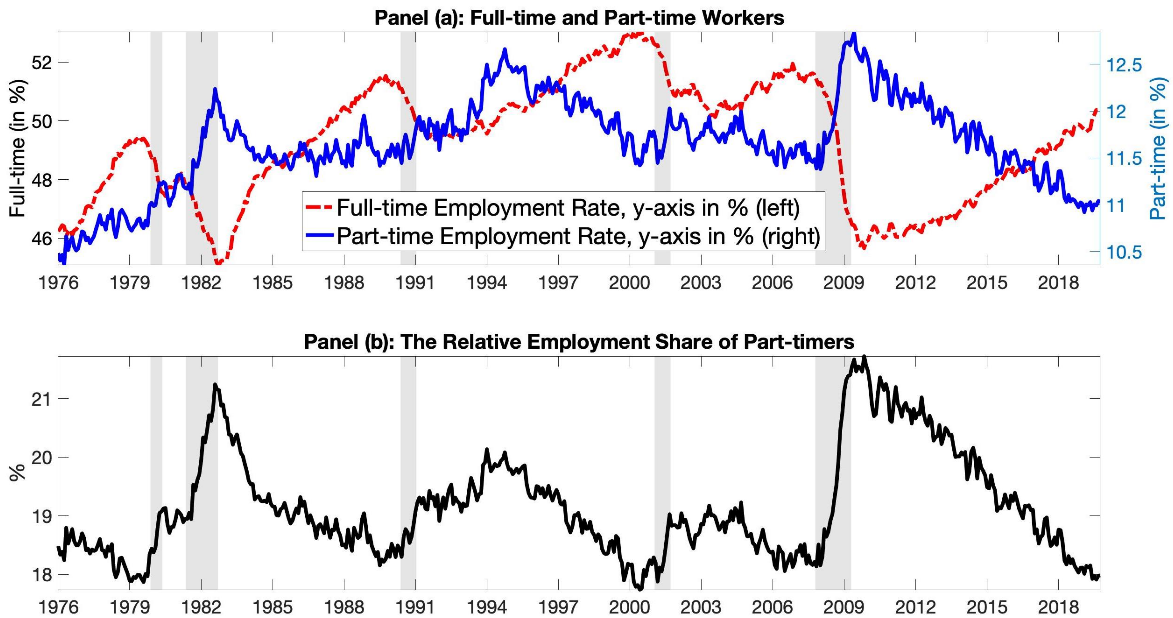
I use full-time and part-time workers as proxies for broader notions of regular and irregular types in the model I develop in Section 1.3. These proxies give me sufficiently long-time series at the business cycle frequency. Moreover, full-time and part-time status has been consistently defined compared to other classifications of work arrangements. (e.g. contingent workers, contract workers, etc.) Because full-time and part-time status does not represent all the regular and irregular jobs, I validate the use of full-time and part-time classification by comparing the relative shares of full-time and part-time workers with those of permanent and temporary workers in other countries where data on both full-time and part-time workers and permanent and temporary workers are available at the annual frequency (see Appendix A.1.1). I show that the relative shares of the full-time and part-time workers are comparable to those of permanent and temporary workers in other countries where data on both full-time and part-time workers and permanent and temporary workers at the annual frequency are available: Germany, Italy, Greece, and the United Kingdom. Moreover, I show that the evolution of the share of involuntary part-timers is in line with that of temporary workers in these countries.

Fact 1. Regular (irregular) jobs are procyclical (countercyclical).

First, I show that the relative size of full-time employment to part-time employment in the United States changes over the business cycle. Panel (a) of Figure 1.1 plots the time series of full-time and part-time employment (e^{FT} and e^{PT}) as a share of population of age over fifteen, and Panel (b) plots the *relative* share of the part-time workers out of total employment. While the full-time employment is *procyclical* (the red dash-dot line), the part-time employment is *countercyclical* (the blue solid line). Countercyclicality of part-time employment particularly stands out during longer recessions of the early 1980s and the Great Recession. Panel (b) shows that the relative employment share of part-time workers increases during recessions. This is consistent with one of the findings in Katz and Krueger (2017) that weak labor market conditions lead to an increase in irregular jobs.[5] The fact that the relative size of the two labor markets changes over the business cycle implies that the composition of workers significantly varies over the business cycle.

[5] Their "non-traditional" work corresponds to irregular jobs in my paper.

Figure 1.1: The Size of Full-time and Part-time Workers out of Population (age> 15) and The Relative Employment Share of the Part-time Workers.



Note: In Panel (a), the red dash-dot line is the size of full-time workers out of population of age greater than fifteen in percentage with the y-axis on the left, and the blue solid line is the size of part-time workers in percentage with the y-axis on the right. In Panel (b), the black solid line denotes the relative employment share of part-time workers, that is, $e^{PT}/(e^{FT}+e^{PT})$. Recession periods from NBER classifications are denoted as grey shaded areas. *Source*: CPS microdata from January 1976 to December 2019.

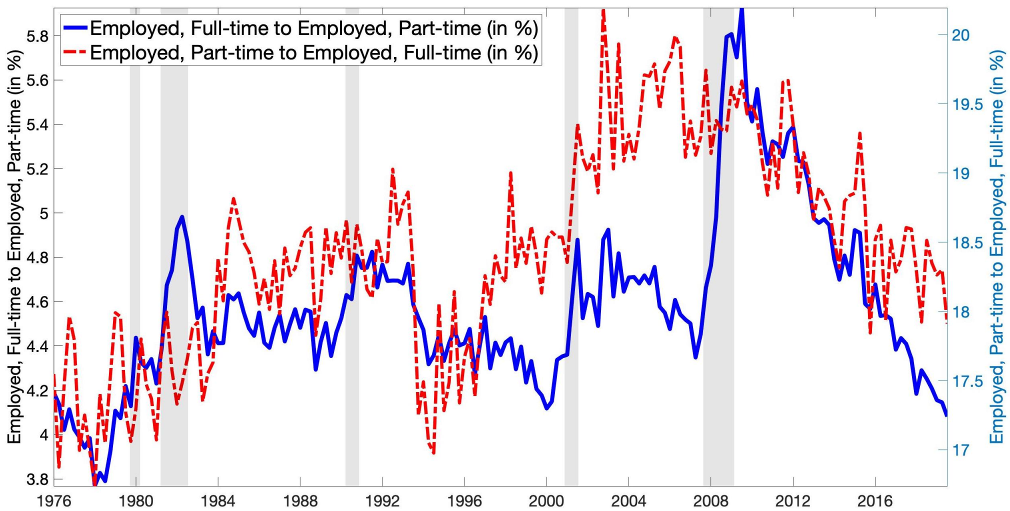
Fact 2: Flows from the regular employment to the irregular employment are countercyclical.

The changes in the relative size of the two labor markets stem from the flows of workers between them.[6] The transition from part-time employment to full-time employment, $f_{e^{PT},e^{FT}}$ (red dash-dot line), does not exhibit any cyclicality. The transition from full-time employment to part-time employment, $f_{e^{FT},e^{PT}}$ (blue solid line), is clearly countercyclical. It sharply rises during recessions and gradually decreases during booms. This countercyclical flow is not entirely driven by firms' adjustment of intensive margins. To show this, I compare $f_{e^{FT},e^{PT}}$ for hourly-paid workers with those who are not hourly-paid and non-respondents. I also compare this flow for those who work for the same employer and do the same job and for those who either do not work for the same employer or do different jobs than before. See Appendix A.1.2. Moreover, the magnitude of the flow, $f_{e^{FT},e^{PT}}$ is significantly large. For example, in recessions, the number of workers who move from

[6]See Appendix A.1 for the construction of the series.

full-time employment to part-time employment is more than twice the number of workers who move from the employed to the unemployed.[7]

Figure 1.2: Flows between Full-Time Employment and Part-Time Employment



Note: This figure shows the gross flows between full-time employment, e^{FT} with the blue solid line in percentage (y-axis on the left) and part-time employment, e^{PT} with the red dash-dot line in percentage (y-axis on the right). Recession periods from the NBER classifications are denoted as grey shaded areas. All the flows are margin error adjusted, "deNUNfied" (their suggested method of correcting for classification errors) following Elsby, Hobijn, and Şahin (2015), corrected for temporal aggregation bias following Shimer (2012) and seasonally-adjusted using X-13-ARIMA. *Source*: CPS microdata from January 1976 to December 2019.

Fact 3: Flows from regular to irregular employment explain most of the changes in the composition of workers.

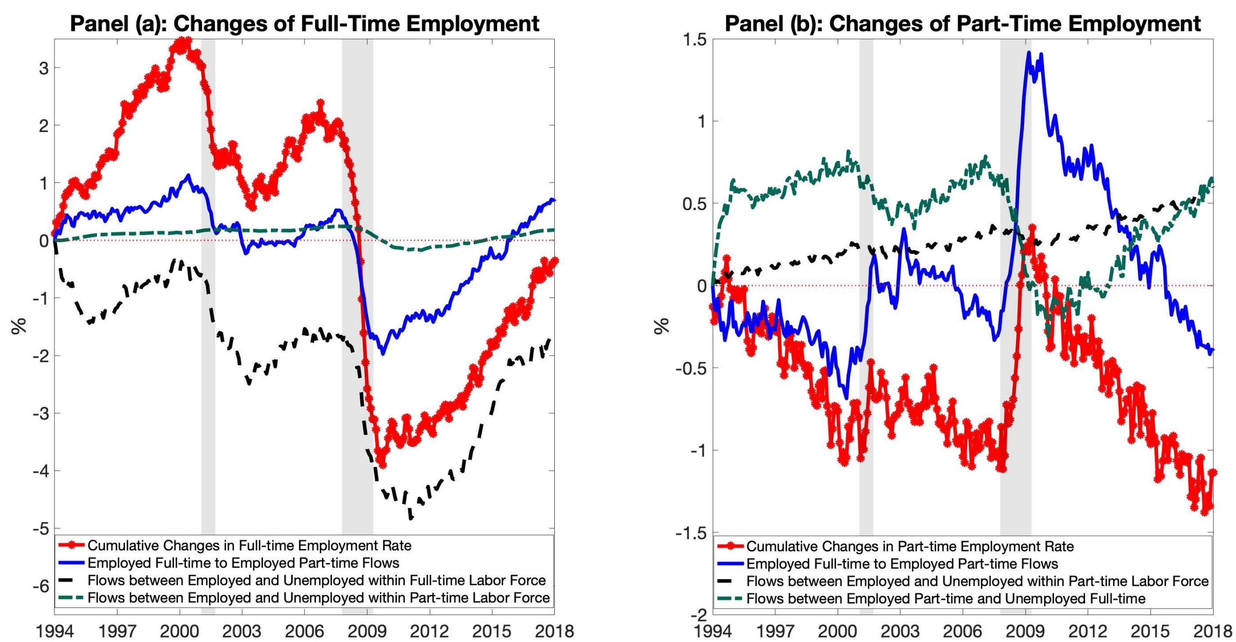
Following the method developed by Elsby et al. (2019), I calculate the contributions of the flows *across* the two labor markets to explain the evolution of full-time employment and part-time employment.[8] Panel (a) of Figure 1.3 shows the contribution of each flow to explain full-time employment rate changes. The red line with circles shows the *cumulative* changes of full-time employments from January 1994 to December 2019. This shows that the stock of full-time employment significantly drops during recessions and gradually rises during expansions. As expected, the E to U and U to E transitions *within* the full-time labor market, $\Delta p_{e^{FT},u^{FT}}$ and $\Delta p_{u^{FT},e^{FT}}$, explain most of the changes of full-time

[7]For the other flows, $f_{e^{FT},u^{FT}}$, $f_{e^{PT},u^{PT}}$, $f_{e^{FT},u^{PT}}$, $f_{e^{PT},u^{FT}}$, $f_{e^{FT},n}$, $f_{n,e^{FT}}$, $f_{e^{PT},n}$, and $f_{n,e^{PT}}$, see Appendix A.1.3 and A.1.5.

[8]For the flow decomposition of full-time labor force, $e^{FT}+u^{FT}$, part-time labor force, $e^{PT}+u^{PT}$, the total labor force, $e^{FT}+u^{FT}+e^{PT}+u^{PT}$ and not-in-the labor force, n, see Appendix A.1.4, A.1.7, and the online appendix of Elsby et al. (2019).

employment, Δe^{FT}. We can observe this from the black dash line, which closely tracks the changes in full time employment rates denoted as the red line with circles. On the other hand, the contribution of the flows from the full-time employment rate to the part-time employment rate, $\Delta p_{e^{FT},e^{PT}}$, is plotted with the blue solid line. While these are not as big as the contribution shown in the black dash line, which represents the E to U and U to E transitions within the full-time labor market, the flows of $\Delta p_{e^{FT},e^{PT}}$ still explain significant portions of the changes in full-time employment. For example, the full-time employment rate could have dropped by four percentage point during Great Recessions, if only the E to U and U to E transitions within the full-time labor market had changed. However, the transition rates from full-time to part-time employment have risen sharply during the Great Recession. This has accelerated the drop in full-time employment rates by two percentage point more.

Figure 1.3: Flow Decomposition of Full-Time and Part-Time Employment Rates



Note: In Panel (a) (Panel (b)), the red line with circles plots the cumulative changes of the full-time employment rates (the part-time employment rates); the blue solid line is the cumulative contributions of the changes of the flows from full-time employment to part-time employment to explain the full-time employment rate changes (to explain the part-time employment rate changes); The black dash line is the cumulative contributions of the changes of the E to U and U to E transitions within the full-time labor market (within the part-time labor market), and the green dash-dot line is the contributions of the changes of the flows between part-time employment and full-time unemployment rate. Recession periods according to NBER classifications are denoted as grey shaded area. All the flows are margin error adjusted, "deNUNfied" (their suggested method of correcting for classification errors) following Elsby, Hobijn, and Şahin (2015), and seasonally-adjusted using X-13-ARIMA. *Source*: CPS microdata from Januaray 1994 to December 2019.

The importance of the flows *across* the two labor markets stand out even more in the flow decomposition of the part-time employed. Panel (b) of Figure 1.3, the red line with circles shows the cumulative changes in the part-time employments, Δe^{PT} from January 1994 to December 2019. Consistent with Figure 1.1, the stock of part-time employment sharply rose during the Great Recession. What contributes the most to the changes in part-time employment rate, Δe^{PT}, is the flow from the full-time employment rate to the part-time employment rate, $\Delta p_{e^{FT},e^{PT}}$ (blue solid line). This time series closely tracks Δe^{PT}, suggesting that most of the changes in part-time employment are explained by higher transitions from full-time to part-time employment. Other important sources explaining the evolution of the part-time employment rates are the flows between part-time and full-time unemployment, $\Delta p_{e^{PT},u^{FT}}$ and $\Delta p_{u^{FT},e^{PT}}$ (the green dash-dot line). For example, during the Great Recession, the part-time employment rate could have risen by two percent, if only the transition probabilities from the full-time employment to the part-time employment, $\Delta p_{e^{FT},e^{PT}}$ had risen. However, the part-time employment rate has risen by one and a half percent, because more part-time workers exit to full-time unemployment (looking for full-time jobs), and fewer full-time unemployed workers enter part-time employment. Due to these changes, the increases in the part-time employment rate during the Great Recession were attenuated. In contrast to these *cross*-market flows, flows *within* the part-time labor market, $\Delta p_{e^{PT},u^{PT}}$ and $\Delta p_{u^{PT},e^{PT}}$, do not explain the evolution of part-time employment, as is clear from the black dash line.

In summary, the flows of workers across the two labor markets rather than the flows between not-in-the-labor-force and each labor market explain the changes in the composition of worker types. This further illustrates that a significant portion of the workforce may experience fluctuations in their labor income from switching job-types over the business cycle. In other words, switching job types could be an important source of income risks for a large number of workers over the business cycle.

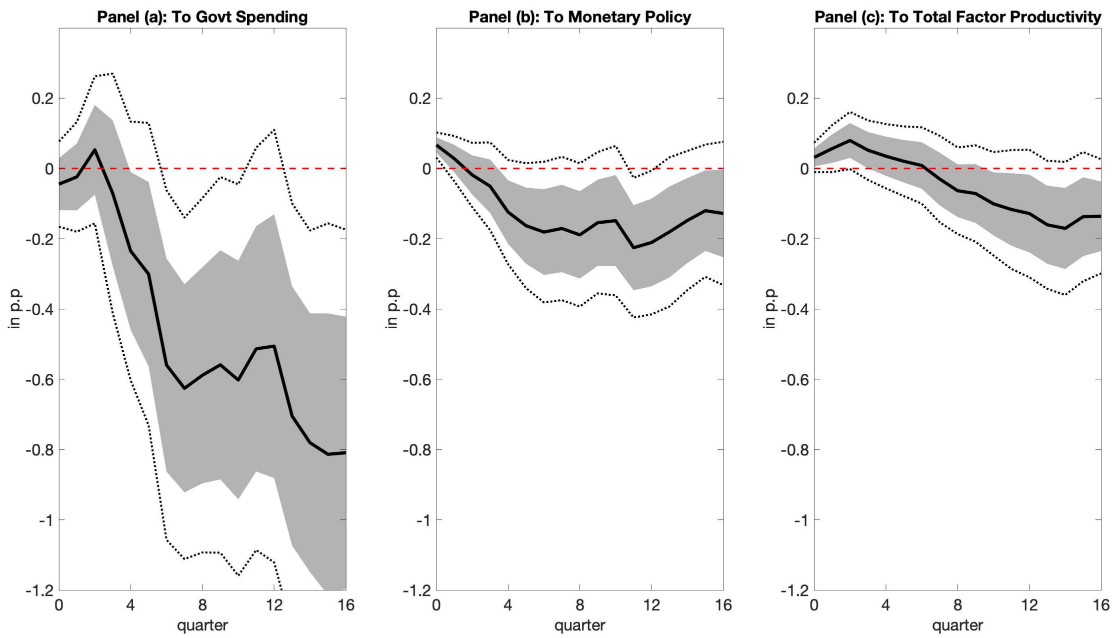
Fact 4: The two labor markets behave differentially over the business cycle.

To study the behavior of labor markets, I now estimate impulse responses of each labor market variables to observable structural shocks: **?** monetary policy shocks; utilization rate adjusted total factor productivity shocks from Basu, Fernald, and Kimball (2006); government spending shocks from Ben Zeev and Pappa (2017). For impulse response estimations, I use Jordà (2005) local projections:

$$y_{t+h} - y_{t-1} = \alpha_h + \beta_h x_t + \psi_h(L) z_{t-1} + \epsilon_{t+h}, \quad \text{for } h = 0, 1, 2, \cdots, \tag{1.1}$$

where y denotes an endogenous dependent variable of interest, x_t denotes a structural shock series, z_{t-1} is a vector of control variables, and L denotes lag operator. $\{\beta_h\}$ gives the responses of x at time $t+h$ to the shock at time t for $h = 0, 1, 2, \cdots$.

Figure 1.4: The Impulse Responses of the Share of Part-time Workers out of Total Employment



Note: This figure shows the responses of the share of part-time workers out of total employment ($e^{PT}/(e^{FT}+e^{PT})$) to structural shocks, estimated from the local projection of Equation (1.1). Panel (a) shows the responses to a one-standard deviation of positive government spending shocks, Panel (b) shows those to a one-standard deviation of expansionary monetary policy shocks, and Panel (c) shows those to a one-standard deviation of total factor productivity shocks. The point estimates are denoted as black solid lines with black dotted lines as 90 percent confidence intervals and with the grey shaded areas as 68 confidence intervals. *Sources:* The relative share of part-time workers is calculated using the microdata from the CPS. Panel (a) uses the government spending shocks constructed from Ben Zeev and Pappa (2017) between the second quarter of 1976 to the fourth quarter of 2000. Panel (b) uses quarterly-aggregated **?** monetary policy shock series from the second quarter of 1976 to the fourth quarter of 1996. Panel (c) uses utilization rate adjusted total factor productivity shocks constructed from Fernald (2014) between the second quarter of 1976 and the third quarter of 2015.

Figure 1.4 first presents the estimated impulse response functions of the composition of workers, that is, the share of part-time workers out of the total employment. This figure further corroborates the previous finding that the composition of workers changes over the business cycle. In response to a one standard deviation of positive government spending shock, the share of part-time workers out of the total employment significantly decreases by 1 percentage points at the peak. Similarly, in response to a one standard deviation of an expansionary monetary policy shock, the share of part-timers drops by 0.2 percentage points at the peak. Lastly, the share decreases by 0.15 percentage points in response to one standard deviation of utilization-rate adjusted TFP shock.

Figure 1.5: The Responses of Each Labor Market Variables to Expansionary Monetary Policy Shocks

Note: This figure shows the responses of total labor market variables and each labor market's variables: employment rates, the size of labor forces, and unemployment rates to a one standard deviation of expansionary monetary policy shocks. The estimates for the total labor market variables' responses are denoted as black solid lines with black dotted lines as 90 percent confidence bands and with grey shaded area as 68 percent confidence bands. The estimates for the full-time labor market variables' responses are denoted as red solid lines with red dotted lines as 90 percent confidence bands and with the pink shaded area as 68 percent confidence bands. Those for the part-time labor market variables' responses are denoted as blue solid lines with blue dotted lines as 90 percent confidence bands and with the sky-blue shaded area as 68 percent confidence bands. *Sources:* All the labor market variables are calculated from the CPS from March 1976 to December 1996. I use quarterly-aggregated **?** monetary policy shocks from March 1976 to December 1996.

Moreover, Figure 1.5 shows that each labor market variables exhibit *differential* responses to a monetary policy shock. Estimated responses to other structural shocks paint the same picture that the two labor market exhibit differential dynamics (See Appendix A.1.8). The employment rates in each labor market move to the opposite directions in response to an expansionary monetary policy shock. For example, while the full-time employment rates significantly increase, the part-time employment rates decrease.

In contrast, the unemployment rates in general move in the same direction. For example, both full-time and part-time unemployment rates significantly decrease in response to an expansionary monetary policy shock. The magnitudes of the responses are, however, different. The unemployment rate in the full-time labor market responds to the most, while the part-time unemployment rate moves only modestly to the shocks. The responses of the total unemployment rate are in between the two, while closer to the responses of the full-time unemployment rates.

These differences in the magnitudes of the unemployment rates' responses in each labor market stem from the differential responses of the labor force sizes in each labor market. First, the total labor force participation rates respond only modestly, which is consistent with the modest procyclicality of the total labor force participation rates documented in the literature (see, for example, Elsby, Hobijn, and Şahin, 2015). This, however, masks heterogeneous responses of each labor market's labor force. For example, in response to one standard deviation of positive monetary policy shocks, the total labor force participation rate increases by 0.25 percent. In contrast to this, the full-time labor force significantly rises by 0.45 percent at the peak and the part-time labor force significantly decreases by 0.5 percent at the peak. The magnitudes of the changes in the full-time labor force are, however, smaller compared to the magnitudes of the changes in the full-time employment rates, while the changes in the part-time labor force are greater than the changes in the part-time employment rates. This generates different magnitudes of each labor market's unemployment rate changes but, in general, in the same direction.

To summarize, this section documents the differential experiences of the full-time and the part-time labor markets over the business cycle. While full-time employment is procyclical, part-time employment is strongly countercyclical. This is largely due to the countercyclical flows from full-time employment to part-time employment. Flow decompositions show that these countercyclical flows explain a large portion of the falls of the full-time employment rate and the rise of part-time employment rate during recessions. These findings imply that the *composition* of workers significantly varies over the business cycle, and these changes in the composition of workers largely come from the workers who move *between* the two labor markets. The two labor markets also exhibit differential dynamics to structural shocks. Motivated by these facts, the next section introduces a model that can generate these facts and studies implications of the changes in the *composition* of work arrangements over the business cycle.

1.3 A New Keynesian Model with Regular and Irregular Work Arrangements

This section presents a New Keynesian model featuring *two types* of labor.[9] Unlike the standard New Keynesian model embedding a theory of unemployment (Galí, 2011; Galí, Smets, and Wouters, 2011; Christiano, Trabandt, and Walentin, 2021), my model features two types of labor: (i) regular jobs with higher productivity but are difficult to create or destruct and (ii) irregular jobs with lower productivity but are easier to create or de-

[9] Because the model builds on Christiano, Trabandt, and Walentin (2021), notations and functional specifications closely follow theirs.

struct.[10]

On the labor supply side, workers endogenously decide which type of jobs they would like to have. On the labor demand side, I introduce multiple margins of adjustment for firms to change the total amount of labor input, particularly by changing the composition of the two types: not just creating or destructing each type of jobs, but transferring one type to the other via promotions and/or demotions. Because there is more than one type of labor, firms now can adjust the total amount of labor input by changing the composition of job types by creating one type of jobs but destroying the other type of jobs and/or via transferring one type to the other type. I call this "*composition margin*,"[11] and examine the relevance of this composition margin for understanding labor market dynamics and welfare and policy implications over the business cycle.

The model has the following agents: (i) Infinitely many workers within a representative family, who decide whether to participate in the labor market or not. If participating, they decide *which* labor market to enter and how much job search effort to exert; (ii) a representative family that supplies *two types* of labor, consumes, invests in physical capital, chooses the degree of capital utilization, leases capital services to intermediate-goods-producing firms, and saves using nominal bonds,[12] (iii) intermediate-goods-producing-firms that use capital services and two types of labor to produce differentiated intermediate goods and set prices by paying price adjustment costs, (iv) final-goods-producing-firms that bundle differentiated intermediate goods into a final good; (v) a central bank conducting a Taylor-rule-type monetary policy and a fiscal authority, which does spending exogenously and finances it with lump-sum taxes and debt. The next subsections explain the environments of each economic agent and their optimization problems.

1.3.1 Workers

There are infinitely many identical families $h \in [0,1]$ consisting of infinitely many workers. Figure 1.6 summarizes the environment of a worker within each family. Each worker within a family has different level of work aversion, $\ell \sim \mathcal{U}[0,1]$, which summarizes her inherent characteristics such as gender, education level, the number of kids, and so on. Given ℓ, a worker decides to either participate in a labor market or not. If she decides to participate in any labor market, she simultaneously determines which market to enter: (i)

[10] Difficulty of adjustment comes from a number of sources: (i) firms need to pay high physical costs to hire and fire regular workers, (ii) regular workers are more attached to firms as exogenous separation rates are lower, (iii) the fact that they have higher productivity makes firms reluctant to fire them, and (iv) it takes time for promoted irregular workers to exhibit full productivity as other regular workers.

[11] It is true that the composition margin is not separable from the other two laobr adjustment margins, extensive (the number of workers) and intensive margins (hours worked per worker). The composition margin is, rather, closely connected to the other two margins and helps to better understand the behaviors of the other two margins over the business cycle. See Appendix A.2.7 for the relationships between the composition margin and the other two margins.

[12] It is easy to extend the model to incorporate wage rigidity for regular workers. Appendix A.2.11 presents such extensions.

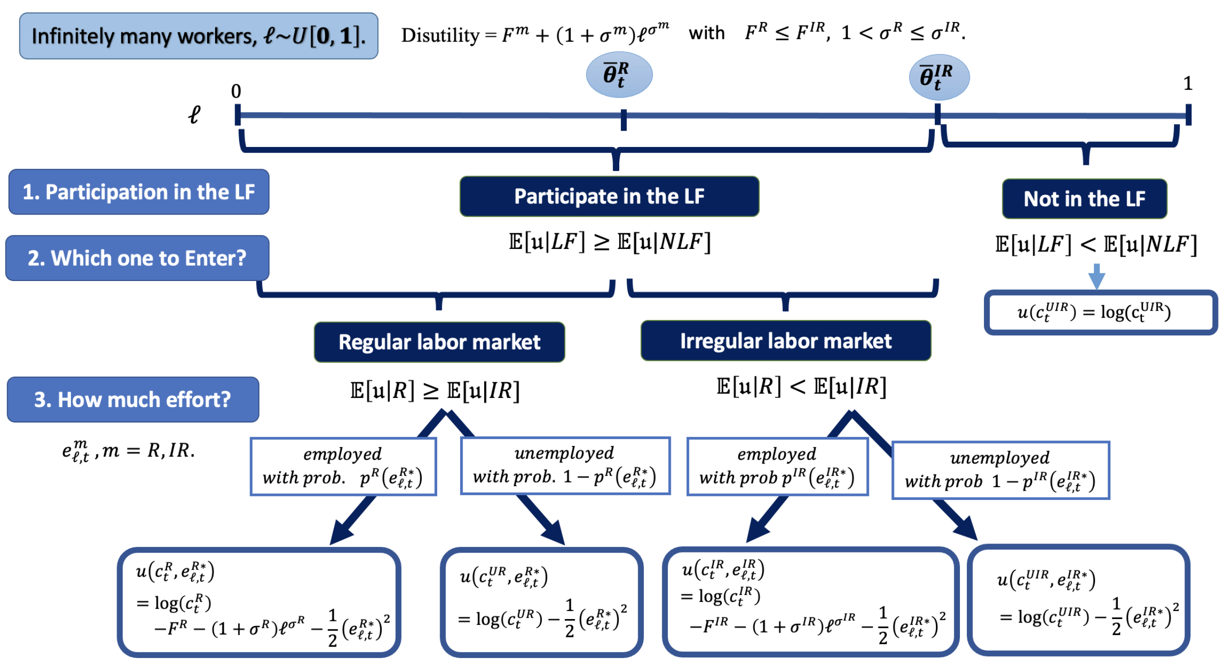
Regular labor market ($m = R$) and (ii) Irregular labor market ($m = IR$). Upon entering a labor market, she chooses the optimal level of effort to exert to find a job.

A worker with an inherent work aversion level ℓ faces the following disutility of working when she decides to enter a labor market $m = R$ or IR:

$$F^m + (1+\sigma^m)\ell^{\sigma^m}, \quad m = R,\ IR. \tag{1.2}$$

I assume that $F^R \leq F^{IR}$, so that working in the irregular labor market generates higher disutility. I also assume $1 < \sigma_L^R \leq \sigma_L^{IR}$, implying that disutility from working increases at a higher rate in the regular labor market than in the irregular labor market.[13] When workers decide whether to work or not and which labor market to enter, they know the values of these parameters.

Figure 1.6: Summary of Workers' Problems



Note: This figure illustrates an individual worker's problem within the representative family. Each worker in the family has three problems to solve. Given ℓ, she decides whether to participate in the labor force, by comparing expected utility from participating in any labor market and from not participating. If she decides to enter any labor market, she needs to choose *which* labor market to enter, again by comparing ex-ante expected utility from entering each labor markets. Upon entering a labor market, she determines the level of effort to exert by taking into account the fact that it increases the probability to find a job at the quadratic utility costs.

[13] The assumption of $1 < \sigma_L^R \leq \sigma_L^{IR}$ reflects the fact that regular workers tend to work overtime and have much more responsibilities than irregular workers (see Mas and Pallais, 2020). Therefore the higher the inherent level of private cost of working is, the higher their disutility becomes when participating in the regular labor market.

If a worker with ℓ chooses to enter a labor market m, she decides how much effort, $e^m_{\ell,t}$ to exert to find a job. The more effort she exerts, the higher the probability that she finds a job, but at some quadratic utility costs defined as $(e^m_{\ell,t})^2/2$, for $m = R,\ IR$. The probability of finding a job in a labor market m when a worker exerts effort, $e^m_{\ell,t}$ is given by the following linear function:[14]

$$p(e_{\ell,t}) = \eta^m + a^m e^m_{\ell,t},\ \ \eta^m, a^m \geq 0,\ \ m = R, IR. \tag{1.3}$$

I assume $0 \leq \eta^R \leq \eta^{IR}$, and $0 \leq a^R \leq a^{IR}$, to capture the fact that it is easier to find an irregular job than a regular job, given the same level of effort.[15]

Ex-ante, a worker with a work aversion of $\ell \in [0,1]$ considering to enter a labor market m has the following expected utility:

$$p(e^m_{\ell,t}) \times \left\{ \overbrace{\log(c^m_t)}^{\substack{\text{utility from consumption}\\ \text{when she finds a job}}} - \overbrace{\left(F^m + (1+\sigma^m_L)\ell^{\sigma^m_L}\right)}^{\text{Disutility from working}} \right\} + \left(1 - p(e^m_{\ell,t})\right) \times \left\{ \overbrace{\log(c^{Um}_t)}^{\substack{\text{utility from consumption}\\ \text{when she cannot find a job}}} \right\} - \overbrace{\frac{1}{2}(e^m_{\ell t})^2}^{\text{effort cost}}, \tag{1.4}$$

where c^m_t is the level of consumption when this worker gets a job in a labor market m, and c^{Um}_t is the level of consumption if she fails to find a job in a labor market, $m = R,\ IR$. Here, I allow for the possibility of $c^{UR}_t \geq c^{UIR}_t$ to capture the fact that unemployed workers in the irregular labor market typically do not have access to unemployment benefits. While the level of c^R_t, c^{IR}_t, c^{UR}_t do vary over time, I assume that c^{UR}_t / c^{UIR}_t is a fixed fraction to derive a tractable functional form for the family-level utility (See Section 1.3.2). A worker with inherent work aversion ℓ, considering to enter a labor market m, then optimally chooses the effort level $e^m_{\ell,t}$ which maximizes her *ex-ante* utility given by Equation (1.4).[16]

Meanwhile, if a worker decides not to enter any labor market, her utility is simply given by

$$\log(c^{UIR}_t), \tag{1.5}$$

[14]Parameter values in the job finding probability functions are not time-varying. Job search effort, however, is procyclical, which makes job finding probabilities in recessions lower than those in booms.

[15]These parameter values capture the fact that firms might be more cautious to hire regular workers than irregular workers, as the former tends to be more attached to firms than the latter. Firing costs of regular workers are much higher than those of irregular workers due to this firm-specific human capital and severance payment. In fact, firms tend to have more screening processes for hiring regular workers than hiring irregular workers. (see, for example, Houseman, Kalleberg, and Erickcek, 2003) It is more likely to *randomly* encounter vacancy postings for irregular jobs than those for regular jobs. For example, it is easy to encounter job postings for restaurant/cafe servers when we visit those restaurants/cafes, and servers are one of the popular irregular jobs. (e.g. part-timers) This parameter choice is consistent with the estimated transition probabilities in Hall and Kudlyak (2019) that the transition probability from unemployment to the short-term job is higher than the transition probability from unemployment to the longer-term job for men.

[16]For the functional form of the optimal level of effort, see Appendix A.2.1.

regardless of the value of $\ell \in [0,1]$. I assume that the level of consumption for those out of the labor force is the same as the level of consumption for the unemployed in the irregular labor market.

A worker with ℓ that considers entering a labor market m compares the ex-ante expected utilities of participating in any labor market as in Equation (1.4) given the optimal level of effort $e^{m*}_{\ell,t}$, and the utility of being out of labor force given by Equation (1.5). Upon deciding to participate in any labor market, she compares her expected utility of participating in the regular labor market and that of participating in the irregular labor market. If the former is higher, then she enters the regular labor market, and if the latter is higher, she enters the irregular labor market.

Remark 1 *For $1 \le \sigma^R_L \le \sigma^{IR}_L$,*[17] *we can define the following two thresholds, $\bar{\theta}^R_t$ and $\bar{\theta}^{IR}_t$ with the following rankings:*

$$0 < \bar{\theta}^R_t \le \bar{\theta}^{IR}_t < 1, \tag{1.6}$$

where $\bar{\theta}^{IR}_t$ is the value of ℓ at which the ex-ante expected utility of a worker with $\ell = \bar{\theta}^{IR}_t$ when entering the irregular labor market is the same as the utility of being out of labor force, and $\bar{\theta}^R_t$ is the value of ℓ at which the ex-ante *expected utilities of participating in the regular and in the irregular labor market for a worker with $\ell = \bar{\theta}^R_t$ are the same.*[18]

Given the two thresholds, then a worker with ℓ makes the following decision regarding her labor market status:

$$\begin{cases} \text{if } 0 \le \ell \le \bar{\theta}^R_t & \Rightarrow \text{ Participate in the regular labor market,} \\ \text{if } \bar{\theta}^R_t < \ell \le \bar{\theta}^{IR}_t, & \Rightarrow \text{ Participate in the irregular labor market,} \\ \text{if } \bar{\theta}^R_t < \ell \le 1, & \Rightarrow \text{ Out of the labor force.} \end{cases}$$

Therefore, workers with ℓ close to $\bar{\theta}^{IR}_t$ are marginally attached to the labor market (mostly irregular labor market), in a sense that a slight decrease of $\bar{\theta}^{IR}_t$[19] pushes her out of the labor force.[20] Similarly, a slight decrease in $\bar{\theta}^R_t$ makes workers with ℓ close to $\bar{\theta}^R_t$ move from the regular labor market to the irregular labor market. Therefore, these workers are

[17] This assumption makes sure to generate that those tend to participate in the irregular labor market are the ones who sometimes are out of labor force. Therefore, they are the ones who are *marginally* attached to the total labor market, which is consistent with one of the findings from Hall and Kudlyak (2019) that "circlings" happen between unemployment, short-term jobs, and out of labor force, considering that short-term jobs are close to irregular jobs than regular jobs.

[18] See Appendix A.2.1 for the formal equations related to these two thresholds, *i.e.* incentive compatibility conditions.

[19] For example, in response to a negative demand shock (a contractionary monetary policy shock or a negative government spending shock), labor demand decreases. This in turn decreases $\bar{\theta}^{IR}_t$ with lower wages. See Section 1.5.

[20] This is consistent with empirical analysis using the microdata from the CPS. Flows between the part-time labor market and out-of-the-labor-force are cyclically more important than those between the full-time labor market and out-of-the-labor-force. See Appendix A.1.5.

marginally attached to the *regular* labor market because they move between the two labor markets, as $\bar{\theta}_t^R$ changes over time.

Because of the unit measure of workers ($\ell \sim \mathcal{U}[0,1]$) within the representative family, these two thresholds correspond to the size of each type's labor forces: $\bar{\theta}_t^{IR}$ corresponds to the size of the total labor force; $\bar{\theta}_t^R$ corresponds to the size of the regular labor force; $\bar{\theta}_t^{IR} - \bar{\theta}_t^R$ is the size of the irregular labor force.

The two thresholds $\bar{\theta}_t^{IR}$ and $\bar{\theta}_t^R$ and hence each labor market's labor forces vary as consumption premiums (c_t^R/c_t^{UR}, c_t^{IR}/c_t^{UIR}, c_t^R/c_t^{IR}) change over time.[21] Intuitively, a worker's labor market participation decision, given ℓ, depends on the level of consumption which differs across the labor markets and the labor market status. If the level of consumption for a regular worker is much higher than the level of consumption for an irregular worker, more workers want to enter the regular market, which would increase $\bar{\theta}_t^R$. Similarly, if the gap between the levels of consumption for any worker and for a non-worker is much higher, then more workers are willing to participate in the labor force. The next section explains how the representative family sets consumption premiums, which in turn determines the sizes of labor forces in each labor market.

1.3.2 A Representative Family's Problem

In this section, I describe a representative family's problems. The representative family's problem is summarized in Figure 1.7. This extends Section 2.3. of CTW to the case of two types of jobs. I start this section by deriving a family's indirect utility function. Given this family-level indirect utility function, I present the problem that the representative family solves to select the level of family-wide consumption, the amount of regular and irregular workers to supply, and the amount of savings from the family-level optimization problem.

Consider first the relationship between the two thresholds, $\bar{\theta}_t^R$ and $\bar{\theta}_t^{IR}$, and the number of each type of worker to supply at the family level. The number of regular workers, n_t^R, and the number of irregular workers, n_t^{IR}, are obtained by integrating the probability of finding a job given the optimal level of effort up to the thresholds determining its labor market's sizes as follows:

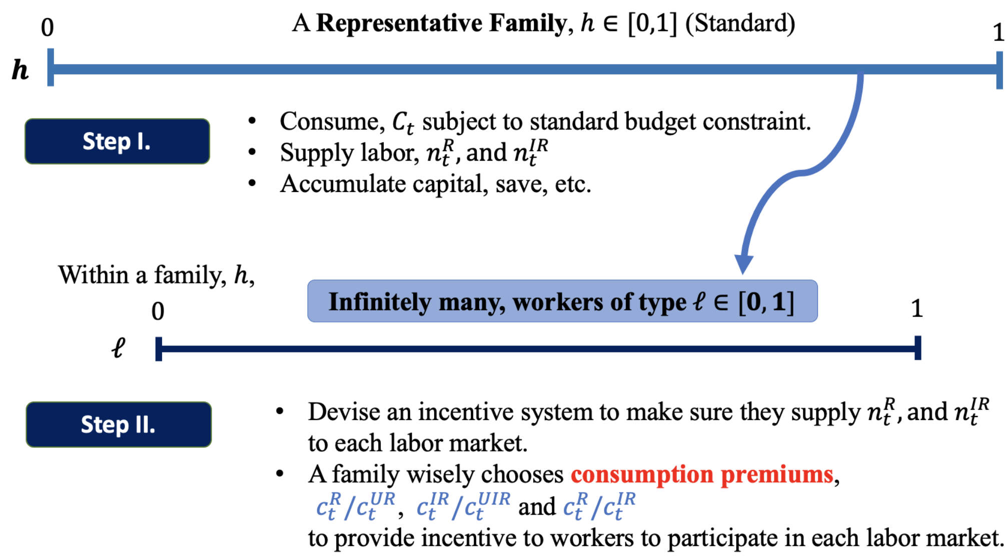
$$n_t^R = \int_0^{\bar{\theta}_t^R} p(e_{\ell,t}^{R,*})\mathrm{d}\ell, \quad n_t^{IR} = \int_{\bar{\theta}_t^R}^{\bar{\theta}_t^{IR}} p(e_{\ell,t}^{IR,*})\mathrm{d}\ell. \tag{1.7}$$

From the two thresholds and the number of workers of each type, the unemployment rates for each labor market, and the total unemployment rate can be written as follows:

$$u_t^R = \frac{\bar{\theta}_t^R - n_t^R}{\bar{\theta}_t^R}, \quad u_t^{IR} = \frac{(\bar{\theta}_t^{IR} - \bar{\theta}_t^R) - n_t^{IR}}{(\bar{\theta}_t^{IR} - \bar{\theta}_t^R)}, \quad u_t = \frac{\bar{\theta}_t^{IR} - n_t^R - n_t^{IR}}{\bar{\theta}_t^{IR}}.$$

[21] Consumption premiums are derived explicitly in Appendix A.2.1.

Figure 1.7: Summary of a Representative Family's Problem



Note: This figure illustrates the representative family's problem. The representative family selects the level of family-wide consumption, saves, supplies regular and irregular subject to budget constraint, and capital accumulation process. For infinitely many workers within the family, she devises an incentive system by setting corresponding consumption premiums in each labor market to supply the desired amount of regular and irregular labor.

The close relationship between the number of workers and the two thresholds shows that if the representative family wants to supply n_t^R (n_t^{IR}) number of workers to the regular (irregular) labor market, the family needs to adjust the two thresholds, $\bar{\theta}_t^R$ and $\bar{\theta}_t^{IR}$, accordingly. Because $\bar{\theta}_t^R$ and $\bar{\theta}_t^{IR}$ change in response to the changes of consumption premiums in each labor markets (c_t^m/c_t^{Um} for $m = R, IR$ and c_t^R/c_t^{IR}), the family needs to set consumption premiums correspondingly, to make sure $\bar{\theta}_t^{IR}$ amount of workers participate in the labor market, of whom $\bar{\theta}_t^R$ amount of workers enter the regular labor market, and the rest participate in the irregular labor market.[22]

[22]I assume that the family is not benevolent enough to provide perfect consumption insurance to workers. She wants to differentiate the level of consumption for workers with different labor market outcomes. By doing so, she can make sure the right workers participate in the right labor markets. That is those who have lower (higher) inherent work aversion level participate in the regular (irregular) labor market. One way to rationalize this behavior of a representative family is to introduce information asymmetry between the family and workers as in CTW. CTW assumes that each worker within the family draws her inherent work aversion level $\ell \overset{i.i.d.}{\sim} \mathcal{U}[0,1]$ at the beginning of *every* period. They then assume that each worker's type, ℓ which determines her private cost of working, and the level of effort, $e_{\ell,t}^m$ each worker exerts to find a job, are *private* information. Under this environment, the family cannot perfectly insure against each worker's idiosyncratic risk about her labor market status. Therefore, the family needs to devise an alternative insurance arrangement for workers to provide incentives.

When setting these consumption premiums, the family has to consider the following feasibility conditions as well:[23]

$$n_t^R c_t^R + (\bar{\theta}_t^R - n_t^R) c_t^{UR} + n_t^{IR} c_t^{IR} + (1 - \bar{\theta}_t^R - n_t^{IR}) c_t^{UIR} = C_t, \tag{1.8}$$

where C_t is total amount of family-wide consumption, which will be determined later from the family's optimization problem. Formal derivations of insurance provision problem of the representative family can be found in Appendix A.2.2.

Combining all, we can derive the indirect utility function for the representative family, which extends the indirect utility function for a representative family in CTW to the case with two types of labor:

Proposition 1 *The representative family's indirect utility is reduced to the following simple expression:*

$$u(C_t, n_t^R, n_t^{IR}) = \log C_t - z(n_t^R, n_t^{IR}), \tag{1.9}$$

with $z(n_t^R, n_t^{IR})$ summarizing disutility generated from supplying n_t^R and n_t^{IR} to each labor market. Appendix A.2.3 provides the functional form of $z(n_t^R, n_t^{IR})$ with proofs.

Provided this indirect utility function for the representative family, we can write the representative family's optimization problem. The representative family determines the level of family-wide consumption C_t; the amount of regular-type labor n_t^R to supply; the amount of irregular-type labor n_t^{IR} to supply; capital utilization rate, v_t; next period's capital stock, K_{t+1}; the amount of investment, I_t; the amount of a nominal bond, B_{t+1}, subject to budget constraint and capital accumulation process, and the cost of capital utilization rates which accelerates the depreciation of existing capital, $\delta(v_t)$.

Formally, the family's problem can be written as follows:

$$\max_{\{C_t, n_t^R, n_t^{IR}, B_{t+1}, v_t, K_{t+1}, I_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \underbrace{u(C_t, n_t^R, n_t^{IR})}_{=\log C_t - z(n_t^R, n_t^{IR})}$$

subject to

$$P_t C_t + P_t I_t + B_{t+1} \leq (1 + i_{t-1}) B_t + W_t^R n_t^R + W_t^{IR} n_t^{IR} + R_t K_t v_t + \text{Profits, Taxes, and Transfers}_t,$$

$$K_{t+1} = \left[1 - \frac{\tau}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t + (1 - \delta(v_t)) K_t,$$

where P_t denotes the aggregate price level and W_t^R (W_t^{IR}) is nominal wage of regular (irregular) workers, and R_t is nominal rental rate of the capital services. The representative family takes these prices as given when solving the problem.

[23] Again for $c_t^{UR} \geq c_t^{UIR}$, it needs to be that $c_t^{UR}/c_t^{UIR} > 1$, a fixed fraction which does not vary over time. c_t^R, c_t^{UR}, c_t^{IR}, however, do vary over time in response to aggregate shocks.

Because the indirect utility function for the representative family reduces to the standard functional form often used in dynamic macroeconomic models, it is easy to embed the rich structure of the labor supply decisions into the standard (medium-scale) New Keynesian model, yet it can still explain various labor market variables such as labor force participation rates, unemployment rates, job search efforts in *each* labor market, and so on. Moreover, we can easily extend the model to incorporate more complicated structures such as habit formation, and nominal wage rigidity for a particular type of labor. Appendix A.2.11 presents these extensions.

1.3.3 Production

As is standard in New Keynesian models, there are two production sectors: intermediate goods sector and final goods sector. This section describes the firms' decisions in each sector. Final goods firms are standard. Intermediate-goods-producing firms are non-standard, as they face multiple ways of adjusting the total amount of labor input, given the two types of labor.

1.3.3.1 Final Goods Production

A final good, Y_t, is produced using a continuum of intermediate goods using Dixit and Stiglitz (1977) aggregator as follows:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \mathrm{d}j, \;\; \epsilon_p > 1,$$

with ϵ_p denoting the degree of substitutability between intermediate goods $j \in [0, 1]$. Final goods are produced by a competitive, representative final-goods-producing firm. Profit maximization by this firm generates the following downward-sloping demand curve for each intermediate good, $j \in [0, 1]$:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_p} Y_t, \quad P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_p} \mathrm{d}j \right)^{\frac{1}{1-\epsilon_p}},$$

where $P_t(j)$ is the price for an intermediate good $j \in [0, 1]$ and P_t is the aggregate price index.

1.3.3.2 Intermediate Goods Production

There are infinitely many firms, $j \in [0, 1]$, in the intermediate goods sector and each firm produces a fixed variety under the monopolistic competition. Intermediate-goods-producing firms use *two types* of labor: (i) regular types and (ii) irregular types, and use an effective unit of capital to produce intermediate goods using the following production technology:

$$Y_t(j) = \left(A_t^P A_t^T n_t(j)\right)^{1-\alpha} \hat{K}_t(j)^{\alpha}, \tag{1.10}$$

where A_t^P denotes economy-wide persistent productivity process, A_t^T denotes transitory productivity shock, $\hat{K}_t(j)$ is effective unit of capital, and $n_t(j)$ is effective unit of labor employed by a firm j which is given by:

$$n_t(j) = \left((\eta_n)^{\frac{1}{\epsilon_n}} \left(n_t^R(j)\right)^{\frac{\epsilon_n - 1}{\epsilon_n}} + (1-\eta_n)^{\frac{1}{\epsilon_n}} \left(n_t^{IR}(j)\right)^{\frac{\epsilon_n - 1}{\epsilon_n}}\right)^{\frac{\epsilon_n}{\epsilon_n - 1}}, \ \ \eta_n > 0.5. \tag{1.11}$$

This functional form reflects the fact that the two types of labor are imperfectly substitutable with the degree of substitutability represented as $\epsilon_n > 0$. I assume that regular types are more productive than irregular types, *i.e.* $\eta_n > 0.5$.

Because there is more than one type of labor, firms can adjust the total labor input in a number of ways. Firms can create or destroy each type of job.[24] On top of this, firms can transfer one type of labor to the other type, by promoting some portion of irregular jobs to regular jobs or by demoting some portion of regular jobs to irregular jobs.[25] Hence, firms can adjust the composition of job types. This "*composition margin*" differentiates my model from most of the other dynamic models with more than one type of labor. This transfer is the key to generate differential responses of the two labor markets documented in Section 1.2.

Given these *multiple* ways of adjusting the total amount of labor, the evolution of the stock of regular and irregular jobs for a firm j can be written as follows:

$$n_t^R(j) = (\rho^R + x_t^R(j) - p_t^{R\to IR}(j))n_{t-1}^R(j) + p_t^{IR\to R}(j)n_{t-1}^{IR}(j), \ \ \rho^R > 0, \tag{1.12}$$

$$n_t^{IR}(j) = (x_t^{IR}(j) - p_t^{IR\to R}(j))n_{t-1}^{IR}(j) + p_t^{R\to IR}(j)n_{t-1}^R(j), \tag{1.13}$$

where x_t^R (x_t^{IR}) is the net hiring rate of regular (irregular) workers with the net creation of regular (irregular) jobs and $p_t^{IR\to R}$ ($p_t^{R\to IR}$) is the promotion rate (demotion rate), as

[24] For the intermediate goods producing firms, they consider each type of labor in terms of "jobs," not in terms of workers. For example, when firms keep some portion, ρ^R of regular jobs after one period, firms in my model do not care if those are taken by the same workers from the previous period or not. This distinction is due to the assumption that workers in my model draw their work aversion level, l every period, which enables the differences in consumption according to labor market status and makes my model tractable and solvable.

[25] Here are some examples of demotions: If firms furlough full-time workers and recall them but as part-timers during economic downturns, or as fixed-term contract workers, then this could be considered as demotion, in particular, if this happens within a quarter. (For instance, Fujita and Moscarini (2017) document that the average duration from the first separation to the first recall is two and a half months.) As another example, consider the case where firms suggest workers who are close to the retirement clock to retire early during economic downturns and re-hire them as temporary/fixed-term part-time workers or short-time workers.

a share of previous period's regular (irregular) jobs. Regular jobs are destroyed at rate $1-\rho^R$, while all the non-transferred irregular jobs are destroyed after one period. This reflects *stickier* nature of regular types and *flexible* nature of irregular types.

All the adjustment margins are subject to costs. Importantly, I assume that the stock of regular jobs is costlier to adjust than irregular jobs. Specifically, in order to create/destruct regular jobs, firms need to pay *large* adjustment costs. In contrast to this, firms can easily create/destruct irregular jobs and promote/demote each type to the other type, by paying only *negligible* cost. I assume the following quadratic functional forms for the adjustment costs:

$$\mathcal{C}(x_t^R; n_{t-1}^R) = \frac{\tilde{\kappa}}{2}\left(x_t^R\right)^2 n_{t-1}^R Y_t, \;\; \mathcal{C}(x_t^{IR}; n_{t-1}^{IR}) = \frac{\tilde{\gamma}}{2}\left(x_t^{IR}\right)^2 n_{t-1}^{IR} Y_t, \quad \tilde{\kappa},\ \tilde{\gamma} > 0,$$
$$\mathcal{C}(p_t^{R\to IR}; n_{t-1}^R) = \frac{\tilde{\theta}}{2}\left(p_t^{R\to IR}\right)^2 n_{t-1}^R Y_t, \;\; \mathcal{C}(p_t^{IR\to R}; n_{t-1}^{IR}) = \frac{\tilde{\nu}}{2}\left(p_t^{IR\to R}\right)^2 n_{t-1}^{IR} Y_t, \;\; \text{for } p_t^{R\to IR},\ p_t^{IR\to R} > 0.$$

I assume that it is relatively costlier to create or destruct regular jobs. Meanwhile, I assume that costs for creating or destructing irregular jobs or for promotions and demotions are all negligible.

On top of these labor adjustment frictions, intermediate-goods-producing firms are subject to Rotemberg (1982)-type nominal frictions.[26] Each intermediate-goods-producing firms can freely adjust its prices every period, but instead, they need to pay quadratic price-adjustment-costs defined as below:

$$\frac{\phi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t,$$

where ϕ_p governs the degree of nominal frictions. Full representation of intermediate-goods-producing firms' problems and first order conditions are in Appendix A.2.4.2.

1.3.4 Policy, Exogenous Shocks, and Market Clearing Conditions

Monetary policy follows an inertial interest rate rule with zero inflation rate in the steady-state as follows:

$$i_t = (1-\rho_i)i^* + \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi \pi_t + \phi_u u_t\right) + \epsilon_{i,t}.$$

The government consumes an exogenous share ω_t^g of output,

$$G_t = \omega_t^g Y_t, \;\; \omega_t^g = (1-\rho_g)\omega^g + \rho_g \omega_{t-1}^g + \epsilon_{g,t}.$$

The government balances its budget each period with lump sum taxes, $T_t = G_t$.

[26] As documented in Woodford (2005), because intermediate-goods-producing firms have firm-specific state variables, it would be tricky to use Calvo (1983)-type frictions.

The exogenous process for the persistent technology shock, A_t^P is unit-root process with a zero mean in logs:

$$\log A_t^P = \log A_{t-1}^P + \epsilon_{A^P,t}, \quad \mathbb{E}[\epsilon_{A^P,t}^2] = \left(\sigma^A\right)^2$$

Finally, clearing in the loan market requires $B_{t+1} = 0$, for all periods, and clearing in the market for final goods requires:

$$C_t + I_t + G_t + \underbrace{\mathcal{C}(x_t^R; n_{t-1}^R) + \mathcal{C}(x_t^{IR}; n_{t-1}^{IR}) + \mathcal{C}(p_t^{R\to IR}; n_{t-1}^R) + \mathcal{C}(p_t^{IR\to R}; n_{t-1}^{IR}) + \frac{\phi_p}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - 1\right)^2 Y_t}_{=\text{Sum of all the adjustment costs}} = Y_t.$$

1.4 Calibration

In this section, I describe the parameterization of the model. Table 1.1 summarizes the choice of structural parameter values set to the standard values.

The remaining parameters are non-standard. On the firm side, I first set the exogenous separation rate for the regular type of jobs, $1 - \rho^R$, to 2 percent. This choice follows from the fact that firms without employment growth lose 0.891 percent of its workforce with reasons other than layoff and discharges per month according to Job Openings and Labor Turnover Survey (see Baydur, 2017). This corresponds to 2.67 percent in a quarter. Because $1 - \rho^R$ is the exogenous separation rate for *regular* types who are more attached to firms than the other type, I set this value to be 2 percent which is slightly less than 2.67.[27] The relative productivity between the regular and irregular types, η^n, is set to 0.75 so that the relative marginal productivity between the two types captures the relative median earnings between the full-time and the part-time workers (of men of age over fifteen), given the substitutability parameter between regular and irregular workers, ϵ_n as 2.[28]

On the family side, the ratio of the consumption for the unemployed in the irregular labor market and of those in the regular labor market, c^{UIR}/c^{UR} is set to 0.85, *i.e.* the level of consumption for the unemployed who do not have an access to unemployment benefits is about 85 percent of the level of consumption for the unemployed with unemployment insurance (UI) in the regular labor market. This number reflects the analysis from Ganong and Noel (2019) showing that household spending on non-durables after UI exhaustion drops by 15.1 percent compared to the pre-exhaustion period.[29]

[27] Results presented in the next section are robust to the value of this parameter from 1 percent to 3 percent.

[28] The results in Section 1.5 are robust to $\epsilon_n \in [1, 4]$.There is no empirical estimate for the elasticity of substitution between regular and irregular workers. As a proxy, I examine the elasticity of substitution between workers with education levels of high school equivalents and workers with college equivalents. The estimates for this value in the literature range from $1/0.7$ to $1/0.3$ (see, for example, Card, 2009).

[29] Results presented in the next section are robust to the value of this parameter from 0.75 to 1. Note however that in the model, other consumption premiums, c_t^R/c_t^{UR}, c_t^{IR}/c_t^{UIR}, and c_t^R/c_t^{IR} are time-varying.

Table 1.1: Parameter Values (1)

Parameter	Description	Value
β	Discount rate	0.9987
α	Power on capital in production function	1/3
ϵ_p	Substitutability across the intermediate goods	10
ϕ_p	Price Stickiness (Price Adjustment Cost)	$\frac{(\epsilon_p-1)0.75}{(1-0.75)(1-0.75\beta)}$
τ	Investment Adjustment Cost	5
δ_0, δ_1, δ_2	Capital Depreciation Function Parameters	$\delta(v) = \delta_0 + \delta_1(v-1) + \frac{\delta_2}{2}(v-1)^2$
δ_0	steady-state value of depreciation rates	0.025
δ_1	steady-state value of utilization rate = 1	$1/\beta - (1-\delta_0)$
δ_2	Curvature on Capital Depreciation Funtion	0.0361
ω_g	Steady-state government spending-GDP ratio	0.2
ϕ_π	Policy weight on inflation	1.5
ϕ_u	Policy weight on unemployment gap	-0.1
ρ_g	Autocorrelation, government spending shock	0.97
ρ_i	Autocorrelation, monetary policy shock	0.87
σ_{A^T}	Standard deviations, transitory technology shock	0.0056
σ_{A^P}	Standard deviations, persistent technology shock	0.01
σ_g	Standard deviations, government spending shock	0.015
σ_i	Standard deviations, monetary policy shock	0.002

Note: This table summarizes parameter values related to non-labor market variables that are set to conventional values following the literature: β is set so that the risk-free rate is 1%; α and v are set so that the labor share becomes 60%; ϵ_p is set to be 10 following Christiano, Trabandt, and Walentin (2010); ϕ_p is set so that it corresponds to the Calvo parameter of 0.75 which corresponds to the expected price duration of 4 quarters; for the parameter in the adjustment cost for investment and the parameters in the capital depreciation function, I follow Sims and Wolff (2018); for the steady-state value of the government spending to GDP ratio, policy weight on inflation and output gap, I use those from Sims and Wolff (2017). The choice of parameter values related to exogenous shock processes are loosely based on those estimated in the literature (see, for example, Sims and Wolff, 2017, 2018; Ireland, 2001, 2004)

The remaining parameters are those related to disutility from working, F^m, σ^m, those in the probability of finding jobs, η^m, a^m, η^m, and adjustment costs parameters. They jointly determine the relative size of labor forces in each labor market, the total labor force participation rates, the number of workers in each labor market, the unemployment rates in each labor market, and the total unemployment rate.

I calibrate the remaining parameter values by moment matching. Table 1.2 reports the calibrated parameter values. Specifically, I target means and standard deviations of each labor market's participation rates ($\mathbb{E}[\bar{\theta}_t^{IR}]$, $\mathbb{E}_t[\bar{\theta}_t^{R}]$, $\sigma\left(\bar{\theta}_t^{IR}\right)$, and $\sigma\left(\bar{\theta}_t^{R}\right)$), means and standard deviations of the total unemployment rate and of each labor market's unemployment rates ($\mathbb{E}[u_t]$, $\mathbb{E}[u_t^R]$, $\mathbb{E}[u_t^{IR}]$, $\sigma\left(u_t\right)$, $\sigma\left(u_t^R\right)$, and $\sigma\left(u_t^{IR}\right)$), and means and standard deviations of the share of workers in each labor market ($\mathbb{E}[n_t^R]$, $\mathbb{E}[n_t^{IR}]$, $\sigma\left(n_t^R\right)$, and $\sigma\left(n_t^{IR}\right)$).[30]

[30]For the United States, there is no good-quality data for net hiring rates for regular and irregular work-

Table 1.2: Parameter Values related to Labor Market Variables

Parameter	Description	Value
F^{IR}	Disutility for irregular workers	0.60
σ^{IR}	Disutility for irregular workers, curvature	10.04
η^{IR}	Probability to find a job in the irregular labor market	0.93
a^{IR}	Probability to find a job in the irregular labor market	0.51
F^{R}	Disutility for irregular workers	0.75
σ^{R}	Disutility for irregular workers, curvature	3.07
η^{R}	Probability to find a job in the regular labor market	0.83
a^{R}	Probability to find a job in the regular labor market	0.51
$\tilde{\kappa}$	Adjustment cost of hiring/firing regular workers	19.96
$\tilde{\nu}$	Adjustment cost for promotion	1.01
$\tilde{\theta}$	Adjustment cost for demotion	0.50

Note: This table summarizes parameter values calculated by minimizing the distance between the moments from the baseline model and the targeted data counterparts. Targeted moments are the mean of total labor force participation rates, mean of full-time labor forces, means and standard deviations of the total unemployment rates, means and standard deviations of the full-time and part-time unemployment rates, means and standard deviations of the full-time and the part-time employment rates, and the means of promotion and demotion rates.

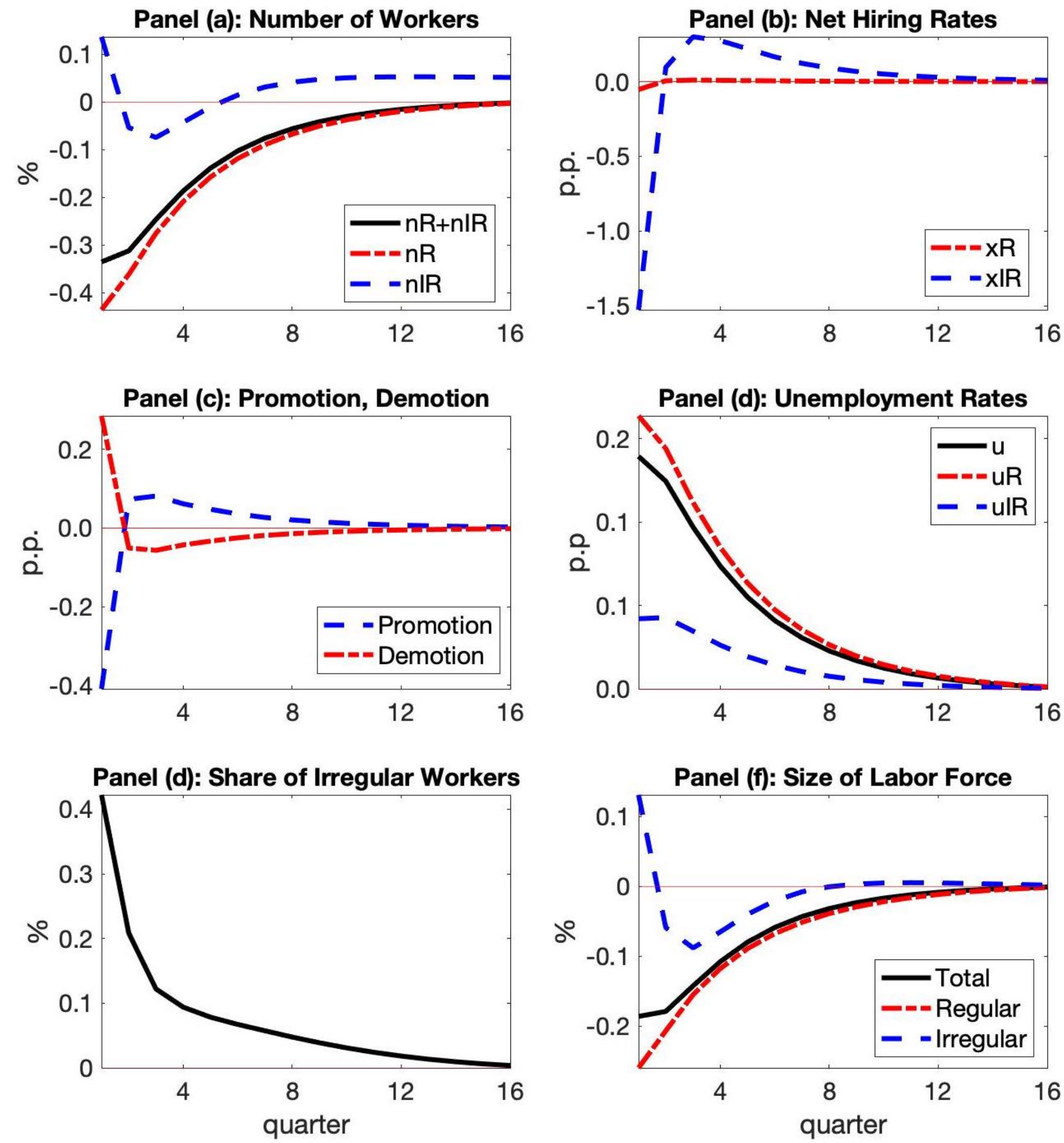
For the data counterparts, I use full-time workers and part-time workers in the CPS as proxies for regular workers and irregular workers, respectively, as described in Section A.1. I use the share of full-time workers (part-time workers) out of the population of age over fifteen as a proxy for n_t^R (n_t^{IR}). To get proxies for u_t^R and u_t^{IR} from the data, I use the unemployment rates of the full-time and the part-time labor markets, respectively. I then calculate $\bar{\theta}_t^R$ by dividing the sum of full-time workers and the unemployed mostly seeking full-time jobs by population of age greater than fifteen. $\bar{\theta}_t^{IR}$ corresponds to the total labor force participation rates.

1.5 Labor Market Dynamics

This section studies the labor market dynamics in the baseline model with two types of labor. Specifically, I show that my model replicates the changes in the composition of labor types and differential responses of regular and irregular labor market variables to structural shocks, as documented in Section 1.2. I focus on monetary policy shocks and relegate impulse responses to other shocks to Appendix A.2.6.

ers. In this regard, I did not target moments of x_t^R and x_t^{IR}. Another reason is due to the assumption that I made that irregular workers are all separated after one period, which makes x_t^{IR} higher in the model.

Figure 1.8: Responses of Labor Market Variables to a Contractionary Monetary Policy Shock



Note: This figure shows the responses of labor market related variables to a one-standard deviation of contractionary monetary policy shock from the baseline model. The first panel in the first row shows the responses of the number of workers as black solid lines, those of the number of regular workers as red dash-dot lines, and those of the number of irregular workers as blue dash lines. The second panel shows the responses of net hiring rates of regular (red dash-dot lines) and irregular workers (blue dashed lines). The third panel shows the responses of promotion and demotion rates with the blue dash line and the red dash-dot line, respectively. In the second row, the first panel shows the responses of the share of irregular workers, the second panel shows the responses of the aggregate unemployment rate (black solid line), those of the unemployment rate in the regular labor market (red dash-dot line), and those of the unemployment rate in the irregular labor market (blue dash line). The last panel shows the responses of the sizes of the labor force. The black solid line shows the responses of the total labor force, the red dash line shows the responses of the regular labor force, and the blue dash line shows the responses of the irregular labor force.

Figure 1.8 shows that my model successfully replicates the differential responses of the two labor markets. In response to a contractionary monetary policy shock, the total number of workers and the number of regular workers decrease, while the number of irregular workers increases. At first glance, this could be puzzling because it is much easier for firms to create or destruct irregular jobs than regular jobs, given that the adjustment costs for creating or destructing irregular jobs are much smaller. As can be seen from the second panel of Figure 1.8, this is actually the case. The responses of net hiring rates for irregular types, x_t^{IR} are much larger than those for regular types, x_t^R, and they both decrease in response to a contractionary monetary policy shock.

What derives the differential responses of the two labor markets is the ability of firms to transfer one type of labor to the other type. Firms actively exploit this margin to adjust the total amount of labor input at the time of the shock.[31] For example, in response to a contractionary monetary policy shock, firms want to decrease the total labor input. This reaction could be achieved by firing regular workers. To do so, however, firms need to pay high firing costs. Therefore, firms instead increase demotion rates and decrease promotion rates, which increases the share of irregular types with lower productivity, and therefore decreases the total amount of labor input to use for production at relatively lower costs.[32] Utilization of this "composition margin" explains the differential responses of the number of workers in each labor type.

One implication from firms' use of composition margin is that conventional aggregate labor market variables may not be enough to understand how firms adjust the total labor input in response to shocks. If firms mostly utilize the composition margin, then the total amount of labor input, $n_t = \left((\eta^n)^{\frac{1}{\epsilon}} \left(n_t^R\right)^{\frac{\epsilon_n - 1}{\epsilon_n}} + (1-\eta^n)^{\frac{1}{\epsilon}} \left(n_t^{IR}\right)^{\frac{\epsilon_n - 1}{\epsilon_n}} \right)^{\frac{\epsilon_n}{\epsilon_n - 1}}$, changes substantially, even if there is no change in the number of workers, $L_t \equiv n_t^R + n_t^{IR}$. For instance, if the number of newly hired irregular workers and the number of fired regular workers are the same or if firms do not create or destruct either type of jobs, but use promotion and/or demotion to change the composition of job types, there is no change in the headcount of workers, but this stability of headcounts masks a change in the total labor input. This implies that the headcount of workers can underestimate the cyclicality of the actual amount of labor input used by firms. Table 1.3 illustrates this point by showing that the correlation of the headcount of workers with output is much smaller than the correlation of actual labor input with output. The changes in the composition of worker types help explain the gap between the two. This insight further suggests the importance of accounting for the composition of worker types when measuring the total

[31]In order to generate differential responses of each labor market, *either* promotion *or* demotion is necessary. As long as firms can either promote irregular workers to be regular workers or demote regular workers to be irregular workers, the numbers of regular and irregular workers move in the opposite direction to structural shocks.

[32]Active utilization of this transfer via promotion and/or demotion is consistent with Borowczyk-Martins and Lalé (2019) that flows from full-time to part-time employment are significant within the sample employees in the United States and in the United Kingdom.

factor productivity (TFP).[33].

Table 1.3: Correlation of Labor Market Variables with Output

Correlation with	Headcount L_t	Total Labor Input n_t	Share of Irregular Types s_t^{IR}	Total LFP $\bar{b}_t$
Output (Y_t)	0.69	0.87	-0.88	0.49
Correlation with	Headcount Growth g_{L_t}	Total Labor Input Growth g_{n_t}	Share of Irregular Types Growth $g_{s_t^{IR}}$	Total LFP Growth $g_{\bar{b}_t}$
Output Growth (g_{Y_t})	0.59	0.89	-0.96	0.37

Note: This table calculates the correlations of labor market variables with output. The first row calculates the correlations of HP-filtered labor market variables with the HP-filtered output (smooth parameter, $\lambda = 1600$). The second row calculates the correlations of log-changes in labor market variables with log-changes of output. Labor market variables of interest are the total number of workers ($L_t = n_t^R + n_t^{IR}$), the total amount of labor input to use (n_t), the share of irregular workers out of the total number of workers ($s_t^{IR} = n_t^{IR}/(n_t^R + n_t^{IR})$), and the total labor force participation ($\bar{b}_t$).

Moreover, the baseline model successfully replicates the relative rankings of the magnitudes of the changes in the unemployment rates: the responses of the unemployment rate in the regular labor market are the largest and those in the irregular labor market are the smallest, which is consistent with the empirical analysis in Section 1.2. This pattern is due to the responses of labor force participation rates. As can be seen from the last panel in Figure 1.8, the total labor force participation rate responds only modestly to a contractionary monetary policy shock. It exhibits slight procyclicality. This mild response, however, masks heterogeneous responses of the two labor market's labor forces, which move in opposite directions, with substantially larger amplitudes. The size of the regular labor force moves in the same direction as the total labor force.

Through the lens of the model, this is because $\bar{\theta}_t^R$, the threshold determining the regular and the irregular labor markets, responds much more to aggregate shocks than $\bar{\theta}_t^{IR}$, the threshold determining labor force participation. Because $\bar{b}_t$ moves only modestly to exogenous shocks, the total labor force responds only modestly to shocks. Meanwhile, larger movements of $\bar{\theta}_t^R$ contributes to the differential responses of the size of each type's labor force. For instance, a contractionary monetary policy shock decreases $\bar{\theta}_t^R$, increasing the number of workers participating in the irregular labor market and decreasing the size of the regular labor force. Consider the role of the changes of $\bar{\theta}_t^{IR}$ and the changes of $\bar{\theta}_t^R$ in the model. If only $\bar{\theta}_t^{IR}$ changes, then both the number of irregular types and regular types move in the same direction. Meanwhile, if $\bar{\theta}_t^R$ changes with no change in $\bar{\theta}_t^{IR}$, then n_t^R and n_t^{IR} move in opposite directions. See Appendix A.2.5 for formal comparative statics analysis. As the response of the share of irregular workers to a contractionary monetary policy shock illustrates, firms actively utilize the composition margin by changing the

[33]Fernald (2014) emphasizes the importance of controlling for the composition of workers with different skills for measured TFP.

relative share of regular and irregular workers to adjust the total amount of labor input. These changes in the relative labor demand between the two then generate more volatile $\bar{\theta}_t^R$ over the business cycle. This result is consistent with the fact that the total labor force participation rates are modestly procyclical (see Elsby, Hobijn, and Şahin, 2015) and with my analysis from the CPS that gross worker flows between the full-time labor market and the part-time labor market are much larger and more cyclical than gross flows between the part-time labor market and out of labor force (see Section 1.2 and Appendix A.1.5).

Because aggregate variables mask heterogeneous experiences of each type of labor, aggregate labor market variables may underestimate the volatility that individual workers experience within the total labor force over the business cycle. As documented in Section 1.2, many workers move between the two labor markets within the total labor force. This implies that a lot more workers experience changes in their labor income by moving across the two labor markets and hence by switching their job types than those who move in and out of the labor force. The variation in aggregate labor market variables fails to capture this important source of income fluctuations generated by switching job types. In contrast, the changes in the composition of worker types can capture these risks in labor income fluctuations of workers, in particular, of those who are marginally attached to the regular labor market and hence frequently change their job types by moving across the two labor markets. These switchers manifest themselves as frequent and large changes in $\bar{\theta}_t^R$ in my model, which leads to many workers with inherent work aversion of ℓ near $\bar{\theta}_t^R$ (marginally attached "regular workers") frequently switch their job types. Section 1.6 explores the implications of this to the welfare and monetary policy.

1.6 Welfare and Policy Implications

This section explores welfare and policy implications by building on the insights from empirical and model analysis. I first calculate the welfare costs of economic fluctuations by individual workers and see if a certain group of workers tend to pay larger costs of economic fluctuations. It shows that switchers who frequently move between the regular and the irregular labor markets and move in and out total labor force are the most vulnerable to economic fluctuations. Based on the welfare cost calculations, I then discuss what these findings imply for optimal monetary policies, in particular, towards more *inclusive* monetary policy.

1.6.1 Who Bears the Cost of Business Cycle?

Following the approach in Lucas (1987), I calculate welfare costs of the business cycle by individual workers for each ℓ's, Λ^ℓ, as follows:

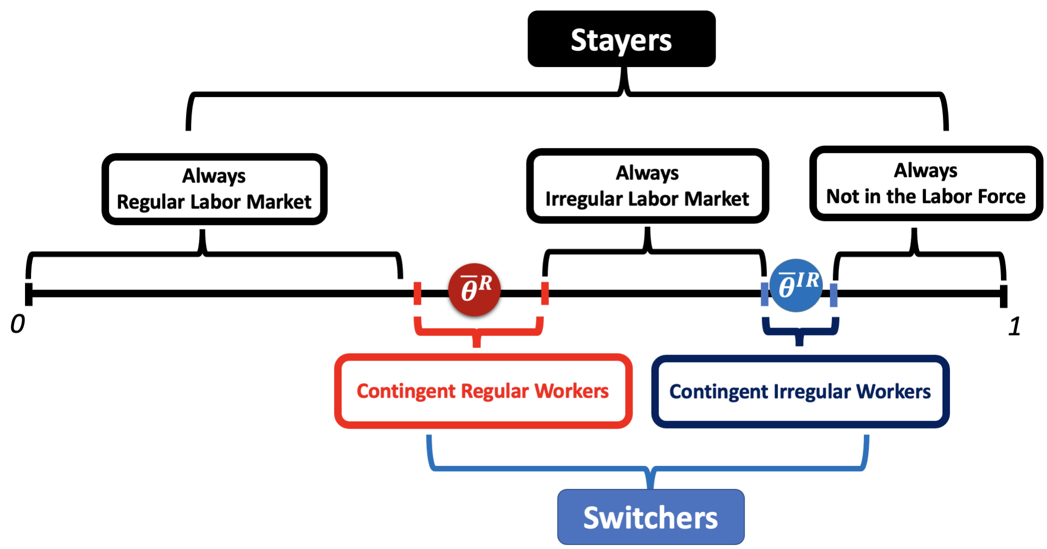
$$\mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t U\left((1+\Lambda^\ell)c_t(\ell),\ \ell\right)\right] = \sum_{t=0}^{\infty}\beta^t \mathbb{E}\left[U\left(\bar{c}(\ell),\ \ell\right)\right], \tag{1.14}$$

where $\overline{x}$ denotes the steady-state value of a variable, x. For logarithmic utility, we have

$$\Lambda^{\ell} = \exp\left(-(1-\beta)\left(\mathbb{V}^{\ell} - \overline{\mathbb{V}^{\ell}}\right)\right) - 1, \tag{1.15}$$

with $\mathbb{V}^{\ell} \equiv \mathbb{E}_0\left[\sum_{t=0}^{\infty} \beta^t U\left(c_t(\ell),\ \ell\right)\right]$ and $\overline{\mathbb{V}^{\ell}} = \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[U\left(\bar{c}(\ell),\ \ell\right)\right]$. Here, Λ^{ℓ} representes the welfare costs of business cycle for individual workers with $\ell \in [0,1]$ in consumption equivalents, as it denotes the value that needs to be in order to make utility in the stochastic equilibrium equal to the utility in the steady state (that is non-stochastic equilibrium). In other words, Λ^{ℓ} denotes how much consumption an individual worker with a work aversion ℓ is willing to forgo in order to avoid economic fluctuations.

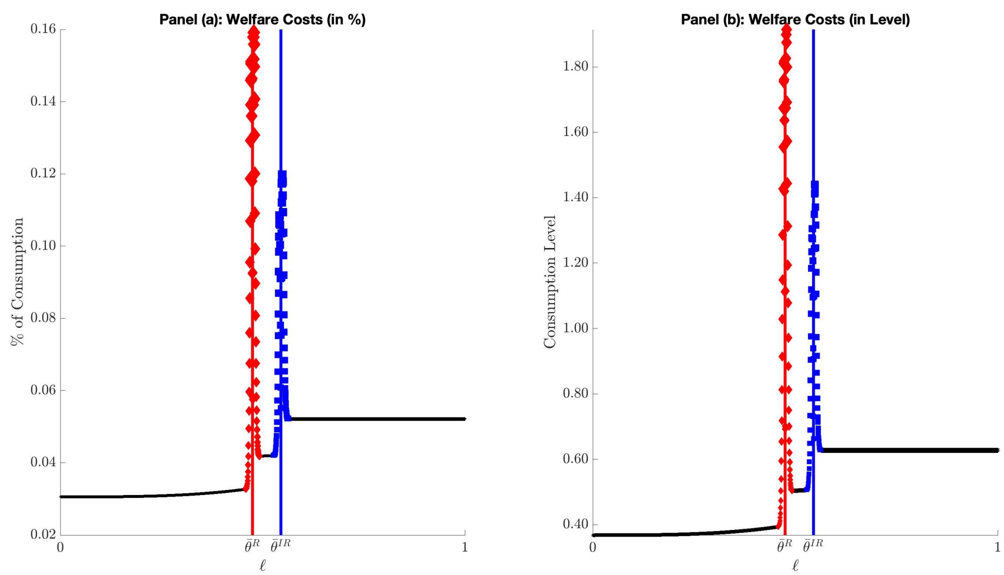
Figure 1.9: Five Groups of Types in the Model



Note: This figure illustrates the five groups of types in the baseline model: i) Those who always participate in the regular labor market, *i.e.* workers with $\ell \in [0, \bar{\theta}^R - \epsilon)$, (ii) Those who always participate in the irregular labor market ,*i.e.* workers with $\ell \in (\bar{\theta}^R + \epsilon, \bar{\theta}^{IR} - \epsilon)$, (iii) Those who always not in the labor force, *i.e.* workers with $\ell \in (\bar{\theta}^{IR} + \epsilon, 1]$, (iv) contingent "regular workers," *i.e.* workers with $\ell \in [\bar{\theta}^R - \epsilon, \bar{\theta}^R + \epsilon]$, and (v) contingent "irregular workers," *i.e.* workers with $\ell \in [\bar{\theta}^{IR} - \epsilon, \bar{\theta}^{IR} + \epsilon]$.

Types in my model can be divided into five groups for some positive number $\epsilon > 0$: (i) Those who always participate in the regular labor market, *i.e.* workers with $\ell \in [0, \bar{\theta}^R - \epsilon)$, (ii) Those who always participate in the irregular labor market, *i.e.* workers with $\ell \in (\bar{\theta}^R + \epsilon, \bar{\theta}^{IR} - \epsilon)$, (iii) Those who are always not in the labor force, *i.e.* workers with $\ell \in (\bar{\theta}^{IR} + \epsilon, 1]$, (iv) contingent "regular workers," *i.e.* workers with $\ell \in [\bar{\theta}^R - \epsilon, \bar{\theta}^R + \epsilon]$, and (v) contingent ""irregular workers," *i.e.* workers with $\ell \in [\bar{\theta}^{IR} - \epsilon, \bar{\theta}^{IR} + \epsilon]$. The first three groups are "stayers," in a sense that they always stay in one labor market or in not-in-the-labor-force (NLF). On the other hand, the last two groups are "switchers" or "contingent workers," in the sense that their current labor types are not expected to last for a long time. For example, contingent *regular* workers move between the regular and the irregular labor markets over the business cycle. One can say that they are marginally attached to the *regular* labor market. On the other hand, contingent *irregular* workers move frequently in and out of the labor force, so they are marginally attached to the *irregular* labor market.

Figure 1.10: Welfare Costs of the Business Cycle by Workers' Type, ℓ, Λ^{ℓ} in %



Note: Panel (a) shows the scatter plot of the welfare costs of business cycles by individual workers' type, ℓ, Λ^{ℓ} in the percentage of consumption equivalent when I feed in transitory technology shocks to the economy. That is, Λ^{ℓ} is how much consumption in percentage an individual worker is willing to forgo in order to avoid economic fluctuations due to technology shocks for each type ℓ. x-axis denotes individual workers' type, ℓ which determines the degree of work aversion, and therefore the type of labor market that these workers are likely to choose, and y-axis denotes the welfare costs, Λ^{ℓ}. Panel (b) shows the scatter plot of the welfare costs in the level (in the unit of real consumption) with all the shocks in the model. To calculate this, I multiply the average level of consumption per each worker, $\mathbb{E}[c_t(\ell)]$ to Λ^{ℓ}. The two vertical lines in the two panels correspond to the two thresholds. The red vertical line is $\overline{\theta}^R$ and the blue vertical line is $\overline{\theta}^{IR}$.

Figure 1.10 shows the welfare costs of economic fluctuations borne by each worker with $\ell \in [0,1]$.[34] Among the stayers, those who always participate in the regular labor market, with lower values of ℓ, pay the smallest welfare costs of economic fluctuations; workers who are always NLF pay the largest welfare costs; workers who always participate in the irregular labor market pay the amount between the two. This ranking of

[34]Figure 1.10 plots the welfare costs of business cylces when the economy is hit by transitory technology shocks. The cases with other exogenous shocks paint the same picture.

the costs between the workers among the stayers is associated with the differences in the marginal utilities. Limited consumption insurance generates that the average level of consumption for those who are always in the regular labor market is the highest, that for those who are always NLF is the smallest, and that for workers who are always in the irregular labor market is in between the two. Logarithmic utility from consumption implies the reverse ranking for the marginal utilities, making consumption volatilities for those always NLF more costly.[35]

More importantly, there are spikes near $\overline{\theta}^R$ and $\overline{\theta}^{IR}$, the steady state values of the two thresholds. This illustrates that "switchers" pay significantly higher welfare costs of business cycles than stayers. Because switchers are marginally attached to either the regular or the irregular labor market, they frequently move between the regular and the irregular labor market, or between the irregular labor market and NLF. This makes their consumption and disutility from supplying labor substantially volatile over the business cycle, making them pay the largest costs of economic fluctuations.[36] For instance, as $\bar{\theta}_t^{IR}$ decreases in response to a contractionary monetary policy shock ($\bar{\theta}_t^{IR} < \bar{\theta}^{IR}$), contingent irregular workers with ℓ right below $\bar{\theta}^{IR}$ exit the labor force. The changes in $\bar{\theta}_t^{IR}$ in response to aggregate shocks make these contingent *irregular* workers' labor market status fluctuates over the three states: (i) employed in the irregular labor market and consume c_t^{IR}, (ii) unemployed in the irregular labor market and consume c_t^{UIR}, and (iii) NLF and consume c_t^{UIR} where the levels of consumption, c_t^{IR} and c_t^{UIR} themselves vary over time. As they are marginally attached to the labor market, they experience larger fluctuations in their consumption with imperfect consumption insurance and disutility from working. This captures one of the findings from Hall and Kudlyak (2019) that frequent "circling" happens between unemployment, short-term jobs, and out of labor force, considering that short-term jobs are much closer to the irregular type than the regular type in my model.

Similarly, the decreases of $\bar{\theta}_t^R$ in response to a contractionary monetary policy shock ($\bar{\theta}_t^R < \bar{\theta}^R$) make workers with ℓ right below $\bar{\theta}^R$ move from the regular labor market to the irregular labor market. With the changes in $\bar{\theta}_t^R$, contingent regular workers face the largest risks over their labor market status: (i) employed in the regular labor market and consume c_t^R, (ii) unemployed in the regular labor market and consume c_t^{UR}, (iii) employed in the irregular labor market and consume c_t^{IR}, and (iv) unemployed in the irregular labor market and consume c_t^{UIR}, where the levels of consumption, c_t^R, c_t^{UR}, c_t^{IR}, and c_t^{UIR} themselves change in response to structural shocks. Because contingent *regular* workers' risk regarding labor market status is the largest, they pay the largest costs of economic fluctuations over the business cycle with larger volatilities of their consumption and disutility from supplying labor.[37] Not only the magnitude but also the mass of contingent workers are larger near the threshold $\bar{\theta}_t^R$ than $\bar{\theta}_t^{IR}$. That is, the number of contingent "regular"

[35]For the simulated stream of consumption and disutility from supplying labor for a worker in each group, see Appendix A.2.8.

[36]For the stream of consumption and disutility from supplying labor for a worker in each group, see Appendix A.2.8.

[37]See again Appendix A.2.8 for the comparison of the volatilities of consumption and disutility from

workers is much greater than the number of contingent "irregular" workers. This is because $\bar{\theta}_t^R$ is about three times more volatile than $\bar{\theta}_t^{IR}$ in the baseline model, making those workers with ℓ close to $\bar{\theta}^R$ move between the regular and the irregular labor markets much more often than those with ℓ close to $\bar{\theta}^{IR}$ in and out the irregular labor market.[38] Moreover, larger changes in $\bar{\theta}_t^R$ generate many more contingent regular workers than contingent irregular workers.[39] In the next subsection, I examine if the central bank can achieve higher overall welfare by targeting these vulnerable groups of workers (that is, contingent workers).

1.6.2 Towards More Inclusive Monetary Policy: An Alternative Interest Rate Rule

To study if monetary policy can be more inclusive, I examine if the monetary authority can do better by targeting a specific group of workers (switchers or contingent workers) who are more exposed to economic fluctuations. Specifically, I consider alternative specifications of the Taylor (1993) rule that considers deviations of the two thresholds, the threshold determining labor force participation ($\bar{\theta}_t^{IR}$) and the threshold determining participation of either the regular or the irregular labor markets ($\bar{\theta}_t^R$) from their steady-state values, and hence stabilizing the composition of labor types:

$$i_t = (1-\rho_i)i^* + \rho_i i_{t-1} + (1-\rho_i)\left[\phi_\pi \pi_t + \phi_{\theta^R}\left(\bar{\theta}_t^R - \bar{\theta}^R\right) + \phi_{\theta^{IR}}\left(\bar{\theta}_t^{IR} - \bar{\theta}^{IR}\right)\right]. \quad (1.16)$$

The results from the previous section show that contingent workers face larger risks regarding their labor market status as the two thresholds, $\bar{\theta}_t^R$ and $\bar{\theta}_t^{IR}$ change over the business cycle. In this regard, if the central bank can stabilize the two thresholds, and hence the composition of job types, then it can substantially reduce the labor market risks that these contingent workers face. For example, the stabilization of $\bar{\theta}_t^{IR}$ can minimize the movement of contingent irregular workers who frequently enter and exit the labor force. More importantly, the stabilization of the composition of labor types, and hence the stabilization of $\bar{\theta}_t^R$, can minimize the movement of contingent regular workers across the two labor markets and can decrease the number of contingent *regular* workers, which is significantly larger than the number of contingent irregular workers.

supplying labor between the groups of workers.

[38] See, for example, the last panel in the second row of Figure 1.8. Responses of total labor force corresponds to responses of $\bar{\theta}_t^{IR}$ and responses of regular labor force corresponds to responses of $\bar{\theta}_t^R$.

[39] This is consistent with the fact that the total labor force participation rates are only modestly procyclical (see Elsby, Hobijn, and Şahin, 2015) and with the analysis from the CPS that gross flows between the full-time labor market and the part-time labor market are much higher and more cyclical than gross flows between the part-time labor market and out of labor force. Analysis from the CPS in Section A.1 also shows that the *across*-market flows are strongly cyclical and of large magnitudes at the quarterly frequency. The baseline model in Section 1.3 generates that the responses of the total labor force participation rate, $\bar{\theta}_t^{IR}$ are smaller than the responses of the regular labor force, $\bar{\theta}_t^R$.

Therefore, I consider the variant of Taylor (1993) rule of Equation (1.16). The two thresholds correspond to the size of labor forces: $\bar{\theta}_t^{IR}$ is the size of the total labor force and $\bar{\theta}_t^{R}$ is the size of the regular labor force, both of which are based on the observable variables. I then compare the maximum welfare of the representative family (or equivalently, minimum welfare loss compared to the non-stochastic equilibrium) achieved from the alternative specification for the interest rate rule of Equation (1.16) with those achieved from the other two policy rules:

$$i_t = (1-\rho_i)i^* + \rho_i i_{t-1} + (1-\rho_i)\left[\phi_\pi \pi_t + \phi_u \left(u_t - \overline{u}\right)\right], \tag{1.17}$$

which is the conventional Taylor (1993) rule, which targets the inflation rate and the aggregate unemployment gap, and

$$i_t = (1-\rho_i)i^* + \rho_i i_{t-1} + (1-\rho_i)\left[\phi_\pi \pi_t + \phi_{u^R}\left(u_t^R - \overline{u^R}\right) + \phi_{u^{IR}}\left(u_t^{IR} - \overline{u^{IR}}\right)\right], \tag{1.18}$$

which targets each labor market's unemployment gap, separately as in Equation (1.18).

Table 1.4: Comparison of the Welfare Losses across Different Interest Rate Rules

	Aggregate Unemployment Rate u	Two Unemployment Rates u^R and u^{IR}	Two Thresholds $\overline{\theta}^{IR}$ and $\overline{\theta}^{R}$
Consumption Equivalent (%)	1.231	1.230	1.098
Relative Costs (%)	100	99	89

Note: This table shows the minimum welfare losses compared to the non-stochastic equilibrium that are achieved from different specifications for Taylor (1993) rules. The minimum welfare losses are represented in terms of consumption equivalent in percentage in the first row, in a similar fashion to the welfare cost calculations when all the shocks are fed in the baseline model, but at the family level. Recall that the family's welfare is defined by integrating all individual workers' utilities. This utilitarian family puts the same weights on each workers' welfare. The second row calculates the relative welfare losses with the other two interest rate rules compared to the minimum welfare loss achieved from the conventional Taylor rule with the aggregate unemployment gap.

Table 1.4 compares family-level welfare under the different monetary policy rules.[40] It shows that the alternative interest rate rule which reacts to the deviations of the two thresholds from their steady-state values significantly reduces the overall welfare loss from aggregate fluctuations.[41] Compared to the family-level welfare in the non-stochastic equilibrium, the minimum welfare loss achieved from the conventional interest rate rule

[40]Indirect utility of the representative family is derived by integrating all the individual workers' utilities. This utilitarian family puts the same weights on each workers' welfare.

[41]This holds true regardless of the nature of shocks. When the economy is shocked with only technology shocks or monetary policy shocks, or government spending shocks, the minimum welfare losses obtained from the alternative specification of the policy rule are the lowest.

with the aggregate unemployment gap is the largest.[42] That is, the cost of the business cycle in terms of consumption equivalent with the optimal weight on the aggregate unemployment gap in the conventional monetary policy rule is the largest among the three specifications. When the central bank instead stabilizes each labor market's unemployment rate separately, the minimum welfare loss in consumption equivalent decreases, but by a modest 1 percent.[43] On the other hand, the minimum welfare loss achieved from the alternative Taylor rule specification with the stabilization of the two thresholds is about 11 percent lower than that with the conventional Taylor rule.

Compared to the specification of Equation (1.18), which considers the overall stance of each labor market, this alternative interest rate rule of Equation (1.16) directly targets contingent workers who are the most vulnerable to aggregate fluctuations. Because the labor market risks of those contingent workers are related to the changes in the two thresholds, the stabilization of the two thresholds can directly lower the burden of them who pay the largest costs of the business cycle. Therefore, targeting contingent workers with the stabilization of the two thresholds generates the largest welfare gains for those contingent workers. Moreover, Table 1.4 shows that the alternative specification of monetary policy not only improves these vulnerable groups' welfare, but also increases the economy-wide welfare as well.

The maximum welfare is achieved with $\phi^*_{\bar{\theta}^{IR}} = -0.89$ and $\phi^*_{\bar{\theta}^{R}} = 1.24$. (See Appendix A.2.9.) That is, the central bank wants to put higher weight on stabilizing $\bar{\theta}^R_t$ than stabilizing $\bar{\theta}^{IR}_t$. This is because contingent *regular* workers who are marginally attached to the *regular* labor market pay the highest welfare costs of economic fluctuations according to the analysis from the previous subsection. Additionally, there is a larger mass of contingent regular workers than contingent irregular workers with more volatile $\bar{\theta}^R_t$. Therefore, the central bank wants to put higher weight on the stabilization of $\bar{\theta}^R_t$ than the stabilization of $\bar{\theta}^{IR}_t$. To further understand the opposite signs of the optimal weights, I re-write the alternative Taylor rule as follows:

$$\begin{aligned} i_t &= (1-\rho_i)i^* + \rho_i i_{t-1} + (1-\rho_i)\left[\phi_\pi \pi_t + \phi_{\theta^R}\left(\bar{\theta}^R_t - \bar{\theta}^R\right) + \phi_{\theta^{IR}}\left(\bar{\theta}^{IR}_t - \bar{\theta}^{IR}\right)\right], \\ &= (1-\rho_i)i^* + \rho_i i_{t-1} + (1-\rho_i)\left[\phi_\pi \pi_t + (\phi_{\bar{\theta}^R} + \phi_{\bar{\theta}^{IR}})\left(\bar{\theta}^R_t - \bar{\theta}^R\right) + \phi_{\theta^{IR}}\left(\left(\bar{\theta}^{IR}_t - \bar{\theta}^R_t\right) - \left(\bar{\theta}^{IR} - \bar{\theta}^{IR}\right)\right)\right]. \end{aligned} \tag{1.19}$$

From this, it is clear that central banks would want to reduce interest rates when the share of workers participating in the irregular labor market rises ($\phi^*_{\bar{\theta}^{IR}} < 0$). This reflects the fact that the size of the irregular labor force increases in recessions. On top of this, central banks would want to further adjust interest rates with the changes of the regular

[42]This is associated with different cyclicalities of headcount and total labor input in Section 1.5. While the actual output gap is associated with actual labor input used by firms, the aggregate unemployment rate is calculated from the headcount of workers, which underestimates the cyclicalities of total labor input. Appendix A.2.10 derives the second-order approximation of the family's welfare in terms of the volatilities of the output gap and compares this with the formulation with the aggregate unemployment gap.

[43]This is because the volatility of u^{IR}_t is substantially small in the data.

labor force. Because $\phi^*_{\bar{\theta}^R} + \phi^*_{\bar{\theta}^{IR}} > 0$, central banks would want to further reduce interest rates when the regular labor force decreases.

This is consistent with the analytical formulation of the representative family's life-time utility. As is standard in New Keynesian models, the representative family's utility, in a simpler version of the model, can be approximated up to the second order as the welfare loss associated with the variance of inflation and output gap. Because the output gap is a function of the two types of labor input and the two thresholds are functions of the two types of labor, the output gap, in turn, can be represented in terms of the two thresholds as follows:[44]

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - \overline{U}}{\overline{U_C C}} \approx -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi_\pi \pi_t^2 + \left(\Phi_{\theta^{IR}} \check{\bar{\theta}}_t^{IR} + \Phi_{\theta^R} \check{\bar{\theta}}^R \right)^2 \right\} + t.i.p. + h.o.t., \quad (1.20)$$

where a variable with check denotes log-deviation from its value in the steady state, t.i.p. stands for terms independent from policy, and h.o.t. means higher order terms. Under the baseline calibration, $\Phi_{\theta^R} > 0$ and $\Phi_{\theta^{IR}} > 0$. This means the volatilities of the two thresholds generate larger welfare losses. Therefore, the central bank wants to stabilize the two thresholds to achieve smaller welfare losses. Moreover, Φ_{θ^R} is greater than $\Phi_{\theta^{IR}}$ under the baseline calibration, meaning that the volatility of $\bar{\theta}_t^R$ generates larger welfare losses than the volatility of $\bar{\theta}_t^{IR}$. This translates into higher weights on the stabilization of $\bar{\theta}_t^R$. Meanwhile, because $\Phi_{\theta^R} > 0$ and $\Phi_{\theta^{IR}} > 0$, the opposite direction of the changes in the two thresholds may generate smaller welfare losses. The signs of the optimal weight, $(\phi^*_{\theta^{IR}}, \phi^*_{\theta^R})$, reflect this point. The optimal weight suggests that the central bank can achieve higher welfare by raising interest rates when the size of the regular labor force, $\bar{\theta}_t^R$, increases and by lowering nominal interest rates when the total labor force size, $\bar{\theta}_t^{IR}$, increases. The opposite responses to the two thresholds contributes to the stabilization of irregular labor force size, $\bar{\theta}_t^{IR} - \bar{\theta}_t^R$.

This formulation also suggests that the stabilization of the aggregate unemployment rate is not sufficient to achieve higher overall welfare. The second-order approximation of the life-time utility of the representative family can be re-written with the aggregate unemployment gap as follows:[45]

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - \overline{U}}{\overline{U_C C}} \approx -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi_\pi \pi_t^2 + \left(\Phi_u \check{u}_t + \Delta^{u,n^R} \check{n}_t^R + \Delta^{u,n^{IR}} \check{n}_t^{IR} \right)^2 \right\} + t.i.p. + h.o.t., \quad (1.21)$$

or equivalently, in terms of $\check{\bar{\theta}}_t^{IR}$ and $\check{\bar{\theta}}_t^R$,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - \overline{U}}{\overline{U_C C}} \approx -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi_\pi \pi_t^2 + \left(\Phi_u \check{u}_t + \Delta^{u,\bar{\theta}^{IR}} \check{\bar{\theta}}_t^{IR} + \Delta^{u,\bar{\theta}^R} \check{\bar{\theta}}_t^R \right)^2 \right\} + t.i.p. + h.o.t. \quad (1.22)$$

[44]See Appendix A.2.10 for formal derivations.

[45]See Appendix A.2.10 for formal derivations.

The presence of the extra-terms, such as, $\Delta^{u,n^R}\check{n}_t^R + \Delta^{u,n^{IR}}\check{n}_t^{IR}$ or $\Delta^{u,\bar{\theta}^{IR}}\check{\bar{\theta}}_t^{IR} + \Delta^{u,\bar{\theta}^R}\check{\bar{\theta}}_t^R$ illustrates that central banks need to account for more than the stabilization of aggregate unemployment rates to achieve higher welfare. Because aggregate unemployment rates do not account for the differential dynamics of the two labor markets, which tend to be more volatile, the stabilization of the aggregate unemployment rate does not translate into the stabilization of the two labor markets.[46] Moreover because unemployment rates are calculated based on the headcount of the labor, which tends to underestimate the cyclicality of the total labor input (see Section 1.5), the stabilization of aggregate unemployment rates is not enough to stabilize the economy.

How does the current policy look like compared to the optimal inclusive monetary policy specified as above? To answer this question, I estimate the interest rate rule with the two thresholds, $\bar{b}_t$ and $\bar{d}_t$, *i.e.* with the size of total labor force and the size of the full-time labor force from the CPS.[47] Specifically, I consider the following three regression equations:

(i) $i_t = \alpha + \rho_1 i_{t-1} + \rho_2 i_{t-2} + (1-\rho_1-\rho_2)\left[\phi_\pi \pi_t + \phi_u\left(u_t - u_t^*\right)\right] + \epsilon_i,$

(ii) $i_t = \alpha + \rho_1 i_{t-1} + \rho_2 i_{t-2} + (1-\rho_1-\rho_2)\left[\phi_\pi \pi_t + \phi_{\theta^{IR}}\left(\bar{\theta}_t^{IR} - \bar{\theta}^{IR}\right) + \phi_{\theta^R}\left(\bar{\theta}_t^R - \bar{\theta}^R\right)\right] + \epsilon_i,$

(iii) $i_t = \alpha + \rho_1 i_{t-1} + \rho_2 i_{t-2} + (1-\rho_1-\rho_2)\left[\phi_\pi \pi_t + \phi_u\left(u_t - u_t^*\right) + \phi_{\theta^{IR}}\left(\bar{\theta}_t^{IR} - \bar{\theta}^{IR}\right) + \phi_{\theta^R}\left(\bar{\theta}_t^R - \bar{\theta}^R\right)\right] + \epsilon_i.$

I estimate the above regressions by using nowcasts of GDP-deflator based inflation rates (for π_t), nowcasts of unemployment rates (for u_t) from the Greenbook dataset maintained by the Federal Reserve Bank of Philadelphia, and short-term natural rate of unemployment (NAIRU, for u_t^*) retrieved from the Federal Reserve Economic Data (FRED).[48] To calculate the deviations of the two thresholds from their steady-state values, I calculate their sample means across the sample horizon from the first quarter of 1976 to the last quarter of 2007 and subtract them from the series of total labor force participation rates and the share of the full-time labor force out of the total population. As alternatives, I also consider the HP-filtered series of them with a large enough smoothing parameter of 10^7.[49]

Table 1.5 presents the estimates of the three interest rate rules considered above. Compared to the optimal weights on each threshold calculated above, the Federal Reserve has been putting either the wrong sign on the stabilization of $\bar{\theta}_t^{IR}$ and $\bar{\theta}_t^R$, and/or wrong relative weights on each threshold. This could have decreased not only the economy-wide

[46] See Section 1.5 for the comparison of impulse responses of aggregate labor market variables and each labor market's variables

[47] The Federal Reserve does not explicitly take into account the compositional changes of worker types when adjusting nominal interest rates. In this regard, this exercise backs out the weights that the Federal Reserve could have put on if it follows the alternative specification of interest rate rule of Equation (1.16).

[48] Taylor rule estimates in Table 1.5 are robust to the forecasts of inflation rates in other horizons (one to three-quarters ahead forecasts of GDP-deflator based inflation rates and the four-quarter average of expected inflation rates. They are also robust to the use of the long-run natural rate of unemployment from the FRED.

[49] Taylor rule estimates in Table 1.5 are robust to a range of smoothing parameters from 10^5 to 10^9.

Table 1.5: Taylor Rule Estimation from Data

	Optimal Monetary Policy	Data				
		(i)	Deviation from Mean		HP filtered	
			(ii)	(iii)	(ii)	(iii)
ϕ_π	1.5	1.51***	1.88***	1.81***	1.47***	1.63***
		(0.13)	(0.19)	(0.22)	(0.15)	(0.15)
ϕ_u		-0.82***		-0.83***		-0.71***
		(0.08)		(0.14)		(0.15)
$\phi_{\theta^{IR}}$	-0.89		1.03***	2.00***	1.99***	1.89***
			(0.26)	(0.25)	(0.27)	(0.24)
ϕ_{θ^R}	1.24		-0.18	-2.44***	-1.83***	-1.08**
			(0.22)	(0.19)	(0.22)	(0.30)
R^2		0.93	0.93	0.93	0.93	0.93
RMSE		0.96	0.97	0.96	0.96	0.96
Welfare Loss (in %)	1.10	3.23	2.17	1.42	2.65	3.93
std. dev of $\bar{\theta}_t^{IR}$ (%)	0.59	0.88	1.24	1.07	1.27	0.91
std. dev of $\bar{\theta}_t^{R}$ (%)	0.63	0.89	1.25	1.24	1.43	1.11
Welfare Loss (in %, $\phi_\pi = 1.5$)	1.10	3.36	2.20	2.24	2.21	2.57
std. dev of $\bar{\theta}_t^{IR}$ (%, $\phi_\pi = 1.5$)	0.59	0.89	0.95	1.01	1.38	1.01
std. dev of $\bar{\theta}_t^{R}$ (%, $\phi_\pi = 1.5$)	0.63	0.91	0.96	1.24	1.43	1.22

Note: This table presents the estimates of interest rate rules (i)-(iii). I use nowcasts of GDP-deflator based inflation rates and unemployment rates. I use total labor force participation rates and the share of the full-time labor force out of population from the CPS for $\bar{\theta}_t^R$ and $\bar{\theta}_t^{IR}$, respectively. To calculate the deviations of the two thresholds from their steady-state values, I subtract their sample means from their time series (the fourth and the fifth columns). Alternatively, I consider the HP-filtered series of them with the smoothing parameter of 10^7 (the last two columns). The first column presents the estimates of the weight on the inflation rate and unemployment gap for the conventional Taylor rule of (ii). The second and the fourth columns show the estimates of the weights on inflation rates and the two thresholds from the interest rate rule of (ii). The third and the fifth columns show the estimates of the weights on inflation rates, unemployment rate gaps, and on the two thresholds from the interest rate rule of (iii). The sample period is from the first quarter of 1976 to the last quarter of 2007. Statistical significance at the 90/95/99% confidence level indicated with */**/***, respectively. Newey-West robust standard errors are reported in parenthesis. I also calculate welfare loss in terms of consumption equivalent in percentage with all the shocks and standard deviations of $\bar{\theta}_t^R$ and $\bar{\theta}_t^{IR}$ in percentages calculated from the baseline model under different policy rules with the estimates of weights on inflation, unemployment, and the two thresholds, $\hat{\phi}_\pi$, $\hat{\phi}_u$, $\hat{\phi}_b$, and $\hat{\phi}_d$. The last three rows present the case when I fix $\phi_\pi = 1.5$, but use the estimates from the data for other weights, $\hat{\phi}_u$, $\hat{\phi}_b$, and $\hat{\phi}_d$.

welfare (see welfare loss in Table 1.5) but also the contingent workers' welfare, by making the two thresholds more volatile (see standard deviations of $\bar{\theta}_t^{IR}$ and $\bar{\theta}_t^{R}$ in Table 1.5). For example, welfare costs under the current policy rules are greater than the welfare cost under the optimal inclusive monetary policy rule. Under the current policy rules, $\bar{\theta}_t^{IR}$ and $\bar{\theta}_t^{R}$ are more volatile than $\bar{\theta}_t^{IR}$ and $\bar{\theta}_t^{R}$ under the optimal policy, respectively. In particular, the current policy could have generated a lot more contingent workers and made labor market risks of the contingent workers greater. Therefore, contingent workers who frequently move either between the regular and irregular labor markets or between the irregular labor markets and NLF would have experienced even greater volatility in their consumption and disutility from supplying labor. In sum, the current Federal Reserve's policy may be harsher on contingent workers, making them even more vulnerable to economic fluctuations. If it instead considers the alternative specification of the policy rule, which targets these contingent workers, then it can substantially ease the burden of the vulnerable, and achieve higher overall welfare.

1.7 Concluding Remarks

Central bankers traditionally emphasize that their job is to take care of aggregate fluctuations while redistributive aspects of the business cycles should be addressed by fiscal policy instruments. However, there is a growing consensus that monetary policy can have important redistributive effects (in fact, monetary policy transmission can rely on redistribution, see, for example, Auclert, 2019), and in the current environment of rising inequality and polarization, central banks are under increasing pressure to explore new ways to help the "average" citizen and especially the least-protected groups in the economy.

Mainstream monetary models are poorly equipped to shed light on these new tasks because these models largely rely on representative agents or (nearly) perfect insurance and so have little (if any) heterogeneity. To make progress, I depart from this tradition and develop a tractable New Keynesian model featuring two types of labor where workers and firms make endogenous decisions over labor types. My model generates that "switchers" or "contingent workers," who frequently move between the regular and the irregular labor market or between the irregular labor market and not-in-the-labor-force, incur substantially larger costs of economic fluctuations. In particular, contingent regular workers who are marginally attached to the regular labor market pay the largest cost. These workers face the largest idiosyncratic risks regarding their labor market status, which generates larger volatilities in their consumption and disutility from labor supply. This makes them more vulnerable to business cycles. Can monetary policy ease the burden of these contingent workers? I show that an alternative interest rate rule to stabilize the composition of workers, based on the size of each labor force, can achieve a significantly higher aggregate welfare.

While my model focuses on a specific dimension of heterogeneity, labor market arrangements, the lesson is broader. Welfare and policy implications from my model raise a warning flag on monetary policy frameworks that focus only on "aggregate" labor market variables. The distribution of business cycle costs is unevenly distributed in the economy. By understanding the welfare implications of this heterogeneity and fine-tuning policies to respond to this heterogeneity, monetary policy can have disproportionally large benefits for the vulnerable groups such as contingent workers and thus deliver superior welfare at the aggregate level.

Much work remains ahead to understand and develop inclusive monetary policy. To gain tractability, my model makes several simplifying assumptions and future work can introduce additional realistic features. For example, in my model, workers gather into the family and they do not have access to any credit markets other than through the arrangements with the representative family, who serves as a stand-in for all the possible arrangements that workers deal with their idiosyncratic labor market risks. While convenient, this assumption misses the heterogeneity in the ability of different workers to insure against their labor market risks. It is plausible that irregular types and those not in the labor force build up smaller amounts of wealth and their borrowing constraints may be more binding than regular workers, and hence they have less opportunity to self-insure. Accounting for this heterogeneity in asset holdings and the interaction of this heterogeneity with the different labor market risks for various worker types can generate even larger costs of economic fluctuations for a certain group. Incorporating this dimension of heterogeneity is left for future work.

Chapter 2

Unemployment Effects of Stay-at-Home Orders: Evidence from High Frequency Claims Data

Introductory Comments

This chapter is co-authored with Peter McCrory, Todd Messer, and Preston Mui. This chapter studies labor market dynamics during a particular episode of the business cycle: the early COVID-19 periods. Specifically, we try to disentangle the labor market effects of stay-at-home (SAH) orders from the general economic disruption wrought by the COVID-19 pandemic. We do so by studying the impact of highly decentralized implementation of SAH orders on initial claims for unemployment insurance, a high-frequency, regionally disaggregated indicator of real economic activity in the United States. We use a currency union model to map cross-sectional estimates to aggregate employment losses. This chapter is forthcoming in *Review of Economics and Statistics*.

2.1 Introduction

To limit the spread and severity of the COVID-19 pandemic, officials around the globe turned to non-pharmaceutical interventions (NPIs), such as shutting down schools, restricting economic activities to those deemed essential, and requiring people to remain at home whenever possible. In mid-March 2020, Ferguson et al. (2020) issued a report projecting that, in the absence of the effective implementation of NPI mitigation strategies, more than 2 million Americans were potentially at risk of death from the COVID-19 respiratory disease, with many more facing uncertain medical complications in the near- and long-run.

Soon after, state and local officials in the United States began announcing Stay-at-Home (SAH) orders, which restricted residents from leaving their homes except for essential activities. The earliest SAH order was implemented in the Bay Area, California on March 16th, 2020. Three days later, the governor of California issued a state-wide SAH order. By March 24th, more than 50% of the U.S. population was under a SAH order (see Figure 2.1). By April 4th, 95% of the U.S. population was under a state or local SAH order, likely substantially reducing the supply of and demand for locally produced goods and services.

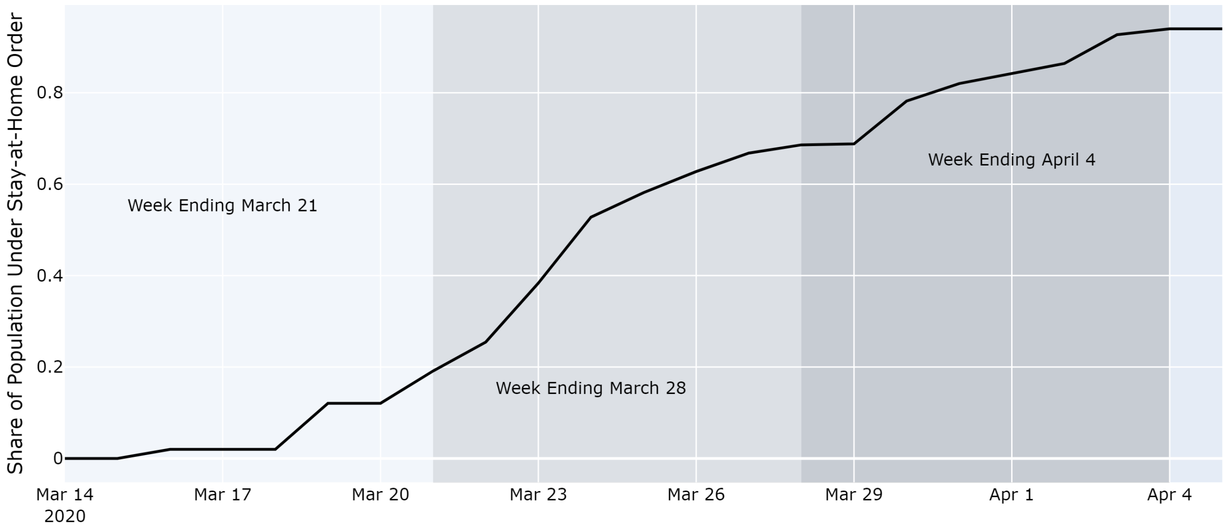
At the same time, there was mounting evidence of substantial disruption to labor markets in the United States. For the week ending March 21st, 2020, the Department of Labor (DOL) reported that more than 3.3 million individuals filed for unemployment benefits.[1] In the subsequent weeks ending March 28th and April 4th, initial claims for unemployment once again hit unprecedented highs of more than 6.9 million claims and 6.7 million claims, respectively. Taken together, total unemployment insurance (UI) claims over this three week period was almost 17 million.

How much of the initially observed increase in UI claims was attributable to the newly implemented SAH orders? This is not a straightforward question to answer since the increase in unemployment claims could plausibly be attributed to a multitude of factors other than SAH orders that occurred at the same time. For example, consumer and business sentiment both declined and economic uncertainty rose as the pandemic worsened. One stark example of this economic uncertainty was the swift drop in the value of the S&P 500 stock market index, which lost roughly 30% of its value between February 20 and March 16, the first day a SAH order was announced in the United States.

In this chapter, we disentangle the local effects of SAH orders from the broader economic disruption brought on by the COVID-19 pandemic and other factors affecting all states equally. We do so by providing evidence of a direct causal link between the implementation of SAH orders and the observed increase in UI claims. To the best of our knowledge, this chapter is the first systematic study of the causal link between SAH orders and UI claims in the United States. This is our main contribution.

[1]For comparison, in this week one year prior, there were just over 200 thousand initial claims for unemployment insurance. This was also the first time since the DOL began issuing these reports that the flow into unemployment insurance exceeded the number of individuals with continuing claims.

Figure 2.1: Cumulative Share of Population under Stay-at-Home Order in the U.S.



Sources: Census Bureau, the *New York Times*; Authors' Calculations

We show that the decentralized implementation of SAH orders across the U.S. induced high-frequency regional variation as to when and to what degree local economies were subject to such orders. We leverage the cross-sectional variation in the length of time that states were exposed to such orders to estimate its effect on UI claims.[2,3]

We find that an additional week of exposure to SAH orders increased UI claims by approximately 1.9% of a state's employment level, relative to unexposed states. The effect is precisely estimated and robust to the inclusion of a battery of controls one might suspect are correlated with both local labor market disruption and SAH implementation, lending it a causal interpretation. The set of controls we consider include the severity of the local exposure to the coronavirus pandemic, state-level political economy factors, and each state's industry composition.

We use our cross-sectional estimate to calculate the implied aggregate effect of SAH orders on the number of new unemployment claims. This exercise yields an estimate of approximately 4 million UI claims attributable to SAH orders through April 4, comprising roughly 24% of total claims over the time period. We refer to this calculation as the relative-implied aggregate estimate of employment losses from SAH orders.

[2]Our variable of interest pertains to the *government* implementation of SAH orders. Our design does not aim to capture the effects of, for example, social distancing behaviors that may have taken place in the absence of a government order.

[3]In this chapter, we principally focus on UI claims for three reasons: (1) UI claims are among the highest frequency indicators of real economic activity—especially as it relates to the labor market; (2) These data are consistently reported at a subnational level; (3) The data are publicly and readily available.

We then investigate whether the change in unemployment claims is due to demand or supply side factors using proxies for local economic activity. Using daily data from Google on local mobility trends by county, we estimate the on-impact effect of SAH orders on visits to retail and workplace locations, the former capturing demand shocks and the latter capturing supply shocks. We find sharp, on-impact, declines in mobility in both the retail and mobility indices, suggesting that SAH orders worked through both channels. The decline in both mobility indices persists for at least two and a half weeks, the horizon over which we estimate the event studies.

It is well known that cross-sectional research designs, such as the one employed in this chapter, hold constant general equilibrium effects as well as other aggregate factors. Simply scaling up our cross-sectional estimate may therefore give a biased impression of the aggregate effect of SAH orders on UI claims in the United States.

To understand the nature of these general equilibrium forces, we present a simplified currency union model to provide conditions under which the relative-implied estimate represents an upper or lower bound on aggregate employment losses. When the SAH shock is viewed primarily as a technology shock—and in the empirically relevant case with flexible prices—our estimate represents an *upper bound* on the aggregate effect. However, when SAH orders are treated as a local demand shock, the interpretation is a bit more subtle and depends upon the persistence of the shock and degree of price flexibility. Across all combinations of price rigidity and persistence, we find that our back-of-the-envelope estimate, at most, understates aggregate employment losses by a factor of approximately two. With sticky prices and a zero-persistence shock, the relative-implied estimate associated with the SAH-induced local demand shock understates aggregate employment losses by 12%.

Our evidence from the mobility indices suggests that the SAH shock should be viewed as a combination of both local supply and demand shocks. The model results then imply a (non-binding) *upper bound* on UI claims from SAH orders through April 4, 2020 of approximately 8 million. Thus, relative to the total rise of around 16.5 million, at most around 50% of the total rise in UI claims over this period can be attributed to SAH orders.

Finally, we document the robustness of our empirical results by considering two alternative research designs. First, we consider a panel design that allows us to control separately for week fixed effects and state fixed effects. The inclusion of such fixed effects controls for time-varying aggregate effects and time-invariant state effects. Second, we estimate county-level specifications which allow us to control for unobserved state-level factors, such as each state's ability to respond to and process unprecedented numbers of unemployment claims. We find similar results in both cases.

Related Literature

This chapter relates most obviously to the rapidly growing economic literature studying the COVID-19 pandemic, its economic implications, and the policies used to address the simultaneous public health and economic crises. The epidemiology literature has focused on the health effects of NPIs. In a notable study, Hsiang et al. (2020) estimate that, in six major countries, NPI interventions prevented or delayed over 62 million COVID-19 cases.[4] Our focus is, instead, on the macroeconomic effects of the coronavirus pandemic. Broadly speaking, the macroeconomic literature on COVID-19 has split into two distinct yet highly related strands. Here we provide a representative, albeit not exhaustive, review.

The first strand of research focuses on the relationship between macroeconomic activity, policy, and the unfolding pandemic. Gourinchas (2020) and Atkeson (2020) are early summaries of how the public health crisis and associated policy interventions interact with the economy. Both emphasize the trade-off between flattening the pandemic curve while steepening the recession curve. Similarly, Faria–e–Castro (2020) studies the effect of a pandemic-like event in a quantitative DSGE model in order to assess the economic damage associated with the pandemic along with the fiscal interventions employed in the U.S. to attempt to flatten the recession curve. Eichenbaum, Rebelo, and Trabandt (2020) derive an extension of the standard Susceptible-Infected-Recovered (SIR) epidemiological model to incorporate macroeconomic effects, formalizing the relationship between the flattening the pandemic curve and amplifying the recession curve. We view this chapter as providing causally identified, empirical support for the claim that flattening the pandemic curve requires steepening the recession curve.

The second strand of research uses high-frequency data to understand the economic fallout wrought by the COVID-19 pandemic. This chapter aligns more closely with this strand of the literature. Baker et al. (2020) show that economic uncertainty measured by stock market volatility, newspaper-based economic uncertainty, and subjective uncertainty in business expectation surveys rose sharply as the pandemic worsened. Lewis, Mertens, and Stock (2020) derive a weekly national economic activity index and show that the COVID-19 outbreak had already had a substantial negative effect on the United States economy in the early weeks of the crisis. Hassan et al. (2020) use firm earnings calls to quantify the risks to firms as a result of the COVID-19 crisis. Coibion, Gorodnichenko, and Weber (2020b) examine how the pandemic affected the labor market in general. Using a repeated large-scale household survey, they show that by April 6th, 2020, 20 millions jobs were lost and the labor market participation rate had fallen sharply.

This chapter also relates to empirical work studying the effect of lockdown policies more specifically. For example, Hartl, Wälde, and Weber (2020) study the effect of lockdowns in Germany on the spread of the COVID-19. In contrast to these papers, we use geographic variation to understand the effect of COVID-19 on economic activity. In that respect, this chapter can be thought of a high frequency version of Correia et al. (2020), who find that over the long term, NPI policies implemented in response to the 1918 Influenza Pandemic ultimately resulted in faster growth during the recovery following the pandemic.

Other papers employing geographic variation in NPI implementation to understand

[4] The six countries are China, South Korea, Italy, Iran, France, and the United States.

their contribution to the economic fallout associated with COVID-19 pandemic include the following: Kong and Prinz (2020) use high-frequency Google search data as a proxy for UI claim activity to study the labor market effects of various NPIs; Coibion, Gorodnichenko, and Weber (2020a) study the effect of lockdowns on employment and macroeconomic expectations; Kahn, Lange, and Wiczer (2020) document broad declines job market openings in mid-March prior to implementation of SAH orders; Kudlyak and Wolcott (2020) provide evidence that the bulk of UI claims over this period were classified as temporary, suggesting that the long-run costs of lockdowns may be mitigated, so long as worker-firm matches persist until the recovery; and, Sauvagnat, Barrot, and Grassi (2020) document regional lockdowns depressed the market value of affected firms.

A closely related paper is Friedson et al. (2020), which uses the state-wide SAH order implementation in California along with high frequency data on confirmed COVID-19 cases and deaths to estimate the effect of this policy on flattening the pandemic curve. Unlike our approach, however, the authors in this paper use a synthetic control research design to identify the causal effects on this policy. The authors argue that the SAH order in California reduced the number of cases by 150K over three weeks; the authors perform a back-of-the-envelope calculation to calculate roughly 2-4 jobs lost over a three week period in California per case saved. In contrast to Friedson et al. (2020), we are able to directly estimate the causal effect of SAH orders on UI claims. Taking their benchmark number of cases saved over three weeks, we find that a SAH order implemented over three weeks in California would increase UI claims by 6.4 per case saved.

2.2 Data

2.2.1 State-Level Stay-at-Home Exposure

We construct a county-level dataset of SAH order implementation based on reporting by the *New York Times*. On March 24th, 2020, the *New York Times* began tracking all cities, counties, and states in the United States that had issued SAH orders and the dates that those orders became effective.[5]

We calculate the number of weeks that each county c in the U.S. had been under a SAH order between day $t-k$ and day t (and counting the day that the policy became

[5]The most recent version of this page is available at `https://www.nytimes.com/interactive/2020/us/coronavirus-stay-at-home-order.html`. In a few instances, states implemented the closure of non-essential businesses prior to broader SAH orders that affected businesses and households alike. We show that our results are qualitatively and quantitatively robust to accounting for this occasional discrepancy in timing in Appendix B.1.3. We choose to rely upon the *New York Times* reporting since it provides sub-state variation. Over time, the *New York Times* stopped separately reporting sub-state orders when a state-wide SAH order was issued. We used the *Internet Archive* to verify the timing and location of SAH orders as reported in the *New York Times*.

effective).[6] We denote this variable with $SAH_{c,s,t,t-k}$, where s indicates the state in which the county is located. Except when explicitly stated, we drop the $t-k$ subscript and set k to be large enough so that this variable records the total number of weeks of SAH implementation in county c through time t.

As an example, consider Alameda County, California. Alameda County was among the first counties to be under a SAH order when one was issued on March 16th, 2020. Here, $SAH_{Alameda,CA,Mar.28} = 13/7$, as Alameda County had been under Stay-at-Home policies for thirteen days. Los Angeles County, California, on the other hand, did not issue a SAH order before the State of California did so. We therefore set $SAH_{LosAngeles,CA,Mar.28} = 10/7$ since the state-wide order was issued in California on March 19th, 2020.

The previous two examples illustrate how, in some instances, county officials took action before the state in which they were located did. While we are able to use this county-level variation to study the impact of SAH orders on retail and workplace mobility, as measured by the Google mobility index, our main outcome of interest, new unemployment claims, is available to us only at the state-level.[7]

To aggregate county-level SAH orders to the state level, we construct a state-level measure of the duration of exposure to SAH orders by taking an employment-weighted average across counties in a given state. Formally, we calculate:

$$SAH_{s,t} \equiv \sum_{c \in s} \frac{Emp_{c,s}}{Emp_s} \times SAH_{c,s,t} \tag{2.1}$$

Employment for each county is the average level of employment in 2018 as reported by the BLS in the Quarterly Census of Employment and Wages (QCEW).[8] One can think of $SAH_{s,t}$ as the average number of weeks a worker in state s was subject to SAH orders by time t.

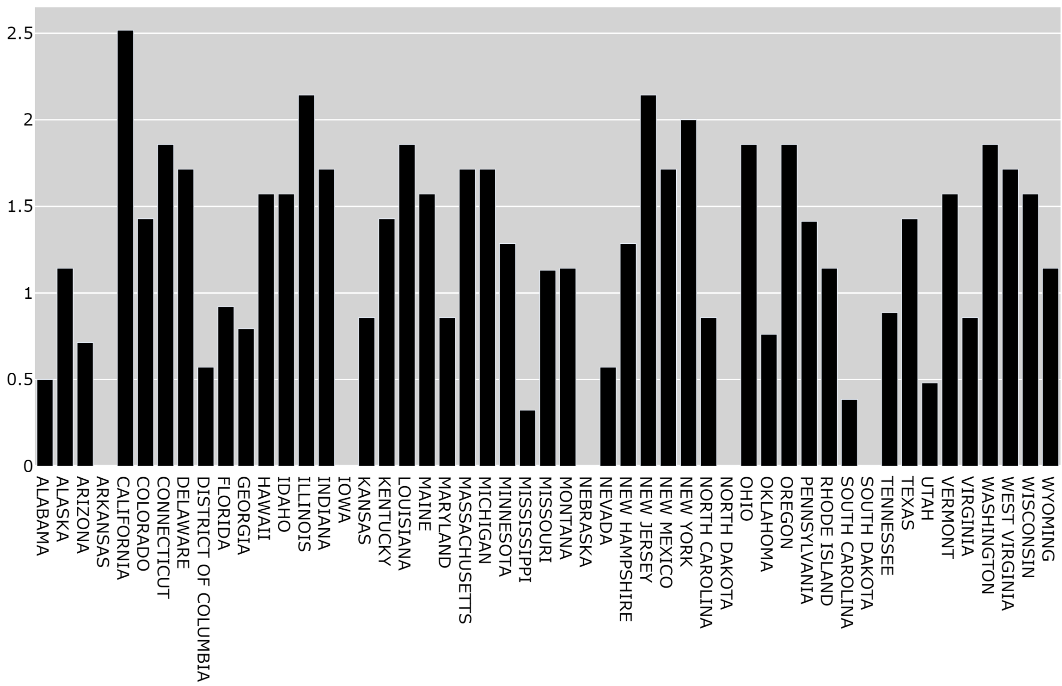
Figure 2.2 reports $SAH_{s,Apr.4}$ for each state in the U.S. and the District of Columbia. California had the highest exposure to SAH orders at 2.5, indicating that Californian workers were on average subject to SAH orders for two and a half weeks. Conversely, five states (Arkansas, Iowa, Nebraska, Northa Dakota, and South Dakota) had no counties under SAH orders by April 4. The average value across all states of $SAH_{s,Apr.4}$ is 1.2.

[6] When a city implements a SAH order, we assign that date to all counties in which that city is located—unless of course the county had already issued a SAH order.

[7] While we lack sufficient data to estimate county-level effects on UI claims, in Section 2.6 we consider county-level regressions in which we estimate the March to April change in log employment and the unemployment rate using data published by the Bureau of Labor Statistics. We find quantitatively similar results even after conditioning on state-level fixed effects.

[8] The annual averages by county in 2019 were, at the time of writing, not yet publicly available.

Figure 2.2: Employment-Weighted State Exposure to Stay-at-Home Policies Through Week Ending April 4



The Employment-Weighted exposure to SAH policies for a particular state is calculated by multiplying the number of weeks through April 4, 2020 that each county in the state was subject to SAH orders by the 2018 QCEW average employment share of that county in the state, and summing over each states' counties. Sources: Bureau of Labor Statistics, the *New York Times*; Authors' Calculations

2.2.2 Main Outcome Variable: State Initial Claims for Unemployment Insurance

Our main outcome of interest is initial unemployment insurance claims. Initial UI claims is among the highest-frequency real economic activity indicators available. As discussed in the introduction, initial claims for unemployment insurance for the week ending March 21st, 2020 were unprecedented, with more than 3 million workers claiming benefits. By the end of that week, very few states or counties had issued SAH orders. Figure 2.1 shows that by March 21st, only around 20% of the U.S. population was under such directives. This suggests that a substantial portion of the initial economic disruption associated with the COVID-19 crisis may have occurred in the absence of SAH orders.

Let $UI_{s,t}$ indicate new unemployment insurance claims for state s at time t and UI_{s,t_0,t_1} denote cumulative unemployment claims for state s from time t_0 to t_1. In our baseline specification, we consider the effect of SAH orders on cumulative weekly unemployment insurance claims by state from March 14th, 2020 to April 4th, 2020:

$$UI_{s,Mar.21,Apr.4} = UI_{s,Mar.21} + UI_{s,Mar.28} + UI_{s,Apr.4} \tag{2.2}$$

We then normalize this variable by employment for each state, as reported in the 2018 QCEW, to construct our outcome variable of interest:

$$\frac{UI_{s,Mar.21,Apr.4}}{Emp_s} \tag{2.3}$$

Our choice of April 4th, 2020 as the end date for this regressions is driven by the observation that, by April 4th, 2020, approximately 95% of the U.S. population was under a SAH order. In Section 2.6, we consider 2-week and 4-week horizon specifications and find quantitatively similar results.

2.3 Empirical Specification

We now turn to our research design. Our main design is a state-level, cross-sectional regression:

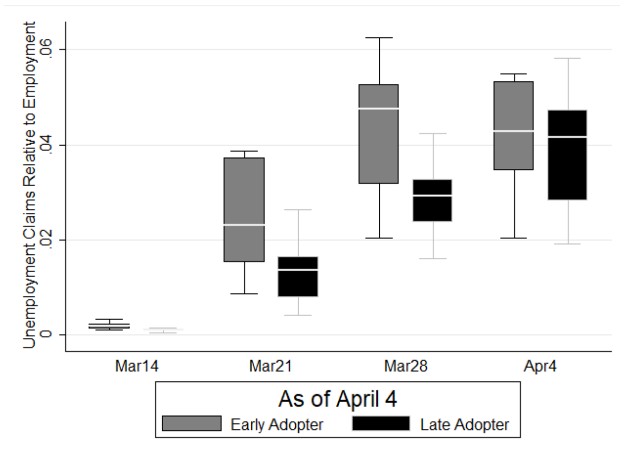
$$\frac{UI_{s,Mar.21,Apr.4}}{Emp_s} = \alpha + \beta_C \times SAH_{s,Apr.4} + X_s\Gamma + \epsilon_s \tag{2.4}$$

where α is a constant, β_C is the coefficient on state-level exposure to SAH orders, X_s is a vector of controls with associated vector of coefficients Γ, and ϵ_s represents the error term in this equation.

To illustrate the motivation for our empirical design, in Figure 2.3 we compare the evolution of UI claims to state employment of "early adopters," defined as those states being in the top quartile of SAH exposure through April 4, 2020, to that of "late adopters," defined as those states being in the bottom quartile.[9] This figure provides *prima facie* graphical evidence of the main result of this chapter: in the first few weeks, early adopters initially had a higher rise in unemployment claims relative to late adopters. By the week ending April 4th, 2020, the relative effect of adopting SAH orders early largely disappears, reflecting the fact that by this point approximately 95% of the U.S. population was under a SAH order, with most having been under the order for the full week ending April 4th.

[9] The upper and lower edges of the boxes denote the interquartile range of each group, with the horizontal line denoting the median. As is standard, the "whiskers" denote the value representing 1.5 times the interquartile range boundaries.

Figure 2.3: Box Plots by Week of Initial UI Claims Relative to Employment for Early and Late Adopters of SAH orders



For each state we calculate SAH exposure through April 4th by multiplying the number of weeks each county was subject to SAH through April 4 by the 2018 QCEW average employment share of that county in the state, and summing over each state's counties. Early adopters are those states in the top quantile of SAH exposure and late adopters are those states in the bottom quartile. Sources: Bureau of Labor Statistics, Department of Labor, and the *New York Times*; Authors' Calculations.

This figure also suggests that SAH orders alone likely do not account for all of the rise in unemployment claims.[10] In the early weeks, late adopters also experienced historically unprecedented levels of UI claims even though early adopters had higher claims on average. For example, consider the week ending March 28. Here the difference between the median value of the two groups was approximately 1% of state employment; in that week, the median value of initial claims to employment for late adopters was roughly 3%, despite close to zero SAH exposure by this point. By April 4th, this difference almost completely disappears. Late adopters, who were under SAH orders for a much shorter period of time (or not at all, in some cases), converged to similar levels of unemployment claims relative to employment.

Confounding Factors

In order for our estimate $\hat{\beta}_C$ to have a causal interpretation, it must be the case that the timing of SAH orders implemented at the state and sub-state-level be orthogonal with unobserved factors affecting reported state-level UI claims.[11]

[10]We thank an anonymous referee for pointing out that this could have the alternative interpretation that local SAH order implementation had substantial negative spillover effects on the rest of the country. See Section 2.5 for a model-driven discussion of such potential spillover effects between states.

[11]An additional reason for preferring April 4th is that over longer horizons, there is greater risk of omitted variable bias (i.e. $\mathbb{C}ov[\epsilon_s SAH_{s,Apr.4}] \neq 0$). A salient example is the rollout of the Paycheck Protection

We provide further support for our causal interpretation by testing the magnitude and significance of the estimate $\hat{\beta}_C$ against the inclusion of three sets of important controls. The first set of controls considers the impact that the COVID-19 outbreak itself had on local labor markets. States that chose to implement SAH orders earlier may have done so simply because of the intensity, perceived or otherwise, of the local outbreak. In most macro-SIR models, a larger real outbreak would directly result in a larger drop in consumption due to a higher risk of contracting the virus associated with consumption activity (e.g. Eichenbaum, Rebelo, and Trabandt (2020)). To account for this concern, we control for the number of excess deaths, as reported by the Centers for Disease Control and Prevention (CDC), relative to population. We also include the share of the population over 60, as this demographic was more at risk of serious health complications arising from contracting COVID-19.

Additionally, one may be concerned that consumers' perceptions of the outbreak differed from its actual severity. During this time period, the reported number of new confirmed cases was an important statistic reported by the media. This statistic, which suffers from differential testing capability and definitions across states, differs from the measure of excess deaths as it focuses on how local labor markets may have interpreted the severity of the outbreak.[12] We therefore also include the total confirmed cases relative to population.[13] Note that the severity of the outbreak would lead to an upward bias in our estimate $\hat{\beta}_C$ if states were more likely to enact SAH orders when the local outbreak was worse or perceived to have been worse, which may itself have led to labor market disruptions.[14]

The second set of controls we consider relates to the political economy of the state government. Some states may have had more generous social safety nets that led workers to separate from firms earlier than in states with less generous policies. Moreover, states with generous policies may also have been more likely to respond earlier to the pandemic, thereby generating bias. To account for this concern, we consider two political

Program (PPP) on April 3rd. (The PPP was a central component of the CARES Act, a two trillion fiscal relief package signed into law on March 27, 2020. The PPP authorized $350 billion dollars in potentially forgivable SBA guaranteed loans.) This program provided forgivable loans to small businesses affected by the economic fallout of the pandemic, so long as those loans were used to retain workers. On the margin, PPP incentivizes firms to not lay off their workers, which would tend to lower UI claims for the week after April 4th. Depending upon how this interacts with the differential timing of SAH implementation, the bias could go in either direction.

[12] Evidence from Fetzer et al. (2020) suggests that the arrival of confirmed COVID-19 cases leads to a sharp rise in measures of economic anxiety, which would have an effect on real economic activity through the change in household and firm beliefs about the future state of the economy.

[13] We rely upon confirmed COVID-19 cases as compiled at the county-by-day frequency by USAFacts. USAFacts is a non-profit organization that compiles these data from publicly available sources, typically from daily reports issued by state and local officials. See `https://usafacts.org/visualizations/coronavirus-covid-19-spread-map/` for more details.

[14] Our controls for excess deaths and confirmed cases are taken as cumulative sums as of the end of the sample period, which is April 4th in the benchmark analysis. We experimented with using lagged values of these measures as pre-period controls, and they had no effect on the magnitude or significance of our coefficient of interest. These results are available upon request.

economy controls. First, we include the average UI replacement rate in 2019, as reported by the Department of Labor's Employment and Training Administration.[15] Second, we include the Republican vote share in the 2016 presidential election.[16] The first measure is designed to capture the generosity of the social safety net, while the latter is meant to capture political constraints on state and local officials to implement various public health NPIs.

Finally, our last set of controls is intended to address the concern that the timing of SAH implementation may be related to the sectoral composition within each state, and therefore the magnitude of job losses experienced by that state irrespective of SAH orders. To address this concern, we use a measure of predicted state-level UI claims as determined by industry composition within each state and the monthly change in jobs as reported in the national jobs report in March by the BLS. These numbers are based on a survey reference period that concluded on March 14th, 2020—fortuitously for us, two days before any SAH order was announced. Specifically we construct a Bartik-style control:

$$B_s = \sum_i \Delta \ln Emp_{i,March} \times \omega_{i,s} \tag{2.5}$$

where $\Delta \ln Emp_{i,March}$ is the monthly percentage change in employment in industry i (3-digit NAICs) for the month of March. $\omega_{i,s}$ is the share of employment in industry i in the state, as reported in the QCEW for 2018.

We also control for the extent of work-at-home capacity at the state-level. Dingel and Neiman (2020) construct an index denoting the share of jobs that can be done at home by cities, industries, and countries. We construct a state-level index by taking an state employment-weighted average of the Dingel and Neiman (2020) industry-level (2-digit NAICS) work-at-home index. It may be the case that states with a higher capacity to work from home may have been willing to implement SAH orders earlier if the labor market disruption of such policies was perceived to be lower when more workers are able to work from home. If this index is correlated with the number of initial UI claims received by the state in the absence of implementing SAH orders, then failing to include this control would introduce bias.[17]

Causal interpretations aside, the cross-sectional framework is nevertheless constrained in only answering the following question: By how much did UI claims increase in a state that implemented SAH orders *relative* to a state that did not? The constant term absorbs, for example, the general equilibrium effects of stay-at-home orders which would affect all states within the U.S.—not just those implementing SAH orders. To the extent that other

[15] See `https://oui.doleta.gov/unemploy/ui_replacement_rates.asp` for more details.

[16] As reported by the *New York Times* at `https://www.nytimes.com/elections/2016/results/president`.

[17] In unreported regressions, we study whether the effect of SAH orders differentially depends upon the value of the work-at-home index; we find no evidence that this is the case.

states' labor markets were affected in any way by the local imposition of SAH orders, then $\hat{\beta}_C$ will fail to capture the *entire* effect of such policies. We postpone discussion of the mapping between the relative effect of SAH orders and their aggregate effect until after presenting our cross-sectional results.

Figure 2.4: Scatterplot of SAH Exposure to Cumulative Initial Weekly Claims for Weeks Ending March 21 thru April 4

The Employment-Weighted exposure to SAH policies for a particular state is calculated by multiplying the number of weeks through April 4, 2020 that each county in the state was subject to SAH orders by the 2018 QCEW average employment share of that county in the state, and summing over each states' counties. UI claims are cumulative new claims between weeks ending March 21, 2020 and April 4, 2020, divided by average 2018 QCEW average employment in the state. The size of each bubble is proportional to state population; The color gradient of each observation is determined by the number of confirmed COVID-19 cases per thousand people.

Sources: Bureau of Labor Statistics, Census Bureau, the *New York Times*, USAFacts.org, Department of Labor; Authors' Calculations

2.4 Results

2.4.1 Effects of SAH Orders on State-Level UI Claims

In Table 2.1, we present results from estimating Equation (2.4). Column (1) shows the univariate specification, with no controls. The point estimate of approximately 1.9% (SE: 0.67%) implies that a one-week increase in exposure to SAH orders raises the number of claims as a share of state employment by 1.9% relative to states that did not implement SAH orders. Figure 2.4 displays this result graphically. The bubbles are shaded according to the intensity of the confirmed COVID-19 cases per thousand people and the size of the bubbles are proportional to state population.

In Column (2), we control for the number of confirmed COVID-19 cases per one thousand people, excess deaths by state, and the share of state population over the age of 60. As discussed, these are intended to control for factors related to the pandemic that might simultaneously affect both the timing of SAH implementation and the severity of state labor market disruptions. The change in the coefficient is immaterial—economically and statistically. In Column (3) we control for political economy factors: the state's UI replacement rate in 2019 and the 2016 Trump vote share. Our estimate $\hat{\beta}_C$ falls only slightly to 1.8%. In Column (4) we include controls for each state's sectoral composition (and in turn its sensitivity to both the pandemic-induced crisis and timing of SAH implementation). Our point estimate is again largely unchanged. Finally, in column (5), we select a parsimonious specification that captures dimensions of each set of controls. We control for confirmed cases, excess deaths, the UI replacement rate, and the WAH index (the only significant variable). In this specification, which is our preferred specification, the estimate of β_C is still 1.9%.[18,19]

Our results support the idea that policies that work to flatten the pandemic curve also imply a steepening of the recession curve (Gourinchas, 2020). To quantify this steepning of the recession curve, we use our point estimate of the relative effect on state-level UI claims of SAH orders to calculate a back-of-the-envelope estimate of the total implied number of UI claims between March 14 and April 4 attributable to SAH orders. We cal-

[18] In the appendix, we consider three additional robustness exercises at the state-level. We alternate the horizon over which the model is estimated (2 and 4 weeks), estimate the model by weighted least squares, and re-estimate the model dropping one state at a time. The results are quantitatively and qualitatively similar.

[19] In unreported regressions, we find that, when including all regressors, $\hat{\beta}_C$ is somewhat attenuated—albeit statistically indistinguishable from our baseline estimate; however, this attenuation is largely driven by the parametric assumption of linearity on the share of votes for Trump in 2016, which places substantial leverage on Wyoming and West Virginia. Dropping these states from the full specification with all control variables yields a point estimate of 1.8% (SE: 0.75%). These regressions are available upon request.

culate the relative-implied estimate as follows:[20]

$$\text{Relative-Implied-Aggregate-Claims} = \sum_{s} \hat{\beta}_C \times SAH_{s,Apr.4} \times Emp_s \tag{2.6}$$

where s indexes a particular state. This is a back-of-the-envelope calculation as it simply scales up the cross-sectional coefficient $\hat{\beta}_C$ according to each state's SAH exposure through April 4, 2020 and each state's level of employment.

This back-of-the-envelope calculation yields an estimate of 4 million UI claims attributable to SAH orders through April 4. Ignoring cross-regional spillovers, this relative-implied estimate suggests that approximately 24% of total claims through April 4, 2020 were attributable to such orders.

This calculation does not incorporate general equilibrium effects or spillovers that may have arisen as a result of local SAH implementation. As we discuss in Section 2.5, when the SAH order is interpreted as a local productivity shock, this represents an upper bound on aggregate employment losses; when, however, the SAH implementation is treated as a local demand shock, the analysis is a bit subtler. Yet, even in this case, we find that at most the relative-implied aggregate multiplier understates true employment aggregate employment losses by a factor of 2. Through the lens of the model, this provides an upper bound on total employment losses attributable SAH orders: 8 million UI claims through April 4, or approximately half of the overall spike in claims during the initial weeks of the economic crisis induced by the COVID-19 pandemic.

An alternative back-of-the-envelope calculation to assess the magnitude of our estimate is to instead focus the relative contribution of SAH orders in terms of typical cross-sectional variation in UI claims in our sample. Our estimates imply that a state which implemented SAH orders one week earlier saw an increase in UI claims by 1.9% of its 2018 employment level relative to a state one week later, which is slightly less than 50% of the cross-sectional standard deviation of employment-normalized claims between weeks ending March 21 and April 4.[21]

2.4.2 High Frequency Effects on Proxies for Local Economic Activity

In this subsection, we provide additional evidence that the SAH orders had immediate and highly localized effects on daily indicators of economic activity. This exercise is important because of concerns that the state-level effects we estimate above simply reflect differential labor market disruptions that would have occurred in the absence of SAH orders in precisely those places most likely to implement SAH orders earliest.

We estimate the local effect of SAH using high frequency proxies for economic activity from Google's Community Mobility Report, which measures changes in visits to

[20]We use the terminology "relative-implied" because in the cross-section we are only able to identify effects of SAH orders relative to states not implementing SAH orders. We discuss this issue at greater length in Section 2.5.

[21]We thank an anonymous referee for this particular recommendation.

Table 2.1: Effect of Stay-at-Home Orders on Cumulative Initial Weekly Claims Relative to State Employment for Weeks Ending March 21 thru April 4, 2020

	(1) Bivariate	(2) Covid	(3) Pol. Econ.	(4) Sectoral	(5) All
SAH Exposure thru Apr. 4	0.0194***	0.0192**	0.0178**	0.0209***	0.0187**
	(0.00664)	(0.00742)	(0.00818)	(0.00637)	(0.00714)
COVID-19 Cases per 1K		-0.00213			0.00194
		(0.00621)			(0.00676)
Excess Deaths per 1K		0.0446			0.0480
		(0.109)			(0.113)
Share Age 60+		0.237			
		(0.281)			
Avg. UI Replacement Rate			0.0719		0.0726
			(0.0794)		(0.0787)
2016 Trump Vote Share			-0.0225		
			(0.0508)		
Work at Home Index				-0.331+	-0.388+
				(0.192)	(0.229)
Bartik-Predicted Job Loss				-2.401	
				(7.528)	
Constant	0.0815***	0.0357	0.0621	0.181**	0.182**
	(0.00848)	(0.0543)	(0.0481)	(0.0742)	(0.0821)
Adj. R-Square	0.0829	0.0434	0.0618	0.0966	0.0763
No. Obs.	51	51	51	51	51

This table reports results from estimating equation (2.4): $\frac{UI_{s,Mar.21,Apr.4}}{Emp_s} = \alpha + \beta_C \times SAH_{s,Apr.4} + X_s\Gamma + \epsilon_s$, where each column considers a different set of controls X_s. Column (5)—a parsimonious model controlling for pandemic severity, political economy factors, and state sectoral composition—is our benchmark specification. The dependent variable in all columns is our measure of cumulative new unemployment claims as a fraction of state employment, as calculated in Equation (2.3). The interpretation of the SAH Exposure coefficient ($\hat{\beta}_C$; top row) is the effect on normalized new UI claims of one additional week of state exposure to SAH. The Employment-Weighted exposure to SAH for a particular state is calculated by multiplying the number of weeks through April 4, 2020 that each county in the state was subject to SAH with the 2018 QCEW average employment share of that county in the state, and summing over each states' counties.

Robust Standard Errors in Parentheses
$+ \; p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

establishments in various categories, such as retail and work.[22] Early on in the COVID-19 pandemic, Google began publishing data documenting how often its users were visiting

[22] `https://www.google.com/covid19/mobility/`

different types of establishments. The data are reported as values relative to the median visitation rates by week-day between January 3, 2020 and February 6, 2020.[23],[24]

We use the retail and workplace mobility indices because these two indices are consistently recorded for the time sample we study. Failing to find an effect on these proxies for local economic activity would call into question the results we find in the aggregate, at the state-level. We interpret retail mobility as broadly representing "demand" responses to SAH orders and workplace mobility as broadly representing "supply," at least on-impact.[25] Over longer-horizons, workers laid off because of demand-side disruptions will, naturally, cease commuting to and from work.

Formally, we estimate event studies of the following form:

$$Mobility_{c,t} = \alpha_c + \phi_{CZ(c),t} + \sum_{k=\underline{K}}^{\overline{K}} \beta_k SAH_{c,t+k} + X_{c,t} + \underline{D}_{c,t} + \overline{D}_{c,t} + \varepsilon_{c,t} \tag{2.7}$$

where $Mobility_{c,t}$ represents either the retail or workplace mobility index published by Google for county c on day t, and $SAH_{c,t}$ is a dummy variable equal to 1 on the day a county imposes SAH orders. We set $\underline{K} = -17$ and $\overline{K} = 21$ so that the analysis examines three weeks prior and two and a half weeks following the imposition of SAH orders.[26] The event study is estimated over the period February 15th through April 24th, 2020. We non-parametrically control for county size by discretizing county employment into fifteen equally sized bins and interacting each bin with time fixed effects. α_c refers to the inclusion of county fixed effects. To isolate the local effect of SAH orders on economic activity, we also include commuting zone-by-time fixed effects.[27] This implies that our event-study estimates are identified only off of differential timing of SAH implementation among counties contained within the same commuting zone.

Results for retail mobility are presented in Figure 2.5. The day SAH orders went into effect, there was an immediate decline of approximately 2% in retail mobility. This falls

[23]One possible limitation of this data is that the sample of accounts included in the surveys is derived from only those with Google Accounts who opt into location services. We believe sample selection bias is unlikely to be a major concern given Google's broad reach (there are over 1.5 billion Gmail accounts, for example).

[24]Note that for privacy reasons, data is missing for some days for some counties. When possible, we carry forward the last non-missing value. Excluding counties with missing values yields the same result; this figure is available from the authors upon request.

[25]Of course, both indicators are equilibrium outcomes of both supply and demand shocks. The on-impact effect on work-place mobility at the very least reflects disruptions to each firm's ability to produce. Similarly, the on-impact effect on retail mobility is indicative of a decline in retail demand by consumers since, presumably, the supply of retail goods is at least fixed in the very short-run.

[26]Because our sample is necessarily unbalanced in event-time, we also include "long-run" dummy variables, $\underline{D}_{c,t}$ and $\overline{D}_{c,t}$. $\underline{D}_{c,t}$ is equal to 1 if a county imposed SAH orders at least $\overline{K}$ days prior. $\overline{D}_{c,t}$ is equal to 1 if a county will impose SAH at least $\underline{K}$ periods in the future.

[27]We use the United States Department of Agriculture (USDA) 2000 county to commuting zone crosswalk. This is available at `https://www.ers.usda.gov/data-products/commuting-zones-and-labor-market-areas/`.

further to 7% the day after SAH order implementation, before slowly recovering to approximately 2% lower retail mobility two and a half weeks following the SAH order imposition.[28] The large transitory dip may reflect sentiment among consumers to shut-in before revisiting grocery stores and pharmacies. Alternatively, given our inclusion of commuting zone-by-time fixed effects, the transitory nature of the shock may reflect negative, within-labor market spillovers of SAH orders. Regardless, the lack of a pre-trend is noticeable and provides additional support for a causal interpretation.

SAH orders may have affected firms' ability to produce by preventing workers from accessing their places of employment. To investigate whether SAH orders may have affected firms' productive capacity through this channel, we re-estimate our event study using workplace mobility as the outcome variable.[29]

Figure 2.6 shows the result. As with the retail mobility event study, the workplace mobility index exhibits no differential pre-trend prior to the county-level imposition of SAH orders. In the first two days following the imposition of SAH orders, workplace mobility declined sharply relative to non-treated counties within its commuting zone. This relative decline in workplace mobility persists for nearly two and a half weeks following.

We draw three conclusions from these high-frequency event studies. First, the lack of pre-trends in the event studies suggest that the timing of SAH orders can be seen as plausibly randomly assigned with respect to local labor market conditions. This provides corroborating evidence for our cross-sectional identification strategy. In particular, it suggests that there were real effects of the SAH orders on local economies. Second, with the important caveat that both mobility indices are equilibrium objects, SAH orders appear to have had *both* local supply and local demand effects. Both retail mobility and workplace mobility fell substantially on impact and remained persistently low for at least two weeks following implementation of SAH orders. Third, given that overall workplace and retail mobility in the U.S. fell by 48 and 40 percent through April 24th relative to their baseline levels, our results bolster the claim that alternative mechanisms were responsible for the majority of job losses in the early weeks of the crisis; upon SAH implementation, relative workplace and retail mobility fell by, at most, 2 and 7 percent, respectively.

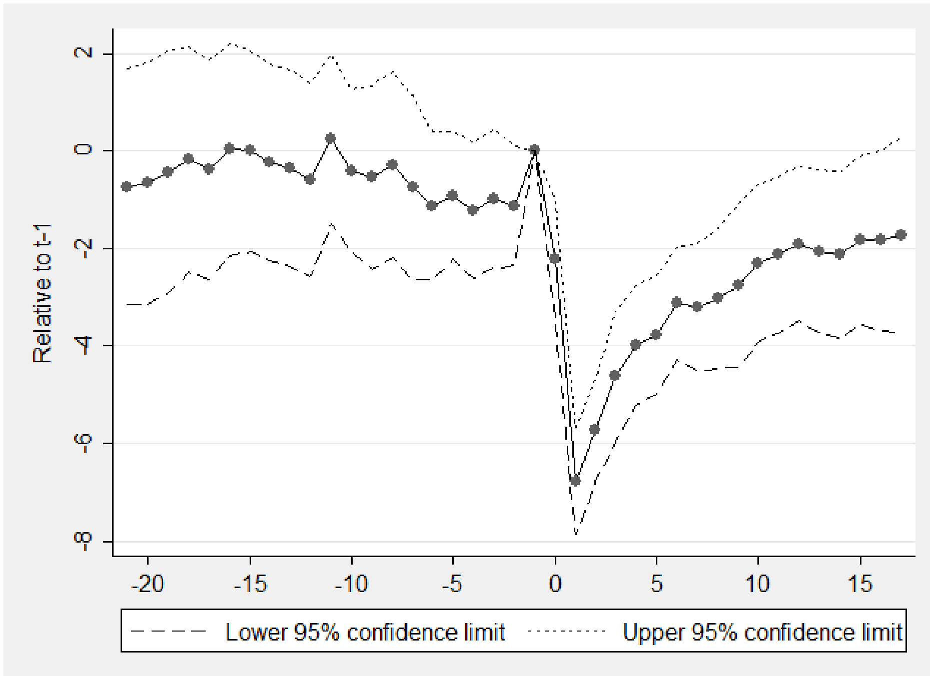
2.5 Aggregate Versus Relative Effects

Our empirical strategy relies on cross-sectional variation in the timing and location of SAH orders to identify the relative effect such policies had on labor markets during the

[28] Restricting the sample to exclude never-takers yields the same result. This design identifies the mobility effects off of counties that ultimately implemented SAH orders but at different times.

[29] An obvious concern with simply replacing the outcome variable is that changes in workplace mobility, unlike retail mobility, is highly dependent on the ability of individuals to work from home. The timing of SAH orders may be partially driven by the ability of workers in some regions to transition to working at home. In unreported regressions, we also non-parametrically control for this possibility by partitioning the WAH variable into 15 equally sized bins and interacting each bin with time fixed effects. The event study is essentially unchanged.

Figure 2.5: County Retail Mobility Event Study



This figure plots estimated coefficients from the county-level, event-study specification in equation (2.7), where coefficients have been normalized relative to one day prior to county-level SAH orders went into effect. The model includes as controls county fixed effects, commuting zone-by-time fixed effects, and indicators for county employment bins interacted with time to non-parametrically control for county size. The outcome variable is the retail mobility index published in Google's Community Mobility Report. This index is constructed using visits and duration of visits to retail establishments. The time unit is days.

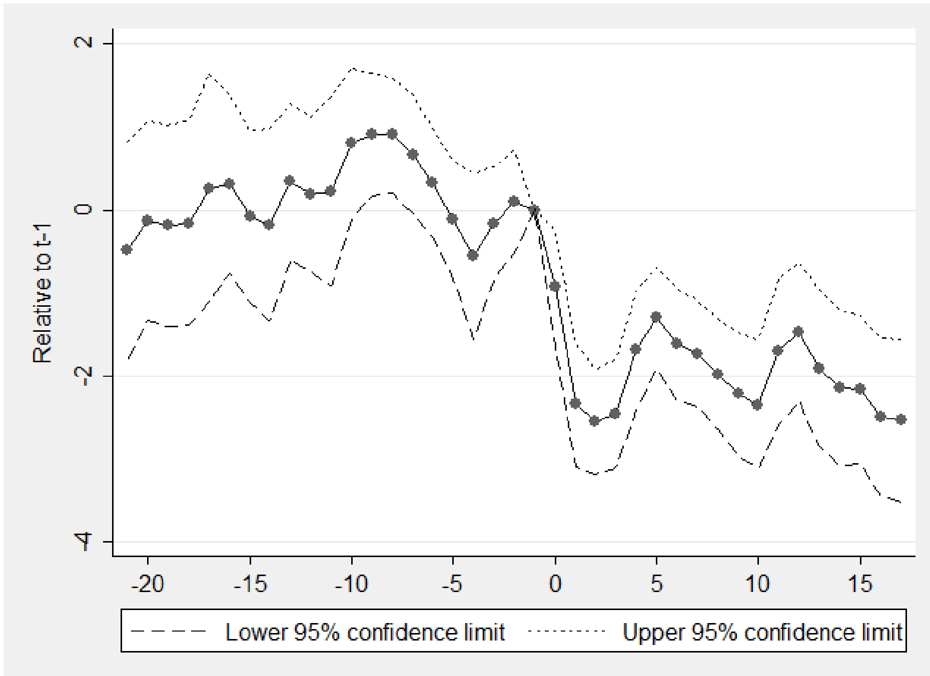
Standard Errors: Two-Way Clustered by County and Day

Sources: Google, the *New York Times*; Census Bureau; United States Department of Agriculture; Authors' Calculations

initial weeks of the COVID-19 outbreak in the United States. In this section, we discuss in greater detail the sorts of spillovers that are likely to be relevant and the conditions under which the relative-implied aggregate estimate (see equation (2.6)) represents a lower or upper bound on the aggregate effects of SAH orders on UI claims. This is important for how one should interpret our back-of-the-envelope calculation that in the early period of the crisis, approximately only 24% of UI claims through April 4, 2020 were related to SAH orders.

To the extent that there are cross-regional (either positive or negative) spillovers of SAH orders, our estimate will not capture the *aggregate* effect of SAH orders. This limitation is related to the stable unit value (SUTVA) assumption in the causal inference literature, which requires that potential outcomes be independent of the treatment status of other observational units. Because of considerable trade between U.S. states, SUTVA is

Figure 2.6: County Workplace Mobility Event Study



This figure plots estimated coefficients from the county-level, event-study specification in equation (2.7), where coefficients have been normalized relative to one day prior to county-level SAH orders went into effect. The model includes as controls county fixed effects, commuting zone-by-time fixed effects, and indicators for county employment bins interacted with time to non-parametrically control for county size. The outcome variable is the workplace mobility index published in Google's Community Mobility Report. This index is constructed using visits and duration of visits to places of employment. The time unit is days.

Standard Errors: Two-Way Clustered by County and Day

Sources: Google, the *New York Times*; Census Bureau; United States Department of Agriculture; Authors' Calculations

likely to be violated in our setting.[30]

To guide our discussion, we use a benchmark currency-union model to study the effects of SAH orders on the local economy, the rest of the currency union, and the entire economy as a whole. We present results for an economy characterized either by sticky prices or flexible prices, with SAH orders modeled as either a pure local demand shock or a pure local productivity/supply shock; the evidence from Subsection 2.4.2 suggests that both channels were operative.[31] We then briefly summarize other important cross-regional spillovers not well-captured by the currency model we study. The most salient

[30]SUTVA violations are likely to be more salient in the cross-section when the model is estimated over longer horizons. This is, in part, why we choose as our baseline the 3-week horizon specification.

[31]Additionally, as is discussed in Brinca, Duarte, and Faria-e Castro (2020), it is appropriate to view the COVID-19 pandemic (and associated policy responses) as some combination of demand and supply shocks. We consider pure demand and supply shocks to illustrate the economic implications of each in isolation.

of these spillovers relate to the *informational* effect of early SAH implementation in some parts of the country.

2.5.1 Currency Union Model: Supply and Demand Shock Implications of SAH Orders

In this section, we consider the implications of local demand or supply shocks in a benchmark currency union model under either sticky or flexible prices. The model we consider is a simpler version of the baseline, separable utility, complete markets model presented in Nakamura and Steinsson (2014), modified to incorporate productivity shocks and discount rate shocks (to model negative local supply and demand shocks, respectively).[32] We follow Nakamura and Steinsson (2014) in calibrating the model to the U.S. setting. The full model specification is relegated to the Appendix; here we present only those aspects of the model modified to study the effects of SAH orders.

2.5.1.1 Modeling SAH Orders

Our first model experiment is to treat the implementation of SAH orders as a pure local demand shock. To incorporate this into the model, we introduce a consumption preference shock, δ_t. This preference shock causes home region households to prefer, all else equal, delaying consumption into the future. This may be a reasonable way to model the SAH shock for a variety of reasons. First, to the extent that the drop in retail mobility, as shown in Figure 2.5, represents a decline in goods consumption, households may simply be delaying such purchases until temporarily closed stores reopen. Second, the inability to purchase locally furnished goods and services may lead households to temporarily save more than they might otherwise choose to do, which would be observationally equivalent to a discount rate shock only to consumption.

Households in the home region maximize the present discounted value of expected utility over current and future consumption C_t and labor supply N_t.

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\delta_t \frac{(C_t)^{1-\sigma}}{1-\sigma} - \chi \frac{(N_t)^{1+\psi}}{1+\psi} \right],$$

where β is the rate of time discounting, σ is the inverse intertemporal elasticity of substitution, ψ is the inverse Frisch elasticity of labor supply, and χ is the weight on labor supply. The discount rate shock process follows

$$\log \delta_t = \rho^{\delta} \log \delta_{t-1} + \epsilon_t^{\delta}. \tag{2.8}$$

[32] Implications from a model with different preference structures (e.g. Greenwood, Hercowitz, and Huffman (1988) preference) and with incomplete market are qualitatively the same. Unlike the original focus of Nakamura and Steinsson (2014), the model we consider does not incorporate government spending shocks, as that is not our focus in this chapter.

We close the household side of the model by assuming preferences for varieties are constant elasticity of substitution (CES), which gives rise to the standard CES demand curve via cost minimization.

Alternatively, the SAH orders may be modeled as a local productivity shock. Even if demand for locally produced goods is unchanged, firms may be constrained in supplying the goods and services demanded by local households or by the rest of the currency union. We model this interpretation as a region-level productivity shock for intermediate-goods-producing firms. A firm i in the home region faces the following production function

$$y_{h,t}(i) = A_t N_{h,t}(i)^{\alpha},$$

where $y_{h,t}(i)$ is the output of a firm i, $N_{h,t}(i)$ is the amount of labor input hired by the firm, and A_t is region-wide technology in the home region. α is the returns to scale parameter on labor. The aggregate supply shock A_t evolves according to the following process:

$$\log A_t = \rho^A \log A_{t-1} + \epsilon_t^A. \tag{2.9}$$

Firms maximize profits subject to demand by households. Nominal rigidities are specified à la **?** with associated price-reset parameter θ.

Finally, we close the model by assuming bond markets are complete, labor markets are perfectly competitive, and, when prices are sticky, the monetary authority follows a union-wide Taylor rule. A full derivation is available in the Appendix.

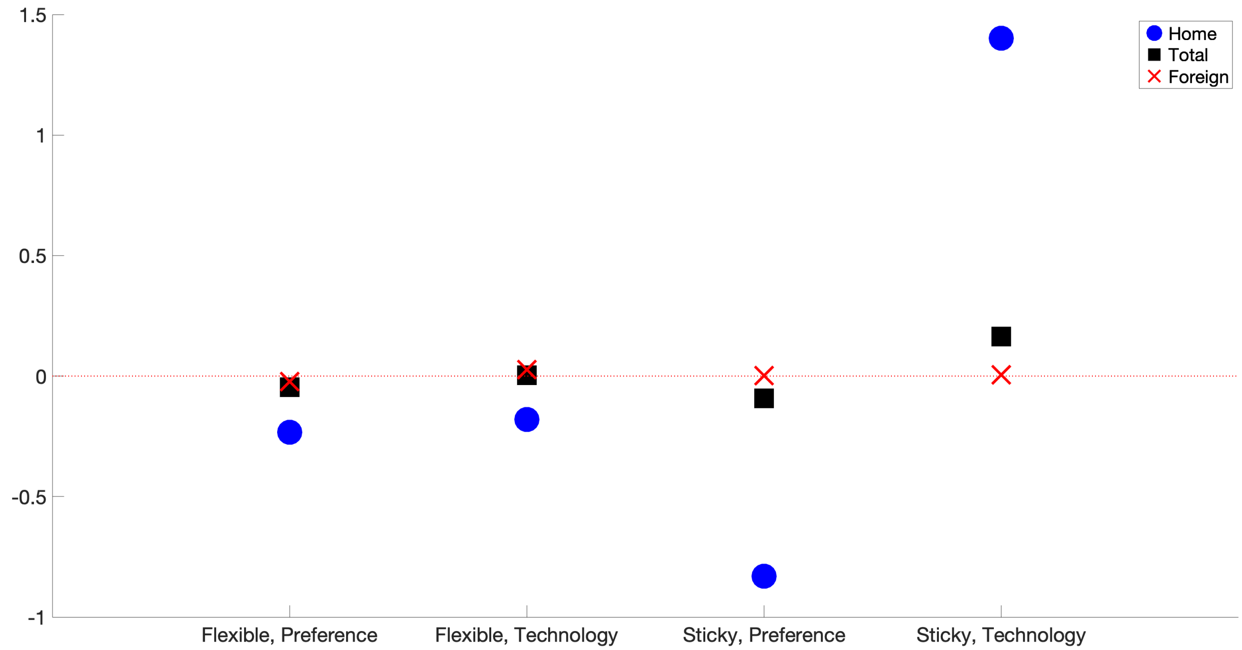
2.5.1.2 Model Results: Modeling SAH Order Shocks under Flexible and Sticky Prices

We model the implementation of SAH orders as a one-time negative shock with either $\epsilon_t^\delta = -1$ (for local demand shocks) or $\epsilon_t^A = -1$ (for local supply shocks). We choose zero decay parameters on the shock series to illustrate the dynamics of the model in settings in which the shock induced by the SAH order is temporary. Specifically, we set $\rho^A = \rho^\delta = 0$. For the purposes of mapping the relative-implied employment losses to aggregate employment losses, this is without loss for the results for the technology shock but not without loss with respect to the demand shock with sticky prices. Below, we discuss what happens when the demand shock exhibits some persistence.

We calibrate the remaining variables according to Nakamura and Steinsson (2014) (see their Section III.D.). When working with the sticky price model, we set the Calvo parameter $\theta = 0.75$. In the flexible price model, we set $\theta = 0$.

We consider each of the two types of shocks in isolation under either sticky prices or fully flexible prices. In each of the four scenarios, we calculate the on-impact responses of home region employment, foreign region employment, and aggregate employment to the local shock. Because the model is calibrated to a quarterly frequency and because our empirical design estimates the relative effect over a short horizon (3-weeks), the relevant horizon for mapping the model to the cross-section is the *on-impact* relative effect between employment in the shocked home region and the non-shocked foreign region.

Figure 2.7: On-Impact Response of Home Employment, Foreign Employment, and Union-Wide Employment to a Local SAH-induced: (i) Technology Shock with Flexible Prices, (ii) Technology Shock with Sticky Prices, (iii) Preference Shock with Flexible Prices, and (iv) Preference Shock with Sticky Prices



This figure shows the on-impact responses of aggregate employment and employment in each region to local demand (preference) and supply (technology) shocks with flexible or sticky prices. Each column represents different scenarios. In both all cases, the shocks persist for a single quarter only ($\rho^{\delta} = \rho^{A} = 0$; see equations (2.8) and (2.9)). The blue circles show the responses of employment in a home region, the red crosses are the responses of employment in a foreign region, and the black squares are the responses of aggregate employment. In the first three scenarios, the on-impact effect of home region employment declines *relative* to employment in the foreign region; this is consistent with our cross-sectional estimates of a positive coefficient on SAH exposure. The final column in which prices are sticky an the SAH orders are modeled as a technology shock produces a counterfactual prediction that employment is higher in the home region relative to the foreign region.

The results from these exercises are reported in Figure 2.7 and Table 2.2. Figure 2.7 shows the *on-impact* responses of employment in a home region (blue circles) and a foreign region (red crosses), and aggregate employment (black squares) under the four different scenarios. Table 2.2 then compares the relative-implied aggregate employment calculated from the differences between the responses of home and foreign employment and the responses of aggregate employment under different scenarios.[33]

Table 2.2: On-Impact Response of Union-Wide Employment and Relative-Implied Aggregate Employment to a Local SAH-induced: (i) Preference Shock with Flexible Prices, (ii) Preference Shock with Sticky Prices, (iii) Technology Shock with Flexible Prices, and (iv) Technology Shock with Sticky Prices

	Flexible				Sticky		
	Total	Implied	Factor		Total	Implied	Factor
Preference Shock	-0.047	-0.021	2.21	$\rho^{\delta} = 0.9$	-0.032	-0.075	0.43
				$\rho^{\delta} = 0.0$	-0.093	-0.083	1.12
Technology Shock	0.003	-0.021	-0.16		0.1642	0.1398	1.18

This table shows the on-impact responses of aggregate employment and the relative-implied employment to a local demand (preference) and supply (technology) shocks with flexible or sticky prices. The columns labeled "Total" correspond to the model-implied on-impact aggregate employment change (i.e. a population-weighted average of the employment change in the home and foreign regions). The columns labeled "Implied" correspond to the relative-implied aggregate change in the model. This is calculated as the difference between the on-impact employment effect in the home region and the on-impact employment effect in the foreign region, together multiplied by the size of the home region. This is the model analog of the relative-implied aggregate estimate in equation (2.6). A negative value for the implied column implies that the model is consistent with our cross-sectional estimate. The columns labeled "Factor" takes the ratio of the on-impact aggregate employment effect to the relative-implied effect. A negative value in this column (Flexible prices and Technology shock) implies that the relative-implied employment effect is of the opposite sign to the aggregate employment effect.

In the model, only three of the four stylized scenarios we consider produce relative effects of SAH orders that are consistent with the positive coefficient we estimate in the data. When the SAH orders are modeled as local productivity shocks, only the flexible price equilibrium produces an immediate, relative decline in employment in the home region subject to the shock. When the SAH orders are instead modeled as local demand shocks, both the sticky price and flexible price economies produce a steeper decline in the shocked home region's employment relative to the rest of the economy, as suggested by the cross-sectional evidence presented above.

When SAH orders are modeled as negative productivity shocks with fully flexible prices, the immediate, relative effect of SAH orders is an *upper bound* on the aggregate employment effect over the same horizon. This is because the decline in local employment arising from the SAH order is offset by an increase in employment in the rest of the economy. The mechanism is that in the flexible price case, the negative productivity shock

[33] Formally, the relative-implied estimate in the model is calculated as $n(\ell_t - \ell_t^*)$, where ℓ_t and ℓ_t^* represent log deviations from steady state of home and foreign region per-capita employment respectively. n is the size of the home-region. This is exactly the model-analog of the relative-implied estimate reported in equation (2.6).

in the home region translates into an improvement in the foreign region's terms of trade. This, in turn, increases labor demand in the foreign region, which increases employment in the foreign region.

In contrast, when prices are fully flexible in response to an SAH-induced home-region demand shock, the relative-implied estimate represents a *lower* bound on aggregate employment losses. This is because employment in both the home and foreign regions fall in response to the shock. With prices being fully flexible, the negative preference shock in the home region leads to a decline in prices for home goods relative to foreign goods, making foreign consumption more expensive. This, in turn, decreases demand for foreign goods, resulting in a decline in foreign employment, which is necessary for market clearing. When prices are fully flexible and the effect of SAH orders is a pure local demand shock, aggregating the relative employment losses understates the aggregate employment losses by a factor of about two (see Table 2.2, Row 1, Column 3).

The case with sticky prices and SAH orders modeled as a pure local demand shock lies in between the previous two scenarios. When the local demand shock is sufficiently persistent, the immediate, relative effect of SAH orders could potentially *overstate* the aggregate employment effect. This is because employment in the foreign region increases on impact. Meanwhile, when the demand shock has essentially no persistence, so that it only affects demand in the home region for a single quarter, employment in the foreign region also falls on impact, implying that the (aggregated) relative employment effect again understates aggregate employment losses, in the quarter of the shock (See Figure 2.7). Regardless, the degree to which this on-impact effect understates aggregate employment losses is bounded above by the response under flexible prices to a local demand shock.

The evidence presented in Subsection 2.4.2 suggests that SAH orders represented a shock to both the supply of and demand for locally produced goods. This on its own implies that the flexible price, preference shock scenario provides a non-binding upper bound on aggregate employment losses. Specifically, in this scenario the relative-implied aggregate estimate would understate employment losses by roughly a factor of two. The distance from this upper bound increases, moreover, with price rigidity and the persistence of the SAH shock. In the baseline calibration, when prices are sticky and the demand shock has no persistence, the relative-implied job losses understates aggregate employment losses by 12%.

2.5.2 Other Cross-Regional Spillovers

The benchmark currency-union model presented in the previous section illustrates how locally implemented SAH orders would affect the local economy, other regions in the currency union and the entire economy as a whole. The spillover forces in the model work through the trade in goods between regions and associated price and expenditure switching effects. However, there may be other important cross-regional spillovers that are not well-captured by the model, but may nevertheless be important for interpreting our empirical results in light of the aggregate effects of SAH orders.

An important example is an *informational effect* of early SAH implementation in some parts of the economy. For example, the early imposition of SAH orders in some regions may signal to the rest of the country that a SAH order is likely to be imposed some time in the near future. This informational channel can be incorporated into the model by assuming that the foreign region learns, on-impact, that a SAH order will be imposed in the foreign region in the subsequent period. We experimented with this specific informational channel of local SAH order implementation and found that the upper and lower bounds provided in the previous subsection continued to hold.[34]

A more subtle informational effect of SAH implementation relates to the credible signal it sends about the severity of the COVID-19 pandemic and the potential economic disruptions it is likely to induce, even in the absence of any additional SAH orders. In this interpretation, the SAH orders have spillover effects on the rest of the economy through the changes they induce to beliefs held by households and firms about the future path of the economy. As opposed to other signals conveyed by public officials about the severity of the pandemic, SAH implementation is a credible signal because it imposes non-trivial costs on the economy. This could, in turn, lead to a reduction in demand as a result of increased economic anxiety and fear of exposure to the COVID-19.

If this second informational effect of local SAH implementation ultimately led to job losses throughout the rest of the country, then our relative-implied estimate would understate the aggregate job losses attributable to SAH orders. Neither the model nor the empirical design takes this particular spillover mechanism into account. We view understanding the role of SAH orders as credibly communicating the severity of the pandemic as an important and interesting avenue for future research.[35]

Another important example is spillovers through firm networks—internal and external.[36] For example, complex supply chains may cause economic activity to decline in parts of the country where SAH orders are not yet enacted if the sourcing of intermediate inputs is affected. Alternatively, national chains may close establishments located in regions without SAH orders due to losses in other major markets with SAH orders. Arguably, these sorts of spillovers would lead our relative-implied estimate of job losses to understate true aggregate employment losses. However, we believe these channels are minor, as the adjustments would need to occur over a very short period time. The horizon of our empirical specifications is three weeks, during which time existing inventories were likely to be sufficient for production.[37]

[34]These results are available upon request.

[35]Coibion, Gorodnichenko, and Weber (2020a) provide evidence that local SAH orders led households in the affected regions to hold more pessimistic views of the future path of the economy. This is a separate, though related, channel than the *aggregate* change in beliefs that may have occurred following the early imposition of SAH orders.

[36]We thank an anonymous referee for pointing this out.

[37]It is a well known observation that inventories generally adjust more slowly to changes in sales, con-

2.6 Alternative Specifications

2.6.1 Panel Specification

One concern with the cross-sectional specifications is that there may be some unobserved aggregate factor that induced large increases in UI claims at the same time that states and local municipalities implemented SAH orders. Alternatively, there may be time-invariant state-specific factors that drove both increases in unemployment claims and SAH orders. To address these concerns, we employ a panel specification, which allows us to control for week and state fixed effects.

We modify the specification so that the outcome variable is the flow value of initial claims on date t and the SAH order treatment is the share of the *current week* that a state was subject to SAH orders, where we take a weighted average of county-level exposure as before.[38]

$$\frac{UI_{s,t}}{Emp_s} = \alpha_s + \phi_t + \beta_P \times SAH_{s,t,t-7} + \mathbf{X}_{s,t}\Gamma + \epsilon_{s,t} \tag{2.10}$$

We consider a variety of state-time controls. We include two lags of $SAH_{s,t,t-7}$ to account for dynamics in the effect of SAH orders on unemployment claims. Additionally, we include the share of the population that works from home, the number of confirmed cases per one thousand people, and the Bartik-style employment control from before. Each of these three controls is interacted with a dummy equal to one for weeks ending March 21st, 2020 and onward.[39] We estimate the following fixed effects panel regression on weekly observations for the week ending January 4 through the week ending April 11.[40]

Table 2.3 provides our estimate of $\hat{\beta}_P$ for the contemporaneous effect and two lags. Column (1) presents the results with no lags. The point estimate of 0.90% (SE: 0.35%) suggests that a full week of SAH order exposure increased unemployment claims by .90% of total state-level employment. In column (2), we include two lags of SAH orders. The point estimate on the contemporaneous effect is little changed, though it rises slightly. Importantly, neither of the coefficients on the first nor the second lag is significant. This result suggests that, in our sample, that SAH orders have constant, contemporaneous effects on UI claims. At longer horizons, we would suspect non-linearities to eventually kick in, with the effect of SAH orders declining. Finally, our point estimates are little changed when including additional controls in Column (3).

sistent with the claim that this particular source of bias is most relevant at lower frequencies and longer horizons. (See Ramey and West, 1999; Bils and Kahn, 2000).

[38]Because in our sample no state or local municipality reopened, once $SAH_{s,t,t-7} = 1$ it remains equal to one for all remaining weeks.

[39]Note that because our measures of work-from-home and employment loss are constant across time, we are controlling for the relative effect of each from before the week ending March 21st.

[40]We drop the first two weeks in all specifications to ensure the sample size is constant throughout.

Table 2.3: Panel Specification: Effect of Stay-at-Home Orders on Initial Weekly Claims Relative to State Employment

	(1)	(2)	(3)	(4)
SAH Exposure Current Week	0.00919**	0.0101***	0.00997***	0.0125***
	(0.00350)	(0.00321)	(0.00329)	(0.00353)
SAH Exposure First Lag		-0.00293	-0.00367	-0.00299
		(0.00359)	(0.00358)	(0.00372)
SAH Exposure Second Lag		0.00245	-0.00115	0.000809
		(0.00230)	(0.00302)	(0.00332)
State FE	Y	Y	Y	N
Week FE	Y	Y	Y	Y
Post-March 21 X Work at Home Index	N	N	Y	Y
Post-March 21 X Excess Deaths per 1K	N	N	Y	Y
Post-March 21 X COVID-19 Cases per 1K	N	N	Y	Y
Post-March 21 X Avg. UI Replacement Rate	N	N	Y	Y
Adj. R-Square	0.826	0.822	0.831	0.801
No. Obs.	765	663	663	663

This table reports results from estimating equation (2.10): $\frac{UI_{s,t}}{Emp_s} = \alpha_s + \phi_t + \beta_P \times SAH_{s,t,t-7} + \mathbf{X}_{s,t}\Gamma + \epsilon_{s,t}$, where each column considers a different set of controls X_s. The dependent variable in all columns is weekly initial unemployment claims as a fraction of state employment. The interpretation of the SAH Exposure coefficient ($\hat{\beta}_P$; top row) is the effect on normalized new UI claims of a full week of state exposure to SAH. The Employment-Weighted exposure to SAH for a particular state is calculated by multiplying the share of the current week each county in the state is subject to SAH by the 2018 QCEW average employment share of that county in the state, and summing over each states' counties. UI claims are cumulative new claims during the period, divided by average 2018 QCEW average employment in the state.

Standard Errors Clustered by State in Parentheses

$+$ $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Our estimates $\hat{\beta}_P$ in the first three columns tend to be somewhat lower than what we find in our benchmark, cross-sectional design. In particular, the panel design implies that each week of SAH exposure increased UI claims by 1% of state employment; in contrast, our estimates of $\hat{\beta}_C$ imply that each week of SAH exposure increased UI claims by approximately 1.9% of state employment. While, at first glance, β_C and β_P aim to estimate the same moment, the inclusion of state and time fixed effects imply that they are not directly comparable.[41] In column (4), we consider the panel specification in which we drop state fixed effects, to make the panel and cross-sectional regressions comparable: the point estimate rises to 1.2% and is statistically indistinguishable from what we find in the cross-section.

[41]See Kropko and Kubinec (2020) for a discussion of the proper interpretation of two-way fixed effect estimators in relation to one-way fixed effect estimators.

2.6.2 County-Level Employment and Unemployment Effects

Another major concern with the estimates of Equation (2.4) is that states may have experienced substantial difficulty in scaling up their systems to process the historically unprecedented numbers of unemployment claims. For example, it is well known that some states' unemployment insurance systems rely on archaic computer programming languages.[42] Thus, it is reasonable to be worried that states with more cumbersome systems may systematically report lower UI claims numbers relative to those states with more efficient systems.

A priori, the induced omitted variable bias could go in either direction. On the one hand, states with stronger UI systems may have also been more inclined to respond aggressively to the COVID-19 pandemic with SAH orders, generating an upward bias in our estimates. On the other hand, the severity of labor market disruptions from the COVID-19 pandemic may have both made it more difficult for states to process new claims *and* made them more likely to impose SAH orders earlier—thus, generating a downward bias. While we have already controlled for measures of COVID-19 in our estimates of Equation (2.4), in this subsection we present an alternative design at the county-level using employment and unemployment as outcomes, albeit at a lower frequency. Using total employment, rather than unemployment insurance claims, allows us to sidestep the issue of whether states could meet demand for UI claims. This design also allows for the inclusion of state fixed effects to identify the relative effect of SAH orders using within-state variation in the timing of SAH implementation.

We analyze the effects of SAH orders at the county-level relying upon local area unemployment and employment statistics constructed by the Bureau of Labor Statistics (BLS). The downside is that this data is constructed at the monthly frequency, rather than the weekly frequency in our main specification.[43] The BLS primarily relies upon the Current Population Survey (CPS) as the primary input into constructing estimates of county-level employment and unemployment.[44] Fortunately, the survey reference periods for the CPS aligns quite nicely with measuring household employment and unemployment just prior to the broad implementation of SAH orders and one month hence. The reference week for the CPS for March 2020 was March 8th through March 14th and the reference week for April was April 12th through April 18th.

We estimate analogs of our state-level regression at the county-level, using as our outcome variable either the log change in employment or the change in the unemployment rate between March 2020 and April 2020. County-level treatment is the weekly SAH expo-

[42]See, for example, "'COBOL Cowboys' Aim To Rescue Sluggish State Unemployment Systems" by NPR (`https://www.npr.org/2020/04/22/841682627/cobol-cowboys-aim-to-rescue-sluggish-state-unemployment-systems`).

[43]In Appendix B.1.4 we estimate event study specifications using high frequency employment statistics at the county-level for a subset of counties in the U.S. for which these data exist. We find no evidence of differential changes in county-level employment prior to SAH implementation while at the same time finding that SAH orders lowered employment on average by 1.9% after one week.

[44]For additional details on the methodology employed by the Bureau of Labor Statistics, see .

sure through April 15, 2020. Formally, we estimate the following regression by ordinary least squares:

$$\Delta y_{c,s,April} = \alpha_s + \beta^{y}_{C,county} \times SAH_{c,s,Apr.15} + X_{c,s}\Gamma + \epsilon_{c,s} \tag{2.11}$$

where $y_{c,s,April}$ indicates the monthly change between March and April in either log employment or the unemployment rate. α_s are state-level fixed effects which control for all state-level policies implemented between mid-March and mid-April that may have been systematically related to observed UI claims during that period. We also report results when constraining $\alpha_s = \alpha$ to provide a natural benchmark against our state-level regression. We also control for the number of confirmed COVID-19 cases per thousand people and the WAH index, which are our only controls available at the county-level.[45]

Because the first outcome variable we consider at the county-level is the log change in county employment, we expect that the estimated relative effect of SAH orders on local employment, $\hat{\beta}^{emp}_{C,county}$, will be comparable to our estimate of the same parameter at the state-level.[46] If the timing of the decentralized implementation of SAH orders was orthogonal to state-level economic conditions and if there were negligible spillovers from treated counties to untreated counties within the same state, then we would expect to see a relatively stable coefficient regardless of whether we include state fixed effects, α_s, or not.

Table 2.4 provides the results for the effects of SAH orders on employment. The first column shows the results restricting $\alpha_s = \alpha$ (e.g., no state fixed effects). The point estimate suggests that the relative effect of SAH exposure on employment at the county-level is to reduce employment by of -1.8% (SE: .57%). That we use a different outcome variable and different level of disaggregation yet obtain a coefficient of similar magnitude is encouraging.

Columns (2) and (3) focus on the 12 states for which there is variation across counties in the timing of SAH orders. The magnitude of the estimate falls by about one third, regardless of whether we include controls—although this difference is not statistically significant. If, as we argue above, the timing of SAH implementation was orthogonal to policies and economic conditions at the state-level[47], then the decline in the point estimate is suggestive evidence of negative spillovers between treated and untreated counties. While this may be the appropriate interpretation, it appears that the bulk of employment losses were nevertheless concentrated within the labor markets in which SAH orders were implemented.

[45]We control for the number of confirmed COVID-19 cases through April 15th to align with the timing of the surveys used by the BLS to construct county-level employment and unemployment statistics.

[46]Note that because we use the 2018 QCEW to normalize UI claims at the state-level, we should expect the county-level estimates to be slightly lower in magnitude since the state-level regressions calculates the percent change off of a smaller base value.

[47]And the average treatment effect among counties in the twelve states appearing in columns (2)-(4) is the same as for counties.

Table 2.4: County-Level Specification: Effect of Stay-at-Home Orders on Local Employment Growth

	(1)	(2)	(3)	(4)
	$\Delta \ln Emp$	$\Delta \ln Emp$	$\Delta \ln Emp$	$\Delta \ln Emp$
SAH Exposure thru Apr. 15	-0.0176***	-0.0124**	-0.0129**	-0.00905**
	(0.00568)	(0.00464)	(0.00453)	(0.00397)
Covid-19 Cases per 1K Emp			-0.0000280	-0.000116
			(0.0000348)	(0.000121)
Work at Home Index			0.0549	0.0547
			(0.0457)	(0.0537)
Constant	-0.0824***	-0.113***	-0.129***	-0.135***
	(0.0147)	(0.00900)	(0.0157)	(0.0139)
Dep Mean	-0.12	-0.14	-0.14	-0.14
States	51.00	12.00	12.00	12.00
State FE	No	Yes	Yes	Yes
CZ FE	No	No	No	Yes
Adj. R-Square	0.10	0.62	0.63	0.74
No. Obs.	3141.00	1116.00	1116.00	453.00

This table reports results from estimating equation (2.11): $\Delta \ln Emp_{c,s,April} = \alpha_s + \beta^{Emp}_{C,county} \times SAH_{c,s,Apr.15} + X_{c,s}\Gamma + \epsilon_{c,s}$, where each column considers a different set of controls X_s. The dependent variable in all columns is $\Delta \ln Emp$, which refers to the log change in county employment between March, 2020 and April, 2020 as estimated by the BLS. SAH exposure for a particular county is calculated as the number of weeks that the county was subject to SAH orders through April 15, 2020. Columns (2) thru (4) include state fixed effects; Column (3) includes fixed effects for USDA defined commuting zones (CZ).

Standard Errors Clustered by State in Parentheses

$+\ p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Finally, in the last column, we include commuting zone fixed effects and find that the coefficient is roughly a third of the effect estimated in column (3). Following a similar logic as in the previous paragraph, this would suggest that not only were the bulk of employment losses concentrated within the labor market, they were moreover concentrated within the specific counties in which the SAH orders were implemented.

Table 2.5 provides the results for the effects of SAH orders on the change in the county-level unemployment rate. As with the employment specification, the first column does not include state fixed effects. In columns (2) and (3) we include state fixed effects; in the final column, we condition further on commuting zone fixed effects. Consider the result reported in column (3), the state fixed effects specification with controls for local COVID-19 pandemic and capacity for the local labor force to work from home: the point estimate is 1.5 (SE: 0.331), implying that each week of SAH exposure at the county-level increased the local unemployment rate by 1.5.

Table 2.5: County-Level Specification: Effect of Stay-at-Home Orders on Local Unemployment Rate

	(1) ΔUR	(2) ΔUR	(3) ΔUR	(4) ΔUR
SAH Exposure thru Apr. 15	1.574***	1.382***	1.570***	0.944***
	(0.400)	(0.331)	(0.331)	(0.216)
Covid-19 Cases per 1K Emp			-0.000239	0.0110
			(0.00468)	(0.00806)
Work at Home Index			-12.29**	-5.437
			(5.336)	(5.089)
Constant	4.114***	4.425***	7.922***	6.689***
	(0.888)	(0.642)	(2.005)	(1.863)
Dep Mean	7.69	7.11	7.11	7.32
States	51.00	12.00	12.00	12.00
State FE	No	Yes	Yes	Yes
CZ FE	No	No	No	Yes
Adj. R-Square	0.13	0.39	0.40	0.59
No. Obs.	3141.00	1116.00	1116.00	453.00

This table reports results from estimating equation (2.11): $\Delta UR_{c,s,April} = \alpha_s + \beta^{UR}_{C,county} \times SAH_{c,s,Apr.15} + X_{c,s}\Gamma + \epsilon_{c,s}$, where each column considers a different set of controls $X_{c,s}$. The dependent variable in all columns is ΔUR, which refers to the change in the county unemployment rate between March, 2020 and April, 2020 as estimated by the BLS. SAH exposure for a particular county is calculated as the number of weeks that the county was subject to SAH orders through April 15, 2020. Columns (2) thru (4) include state fixed effects; Column (3) includes fixed effects for commuting zones (CZ) classified by the USDA in 2000.

Standard Errors Clustered by State in Parentheses

$+\ p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

In sum, we view the panel and county-level results as corroborating evidence of the main result in this chapter: that the cross-sectional effect of SAH orders had real costs to the labor markets in the early weeks of the crisis, but that such costs were likely dwarfed by other factors. While not inconsistent with our state-level analysis, broadly the panel and county-level designs yield somewhat lower point estimates than in our benchmark specification. In this respect, relative to a null that all observed UI claims were attributable to SAH orders, the state-level specification yields the most conservative estimate of the relative effect of such orders on local labor markets. Through the lens of our theoretical model, these cross-sectional estimates imply, at most, a non-binding upper bound of half of total UI claims through April 4, 2020 being attributable to SAH orders.

2.7 Conclusion

While non-pharmaceutical interventions (NPIs) are necessary to slow the spread of viruses such as COVID-19, they likely steepen the recession curve. But to what extent? We provide estimates of how much one prominent NPI disrupted local labor markets in the short run in the U.S. in the early weeks of the coronavirus pandemic.

In particular, we investigate the effect of Stay-at-Home (SAH) orders on new unemployment claims in order to quantify the causal effect of this severe NPI (i.e., flattening the pandemic curve) on economic activity (i.e., steepening the recession curve). The decentralized implementation of SAH orders in the U.S. induced both geographic and temporal variation in when regions were subject to restrictions on economic and social mobility. Between March 14th and April 4th, the share of workers under such orders rose from 0% to almost 95%. This rise was gradual but steady, with new areas implementing SAH orders on a daily basis. We couple this variation in SAH implementation with high-frequency unemployment claims data to quantify the resulting economic disruption.

We find that a one-week increase in stay-at-home orders raised unemployment claims by 1.9% of state-level employment. This estimate is robust to a battery of controls, including the severity of the local COVID-19 pandemic, the local political economy response, and the industry mix of the local economy. Using Google mobility data, we find evidence of both supply and demand driven effects. A back-of-the-envelope calculation using our estimate implies that SAH orders resulted in a rise of 4 million unemployment insurance claims, about a quarter of the total unemployment insurance claims during this period. A stylized currency union model suggests that in some empirically relevant cases, this estimate can be seen as an upper bound. When it instead represents a lower bound, it at most understates job losses by a factor of two.

While it is beyond the scope of this chapter to uncover all determinants of the unprecedented initial rise in unemployment during the COVID-19 pandemic, there is evidence that the economic downturn was already under way by the time that SAH orders were implemented. Even before the national emergency was announced by President Trump on March 13, 2020, households were reallocating their spending away from in-person goods and services.[48] Consistent with this evidence, our estimates imply that a sizeable share of the increase in unemployment in the early weeks of the COVID-19 crisis was due to other channels, such as decreased consumer sentiment, stock market disruptions, and social distancing that would have occurred in the absence of government orders.

Nevertheless, despite representing a minority share of the overall increase in unemployment in the initial three weeks of the crisis, our estimates suggest that over longer horizons SAH orders played a much larger role. Performing an out-of-sample forecast

[48] By March 13, grocery spending was up 44%, restaurant spending was down 10%, and entertainment and recreation spending was down 23%, all relative to their respective levels in January 2020. At about the same time—and preceding any reported SAH orders—both national consumer spending and small business revenue began their precipitous declines. Statistics calculated from data available at `https://tracktherecovery.org/`.

through April 25 of the relative-implied aggregate effect of SAH orders is illustrative: An additional 7.5 million UI claims between April 4 and April 25 are due to SAH orders, little more than half of the additional overall increase in UI claims nationally during that time.[49]

In sum, we see this chapter as providing evidence that undoing SAH orders may relieve only a fraction of the economic disruption arising from the COVID-19 pandemic while at the same time exacerbating the public health crisis. This implies that the economic downturn may persist at least until the pandemic itself is resolved. At the same time, we document a large elasticity of unemployment with respect to such lockdown measures, suggesting that the costs of SAH orders are non-trivial in the long-run.

[49]This helps to reconcile our estimates with Coibion, Gorodnichenko, and Weber (2020a) who find a larger contribution of SAH orders to job losses throughout April than we do. In this exercise, we adjust for whether a state reopened before April 25; not adjusting increases the out-of-sample forecast to 7.6 million claims. See `https://www.nytimes.com/interactive/2020/us/states-reopen-map-coronavirus.html` for state reopening dates.

Chapter 3

Estimating the Effects of Central Bank Communication

Introductory Comments

This chapter is co-authored with Nicholas Sander and continues with the overall theme of business cycles, monetary policy and labor market. This chapter studies the effect of monetary policy on market outcomes including labor market variables over the business cycle. Specifically, we try to estimate how central bank communications affect the dynamic response of aggregate variables to monetary policy shocks – surprise changes in the short term interest rates used by central banks to implement monetary policy. To do so, we compare impulse responses to high-frequency monetary surprises during announcements when the Bank of England also releases a detailed inflation report to those where a simple press statement is released. We find that when a simple press statement is released, policy has conventional signs: unemployment and inflation fall following a surprise tightening. When a detailed inflation report is released, however, surprise tightening raises unemployment and inflation suggesting the information effect can be controlled by central banks.

3.1 Introduction

It has long been an important message of monetary policy that expectations matter and that controlling expectations through transparency can help monetary policy makers better achieve their policy goals.[1] Recent work however has begun to differentiate between transparency about policy implementation - the policy rule the central bank applies - and information about the economy.[2] The traditional expectations channel is supposed to support monetary policy: when people know the central bank wants inflation to be low, they plan for low inflation and the associated changes in behavior reduces the need for central banks to respond dramatically to inflationary pressure. However, when the central bank and the public have different information, surprise policy movements might reveal that information to the public in ways that offset the effects of monetary policy.

For example, an unexpected loosening of monetary policy is expected to raise demand (through the standard savings/spending & exchange rate channels). However, if the public knows that central banks cut rates when the economy is weak, then a surprise cut in policy suggests to the public that the economy is weaker than they believe. A reasonable response to news about a weak economy could be to spend less; lowering demand and offsetting the direct effects of the policy cut to some degree.

The purpose of this chapter is to assess the strength of this "Information Channel" empirically. In particular we want to assess the extent to which varying the information explicitly provided by the central bank affects the economic responses to monetary policy. Central Banks have certainly acted as if the information they send is important for policy outcomes. For instance, when analyzing forward guidance, Campbell et al. (2012) differentiate between "Oddyssean" forward guidance (committing to a low interest rate strategy come what may) from "Delphic forward guidance" (predicting low interest rates due to expected economic weakness) and show that early forward guidance undertaken by the Federal Reserve had a notable Delphic component. After this research, the Federal Reserve then adopted the "Evans Rule": explicit forward guidance broken only if certain future outcomes were achieved. This change in strategy suggests the Federal Reserve thought the information channel might be large in this context. There are also many cases of "open mouth operations" where Central Bank Governors attempt to manipulate market prices through strong statements (see Guthrie and Wright (2000) for one example).

However, work by Nakamura and Steinsson (2018) suggests that this information effect can be strong even in contexts when relatively little information is given. They show that private forecasts of GDP rise in response to surprise tightening of the Federal Funds Rate by the US Federal Reserve. During these announcements by the Federal Reserve, the only formal information released is a short press statement justifying the policy move. This reactions of forecasters suggests that merely adjusting policy rates without context might still have a substantial information effect. Their work puts into question whether

[1] Barro and Gordon (1983) and Clarida, Gali, and Gertler (1999) are notable examples.

[2] Nakamura and Steinsson (2018) and Campbell et al. (2012) and Campbell et al. (2017) in context of forward guidance

Central Banks are able to control this information channel through a carefully crafted policy message.

In this chapter, we assess the extent to which Central Banks can moderate this information channel through their communications by considering a useful reporting structure undertaken by the Bank of England since 1998.[3] Every year the Bank of England makes 8 regularly scheduled policy announcements roughly 6 weeks apart: 4 of these involve the release of an Inflation Report with detailed analysis of the current economic situation in the United Kingdom and outcomes in the absence of policy intervention. During the other 4 announcements (which occur in between the Inflation Report announcements) the Bank of England releases a short press release detailing the policy change with a brief description of the current economic situation.

We show that under two assumptions – namely that the Bank of England's policy rule and the structural economic parameters governing the response to monetary policy remain unchanged when an Inflation Report is released – we can estimate the "treatment effect" of releasing an inflation report in the same month as changing policy on the economic responses to monetary policy. This can be done in a simple differences-in-differences framework interacting a time dummy for an Inflation Report with the interest rate. We show that under these assumptions, the *difference* in impulse responses can even be estimated with an ordinary least squares (OLS). The reason for this is that if the central bank follows the same policy rule in both information regimes, then the biases from their forward looking behaviour cancel out when differencing. [4]

However, to obtain consistent estimates of the *level* effects of policy surprises on economic outcomes (both for when an Inflation Report is included with the announcement and when it is not) we do need to estimate the effects with an instrument. For this we use the high frequency surprise measure of Kuttner (2001) and Gürkaynak, Sack, and Swanson (2005) for our baseline estimates as well as the "hybrid" measure proposed by Miranda-Agrippino (2016) and Miranda-Agrippino and Ricco (2018) where these surprise measures are purged of information contained in forecasts produced by the Central Bank.

Using standard high-frequency measures of monetary policy surprises we first embed the differences-in-differences style interaction into a Jordà (2005) local projections framework to construct the difference in the dynamic responses of economic variables depending on whether an Inflation Report is released or not.

Firstly we document considerable differences in economic responses to the same policy surprise depending on the amount of information accompanying the policy change. When there is an Inflation Report accompanying a given policy tightening we find that the dynamic paths of unemployment is lower and inflation is higher than when there is no In-

[3]Many central banks undertake the same reporting structure as the Bank of England. Future work on this project will be to repeat the analysis presented here for a variety of central banks with the same reporting structure.

[4]This also implies a useful cross-check on our estimation procedure: estimating with valid monetary policy instruments should also give similar estimates of the differences. In Section 3.4.3 we verify that this is the case.

flation Report released along with the policy change.[5] Moreover, the policy rate track after the initial shock remains considerably elevated if an Inflation Report is released relative to an equivalent policy shock with no Inflation Report provided. Given that our framework predicts that we can estimate these differences with OLS or any valid instrument and recover the correct estimates, we confirm that these differences' estimates are the same sign and similar magnitude whether estimating the full system with OLS, or instrumenting with the high-frequency surprise instrument or the hybrid forecast-adjusted-surprise instrument.

Next, we estimate the levels of the impulse responses to policy shocks in these two different information disclosure regimes. To do this a valid instrument is required and we focus on estimates coming from the high-frequency surprise instrument.[6] We find that when no accompanying Inflation Report is released, the effects of policy have the conventional signs: unemployment and inflation fall following a 100 basis point surprise tightening. However, when an accompanying Inflation Report is released, the responses have the opposite signs: unemployment and inflation *rise* following a surprise monetary tightening.

To validate that our findings are likely coming from a genuine response to the content of the Inflation Report and not something specific to the monthly time dummies corresponding to Inflation Report releases, we conduct two types of placebo tests. Firstly, we implement the same analysis on the U.S. which does not release an equivalent document to the Bank of England's Inflation Report at all (although they do release minutes 3 weeks after every policy announcement) and we find no systematic differences in outcomes when we compare policy surprises in the second month of every quarter in the US (the quarter the Inflation Report is released in the United Kingdom) to policy surprises in other months. Next, for the United Kingdom we replace the dummy for the release of the inflation report with two other seasonal dummies and repeat the analysis. One type of dummy variables is set to 1 for the 1^{st} half of the year and the second type is 1 for the 1^{st} month of every quarter instead of the second. In both cases we do not find systematic evidence of differences in the impulse responses suggesting that our methodology is picking up something related to the release of the Inflation Report and not unaccounted for seasonal factors.

We see three implications of these findings: Firstly, the information channel is very strong and can flip the sign of a surprise policy tightening. Secondly, central banks can reduce the impact of the information channel by limiting the amount of information and

[5]To be precise, the Inflation Report is released approximately 1 week after the initial press release but as we are using the monthly data and the press-release is usually in the first week of the second month of each quarter, the Inflation Report is released in the same month as the press release. For simplicity, we will refer to this as "accompanying" the press release.

[6]This is largely because the forecast-adjusted-surprise instrument delivers a low first stage. There is an argument that it is better identified than the high-frequency instrument since it makes sure that the policy surprises used in the instrument are not simply the central banks setting policy in a forward looking manner but in response to their private information.

analysis they provide to the public. Thirdly, the traditional wisdom that central banks should be transparent and accountable seems to not apply to the release of timely knowledge about future economic events. Explaining policy decisions in terms of expected economic trends can undermine the direct effect the policy change has on economic outcomes. The theoretical literature is clear that commitment to a policy rule achieves better outcomes by making policy predictable and credible. In practice however, central banks prefer to set policy under discretion using press statements and detailed discussion of economic trends as tools to achieve credibility. Our work suggests that this might come with some cost of making policy less effective (at least when new information is revealed to the public).

3.1.1 Related Literature

This chapter contributes the three strands of literature. First, there is the literature documenting the existence of the Information Effect. The seminal paper to note the information advantage of the Federal Reserve was Romer and Romer (2000) who document a performance advantage of FOMC forecasts relative to market contexts. Then subsequent work investigated whether the market was aware of this information advantage. This was originally in the context of understanding the effects of forward guidance - signalling publicly that low policy rates will remain into the medium term. Notable papers in this literature are Campbell et al. (2012) and Campbell et al. (2017) who discuss the possibility that the promise of low rates into the future be interpreted by markets as predicting future economic weakness ("Delphic" forward guidance). Nakamura and Steinsson (2018) apply the same principle of interest rate movements containing information to document that the same logic appears to apply to conventional policy surprises - market forecasts for future GDP *rise* following a policy tightening.[7] Subsequent work by Miranda-Agrippino (2016) and Miranda-Agrippino and Ricco (2018) have noted that if there is an Information Effect, it implies that the two popular types of monetary policy identification methodologies: high-frequency policy surprise estimates and the Romer and Romer (2004) method of removing the central bank's forecasts from policy choices are both biased. However they show that their interaction should not be biased. Jarociński and Karadi (2020) and Cieslak and Schrimpf (2019) also try to separate the "pure" policy shock form an information shock by using the response of stock markets to the policy surprise. If a surprise tightening of policy causes the stock market to rise, then it is classified as largely an information shock.

These last two papers are very similar to our approach except that we interpret the direction of the information given from the *central bank's* perspective. If they are raising rates, then we infer the Inflation Report contains information justifying higher interest rates - either a stronger economy or inflationary pressure. Using this method to classify

[7]This finding has been questioned by Bauer and Swanson (2020) who argue that both the Federal Reserve and markets are simply responding to public information.

information content, we show that the release of discussion and analysis about economic events is an important driver of the Information Effect for conventional policy movements. Moreover, we focus not on finding an instrument purged of private central bank information but on understanding the Information Effect induced by this information existing.

The second strand of literature we contribute to is that about the effect of central bank communications and speeches on markets. There is analysis on the longer term outcomes of having more transparent communication strategies (see (Blinder et al., 2008) for a nice summary). There is a literature on the effects of communications on policy outcomes in the short term - "open mouth operations" as dubbed by Guthrie and Wright (2000) - which appears to affect market prices of yeild curves and exchange rates. Nechio and Wilson (2016) find considerable market responses to the release of the Federal Reserve's FOMC minutes in a short window after the release. There are also several papers using machine learning methods to classify the content and tone in central bank minutes or speeches (such as Hansen and McMahon (2016), Apel and Blix (2014) and Lucca and Trebbi (2009)).

We contribute to this literature by showing that providing written analysis and economic projections to the public also appear to have sizeable impacts on economic outcomes. We do this using a procedure that avoids having to objectively classify information using narrative or textual analysis tools. Instead we infer the content of the information from the interest rate movement which allows us to avoid all the complications of having to classify the information content via the words used. Avoiding the need for narrative or textual classification is similar to Jarociński and Karadi (2020) and Cieslak and Schrimpf (2019) but a key difference is that they use the market reaction to infer the information content released by the central bank, whereas we use the central bank's own interest rate movement as a proxy for the information content. The advantage of this is that we do not need to make assumptions about the market interpretation of this information - merely that the central bank will release information supporting higher interest rates when choosing to raise interest rates.

The third strand of literature we contribute to is concerned with investigating optimal signalling. This can be in the context of forward guidance such as Negro, Giannoni, and Patterson (undated) or more generally about public signals such as Morris and Shin (2002) or Kamenica and Gentzkow (2011). These papers solve for whether it is optimal to share information publicly (and in the later case, which information). Tamura (2016) applies the general framework of Kamenica and Gentzkow (2011) to the context of a central bank and finds that in theory Central Bank signals can be a useful tool to the Central Bank's objectives and Melosi (2017) does so in a more standard macroeconomic framework. This chapter is able to validate these findings empirically using the Inflation Report. We find that the information in these reports can affect economic outcomes dramatically.

The remainder of this chapter is structured as follows: Section 3.2 outlines the econometric framework used in this chapter and the assumptions underlying it; Section 3.3 discusses the data used and the differences between the information content of a press-release of a policy change and an Inflation Report; Section 3.4 present the results and placebo tests; Section 3.5 discusses the implications of our findings and outlines future work that could shed more light on this issue and Section 3.6 concludes.

3.2 Econometric Framework

The purpose of this section is to outline an econometric framework with which to discuss the estimation procedure and issues of identification. Since we are interested in the effects of information dissemination it is important to specify the information sets of agents carefully and to specify the feedback of this information to economic outcomes.

In this section we introduce a general framework to discuss the econometric issues we face both in estimating the effects of information on the impulse responses to monetary policy and estimating the monetary policy impulse responses directly.

Let there be a vector of N economic variables $\underline{Y}_t$ which to some extent are driven by expectations, R policy variables $\underline{r}_t$ and economic shocks $\underline{u}_t$. A generic linear structural form that nests most macroeconomic models one might consider for analyzing these issues is a first-order Vector Autoregression, VAR(1) of the following form:

$$\underline{Y}_t = \mathbf{A}_\ell \underline{Y}_{t-1} + \mathbf{A}_f \mathbb{E}_t \left[\underline{Y}_{t+1} | \mathcal{I}_t^M \right] - \mathbf{\Gamma} \underline{r}_t + \underline{u}_t \tag{3.1}$$

where $\mathcal{I}_t^M$ is the information set of market participants up to and including time t, both $\mathbf{A}_\ell$ and $\mathbf{A}_f$ are N x N matrices of structural parameters and $\mathbf{\Gamma}$ is an N x R matrix of structural parameters. We use bolded notation $\mathbf{X}$ to denote matrices and underlined notation $\underline{X}$ to denote a vector.

We are going to be estimating impulse responses via Jordà (2005) local projections and show in Appendix C.1 that Equation 3.1 can be written in the following form:[8]

$$\underline{Y}_{t+k} = \underline{c}^k + \mathbf{\Phi}^k \underline{Y}_{t-1} - \underbrace{\sum_{s>0} \Gamma_s^k \mathbb{E}_t [\underline{r}_{t+s} | \mathcal{I}_t^M]}_{\text{Yield Curve}} + \underbrace{\sum_{j>0} \Theta_j^k \mathbb{E}_t \left[\underline{u}_{t+j} | \mathcal{I}_t^M \right]}_{\substack{\text{Future Economic} \\ \text{Shocks}}} + \underline{\tilde{u}}_{t+k} \tag{3.2}$$

$$\underline{\tilde{u}}_{t+k} \perp \underline{Y}_{t-1}, \mathbb{E}_t [\underline{r}_{t+s} | I_t^M] \mathbb{E}_t \left[\underline{u}_{t+j} | \mathcal{I}_t^M \right],$$

where $\underline{\tilde{u}}_{t+k}$ contains terms unrelated to time t variables (see Appendix C.1 for the functional form of $\underline{\tilde{u}}_{t+k}$). Note here that $\underline{\tilde{u}}_{t+k}$ includes information about future yield curves

[8]This representation imposes two restrictions: 1. The Data Generating Process is linear, 2. The system has a unique solution. Any model with these features can be written in the form above (perhaps with $\underline{Y}$ containing lags of some variables). See Appendix C.1 for more details.

(formed after t) and information about future beliefs about the economy ($\{\mathbb{E}_{t+s}u_{t+j}\}_{j>0}\}_{s=1}^{k}$). Henceforth, let's assume that there is one monetary policy tool: the short rate (i.e. R=1).[9]

We are interested in characterizing the impulse response of $\{\underline{Y}_{t+k}\}_{k>0}$ to an exogenous surprise movement in monetary policy. Note that we need the movement to be both exogenous (i.e. not a response to future inflationary pressure) and a surprise to markets (i.e not already captured in the yield curve). Using total differentiation we get the following impulse response expression:

$$\Delta_{r_t}\underline{Y}_{t+k} = \underbrace{-\sum_{s>0}\Gamma_s^k \frac{\partial \mathbb{E}_t^M r_{t+s}}{\partial r_t}}_{\substack{\text{Direct/ Yield curve effect}\\ \text{of monetary policy}}} + \underbrace{\sum_{j>1}\Theta_j^k \Delta_{r_t}\mathbb{E}_t^M \underline{u}_{t+j}}_{\substack{\text{Information Effect}\\ \text{of monetary policy}}} \;\forall k > 0, \tag{3.3}$$

where $\mathbb{E}_t^M$ denotes $\mathbb{E}_t\left[\cdot|\mathcal{I}_t^M\right]$.

Note that there are two terms: the first represents both the classic channel of monetary policy along with indirect effects by shifting the yield curve. This term excludes the possibility that markets update their information set about future economic shocks $\underline{u}_{t+j}$. The second term represents the "Information Channel" of monetary policy: changing rates might affect economic outcomes indirectly through changing agent's beliefs about future economic shocks $\underline{u}_{t+j}$.[10]

Note that if markets are extracting information about the economy from the central bank's interest rate choices, then market expectations will adjust even to interest rate changes made for exogenous reasons – the second term is a part of the correct impulse response.[11] This is because markets cannot tell with absolute precision exactly what part of the interest rate movement is a response to the economy and what part is exogenous. This is the essence of the argument made in Nakamura and Steinsson (2018). They argue that even exogenous policy movements might have net effects in the opposite direction to the direct effect due to a very strong information effect.

Next we let the interest rate rule and yield curves be determined as follows:

[9]Note that with the single policy instrument assumption, forward guidance is not ruled out, but QE (Quantitative Easing) is.

[10]Because the second effect is an information effect the derivative $\Delta_{r_t}\underline{u}_t = 0$.

[11]For instance consider the simplest signal extraction problem, that if a signal is normally distributed as $s_t = x_t + \epsilon_t$ and prior information is $I_t \sim N(x_t, \sigma_I)$ with $x_t \perp I_t$, then the optimal belief after observing x_t is to combine x_t and I_t as follows: $\frac{\frac{1}{\sigma_s}}{\frac{1}{\sigma_\epsilon}+\frac{1}{\sigma_I}}x_t + \frac{\frac{1}{\sigma_I}}{\frac{1}{\sigma_\epsilon}+\frac{1}{\sigma_I}}I_t$. In this formulation beliefs respond to the noise ϵ_t because there is no way to tell that the movement in s_t is due to x_t moving or ϵ_t moving. The same logic applies to pure monetary shocks: similar to movements in ϵ, the public will infer this policy movement as having information even though in this particular case it does not.

$$r_t = r^* + \rho r_{t-1} + \sum_{\ell>0} \Psi_\ell \mathbb{E}_t^{CB} \underline{u}_{t+l} + \epsilon_t,$$
$$\mathbb{E}_t^M r_{t+s} = \alpha_s + \beta_s r_t + e_{s,t}, \quad e_{s,t} \perp r_t, \tag{3.4}$$

where $\mathbb{E}_t^{CB} = \mathbb{E}_t \left[\cdot|\mathcal{I}_t^{CB}\right]$, CB denotes a central bank, and we write the yield curve in a reduced form manner without any assumptions about the expectations formations process. For example, if the process were rational, then

$$\mathbb{E}_t^M r_{t+s} = \underbrace{\sum_{j=1}^{s} \rho^{s-j} r^* + \rho^s r_t + \sum_{j=1}^{s} \rho^{s-j} \left(\sum_{l>0} \Psi_l \mathbb{E}_t^M \mathbb{E}_{t+j}^{CB} \underline{u}_{t+j+l} + E_t^M \epsilon_{t+j} \right)}_{\text{Expectations Hypothesis}} + \underbrace{\zeta_{t,t+s}}_{\substack{\text{Risk}\\ \text{Premium}}}$$
$$= \alpha_s + \beta_s r_t + e_{s,t}$$
$$\Rightarrow \beta_s \equiv \rho^s + \sum_{j=1}^{s} \rho^{s-j} \left(\sum_{l>0} \Psi_l \frac{Cov\left(r_t, \mathbb{E}_t^M E_{t+j}^{CB} \underline{u}_{t+j+l}\right)}{Var(r_t)} + \frac{Cov\left(r_t, \mathbb{E}_t^M \epsilon_{t+j}\right)}{Var(r_t)} \right) + \frac{Cov\left(r_t, \zeta_{t,t+s}\right)}{Var(r_t)}$$

in order to deliver the result that $e_{s,t} \perp r_t$. Note also that $e_{s,t}$ is only orthogonal to r_t – it might still be correlated with other economic variables $\underline{Y}$ or shocks $\underline{u}$.

With this framework it is possible to have a fairly comprehensive discussion of the various econometric issues around estimating impulse responses to monetary policy correctly. We outline more details in Appendix C.2 and prove formally in Appendix C.2.2 that OLS estimates of the following regression

$$\underline{Y}_{t+k} = \underline{c}^k + \underline{\Lambda}^k r_t + \mathbf{\Phi}^k \underline{Y}_{t-1} + \xi_{t+k} \tag{3.5}$$

recover the following:

$$\hat{\underline{\Lambda}}_{OLS}^k = \underbrace{\mathbf{\Gamma}^k \mathbf{B} - \mathbf{\Theta}^k \mathbf{Y}}_{\text{Correct Impulse Response}} \underbrace{- \tilde{\mathbf{\Theta}}^k \frac{\mathbb{E}_{t+\tau}^M \underline{\mathbf{u}}_{\mathbf{t}} (1_k \otimes \mathbb{E}_t^{CB} \underline{\mathbf{u}}_t)'}{\mathbb{E}\left[(r_t^\perp)^2\right]} (1_k \otimes \mathbf{\Psi})'}_{\text{Central Bank is Forward Looking}} + \underbrace{\mathbf{\Theta}^k \mathbf{Y} \frac{\mathbb{E}\left[\mathbb{E}_{t^-}^M [r_t]^\perp r_t^\perp\right]}{\mathbb{E}\left[(r_t^\perp)^2\right]}}_{\substack{\text{Market already priced}\\ \text{in policy movement}}} \tag{3.6}$$

where the matrices are defined formally in Appendix C.2.

When discussing a structural shock, we typically say it needs to be exogenous (unrelated to other shocks - past, present or future), and a surprise. The second term in the equation reflects the bias induced by OLS because the policy rate is itself responding to current and future economic shocks. The third term represents bias caused by markets being able to predict in advance policy rates – *i.e.* they may not be a surprise. We show in Appendix C.2 similar equations for the high frequency, Romer and Romer (2004) and hybrid instruments and discuss their identification assumptions.

In the following subsection, we introduce the notion of a signal to supplement the setting of interest rates, allow the signal structure to have multiple regimes, and then discuss the econometric problems around exploiting these regime changes to learn about the Information Effect.

3.2.1 Heterogeneous Information Regimes

3.2.1.1 Introducing Signalling as a Central Bank Tool

Thus far, the framework introduced ignores that the interest rate movement is not the only way in which markets can extract the central bank's information.

Many central banks for instance produce a series of detailed reports 4 times a year and host prolonged press conferences where participants can ask questions of the Governor and policy teams. In the US, after every meeting the minutes of the FOMC discussing the policy decision is released 3 weeks after each meeting. These reports or discussions themselves contain a lot of information and often forecasts of future economic variables that might be informative to markets.

Furthermore, information is often used as a justification for policy movements - so it is correlated with the policy decision and potentially with the policy surprises as well.

To introduce this into the framework let there be a set of signals $\underline{s}_t \in \underline{S}$ that the central bank may choose to share with the public. We assume signals are sent in conjunction with policy announcements.[12] Let $\emptyset \in \underline{S}$ denote no signal sent. Other than this there is no structure placed on the form of signals sent. Moreover, without loss of generality, we can say that the effect of receiving a signal s_t on $\mathbb{E}_t^M[\underline{u}_{t+j}]$ (relative to an expected empty signal and $\mathcal{I}_t^M = \emptyset$) is $\underline{\kappa}_{j,s_t,t}$.

Using this notation for the signals sent has several key advantages: Firstly, there is no need to put any structure on the form the signals take: instead they are defined according to their effects on market beliefs. This avoids the difficulties faced in the literature on monetary policy communication involving the need to classify the meaning of statements very carefully.[13]

It also avoids the issue noted in the Bayesian Persuasion literature (starting with Kamenica and Gentzkow (2011)) that the signal content is irrelevant to the optimal reaction. All that matters is the covariance of the particular signal with the outcomes the market cares about. For example, consider the case where a central bank states publicly there is a recession coming whenever it knows a boom is coming. Rational market participants

[12] This is not quite the case for many central banks. For instance the Bank of England releases its inflation report one week after announcing its interest rate decision and the Federal Reserve in the US releases its minutes 3 weeks later. Since most economic outcomes are not measured more frequently than monthly, these can be thought of as occurring at the same time.

[13] Notable examples are Hansen and McMahon (2016), Apel and Blix (2014) and Lucca and Trebbi (2009)

will know that the statement "recession" only comes prior to a boom and will increase the probability they place on there being an upcoming boom (and ignore the literal wording of the signal). In this framework, all the signal extraction complication is summarized by the values of $\underline{\kappa}_{j,s_t}$ without having to be spelled out further.

We make one assumption on the $\underline{\kappa}$ that (other than the regime rotations discussed in the next subsection) the effect for a given j and signal realization s_t is constant over time. *i.e.* $\underline{\kappa}_{j,s_t,t} = \underline{\kappa}_{j,s_t}\ \forall t, \quad \forall s_t$. This is equivalent to assuming that a) the central bank sticks to a similar communication strategy and b) the way markets interpret each signal (holding their information constant) does not change over time. Equation C.3 in Appendix C becomes:

$$\begin{aligned}
\underline{Y}_{t+k} = \underline{c}^k + \mathbf{\Phi}^k \underline{Y}_{t-1} - \sum_{s>0} \Gamma_s^k \mathbb{E}_t[\underline{r}_{t+s}|\mathcal{I}_t^M] \\
+ \sum_{j>0} \Theta_j^k \Big(\underbrace{\mathbb{E}_{t^-}\left[\underline{u}_{t+j}|\mathcal{I}_{t^-}^M\right]}_{\substack{\text{Market Beliefs} \\ \text{Before Policy Change}}} + \underbrace{\Upsilon_j(r_t - \mathbb{E}_{t^-}\left[r_t|\mathcal{I}_{t^-}^M\right])}_{\substack{\text{Belief Update} \\ \text{From Policy}}} + \underbrace{\underline{\kappa}_{j,s_t} - \mathbb{E}_{t^-}^M\left[\underline{\kappa}_{j,s_t}|\mathcal{I}_{t^-}^M, r_t\right]}_{\substack{\text{Belief Update} \\ \text{from Policy Statements}}} \Big) \\
+ \underline{\tilde{u}}_{t+k},
\end{aligned} \tag{3.7}$$

where all terms on the second line reflect different factors contributing to the overall market forecast of future economic shocks: $\mathbb{E}_t^M \underline{u}_{t+j}$.

Comparing Equation 3.7 to Equation C.3, there is an additional term comprising the components that form the market's expectations of future shocks: there is the market's prior beliefs of the shocks, then they update their beliefs based firstly on the interest rate surprise they receive and then secondly on surprise signals sent. Since we are modelling the signal and interest rate as being announced at the same time and both are likely correlated, there is a question as to how to attribute the market response to the pair (r_t, s_t) between the interest rate surprise and the signal surprise. We choose a modelling structure where the interest rate surprise is calculated relative to the expected interest rate given the markets information prior to the release. Next, the signal surprise is calculated relative to what markets would expect *knowing the interest rate but not having yet received the signal*. If market expectations were rational, this would mean that the policy rate surprise and signal surprise would be uncorrelated but in a general framework without this assumption, imperfect expectations formation could lead the surprises to still be correlated.[14]

Repeating the exercises of the previous subsections, we can calculate what we should expect to see running Equation 3.5 with OLS and/or the instruments discussed in Ap-

[14]Note that the signal surprise in Equation 3.7 is the surprise *in excess* the signal expected already knowing the interest rate. Rational forecasters would have accounted for all information from the policy surprise and therefore, any such signal surprise would not be related to the interest rate (or interest rate surprise). For our empirical procedure to find an Information Effect, we need the signal surprise in terms of market *reaction* to be correlated with the interest rate surprise. As such, we could fail to find an Information Effect even if one were present *if market forecasters were rational*.

pendix C.2. The following Equations below summarize this:

$$\underline{\hat{\Lambda}}^k_{OLS,signal} = \underline{\hat{\Lambda}}^k_{OLS} + \underbrace{\boldsymbol{\Theta}^k \frac{\mathbb{E}\left[\left(\underline{\kappa}_{j,s_t} - \mathbb{E}^M_{t^-,r_t}\left[\underline{\kappa}_{j,s_t}\right]\right) r_t^{\perp}\right]}{\mathbb{E}\left[(r_t^{\perp})^2\right]}}_{\text{Signal Surprise Correlated with Interest Rate}},$$

$$\underline{\hat{\Lambda}}^k_{HF,signal} = \underline{\hat{\Lambda}}^k_{HF} + \underbrace{\boldsymbol{\Theta}^k \frac{\mathbb{E}\left[\left(\underline{\kappa}_{j,s_t} - \mathbb{E}^M_{t^-,r_t}\left[\underline{\kappa}_{j,s_t}\right]\right)(r_{t^*} - \mathbb{E}^M_{t^*-1} r_{t^*})^{\perp}\right]}{\mathbb{E}\left[((r_t - \mathbb{E}^M_{t^*-1} r_{t^*})^{\perp})^2\right]}}_{\text{Signal Surprise Correlated with Interest Rate Surprise}},$$

$$\underline{\hat{\Lambda}}^k_{RR,signal} = \underline{\hat{\Lambda}}^k_{RR} + \underbrace{\boldsymbol{\Theta}^k \frac{\mathbb{E}\left[\left(\underline{\kappa}_{j,s_t} - \mathbb{E}^M_{t^-,r_t}\left[\underline{\kappa}_{j,s_t}\right]\right)(r_t - \mathbb{E}^{RR}_t r_t)^{\perp}\right]}{\mathbb{E}\left[((r_t - E^{RR}_t r_t)^{\perp})^2\right]}}_{\text{Signal Surprise Correlated with Exogenous Policy}},$$

$$\underline{\hat{\Lambda}}^k_{M,RR,signal} = \underline{\hat{\Lambda}}^k_{M,RR} + \underbrace{\boldsymbol{\Theta}^k \frac{\mathbb{E}\left[\left(\underline{\kappa}_{j,s_t} - \mathbb{E}^M_{t^-,r_t}\left[\underline{\kappa}_{j,s_t}\right]\right)(r_t - \mathbb{E}^{M,RR}_{t^*-1} r_t)^{\perp}\right]}{\mathbb{E}\left[((r_t - \mathbb{E}^{M,RR}_{t^*-1} r_t)^{\perp})^2\right]}}_{\text{Signal Surprise Correlated with Exogenous Market Surprises}}.$$

where "HF" denotes the high frequency instrument, "RR" denotes the Romer and Romer (2004) instrument and "M,RR" denotes the hybrid instrument proposed by Miranda-Agrippino (2016). $\underline{\hat{\Lambda}}^k_{OLS}$, $\underline{\hat{\Lambda}}^k_{HF}$, $\underline{\hat{\Lambda}}^k_{RR}$ and $\underline{\hat{\Lambda}}^k_{M,RR}$ are the expressions for these estimates in a framework without a separate policy signal s_t and come from Equations (3.6), (C.6), (C.7) and (C.8) respectively. Note that these terms are not unbiased unless the instrument used (or the interest rate in the case of OLS) is both a surprise to markets and devoid of information content. For the purposes of the discussion here it is important to note that OLS will in general be biased and the instrument with the least onerous identification assumptions is the hybrid instrument.[15]

However, the introduction of a signalling component to monetary policy also imposes two additional conditions for these instruments to still correctly identify the monetary policy impulse responses:

1. Market surprises to interest rate movements are not correlated with market surprises to the signal received.

2. Exogenous policy movements (as determined by the Romer and Romer (2004)) are not correlated with market surprises to the signals received

For the hybrid instrument, either one of these two assumptions holding will be sufficient.

[15] Unfortunately for our purposes the first stage on this instrument is very weak so for our baseline results we default to the high frequency instrument.

However, it is not clear that either of these assumptions will hold in reality. There is extensive evidence that market forecasts are far from rational suggesting that they might easily respond strongly to signals with the same information content as the interest rate movement.[16]

Moreover, the second condition might be unlikely to hold in reality either. For example, if the signal identifies the policy movement as exogenous, then we would expect the signal surprise to be negatively correlated with the interest rate surprise. While in practice central banks do not advertise clearly which policy movements are exogenous, one might imagine in such a situation that a less strong signal might be sent for a given interest rate movement. This would also induce a similar correlation between the signal surprise and the exogenous interest rate movement.

3.2.1.2 Measuring the effects of signals on the economy

In this chapter, we want to exploit differences in the informativeness of the signals sent in order to understand the effects communication has on economic outcomes. To see how this would work in the framework outlined here, consider that there are two signal regimes $\{\underline{S}_{Low}, \underline{S}_{High}\}$ where in $\underline{S}_{High}$ the signals sent by the central bank are more informative. This regime will map to the case where the Bank of England releases an Inflation Report. $\underline{S}_{Low}$ then corresponds to the case where the Bank of England does not release an Inflation Report. In our framework this is modelled as $Var_s(\kappa_{j,\underline{S}_{High}}) > Var_s(\kappa_{j,\underline{S}_{Low}})$.[17]

Note also that the reason that central banks send signals is to justify their policy stance. We therefore assume the following:

- A1: *Correlation of Signals and Interest Rates:* $\mathbb{E}[r_t \underline{\kappa}_{j,s_t}] \neq \underline{0}, \quad \forall j > 0, \forall s_t \in \underline{S}$.
- A2: *Positive Correlation of "Demand" Signals and Interest Rates:* Let $u_{i,t+j} \in \underline{u}_{i,t+j}$ – *i.e.* $u_{i,t+j}$ is one of the many structural economic shocks that might affect the economy. We call $u_{i,t+j}$ a "demand" shock if it has a positive correlation with both output and Inflation. Let $\kappa_{i,j,s_t} \in \underline{\kappa}_{j,s_t}$ be the revision to beliefs about a future demand shock $u_{i,t+j}$ coming from some signal s_t. We assume that $\mathbb{E}[r_t \kappa_{i,j,s_t}] > 0$.

The purpose of these assumptions is to put some structure on the likely information being released to the market without having to directly measure this information. A1

[16] Coibion and Gorodnichenko (2015) shows that forecast revisions are themselves correlated over time meaning that if forecasters corrected their forecasts upward in one period, they are likely to in the next. Given this, it is plausible that the same might take place when processing consecutive pieces of information.

[17] To see why compare a regime where the signals communicate the true state of the world $\underline{u}_{t+j}$ to a case where they reveal nothing about the true state of the world. In the former case, $\underline{\kappa}_{j,s_t,\underline{S}_{High}} = \underline{u}_{t+j}$ and in the later case $\underline{\kappa}_{j,s_t,\underline{S}_{Low}} = \underline{0}$. Clearly in the first case $Var_s(\underline{\kappa}_{j,\underline{S}_{High}}) = Var(\underline{u}_{t+j}) > \underline{0} = Var_s(\underline{\kappa}_{j,\underline{S}_{High}})$. This example shows the general point that a set of informative signals has a higher variance *between* signals than an uninformative set of signals. Similarly, informative signals have a *lower within signal* variance (because they are more informative). When signals are processed rationally, these two statements are equivalent.

and A2 allow us to infer from an interest rate rise, that the information released suggests positive demand and vice versa for interest rate cuts. These assumptions are not unreasonable: they would hold for instance if the central bank was both forward looking, set rates in response only to its inflation and output forecasts and used the information content and analysis in their Inflation Reports as a tool to justify their policy decision.

We are interested in understanding the information effect of monetary policy. As such we run the following regressions (based on Equation (3.5)):

$$\underline{Y}_{t+k} = \underline{c}^k + \underline{\Lambda}^k r_t + \underline{\beta}^k r_t \times \mathbb{1}_{\underline{S}_t = \underline{S}_{High}} + \phi^k \underline{Y}_{t-1} + \xi_{t+k}, \tag{3.8}$$

where $\underline{S}_t$ represents the set of signals that can be sent at time t and $\mathbb{1}_{\underline{S}_t = \underline{S}_{High}}$ is an indicator variable that is 1 during times when the central bank also releases an inflation report and 0 if the central bank only changes policy.

Essentially this is a continuous variable equivalent to a differences-in-differences estimation procedure. We are estimating the difference in effect of say a 100 basis point rise in monetary policy when accompanied with an inflation report relative to the case where no inflation report is released.

Naturally, this regression cannot be estimated with OLS because of the issues described above: central banks set policy in response to future shocks and this might bias the OLS estimates somewhat. Instead we estimate this with the High-Frequency surprise instrument. This changes the interpretation of $\hat{\beta}$ to be the effect of information in the typical inflation report on economic outcomes to a given monetary policy *surprise*.

This gives the following estimates:

$$\begin{aligned}
\underline{\hat{\Lambda}}^k &= \underline{\hat{\Lambda}}^k_{HF} + \boldsymbol{\Theta}^k \frac{\mathbb{E}\left[\left\{\left(\underline{\kappa}_{j,s_t} - \mathbb{E}^M_{t^-,r_t}\left[\underline{\kappa}_{j,s_t}\right]\right)(r_{t^*} - \mathbb{E}^M_{t^*-1} r_{t^*})^\perp\right\} | s_t \in \underline{S}_{Low}\right]}{\mathbb{E}\left[((r_t - \mathbb{E}^M_{t^*-1} r_{t^*})^\perp)^2 | s_t \in \underline{S}_{Low}\right]} \\
\underline{\hat{\beta}}^k &= \boldsymbol{\Theta}^k \left(\frac{\mathbb{E}\left[\left\{\left(\underline{\kappa}_{j,s_t} - \mathbb{E}^M_{t^-,r_t}\left[\underline{\kappa}_{j,s_t}\right]\right)(r_{t^*} - \mathbb{E}^M_{t^*-1} r_{t^*})^\perp\right\} | s_t \in \underline{S}_{High}\right]}{\mathbb{E}\left[((r_t - E^M_{t^*-1} r_{t^*})^\perp)^2 | s_t \in \underline{S}_{High}\right]} \right. \\
&\qquad \left. - \frac{\mathbb{E}\left[\left\{\left(\underline{\kappa}_{j,s_t} - \mathbb{E}^M_{t^-,r_t}\left[\underline{\kappa}_{j,s_t}\right]\right)(r_{t^*} - \mathbb{E}^M_{t^*-1} r_{t^*})^\perp\right\} | s_t \in \underline{S}_{Low}\right]}{\mathbb{E}\left[((r_t - E^M_{t^*-1} r_{t^*})^\perp)^2 | s_t \in \underline{S}_{Low}\right]} \right)
\end{aligned} \tag{3.9}$$

As can be seen, the estimate of $\underline{\hat{\Lambda}}^k$ is essentially the same as in the general High-Frequency identification case with the addition of a term related to the release of accompanying signals from the central bank. Because of the interaction term, these parameters are now conditional on the low signal content regime. All requirements for identification of this estimate are the same as in the previously discussed case.

For $\hat{\beta}^k$, none of these identification assumptions are needed: we get a consistent estimate of the average effect of the difference between impulse responses in the high information regime relative to the low. The covariance terms in the numerator

$$\mathbb{E}\left[\left\{\left(\underline{\kappa}_{j,s_t} - \mathbb{E}^M_{t^-,r_t}\left[\underline{\kappa}_{j,s_t}\right]\right)(r_{t^*} - \mathbb{E}^M_{t^*-1} r_{t^*})^\perp\right\}\right]$$

is all variation based on Assumptions 1 and 2 that helps us measure the Information Effect and $\mathbf{\Theta}^k$ is matrix of structural parameters governing how markets react to their beliefs. The intuition behind β being identified with fewer assumptions needed is that any bias from the instrument used affects both information regimes equally and therefore cancels out. This comes from two implicit assumptions we made when specifying this framework:

1. The Central Bank uses the same policy rule to set interest rates regardless of whether it releases an Inflation Report or not.
2. The structural equations governing economic behavior are the same regardless of whether an Inflation Report is released or not.

The first assumption is consistent with most models we would write with a Central Bank – the policy rule has fixed parameters, but in practice this requires no strategic policy choices when an Inflation Report needs to be released. One could imagine that a Central Bank avoiding a controversial policy rise when an Inflation Report is needed because they need to justify the choice (or the opposite could happen instead). The first assumption requires no such behaviour in practice. Unfortunately, this assumption is difficult to test.

The second assumption is a little more simpler to test. Essentially we are using the union of 4 monthly dummy's to identify the effects of information. This assumption requires that there is no seasonal factors that might make monetary policy work differently in the months when Inflation Reports are released relative to those where it isn't. All the placebo tests discussed in Section 3.4.2 are testing this second assumption.

One way to test both these assumptions together is to note that estimates of $\hat{\beta}$ should be the same regardless of any instrument used if any. As such in Section **??** we present estimates from a variety of instruments (as well as OLS) to show that we estimate similar differences in economic responses between the two information regimes for a variety of estimation procedures.

Equation 3.9 also gives us insights as to what might make the estimates of the information effect zero:

1. $S_{low} = S_{high}$ – there is no actual difference in signals sent in each regime. This is unlikely given that the inflation reports details considerably more information than the press statements.
2. $\mathbf{\Theta}^k = \underline{\mathbf{0}}$ – markets are not forward looking when making decisions. This is also unlikely.
3. $\mathbb{E}_{t^-,r_t}[\underline{\kappa}_{j,s_t^*}] = \underline{\kappa}_{j,s_t^*}$ in both regimes. This could be due to two reasons: firstly, markets may not view either the information in the press statements or in the inflation reports as relevant beyond what they infer from the literal policy change (whether true or false). Secondly, markets might be able to determine the true value of $\underline{\kappa}_{j,s_t}$

from only the interest rate and other variables in it's information set. In either case we would be able to write $\underline{\kappa}_{j,s_t} = f(r_t)$ with only interest rates affecting the values of each $\underline{\kappa}$.

4. Market forecasts are rational which would mean that: $(\underline{\kappa}_{j,s_t^*} - \mathbb{E}_{t^-,r_t^*}\underline{\kappa}_{j,s_t^*}) \perp (r_{t^*} - \mathbb{E}^M_{t^*-} r_{t^*})$ for both regimes.[18] In this case market participants would react to information released in both regimes, but there would be no correlation of this reaction to their surprise about the change in the interest rate. Since we are using the interest rate surprise as a way to infer the surprise to markets (and thus avoiding the need to classify the information content in each report manually), we need markets to respond to this information in addition to the policy rates to detect an effect.

Given these factors, we can formally state the null hypothesis of $\underline{\beta}^k = 0$ in the following terms:

$\mathbb{H}$0: Markets are not forward looking <u>or</u> Central Bank statements are ignored by markets beyond interest rate movements <u>or</u> Belief updates to signal surprises are uncorrelated with interest rate surprises.

$\mathbb{H}$A: Markets are forward looking <u>and</u> Central Bank statements affect market beliefs <u>and</u> these belief updates to signal surprises are correlated with interest rate surprises.

Rejecting the null hypothesis implies many interesting things however, relative to previous work, we see the main value added of rejecting the null with our procedure as being able to quantify the size of the response of markets to Central Bank information releases.

3.3 Data

In this section, we focus our attention on the United Kingdom for two reasons: it releases an inflation report 4 times a year and another 8 times a year simply changes policy without a report. Secondly, the high-frequency shock series we wish to use is already constructed and available.

One additional advantage for starting the analysis with the UK and not with other countries is that the UK's inflation report does not include any discussion about the actual policy setting but conditions all forecasts on the market interest rate. Other central banks condition their forecast on their own policy forecasts and therefore communicate their future policy intentions via their equivalents to the Inflation Report. However, the Bank of England's inflation report is not explicitly conditioned on any internal interest rate forecast and therefore likely represents a purer release of information about only future economic events.

[18] Recall that the definition of κ is the response to the signals sent by the Central Bank *over and above the response to the interest rate announcement*.

We show the effects of information on monthly measures of policy rates, industrial production, the unemployment rate, and CPI inflation.[19] When estimating the effects, other than the interest rate, the employment rate and the unemployment rate, we take logs of all outcome variables. All data was obtained from either the Bank of England of the UK Office of National Statistics unless otherwise stated. We also obtain the high frequency policy surprises $r_{t^*} - \mathbb{E}^{M}_{t^*-} r_t$ for the UK from Miranda-Agrippino (2016) along with her hybrid shock measure $r_{t^*} - \mathbb{E}^{M,RR}_{t^*-} r_t$ for the UK. We obtain Romer and Romer (2004)-type monetary policy shock measure from Cloyne and Hürtgen (2016).

The estimation sample for all horizons runs from January 1998 to March 2015 for the baseline specification with high-frequency-identified shock series. Because the Bank of England began publishing forecasts on a consistent basis from February 1998, and the Monetary Policy Committee (MPC) reached fully operational size at this point, the sample starts from 1998. The sample ends in March 2015 due to data limitations on high-frequency-identified shock series.[20] The sample used when using Romer and Romer (2004)-type monetary policy shock is from January 1998 to December 2007, and the sample used when using the narrative adjusted shock series (hybrid shock series) is from January 2001 to December 2014 due to data limitations of the shock series. Because the longest sample is available with the high-frequency-identified monetary policy shocks, our baseline results are with this shock series.[21]

As our measure of increased information being provided to the public we use a dummy variable indicating the release of an inflation report in that month to the public. Inflation Reports are released the second month of every quarter and provide extensive analysis on policy issues and are often considerably more detailed than the press statements issued typically for a policy announcement.[22]

As an example, we consider January and February of 2009 where in January policy was lowered 50 basis points and accompanied with a press statement and in February were policy was dropped an additional 50 basis points but supplemented with the release of an Inflation Report. Appendix C.5 shows the whole press release which fits onto a single page. Consider just the discussion about inflation in the January 2009 press release:

> *CPI inflation fell to 4.1% in November. Inflation is expected to fall further, reflecting waning contributions from retail energy and food prices and the direct impact of the*

[19]In Appendix C.4 we also show the effects on the Employment Rate, two other measures of inflation – the CPIH and Retail Price Index measures as well as the stock market price, FTSE-100 index. The results are consistent with to the baseline results shown.

[20]This is also because from June 2015, the Bank of England started to publish the Inflation Report *at the same time* as the policy rate changes and minutes.

[21]Note that because we have the other data from well before January 1998 and up until Marcy 2020, the lags and leads taken to estimate Equation (3.8) do not lower the number of observations in the sample used for estimation.

[22]Note that because the Inflation Reports are released on such a regular schedule there might be seasonal patterns in the data we inadvertently pick up with our estimation procedure. We therefore seasonally adjust all data before using it to ensure such seasonality is removed where present.

> *temporary reduction in Value Added Tax. Measures of inflation expectations have come down. And pay growth remains subdued. But the depreciation in sterling will boost the cost of imports.*
>
> [...]
>
> *Nevertheless, the Committee judged that, looking through the volatility in inflation associated with the movements in Value Added Tax, there remained a significant risk of undershooting the 2% CPI inflation target in the medium term at the existing level of Bank Rate.*

By contrast consider this quote from beginning of the "Prospects for Inflation" section of Bank of England (2019) - the February 2009 Inflation Report (page 38)[23]:

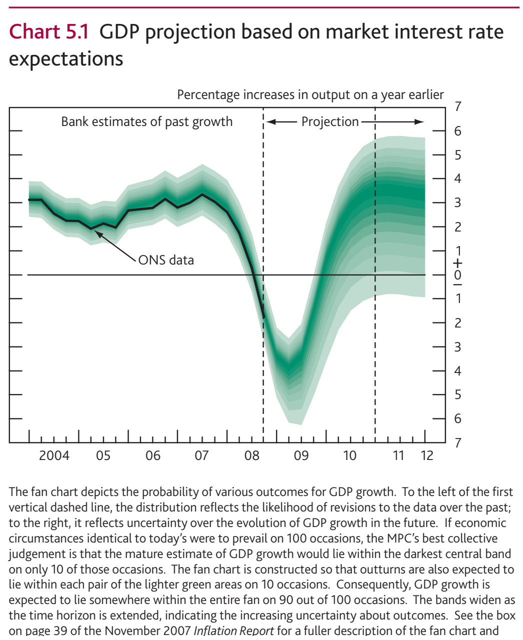
> *On the assumption that Bank Rate follows a path implied by market yields, the central projection is for GDP to contract sharply in the near term, and by more than assumed in the November Report. Further out, growth recovers, reflecting the substantial degree of stimulus from the easing in monetary and fiscal policy, the depreciation in sterling, past falls in commodity prices and actions by authorities at home and abroad to improve the availability of credit. CPI inflation falls well below the 2% target in the medium term, as the drag from the substantial margin of spare capacity more than outweighs the waning impact on import and consumer prices from the lower level of sterling. But the near-term path is uneven, reflecting sharp falls in energy prices, and the temporary reduction in VAT. The risks to growth are weighted heavily to the downside, reflecting in particular uncertainties over the pace at which the availability of credit improves and confidence returns. That also poses downside risks to inflation. But those risks are judged to be broadly matched by upside risks from the substantially lower level of sterling, leaving the overall balance of risks to inflation only modestly to the downside.*

Already there is considerably more detail: inflation projections are rationalized in terms of the various economic pressures and then these factors are weighted together. Furthermore, as Figure 3.1 shows, on the same page there is a chart with inflation forecasts. Here readers can see that the Bank of England is predicting near term falls in inflation of 4% and then by the end of 2010 to have recovered into positive territory. While implied by the January press statement, the exact magnitudes of these dynamics was not clear.

Note that all of this is from *only one page* of the Inflation Report. There is additional discussion of overall cost pressures as well as an in depth discussion of the recent value-added tax (VAT) change and its effect on short run inflation. Clearly the information in the Inflation Report is more substantial than in the press release.

[23] See Figure C.16 in Appendix C.5 for the full page.

Figure 3.1: Inflation Forecast Figure from the February 2009 Bank of England Inflation Report



Note: This Figure was taken from Page 38 of the February 2009 Inflation Report produced by the Bank of England. *Sources*: `https://www.bankofengland.co.uk/-/media/boe/files/inflation-report/2009/february-2009.pdf?la=en&hash=D8B10B7E69D515890540C18F0E095D69E2B67909`

3.4 Results

To produce the baseline results for the information effect, we run the following regression based on Equation (3.5) above:

$$Y_{i,t+k} - Y_{i,t-1} = \alpha_i^k + \lambda_i^k r_t + \beta_i^k r_t \times \mathbb{1}_{\substack{\text{Inflation Report} \\ \text{Released at } t}} + \sum_{l=1}^{12} \left(a_{i,l}^k r_{t-l} + b_{i,l}^k r_{t-l} \times \mathbb{1}_{\substack{\text{Inflation Report} \\ \text{Released at } t-l}} + c_{i,l}^k \Delta y_{i,t-l} \right) + \xi_{i,t+k}, \tag{3.10}$$

where $Y_{i,t}$ represents a single outcome variable of interest (such as Index of Production, CPI and so on) and $\mathbb{1}_x$ is an indicator variable that is one when x is true and zero otherwise. Note that in this formulation we control for 12 lags of the outcome variable Y_i, the policy interest rate r_t as well as lags of the interaction of the policy interest rate with the Inflation Report time dummy.[24]

[24]The results are consistent with varying lags for all controls, from 2 to 24.

Given our differences-in-differences setting, we are interested in the difference in impulse responses of Y_i to r_t depending on whether an inflation report is released or not. As noted by Jordà (2005), the collection of $\{\hat{\beta}^k\}_{k>0}^K$ is a consistent estimate of the difference between the Inflation Report impulse response of Y_i to r_t and the non-Inflation Report impulse response, that is the information effect, under identification assumptions (i.e. Relevance and Exclusion).

To identify the $\{\hat{\lambda}^k\}_{k>0}^K$ collection however, we would then need to discuss the appropriate instrument to use as was discussed in Section 3.2.1.2.

3.4.1 Main Results

Figure 3.2 shows the main results of the estimation along with 68% and 90% confidence bands. Each panel is the collection $\{\hat{\beta}^k\}_{k>0}^K$ from regressions estimated using high frequency policy surprises.

The first panel shows the differential effect of the policy interest rate when an inflation report is released relative to when it is not. A positive coefficient after *k* months indicates that after a 100 basis point policy change, policy rates after an Inflation Report is released tend to be higher than they would be *k* periods after a similar tightening without an accompanying Inflation Report. For interest rates there is some evidence that even though the difference must be zero on impact, afterwards there is evidence that when releasing an Inflation Report policy rates tend to be much higher afterwards. Next we will argue that this is due to the Information Effect creating a rise in demand.

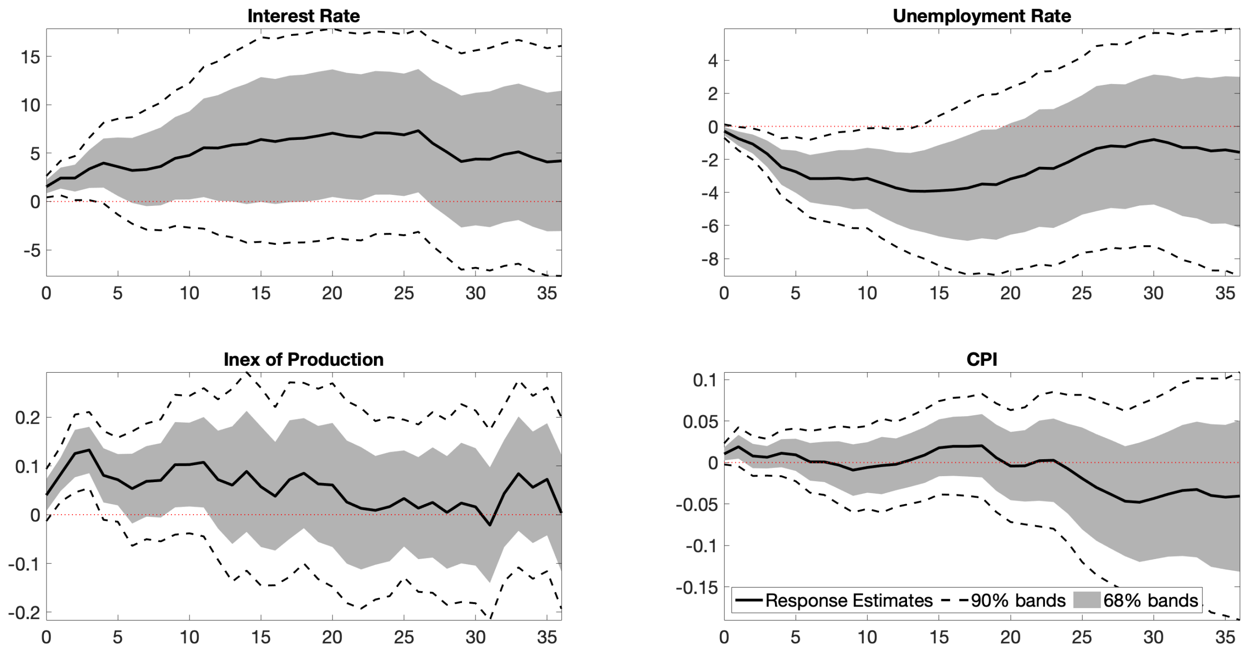
The second panel shows the information effect on the dynamics of unemployment. Here is it clear that unemployment is approximately 4 percentage points lower when policy is accompanied by an Inflation Report relative to a policy shock without it. Similarly, the third panel shows that the index of production is higher when policy is accompanied by an Inflation Report releases relative to that without it. Finally, last panel shows that cumulative inflation is slightly higher in the first three months. Taken together it seems that policy shocks when accompanied with an Inflation Report tend to produce higher demand relative to the absence of Policy Shocks. Consistent with this policy rates end up being much higher after a policy shock done with an Inflation Report in order to keep the additional demand under control.

Note also that the point estimates for unemployment is somewhat large. Figure 3.3 helps to explain how this might be by showing that in many cases the responses have the opposite signs. This Figure shows two separate impulse responses for the *level* effects of the outcome variables to the exogenous policy shocks, along with those from usual linear local projections.

The red lines in the right-end columns show the impulse responses to a 100 basis point monetary policy tightening when *no* Inflation Report is released along with pink shaded area (two red dashed lines) as 68% (90%) confidence bands.[25] Here the signs of policy look

[25] It might appear on first glance that the confidence intervals in Figure 3.2 are not consistent with those

Figure 3.2: Baseline Results for the Information Effect



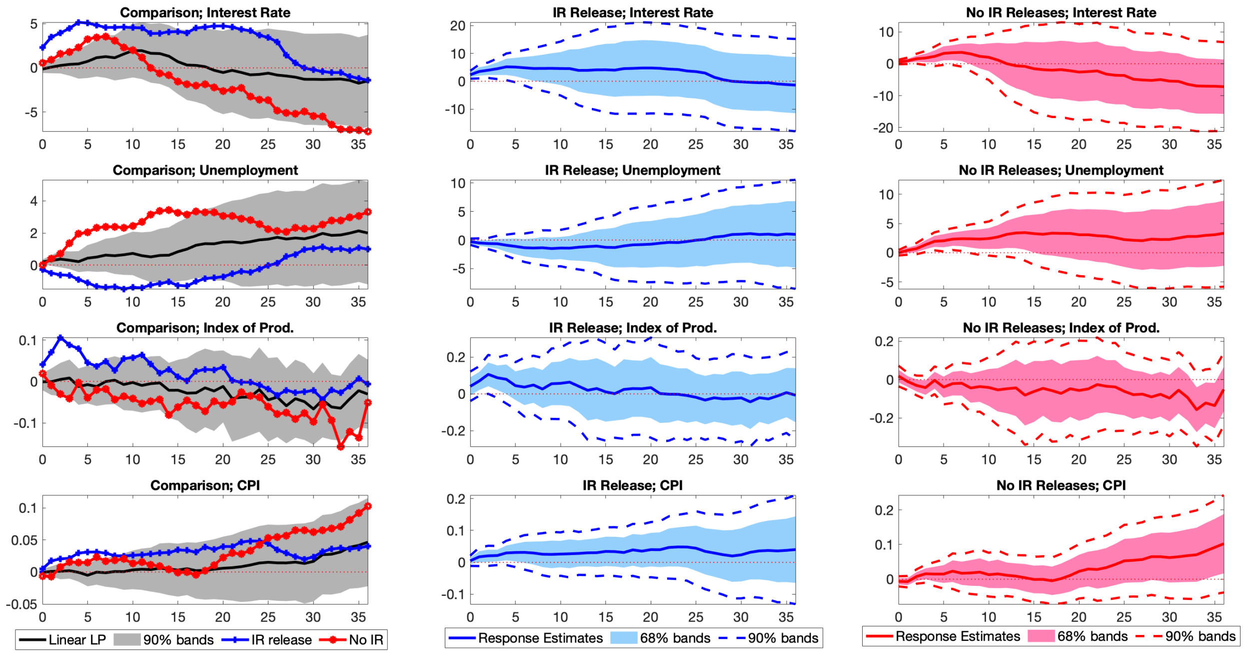
Notes: This figure shows the estimates, and 68% and 90% confidence bands from estimating Jordà (2005) Local Projections of the labelled outcome variable on high frequency monetary policy shock series and the shock series interacted with a time dummy for whether an Inflation Report was released. These graphs all represent the *difference* between outcomes when an inflation report is released relative to when it isn't. Positive estimates indicate that releasing an inflation report makes the equilibrium response of this variable higher. The estimates are denoted as black solid line and 68% (90%) confidence bands are represented as grey shaded area (two black dotted lines).

conventional: the unemployment rate rises and industrial production decreases. There is very limited evidence that the price level falls when estimating with the High Frequency instrument but as with the difference estimates, these estimates are noisy.

The blue lines show the effects of the same 100 basis point policy shock when Inflation Reports are released with sky blue shaded area (two blue dashed lines) as 68% (90%) confidence bands. Here the policy tightening looks *expansionary*. Since the key difference between the events used is the release of the Inflation Report, this suggests that the Information Effect coming from policy changes is tied to the explicit information referenced

in Figure 3.3. However, as is standard in the difference-in-difference regressions, the standard errors on the level and interaction effects in Equation 3.10 (λ^k and β^k respectively) are negatively correlated. This means that the standard error of $\lambda^k + \beta^k$ - the level of the impulse response to monetary policy shocks when Inflation Reports are being released - is not simply the sum of the two standard errors. This is why there are statistically significant differences while it appears the confidence intervals overlap in levels. Note also that the Confidence intervals in Figure 3.2 come directly from the regressions run and so any errors in calculating standard errors would apply to the blue line in Figure3.3.

Figure 3.3: Estimates for Inflation Report and Non-Inflation Report Policy Changes



Notes: This figure shows the different effects of outcomes when there is no inflation report released (the red lines) and when there is an inflation report released (the blue lines). The left-end columns show all the estimates of the effects of outcome. The black lines show the linear effect without considering the effect of inflation report releases along with grey area as 90% confidence bands. The blue line with plus signs (the red line with circles) show the effects when inflation report is released (not released). The middle column shows the estimates of the effects of outcome when inflation report is released with blue solid line along with 68% confidence bands denoted as sky blue shaded area and with 90 % bands with blue dashed lines. The right-end column shows the estimates of the effects of outcome when no inflation report is released with red solid line along with 68% confidence bands denoted as pink shaded area and with 90 % bands with red dashed lines.

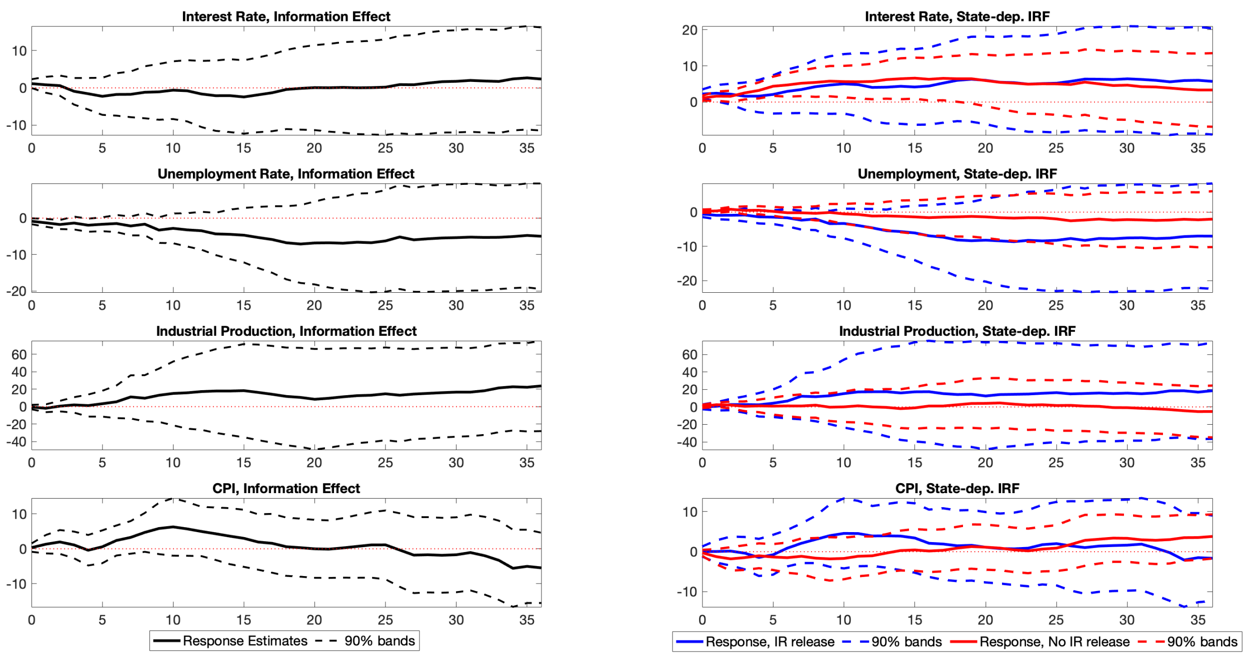
in the Inflation Report. While there might still be some signal extraction coming policy rates directly, the bulk of the Information Effect noted by Nakamura and Steinsson (2018) seems to be linked to the Inflation Report.

We will leave to Section 3.5, the discussion of the implications of these findings and will spend the rest of this section discussing some interesting placebo tests that can be run to make the case that the estimates here are due to the release of the Inflation Report and not some other coincidental factors. We also provide some robustness checks. We show that the results are similar if we use some other types of monetary policy shocks. We also show in Appendix C.4.1 that the results are essentially the same with various methods of incorporating of global financial crisis periods.

3.4.2 Placebo Tests

We have found some very suggestive results that the Information Effect is predominantly driven by the release of explicit discussion about future economic events more than interest rate movements per say, but the identification essentially relies on timing: the months of the year when an Inflation Report is released should (after seasonal adjustments) be similar to the months of the year where there is no inflation report released. In this Subsection we present two placebo tests to make the case that this is likely to be true.

Figure 3.4: Placebo Test: Is there an "Information Effect" for the U.S.?



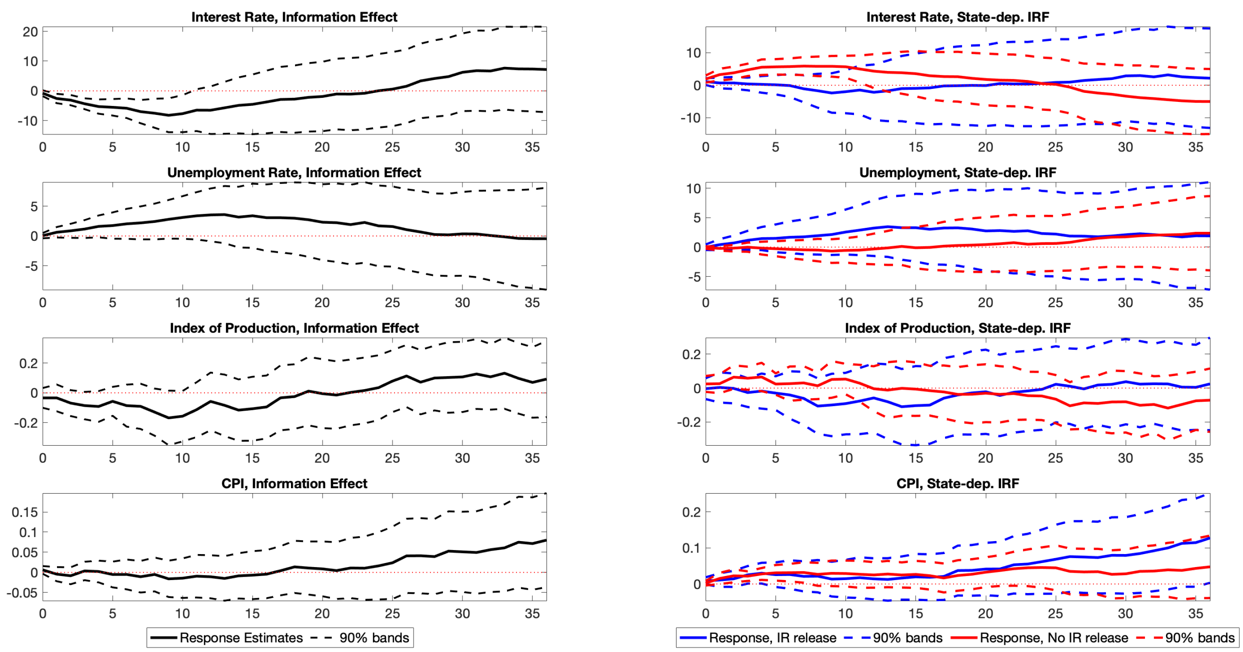
Notes: This figure shows the placebo test results for the U.S. Results are shown in two different ways for each variable. The panels with black lines are showing the coefficient estimates of the difference between the impulse responses of month 2 of each quarter (the month corresponding to an Inflation Report release by the Bank of England) and the other two months of that quarter with the two black lines for 90% confidence bands. Positive numbers mean that the responses from policy announcements in month 2 of each quarter exceed those from announcements in other months in each quarter. The second plot for each variable compares the level of the responses of month 2 surprise announcements to surprise announcements in other months. The blue solid lines are the impulse responses in the second months in each quarter with the two blue dashed lines for 90% confidence bands, and the red solid lines are the response coefficients in every other months in each quarter with the two red dashed lines for 90% confidence bands. All results are estimated using the high frequency surprise instrument from Kuttner (2001) and was obtained from Gertler and Karadi (2015).

The first placebo test is to run the same analysis for a country whose central bank does not release an Inflation Report. We choose to do this for the U.S.. If we run the same analysis for a central bank where nothing special happens in their second announcement in a quarter relative to the first, then this suggests something other than the Inflation Report was driving the results shown above.

The second placebo test involves re-running the analysis for arbitrary splits of the year unrelated to the timing of Inflation Report releases. We show three here: splitting the year in half and comparing the first half to the second half, looking at a random month of the quarter and another month (for example, every third months) of the quarter, and compare those with every other months in each quarter. If we find similar findings in these tests to those we found for the comparison above, then this suggests that the patterns documented are unrelated to the Inflation Report *per se* and rather due to something else.

The U.S. placebo analysis uses data obtained from Gertler and Karadi (2015) with the exception of the unemployment rate which comes from the Federal Reserve Economic Database (FRED). The sample the analysis is run on is from January 1990 to June 2012.

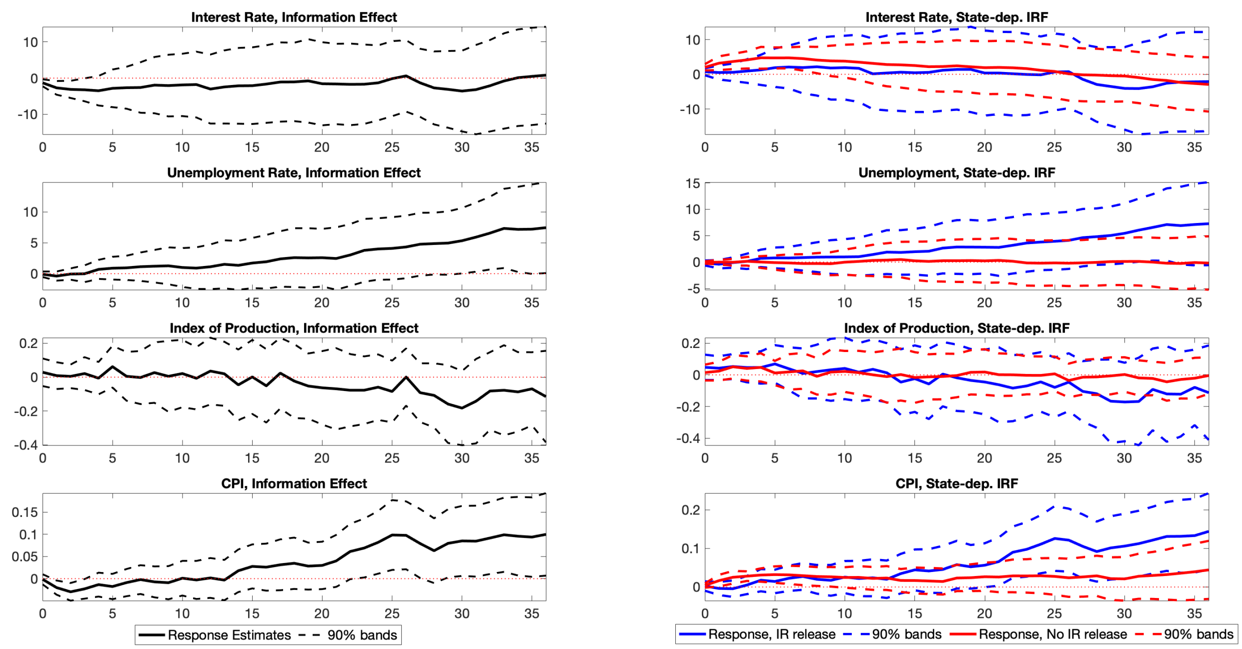
Figure 3.5: Placebo Test: Is the first half of the year different to the second?



Notes: The panels with black lines are showing the coefficient estimates of the difference between the impulse responses of the first six months in each year and the other six months in that year with the two black lines for 90% confidence bands. Positive numbers mean that the responses from policy announcements in the first half of each year exceed those from announcements in other months in the second half of each year. The second plot for each variable compares the level of the responses of the surprise announcements in the first six months to surprise announcements in other months. The blue solid lines are the impulse responses in the first half of each year with the two blue dashed lines for 90% confidence bands, and the red solid lines are the response coefficients in the second half of each year with the two red dashed lines for 90% confidence bands.

Figure 3.4 shows the results for the US using the same high frequency monetary policy surprise instrument as used for the left columns of Figures 3.2 and 3.3. Each variable has two panels showing the responses. The panels with black lines mirror those in Figure 3.2 and show the differences between surprise announcements in the second month of each quarter relative to surprise announcements in every other month in that quarter. The second panel for each variable mirrors those in Figure 3.3 and shows the effects of both surprises separately.

Figure 3.6: Placebo Test: What If We Choose Random Months in Each Quarter?

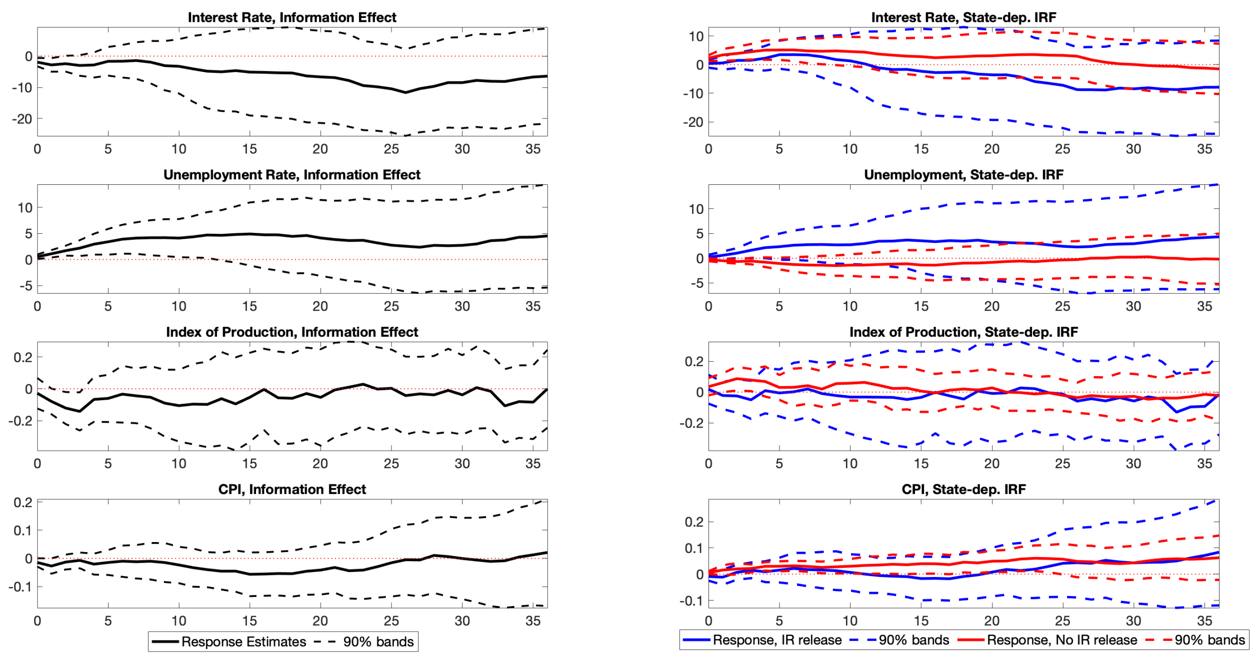


Notes: The panels with black lines are showing the coefficient estimates of the difference between the impulse responses of random months of each quarter and the other two months of that quarter with the two black lines for 90% confidence bands. Positive numbers mean that the responses from policy announcements in random months of each quarter exceed those from announcements in other months in each quarter. The second plot for each variable compares the level of the responses of surprise announcements in random months to surprise announcements in other months. The blue solid lines are the impulse responses in randomly picked months in each quarter with the two blue dashed lines for 90% confidence bands, and the red solid lines are the response coefficients in every other months in each quarter with the two red dashed lines for 90% confidence bands.

As can be seen the differences are essentially never statistically significant from zero. Furthermore, the qualitative behaviour of the levels of each series are very similar in all cases unlike the results for the United Kingdom. If the information effect for the UK was being driven by other seasonal factors, then it seems reasonable to conjecture that the US would experience the same seasonal factors and yet we don't detect the same effects for the US. Overall, this suggests that the information effect isn't present for the US despite using the same timing.

Next we consider a placebo test where we conduct the same differences-in-differences analysis for the United Kingdom but where the timing indicator is set arbitrarily rather than indicating a release of inflation. The first arbitrary timing we consider it to set the indicator dummy to one in the first half of each year and zero otherwise. The estimates from this are shown in Figure 3.5. As Figure 3.5 shows, the estimates are rarely significant. The one exception is the interest rate where it tends to be smaller in the first half. This is in the opposite direction of our main result in Figure 3.2. Moreover, we see from the right columns that the response estimates between the first and the second halves are qualitatively very similar.

Figure 3.7: Placebo Test: Is the Third Month Different From Other Months?



Notes: The panels with black lines are showing the coefficient estimates of the difference between the impulse responses of the third months of each quarter and the other two months of that quarter with the two black lines for 90% confidence bands. Positive numbers mean that the responses from policy announcements in the third months of each quarter exceed those from announcements in other months in each quarter. The second plot for each variable compares the level of the responses of surprise announcements in random months to surprise announcements in other months. The blue solid lines are the impulse responses in third months in each quarter with the two blue dashed lines for 90% confidence bands, and the red solid lines are the response coefficients in every other months in each quarter with the two red dashed lines for 90% confidence bands.

The second arbitrary timing placebo test involves setting the dummy to one in the randomly picked one month of every quarter rather than the second. These results are shown in Figure 3.6. Finally, the third arbitrary timing placebo test is setting the dummy to one in the third months of each quarter, rather than the second. The results are shown in Figure 3.7. Note that this placebo test is a more difficult one to pass because if the information effect has been correctly identified, then the effects of month three of each quarter relative to months one and two will be systematically different even if months one and three have the same mean effect. Nonetheless the estimates for the information effects are rarely significant.

These placebo tests can confirm that the U.S. does not have similar seasonal differences in policy responses to the U.K. and other combinations of seasonal dummies in the U.K. do not seem to lead to similar differences in economic outcomes. Together these point to the most likely explanation for the differences observed in the baseline estimates are that the Inflation Report generates a large Information Effect.

3.4.3 Using Other Policy Measures

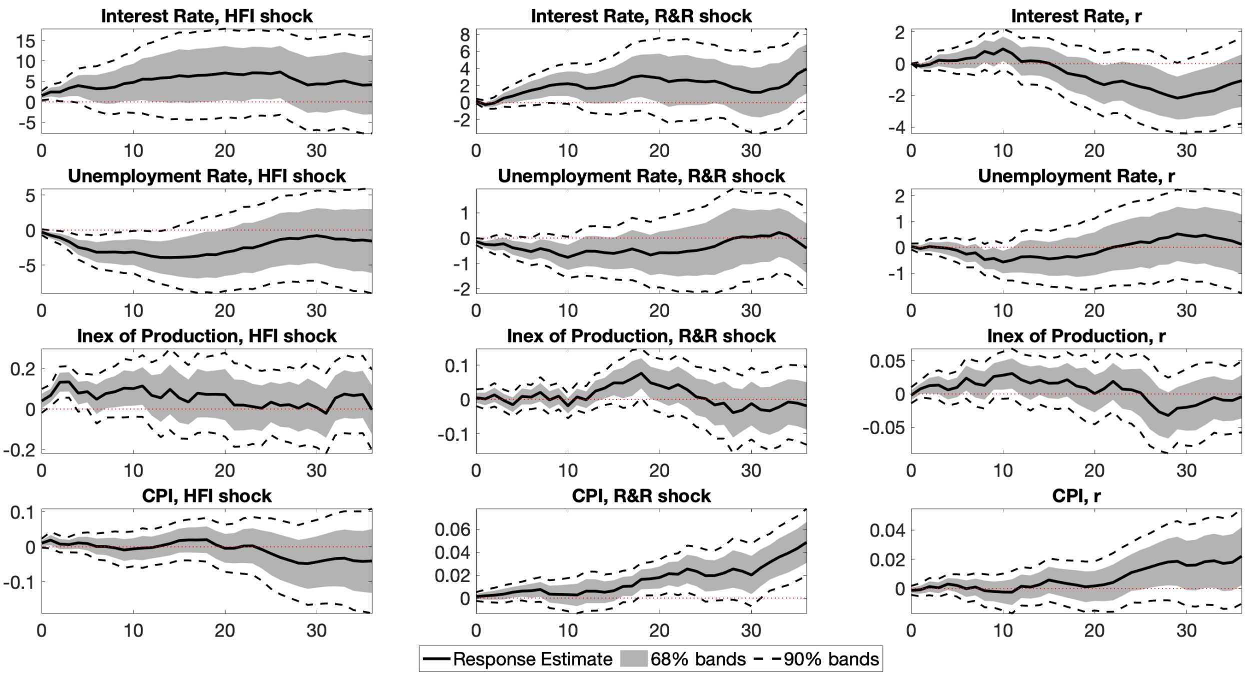
This section presents the results with other types of monetary policy shock series for robustness checks. First, we examine if the information effect survives with Romer and Romer (2004)-type monetary policy shocks and with policy rates without using any instruments. Second, we examine if the responses of interest rates, unemployment rates, index of production, and consumer price index differ when inflation report is released compared to the other months when using Romer and Romer (2004)-type monetary policy shocks and using hybrid forecast-adjusted-surprise instrument. We obtain Romer and Romer (2004)-type monetary policy shocks from Cloyne and Hürtgen (2016) and hybrid shocks from Miranda-Agrippino (2016).

First, Figure 3.8 shows the information effect with different measures of monetary policy. The left-end columns replicate the results in Figure 3.2 for comparison. The estimates here are generated using high-frequency-identified shock series. The middle column show the estimates on the information effects when using Romer and Romer (2004)-type monetary policy shocks that we obtain from Cloyne and Hürtgen (2016). Finally, the right-end columns show the results with policy rates themselves.

According to the theory introduced in Section 3.2.1.2, all three estimation procedures should yield consistent estimates of the information effect for the Inflation Report releases, because all the sources of biases are canceled out. As can be seen from Figure 3.8, the three estimation procedures tend to give similar estimates for each variable being considered. Given the identification argument proposed, this is very reassuring.

Second, we examine if the results of Figure 3.3 survive with different types of monetary policy measures. Figure 3.9 show if the responses of interested variables differ when Inflation Reports are released compared to other cases where no report is released, with different types of moneatry policy shocks. Note that, in this case, for these to be consistent estimates of the true effects, we now do need to use an exogenous measure. Therefore, in addition to Romer and Romer (2004)-type monetary policy shocks, we use hybrid forecast-adjusted-surprise instrument that we obtain from Miranda-Agrippino (2016), as well.

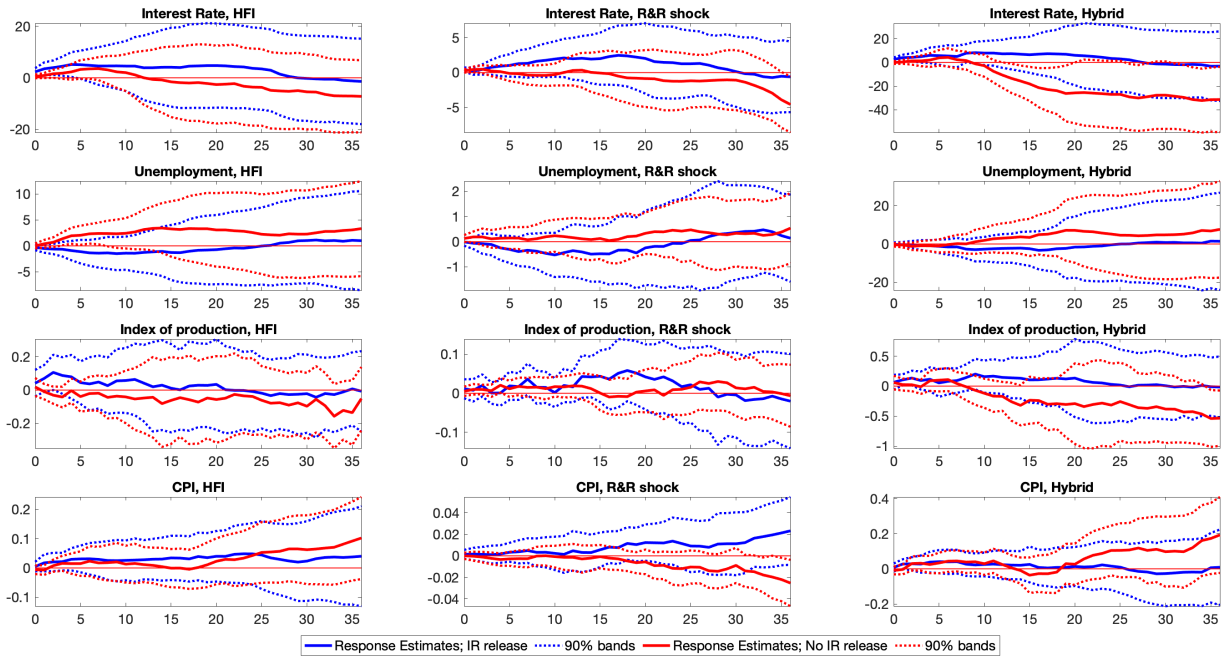
Figure 3.8: Baseline Results for the Information Effect



Notes: This figure shows the estimates, and 68% and 90% confidence bands from estimating Jordà (2005) Local Projections of the labelled outcome variable on various types of monetary policy shock series and the shock series interacted with a time dummy for whether an Inflation Report was released. The left-end columns use high frequency identified shock series for comparison (so the results should be the same as Figure 3.2), the middle columns use Romer and Romer (2004)-type monetary policy shock series that we obtain from Cloyne and Hürtgen (2016), and the last columns use interest rates. These graphs all represent the *difference* between outcomes when an inflation report is released relative to when it isn't. Positive estimates indicate that releasing an inflation report makes the equilibrium response of this variable higher. The estimates are denoted as black solid line and 68% (90%) confidence bands are represented as grey shaded area (two black dotted lines).

In Figure 3.9, the left-end columns again replicate the results with high-frequency-identified monetary policy shocks in Figure 3.3 for comparison. The middle column show the estimates on the information effects when using Romer and Romer (2004)-type monetary policy shocks that we obtain from Cloyne and Hürtgen (2016). Finally, the right-end columns show the results with hybrid monetary policy shocks from Miranda-Agrippino (2016). The blue solid line shows the estimates of the effects of outcome when inflation report is released with two blue dashed lines for 90 % bands. The red solid line shows the estimates of the effects of outcome when no inflation report is released along with two red dashed lines for 90 % bands.

Figure 3.9: Estimates for Inflation Report and Non-Inflation Report Policy Changes



Notes: This figure shows the different effects of outcomes when there is no inflation report released (the red lines) and when there is an inflation report released (the blue lines). The left-end columns show the results with high frequency identified monetary policy shocks for comparison. (Again, the results should be the same as in Figure 3.3.) The middle columns use Romer and Romer (2004)-type monetary policy shocks that we obtain from Cloyne and Hürtgen (2016). The right-end columns use hybrid forecast-adjusted-surprise instrument that we obtain from Miranda-Agrippino (2016). The blue solid line shows the estimates of the effects of outcome when inflation report is released with two blue dashed lines for 90 % bands. The red solid line shows the estimates of the effects of outcome when no inflation report is released along with two red dashed lines for 90 % bands.

As can be seen, the results are similar across the three columns. While the significance is somewhat lower with the smaller samples with narrative shocks and with hybrid shocks, the release of Inflation Report is *expansionary*. When the reports are released, interest rate, index of production, and consumer price index tend to rise, and unemployment rate tends to decrease. Therefore, our results are robust to various types of monetary policy measures.

3.5 Discussion

The results here are suggestive of a very strong information effect stemming largely from the *communications* from the central bank rather than the interest rate announcement. That the information effect is strong is consistent with recent work by Coibion et al. (2019)

showing in a randomized controlled trial large effects on household consumption from changing expectations about inflation.

For central bank, these results have fairly large implications: unlike Nakamura and Steinsson (2018) where it is suggested that the information effect is present regardless of policy communications, these results here suggest that the bulk of the information effect is due to discretionary analysis released by the Central Bank. Furthermore, the release of information as extensive as the Bank of England's Inflation Report seems to flip the effect of monetary policy!

This challenges the current approach Central Banks tend to use to achieve their objectives. In this approach communications are largely a strategy to implement policy under discretion. As put by (Blinder et al., 2001, p. 2):

> *The essential message that any central bank ought to convey to the public is its policy regime: what it is trying to achieve, how it goes about doing so, and its probable reactions to the contingencies that are likely to occur. Of course, no central bank can spell out in advance its reaction to every conceivable contingency; nor is it necessary to reveal every detail of its operations. The guiding principles should be two. First, the bank should reveal enough about its analysis, actions and internal deliberations so that interested observers can understand each monetary policy decision as part of a logical chain of decisions leading to some objective(s).*

This quote describes central bank's communications as an important aspect of achieving its goals because it is dealing with the forward looking public. If the public doesn't believe the central bank will achieve its objectives, they will take actions that make it more costly for the central bank to achieve their objectives.

Essentially one might view the communications aspect of central banking as managing a trade-off: since Barro and Gordon (1983) and Clarida, Gali, and Gertler (1999), it has been known that models with forward looking agents typically suggest it is easier for policymakers to achieve their objectives by committing to a policy rule. However, blindly following a rule does not allow the central bank to respond to unexpected situations that may not have been considered when selecting the rule to follow.[26] In practice central banks have opted to react under discretion while in addition attempting to transparently communicate the reasons for their policy choices in each individual situation.

The end result is that Central Banks feel a pressure to discuss the current state of the economy in order to justify their policy decisions to maintain credibility with the public

[26] This is further complicated when Central Banks do have an imperfect understanding the quantitative strength of many structural relationships in the economy. When the central bank does not understand fully the way the economy operates, it is best to commit to a very simple policy rule - such as a Taylor rule (see Taylor and Williams (2010)). Therefore, to be robust, the rules cannot be too complicated so the risk of not responding optimally ex-post is very high. For example, Mishkin (2007) shows that when output responds to house prices with a delay, following a Taylor rule means the Central Bank would wait too long to respond to a housing crash.

that they can and will achieve their objectives. But the results here suggest that providing too much justification may blunt or even reverse the intended effect of policy movements chosen. As such there is an additional trade-off with a communications strategy: extensive communications with forecasts and analysis may help the Central Bank maintain long-run credibility of its inflation target and other objectives, but at the cost of overwhelming the short run effect of their month-to-month policy changes. The results here support a less information intensive approach. One possibility could be where perhaps once a year there is an extensive analysis released to the public of economic pressures and how the Central Bank's strategy addresses them whereas actual policy announcements could be supplemented with only brief press-statements. However, before Central Banks are encouraged to implement such changes, future work is needed to confirm the results produced here.

3.6 Conclusion

We estimate the sensitivity of the monetary policy "Information Effect" to the release of actual analysis and forecasts released by the Bank of England in their quarterly Inflation Report. Without classifying the information content of the Inflation Report, we exploit that there likely is a positive covariance between the policy decision made by the Bank of England and the information content inside their Inflation Report. Our procedure compares the responses to policy shocks identified by standard high-frequency event study procedures and find that there is a considerable sensitivity to the release of the Inflation Report. We find that the effects of policy look conventional when there is no Inflation Report released suggesting that any Information Effect in operation stemming directly from interest rate changes is not large enough to offset the direct effects of policy.

When a policy tightening is accompanied by an inflation report then we document a very strong Information Effect - strong enough that the net effect is that the policy tightening becomes expansionary. This suggests that by releasing forecasts and analysis of the upcoming economic pressures, Central Banks may be limiting the effectiveness of the policy changes they make.

This suggests that there is a limit to the benefits of Central Bank transparency. In particular, more analysis being provided to justify controversial policy decision may have an unintended consequence - they convince the public of a problem there were not otherwise considering.

Additional work to be done to confirm this falls into three categories: repeating the analysis for more countries with similar disclosure regimes; investigating further the U.S.'s situation where the Information Effect appears strong but there is no Inflation Report equivalent and confirming weaker Information Effects for countries with less detailed disclosure regimes. We leave them for future works.

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Appendix A

Appendix for Chapter 1

Appendix A.1 supplements the analysis from the micro-Current Population Survey (CPS) data from January 1976 to December 2019 in Section A.1, and Appendix B supplements the model in Section 1.3.[1]

A.1 Analysis from the CPS data

This section supplements Section 1.2 which examines the basic macroeconomic facts about the full-time and part-time labor markets, and the changes in the composition of workers over the business cycle, by using the microdata from the Current Population Survey (CPS) from January 1976 to December 2019.[2]

To that end, I first divide the labor market statuses into five groups: (i) Employed in the full-time (e^{FT}), (ii) Employed in the part-time (e^{PT}), (iii) Unemployed in the full-time (u^{FT}), (iv) Unemployed in the part-time (u^{PT}), and (v) Not in the labor force (n), following the CPS distinction of the full-time and the part-time labor forces.[3] I divide the unemployed into the unemployed full-time and the unemployed part-time from the kind of a job current job-seekers are looking for. If they are mainly looking for full-time (part-time) jobs as their "next" jobs, they are classified into the unemployed full-time, u^{FT} (the unemployed part-time, u^{PT}). I then examine the stocks of those labor market statuses and gross flows across them. I use full-time workers as a proxy for "regular-type" labor, and part-time workers as a proxy for "irregular-type" labor. Table A.1 presents summary statistics of the stocks and the flows between them.

[1]All the results in this section come from the sample with all workers of age over fifteen. The results are almost identical if I limit the sample to all the *male* workers of age from 15 to 64.

[2]Microdata from the CPS starts from January 1976.

[3]The CPS distinguishes full-time workers who work over 35 hours or more per week, and part-time workers who work less than 35 hours per week.

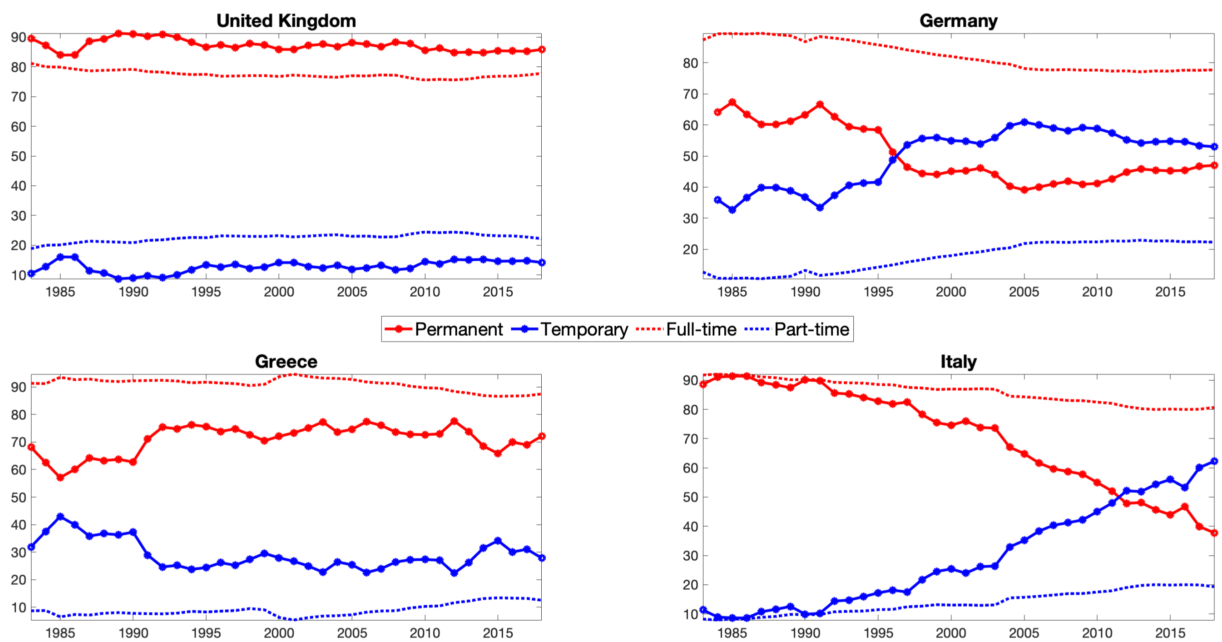
Table A.1: Average Statistics of the Labor Market Statuses in CPS

	e^{FT}	e^{PT}	u^{FT}	u^{PT}	n	stock
e^{FT}	86.62	3.87	0.91	0.07	1.50	49.31
e^{PT}	16.99	65.47	3.29	0.92	8.04	11.66
u^{FT}	14.83	8.04	53.68	0.93	11.73	4.03
u^{PT}	5.00	20.49	5.23	35.04	25.73	0.68
n	1.71	3.49	1.10	0.52	87.64	35.00

Source: CPS microdata from Januaray 1976 to December 2019.

A.1.1 Are Full-time and Part-time Workers Good Proxies for Regular and Irreguar Workers?

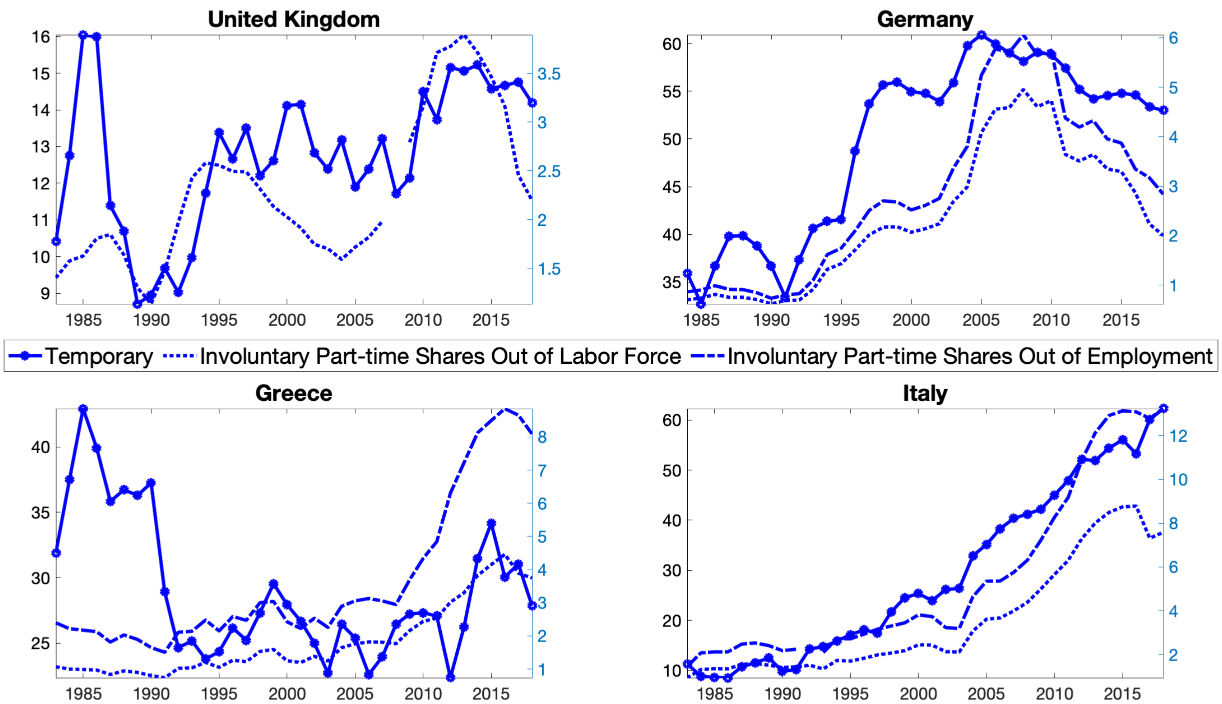
Figure A.1: The Share of Full-time and Part-time Workers and Permanent and Temporary Workers



Note: This figure shows the share of full-time and part-time workers and the share of permanent and temporary workers in four countries, the United Kingdom, Germany, Greece, and Italy, from 1983 to 2019. The shares of full-time workers are denoted as red dotted lines and the shares of part-time workers are denoted as blue dotted lines. The shares of permanent workers are denoted as the red solid lines with circles, while the shares of temporary workers are represented as the blue solid lines with circles. *Sources*: OECD Statistics

Before moving on, this section evaluates if it is okay to use full-time and part-time workers as proxies for regular and irregular types in the model. To preview the results, this section concludes that it makes sense to use the full-time and part-time classifications which are only available at the business cycle frequency from the publicly available data including the CPS.

Figure A.2: The Share of Temporary Workers and Involuntary Part-Timers, Raw Series

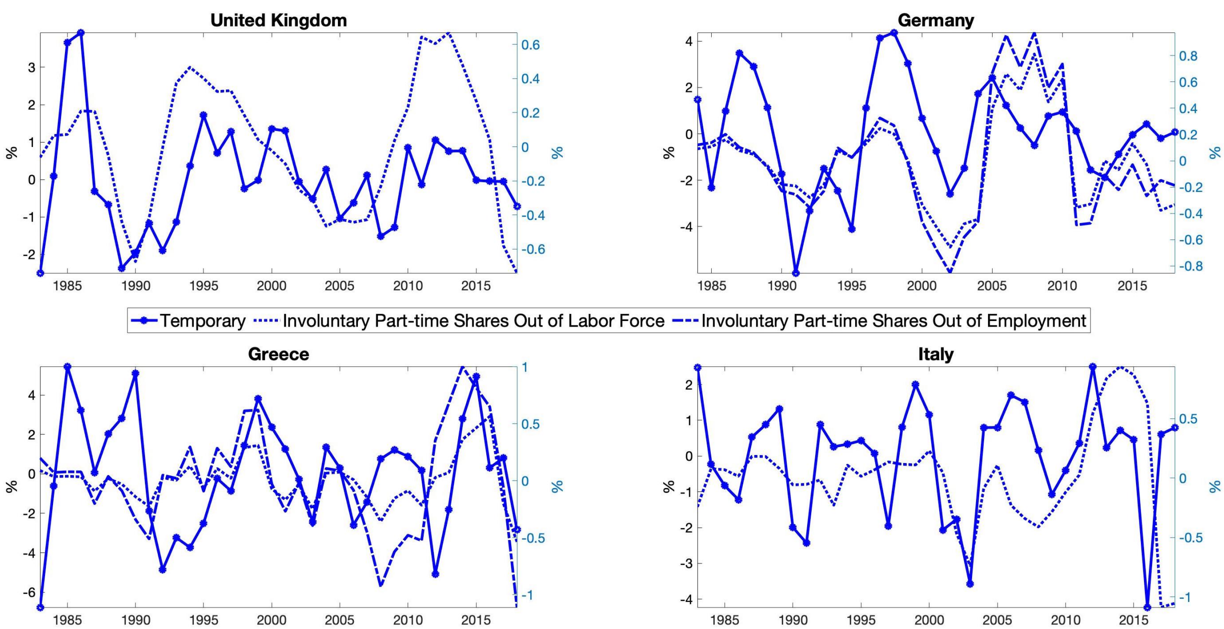


Note: This figure shows the share of temporary workers (y-axis on the left), involuntary part-timers out of the labor force, and out of total employment (y-axis on the right) in four countries, the United Kingdom, Germany, Greece, and Italy, from 1983 to 2019. The shares of temporary workers are represented as the blue solid lines with circles, the shares of involuntary part-time workers out of the total labor force are denoted as blue dotted lines, and the shares of involuntary part-time workers out of total employment are denoted as the blue dash lines. *Sources*: OECD Statistics

For the United States, I use full-time jobs as proxies for regular jobs and part-time jobs as proxies for irregular jobs in the model developed in Section 1.3. In the model, regular and irregular jobs are different in two aspects. First, regular jobs have higher productivity than irregular jobs. Second, regular jobs are more attached to firms than irregular jobs. While the exogenous separation rates for regular jobs are low, irregular jobs are all separated after one period. Moreover, creating/destructing regular jobs are much more expensive than creating/destructing irregular jobs. This implies that I might use some other classifications of labor types. For example, I might use permanent and temporary jobs, instead of full-time and part-time jobs. Alternatively, I can classify all the full-time jobs that are subject to social security benefits into regular jobs and all the other jobs into irregular workers. All the publicly available data that are available at the business cycle frequency (at least quarterly) including the CPS, however, do not have such classifications. Therefore, I instead use full-time and part-time classifications.

To figure out if using full-time and part-time jobs as proxies for regular and irregular types, I first compare the relative share of the full-time workers and the part-time workers with those of permanent and temporary workers in the countries where both series are available at the annual frequency. Figure A.1 shows the shares of workers according to each classification for the United Kingdom, Germany, Greece, and Italy. As can be seen from the figure, while the share of permanent and temporary workers fluctuate much more than the share of full-time and part-time workers, they are of similar magnitudes.

Figure A.3: The Share of Temporary Workers and Involuntary Part-Timers, HP-Filtered ((smoothing parameter of 100)

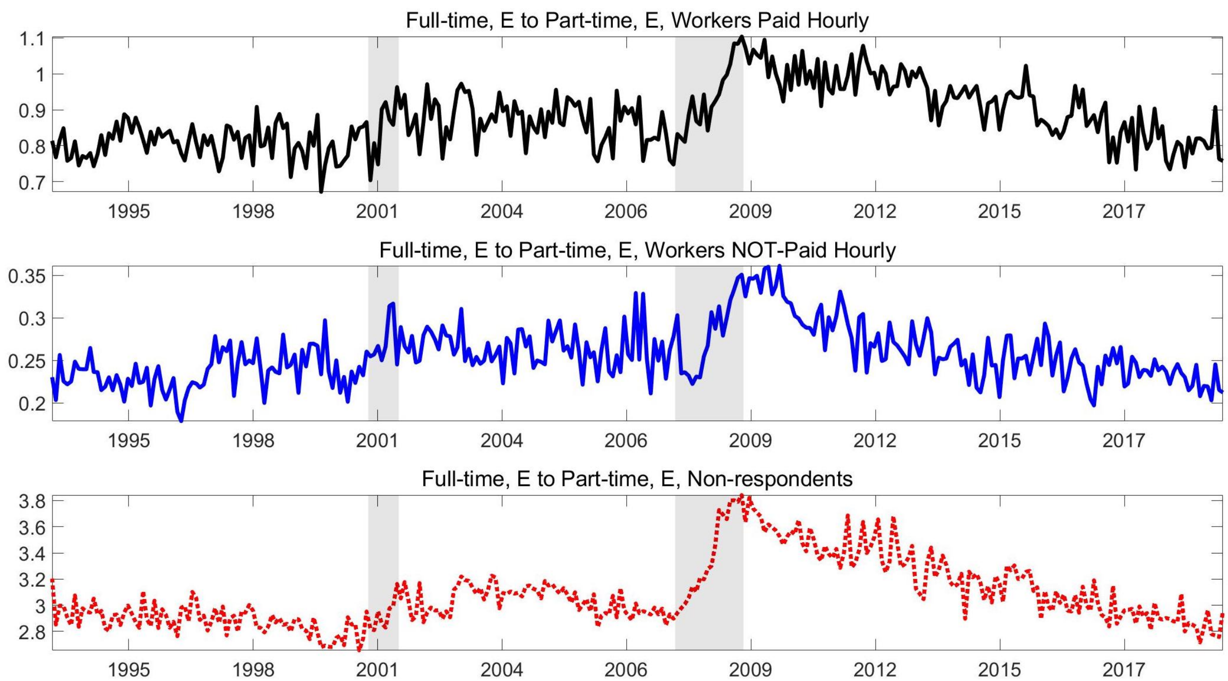


Note: This figure shows the HP-filtered (smoothing parameter of 100) series of the share of temporary workers (y-axis on the left), involuntary part-timers out of the labor force, and out of total employment (y-axis on the right) in four countries, the United Kingdom, Germany, Greece, and Italy, from 1983 to 2019. The shares of temporary workers are represented as the blue solid lines with circles, the shares of involuntary part-time workers out of total labor force are denoted as blue dotted lines, and the shares of involuntary part-time workers out of total employment are denoted as the blue dash lines. *Sources*: OECD Statistics

When I focus on *involuntary* part-time workers, they are even more in line with temporary workers. Figure A.3 illustrates this. As can be seen from this figure, the share of involuntary part-timers out of the total labor force and/or out of total employment closely tracks the changes in the share of temporary workers. Given that these involuntary part-time workers are those who tend to experience fluctuations over the business cycles, this figure and the similar magnitudes of the shares according to the two classifications in Figure A.1 at least partly confirm the quality of using full-time and part-time workers as proxies for regular and irregular workers.

A.1.2 Flows from Full-Time Employment to Part-Time Employment

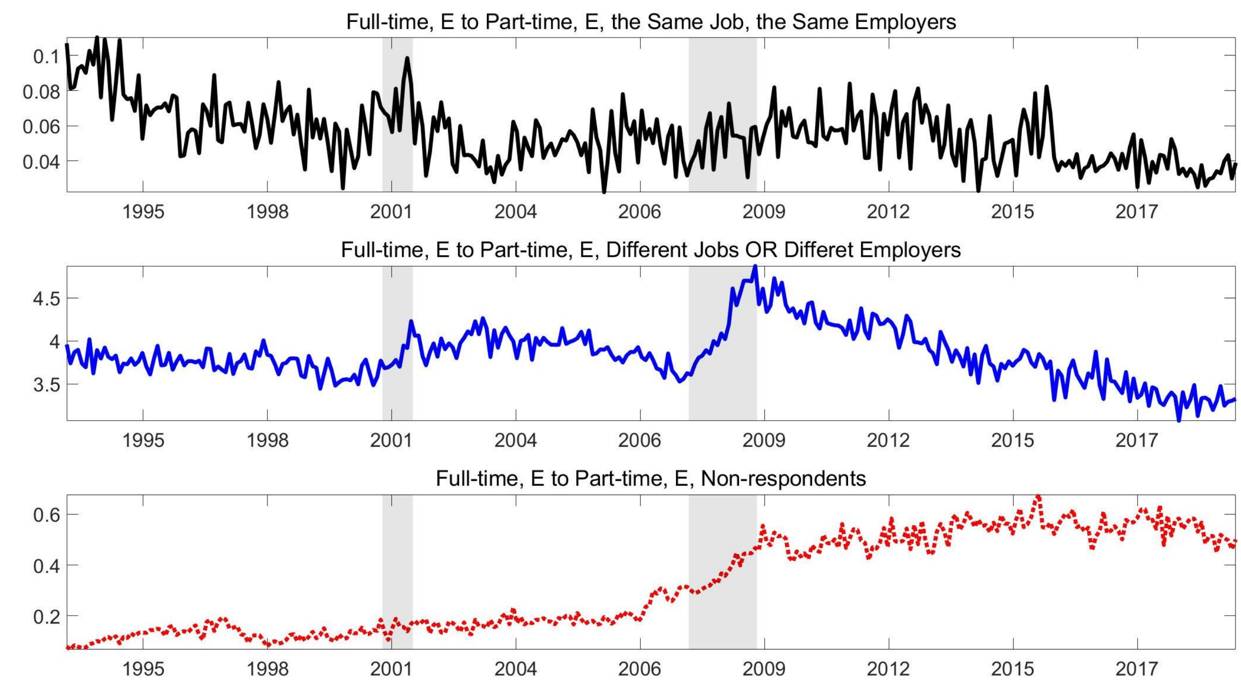
Figure A.4: $f_{e^{FT},e^{PT}}$ of Hourly-Paid Workers, Not-Hourly-Paid Workers, and Non-respondents



Note: The first panel shows the flow from full-time employment to part-time employment for hourly-paid workers with the black solid line in %. The second panel shows the same flows for those who are not paid hourly with the blue solid line. The last panel plots the same flows for the non-respondents to this question. *Source*: CPS microdata from January 1994 to December 2019. Flows are seasonally adjusted.

This section examines if the flows from the full-time employment to part-time employment, $f_{e^{FT},e^{PT}}$ are entirely driven by firms' adjustments of *intensive* margins or not. To this end, I first compare the flows of $f_{e^{FT},e^{PT}}$ for hourly-paid workers, and those who are not paid hourly. Second, I compare the flows of $f_{e^{FT},e^{PT}}$ for workers who work for the same employer, and do the same job with those who either work for different employers *or* do different jobs. Figure A.4 and A.5 illustrate that the flows from the full-time employment to part-time employment are not entirely driven by firms' adjustments of intensive margins, but show the compositional changes of jobs over the business cycle.

Figure A.5: $f_{e^{FT},e^{PT}}$ of Workers Who Do the Same Job and Work for the Same Employer, and Those Who Either a Do Different Job or Work for a Different Employer, and Non-respondents



Note: The first panel shows the flows of workers who do exactly the same job and work for the same employer with the black solid line in %. The second panel shows the same flows for those who either do a different job or work for a different employer with the blue solid line. The last panel plots the same flows for the non-respondents to this question. *Source*: CPS microdata from Januaray 1994 to December 2019. Flows are seasonally adjusted.

After the re-design of the CPS in 1994, there is a question asking if a worker is paid hourly or not. Because firms can easily adjust labor hours each worker works if they are paid hourly, if the countercyclicality of the flows from the full-time employment to the part-time employment is entirely driven by hourly-paid workers, then this does not necessarily mean the compositional changes of worker types over the business cycle. In this regard, I examine if the countercyclicality of $f_{e^{FT},e^{PT}}$ holds for workers who are not paid hourly. Figure A.4 shows that $f_{e^{FT},e^{PT}}$ is countercyclical for not only workers who are paid hourly but also for those who are not paid hourly and for non-respondents.[4]

I can also examine if the flows from full-time employment to part-time employment are driven by the same employers with the same job. If workers who moved from the full-time employment status to the part-time employment status do exactly the same job for the same employer, then it could be considered as the adjustment of intensive margins. To examine this, I compare the flows, $f_{e^{FT},e^{PT}}$ for workers who do the same job for the same employer with those who either do a different job or work for the different employers. Figure A.5 shows that not only the flows of $f_{e^{FT},e^{PT}}$ for workers who work for the same employers and do exactly the same job are small, but also they do not exhibit strong countercyclicality. Most of the flows, $f_{e^{FT},e^{PT}}$ are from workers who either do not work for the same employer or do different jobs than before, and the flows of these workers show strong countercyclicality.

A.1.3 Evidence from CPS: Flows within the Labor Force

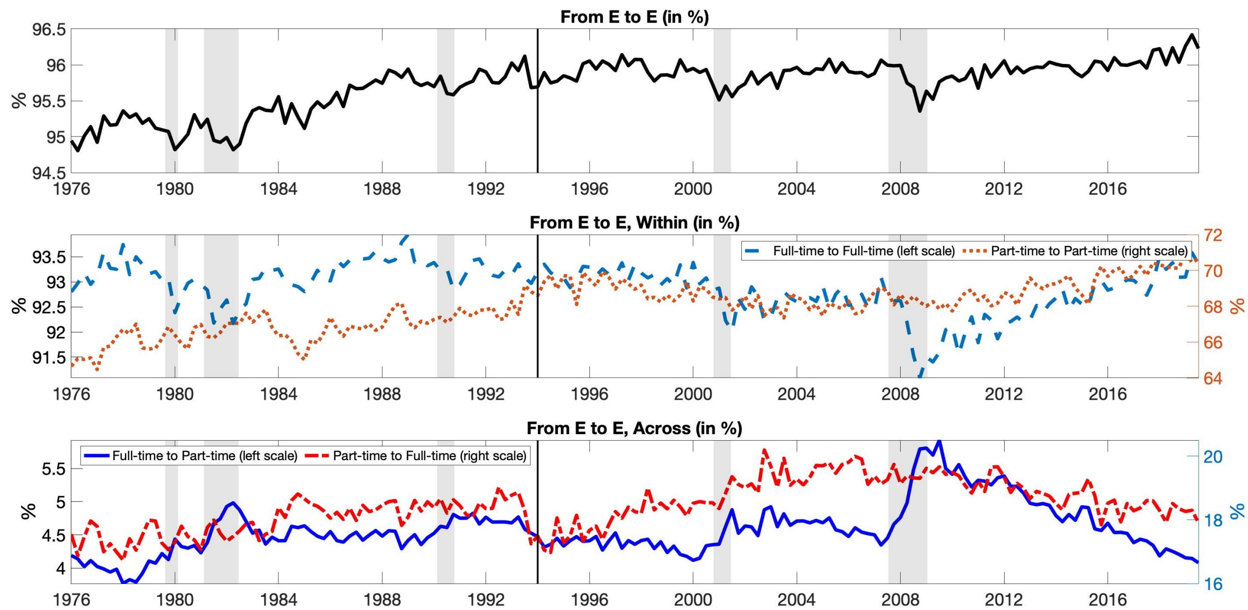
This section examines flows between more granular labor market states. Conventional studies have looked at labor market flows between employment and unemployment over the business cycle. This section investigates labor market flows among more *granular* labor market states. In particular, I focus on the importance of labor market flows *across* the two different labor markets: the full-time labor market and the part-time labor market.

All the flows calculated using the microdata from the CPS denote the gross flows from one state to the other state from the previous month to the next month. These flows are obtained by using a rotating-panel element in the CPS design that those who participated in the survey remain in the sample for four months in a row, rotate out for eight months, and then rotate back in for another four months. This feature allows some samples in a given month to be linked longitudinally to those in the subsequent month.[5] From this longitudinally linked microdata, the estimated transition probabilities are then simply calculated by the fraction of those in one state in a given month who report that they are in the other state. This becomes the transition probabilities from one state to the other state. All the flows reported here are adjusted for margin errors, classification errors, temporal aggregation errors, seasonally adjusted, and converted to quarterly series by averaging out for three months in a given quarter. Details about the margin error adjustments, the classification error adjustments using “deNUNified" following Elsby, Hobijn, and Şahin (2015), and temporal aggregation bias correction following Shimer (2012) are laid out in subsequent subsections of A.1.6.

[4]Because there are many missing values for the question asking if he or she is paid hourly or not, I also report the case for non-respondents.

[5]As is documented by Shimer (2012), it is impossible to match data for Dec. 1975/Jan. 1976, Dec. 1977/Jan. 1978, Jun. 1985/Jul. 1985, Sep. 1985/Oct. 1985, Dec. 1993/Jan. 1994, and May 1995/Jun. 1995 to Aug. 1995/Sep. 1995.

Figure A.6: Flows from Employment to Employment

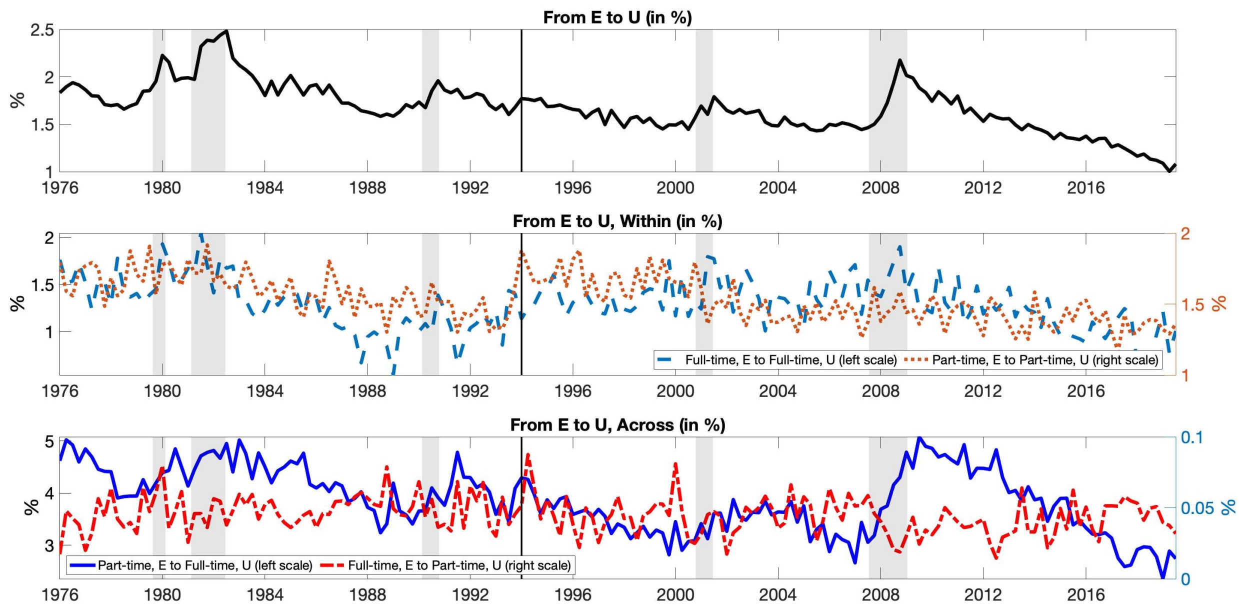


Note: The first panel shows the flow from employment to employment with the black solid line in %. The second panel shows the flow from employment to employment within the same labor market. The light blue dash line is the flows from full-time employment to full-time employment with the y-axis on the left in % and the light red dotted line plots the flows from part-time employment to part-time employment with the y-axis on the right in %. The last panel plots the flow from the employment to employment *across* the labor market. The blue solid line shows the flow from full-time employment to part-time employment with the y-axis on the left in % and the red dash-dot line is the flow from part-time employment to full-time employment with the y-axis on the right in %. All the flows are margin error adjusted,"deNUNfied" following Elsby, Hobijn, and Şahin (2015), corrected for temporal aggregation bias following Shimer (2012) and seasonally-adjusted using X-13-ARIMA. *Source*: CPS microdata from January 1976 to December 2019. The black vertical line is January 1994 to denote the re-design of CPS.

Figure A.6 shows the flows from the employment (E) to the employment within the same labor market and *across* the labor markets. As is well known, E to E transition is *procyclical*. This procyclicality comes from the full-time labor market, as the middle panel shows. While the cyclicality of the E to E transition within the part-time labor market (the light red dotted line) does not stand out (It is almost acyclical), the E to E transition within the full-time labor market (the light blue dash line) is procyclical.

The E to E transitions *across* the two labor markets are shown in the last panel. The flows from full-time employment to part-time employment are denoted as the blue solid line and the flows from part-time employment to full-time employment are denoted as the red dash-dot line. Among the two, the transition from full-time employment to part-time employment is important over the business cycle. While the transition from part-time employment to full-time employment does not exhibit any regular cyclicality, the transition from full-time employment to part-time employment shows strong *countercyclicality.* It sharply rises during recessions and gradually decreases during booms.

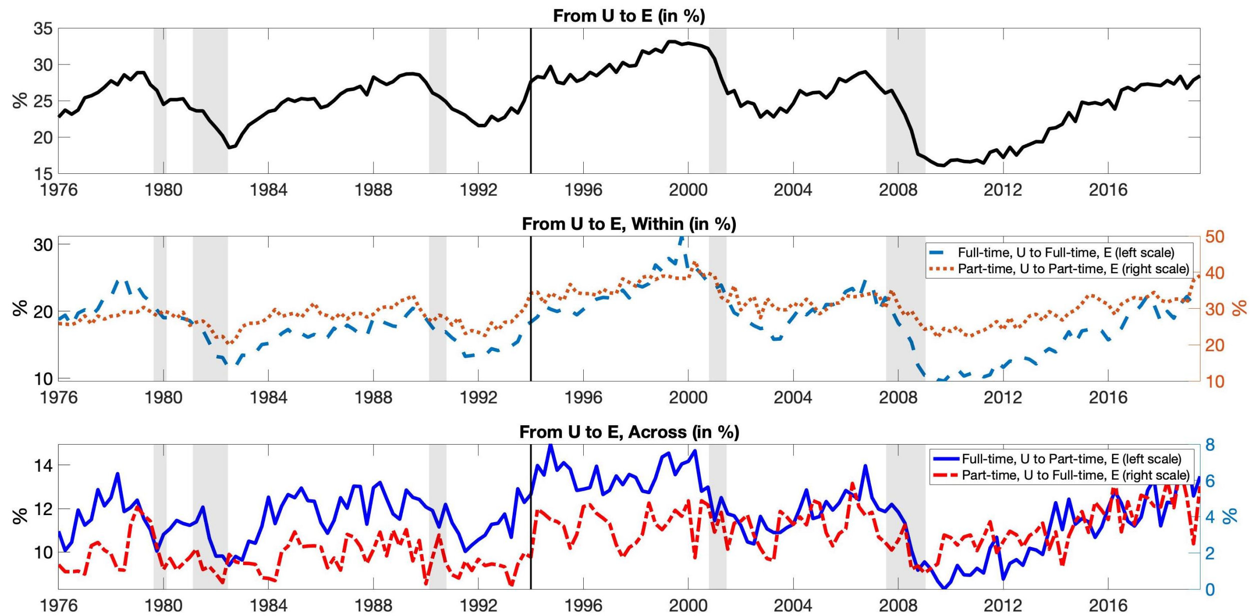
Figure A.7: Flows from Employment to Unemployment



Note: The first panel shows the flow from employment to unemployment with the black solid line in %. The second panel shows the flow from employment to unemployment within the same labor market. The light blue dash line is the flows from full-time employment to full-time unemployment with the y-axis on the left in % and the light red dotted line plots the flows from part-time employment to part-time unemployment with the y-axis on the right in %. The last panel plots the flow from the employment to unemployment *across* the labor market. The blue solid line shows the flow from part-time employment to full-time unemployment with the y-axis on the left in % and the red dash-dot line is the flows from full-time employment to part-time unemployment with the y-axis on the right in %. All the flows are margin error adjusted, “deNUNfied" following Elsby, Hobijn, and Şahin (2015), corrected for temporal aggregation bias following Shimer (2012) and seasonally-adjusted using X-13-ARIMA. *Source*: CPS microdata from January 1976 to December 2019. The black vertical line is January 1994 to denote the re-design of CPS.

The magnitudes of these flows are also large. One important countercyclical flow over the business cycle is the transition from employment to unemployment, which is about one percent of the full-time workers. The transition from full-time employment to part-time employment is about five percent of the full-time workers, which is four times larger in terms of the magnitude, showing the importance of this flow in terms of magnitudes.

Figure A.8: Flows from Unemployment to Employment



Note: The first panel shows the flow from unemployment to employment with the black solid line in %. The second-panel shows the flow from unemployment to employment within the same labor market. The light blue dash line is the flows from full-time unemployment to full-time employment with the y-axis on the left in % and the light red dotted line plots the flows from part-time unemployment to part-time employment with the y-axis on the right in %. The last panel plots flow from the unemployment to employment *across* the labor market. The blue solid line shows the flows from full-time unemployment to part-time employment with the y-axis on the left in % and the red dash-dot line is the flows from part-time unemployment to full-time employment with the y-axis on the right in %. All the flows are margin error adjusted, “deNUNfied" following Elsby, Hobijn, and Şahin (2015), corrected for temporal aggregation bias following Shimer (2012) and seasonally-adjusted using X-13-ARIMA.*Source*: CPS microdata from January 1976 to December 2019. The black vertical line is January 1994 to denote the re-design of CPS.

Figure A.7 shows the flows from employment to unemployment within and across the labor markets. As is well documented in the literature, the transition from employment (E) to unemployment (U) is strongly *countercyclical*. It sharply rises during recessions and gradually drops during booms. Similar to E to E transitions, this cyclicality stands out only *within* the full-time labor market. The E to U transition *within* the part-time labor market does not show any cyclicality. For the flows *across* the labor markets, only flows from part-time employment to full-time unemployment show similar countercyclicality, but not for the flows from full-time employment to part-time unemployment.

We can observe similar patterns, but in the opposite directions for the flows from the unemployment (U) to the employment within and across the labor markets. Figure A.8 shows that the U to E transition is strongly *procyclical*. This procyclicality comes mostly from the U to E transition within the full-time labor market, while the transition within the part-time labor market shows weak procyclicality. Similar patterns are observed for the flows across the two labor markets.

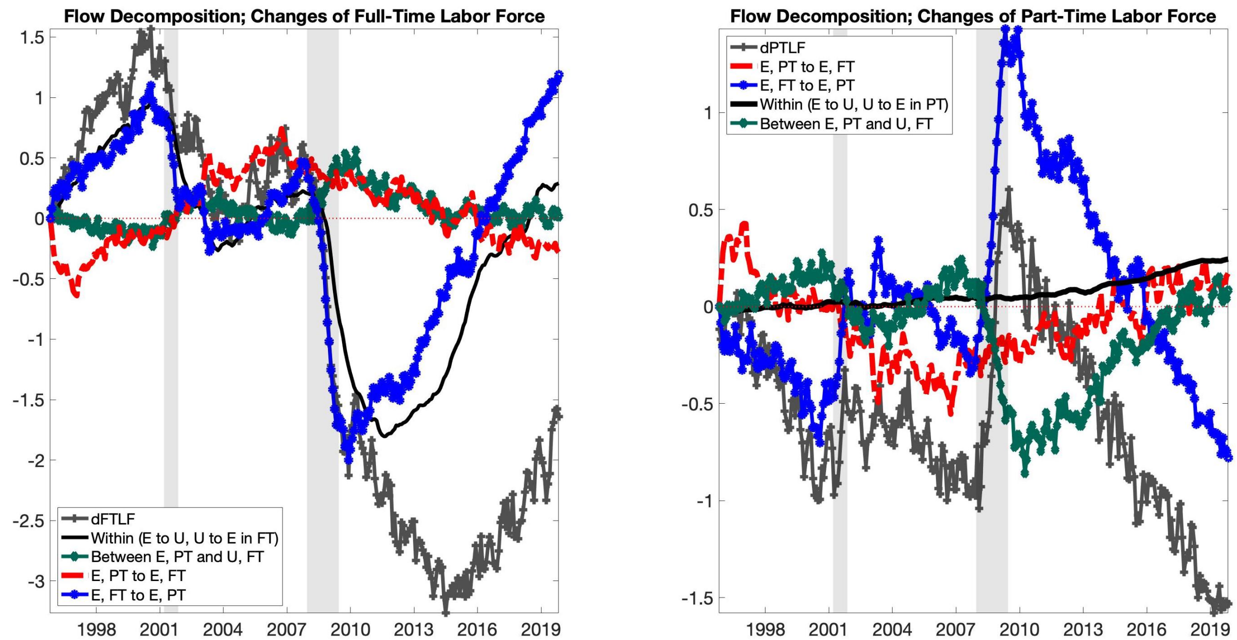
To summarize the findings for the flows from the CPS microdata flows *across* the full-time labor market and the part-time labor market are important in terms of the magnitude and cyclicality. Among the flows *across* the two labor market, the flow from full-time employment to part-time employment seem to be the most interesting and important. It is significantly large as it is five times larger than the flow from employment to unemployment within the full-time labor market. It is strongly countercyclical over the business cycle. The other flows *across* the labor market which is notable are the flows between the part-time employment and full-time unemployment. They behave similarly to E to U and U to E transitions within the full-time labor market. Flows from part-time employment to full-time unemployment are countercyclical and flows from full-time unemployment to part-time employment are procyclical.

A.1.4 Evidence from CPS: Flow Decomposition within the Labor Force

Previous section and SectionA.1.3 document the importance of the flows *across* the two labor markets. Among them, in particular, I show the importance of the flow from full-time employment to part-time employment. As a way of examining the importance of this *across*-market flow, this section investigates its contribution to explain changes in the stock of each labor market states. Following Elsby et al. (2019), this section calculates the contribution of the flow *across* the labor markets to explain the behaviors of the full-time labor force, the part-time labor force, and the total labor force, which are not presented in Section 1.2. For details about the method regarding flow decomposition, see Appendix A.1.7.

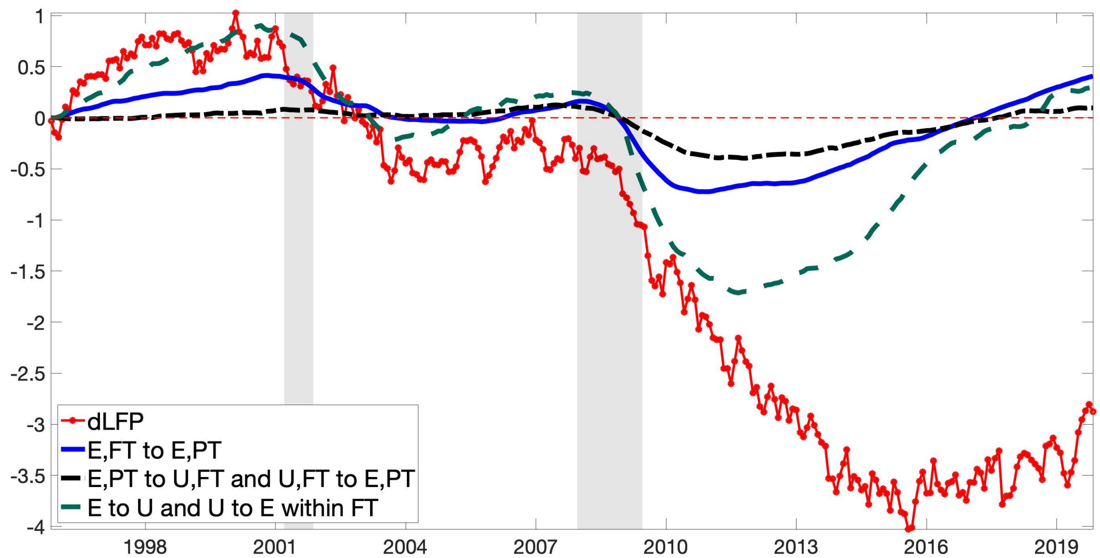
The left panel of Figure A.9 shows the contribution of the changes in the flows *within* and *across* the labor markets to explain changes in the full-time labor force. The grey line with crosses shows the *cumulative* changes in the stock of the full-time labor force, $e^{FT} + u^{FT}$ from January 1996 to December 2019. As is expected, the E to U and U to E transitions within the full-time labor market explain the significant amount of the changes in the full-time employment labor force. We can observe this pattern from the black solid line, which closely tracks the full-time labor force denoted as the grey line with crosses. The contributions of the flows from full-time employment to part-time employment denoted as the blue solid line with circles are as large as E to U and U to E transitions within the full-time labor market, showing the importance of this *across*-market flows. For example, the full-time employment rate could have dropped by about two and a half percent during the Great Recession, if *only* E to U and U to E transitions within the full-time labor market have changed. As the transition rates from full-time employment to part-time employment sharply rose during the Great Recession, however, the size of the full-time labor force has dropped even further. Meanwhile, the green solid line with diamonds denotes that because the transition rate from full-time unemployment to part-time employment is *procyclical*, the decreases in the full-time labor force size are partially offset by the changes in these flows. That is, had the flows between full-time unemployment to part-time employment not dropped, the size of the full-time labor force could have dropped even further.

Figure A.9: Flow Decomposition of the Full-Time and the Part-Time Labor Forces



Note: In the left panel, the grey line with crosses plots the cumulative changes of the full-time labor force. The blue solid line with circles is the cumulative contributions of the changes in the flows from full-time employment to part-time employment to explain the full-time labor force changes. The black solid line is the cumulative contributions of the changes in E to U and U to E transitions within the full-time labor market, and the green solid line with diamonds is the cumulative contribution of the changes in the flows between part-time employment and full-time unemployment. Lastly, the red dash line is the cumulative contribution of the changes in the flow from part-time employment to full-time employment. In the right panel, the grey line with crosses plots the cumulative changes in the part-time labor force. The blue solid line with circles is the cumulative contribution of the changes in the flow from full-time employment to part-time employment to explain the changes in the part-time labor forces. The black solid line is the cumulative contribution of the changes in E to U and U to E transitions within the full-time labor market, and the green solid line with diamonds is the cumulative contribution of the changes in the flows between part-time employment and full-time unemployment. Lastly, the red dash line is the cumulative contribution of the changes in the flow from part-time employment to full-time employment. Recession periods following the NBER classifications are denoted as grey shaded areas. All the flows are margin error adjusted, “deNUNfied" following Elsby, Hobijn, and Şahin (2015), corrected for temporal aggregation bias following Shimer (2012), and seasonally-adjusted using X-13-ARIMA. *Source*: CPS microdata from January 1996 to December 2019.

Figure A.10: Flow Decomposition of the Total Labor Force Participation: Contributions of Churn



Note: The red line with circles plots the cumulative changes in the total labor force participation rates. The blue solid line is the cumulative contribution of the changes in the flow from full-time employment to part-time employment to explain the changes in the total labor force. The green dash line is the cumulative contribution of the changes in E to U and U to E transitions within the full-time labor market, and the green dash-dot line is the cumulative contribution of the changes in the transition probabilities between part-time employment and full-time unemployment. Recession periods following the NBER classifications are denoted as grey shaded areas. All the flows are margin error adjusted, "deNUNfied" following Elsby, Hobijn, and Şahin (2015), corrected for temporal aggregation bias following Shimer (2012), and seasonally-adjusted using X-13-ARIMA. *Source*: CPS microdata from January 1996 to December 2019.

The importance of the flows *across* the two labor markets stand out more in the flow decomposition of part-time employment. In the right panel of Figure A.9, the grey line with crosses show the cumulative changes in the part-time labor forces, $e^{PT} + u^{PT}$ from January 1996 to December 2019. What contributes the most to the changes in the part-time labor force is the flow from full-time employment to part-time employment, the contribution of which is denoted as the blue solid line with circles. The blue solid line with circles closely tracks the changes in part-time labor force. Another important source to explain the evolution of the part-time labor force is the changes in the flow between part-time employment and full-time unemployment, which is denoted as the green solid line with diamonds. For example, during the Great Recession, the part-time labor force could have risen by two and a half percent, if only the transition probabilities from full-time employment to part-time employment have risen. However, it has risen by one and a half percent. This is because more part-time workers exit the full-time unemployment state (they are now looking for full-time jobs), and fewer full-time unemployed workers enter the part-time employment state. Due to these flow rate changes, the increases in part-time employment during the Great Recession have been partially muted. In contrast to these flows *across* the two labor market, flows within the part-time labor market do not explain the changes in part-time employment, as is shown with the black solid line.

More broadly, Figure A.10 shows that the flows *across* the two labor markets significantly contribute to the evolution of the total labor force participation rates. The red line with circles plots the cumulative changes in the total labor force participation rates from January 1996 to December 2019. As is documented in Elsby et al. (2019), “churn," that is, E to U and U to E transitions contributes significantly to the behaviors of the total labor force participation rates. In particular, the green dash line shows the contribution of such flows within the full-time labor market to explain the changes in the total labor force participation rates. It closely tracks the red line with circles, showing the importance of this flow. The magnitudes of the other two lines showing the contributions of the flows *across* the two labor markets in explaining total labor force participation rate changes are quite significant as well. The blue solid line shows the contributions of the flows from full-time employment to part-time employment, and the black dash-dot line shows the flows between part-time employment and full-time unemployment. Even though they are not as large as the green dash line, their contributions are quite large as well. For instance, the changes in the transition probabilities from full-time employment to part-time employment additionally drop about one more percent of the total labor force participation rate during the Great Recession.

A.1.5 Evidence from CPS: Flows between Labor Force and Not in the Labor Force

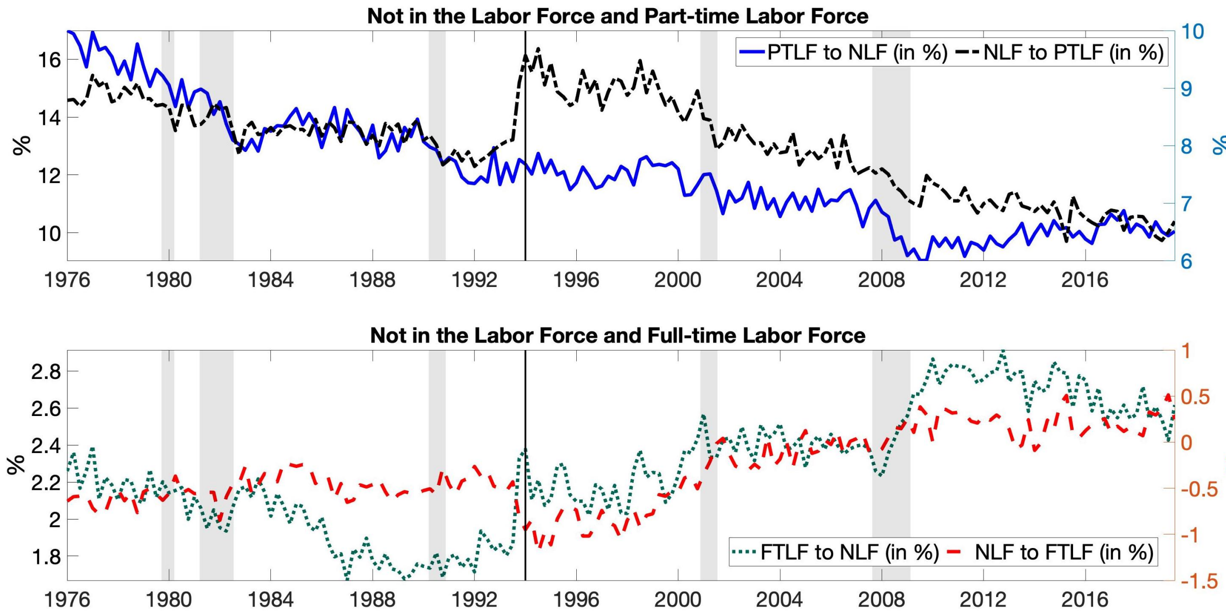
So far, we have examined stocks and flows of the employed and the unemployed in the two labor markets. This subsection investigates flows between the labor force and not-in-the labor force. In particular, this section examines *which* labor market is more important to understand the behavior of not-in-the labor force over the business cycle.

Table A.2: Correlation between Real GDP and Flows between the Full-time, the Part-time Labor Force and Out of Labor Force

	Full-time		Part-time	
	Exit to NLF	Entry from NLF	Exit to NLF	Entry from NLF
Correlation	-0.0187	0.0803	0.5572	0.2731

Note: This table calculates the correlation between real GDP and the flows in and out to not-in-the-labor-force. *Source*: CPS microdata from January 1996 to denote the re-design of CPS and Real GDP from FRED. All series are at the quarterly frequency and band-pass filtered.

Figure A.11: Flows between the Labor Force and Not-in-the-Labor Force

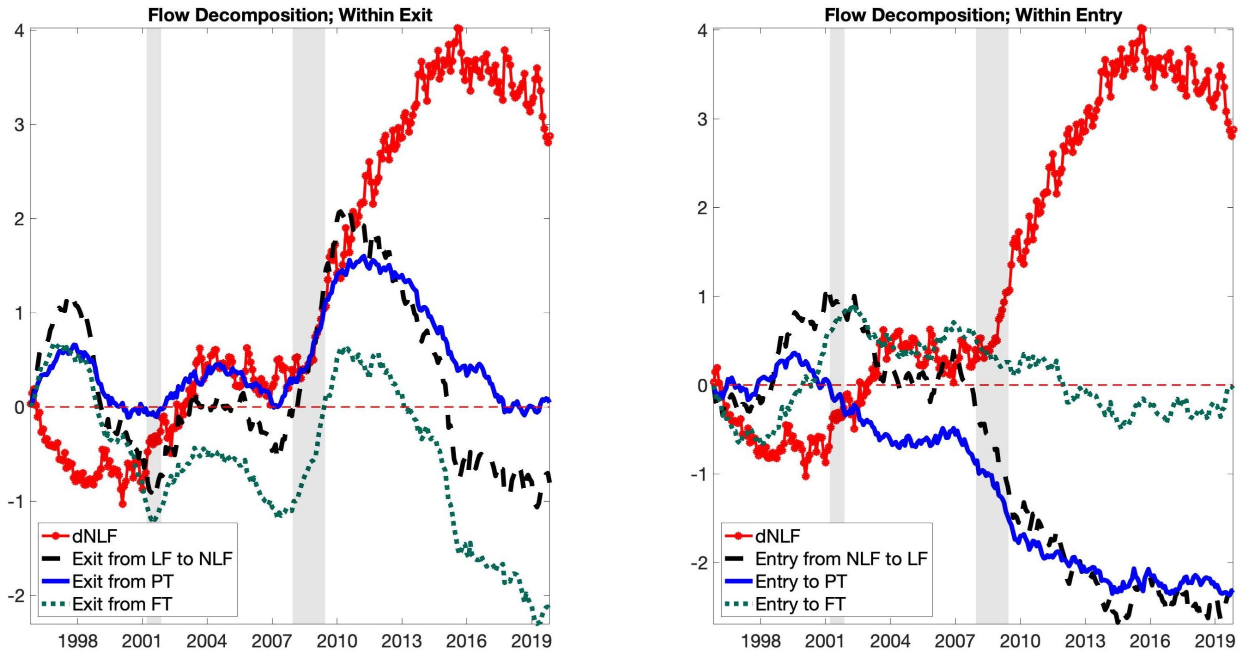


Note: The upper panel shows the flow between the part-time labor force and not-in-the labor force. The black dashed line is the transition from not-in-the labor force to the part-time labor force with the y-axis on the left in %, and the blue solid line is the transition from the part-time labor force to not-in-the labor force with the y-axis on the right in %. The lower panel shows the flow between the full-time labor force and not-in-the labor force. The green dotted line is the flows from the full-time labor force to not-in-the labor force with the y-axis on the left in % and the red dash line plots the flows from not-in-the labor force to the full-time labor force with the y-axis on the right in %. All the flows are margin error adjusted, deNUNfied following Elsby, Hobijn, and Şahin (2015), corrected for temporal aggregation bias following Shimer (2012) and seasonally-adjusted using X-13-ARIMA. *Source*: CPS microdata from January 1976 to December 2019. The black vertical line is January 1994 to denote the re-design of CPS.

Figure A.11 first shows flows between each labor market's labor force and not-in-the-labor force. The upper panel shows flows between the part-time labor force and not-in-the labor force with the blue solid line. The blue solid line shows the exit to not-in-the labor force with the *y*-axis on the left and the black dash line shows the entry from not-in-the labor force with the *y*-axis on the right. The two lines exhibit procyclical behaviors. Both entry from and exit to not-in-the labor force decrease during recessions. Meanwhile, the lower panel shows those between the full-time labor force and not-in-the labor force with the green dotted line as the exit to the not-in-the labor force with the y-axis on the left, and with the red dash line as the entry from not-in-the labor force with y-axis on the right. In contrast to the upper panel, the two lines in the lower panel do not exhibit strong cyclicality. Moreover, the correlations of the real gross domestic product (GDP) with the flows between the part-time labor force and not-in-the labor-force are higher than those with the flows between the full-time labor force and not-in-the-labor-force (Table A.2).

Therefore, the part-time labor force is *cyclically* more important to explain the exit to and entry from the not-in-the-labor force.

Figure A.12: Flow Decomposition of Not-in-the-Labor Force: Contributions of Exit and Entry



Note: The two panels show the flow decomposition of the total labor force. In both panels, the red line with circles shows cumulative changes in not-in-the-labor force. The left panel shows the contribution of the flows related to exits to not-in-the labor force and the right panel shows the contribution of the flows related to entries from not-in-the labor force to explain the evolution of the total labor force. In the left panel, the black dash line shows the contribution of the total exit, *i.e.* those of the exits from both the full-time and part-time labor markets, the blue solid line is the contribution of the exit flows from the part-time labor force, and the green dotted line is those from the full-time labor force. In the right panel, the black dash line shows the contribution of the total entry, *i.e.* those of the entries from not-in-the labor force to both the full-time and the part-time labor markets, the blue solid line is the contributions of the entry flows to the part-time labor force, and the green dotted line is those to the full-time labor force. Recession periods according to the NBER classifications are denoted as grey shaded areas. All the flows are margin error adjusted, "deNUNfied" following Elsby, Hobijn, and Şahin (2015), corrected for temporal aggregation bias following Shimer (2012), and seasonally-adjusted using X-13-ARIMA. *Source*: CPS microdata from January 1996 to denote the re-design of CPS.

Figure A.12 further corroborates this finding that the part-time labor force seems to be more important in explaining the behavior of not-in-the-labor force over the business cycle. In both panels, cumulative changes in not-in-the-labor-force-state are plotted as the red lines with circles. The left panel shows the contribution of the exits from both labor markets to not-in-the-labor force to explain the total labor force participation rates, and the right panel shows the contribution of entries to both labor markets. The black dash line in the left panel is the contribution of total exits from both labor markets, and the black dash line in the right panel is the contribution of total entries to both labor markets. The blue solid lines in each panel, which plot exit from and entry to the part-time labor force, respectively, closely track the black dash lines, showing the importance of the part-time labor force in explaining the evolution of not-in-the labor force. Meanwhile, the green dotted lines in each panel that show exit from and entry to the full-time labor force are apart from the black solid lines.

To summarize the findings from the micro CPS data: (i) flows *across* the two labor markets are important in terms of the magnitudes and cyclicalities; (ii) Among them, the flows from full-time employment to part-time employment are strongly countercyclical and four times larger than the E to U transitions within the full-time labor market; (iii) The flows between part-time employment and full-time unemployment are the other important flows *across* the two labor markets; (iv) The part-time labor force is more important in explaining the evolution of not-in-the-labor force over the business cycle than the full-time labor force.

A.1.6 Adjustments and Decompositions

This section lays out methods for margin-error adjustments, classification-error adjustments, temporal aggregation bias adjustments, and flow decompositions.

A.1.6.1 Margin Error Adjustments

Let $\mathbf{s}_t$ be the vector of labor market statuses:

$$\mathbf{s}_t = [e_{FT,t},\ e_{PT,t},\ u_{FT,t},\ u_{PT,t}],' \tag{A.1}$$

where $e_{FT,t}$ ($e_{PT,t}$) is the full-time (part-time) employment to population (of age higher than 16) ratio, $u_{t,FT}$ ($u_{t,PT}$) is the full-time (part-time) unemployment to population ratio. Then not in the labor force participation rate, n_t is simply the residual, $n_t = 1 - e_{FT,t} - e_{PT,t} - u_{FT,t} - u_{PT,t}$.

Now, let $p_{i,j,t}$ be the probability of transitioning from state i to state j. Then we can write the evolution of the labor market states as follows:

$$\Delta\mathbf{s}_t = \mathbf{s}_t - \mathbf{s}_{t-1} = \tilde{\mathbf{P}}_t \begin{bmatrix} e_{FT,t-1} \\ e_{PT,t-1} \\ u_{FT,t-1} \\ u_{PT,t-1} \\ n_{t-1} \end{bmatrix} = \mathbf{X}_{t-1}\mathbf{p}_t,$$

where $\tilde{\mathbf{P}}_t$ equals to

$$
\begin{aligned}
\tilde{\mathbf{P}}_t[1,1:5] &= [-p_{e_{FT},e_{PT},t} - p_{e_{FT},u_{FT},t} - p_{e_{FT},u_{PT},t} - p_{e_{FT},n,t} \quad p_{e_{PT},e_{FT},t} \quad p_{u_{FT},e_{FT},t} \quad p_{u_{PT},e_{FT},t} \quad p_{n,e_{FT},t}]\,, \\
\tilde{\mathbf{P}}_t[2,1:5] &= [p_{e_{FT},e_{PT},t} \quad -p_{e_{PT},e_{FT},t} - p_{e_{PT},u_{FT},t} - p_{e_{PT},u_{PT},t} - p_{e_{PT},n,t} \quad p_{u_{FT},e_{PT},t} \quad p_{u_{PT},e_{PT},t} \quad p_{n,e_{PT},t}]\,, \\
\tilde{\mathbf{P}}_t[3,1:5] &= [p_{e_{FT},u_{FT},t} \quad p_{e_{PT},u_{FT},t} \quad -p_{u_{FT},e_{FT},t} - p_{u_{FT},e_{PT},t} - p_{u_{FT},u_{PT},t} - p_{u_{FT},n,t} \quad p_{u_{PT},u_{FT},t} \quad p_{n,u_{FT},t}]\,, \\
\tilde{\mathbf{P}}_t[4,1:5] &= [p_{e_{FT}.u_{PT},t} \quad p_{e_{PT},u_{PT},t} \quad p_{u_{FT},u_{PT},t} \quad -p_{u_{PT},e_{FT},t} - p_{u_{PT},e_{PT},t} - p_{u_{PT},u_{FT},t} - p_{u_{PT},n,t} \quad p_{n,u_{PT},t,}]
\end{aligned}
$$

and $\mathbf{X}_{t-1}$ equals to

$$
\begin{aligned}
\mathbf{X}_{t-1}[1,1:20] &= [\; -e_{FT,t-1} \quad -e_{FT,t-1} \quad -e_{FT,t-1} \quad -e_{FT,t-1} \quad e_{PT,t-1} \quad 0 \quad 0 \quad 0 \quad u_{FT,t-1} \quad 0 \quad 0 \quad 0 \quad u_{PT,t-1} \quad 0 \quad 0 \quad 0 \quad n_{t-1} \quad 0 \quad 0 \quad 0 \;]\,, \\
\mathbf{X}_{t-1}[2,1:20] &= [\; e_{FT,t-1} \quad 0 \quad 0 \quad 0 \quad -e_{PT,t-1} \quad -e_{PT,t-1} \quad -e_{PT,t-1} \quad -e_{PT,t-1} \quad 0 \quad u_{FT,t-1} \quad 0 \quad 0 \quad 0 \quad u_{PT,t-1} \quad 0 \quad 0 \quad 0 \quad n_{t-1} \quad 0 \quad 0 \;]\,, \\
\mathbf{X}_{t-1}[3,1:20] &= [\; 0 \quad e_{FT,t-1} \quad 0 \quad 0 \quad 0 \quad e_{PT,t-1} \quad 0 \quad 0 \quad -u_{FT,t-1} \quad -u_{FT,t-1} \quad -u_{FT,t-1} \quad -u_{FT,t-1} \quad 0 \quad 0 \quad u_{PT,t-1} \quad 0 \quad 0 \quad 0 \quad n_{t-1} \quad 0 \;]\,, \\
\mathbf{X}_{t-1}[4,1:20] &= [\; 0 \quad 0 \quad e_{FT,t-1} \quad 0 \quad 0 \quad 0 \quad e_{PT,t-1} \quad 0 \quad 0 \quad 0 \quad u_{FT,t-1} \quad 0 \quad -u_{PT,t-1} \quad -u_{PT,t-1} \quad -u_{PT,t-1} \quad -u_{PT,t-1} \quad 0 \quad 0 \quad 0 \quad n_{t-1} \;]\,,
\end{aligned}
$$

and $\mathbf{p}_t$ is

$$
\begin{aligned}
\mathbf{p}_t[1:10,1] &= [\; p_{e_{FT},e_{PT},t} \quad p_{e_{FT},u_{FT},t} \quad p_{e_{FT},u_{PT},t} \quad p_{e_{FT},n,t} \quad p_{e_{PT},e_{FT},t} \quad p_{e_{PT},u_{FT},t} \quad p_{e_{PT},u_{PT},t} \quad p_{e_{PT},n,t} \quad p_{u_{FT},e_{FT},t} \quad p_{u_{FT},e_{PT},t} \;]'\,, \\
\mathbf{p}_t[11:20,1] &= [\; p_{u_{FT},u_{PT},t} \quad p_{u_{FT},n,t} \quad p_{u_{PT},e_{FT},t} \quad p_{u_{PT},e_{PT},t} \quad p_{u_{PT},u_{FT},t} \quad p_{u_{PT},n,t} \quad p_{n,e_{FT},t} \quad p_{n,e_{PT},t} \quad p_{n,u_{FT},t} \quad p_{n,u_{PT},t} \;]'.
\end{aligned}
$$

Then following Elsby, Hobijn, and Şahin (2015), I use a weighted-restricted-least-squares method to choose the vector of transition probabilities subject to the labor market statuses vector. In specific, I choose $\mathbf{p}_t$ which solves

$$\min_{\mathbf{p}_t} \; (\mathbf{p}_t - \hat{\mathbf{p}}_t)'\mathbf{W}_t(\mathbf{p}_t - \hat{\mathbf{p}}_t), \;\text{ subject to }\; \Delta \mathbf{s}_t = \mathbf{X}_{t-1}\mathbf{p}_t.$$

The solution of the above could be calculated by solving

$$\begin{bmatrix} \mathbf{p}_t \\ \mu_t \end{bmatrix} = \begin{bmatrix} \mathbf{W}_t & \mathbf{X}'_{t-1} \\ \mathbf{X}_{t-1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{W}_t \hat{\mathbf{p}}_t \\ \Delta \mathbf{s}_t \end{bmatrix},$$

with μ_t as the associated Lagrangian multiplier on the constraint. For the weighting matrix, I use the following where non-zero entries are:

$$
\mathbf{W}_t[1:4,1:4] = \begin{bmatrix}
\frac{\hat{p}_{e_{FT},e_{PT},t}(1-\hat{p}_{e_{FT},e_{PT},t})}{e_{FT,t-1}} & -\frac{\hat{p}_{e_{FT},e_{PT},t}\hat{p}_{e_{FT},u_{FT},t}}{e_{FT,t-1}} & -\frac{\hat{p}_{e_{FT},e_{PT},t}\hat{p}_{e_{FT},u_{PT},t}}{e_{FT,t-1}} & -\frac{\hat{p}_{e_{FT},e_{PT},t}\hat{p}_{e_{FT},n,t}}{e_{FT,t-1}} \\
-\frac{\hat{p}_{e_{FT},e_{PT},t}\hat{p}_{e_{FT},u_{FT},t}}{e_{FT,t-1}} & \frac{\hat{p}_{e_{FT},u_{FT},t}(1-\hat{p}_{e_{FT},u_{FT},t})}{e_{FT,t-1}} & -\frac{\hat{p}_{e_{FT},u_{FT},t}\hat{p}_{e_{FT},u_{PT},t}}{e_{FT,t-1}} & -\frac{\hat{p}_{e_{FT},u_{FT},t}\hat{p}_{e_{FT},n,t}}{e_{FT,t-1}} \\
-\frac{\hat{p}_{e_{FT},e_{PT},t}\hat{p}_{e_{FT},u_{PT},t}}{e_{FT,t-1}} & -\frac{\hat{p}_{e_{FT},u_{PT},t}\hat{p}_{e_{FT},u_{FT},t}}{e_{FT,t-1}} & \frac{\hat{p}_{e_{FT},u_{PT},t}(1-\hat{p}_{e_{FT},u_{PT},t})}{e_{FT,t-1}} & -\frac{\hat{p}_{e_{FT},u_{PT},t}\hat{p}_{e_{FT},n,t}}{e_{FT,t-1}} \\
-\frac{\hat{p}_{e_{FT},e_{PT},t}\hat{p}_{e_{FT},n,t}}{e_{FT,t-1}} & -\frac{\hat{p}_{e_{FT},u_{FT},t}\hat{p}_{e_{FT},n,t}}{e_{FT,t-1}} & -\frac{\hat{p}_{e_{FT},u_{PT},t}\hat{p}_{e_{FT},n,t}}{e_{FT,t-1}} & \frac{\hat{p}_{e_{FT},n,t}(1-\hat{p}_{e_{FT},n,t})}{e_{FT,t-1}}
\end{bmatrix},
$$

$$
\mathbf{W}_t[5:8,5:8] = \begin{bmatrix}
\frac{\hat{p}_{e_{PT},e_{FT},t}(1-\hat{p}_{e_{PT},e_{FT},t})}{e_{PT,t-1}} & -\frac{\hat{p}_{e_{PT},e_{FT},t}\hat{p}_{e_{PT},u_{FT},t}}{e_{PT,t-1}} & -\frac{\hat{p}_{e_{PT},e_{FT},t}\hat{p}_{e_{PT},u_{PT},t}}{e_{PT,t-1}} & -\frac{\hat{p}_{e_{PT},e_{FT},t}\hat{p}_{e_{PT},n,t}}{e_{PT,t-1}} \\
-\frac{\hat{p}_{e_{PT},e_{FT},t}\hat{p}_{e_{PT},u_{FT},t}}{e_{PT,t-1}} & \frac{\hat{p}_{e_{PT},u_{FT},t}(1-\hat{p}_{e_{PT},u_{FT},t})}{e_{PT,t-1}} & -\frac{\hat{p}_{e_{PT},u_{FT},t}\hat{p}_{e_{PT},u_{PT},t}}{e_{PT,t-1}} & -\frac{\hat{p}_{e_{PT},u_{FT},t}\hat{p}_{e_{PT},n,t}}{e_{PT,t-1}} \\
-\frac{\hat{p}_{e_{PT},e_{FT},t}\hat{p}_{e_{PT},u_{PT},t}}{e_{PT,t-1}} & -\frac{\hat{p}_{e_{PT},u_{PT},t}\hat{p}_{e_{PT},u_{FT},t}}{e_{PT,t-1}} & \frac{\hat{p}_{e_{PT},u_{PT},t}(1-\hat{p}_{e_{PT},u_{PT},t})}{e_{PT,t-1}} & -\frac{\hat{p}_{e_{PT},u_{PT},t}\hat{p}_{e_{PT},n,t}}{e_{PT,t-1}} \\
-\frac{\hat{p}_{e_{PT},e_{FT},t}\hat{p}_{e_{PT},n,t}}{e_{PT,t-1}} & -\frac{\hat{p}_{e_{PT},u_{FT},t}\hat{p}_{e_{PT},n,t}}{e_{PT,t-1}} & -\frac{\hat{p}_{e_{PT},u_{PT},t}\hat{p}_{e_{PT},n,t}}{e_{PT,t-1}} & \frac{\hat{p}_{e_{PT},n,t}(1-\hat{p}_{e_{PT},n,t})}{e_{PT,t-1}}
\end{bmatrix},
$$

$$\mathbf{W}_t[9:12,9:12] = \begin{bmatrix} \frac{\hat{p}_{u_{FT},e_{FT},t}(1-\hat{p}_{u_{FT},e_{FT},t})}{u_{FT,t-1}} & -\frac{\hat{p}_{u_{FT},e_{FT},t}\hat{p}_{u_{FT},e_{PT},t}}{u_{FT,t-1}} & -\frac{\hat{p}_{u_{FT},e_{FT},t}\hat{p}_{u_{FT},u_{PT},t}}{u_{FT,t-1}} & -\frac{\hat{p}_{u_{FT},e_{FT},t}\hat{p}_{u_{FT},n,t}}{u_{FT,t-1}} \\ -\frac{\hat{p}_{u_{FT},e_{PT},t}\hat{p}_{u_{FT},e_{FT},t}}{e_{FT,t-1}} & \frac{\hat{p}_{u_{FT},e_{PT},t}(1-\hat{p}_{u_{FT},e_{PT},t})}{u_{FT,t-1}} & -\frac{\hat{p}_{u_{FT},e_{PT},t}\hat{p}_{u_{FT},u_{PT},t}}{u_{FT,t-1}} & -\frac{\hat{p}_{u_{FT},e_{PT},t}\hat{p}_{u_{FT},n,t}}{u_{FT,t-1}} \\ -\frac{\hat{p}_{u_{FT},u_{PT},t}\hat{p}_{u_{FT},e_{FT},t}}{u_{FT,t-1}} & -\frac{\hat{p}_{u_{FT},u_{PT},t}\hat{p}_{u_{FT},e_{PT},t}}{u_{FT,t-1}} & \frac{\hat{p}_{u_{FT},u_{PT},t}(1-\hat{p}_{u_{FT},u_{PT},t})}{u_{FT,t-1}} & -\frac{\hat{p}_{u_{FT},u_{PT},t}\hat{p}_{u_{FT},n,t}}{u_{FT,t-1}} \\ -\frac{\hat{p}_{u_{FT},e_{FT},t}\hat{p}_{u_{FT},n,t}}{u_{FT,t-1}} & -\frac{\hat{p}_{u_{FT},e_{PT},t}\hat{p}_{u_{FT},n,t}}{u_{FT,t-1}} & -\frac{\hat{p}_{u_{FT},u_{PT},t}\hat{p}_{u_{FT},n,t}}{u_{FT,t-1}} & \frac{\hat{p}_{u_{FT},n,t}(1-\hat{p}_{u_{FT},n,t})}{u_{FT,t-1}} \end{bmatrix},$$

$$\mathbf{W}_t[13:16,13:16] = \begin{bmatrix} \frac{\hat{p}_{u_{PT},e_{FT},t}(1-\hat{p}_{u_{PT},e_{FT},t})}{u_{PT,t-1}} & -\frac{\hat{p}_{u_{PT},e_{FT},t}\hat{p}_{u_{PT},e_{PT},t}}{u_{PT,t-1}} & -\frac{\hat{p}_{u_{PT},e_{FT},t}\hat{p}_{u_{PT},u_{FT},t}}{u_{PT,t-1}} & -\frac{\hat{p}_{u_{PT},e_{FT},t}\hat{p}_{u_{PT},n,t}}{u_{PT,t-1}} \\ -\frac{\hat{p}_{u_{PT},e_{PT},t}\hat{p}_{u_{PT},e_{FT},t}}{u_{PT,t-1}} & \frac{\hat{p}_{u_{PT},e_{PT},t}(1-\hat{p}_{u_{PT},e_{PT},t})}{u_{PT,t-1}} & -\frac{\hat{p}_{u_{PT},e_{PT},t}\hat{p}_{u_{PT},u_{FT},t}}{u_{PT,t-1}} & -\frac{\hat{p}_{u_{PT},e_{PT},t}\hat{p}_{u_{PT},n,t}}{u_{PT,t-1}} \\ -\frac{\hat{p}_{u_{PT},u_{FT},t}\hat{p}_{u_{PT},e_{FT},t}}{u_{PT,t-1}} & -\frac{\hat{p}_{u_{PT},u_{FT},t}\hat{p}_{u_{PT},e_{PT},t}}{u_{PT,t-1}} & \frac{\hat{p}_{u_{PT},u_{FT},t}(1-\hat{p}_{u_{PT},u_{FT},t})}{u_{PT,t-1}} & -\frac{\hat{p}_{u_{PT},u_{FT},t}\hat{p}_{u_{PT},n,t}}{u_{PT,t-1}} \\ -\frac{\hat{p}_{u_{PT},e_{FT},t}\hat{p}_{u_{PT},n,t}}{u_{PT,t-1}} & -\frac{\hat{p}_{u_{PT},e_{PT},t}\hat{p}_{u_{PT},n,t}}{u_{PT,t-1}} & -\frac{\hat{p}_{u_{PT},u_{FT},t}\hat{p}_{u_{PT},n,t}}{u_{PT,t-1}} & \frac{\hat{p}_{u_{PT},n,t}(1-\hat{p}_{u_{PT},n,t})}{u_{PT,t-1}} \end{bmatrix},$$

$$\mathbf{W}_t[17:20,17:20] = \begin{bmatrix} \frac{\hat{p}_{n,e_{FT},t}(1-\hat{p}_{n,e_{FT},t})}{n_{t-1}} & -\frac{\hat{p}_{n,e_{FT},t}\hat{p}_{n,e_{PT},t}}{n_{t-1}} & -\frac{\hat{p}_{n,e_{FT},t}\hat{p}_{n,u_{FT},t}}{n_{t-1}} & -\frac{\hat{p}_{n,e_{FT},t}\hat{p}_{n,u_{PT},t}}{n_{t-1}} \\ -\frac{\hat{p}_{n,e_{PT},t}\hat{p}_{n,e_{FT},t}}{n_{t-1}} & \frac{\hat{p}_{n,e_{PT},t}(1-\hat{p}_{n,e_{PT},t})}{n_{t-1}} & -\frac{\hat{p}_{n,e_{PT},t}\hat{p}_{n,u_{FT},t}}{n_{t-1}} & -\frac{\hat{p}_{n,e_{PT},t}\hat{p}_{n,u_{PT},t}}{n_{t-1}} \\ -\frac{\hat{p}_{n,u_{FT},t}\hat{p}_{n,e_{FT},t}}{n_{t-1}} & -\frac{\hat{p}_{n,u_{FT},t}\hat{p}_{n,e_{PT},t}}{n_{t-1}} & \frac{\hat{p}_{n,u_{FT},t}(1-\hat{p}_{n,u_{FT},t})}{n_{t-1}} & -\frac{\hat{p}_{n,u_{FT},t}\hat{p}_{n,u_{PT},t}}{n_{t-1}} \\ -\frac{\hat{p}_{n,e_{FT},t}\hat{p}_{n,u_{PT},t}}{n_{t-1}} & -\frac{\hat{p}_{n,e_{PT},t}\hat{p}_{n,u_{PT},t}}{n_{t-1}} & -\frac{\hat{p}_{n,u_{FT},t}\hat{p}_{n,u_{PT},t}}{n_{t-1}} & \frac{\hat{p}_{n,u_{PT},t}(1-\hat{p}_{n,u_{PT},t})}{n_{t-1}} \end{bmatrix}.$$

A.1.6.2 Classification Error Adjustments

Following Elsby, Hobijn, and Şahin (2015), I recode unemployment-nonparticipation cyclers, *i.e.* I “deNUNified" flows. To that end, I first identify the transitions that involve the reversal of a transition from nonparticipation to unemployment and denote them as “NUN"s, and identify the reversal of a transition from unemployment to nonparticipation and denote them as “UNU"s. Then I recode “NUN"s as “NNN"s and “UNU"s as “UUU"s. For the details, please see Table 2 of Elsby, Hobijn, and Şahin (2015).

A.1.6.3 Temporal Aggregation Bias Adjustments

The evolution of the labor market states, $\mathbf{s}_t$ can be written using the discrete-time transition probabilities as follows:

$$\mathbf{s}_t = \widehat{\mathbf{P}}_t \mathbf{s}_{t-1} + \mathbf{d}_t,$$

where $\widehat{\mathbf{P}}_t$ equals to

$$\widehat{\mathbf{P}}_t[1,1:4] = \left[1 - p_{e_{FT},e_{PT},t} - p_{e_{FT},u_{FT},t} - p_{e_{FT},u_{PT},t} - p_{e_{FT},n,t} - p_{n,e_{FT},t} \quad p_{e_{PT},e_{FT},t} - p_{n,e_{FT},t} \quad p_{u_{FT},e_{FT},t} - p_{n,e_{FT},t} \quad p_{u_{PT},e_{FT},t} - p_{n,e_{FT},t}\right],$$

$$\widehat{\mathbf{P}}_t[2,1:4] = \left[p_{e_{FT},e_{PT},t} - p_{n,e_{PT},t} \quad 1 - p_{e_{PT},e_{FT},t} - p_{e_{PT},u_{FT},t} - p_{e_{PT},u_{PT},t} - p_{e_{PT},n,t} - p_{n,e_{PT},t} \quad p_{u_{FT},e_{PT},t} - p_{n,e_{PT},t} \quad p_{u_{PT},e_{PT},t} - p_{n,e_{PT},t}\right],$$

$$\widehat{\mathbf{P}}_t[3,1:4] = \left[p_{e_{FT},u_{FT},t} - p_{n,u_{FT},t} \quad p_{e_{PT},u_{FT},t} - p_{n,u_{FT},t} \quad 1 - p_{u_{FT},e_{FT},t} - p_{u_{FT},e_{PT},t} - p_{u_{FT},u_{PT},t} - p_{u_{FT},n,t} - p_{n,u_{FT},t} \quad p_{u_{PT},u_{FT},t} - p_{n,u_{FT},t}\right],$$

$$\widehat{\mathbf{P}}_t[4,1:4] = \left[p_{e_{FT},u_{PT},t} - p_{n,u_{PT},t} \quad p_{e_{PT},u_{PT},t} - p_{n,u_{PT},t} \quad p_{u_{FT},u_{PT},t} - p_{n,u_{PT},t} \quad 1 - p_{u_{PT},e_{FT},t} - p_{u_{PT},e_{PT},t} - p_{u_{PT},u_{FT},t} - p_{u_{PT},n,t} - p_{n,u_{PT},t}\right].$$

Similarly, we can write the analogous continuous-time Markov chain can be written as:

$$\dot{\mathbf{s}}_t = \widehat{\mathbf{F}}_t \mathbf{s}_{t-1} + \mathbf{q}_t.$$

Shimer (2012) shows that the eigenvectors of $\widehat{\mathbf{P}}_t$ are the same as those of $\widehat{\mathbf{F}}_t$, and that the eigenvalues of $\widehat{\mathbf{P}}_t$ are equal to the exponentiated eigenvalues of $\widehat{\mathbf{F}}_t$. Therefore, we can infer the matrix of flow hazard rates $\widehat{\mathbf{F}}_t$ from the eigen decomposition.

A.1.7 Flow Decompositions

This section summarizes the flow decomposition of labor market states in Elsby et al. (2019). The evolution of changes of labor market states can be written as follows:

$$\Delta \mathbf{s}_t = \mathbf{P}_t \mathbf{s}_{t-1} + \mathbf{d}_t, \tag{A.2}$$

where

$$\mathbf{P}_t[1,1:4] = \left[-p_{e_{FT},e_{PT},t} - p_{e_{FT},u_{FT},t} - p_{e_{FT},u_{PT},t} - p_{e_{FT},n,t} - p_{n,e_{FT},t},\quad p_{e_{PT},e_{FT},t} - p_{n,e_{FT},t},\quad p_{u_{FT},e_{FT},t} - p_{n,e_{FT},t},\quad p_{u_{PT},e_{FT},t} - p_{n,e_{FT},t}\right] \tag{A.3}$$

$$\mathbf{P}_t[2,1:4] = \left[p_{e_{FT},e_{PT},t} - p_{n,e_{PT},t},\quad -p_{e_{PT},e_{FT},t} - p_{e_{PT},u_{FT},t} - p_{e_{PT},u_{PT},t} - p_{e_{PT},n,t} - p_{n,e_{PT},t},\quad p_{u_{FT},e_{PT},t} - p_{n,e_{PT},t},\quad p_{u_{PT},e_{PT},t} - p_{n,e_{PT},t}\right] \tag{A.4}$$

$$\mathbf{P}_t[3,1:4] = \left[p_{e_{FT},u_{FT},t} - p_{n,u_{FT},t},\quad p_{e_{PT},u_{FT},t} - p_{n,u_{FT},t},\quad -p_{u_{FT},e_{FT},t} - p_{u_{FT},e_{PT},t} - p_{u_{FT},u_{PT},t} - p_{u_{FT},n,t} - p_{n,u_{FT},t},\quad p_{u_{PT},u_{FT},t} - p_{n,u_{FT},t}\right] \tag{A.5}$$

$$\mathbf{P}_t[4,1:4] = \left[p_{e_{FT},u_{PT},t} - p_{n,u_{PT},t},\quad p_{e_{PT},u_{PT},t} - p_{n,u_{PT},t},\quad p_{u_{FT},u_{PT},t} - p_{n,u_{PT},t},\quad -p_{u_{PT},e_{FT},t} - p_{u_{PT},e_{PT},t} - p_{u_{PT},u_{FT},t} - p_{u_{PT},n,t} - p_{n,u_{PT},t}\right] \tag{A.6}$$

and $\mathbf{d}_t = \left[p_{n,e_{FT},t},\ p_{n,e_{PT},t},\ p_{n,u_{FT},t},\ p_{n,u_{PT},t}\right]'$.

Elsby et al. (2019) shows that the evolution of the labor market states $\mathbf{s}_t$ can be decomposed using the following equation:

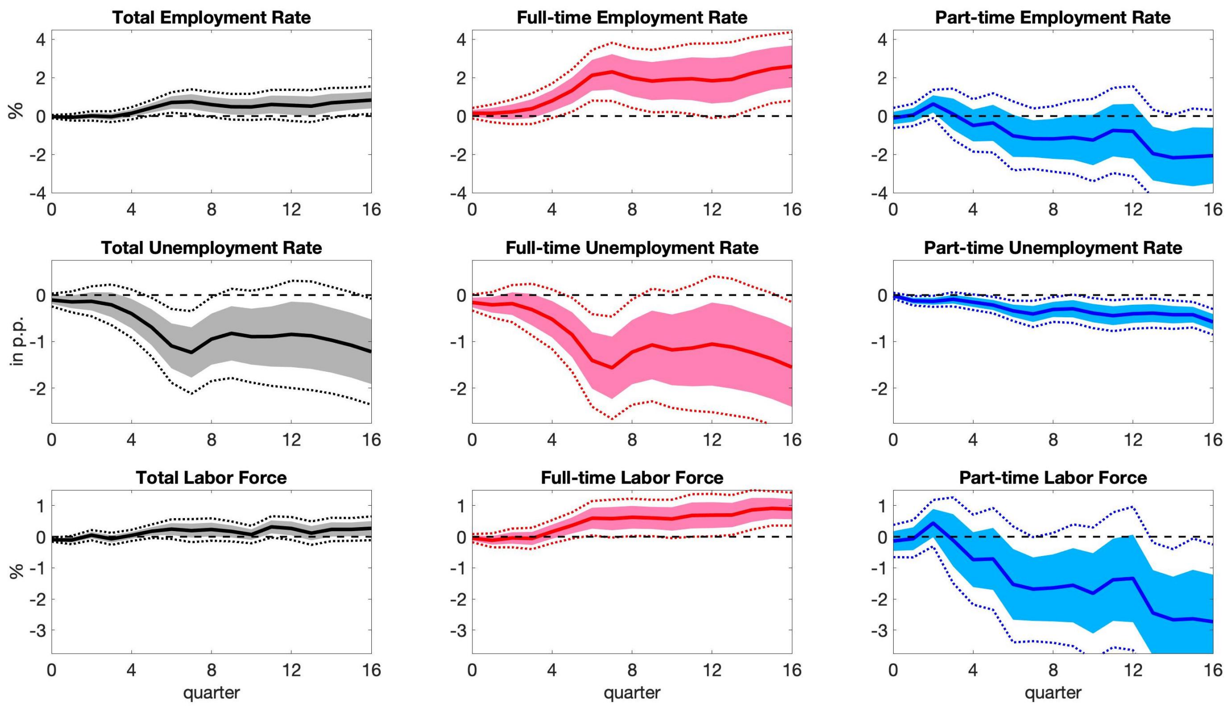
$$\Delta \mathbf{s}_t = \mathbf{P}_t(\mathbf{I} + \mathbf{P}_{t-1})\mathbf{P}_{t-1}^{-1}\Delta \mathbf{s}_{t-1} + \mathbf{P}_t(\mathbf{P}_t + \mathbf{P}_{t-1})^{-1} \times \left[2\Delta \mathbf{d}_t + \Delta \mathbf{P}_t(\bar{\mathbf{s}}_t + \bar{\mathbf{s}}_{t-1})\right],$$

with $\bar{\mathbf{s}}_t = -\mathbf{P}_t^{-1}\mathbf{d}_t$ calculated from Equation (A.2). From this, we can calculate the contributions of each entries in $\Delta \mathbf{P}_t$ and $\Delta \mathbf{d}_t$ to $\Delta \mathbf{s}_t$.

A.1.8 Empirical IRFs to Other Shocks

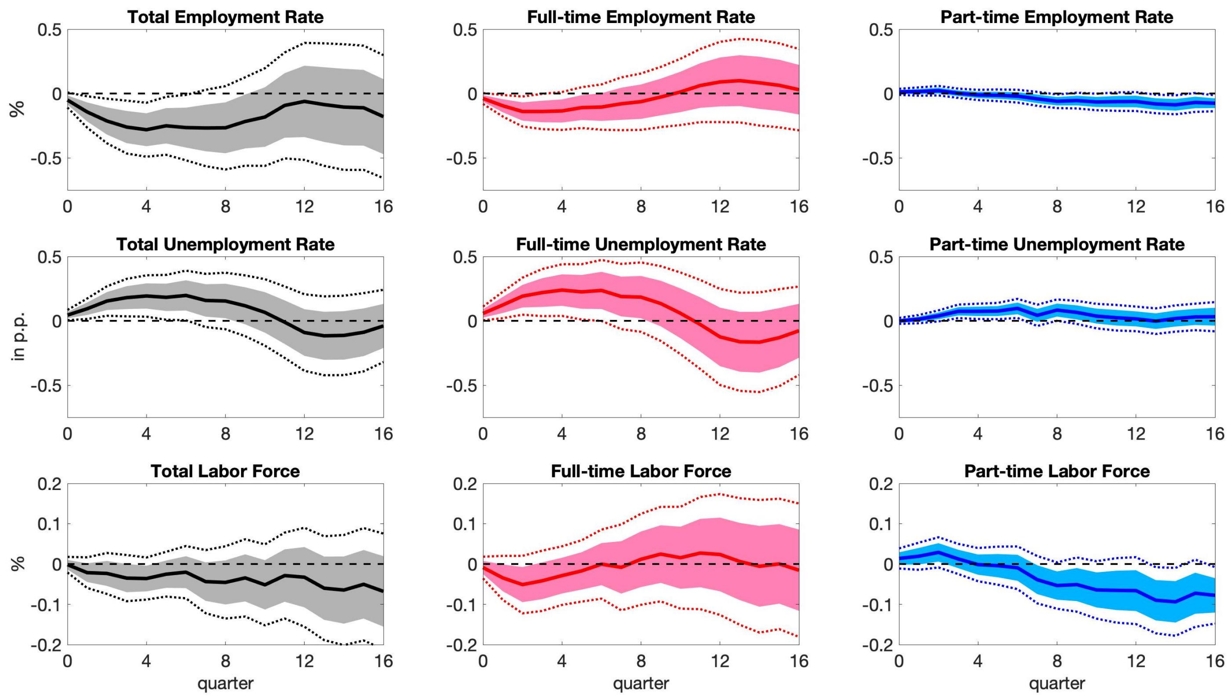
This section presents the estimated responses of total labor market variables and each labor market's variables to other structural shocks of government spending shocks from Ben Zeev and Pappa (2017) (Figure A.13) and utilization-rate adjusted total factor productivity shocks from Fernald (2014) (Figure A.14), using Equation (1.1) with $L = 8$.

Figure A.13: The Responses of Each Labor Market Variables to Government Spending Shocks



Note: This figure shows the responses of total labor market variables and each labor market's variables: employment rates, the size of labor forces, and unemployment rates to positive government spending shocks. The estimates for the total labor market variables' responses are denoted as black solid lines with black dotted lines as 90 percent confidence bands and with grey shaded area as 68 percent confidence bands. The estimates for the full-time labor market variables' responses are denoted as red solid lines with red dotted lines as 90 percent confidence bands and with the pink shaded area as 68 percent confidence bands. Those for the part-time labor market variables' responses are denoted as blue solid lines with blue dotted lines as 90 percent confidence bands and with the sky-blue shaded area as 68 percent confidence bands. *Sources:* All the labor market variables are calculated from the CPS from March 1976 to December 2013. Government spending shocks from the second quarter of 1976 to the fourth quarter of 2000 are taken from Ben Zeev and Pappa (2017).

Figure A.14: The Responses of Each Labor Market Variables to Total Factor Productivity Shocks



Note: This figure shows the responses of total labor market variables and each labor market's variables: employment rates, the size of labor forces, and unemployment rates to one standard deviation of utilization rate adjusted total factor productivity shock. The estimates for the total labor market variables' responses are denoted as black solid lines with black dotted lines as 90 percent confidence bands and with grey shaded area as 68 percent confidence bands. The estimates for the full-time labor market variables' responses are denoted as red solid lines with red dotted lines as 90 percent confidence bands and with the pink shaded area as 68 percent confidence bands. Those for the part-time labor market variables' responses are denoted as blue solid lines with blue dotted lines as 90 percent confidence bands and with the sky-blue shaded area as 68 percent confidence bands. *Sources:* All the labor market variables are calculated from the CPS from March 1976 to September 2015. Utilization rate-adjusted total factor productivity shocks from the second quarter of 1976 to the third quarter of 2015 are taken from Fernald (2014).

A.2 Model Supplements

A.2.1 Solving for Consumption Premiums

First, the optimal level of effort derived by maximizing *ex-ante* utility of participating in each labor market, Equation (1.4) is given by

$$e^{m,*}_{\ell,t} = \max\left\{a^m \left(\log c^m_t - \log c^{Um}_t - F^m - (1+\sigma^m)\ell^{\sigma^m}\right),\ 0\right\}, \tag{A.7}$$

so that a worker with higher ℓ exerts less effort, while she exerts more efforts with higher incentive, c^m_t / c^{Um}_t, for $m = R, IR$.

Given the optimal effort level in Equation (A.7), the *ex-ante* utility that a worker with a type l considering to enter a labor market m given by Equation(1.4) can be written as follows:

$$\begin{aligned}
&\left[\eta^m + (a^m)^2 \max\left\{\left(\log\left(\frac{c^m_t}{c^{Um}_t}\right) - F^m - (1+\sigma^m)\ell^{\sigma^m}\right),\ 0\right\}\right] + \log c^{Um}_t \\
&\times \left[\log\left(\frac{c^m_t}{c^{Um}_t} - F^m - (1+\sigma^m)\ell^{\sigma^m}\right)\right] \\
&-\frac{1}{2}\left[\max\left\{a^m\left(\log\left(\frac{c^m_t}{c^{Um}_t}\right) - F^m - (1+\sigma^m)\ell^{\sigma^m}\right),\ 0\right\}\right]^2,\ \ m = R, IR.
\end{aligned} \tag{A.8}$$

Let $\bar{\theta}^{IR}_t$ be the threshold at which a worker with a type $\ell = \bar{\theta}^{IR}_t$ is indifferent between participating in the irregular labor market and being out of labor force, and $\bar{\theta}^{R}_t$ be the threshold at which a worker with a type $\ell = \bar{\theta}^{R}_t$ is indifferent between entering to the regular and irregular labor markets. That is, the ex-ante expected utility of entering the irregular labor market is the same as the utility of being in the not-in-the-labor force for a worker with $\ell = \bar{\theta}^{IR}_t$, and the ex-ante expected utilities of participating in the regular and irregular labor market are the same for a worker who draws $\ell = \bar{\theta}^{R}_t$.

We can write the incentive compatibility conditions related to the two thresholds in equations as follows:

$$\log\left(\frac{c^{IR}_t}{c^{UIR}_t}\right) - F^{IR} = (1+\sigma^{IR})\left(\bar{\theta}^{IR}_t\right)^{\sigma^{IR}}. \tag{A.9}$$

The second incentive compatibility condition equating the ex-ante expected utilities of entering to regular and irregular labor markets can be written as follows:

$$
\begin{aligned}
& \left[\eta^R + (a^R)^2 \max\left\{\left(\log\left(\frac{c_t^R}{c_t^{UR}}\right) - F^R - (1+\sigma^R)\ell^{\sigma^R}\right), 0\right\}\right] \\
& \times \left\{\log(c_t^R) - F^R - (1+\sigma^R)\ell^{\sigma^R} - \frac{1}{2}\left(\max\left\{a^R\left(\log\left(\frac{c_t^R}{c_t^{UR}}\right) - F^R - (1+\sigma^R)\ell^{\sigma^R}\right), 0\right\}\right)^2\right\} \\
& + \left[1 - \eta^R - (a^R)^2 \max\left\{\left(\log\left(\frac{c_t^R}{c_t^{UR}}\right) - F^R - (1+\sigma^R)\ell^{\sigma^R}\right), 0\right\}\right] \\
& \times \left\{\log(c_t^{UR}) - \frac{1}{2}\left(\max\left\{a^R\left(\log\left(\frac{c_t^R}{c_t^{UR}}\right) - F^R - (1+\sigma^R)\ell^{\sigma^R}\right), 0\right\}\right)^2\right\} \\
= \quad & \left[\eta^{IR} + (a^{IR})^2 \max\left\{a^{IR}\left(\log\left(\frac{c_t^{IR}}{c_t^{UIR}}\right) - F^{IR} - (1+\sigma^{IR})\ell^{\sigma^{IR}}\right), 0\right\}\right] \\
& \times \left\{\log(c_t^{IR}) - F^{IR} - (1+\sigma^{IR})l^{\sigma^{IR}} - \frac{1}{2}\left(\max\left\{a^{IR}\left(\log\left(\frac{c_t^{IR}}{c_t^{UIR}}\right) - F^{IR} - (1+\sigma^{IR})\ell^{\sigma^{IR}}\right), 0\right\}\right)^2\right\} \\
& + \left[1 - \eta^{IR} - (a^{IR})^2 \max\left\{a^{IR}\left(\log\left(\frac{c_t^{IR}}{c_t^{UIR}}\right) - F^{IR} - (1+\sigma^{IR})\ell^{\sigma^{IR}}\right), 0\right\}\right] \\
& \times \left\{\log(c_t^{UIR}) - \frac{1}{2}\left(\max\left\{a^{IR}\left(\log\left(\frac{c_t^{IR}}{c_t^{UIR}}\right) - F^{IR} - (1+\sigma^{IR})\ell^{\sigma^{IR}}\right), 0\right\}\right)^2\right\} \qquad \text{(A.10)}
\end{aligned}
$$

Equation (A.9) can be obtained by equating the ex-ante expected utility of participating in the irregular labor market and the utility from being out of labor force and equation (A.10) can be obtained by equating the ex-ante expected utility of participating in the regular labor market and that of participating in the irregular labor market.

The incentive compatibility conditions above show that because the optimal level of effort is proportional to logarithmic consumption premiums in each labor market (c_t^m / c_t^{Um} for $m = R,\ IR$) per each type of workers, ℓ, the two thresholds $\bar{\theta}_t^{IR}$ and $\bar{\theta}_t^R$ are the functions of consumption premiums given the parameter values of $F^m, \sigma^m, a^m, \eta^m,\ m = R, IR$. Next subsections show this relationships under some cases.

B1-1. Special Case

Under the special case of $a^R = a^{IR} = a$, $\eta^R = \eta^{IR} = \eta$, and $c_t^{UR} = c_t^{UIR} \equiv c_t^U$ so that the probability of finding a job in each regular market is the same with the same level of effort, we can derive a simple relationship between the consumption premium of working as a regular-type, c_t^R / c_t^{UR} and the two thresholds, $\bar{\theta}_t^{IR}$ and $\bar{\theta}_t^R$.

To see this, consider Equation (A.10). The left-hand-side of Equation (A.10) is

$$\eta\left(\log\left(\frac{c_t^R}{c_t^U}\right)-F^R-(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}\right)+\frac{a^2}{2}\left(\log\left(\frac{c_t^R}{c_t^U}\right)-F^R\right)^2$$
$$-a^2\left(\log\left(\frac{c_t^R}{c_t^U}\right)-F^R\right)(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}+\frac{a^2}{2}(1+\sigma^R)^2\left(\bar{\theta}_t^R\right)^{2\sigma^R},$$

and the right-hand-side of Equation (A.10) is

$$\eta(1+\sigma^{IR})(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}-\left(\bar{\theta}_t^R\right)^{\sigma^{IR}})+\frac{a^2}{2}(1+\sigma^{IR})^2(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}-\left(\bar{\theta}_t^R\right)^{\sigma^{IR}})^2.$$

Equating the above two equations and defining $x\equiv\log\left(\frac{c_t^R}{c_t^U}\right)-F^R$ then generates the following equation:

$$\frac{a^2}{2}x^2-(a^2(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}-\eta)x+\frac{a^2}{2}(1+\sigma^R)^2\left(\bar{\theta}_t^R\right)^{2\sigma^R}$$
$$-\eta(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}-\eta(1+\sigma^{IR})(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}-\left(\bar{\theta}_t^R\right)^{\sigma^{IR}})-\frac{a^2}{2}(1+\sigma^{IR})^2(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}-\left(\bar{\theta}_t^R\right)^{\sigma^{IR}})=0.$$

Solving for x generates

$$x^*=(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}-\frac{\eta}{a^2}\pm\sqrt{\left((1+\sigma^{IR})((\bar{\theta}_t^{IR})^{\sigma^{IR}}-(\bar{\theta}_t^R)^{\sigma^{IR}})+\frac{\eta}{a^2}\right)^2}.$$

Among the two roots, the reasonable solution for x (because consumption for workers should be higher than consumption for the unemployed) is

$$x\equiv\log\left(\frac{c_t^R}{c_t^U}\right)-F^R=(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}+(1+\sigma^{IR})(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}-\left(\bar{\theta}_t^R\right)^{\sigma^{IR}}).$$

Combining the above expression with the incentive compatibility condition for $\left(\bar{\theta}_t^{IR}\right)$ gives

$$\log\left(\frac{c_t^R}{c_t^U}\right)-F^R=(1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}-\left(\bar{\theta}_t^R\right)^{\sigma^{IR}}\right)+(1+\sigma^R)\left(\bar{\theta}_t^{IR}\right)^{\sigma^R}. \tag{A.11}$$

B1-2. General Case

Under more general case where $a^R\leq a^{IR}$, $\eta^R\leq\eta^{IR}$, $c_t^{UR}\neq c_t^{UIR}$, and $\mathcal{C}\equiv c_t^{UR}/c_t^{UIR}$, the left-hand-side of Equation (A.10) is

$$\eta^R\left(\log\left(\frac{c_t^R}{c_t^{UR}}\right)-F^R-(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}\right)+\frac{(a^R)^2}{2}\left(\log\left(\frac{c_t^R}{c_t^{UR}}\right)-F^R\right)^2$$
$$-(a^R)^2\left(\log\left(\frac{c_t^R}{c_t^{UR}}\right)-F^R\right)(1+\sigma^R)\left(\bar{\theta}_t^R\right)\sigma^R+\frac{(a^R)^2}{2}(1+\sigma^R)^2\left(\bar{\theta}_t^R\right)^{2\sigma^R}+\log c_t^{UR},$$

and the right-hand-side of Equation (A.10) is

$$\eta^{IR}(1+\sigma^{IR})(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}) + \frac{(a^{IR})^2}{2}(1+\sigma^{IR})^2(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}})^2 + \log c_t^{UIR}.$$

Equating the above two equations and defining $x \equiv \log\left(\frac{c_t^R}{c_t^{UR}}\right) - F^R$ then generates the following equation:

$$\frac{(a^R)^2}{2}x^2 - ((a^R)^2(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R} - \eta^R)x + \frac{(a^R)^2}{2}(1+\sigma^R)^2\left(\bar{\theta}_t^R\right)^{2\sigma^R} + \log\frac{c_t^{UR}}{c_t^{UIR}}$$
$$-\eta^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R} - \eta^{IR}(1+\sigma^{IR})(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}) - \frac{(a^{IR})^2}{2}(1+\sigma^{IR})^2(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}) = 0.$$

Solving for x generates

$$x^* = (1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R} - \frac{\eta^R}{(a^R)^2} \pm \frac{1}{a^R}\cdot\sqrt{\left(a^{IR}(1+\sigma^{IR})((\bar{\theta}_t^{IR})^{\sigma^{IR}} - (\bar{\theta}_t^{R})^{\sigma^{IR}}) + \frac{\eta^{IR}}{a^{IR}}\right)^2 + \left(\left(\frac{\eta^R}{a^R}\right)^2 - \left(\frac{\eta^{IR}}{a^{IR}}\right)^2\right) - 2\log\mathcal{C}}.$$

Among the two roots, the reasonable value (again because the consumption of workers needs to be higher than the level of consumption for the unemployed) is

$$\begin{aligned} x &\equiv \log\left(\frac{c_t^R}{c_t^{UR}}\right) - F^R \\ &= (1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R} - \frac{\eta^R}{(a^R)^2} + \frac{1}{a^R}\cdot\sqrt{\left(a^{IR}(1+\sigma^{IR})((\bar{\theta}_t^{IR})^{\sigma^{IR}} - (\bar{\theta}_t^{R})^{\sigma^{IR}}) + \frac{\eta^{IR}}{a^{IR}}\right)^2 + \left(\left(\frac{\eta^R}{a^R}\right)^2 - \left(\frac{\eta^{IR}}{a^{IR}}\right)^2\right) - 2\log\mathcal{C}}. \end{aligned} \tag{A.12}$$

A.2.2 The Representative Family's Incentive Provision Problem

The number of regular workers and irregular workers are obtained by integrating out the probability of finding a job in each labor market given the optimal level of effort as follows:

$$n_t^R = \int_0^{(\bar{\theta}_t^R)}\left[\eta^R + (a^R)^2\max\left\{\left(\log\left(\frac{c_t^R}{c_t^{UR}}\right) - F^R - (1+\sigma^R)\ell^{\sigma^R}\right), 0\right\}\right]\mathrm{d}\ell,$$
$$n_t^{IR} = \int_{(\bar{\theta}_t^R)}^{(\bar{\theta}_t^{IR})}\left[\eta^{IR} + (a^{IR})^2\max\left\{\left(\log\left(\frac{c_t^{IR}}{c_t^{UIR}}\right) - F^{IR} - (1+\sigma^{IR})\ell^{\sigma^{IR}}\right), 0\right\}\right]\mathrm{d}\ell.$$

So the number of regular workers, n_t^R can be represented as follows:

$$n_t^R = \eta^R\left(\bar{\theta}_t^R\right) + (a^R)^2\left(\log\left(\frac{c_t^R}{c_t^{UR}}\right) - F^R\right)\left(\bar{\theta}_t^R\right) - (a^R)^2\left(\bar{\theta}_t^R\right)^{\sigma^R+1}. \tag{A.13}$$

Under the special case of $a^R = a^{IR} = a$, $\eta^R = \eta^{IR} = \eta$, and $c_t^{UR} = c_t^{UIR} \equiv c_t^U$, plugging in Equation (A.11) generates

$$n_t^R = \eta\left(\bar{\theta}_t^R\right) + a^2(1+\sigma^{IR})\left(\bar{\theta}_t^R\right)(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^R\right)^{\sigma^{IR}}) + a^2\sigma^R\left(\bar{\theta}_t^R\right)^{\sigma^R+1}, \quad \text{(A.14)}$$

and under more general case, the number of regular workers becomes

$$\begin{aligned} n_t^R =& (a^R)^2\sigma^R\left(\bar{\theta}_t^R\right)^{\sigma^R+1} \\ &+ a^R\left(\bar{\theta}_t^R\right)\sqrt{\left(a^{IR}(1+\sigma^{IR})((\bar{\theta}_t^{IR})^{\sigma^{IR}} - (\bar{\theta}_t^R)^{\sigma^{IR}}) + \frac{\eta^{IR}}{a^{IR}}\right)^2 + \left(\left(\frac{\eta^R}{a^R}\right)^2 - \left(\frac{\eta^{IR}}{a^{IR}}\right)^2\right) - 2\log\mathcal{C}}. \end{aligned} \quad \text{(A.15)}$$

Similarly, we can write the number of irregular workers, n_t^{IR} by combining the above with the incentive compatibility condition in Equation (A.9) as :

$$n_t^{IR} = \eta^{IR}(\left(\bar{\theta}_t^{IR}\right) - \left(\bar{\theta}_t^R\right)) + (a^{IR})^2\sigma^{IR}\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}+1} - (a^{IR})^2(1+\sigma^{IR})\left(\bar{\theta}_t^R\right)\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} + (a^{IR})^2\left(\bar{\theta}_t^R\right)^{\sigma^{IR}+1}. \quad \text{(A.16)}$$

The expressions for the number of regular and irregular workers in Equations (A.15) and (A.16) show that the two thresholds are the functions of the two number of workers, n_t^R and n_t^{IR}:

$$\bar{\theta}_t^{IR} = f(n_t^R, n_t^{IR}), \quad \bar{\theta}_t^R = g(n_t^R, n_t^{IR}).$$

Combining all, if the family wants to supply n_t^R number of workers to regular labor market and n_t^{IR} number of workers to irregular labor market, then they need to set $\bar{\theta}_t^R$ and $\bar{\theta}_t^{IR}$ accordingly as in Equation (A.15) (or under the simple case, Equation (A.14)) and in Equation (A.16). To ensure that the family lets $\bar{\theta}_t^R$ number of workers participate in regular labor market and $\bar{\theta}_t^{IR} - \bar{\theta}_t^R$ number of workers participate in irregular labor market, the family then needs to set consumption premiums correspondingly as in Equation (A.9) and Equation (A.12) (under the simple case, Equation (A.11)). At the same time, this adjustment of consumption premiums needs to be feasible. Hence, it needs to satisfy the following feasibility condition:

$$n_t^R c_t^R + (g(n_t^R, n_t^{IR}) - n_t^R)c_t^{UR} + n_t^{IR}c_t^{IR} + (1 - g(n_t^R, n_t^{IR}) - n_t^{IR})c_t^{UIR} = C_t. \quad \text{(A.17)}$$

A.2.3 Indirect Utility Function for a Representative Family

This section derives the indirect utility function for a representative family. This follows from integrating the utility of individual workers within the representative family. That is, the summation of the integration of the regular labor forces' expected utilities from zero to $\bar{\theta}_t^R$, the threshold determining the size of the regular labor force, the integration of irregular workers' expected utilities from $\bar{\theta}_t^R$ to $bar\theta_t^{IR}$, the size of the irregular labor force, and the integration of the utilities for those out of labor force generates the indirect utility function for the representative family. Formally, this can be written as follows:

$$
\begin{aligned}
&u(C_t, n_t^R, n_t^{IR}) = \\
&\int_0^{\bar{\theta}_t^R} \overbrace{\left\{ p(e_{\ell,t}^{R,*}) \left(\log c_t^R - F^R - (1+\sigma^R)\ell^{\sigma^R} - \frac{1}{2}(e_{\ell,t}^{R,*})^2 \right) + (1 - p(e_{\ell,t}^{R,*})) \left(\log c_t^{UR} - \frac{1}{2}(e_{\ell,t}^{R,*})^2 \right) \right\}}^{\text{Expected utility from participating in the regular labor market}} \mathrm{d}\ell \\
&+ \int_{\bar{\theta}_t^R}^{\bar{\theta}_t^{IR}} \overbrace{\left\{ p(e_{\ell,t}^{IR,*}) \left(\log c_t^{IR} - F^{IR} - (1+\sigma^{IR})\ell^{\sigma^{IR}} - \frac{1}{2}(e_{\ell,t}^{IR*})^2 \right) + (1 - p(e_{\ell,t}^{IR,*})) \left(\log c_t^{UIR} - \frac{1}{2}(e_{\ell,t}^{IR,*})^2 \right) \right\}}^{\text{Expected utility from participating in the irregular labor market}} \mathrm{d}\ell \\
&+ \int_{\bar{\theta}_t^{IR}}^1 \underbrace{\log c_t^{UIR}}_{\text{utility when not in the labor force}} \mathrm{d}\ell. \qquad \text{(A.18)}
\end{aligned}
$$

If we combine the above with the expression for the optimal efforts in each labor market in Equation (A.7), incentive compatibility conditions in Equation (A.9) and (A.11), and the two thresholds as functions of the number of workers, n_t^R and n_t^{IR} in Equation (A.15) and (A.16), and the resource constraint of Equation (A.17), then we have the following simplified expression for the above equation for indirect utility (under the special case):

$$u(C_t, n_t^R, n_t^{IR}) = \log C_t - z(n_t^R, n_t^{IR}), \qquad \text{(A.19)}$$

where

$$z(n_t^R, n_t^{IR}) = \log N_t - H_t, \qquad \text{(A.20)}$$

with

$$
\begin{aligned}
N_t = n_t^R \cdot &\left(e^{F^R + (1+\sigma^{IR})\left(f(n_t^R, n_t^{IR};)^{\sigma^{IR}} - g(n_t^R, n_t^{IR})^{\sigma^{IR}} \right) + (1+\sigma^R) g(n_t^R, n_t^{IR})^{\sigma^R}} - 1 \right) \\
&+ n_t^{IR} \left(e^{F^{IR} + (1+\sigma^{IR}) f(n_t^R, n_t^{IR})^{\sigma^{IR}}} - 1 \right) + 1, \qquad \text{(A.21)}
\end{aligned}
$$

and

$$
\begin{aligned}
H_t = \eta\sigma^{IR} &\left(f(n_t^R, n_t^{IR})^{\sigma^{IR}+1} - g(n_t^R, n_t^{IR};)^{\sigma^{IR}+1} \right) + \eta\sigma^R g(n_t^R, n_t^{IR})^{\sigma^R+1} \\
&+ \frac{a^2(1+\sigma^{IR})(\sigma^{IR})^2}{2\sigma^{IR}+1} \left(f(n_t^R, n_t^{IR};)^{2\sigma^{IR}+1} - g(n_t^R, n_t^{IR})^{2\sigma^{IR}+1} \right) \\
&+ \frac{a^2(1+\sigma^R)(\sigma^R)^2}{2\sigma^R+1} g(n_t^R, n_t^{IR})^{2\sigma^R+1} \\
&+ a^2(1+\sigma^{IR}) \left(f(n_t^R, n_t^{IR})^{\sigma^{IR}} - g(n_t^R, n_t^{IR};)^{\sigma^{IR}} \right) \\
&\times \left(\sigma^R g(n_t^R, n_t^{IR})^{\sigma^R+1} - \sigma^{IR} g(n_t^R, n_t^{IR})^{\sigma^{IR}+1} \right). \qquad \text{(A.22)}
\end{aligned}
$$

Under more general case, we can simplify the indirect utility of a family as follows:

$$u(C_t, n_t^R, n_t^{IR}) = \log C_t - z(n_t^R, n_t^{IR}), \tag{A.23}$$

where

$$z(n_t^R, n_t^{IR}) = \log N_t - (\log \mathcal{C}) \cdot g(n_t^R, n_t^{IR}) - H_t, \tag{A.24}$$

with $\mathcal{C} = \frac{c_t^{UR}}{c_t^{UIR}}$,

$$\begin{aligned} N_t = {} & n_t^R \cdot \mathcal{C} \cdot \left(e^{F^R + (1+\sigma^R) g(n_t^R, n_t^{IR})^{\sigma^R} - \frac{\eta^R}{(a^R)^2} + \frac{1}{a^R}\sqrt{\overline{X}_t}} - 1 \right) \\ & + n_t^{IR} \left(e^{F^{IR} + (1+\sigma^{IR}) f(n_t^R, n_t^{IR})^{\sigma^{IR}}} - 1 \right) + (\mathcal{C} - 1) \cdot g(n_t^R, n_t^{IR}) + 1, \end{aligned} \tag{A.25}$$

$$\begin{aligned} H_t = {} & \eta^{IR} (1+\sigma^{IR}) f(n_t^R, n_t^{IR})^{\sigma^{IR}} \left(f(n_t^R, n_t^{IR}) - g(n_t^R, n_t^{IR}) \right) \\ & - \eta^{IR} \left(f(n_t^R, n_t^{IR})^{\sigma^{IR}+1} - g(n_t^R, n_t^{IR})^{\sigma^{IR}+1} \right) - \frac{(\eta^R)^2}{2(a^R)^2} g(n_t^R, n_t^{IR}) \\ & + \frac{(a^{IR})^2 (1+\sigma^{IR})(\sigma^{IR})^2}{2\sigma^{IR}+1} f(n_t^R, n_t^{IR})^{2\sigma^{IR}+1} + \frac{(a^R)^2 (1+\sigma^R)(\sigma^R)^2}{2\sigma^R+1} g(n_t^R, n_t^{IR})^{2\sigma^R+1} \\ & - \frac{(a^{IR})^2}{2} (1+\sigma^{IR})^2 f(n_t^R, n_t^{IR})^{2\sigma^{IR}} g(n_t^R, n_t^{IR}) \\ & + (a^{IR})^2 (1+\sigma^{IR}) f(n_t^R, n_t^{IR})^{\sigma^{IR}} g(n_t^R, n_t^{IR})^{\sigma^{IR}+1} \\ & - \frac{(a^{IR})^2}{2} \frac{(1+\sigma^{IR})^2}{2\sigma^{IR}+1} g(n_t^R, n_t^{IR})^{2\sigma^{IR}+1} \\ & + a^R \sigma^R \sqrt{\overline{X}_t} g(n_t^R, n_t^{IR})^{\sigma^R+1} + \frac{\overline{X}_t}{2} g(n_t^R, n_t^{IR}), \end{aligned} \tag{A.26}$$

where $\overline{X}_t$ is defined as below:

$$\overline{X}_t \equiv \left(a^{IR}(1+\sigma^{IR}) \left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}} \right) + \frac{\eta^{IR}}{a^{IR}} \right)^2 + \left(\left(\frac{\eta^R}{a^R} \right)^2 - \left(\frac{\eta^{IR}}{a^{IR}} \right)^2 \right) - 2 \log \mathcal{C}. \tag{A.27}$$

A.2.4 Optimizing Conditions for Families and Firms

A.2.4.1 Family's Problem

Recall that the family's problem is given by:

$$\max_{\{C_t, n_t^R, n_t^{IR}, B_{t+1}, v_t, K_{t+1}\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \underbrace{u(C_t, n_t^R, n_t^{IR})}_{=\log C_t - z(n_t^R, n_t^{IR})} \tag{A.28}$$

subject to

$$P_t C_t + P_t I_t + B_{t+1} \leq (1 + i_{t-1}) B_t + W_t^R n_t^R + W_t^{IR} n_t^{IR} + R_t K_t v_t + \text{Profits, Taxes, and Transfers}_t,$$

$$K_{t+1} = \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t + (1 - \delta(v_t)) K_t,$$

$$\delta(v_t) = \delta_0 + \delta_1 (v_t - 1) + \frac{\delta_2}{2}(v_t - 1)^2,$$

The necessary conditions for the above optimization problem are

$$\lambda_t \equiv P_t \Lambda_t = \frac{1}{C_t},$$

with Λ_t as a Lagrangian multiplier on a budget constraint,

$$z_{n^R}(n_t^R, n_t^{IR}) = \Lambda_t W_t^R,$$

$$z_{n^{IR}}(n_t^R, n_t^{IR}) = \Lambda_t W_t^{IR},$$

where $n_{n^R} \equiv \partial n(n_t^R, n_t^{IR}) / \partial n_t^R$ and $n_{n^{IR}} \equiv \partial n(n_t^R, n_t^{IR}) / \partial n_t^{IR}$.

$$\Lambda_t R_t K_t = \mu_t \delta'(v_t) K_t,$$

with μ_t as a lagrangian multiplier on a capital accumulation process,

$$\Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} (1 + i_t),$$

$$\lambda_t = \mu_t \left(1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \kappa \left(\frac{I_t}{I_{t-1}} - 1\right) \frac{I_t}{I_{t-1}}\right) + \beta \mathbb{E}_t \left[\mu_{t+1} \kappa \left(\frac{I_{t+1}}{I_t} - 1\right) \left(\frac{I_{t+1}}{I_t}\right)^2\right],$$

$$\mu_t = \beta \mathbb{E}_t \left[\Lambda_{t+1} R_{t+1} v_{t+1} + \mu_{t+1} (1 - \delta(v_{t+1}))\right].$$

A.2.4.2 Intermediate Firms' Problem

Each intermediate firm chooses $\{n_t^{IR}(j), n_t^R(j), x_t^R(j), x_t^{IR}(j), P_t(j), \hat{K}_t(j), p_t^{R\to IR}(j), p_t^{IR\to R}(j)\}$ by solving the following profit maximization problem:

$$
\begin{aligned}
\max\ \mathbb{E}_t \sum_{k=0}^{\infty} Q_{t,t+k} \Bigg[& \frac{P_{t+k}(j)}{P_{t+k}} Y_{t+k}(j) - \frac{W_{t+k}^R}{P_{t+k}} n_{t+k}^R(j) - \frac{W_{t+k}^{IR}}{P_{t+k}} n_{t+k}^{IR}(j) - \frac{R_{t+k}}{P_{t+k}} \hat{K}_{t+k}(j) \\
& - \frac{\phi}{2}\left(\frac{P_{t+k}(j)}{P_{t+k-1}(j)} - 1\right)^2 Y_{t+k} - \mathcal{C}(x_{t+k}^R(j); n_{t+k-1}^R(j)) - \mathcal{C}(x_{t+k}^{IR}(j); n_{t+k-1}^{IR}(j)) \\
& - \mathcal{C}(p_{t+k}^{R\to IR}(j); n_{t+k-1}^R(j)) - \mathcal{C}(p_{t+k}^{IR\to R}(j); n_{t+k-1}^{IR}(j)) \Bigg]
\end{aligned}
$$

subject to

$$
\begin{aligned}
Y_{t+k}(j) &= (A_{t+k}\epsilon_{t+k} n_{t+k}(j))^{1-\alpha} \left(\widehat{K}_{t+k}(j)\right)^{\alpha}, \quad \text{with } \widehat{K}_{t+k} = v_{t+k}K_{t+k}, \\
n_{t+k}(j) &= \left((\eta^n)^{\frac{1}{\epsilon_n}} \left(n_{t+k}^R(j)\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\eta^n)^{\frac{1}{\epsilon_n}} \left(n_{t+k}^{IR}(j)\right)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \ \ \eta^n > 0.5, \\
n_{t+k}^R(j) &= \left(\rho^R + x_{t+k}^R(j) - p_t^{R\to IR}(j)\right) n_{t+k-1}^R(j) + p_{t+k}^{IR\to R}(j) n_{t+k-1}^{IR}(j), \ \ \lambda_1 < 1, \\
n_{t+k}^{IR}(j) &= \left(x_{t+k}^{IR}(j) - p_{t+k}^{IR\to R}(j)\right) n_{t+k-1}^{IR}(j) + p_{t+k}^{R\to IR}(j) n_{t+k-1}^R(j), \ \ \lambda_2 > 1. \\
Y_{t+k}(j) &= \left(\frac{P_{t+k}(j)}{P_{t+k}}\right)^{-\epsilon_p} Y_{t+k}, \ \ \forall\, k.
\end{aligned}
$$

The first order necessary conditions are

$$
\begin{aligned}
\frac{W_{t+k}^{IR}}{P_{t+k}} + \lambda_{t+k}^{IR}(j) =& \varphi_{t+k}(j) MPN_{t+k}^{IR}(j) \\
& - \mathbb{E}_{t+k}\left[Q_{t+k,t+k+1} \cdot \left\{\frac{\gamma}{2}\left(x_{t+k+1}^{IR}\right)^2 + \frac{\tilde{\nu}}{2}\left(p_{t+k+1}^{IR\to R}(j)\right)^2\right\} Y_{t+k+1}\right] \\
& + \mathbb{E}_{t+k}\left[Q_{t+k,t+k+1}\left\{x_{t+k+1}^{IR}(j)\lambda_{t+k+1}^{IR}(j) + \left(\lambda_{t+k+1}^R(j) - \lambda_{t+k+1}^{IR}(j)\right) p_{t+k+1}^{IR\to R}(j)\right\}\right],
\end{aligned}
$$

$$
\begin{aligned}
\frac{W_{t+k}^{R}}{P_{t+k}} + \lambda_{t+k}^{R}(j) =& \varphi_{t+k}(j) MPN_{t+k}^{R}(j) \\
& - \mathbb{E}_{t+k}\left[Q_{t+k,t+k+1} \cdot \left\{\frac{\kappa}{2}\left(x_{t+k+1}^{R}(j)\right)^2 + \frac{\tilde{\theta}}{2}\left(p_{t+k+1}^{R\to IR}(j)\right)^2\right\} Y_{t+k+1}\right] \\
& + \mathbb{E}_{t+k}\left[Q_{t+k,t+k+1}\left\{\left(\rho^R + x_{t+k+1}^R(j)\right)\lambda_{t+k+1}^R(j) + \left(\lambda_{t+k+1}^{IR}(j) - \lambda_{t+k+1}^{R}(j)\right) p_{t+k+1}^{R\to IR}(j)\right\}\right],
\end{aligned}
$$

where MPN^R and MPN^{IR} denote marginal productivity of regular and irregular types, respectively. That is, $MPN_t^R = \partial Y_t(j)/\partial n_t^R(j)$ and $MPN^{IR} = \partial Y_t(j)/\partial n_t^{IR}(j)$.

$$\begin{aligned}
\lambda_{t+k}^{IR}(j) &= \gamma x_{t+k}^{IR} Y_{t+k}, \\
\lambda_{t+k}^{R}(j) &= \kappa x_{t+k}^{R} Y_{t+k} \\
\lambda_{t+k}^{R}(j) - \lambda_{t+k}^{IR}(j) &= \tilde{\nu} p_{t+k}^{IR\to R}(j) Y_{t+k}, \\
\lambda_{t+k}^{IR}(j) - \lambda_{t+k}^{R}(j) &= \tilde{\theta} p_{t+k}^{R\to IR}(j) Y_{t+k},
\end{aligned}$$

$$\begin{aligned}
0 =& (1-\epsilon_p)\frac{Y_{t+k}(j)}{P_{t+k}} - \phi\left(\frac{P_{t+k}(j)}{P_{t+k-1}(j)} - 1\right)\frac{Y_{t+k}}{P_{t+k-1}(j)} + \epsilon_p \varphi_{t+k}(j)\frac{Y_{t+k}(j)}{P_{t+k}(j)} \\
&+ \mathbb{E}_{t+k}\left[Q_{t+k,t+k+1}\phi\left(\frac{P_{t+k+1}(j)}{P_{t+k}(j)} - 1\right)\frac{P_{t+k+1}(j)}{P_{t+k}(j)}\frac{Y_{t+k+1}}{P_{t+k}(j)}\right].
\end{aligned}$$

A.2.5 Some Comparative Statics

This section examines the role of the two thresholds in terms of changing the number of regular and irregular workers. Under the parameterizations of this paper, when $\bar{\theta}_t^{IR}$ increases, both the number of regular and irregular workers. In contrast to this, when $\bar{\theta}_t^{R}$ increases, while the number of regular workers increases, the number of irregular workers decreases. Because firms adjust the total amount of labor by changing the composition of workers, that is, by increasing one type of labor and decreasing the other type of labor, it is $\bar{\theta}_t^{R}$ that frequently moves over the business cycle, not $\bar{\theta}_t^{IR}$.

Under the special case of $a^R = a^{IR} = a$, $\eta^R = \eta^{IR} = \eta$, and $c_t^{UR} = c_t^{UIR}$,

$$\begin{aligned}
\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} &= a^2\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\bar{\theta}_t^R > 0, \\
\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} &= \eta + a^2\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}(\bar{\theta}_t^{IR} - \bar{\theta}_t^R) > 0, \\
\frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} &= \eta + a^2(1+\sigma^{IR})(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}) + \underbrace{a^2\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^{R}\right)^{\sigma^R} - a^2\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}}_{>0,\ \text{with}\ \sigma^R > \sigma^{IR}} > 0 \\
\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}} &= -\eta - a^2(1+\sigma^{IR})(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}) < 0.
\end{aligned}$$

Under more general case of $a^R \neq a^{IR}$, $\eta^R \neq \eta^{IR}$, and $c_t^{UR} \neq c_t^{UIR}$,

$$\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} = a^{IR}\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\left(\frac{a^R\bar{\theta}_t^R}{\sqrt{\overline{X}_t}}\underbrace{\left(a^{IR}(1+\sigma^{IR})(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}-\left(\bar{\theta}_t^R\right)^{\sigma^{IR}})\right)}_{>0,\ \text{with}\ \bar{\theta}_t^{IR}\geq\bar{\theta}_t^R}\right) > 0,$$

$$\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} = \eta^{IR} + \left(a^{IR}\right)^2\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}(\bar{\theta}_t^{IR}-\bar{\theta}_t^R) > 0,$$

$$\frac{\partial n_t^R}{\partial \bar{\theta}_t^R} = \underbrace{a^R\sqrt{\overline{X}_t} + \left(a^R\right)^2\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R} - \left(a^Ra^{IR}\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^R\right)^{\sigma^{IR}}\right)\underbrace{\frac{\left(a^{IR}(1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}-\left(\bar{\theta}_t^R\right)^{\sigma^{IR}}+\frac{\eta^{IR}}{a^{IR}}\right)\right)}{\sqrt{\overline{X}_t}}}_{\approx 1,\ \text{with parameterizations in this paper}}}_{>0,\ \text{with}\ \sigma^R<<\sigma^{IR}}$$
$$> 0,$$

$$\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} = -\eta^{IR} - \left(a^{IR}\right)^2(1+\sigma^{IR})(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}-\left(\bar{\theta}_t^R\right)^{\sigma^{IR}}) < 0,$$

where

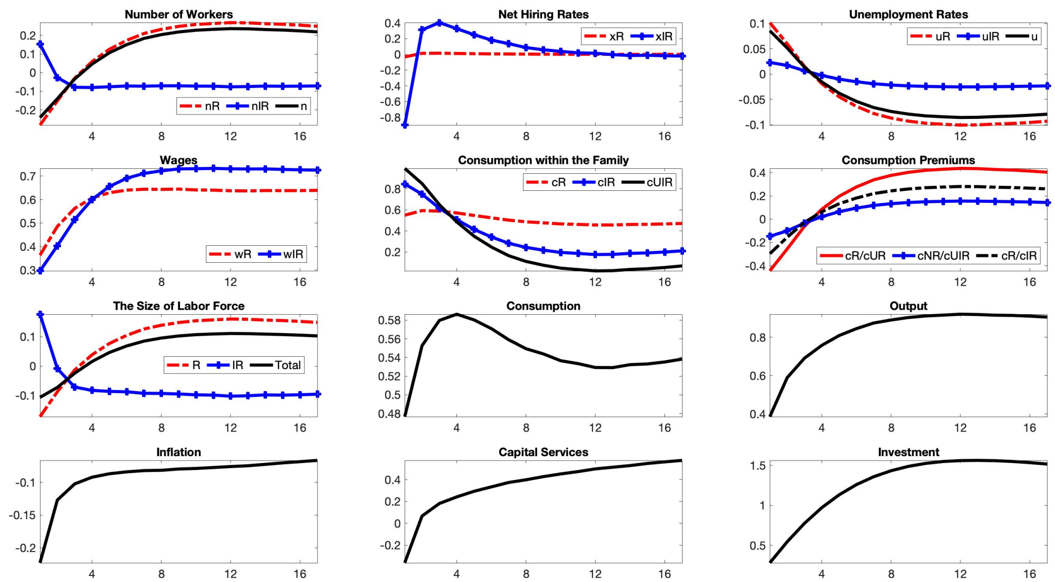
$$\begin{aligned}
\overline{X}_t &\equiv \left(a^{IR}(1+\sigma^{IR})(\left(\bar{\theta}_t^{IR}]\right)^{\sigma^{IR}}-\left(\bar{\theta}_t^R\right)^{\sigma^{IR}})+\frac{\eta^{IR}}{a^{IR}}\right)^2+\left(\frac{\eta^R}{a^R}\right)^2-\left(\frac{\eta^{IR}}{a^{IR}}\right)^2-2\log\mathcal{C}\\
&=\left(a^{IR}(1+\sigma^{IR})(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}-\left(\bar{\theta}_t^R\right)^{\sigma^{IR}})\right)^2+2\eta^{IR}(1+\sigma^{IR})\underbrace{(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}-\left(\bar{\theta}_t^R\right)^{\sigma^{IR}})}_{>0,\ \text{with}\ \bar{\theta}_t^{IR}>\bar{\theta}_t^R}\\
&+\underbrace{\left(\frac{\eta^R}{a^R}\right)^2-2\log\mathcal{C}}_{\geq 0,\ \text{with parameterizations of this paper}}\\
&\geq 0,
\end{aligned}$$

under the parameterizations of this paper.

A.2.6 Impulse Responses

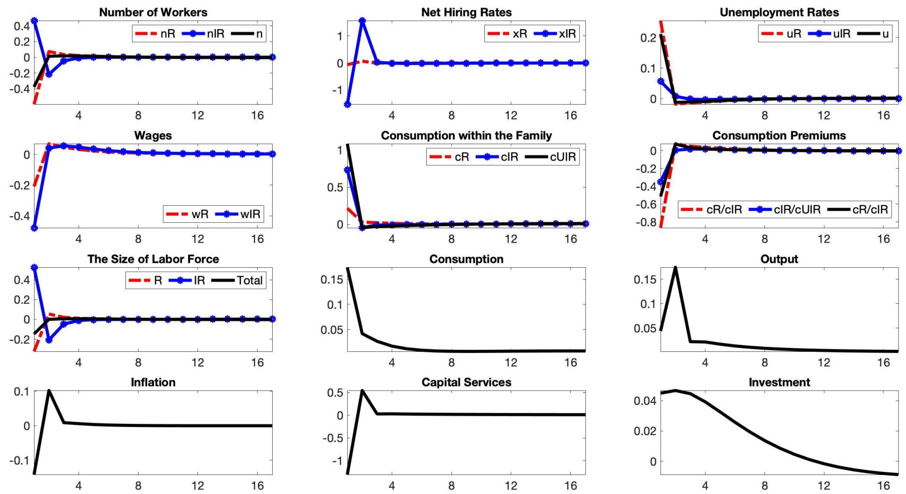
This section shows the responses of model variables to a one-standard deviation of persistent and technology shocks, monetary policy shock, and government spending shock.

Figure A.15: Responses to Persistent Technology Shocks



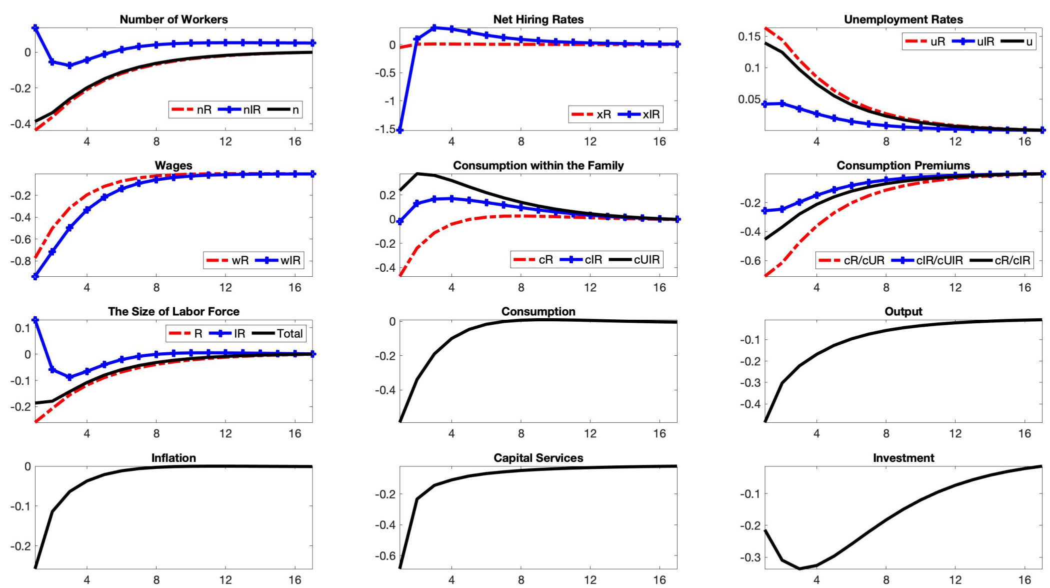
Note: This figure plots the responses of the variables in the baseline model to a one standard deviation of persistent technology shock, ϵ^{A^P} from the baseline model.

Figure A.16: Responses to Transitory Technology Shocks



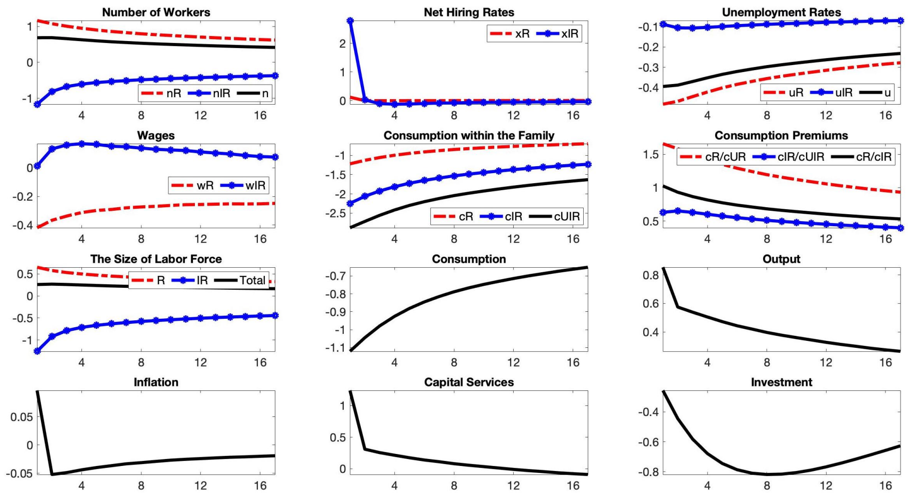
Note: This figure plots the responses of the variables in the baseline model to a one standard deviation of transitory technology shock, A^T from the baseline model.

Figure A.17: Responses to Monetary Policy Shocks



Note: This figure plots the responses of the variables in the baseline model to a one standard deviation of monetary policy shock, ϵ^i from the baseline model.

Figure A.18: Responses to Government Spending Shocks



Note: This figure plots the responses of the variables in the baseline model to a one standard deviation of government spending shock, ϵ^g from the baseline model.

A.2.7 In Relation to Extensive and Intensive Margins

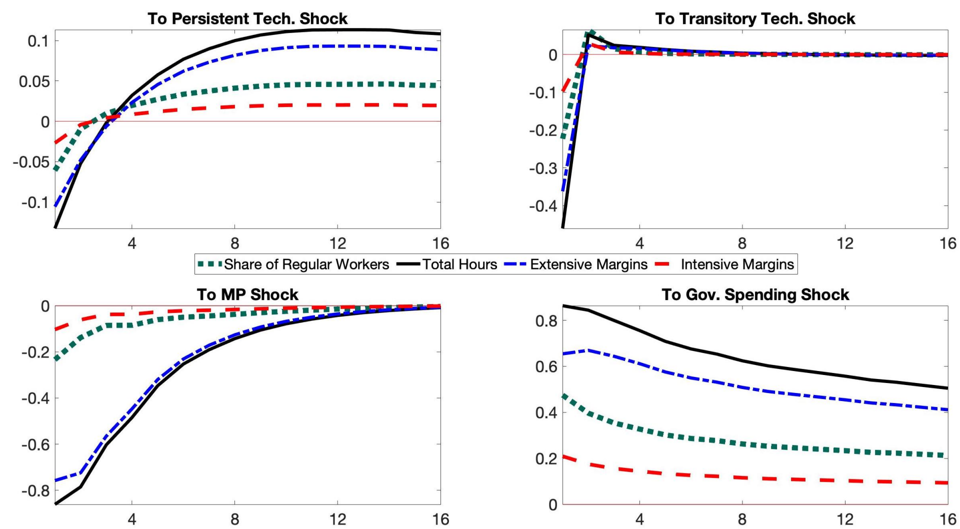
This section examines the relationship between the "composition" margin and the other two margins, extensive and intensive margins that the literature has focused on. Analysis from the CPS and the baseline model has shown the importance of the composition margin over the business cycle. When there is more than one type of labor, firms adjust the total amount of labor input by *changing the composition of workers,* which I call this "the composition margin."

It is, however, true that the composition margin is not separable from the other two margins, extensive (the *number* of workers) and intensive margins (*hours* worked per worker). The composition margin is, rather, closely connected to the other two margins. In fact, this composition margin helps to better understand the behaviors of the other two margins over the business cycle. For example, consider the case where a firm decides to fire two full-time workers and hire two part-timers. As the total number of workers is fixed, there is no change in the extensive margin. The intensive margin, however, is likely to decrease. As another example, if a firm decides to replace two full-time workers with four part-time workers, then there is no change in total labor hours, but the number of workers increases, and the hours per worker decreases. In these cases, it is helpful to examine *the composition margin* to better understand the behaviors of extensive and intensive margins. Moreover, there are some cases where there is no change in both extensive and intensive margins, but the composition margins. For example, if a firm decides to replace two permanent full-time workers with two full-time interns, it is likely that there is no change in the number of workers, total hours of work, and hours per worker. But this would decrease the total amount of labor input, as interns have lower productivity than regular workers. This could have important welfare implications for workers. Section **??** examines this implication for welfare.

To illustrate this point, Figure A.19 plots the responses of extensive, intensive, the composition margins, and the total labor hours to each exogenous shocks. In fact, the model assumes "indivisible labor." Workers can either work or not in each labor market. Therefore, there is no intensive margin. However, if we assume that regular workers work full time and irregular workers work part-time, then we can roughly examine the responses of "hours per worker (intensive margin)," and how this intensive margin is related to the composition margin.

From the figure above, it is first noticeable that most variations in the total amount of labor hours reflect the variations in *extensive* margins, but not completely. This is consistent with previous studies on indivisible labor that variations in the total hours worked are mostly explained by the variations in the extensive margin. (see, for example, Hansen, 1985; Rogerson, 1988; Krusell et al., 2008) The extensive margin, however, does not completely explain the changes in the total labor hours. The gaps between the two are explained by the changes in the composition margin: when the share of regular workers rises, the total amount of labor input increases much more than the total number of workers, and vice versa. Another interesting observation is the close relationship between the composition margin and intensive margins. The responses of intensive margins are largely explained by the changes in the share of regular workers, the composition margin. This is consistent with the finding from the recent study of Borowczyk-Martins and Lalé (2019) that fluctuations in part-time employment play a major role in variations of intensive margins over the business cycles in the United States and the United Kingdom. The baseline model successfully replicates this empirical observation.

Figure A.19: Responses of Three Margins of Labor Adjustment to Exogenous Shocks

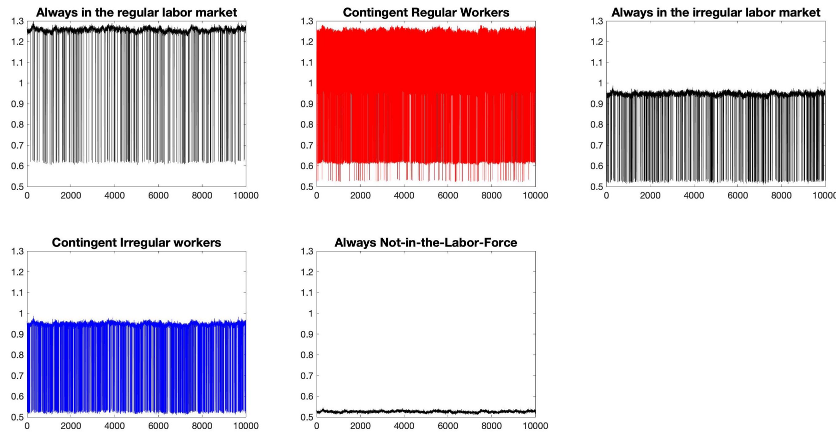


Note: This figure shows the responses of the three margins: (i) extensive margins (the number of workers), (ii) intensive margins (hours worked per workers), and (iii) the composition margin (the share of regular workers) to each exogenous shocks in the baseline model. Each panel shows the responses to each exogenous shocks: persistent technology shock, ϵ^{A^P}, transitory technology shock, ϵ^{A^T}, monetary policy shocks, ϵ^i, and government spending shocks, ϵ^g. The responses of the total labor hours are represented as black solid lines, those of extensive margins are denoted as blue dash-dot lines, those of intensive margins are shown as red dash lines, and those of the composition margins are denoted as green dotted lines.

A.2.8 Welfare Costs of the Business Cycle per Worker

Section 1.6 shows that contingent workers pay substantially larger costs of economic fluctuations than stayers, and among contingent workers, contingent "regular" workers pay substantially larger costs of economic fluctuations. This is because contingent workers face larger risks regarding their labor market status over the business cycle. This section illustrates this by showing the stream (time-series of a random 2,000 consecutive periods) of consumption and disutility from working of a worker in each group from the simulation.

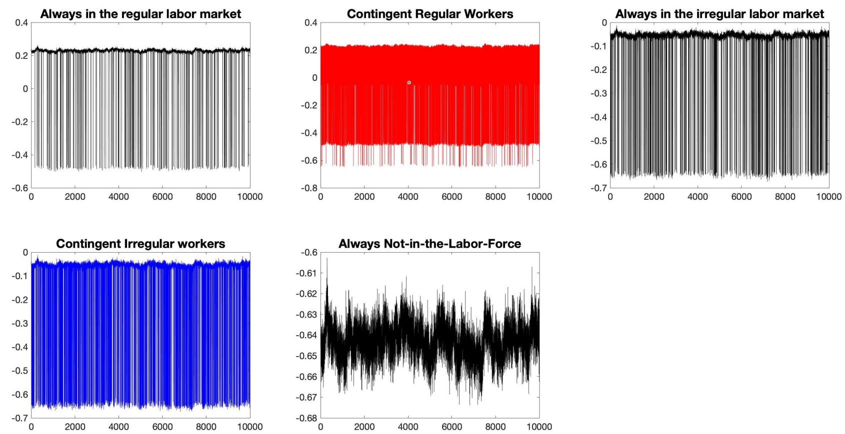
Figure A.20: The Stream of Consumption for a Worker in Each Group



Note: This figure illustrates streams of consumption of a worker in each five groups: (i) Workers who are always in the regular labor market, (ii) Contingent regular workers who frequently move between the regular and irregular labor market, (iii) Workers who are always in the irregular labor market, (iv) Contingent irregular workers who frequently in and out of the labor force, and (v) Workers who are always out of the labor force. This figure illustrates that contingent workers experience larger volatility of consumption than stayers.

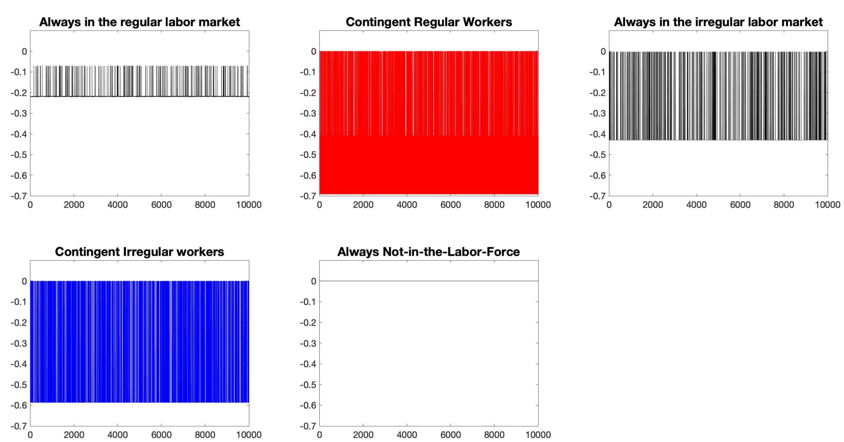
Figure A.20 - A.22 illustrate the streams (time-series of a random 2000 consecutive periods) of consumption, utility from consumption, and disutility from supplying labor of a worker in each five group for 2,000 periods: (i) Workers who are always in the regular labor market, (ii) Contingent regular workers who frequently move between the regular and irregular labor market, (iii) Workers who are always in the irregular labor market, (iv) Contingent irregular workers who frequently in and out of the labor force, and (v) Workers who are always out of the labor force. All panels in Figures A.20 and A.22 share the same y-axes. These figures illustrate that contingent regular and irregular workers experience substantially larger volatility of consumption and disutilities from supplying labor than stayers. Among the two contingent workers, contingent regular workers experience the largest volatility in consumption and labor supply disutility.

Figure A.21: The Stream of Utility from Consumption for a Worker in Each Group



Note: This figure illustrates streams of consumption of a worker in each five group: (i) Workers who are always in the regular labor market, (ii) Contingent regular workers who frequently move between the regular and irregular labor market, (iii) Workers who are always in the irregular labor market, (iv) Contingent irregular workers who frequently in and out of the labor force, and (v) Workers who are always out of the labor force. This figure illustrates that contingent workers experience larger volatility of consumption than stayers.

Figure A.22: The Stream of Disutility from Supplying Labor for a Worker in Each Group

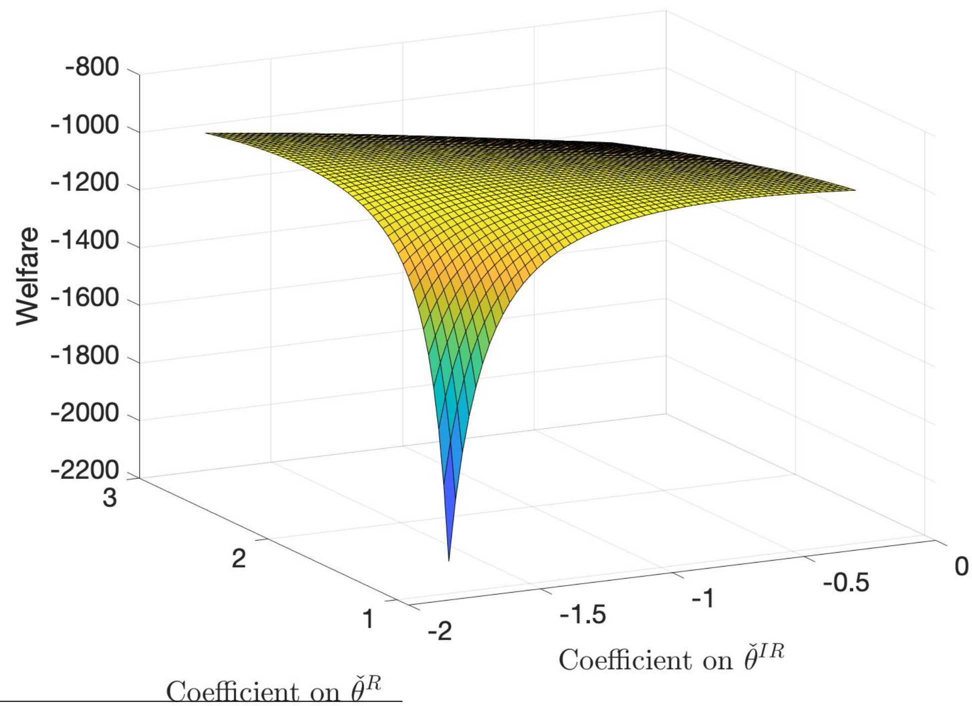


Note: This figure illustrates streams of disutilities from supplying labor (the sum of disutility from working and costs of exerting efforts to find a job) for a worker in each five groups: (i) Workers who are always in the regular labor market, (ii) Contingent regular workers who frequently move between the regular and irregular labor market, (iii) Workers who are always in the irregular labor market, (iv) Contingent irregular workers who frequently in and out of the labor force, and (v) Workers who are always out of the labor force. This figure illustrates that contingent workers experience larger volatility of disutilities from supplying labor than stayers.

A.2.9 Optimal Monetary Policy

This section presents how the weights, $\phi_{\bar{\theta}^R}$ and $\phi_{\bar{\theta}^{IR}}$, affect welfare (holding $\phi_\pi = 1.5$). The figure below shows the level of the representative family's welfare for each combination of $(\phi_{\bar{\theta}^{IR}}, \phi_{\bar{\theta}^R})$. Maximum welfare is achieved with $\phi^*_{\bar{\theta}^{IR}} = -0.89$ and $\phi^*_{\bar{\theta}^R} = 1.24$. That is, the central bank wants to put higher weight on stabilizing $\bar{\theta}^R_t$ than stabilizing $\bar{\theta}^{IR}_t$. This is because contingent *regular* workers who are marginally attached to the *regular* labor market pay the highest welfare costs of economic fluctuations according to the analysis from Section 1.6. Moreover, there is a larger mass of contingent regular workers than contingent irregular workers with more volatile $\bar{\theta}^R_t$. Therefore, the central bank wants to put higher weight on the stabilization of $\bar{\theta}^R_t$ than the stabilization of $\bar{\theta}^{IR}_t$.

Figure A.23: Optimal Monetary Policy



Note: This figure shows the level of the representative family's welfare per each combination of $(\phi_{\bar{\theta}^{IR}}, \phi_{\bar{\theta}^R})$ in the policy rule of Equation (1.16).

A.2.10 Second-order Approximation of Welfare Function

This section presents the second-order approximation of the welfare function which is the life-time expected utility of the representative family's utility function. To that end, consider a “simpler" version of the model in the main text where there is no capital, no promotion and demotion, no adjustment costs for regular and irregular workers, and constant returns to scale of production with only productivity shocks as an exogenous force. Then the economy reduces to the following system of optimal conditions, law of motions, and market clearing conditions:

$$C_t z_{n_t^R} = w_t^R, \tag{A.29}$$

$$C_t z_{n_t^{IR}} = w_t^{IR}, \tag{A.30}$$

$$1 = \beta \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \frac{(1+i_t)}{1+\pi_{t+1}} \right], \tag{A.31}$$

$$w_t^R = \varphi_t A_t \eta^{\frac{1}{\epsilon}} \left(\frac{n_t^R}{n_t} \right)^{-\frac{1}{\epsilon}}, \tag{A.32}$$

$$w_t^{IR} = \varphi_t A_t (1-\eta)^{\frac{1}{\epsilon}} \left(\frac{n_t^{IR}}{n_t} \right)^{-\frac{1}{\epsilon}}, \tag{A.33}$$

$$n_t = \left(\eta^{\frac{1}{\epsilon}} \left(n_t^R \right)^{\frac{\epsilon-1}{\epsilon}} + (1-\eta)^{\frac{1}{\epsilon}} \left(n_t^{IR} \right)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{A.34}$$

$$(1-\epsilon_p) Y_t + \epsilon_p \varphi_t Y_t = \phi_p (1+\pi_t)\pi_t - \beta \phi_p \mathbb{E}_t \left[\frac{C_t}{C_{t+1}} \pi_{t+1} (1+\pi_{t+1}) \right], \tag{A.35}$$

$$Y_t = A_t n_t, \tag{A.36}$$

$$Y_t = C_t + \frac{\phi_p}{2} \pi_t^2, \tag{A.37}$$

$$i_t = \phi_\pi \pi_t + \phi_u (\log Y_t - \log \bar{Y}), \tag{A.38}$$

$$\log A_t = \rho_A \log A_{t-1} + \epsilon_t^A. \tag{A.39}$$

Households' welfare is defined as follows:

$$\mathcal{U} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \underbrace{\left(\log C_t - Z(n_t^R, n_t^{IR}) \right)}_{=U_t}. \tag{A.40}$$

First, the second-order approximation of the period-by-period utility, U_t can be written as follows:

$$\frac{U_t-\overline{U}}{U_{\bar{C}}\bar{C}} \approx \check{C}_t - \overline{Z_{n^R}}\,\overline{n^R}\left(\check{n}_t^R + \frac{1+\omega_{n^R}}{2}\left(\check{n}_t^R\right)^2\right) - \overline{Z_{n^{IR}}}\,\overline{n^{IR}}\left(\check{n}_t^{IR} + \frac{1+\omega_{n^{IR}}}{2}\left(\check{n}_t^{IR}\right)^2\right)$$
$$- \overline{Z_{n^R n^{IR}}}\,\overline{n^R}\,\overline{n^{IR}}\check{n}_t^R\check{n}_t^{IR}$$

where $\check{x}_t$ denotes the percentage deviations from its steady-state value of x, $\overline{x}$ is the steady state value of x, and $\omega_{n^R} = \frac{\overline{Z_{n^R n^R}}\,\overline{n^R}}{\overline{Z_{n^R}}}$, $\omega_{n^{IR}} = \frac{\overline{Z_{n^{IR} n^{IR}}}\,\overline{n^{IR}}}{\overline{Z_{n^{IR}}}}$.

For the disutility from each-type of labor, first, we need to combine each type of labor demand and supply. The log-linearized versions of the two can be then written as

$$\check{C}_t + \omega_{n^R}\check{n}_t^R = \check{\varphi}_t + \check{A}_t - \frac{1}{\epsilon}\left(\check{n}_t^R - \check{n}_t\right), \tag{A.41}$$

$$\check{C}_t + \omega_{n^{IR}}\check{n}_t^{IR} = \check{\varphi}_t + \check{A}_t - \frac{1}{\epsilon}\left(\check{n}_t^{IR} - \check{n}_t\right), \tag{A.42}$$

Combining the two then generates the relationship between $\check{n}_t^R$ and $\check{n}_t^{IR}$:

$$\check{n}_t^{IR} = \frac{1+\epsilon\omega_{n^R}}{1+\epsilon\omega_{n^{IR}}}\check{n}_t^R. \tag{A.43}$$

Then from Equation (A.34), $\overline{CZ_{n^R}}\,\overline{n^R} = \frac{\epsilon_p-1}{\epsilon_p}\eta^{\frac{1}{\epsilon}}\left(\frac{\overline{n^R}}{\overline{n}}\right)^{\frac{\epsilon-1}{\epsilon}}\overline{Y}$, and $\overline{CZ_{n^{IR}}}\,\overline{n^{IR}} = \frac{\epsilon_p-1}{\epsilon_p}(1-\eta)^{\frac{1}{\epsilon}}\left(\frac{\overline{n^{IR}}}{\overline{n}}\right)^{\frac{\epsilon-1}{\epsilon}}\overline{Y}$, we have

$$\check{n}_t^R = \frac{\epsilon_p-1}{\epsilon_p}\left(\frac{1+\epsilon\omega_{n^{IR}}}{(1+\epsilon\omega_{n^{IR}})\overline{Z_{n^R}}\,\overline{n^R} + (1+\epsilon\omega_{n^R})\overline{Z_{n^{IR}}}\,\overline{n^{IR}}}\right)\check{n}_t, \tag{A.44}$$

$$\check{n}_t^{IR} = \frac{\epsilon_p-1}{\epsilon_p}\left(\frac{1+\epsilon\omega_{n^R}}{(1+\epsilon\omega_{n^{IR}})\overline{Z_{n^R}}\,\overline{n^R} + (1+\epsilon\omega_{n^R})\overline{Z_{n^{IR}}}\,\overline{n^{IR}}}\right)\check{n}_t. \tag{A.45}$$

Moreover, if we comebine (A.41) after multiplying both sides by $\frac{\epsilon_p-1}{\epsilon_p}\eta^{\frac{1}{\epsilon}}\left(\frac{\overline{n^R}}{\overline{n}}\right)^{\frac{\epsilon-1}{\epsilon}}\overline{Y} = \overline{CZ_{n^R}}\,\overline{n^R}$ and (A.42) after multiplying both sides by $\frac{\epsilon_p-1}{\epsilon_p}(1-\eta)^{\frac{1}{\epsilon}}\left(\frac{\overline{n^{IR}}}{\overline{n}}\right)^{\frac{\epsilon-1}{\epsilon}}\overline{Y} = \overline{CZ_{n^{IR}}}\,\overline{n^{IR}}$, then we have

$$\check{C}_t + \frac{\epsilon_p}{\epsilon_p-1}\left(\overline{Z_{n^R}}\,\overline{n^R}\omega_{n^R}\check{n_t^R} + \overline{Z_{n^{IR}}}\,\overline{n^{IR}}\omega_{n^{IR}}\check{n_t^{IR}}\right) = \check{\varphi}_t + \check{A}_t. \tag{A.46}$$

Then from Equations (A.44) and (A.45), we have

$$\overline{Z_{n^R}}\,\overline{n^R}\omega_{n^R}\check{n}_t^R + \overline{Z_{n^{IR}}}\,\overline{n^{IR}}\omega_{n^{IR}}\check{n}_t^{IR} = \frac{\epsilon_p - 1}{\epsilon_p}\underbrace{\left(\frac{\overline{Z_{n^R}}\,\overline{n^R}\omega_{n^R}\left(1+\epsilon\omega_{n^{IR}}\right) + \overline{Z_{n^{IR}}}\,\overline{n^{IR}}\omega_{n^R}\left(1+\epsilon\omega_{n^R}\right)}{(1+\epsilon\omega_{n^{IR}})\overline{Z_{n^R}}\,\overline{n^R} + (1+\epsilon\omega_{n^R})\overline{Z_{n^{IR}}}\,\overline{n^{IR}}}\right)}_{\equiv\mathcal{C}_\omega}\check{n}_t.$$

Now, combining this with the first-order approximation of the market clearing condition (Equation (A.37): $\check{C}_t + \frac{\overline{g}}{1-\overline{g}}\check{g}_t = \check{Y}_t$) and the first-order approximation of the production function (Equation (A.36):$\check{n}_t = \check{Y}_t - \check{A}_t$) generates

$$\begin{aligned}\check{\varphi}_t &= \check{C}_t - \check{A}_t + \mathcal{C}_\omega\left(\check{Y}_t - \check{A}_t\right),\\ &= \left(1+\mathcal{C}_\omega\right)\left(\check{Y}_t - \check{A}_t\right).\end{aligned} \tag{A.47}$$

If we combine Equation (A.41) and Equation (A.42) with the first-order approximation of the market clearing condition (Equation (A.37): $\check{C}_t = \check{Y}_t$) and the first-order approximation of the production function (Equation (A.36): $\check{n}_t = \check{Y}_t - \check{A}_t$), then we have

$$\begin{aligned}\check{n}_t^R &= \frac{\epsilon}{1+\epsilon\omega_{n^R}}\left(\check{\varphi}_t - \check{C}_t + \check{A}_t + \frac{1}{\epsilon}\left(\check{Y}_t - \check{A}_t\right)\right)\\ &= \frac{1+\epsilon\mathcal{C}_\omega}{1+\epsilon\omega_{n^R}}\left(\check{Y}_t - \check{A}_t\right),\end{aligned} \tag{A.48}$$

$$\begin{aligned}\check{n}_t^{IR} &= \frac{\epsilon}{1+\epsilon\omega_{n^{IR}}}\left(\check{\varphi}_t - \check{C}_t + \check{A}_t + \frac{1}{\epsilon}\left(\check{Y}_t - \check{A}_t\right)\right),\\ &= \frac{1+\epsilon\mathcal{C}_\omega}{1+\epsilon\omega_{n^{IR}}}\left(\check{Y}_t - \check{A}_t\right).\end{aligned} \tag{A.49}$$

Consider now the first-order approximation and the second-order approximation of the price-setting equation, Equation (A.35). The log-linearized equation can be written as

$$\pi_t + \beta\mathbb{E}_t[\pi_{t+1}] = \frac{(\epsilon_p - 1)\overline{Y}}{\phi_p}\check{\varphi}_t + \mathcal{O}_2. \tag{A.50}$$

Following Damjanovic and Nolan (2011), if we re-arrange this New Keynesian Phillips Curve (NKPC) with $T_t = \varphi_t Y_t$, we have

$$\phi_p\pi_t - \beta\phi_p\mathbb{E}_t[\pi_{t+1}] = (1-\epsilon_p)\overline{Y}(\check{Y}_t - \check{T}_t) + \mathcal{O}_2. \tag{A.51}$$

Meanwhile, the second-order approximation of the NKPC can be written as

$$\begin{aligned}&\left(\check{Y}_t - \check{T}_t\right) + \frac{\phi_p}{(\epsilon_p - 1)\overline{Y}}\left(\pi_t - \beta\mathbb{E}_t[\pi_{t+1}]\right) + \frac{1}{2}\left(\check{Y}_t^2 - \check{T}_t^2\right)\\ &\quad + \frac{3\phi_p}{2(\epsilon_p - 1)\overline{Y}}\left(\pi_t^2 - \beta\mathbb{E}_t[\pi_{t+1}^2]\right) - \beta\frac{\phi_p}{(\epsilon_p - 1)\overline{Y}}\mathbb{E}_t\left[(\check{C}_t - \check{C}_{t+1})\pi_{t+1}\right] = \mathcal{O}_3.\end{aligned} \tag{A.52}$$

Solving forward the above equation then generates

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\check{Y}_t - \check{T}_t\right) + \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\check{Y}_t^2 - \check{T}_t^2\right) = \beta \frac{\phi_p}{(\epsilon_p - 1)\overline{Y}} \mathbb{E}_t \left[(\check{C}_t - \check{C}_{t+1})\pi_{t+1}\right] + t.i.p. + \mathcal{O}_3, \tag{A.53}$$

where t.i.p. stands for terms independent from policy. Then if we combine the second-order approximation of the market clearing condition and $\check{\varphi}_t = \mathcal{C}_\omega \left(\check{Y}_t - \check{A}_t\right) + \check{C}_t - \check{A}_t$,

$$\begin{aligned} \check{T}_t &= (\mathcal{C}_\omega + 1)\left(\check{Y}_t - \check{A}_t\right) + \check{C}_t \\ &= (\mathcal{C}_\omega + 1)\left(\check{Y}_t - \check{A}_t\right) - \frac{\phi_p}{2\overline{Y}}\pi_t^2 + \check{Y}_t. \end{aligned}$$

Therefore,

$$\check{T}_t - \check{Y}_t = (\mathcal{C}_\omega + 1)\left(\check{Y}_t - \check{A}_t\right) - \frac{\phi_p}{2\overline{Y}}\pi_t^2.$$

Moreover, from the above expression for $\check{T}_t$,

$$\check{Y}_t^2 - \check{T}_t^2 = -\left((\mathcal{C}_\omega + 1)\left(\check{Y}_t - \check{A}_t\right)\right)^2 - 2\left((\mathcal{C}_\omega + 1)\left(\check{Y}_t - \check{A}_t\right)\right)\check{Y}_t + t.i.p. + \mathcal{O}_3.$$

So we have

$$\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\check{Y}_t^2 - \check{T}_t^2\right) - \beta \frac{\phi_p}{(\epsilon_p - 1)\overline{Y}} \mathbb{E}_t \left[(\check{C}_t - \check{C}_{t+1})\pi_{t+1}\right] = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\check{T}_t - \check{Y}_t\right) + t.i.p. + \mathcal{O}_3,$$

which is

$$\begin{aligned} -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left((\mathcal{C}_\omega + 1)\left(\check{Y}_t - \check{A}_t\right)\right)^2 - \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left((\mathcal{C}_\omega + 1)\left(\check{Y}_t - \check{A}_t\right)\right)\check{Y}_t \\ - \beta \frac{\phi_p}{(\epsilon_p - 1)\overline{Y}} \mathbb{E}_t \left[(\check{C}_t - \check{C}_{t+1})\pi_{t+1}\right] + \frac{\phi_p}{2\overline{Y}}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_t^2 \\ = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1 + \mathcal{C}_\omega)\left(\check{Y}_t - \check{A}_t\right) + t.i.p. + \mathcal{O}_3 \end{aligned}$$

Now if we multiply $\check{C}_t$ to the NKPC, Equation (A.51), solve forward, and re-arrange then we have

$$-\beta \frac{\phi_p}{(\epsilon_p - 1)\overline{Y}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left[(\check{C}_t - \check{C}_{t+1})\pi_{t+1}\right] = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left((\mathcal{C}_\omega + 1)\left(\check{Y}_t - \check{A}_t\right)\right)\check{Y}_t + t.i.p. + \mathcal{O}_3,$$

Therefore, the above becomes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\check{Y}_t - \check{A}_t\right) + t.i.p. + \mathcal{O}_3 = \frac{\phi_p}{2\overline{Y}\left(1 + \mathcal{C}_\omega\right)}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_t^2 - \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (1 + \mathcal{C}_\omega)\left(\check{Y}_t - \check{A}_t\right)^2.$$

Combining all, the second-order approximation of the disutility from supplying labor can be written as

$$
\begin{aligned}
&\overline{Z_{n^R}}\,\overline{n^R}\left(\check{n}_t^R+\frac{1+\omega_{n^R}}{2}\left(\check{n}_t^R\right)^2\right)+\overline{Z_{n^{IR}}}\,\overline{n^{IR}}\left(\check{n}_t^{IR}+\frac{1+\omega_{n^{IR}}}{2}\left(\check{n}_t^{IR}\right)^2\right)+\overline{Z_{n^Rn^{IR}}}\,\overline{n^R n^{IR}}\check{n}_t^R\check{n}_t^{IR}\\
&=(1+\epsilon\mathcal{C}_\omega)\left(\frac{\overline{Z_{n^R}}\,\overline{n^R}}{1+\epsilon\omega_{n^R}}+\frac{\overline{Z_{n^{IR}}}\,\overline{n^{IR}}}{1+\epsilon\omega_{n^{IR}}}\right)\left(\check{Y}_t-\check{A}_t\right)\\
&\quad+\frac{1}{2}\left(\frac{\overline{Z_{n^R}}\,\overline{n^R}(1+\omega_{n^R})}{(1+\epsilon\omega_{n^R})^2}+\frac{\overline{Z_{n^{IR}}}\,\overline{n^{IR}}(1+\omega_{n^{IR}})}{(1+\epsilon\omega_{n^{IR}})^2}\right)(1+\epsilon\mathcal{C}_\omega)^2\left(\check{Y}_t-\check{A}_t\right)^2\\
&\quad+\frac{\overline{Z_{n^Rn^{IR}}}\,\overline{n^R n^{IR}}(1+\epsilon\mathcal{C}_\omega)^2}{(1+\epsilon\omega_{n^R})(1+\epsilon\omega_{n^{IR}})}\left(\check{Y}_t-\check{A}_t\right)^2\\
&=\underbrace{\left(\overline{Z}_{n^R}\overline{n^R}+\overline{Z}_{n^{IR}}\overline{n^{IR}}\right)}_{=\frac{\epsilon_p-1}{\epsilon_p}}\left(\check{Y}_t-\check{A}_t\right)\\
&\quad+\frac{1}{2}\left(\frac{\overline{Z_{n^R}}\,\overline{n^R}(1+\omega_{n^R})}{(1+\epsilon\omega_{n^R})^2}+\frac{\overline{Z_{n^{IR}}}\,\overline{n^{IR}}(1+\omega_{n^{IR}})}{(1+\epsilon\omega_{n^{IR}})^2}\right)(1+\epsilon\mathcal{C}_\omega)^2\left(\check{Y}_t-\check{A}_t\right)^2\\
&\quad+\frac{\overline{Z_{n^Rn^{IR}}}\,\overline{n^R n^{IR}}(1+\epsilon\mathcal{C}_\omega)^2}{(1+\epsilon\omega_{n^R})(1+\epsilon\omega_{n^{IR}})}\left(\check{Y}_t-\check{A}_t\right)^2.
\end{aligned}
\tag{A.54}
$$

Observe that

$$
\check{n}_t=(\eta^n)^{\frac{1}{\epsilon}}\left(\frac{\overline{n^R}}{\overline{n}}\right)^{\frac{\epsilon-1}{\epsilon}}\check{n}_t^R+(1-\eta^n)^{\frac{1}{\epsilon}}\left(\frac{\overline{n^{IR}}}{\overline{n}}\right)^{\frac{\epsilon-1}{\epsilon}}\check{n}_t^{IR},
\tag{A.55}
$$

and

$$
\overline{n^R}\check{n}_t^R=\frac{\partial\overline{n^R}}{\partial\bar{\theta}^{IR}}\bar{\theta}^{IR}\bar{\theta}_t^{IR}+\frac{\partial\overline{n^R}}{\partial\bar{\theta}^{R}}\bar{\theta}^{R}\bar{\theta}_t^{IR},\quad \overline{n^{IR}}\check{n}_t^{IR}=\frac{\partial\overline{n^{IR}}}{\partial\bar{\theta}^{IR}}\bar{\theta}^{IR}\bar{\theta}_t^{IR}+\frac{\partial\overline{n^{IR}}}{\partial\bar{\theta}^{R}}\bar{\theta}^{R}\bar{\theta}_t^{R}.
$$

Therefore, we have

$$
\begin{aligned}
\check{Y}_t-\check{A}_t=&\left\{(\eta^n)^{\frac{1}{\epsilon}}\left(\frac{\overline{n^R}}{\overline{n}}\right)^{\frac{\epsilon-1}{\epsilon}}\frac{\partial\overline{n^R}}{\partial\bar{\theta}^{IR}}\frac{\bar{\theta}^{IR}}{\overline{n^R}}+(1-\eta^n)^{\frac{1}{\epsilon}}\left(\frac{\overline{n^{IR}}}{\overline{n}}\right)^{\frac{\epsilon-1}{\epsilon}}\frac{\partial\overline{n^{IR}}}{\partial\bar{\theta}^{IR}}\frac{\bar{\theta}^{IR}}{\overline{n^{IR}}}\right\}\check{\theta}_t^{IR}\\
&+\left\{(\eta^n)^{\frac{1}{\epsilon}}\left(\frac{\overline{n^R}}{\overline{n}}\right)^{\frac{\epsilon-1}{\epsilon}}\frac{\partial\overline{n^R}}{\partial\bar{\theta}^{R}}\frac{\bar{\theta}^{R}}{\overline{n^R}}+(1-\eta^n)^{\frac{1}{\epsilon}}\left(\frac{\overline{n^{IR}}}{\overline{n}}\right)^{\frac{\epsilon-1}{\epsilon}}\frac{\partial\overline{n^{IR}}}{\partial\bar{\theta}^{R}}\frac{\bar{\theta}^{R}}{\overline{n^{IR}}}\right\}\check{\theta}_t^{R}.
\end{aligned}
$$

This implies that

$$
\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\frac{U_t-\overline{U}}{\overline{U_C C}}\approx-\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left\{\Phi_\pi\pi_t^2+\left(\Phi_{\bar{\theta}^{IR}}\check{\theta}_t^{IR}+\Phi_{\bar{\theta}^{R}}\check{\theta}_t^{R}\right)^2\right\}+t.i.p.+\mathcal{O}_3,
$$

where

$$\Phi_{\bar{\theta}^{IR}} \equiv \sqrt{\Phi_y} \times \left\{ (\eta^n)^{\frac{1}{\epsilon}} \left(\frac{\overline{n^R}}{\overline{n}} \right)^{\frac{\epsilon-1}{\epsilon}} \frac{\partial \overline{n^R} / \overline{n^R}}{\partial \bar{\theta}^{IR} / \bar{\theta}^{IR}} + (1-\eta^n)^{\frac{1}{\epsilon}} \left(\frac{\overline{n^{IR}}}{\overline{n}} \right)^{\frac{\epsilon-1}{\epsilon}} \frac{\partial \overline{n^{IR}} / \overline{n^{IR}}}{\partial \bar{\theta}^{IR} / \bar{\theta}^{IR}} \right\},$$

$$\Phi_{\bar{\theta}^{R}} \equiv \sqrt{\Phi_y} \times \left\{ (\eta^n)^{\frac{1}{\epsilon}} \left(\frac{\overline{n^R}}{n} \right)^{\frac{\epsilon-1}{\epsilon}} \frac{\partial \overline{n^R} / \overline{n^R}}{\partial \bar{\theta}^{R} / \bar{\theta}^{R}} + (1-\eta^n)^{\frac{1}{\epsilon}} \left(\frac{\overline{n^{IR}}}{\overline{n}} \right)^{\frac{\epsilon-1}{\epsilon}} \frac{\partial \overline{n^{IR}} / \overline{n^{IR}}}{\partial \bar{\theta}^{R} / \bar{\theta}^{R}} \right\}.$$

This implies that output gap can be written in terms of $\check{u}_t$ and either with $\check{n}_t^R$ and $\check{n}_t^{IR}$ or with $\check{\theta}_t^{IR}$ and $\check{\theta}_t^{R}$:

$$\check{Y}_t - \check{A}_t = \check{u}_t + \Delta^{u,n^R} \check{n}_t^R + \Delta^{u,n^{IR}} \check{n}_t^{IR}, \tag{A.56}$$

or equivalently,

$$\check{Y}_t - \check{A}_t = \check{u}_t + \Delta^{u,\bar{\theta}^{IR}} \check{\theta}_t^{IR} + \Delta^{u,\bar{\theta}^{R}} \check{\theta}_t^{R}, \tag{A.57}$$

where

$$\Delta^{u,n^R} = (\eta^n)^{\frac{1}{\epsilon}} \left(\frac{\overline{n^R}}{\overline{n}} \right)^{\frac{\epsilon-1}{\epsilon}} - \frac{\overline{n^R}}{\bar{\theta}^{IR}} \left(\mathcal{E}_{\bar{\theta}^{IR}, \overline{n^R}} - 1 \right),$$

$$\Delta^{u,n^{IR}} = (1-\eta^n)^{\frac{1}{\epsilon}} \left(\frac{\overline{n^{IR}}}{\overline{n}} \right)^{\frac{\epsilon-1}{\epsilon}} - \frac{\overline{n^{IR}}}{\bar{\theta}^{IR}} \left(\mathcal{E}_{\bar{\theta}^{IR}, \overline{n^{IR}}} - 1 \right),$$

$$\Delta^{u,\bar{\theta}^{IR}} = \mathcal{E}_{\overline{n^R}, \bar{\theta}^{IR}} \left((\eta^n)^{\frac{1}{\epsilon}} \left(\frac{\overline{n^R}}{\overline{n}} \right)^{\frac{\epsilon-1}{\epsilon}} + \frac{\overline{n^R}}{\bar{\theta}^{IR}} \right) + \mathcal{E}_{\overline{n^{IR}}, \bar{\theta}^{IR}} \left((1-\eta^n)^{\frac{1}{\epsilon}} \left(\frac{\overline{n^{IR}}}{\overline{n}} \right)^{\frac{\epsilon-1}{\epsilon}} + \frac{\overline{n^{IR}}}{\bar{\theta}^{IR}} \right) - \frac{\overline{n^R} + \overline{n^{IR}}}{\bar{\theta}^{IR}},$$

$$\Delta^{u,\bar{\theta}^{R}} = \mathcal{E}_{\overline{n^R}, \bar{\theta}^{R}} \left((\eta^n)^{\frac{1}{\epsilon}} \left(\frac{\overline{n^R}}{\overline{n}} \right)^{\frac{\epsilon-1}{\epsilon}} + \frac{\overline{n^R}}{\bar{\theta}^{R}} \right) + \mathcal{E}_{\overline{n^{IR}}, \bar{\theta}^{R}} \left((1-\eta^n)^{\frac{1}{\epsilon}} \left(\frac{\overline{n^{IR}}}{\overline{n}} \right)^{\frac{\epsilon-1}{\epsilon}} + \frac{\overline{n^{IR}}}{\bar{\theta}^{R}} \right).$$

This implies that

$$\begin{aligned}
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - \overline{U}}{\overline{U_C C}} &\approx -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi_\pi \pi_t^2 + \left(\Phi_u \check{u}_t + \Delta^{u,n^R} \check{n}_t^R + \Delta^{u,n^{IR}} \check{n}_t^{IR} \right)^2 \right\} + t.i.p. + \mathcal{O}_3, \\
&\approx -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi_\pi \pi_t^2 + \left(\Phi_u \check{u}_t + \Delta^{u,\bar{\theta}^{IR}} \check{\theta}_t^{UR} + \Delta^{u,\bar{\theta}^{R}} \check{\theta}_t^{R} \right)^2 \right\} + t.i.p. + \mathcal{O}_3.
\end{aligned}$$

This shows that the stabilization of the aggregate unemployment rate is not sufficient to minimize welfare losses from aggregate fluctuations, with extra other therms, such as, $\Delta^{u,n^R} \check{n}_t^R + \Delta^{u,n^{IR}} \check{n}_t^{IR}$ or$\Delta^{u,\bar{\theta}^{IR}} \check{\theta}_t^{IR} + \Delta^{u,\bar{\theta}^{R}} \check{\theta}_t^{R}$.

A.2.10.1 Calculating Disutility from Supplying Two Types of Labor

Note that disutility from supplying two types of labor (in the case of simple one) is

$$z(n_t^R, n_t^{IR}) = \log K(n_t^R, n_t^{IR}) - H_t(n_t^R, n_t^{IR}),$$

where

$$\begin{aligned}
K(n_t^R, n_t^{IR}) =& n_t^R \left(e^{F^R + (1+\sigma^{IR})((\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \bar{\theta}_t^{R\sigma^{IR}}) + (1+\sigma^R)\left(bar\theta_t^R\right)^{\sigma^R}} - 1 \right) + n_t^{IR} \left(e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} - 1 \right) + 1, \\
H_t(n_t^R, n_t^{IR}) =& \eta\sigma^{IR}(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}+1} - \left(\bar{\theta}_t^R\right)^{\sigma^{IR}+1}) \\
& + \eta\sigma^R \left(\bar{\theta}_t^R\right)^{\sigma^R+1} + \frac{a^2(1+\sigma^{IR})\left(\sigma^{IR}\right)^2}{2\sigma^{IR}+1} \left(\left(\bar{\theta}_t^{IR}\right)^{2\sigma^{IR}+1} - \left(\bar{\theta}_t^R\right)^{2\sigma^{IR}+1} \right) \\
& + \frac{a^2(1+\sigma^R)\left(\sigma^R\right)^2}{2\sigma^R+1} \left(\bar{\theta}_t^R\right)^{2\sigma^R+1} \\
& + a^2(1+\sigma^{IR}) \left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^R\right)^{\sigma^{IR}} \right) \left(\sigma^R \left(\bar{\theta}_t^R\right)^{\sigma^R+1} - \sigma^{IR} \left(\bar{\theta}_t^R\right)^{\sigma^{IR}+1} \right),
\end{aligned}$$

with $\bar{\theta}_t^{IR} = f(n_t^R, n_t^{IR})$, $\bar{\theta}_t^R = g(n_t^R, n_t^{IR})$.

To calculate this, we first need

$$\begin{aligned}
\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} &= a^2\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \bar{\theta}_t^R, \\
\frac{\partial n_t^R}{\partial \bar{\theta}_t^R} &= \eta + a^2(1+\sigma^{IR})(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^R\right)^{\sigma^{IR}}) + a^2\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R} - a^2\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^R\right)^{\sigma^{IR}}, \\
\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} &= \eta + a^2\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} (\bar{\theta}_t^{IR} - \bar{\theta}_t^R), \\
\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} &= \eta + a^2(1+\sigma^{IR}) \left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^R\right)^{\sigma^{IR}} \right).
\end{aligned}$$

This gives that

$$\frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^R} = \frac{\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R}}{\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}}\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} - \frac{\partial n_t^R}{\partial \bar{\theta}_t^R}\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}}, \quad \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}} = -\frac{\frac{\partial n_t^R}{\partial \bar{\theta}_t^R}}{\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}}\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} - \frac{\partial n_t^R}{\partial \bar{\theta}_t^R}\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}},$$

$$\frac{\partial \bar{\theta}_t^R}{\partial n_t^R} = -\frac{\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}}{\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}}\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} - \frac{\partial n_t^R}{\partial \bar{\theta}_t^R}\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}}, \quad \frac{\partial \bar{\theta}_t^R}{\partial n_t^{IR}} = \frac{\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}}}{\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}}\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} - \frac{\partial n_t^R}{\partial \bar{\theta}_t^R}\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}}.$$

$$\frac{\partial^2 \bar{\theta}_t^{IR}}{\partial \left(n_t^R\right)^2} = \frac{-\left(\left(\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}\right)^2 H + \left(\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}}\right)^2 G\right)}{\left(\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}} - \frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}\right)^3 \left(\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}} + \frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}\right)},$$

$$\frac{\partial^2 \bar{\theta}_t^{IR}}{\partial \left(n_t^{IR}\right)^2} = \frac{-\left(\left(\frac{\partial n_t^{R}}{\partial \bar{\theta}_t^{IR}}\right)^2 H + \left(\frac{\partial n_t^{R}}{\partial \bar{\theta}_t^{R}}\right)^2 G\right)}{\left(\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}} - \frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}\right)^3 \left(\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}} + \frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}\right)},$$

$$\frac{\partial^2 \bar{\theta}_t^{R}}{\partial \left(n_t^R\right)^2} = \frac{-\left(\left(\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}}\right)^2 J + \left(\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}\right)^2 I\right)}{\left(\frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} - \frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}}\right)^3 \left(\frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} + \frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}}\right)},$$

$$\frac{\partial^2 \bar{\theta}_t^{R}}{\partial \left(n_t^{IR}\right)^2} = \frac{-\left(\left(\frac{\partial n_t^{R}}{\partial \bar{\theta}_t^{R}}\right)^2 J + \left(\frac{\partial n_t^{R}}{\partial \bar{\theta}_t^{IR}}\right)^2 I\right)}{\left(\frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} - \frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}}\right)^3 \left(\frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} + \frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}}\right)},$$

$$\frac{\partial^2 \bar{\theta}_t^{IR}}{\partial n_t^R n_t^{IR}} = -\frac{\frac{\partial^2 \bar{\theta}_t^{IR}}{\partial \left(n_t^R\right)^2} \frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} + \frac{\partial^2 \bar{\theta}_t^{IR}}{\partial \left(n_t^{IR}\right)^2} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}}}{\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}} + \frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}},$$

$$\frac{\partial^2 \bar{\theta}_t^{R}}{\partial n_t^R \partial n_t^{IR}} = -\frac{\frac{\partial^2 \bar{\theta}_t^{R}}{\partial \left(n_t^R\right)^2} \frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} + \frac{\partial^2 \bar{\theta}_t^{R}}{\partial \left(n_t^{IR}\right)^2} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}} + \frac{\partial \bar{\theta}_t^{R}}{\partial n_t^{R}} \frac{\partial^2 n_t^{R}}{\partial \bar{\theta}_t^{IR} \partial \bar{\theta}_t^{R}} + \frac{\partial \bar{\theta}_t^{R}}{\partial n_t^{IR}} \frac{\partial^2 n_t^{IR}}{\partial \bar{\theta}_t^{IR} \partial \bar{\theta}_t^{R}}}{\frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} + \frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}}}.$$

where

$$H \equiv \left(\frac{\partial^2 n_t^R}{\partial \left(\bar{\theta}_t^R\right)^2} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} - \frac{\partial^2 n_t^{IR}}{\partial \left(\bar{\theta}_t^R\right)^2} \frac{\partial n_t^{R}}{\partial \bar{\theta}_t^R}\right) \left(\frac{\partial n_t^R}{\bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} + \frac{\partial n_t^R}{\partial \bar{\theta}_t^R} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}\right) - 2 \frac{\partial n_t^R}{\partial \bar{\theta}_t^R} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} \left(\frac{\partial^2 n_t^R}{\partial \left(bar\theta_t^{IR}\right) \partial \bar{\theta}_t^R} + \frac{\partial^2 n_t^{IR}}{\partial \bar{\theta}_t^{IR} \partial \bar{\theta}_t^R}\right)$$

$$G \equiv \left(\frac{\partial^2 n_t^R}{\partial \left(\bar{\theta}_t^{IR}\right)^2} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} - \frac{\partial^2 n_t^{IR}}{\partial \left(\bar{\theta}_t^{IR}\right)^2} \frac{\partial n_t^{R}}{\partial \bar{\theta}_t^R}\right) \left(\frac{\partial n_t^R}{\bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} + \frac{\partial n_t^R}{\partial \bar{\theta}_t^R} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}}\right) - 2 \frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} \left(\frac{\partial^2 n_t^R}{\partial \bar{\theta}_t^{IR} \partial \bar{\theta}_t^R} + \frac{\partial^2 n_t^{IR}}{\partial \bar{\theta}_t^{IR} \partial \bar{\theta}_t^R}\right),$$

$$I = \left(\frac{\partial^2 n_t^R}{\partial \left(\bar{\theta}_t^R\right)^2} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} - \frac{\partial^2 n_t^{IR}}{\partial \left(\bar{\theta}_t^R\right)^2} \frac{\partial n_t^{R}}{\partial \bar{\theta}_t^{IR}}\right) \left(\frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} + \frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}}\right) - 2 \frac{\partial n_t^R}{\partial \bar{\theta}_t^R} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} \left(\frac{\partial^2 n_t^R}{\partial \bar{\theta}_t^{IR} \partial \bar{\theta}_t^R} + \frac{\partial^2 n_t^{IR}}{\partial \bar{\theta}_t^{IR} \partial \bar{\theta}_t^R}\right),$$

$$J = \left(\frac{\partial^2 n_t^R}{\partial \left(\bar{\theta}_t^{IR}\right)^2} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} - \frac{\partial^2 n_t^{IR}}{\partial \left(\bar{\theta}_t^{IR}\right)^2} \frac{\partial n_t^{R}}{\partial \bar{\theta}_t^{IR}}\right) \left(\frac{\partial n_t^R}{\partial \bar{\theta}_t^{R}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} + \frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{R}}\right) - 2 \frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} \frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} \left(\frac{\partial^2 n_t^R}{\partial \bar{\theta}_t^{IR} \partial \bar{\theta}_t^R} + \frac{\partial^2 n_t^{IR}}{\partial \bar{\theta}_t^{IR} \partial \bar{\theta}_t^R}\right).$$

$$
\begin{aligned}
\frac{\partial K(n_t^R, n_t^{IR})}{\partial n_t^R} =& \left(e^{F^R+(1+\sigma^{IR})((\bar{\theta}_t^{IR})^{\sigma^{IR}}-(\bar{\theta}_t^{R})^{\sigma^{IR}})+(1+\sigma^R)(\bar{\theta}_t^R)^{\sigma^R}} - 1\right) \\
&+ n_t^R e^{F^R+(1+\sigma^{IR})((\bar{\theta}_t^{IR})^{\sigma^{IR}}-(\bar{\theta}_t^{R})^{\sigma^{IR}})+(1+\sigma^R)(\bar{\theta}_t^R)^{\sigma^R}} \\
&\quad \times \left(\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\frac{\partial\bar{\theta}_t^{IR}}{\partial n_t^R} + \left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1} - \sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^R\right)^{\sigma^{IR}-1}\right)\frac{\partial\bar{\theta}_t^R)}{\partial n_t^R}\right) \\
&\quad + n_t^{IR} e^{F^{IR}+(1+\sigma^{IR})(\bar{\theta}_t^{IR})^{\sigma^{IR}}}\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\frac{\partial\bar{\theta}_t^{IR}}{\partial n_t^R}, \\
\frac{\partial K(n_t^R, n_t^{IR})}{\partial n_t^{IR}} =& n_t^R e^{F^R+(1+\sigma^{IR})((\bar{\theta}_t^{IR})^{\sigma^{IR}}-(\bar{\theta}_t^{R})^{\sigma^{IR}})+(1+\sigma^R)(\bar{\theta}_t^R)^{\sigma^R}} \\
&\quad \times \left(\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}\frac{\partial\bar{\theta}_t^{IR}}{\partial n_t^{IR}} + \left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1} - \sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^R\right)^{\sigma^{IR}-1}\right)\frac{\partial\bar{\theta}_t^R}{\partial n_t^{IR}}\right) \\
&\quad + \left(e^{F^{IR}+(1+\sigma^{IR})(\bar{\theta}_t^{IR})^{\sigma^{IR}}} - 1\right) + n_t^{IR} e^{F^{IR}+(1+\sigma^{IR})(\bar{\theta}_t^{IR})^{\sigma^{IR}}}\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\frac{\partial\bar{\theta}_t^{IR}}{\partial n_t^{IR}}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial H(n_t^R, n_t^{IR})}{\partial n_t^R} =& \left\{\eta\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} + a^2(1+\sigma^{IR})\left(\sigma^{IR}\right)^2\left(\bar{\theta}_t^{IR}\right)^{2\sigma^{IR}} + a^2\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\left(\sigma^R\left(\bar{\theta}_t^R\right)^{\sigma^R+1} - \sigma^{IR}\left(\bar{\theta}_t^R\right)^{\sigma^{IR}+1}\right)\right\}\frac{\partial\bar{\theta}_t^{IR}}{\partial n_t^R} \\
&+ \left\{\underbrace{\left(\eta\left(\bar{\theta}_t^R\right) + a^2(1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(bar\theta_t^R\right)^{\sigma^{IR}}\right)\left(\bar{\theta}_t^R\right) + a^2\sigma^R\left(\bar{\theta}_t^R\right)^{\sigma^R+1}\right)}_{=n_t^R}\left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1} - \sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^R\right)^{\sigma^{IR}-1}\right)\right\}\frac{\partial\bar{\theta}_t^R}{\partial n_t^{IR}}, \\
\frac{\partial H(n_t^R, n_t^{IR})}{\partial n_t^{IR}} =& \left\{\eta\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} + a^2(1+\sigma^{IR})\left(\sigma^{IR}\right)^2\left(\bar{\theta}_t^{IR}\right)^{2\sigma^{IR}} + a^2\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\left(\sigma^R\left(\bar{\theta}_t^R\right)^{\sigma^R+1} - \sigma^{IR}\left(\bar{\theta}_t^R\right)^{\sigma_{IR}+1}\right)\right\}\frac{\partial\bar{\theta}_t^{IR}}{\partial n_t^{IR}} \\
&+ \left\{\underbrace{\left(\eta\left(\bar{\theta}_t^R\right) + a^2(1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^R\right)^{\sigma^{IR}}\right)\left(\bar{\theta}_t^R\right) + a^2\sigma^R\left(\bar{\theta}_t^R\right)^{\sigma^R+1}\right)}_{=n_t^R}\left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1} - \sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^R\right)^{\sigma^{IR}-1}\right)\right\}\frac{\partial\bar{\theta}_t^R}{\partial n_t^{IR}}.
\end{aligned}
$$

Then,

$$
\begin{aligned}
\frac{\partial z(n_t^R, n_t^{IR})}{\partial n_t^R} &= \frac{1}{K(n_t^R, n_t^{IR})}\frac{\partial K(n_t^R, n_t^{IR})}{\partial n_t^R} - \frac{\partial H(n_t^R, n_t^{IR})}{\partial n_t^R}, \\
\frac{\partial z(n_t^R, n_t^{IR})}{\partial n_t^{IR}} &= \frac{1}{K(n_t^R, n_t^{IR})}\frac{\partial K(n_t^R, n_t^{IR})}{\partial n_t^{IR}} - \frac{\partial H(n_t^R, n_t^{IR})}{\partial n_t^{IR}}.
\end{aligned}
$$

Now to calculate the second derivatives, we first need

$$
\begin{aligned}
\frac{\partial^2 H}{\partial \left(\bar{\theta}_t^{IR}\right)^2} =& \eta \left(\sigma^{IR}\right)^2 (1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} + 2a^2 \left(\sigma^{IR}\right)^3 (1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{2\sigma^{IR}-1} \\
&+ a^2 \sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-2} \left(\sigma^R \left(\bar{\theta}_t^R\right)^{\sigma^R+1} - \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^R\right)^{\sigma^{IR}-1}\right), \\
\frac{\partial^2 H}{\partial \left(\bar{\theta}_t^{R}\right)^2} =& \left(\eta + a^2(1+\sigma^{IR}) \left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right) + a^2\sigma^R(1+\sigma^R) \left(\bar{\theta}_t^{R}\right)^{\sigma^R} - a^2\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right) \\
&\times \left(\sigma^R(1+\sigma^R) \left(\bar{\theta}_t^{R}\right)^{\sigma^R-1} - \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}-1}\right) \\
&+ \left(\eta \left(\bar{\theta}_t^{R}\right) + a^2(1+\sigma^{IR}) \left(\left(\bar{\theta}_t^{IR}\right)^{IR} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right) \left(\bar{\theta}_t^{R}\right) + a^2\sigma^R \left(\bar{\theta}_t^{R}\right)^{\sigma^R+1}\right) \\
&\times \left(\sigma^R(1+\sigma^R)(\sigma^R-1) \left(\bar{\theta}_t^{R}\right)^{\sigma^R-2} - \sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}-2}\right), \\
\frac{\partial^2 H}{\partial \bar{\theta}_t^{IR} \partial \bar{\theta}_t^{R}} =& a^2\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \left(\sigma^R(1+\sigma^R) \left(\bar{\theta}_t^{R}\right)^{\sigma^R} - \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right).
\end{aligned}
$$

Moreover,

$$
\begin{aligned}
\frac{\partial^2 K(n_t^R, n_t^{IR})}{\partial \left(n_t^R\right)^2} =& 2e^{F^R + (1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right) + (1+\sigma^R)\left(\bar{\theta}_t^{R}\right)^{\sigma^R}} \\
&\times \left(\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} + \left(\sigma^R(1+\sigma^R) \left(\bar{\theta}_t^{R}\right)^{\sigma^R-1} - \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}-1}\right) \frac{\partial \bar{\theta}_t^R}{\partial n_t^R}\right) \\
&+ n_t^R e^{F^R + (1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right) + (1+\sigma^R)\left(\bar{\theta}_t^{R}\right)^{\sigma^R}} \\
&\times \left(\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} + \left(\sigma^R(1+\sigma^R) \left(\bar{\theta}_t^{R}\right)^{\sigma^R} - \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right) \frac{\partial \bar{\theta}_t^R}{\partial n_t^R}\right)^2 \\
&+ n_t^R e^{F^R + (1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right) + (1+\sigma^R)\left(\bar{\theta}_t^{R}\right)^{\sigma^R}} \\
&\times \left(\sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-2} \left(\frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R}\right)^2 + \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \frac{\partial^2 \left(\bar{\theta}_t^{IR}\right)}{\partial \left(n_t^R\right)^2}\right. \\
&+ \left(\sigma^R(1+\sigma^R)(\sigma^R-1) \left(\bar{\theta}_t^{R}\right)^{\sigma^R-2} - \sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}-2}\right) \left(\frac{\partial \bar{\theta}_t^R}{\partial n_t^R}\right)^2 \\
&\left.+ \left(\sigma^R(1+\sigma^R)(\sigma^R-1) \left(\bar{\theta}_t^{R}\right)^{\sigma^R-2} - \sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}-2}\right) \frac{\partial^2 \left(\bar{\theta}_t^R\right)}{\partial \left(n_t^R\right)^2}\right) \\
&+ n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} \left(\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\right)^2 \left(\frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R}\right)^2 \\
&+ n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} \left(\sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-2}\right) \left(\frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R}\right)^2 \\
&+ n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} \left(\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\right) \frac{\partial^2 \left(\bar{\theta}_t^{IR}\right)}{\partial \left(n_t^R\right)^2},
\end{aligned}
$$

$$\begin{aligned}
\frac{\partial^2 K(n_t^R, n_t^{IR})}{\partial \left(n_t^{IR}\right)^2} =& n_t^R e^{F^R + (1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right) + (1+\sigma^R)\left(\bar{\theta}_t^{R}\right)^{\sigma^R}} \\
& \times \left(\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}} + \left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^{R}\right)^{\sigma^R} - \sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right)\frac{\partial \bar{\theta}_t^{R}}{\partial n_t^{IR}}\right)^2 \\
& + n_t^R e^{F^R + (1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right) + (1+\sigma^R)\left(\bar{\theta}_t^{R}\right)^{\sigma^R}} \\
& \times \left(\sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1)\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-2}\left(\frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}}\right)^2 + \sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\frac{\partial^2 \bar{\theta}_t^{IR}}{\partial \left(n_t^{IR}\right)^2}\right. \\
& + \left(\sigma^R(1+\sigma^R)(\sigma^R-1)\left(\bar{\theta}_t^{R}\right)^{\sigma^R-2} - \sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1)\left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}-2}\right)\left(\frac{\partial \left(\bar{\theta}_t^{R}\right)}{\partial n_t^{IR}}\right)^2 \\
& \left. + \left(\sigma^R(1+\sigma^R)(\sigma^R-1)\left(\bar{\theta}_t^{R}\right)^{\sigma^R-2} - \sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1)\left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}-2}\right)\frac{\partial^2 \left(\bar{\theta}_t^{R}\right)}{\partial \left(n_t^{IR}\right)^2}\right) \\
& + 2e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}}\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}} \\
& + n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}}\left(\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\right)^2\left(\frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}}\right)^2 \\
& + n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}}\left(\sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1)\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-2}\right)\left(\frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}}\right)^2 \\
& + n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}}\left(\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\right)\frac{\partial^2 \bar{\theta}_t^{IR}}{\partial \left(n_t^{IR}\right)^2}.
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial^2 K(n_t^R, n_t^{IR})}{\partial n_t^R \partial n_t^{IR}} =& e^{F^R + (1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right) + (1+\sigma^R)\left(\bar{\theta}_t^{R}\right)^{\sigma^R}} \\
& \times \left(\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}} + \left(\sigma^R (1+\sigma^R) \left(\bar{\theta}_t^{R}\right)^{\sigma^R - 1} - \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}-1} \right) \frac{\partial \bar{\theta}_t^{R}}{\partial n_t^{IR}} \right) \\
& + n_t^R e^{F^R + (1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right) + (1+\sigma^R)\left(\bar{\theta}_t^{R}\right)^{\sigma^R}} \\
& \times \left(\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}} + \left(\sigma^R (1+\sigma^R) \left(\bar{\theta}_t^{R}\right)^{\sigma^R - 1} - \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}-1} \right) \frac{\partial \bar{\theta}_t^{R}}{\partial n_t^{IR}} \right) \\
& \times \left(\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{R}} + \left(\sigma^R (1+\sigma^R) \left(\bar{\theta}_t^{R}\right)^{\sigma^R - 1} - \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}-1} \right) \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{R}} \right) \\
& + n_t^R e^{F^R + (1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}}\right) + (1+\sigma^R)\left(\bar{\theta}_t^{R}\right)^{\sigma^R}} \\
& \times \Bigg[\sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-2} \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{R}} \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}} + \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \frac{\partial^2 \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R \partial n_t^{IR}} \\
& + \left(\sigma^R(1+\sigma^R)(\sigma^R - 1) \left(\bar{\theta}_t^{R}\right)^{\sigma^R - 2} - \sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}-2} \right) \frac{\partial \bar{\theta}_t^{R}}{\partial n_t^{R}} \frac{\partial \bar{\theta}_t^{R}}{\partial n_t^{IR}} \\
& + \left(\sigma^R (1+\sigma^R) \left(\bar{\theta}_t^{R}\right)^{\sigma^R - 1} - \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{R}\right)^{\sigma^{IR}-1} \right) \frac{\partial^2 \left(\bar{\theta}_t^{R}\right)}{\partial n_t^R \partial n_t^{IR}} \Bigg] \\
& + e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{R}} \\
& + n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} \left(\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \right)^2 \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{R}} \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}} \\
& + n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \frac{\partial^2 \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R \partial n_t^{IR}}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(n_t^R\right)^2} =& \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{IR}\right)^2} \left(\frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} \right)^2 + 2 \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{IR}\right) \partial \left(\bar{\theta}_t^{R}\right)} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} \frac{\partial \left(\bar{\theta}_t^{R}\right)}{\partial n_t^R} \\
& + \frac{\partial H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{IR}\right)} \frac{\partial^2 \left(\bar{\theta}_t^{IR}\right)}{\partial \left(n_t^R\right)^2} + \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{R}\right)^2} \left(\frac{\partial \left(\bar{\theta}_t^{R}\right)}{\partial n_t^R} \right)^2 + \frac{\partial H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{R}\right)} \frac{\partial^2 \left(\bar{\theta}_t^{R}\right)}{\partial \left(n_t^R\right)^2}, \\
\frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(n_t^{IR}\right)^2} =& \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{IR}\right)^2} \left(\frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}} \right)^2 + 2 \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{IR}\right) \partial \left(\bar{\theta}_t^{R}\right)} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}} \frac{\partial \left(\bar{\theta}_t^{R}\right)}{\partial n_t^{IR}} \\
& + \frac{\partial H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{IR}\right)} \frac{\partial^2 \left(\bar{\theta}_t^{IR}\right)}{\partial \left(n_t^{IR}\right)^2} + \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{R}\right)^2} \left(\frac{\partial \left(\bar{\theta}_t^{R}\right)}{\partial n_t^{IR}} \right)^2 + \frac{\partial H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{R}\right)} \frac{\partial^2 \left(\bar{\theta}_t^{R}\right)}{\partial \left(n_t^{IR}\right)^2}, \\
\frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial n_t^R \partial n_t^{IR}} =& \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{IR}\right)^2} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}} + \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{IR}\right) \partial \left(\bar{\theta}_t^{R}\right)} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} \frac{\partial \left(\bar{\theta}_t^{R}\right)}{\partial n_t^{IR}} + \frac{\partial H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{IR}\right)} \frac{\partial^2 \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R \partial n_t^{IR}} \\
& + \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{R}\right)^2} \frac{\partial \left(\bar{\theta}_t^{R}\right)}{\partial n_t^R} \frac{\partial \left(\bar{\theta}_t^{R}\right)}{\partial n_t^{IR}} + \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{IR}\right) \partial \left(\bar{\theta}_t^{R}\right)} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}} \frac{\partial \left(\bar{\theta}_t^{R}\right)}{\partial n_t^R} + \frac{\partial H(n_t^R, n_t^{IR})}{\partial \left(\bar{\theta}_t^{R}\right)} \frac{\partial^2 \left(\bar{\theta}_t^{R}\right)}{\partial n_t^R \partial n_t^{IR}}.
\end{aligned}
$$

Then the second-order derivatives of the disutility from supplying labor is

$$\frac{\partial^2 z(n_t^R, n_t^{IR})}{\partial \left(n_t^R\right)^2} = \frac{\frac{\partial^2 K(n_t^R, n_t^{IR})}{\partial \left(n_t^R\right)^2} K(n_t^R, n_t^{IR}) - \left(\frac{\partial K(n_t^R, n_t^{IR})}{\partial n_t^R}\right)^2}{K(n_t^R, n_t^{IR})^2} - \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(n_t^R\right)^2},$$

$$\frac{\partial^2 z(n_t^R, n_t^{IR})}{\partial \left(n_t^{IR}\right)^2} = \frac{\frac{\partial^2 K(n_t^R, n_t^{IR})}{\partial \left(n_t^{IR}\right)^2} K(n_t^R, n_t^{IR}) - \left(\frac{\partial K(n_t^R, n_t^{IR})}{\partial n_t^{IR}}\right)^2}{K(n_t^R, n_t^{IR})^2} - \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial \left(n_t^{IR}\right)^2},$$

$$\frac{\partial^2 z(n_t^R, n_t^{IR})}{\partial n_t^R \partial n_t^{IR}} = \frac{\frac{\partial^2 K(n_t^R, n_t^{IR})}{\partial n_t^R \partial n_t^{IR}} K(n_t^R, n_t^{IR}) - \frac{\partial K(n_t^R, n_t^{IR})}{\partial n_t^R} \frac{\partial K(n_t^R, n_t^{IR})}{\partial n_t^{IR}}}{K(n_t^R, n_t^{IR})^2} - \frac{\partial^2 H(n_t^R, n_t^{IR})}{\partial n_t^R \partial n_t^{IR}}.$$

Note that disutility from supplying two types of labor under more general case can be written as

$$z(n_t^R, n_t^{IR}) = \log K(n_t^R, n_t^{IR}) - \log \mathcal{C}\bar{\theta}_t^R - H(n_t^R, n_t^{IR}),$$

with $\mathcal{C} = \frac{c_t^{UR}}{c_t^{UIR}}$, and

$$K(n_t^R, n_t^{IR}) = n_t^R \mathcal{C} \left(e^{F^R + (1+\sigma^R)\bar{\theta}_t^{R\sigma^R} - \frac{\eta^R}{(a^R)^2} + \frac{1}{a^R}\sqrt{\bar{X}_t}} - 1\right) + n_t^{IR}\left(e^{F^{IR} + (1+\sigma^{IR})\bar{\theta}_t^{IR\sigma^{IR}}} - 1\right) + (\mathcal{C} - 1)\,\bar{\theta}_t^R + 1,$$

$$\begin{aligned} H(n_t^R, n_t^{IR}) = {} & \eta^{IR}(1+\sigma^{IR})\bar{\theta}_t^{IR\sigma^{IR}}\left(\bar{\theta}_t^{IR} - \bar{\theta}_t^R\right) - \eta^{IR}\left(\bar{\theta}_t^{IR\sigma^{IR}+1} - \left(\bar{\theta}_t^R\right)^{\sigma^{IR}+1}\right) - \frac{\left(\eta^R\right)^2}{2\left(a^R\right)^2}\bar{\theta}_t^R \\ & + \frac{\left(a^{IR}\right)^2(1+\sigma^{IR})(\sigma^{IR})^2}{2\sigma^{IR}+1}\left(\bar{\theta}_t^{IR}\right)^{2\sigma^{IR}+1} + \frac{\left(a^R\right)^2(1+\sigma^R)(\sigma^R)^2}{2\sigma^R+1}\left(\bar{\theta}_t^R\right)^{2\sigma^R+1} \\ & - \frac{\left(a^{IR}\right)^2}{2}(1+\sigma^{IR})^2\left(\bar{\theta}_t^{IR}\right)^{2\sigma^{IR}}\bar{\theta}_t^R + \left(a^{IR}\right)^2(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}\left(\bar{\theta}_t^R\right)^{\sigma^{IR}+1} \\ & - \frac{\left(a^{IR}\right)^2}{2}\frac{(1+\sigma^{IR})^2}{2\sigma^{IR}+1}\left(\bar{\theta}_t^R\right)^{2\sigma^{IR}+1} + a^R\sigma^R\sqrt{\bar{X}_t}\left(\bar{\theta}_t^R\right)^{\sigma^R+1} + \frac{\bar{X}_t}{2}\bar{\theta}_t^R, \end{aligned}$$

where

$$\bar{X}_t \equiv \left(a^{IR}(1+\sigma^{IR})\left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^R\right)^{\sigma^{IR}}\right) + \frac{\eta^{IR}}{a^{IR}}\right)^2 + \left(\left(\frac{\eta^R}{a^R}\right)^2 - \left(\frac{\eta^{IR}}{a^{IR}}\right)^2\right) - 2\log\mathcal{C}.$$

In this case,

$$\begin{aligned} n_t^R &= \left(a^R\right)^2\sigma^R\left(\bar{\theta}_t^R\right)^{\sigma^R+1} + a^R\left(\bar{\theta}_t^R\right)\sqrt{\bar{X}_t}, \\ n_t^{IR} &= \eta^{IR}\left(\bar{\theta}_t^{IR} - \bar{\theta}_t^R\right) + \left(a^{IR}\right)^2\sigma^{IR}\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}+1} - \left(a^{IR}\right)^2(1+\sigma^{IR})\left(\bar{\theta}_t^R\right)\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} + \left(a^{IR}\right)^2\left(\bar{\theta}_t^R\right)^{\sigma^{IR}+1}, \end{aligned}$$

so we have

$$
\begin{aligned}
\frac{\partial n_t^R}{\partial \bar{\theta}_t^{IR}} &= \frac{a^R}{2\sqrt{\bar{X}_t}} \left(\bar{\theta}_t^R\right) \frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^{IR}}, \\
\frac{\partial n_t^R}{\partial \bar{\theta}_t^R} &= \left(a^R\right)^2 \sigma^R (1+\sigma^R) \left(\bar{\theta}_t^R\right)^{\sigma^R} + a^R \sqrt{\bar{X}_t} + \frac{a^R}{2\sqrt{\bar{X}_t}} \left(\bar{\theta}_t^R\right) \frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^R}, \\
\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^{IR}} &= \eta^{IR} + \left(a^{IR}\right)^2 \sigma^{IR} (1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(a^{IR}\right)^2 \sigma^{IR} (1+\sigma^{IR}) \left(\bar{\theta}_t^R\right) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}, \\
\frac{\partial n_t^{IR}}{\partial \bar{\theta}_t^R} &= -\eta^{IR} - \left(a^{IR}\right)^2 (1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} + \left(a^{IR}\right)^2 \left(\sigma^{IR}+1\right) \left(\bar{\theta}_t^R\right)^{\sigma^{IR}},
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^{IR}} &= 2\left(a^{IR}(1+\sigma^{IR}) \left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^R\right)^{\sigma^{IR}}\right) + \frac{\eta^{IR}}{a^{IR}}\right) \left(a^{IR}\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\right), \\
\frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^{IR}} &= 2\left(a^{IR}(1+\sigma^{IR}) \left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^R\right)^{\sigma^{IR}}\right) + \frac{\eta^{IR}}{a^{IR}}\right) \left(-a^{IR}\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^R\right)^{\sigma^{IR}-1}\right),
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial H(n_t^R, n_t^{IR})}{\partial \bar{\theta}_t^{IR}} =& \eta^{IR}\sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \left(\bar{\theta}_t^{IR} - \bar{\theta}_t^R\right) + \left(a^{IR}\right)^2 (1+\sigma^{IR}) \left(\sigma^{IR}\right)^2 \left(\bar{\theta}_t^{IR}\right)^{2\sigma^{IR}} \\
& - \left(a^{IR}\right)^2 \sigma^{IR}(1+\sigma^{IR})^2 \left(\bar{\theta}_t^{IR}\right)^{2\sigma^{IR}-1} \left(\bar{\theta}_t^R\right) + \left(a^{IR}\right)^2 \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \left(\bar{\theta}_t^R\right)^{\sigma^{IR}+1} \\
& + \frac{a^R}{2\sqrt{\bar{X}_t}} \sigma^R \left(\bar{\theta}_t^R\right)^{\sigma^R+1} \frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^{IR}} + \frac{1}{2}\left(\bar{\theta}_t^R\right) \frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^{IR}}, \\
\frac{\partial H(n_t^R, n_t^{IR})}{\partial \bar{\theta}_t^R} =& -\eta^{IR}(1+\sigma^{IR}) \left(\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} - \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}\right) - \frac{\left(\eta^R\right)^2}{2\left(a^R\right)^2} + \left(a^R\right)^2 (1+\sigma^R) \left(\sigma^R\right)^2 \left(\bar{\theta}_t^R\right)^{2\sigma^R} \\
& - \frac{\left(a^{IR}\right)^2}{2} (1+\sigma^{IR})^2 \left(\bar{\theta}_t^{IR}\right)^{2\sigma^{IR}} + \left(a^{IR}\right)^2 (1+\sigma^{IR})^2 \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}} \left(\bar{\theta}_t^R\right)^{\sigma^{IR}} - \frac{\left(a^{IR}\right)^2}{2} (1+\sigma^{IR})^2 \left(\bar{\theta}_t^R\right)^{2\sigma^{IR}} \\
& + \frac{a^R}{2\sqrt{\bar{X}_t}} \sigma^R \left(\bar{\theta}_t^R\right)^{\sigma^R+1} \frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^R\right)} + a^R\sigma^R(1+\sigma^R) \left(\bar{\theta}_t^R\right)^{\sigma^R} \sqrt{\bar{X}_t} + \frac{\bar{X}_t}{2} + \frac{1}{2}\left(\bar{\theta}_t^R\right) \frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^R},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial K(n_t^R, n_t^{IR})}{\partial \bar{n}_t^R} =& \mathcal{C}\left(e^{F^R + (1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R} - \frac{\eta^R}{(a^R)^2} + \frac{1}{a^R}\sqrt{\bar{X}_t}} - 1\right) \\
& + n_t^R \mathcal{C} e^{F^R + (1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R} - \frac{\eta^R}{(a^R)^2} + \frac{1}{a^R}\sqrt{\bar{X}_t}} \left(\frac{1}{2a^R\sqrt{\bar{X}_t}} \frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^{IR}} \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^R} + \left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1} + \frac{1}{2a^R\sqrt{\bar{X}_t}} \frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^R}\right) \frac{\partial \bar{\theta}_t^R}{\partial n_t^R}\right) \\
& + n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^R} + (\mathcal{C}-1)\frac{\partial \bar{\theta}_t^R}{\partial n_t^R}, \\
\frac{\partial K(n_t^R, n_t^{IR})}{\partial \bar{n}_t^{IR}} =& n_t^R \mathcal{C} e^{F^R + (1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R} - \frac{\eta^R}{(a^R)^2} + \frac{1}{a^R}\sqrt{\bar{X}_t}} \left(\frac{1}{2a^R\sqrt{\bar{X}_t}} \frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^{IR}} \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}} + \left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1} + \frac{1}{2a^R\sqrt{\bar{X}_t}} \frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^R}\right) \frac{\partial \bar{\theta}_t^R}{\partial n_t^{IR}}\right) \\
& + \left(e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} - 1\right) + n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} \sigma^{IR}(1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} \frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}} + (\mathcal{C}-1)\frac{\partial \bar{\theta}_t^R}{\partial n_t^{IR}},
\end{aligned}
$$

Then,

$$\begin{aligned}
\frac{\partial z(n_t^R, n_t^{IR})}{\partial \bar{n}_t^R} &= \frac{1}{K(n_t^R, n_t^{IR})}\frac{\partial K(n_t^R, n_t^{IR})}{\partial n_t^R} - \log \mathcal{C}\frac{\partial \left(bar\theta_t^R\right)}{\partial n_t^R} - \frac{\partial H(n_t^R, n_t^{IR})}{\partial n_t^R},\\
\frac{\partial z(n_t^R, n_t^{IR})}{\partial \bar{n}_t^{IR}} &= \frac{1}{K(n_t^R, n_t^{IR})}\frac{\partial K(n_t^R, n_t^{IR})}{\partial n_t^{IR}} - \log \mathcal{C}\frac{\partial \left(bar\theta_t^R\right)}{\partial n_t^{IR}} - \frac{\partial H(n_t^R, n_t^{IR})}{\partial n_t^{IR}}.
\end{aligned}$$

Now to calculate second derivatives under the general case, we first need to calculate

$$\begin{aligned}
\frac{\partial^2 n_t^R}{\partial \left(\bar\theta_t^{IR}\right)^2} &= -\frac{a^R}{4}\bar{X}_t^{-\frac{3}{2}}\left(\bar\theta_t^R\right)\left(\frac{\partial \bar{X}_t}{\partial \bar\theta_t^{IR}}\right)^2 + \frac{a^R}{2\sqrt{\bar{X}_t}}\left(\bar\theta_t^R\right)\frac{\partial^2 \bar{X}_t}{\partial \left(\bar\theta_t^{IR}\right)^2},\\
\frac{\partial^2 n_t^R}{\partial \left(\bar\theta_t^{R}\right)^2} &= \left(a^R\right)^2\left(\sigma^R\right)^2(1+\sigma^R)\left(\bar\theta_t^R\right)^{\sigma^R-1} + \frac{a^R}{2\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial \bar\theta_t^R} - \frac{a^R}{4}\bar{X}_t^{-\frac{3}{2}}\left(\bar\theta_t^R\right)\left(\frac{\bar{X}_t}{\partial \bar\theta_t^R}\right)^2 + \frac{a^R}{2\sqrt{\bar{X}_t}}\left(\bar\theta_t^R\right)\frac{\partial^2 \bar{X}_t}{\partial \left(\bar\theta_t^R\right)^2},\\
\frac{\partial^2 n_t^R}{\partial \bar\theta_t^{IR}\partial \bar\theta_t^R} &= -\frac{a^R}{4}\bar{X}_t^{-\frac{3}{2}}\left(\bar\theta_t^R\right)\frac{\partial \bar{X}_t}{\partial \bar\theta_t^{IR}}\frac{\partial \bar{X}_t}{\partial \bar\theta_t^R} + \frac{a^R}{2\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial \bar\theta_t^{IR}} + \frac{a^R}{2\sqrt{\bar{X}_t}}\left(\bar\theta_t^R\right)\frac{\partial^2 \bar{X}_t}{\partial \bar\theta_t^{IR}\partial \bar\theta_t^R},\\
\frac{\partial^2 n_t^{IR}}{\partial \left(\bar\theta_t^{IR}\right)^2} &= \left(a^{IR}\right)^2\left(\sigma^{IR}\right)^2(1+\sigma^{IR})\left(\bar\theta_t^{IR}\right)^{\sigma^{IR}-1} - \left(a^{IR}\right)^2\sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1)\left(\bar\theta_t^{IR}\right)^{\sigma^{IR}-2}\bar\theta_t^R,\\
\frac{\partial^2 n_t^{IR}}{\partial \left(\bar\theta_t^{R}\right)^2} &= \left(a^{IR}\right)^2\left(\sigma^{IR}+1\right)\sigma^{IR}\left(\bar\theta_t^R\right)^{\sigma^{IR}-1},\\
\frac{\partial^2 n_t^{IR}}{\partial \bar\theta_t^{IR}\partial \bar\theta_t^R} &= -\left(a^{IR}\right)^2(1+\sigma^{IR})\sigma^{IR}\left(\bar\theta_t^{IR}\right)^{\sigma^{IR}-1}.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \bar{X}_t}{\partial \left(\bar\theta_t^{IR}\right)^2} &= 2\left(a^{IR}\sigma^{IR}(1+\sigma^{IR})\left(\bar\theta_t^{IR}\right)^{\sigma^{IR}-1}\right)^2\\
&\quad + 2\left(a^{IR}\sigma(1+\sigma^{IR})\left(\left(\bar\theta_t^{IR}\right)^{\sigma^{IR}} - \left(\bar\theta_t^R\right)^{\sigma^{IR}}\right) + \frac{\eta^{IR}}{a^{IR}}\right)\left(a^{IR}\sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1)\left(\bar\theta_t^{IR}\right)^{\sigma^{IR}-2}\right),\\
\frac{\partial^2 \bar{X}_t}{\partial \left(\bar\theta_t^{R}\right)^2} &= 2\left(a^{IR}\sigma^{IR}(1+\sigma^{IR})\left(\bar\theta_t^{R}\right)^{\sigma^{IR}-1}\right)^2\\
&\quad - 2\left(a^{IR}(1+\sigma^{IR})\left(\left(\bar\theta_t^{IR}\right)^{\sigma^{IR}} - \left(\bar\theta_t^R\right)^{\sigma^{IR}}\right) + \frac{\eta^{IR}}{a^{IR}}\right)\left(a^{IR}\sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1)\left(\bar\theta_t^{R}\right)^{\sigma^{IR}-2}\right),\\
\frac{\partial^2 \bar{X}_t}{\partial \bar\theta_t^{IR}\partial \bar\theta_t^R} &= -2\left(a^{IR}\sigma^{IR}(1+\sigma^{IR})\left(\bar\theta_t^{R}\right)^{\sigma^{IR}-1}\right)\left(a^{IR}\sigma^{IR}(1+\sigma^{IR})\left(\bar\theta_t^{IR}\right)^{\sigma^{IR}-1}\right).
\end{aligned}$$

With those, we can calculate

$$
\begin{aligned}
\frac{\partial^2 K(n_t^R, n_t^{IR})}{\partial \left(n_t^R\right)^2} = {} & 2\mathcal{C} e^{F^R + (1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R} - \frac{\eta^R}{(a^R)^2} + \frac{1}{a^R}\sqrt{\bar{X}_t}} \left(\frac{1}{2a^R\sqrt{\bar{X}_t}} \frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^{IR}\right)} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} + \left(\sigma^R (1+\sigma^R) \left(\bar{\theta}_t^R\right)^{\sigma^R - 1} + \frac{1}{2a^R\sqrt{\bar{X}_t}} \frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^R\right)} \right) \frac{\partial \left(\bar{\theta}_t^R\right)}{\partial n_t^R} \right) \\
& + n_t^R \mathcal{C} e^{F^R + (1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R} - \frac{\eta^R}{(a^R)^2} + \frac{1}{a^R}\sqrt{\bar{X}_t}} \left(\frac{1}{2a^R\sqrt{\bar{X}_t}} \frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^{IR}\right)} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} + \left(\sigma^R (1+\sigma^R) \left(\bar{\theta}_t^R\right)^{\sigma^R - 1} + \frac{1}{2a^R\sqrt{\bar{X}_t}} \frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^R\right)} \right) \frac{\partial \left(\bar{\theta}_t^R\right)}{\partial n_t^R} \right)^2 \\
& + n_t^R \mathcal{C} e^{F^R + (1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R} - \frac{\eta^R}{(a^R)^2} + \frac{1}{a^R}\sqrt{\bar{X}_t}} \times \Bigg[- \frac{1}{4a^R} \bar{X}_t^{-\frac{3}{2}} \left(\frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^{IR}\right)} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} + \frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^R\right)} \frac{\partial \left(\bar{\theta}_t^R\right)}{\partial n_t^R} \right) \frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^{IR}\right)} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} \\
& + \frac{1}{2a^R\sqrt{\bar{X}_t}} \left(\frac{\partial^2 \bar{X}_t}{\partial \left(\bar{\theta}_t^{IR}\right)^2} \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} + \frac{\partial^2 \bar{X}_t}{\partial \left(\bar{\theta}_t^{IR}\right) \partial \left(\bar{\theta}_t^R\right)} \frac{\partial \left(\bar{\theta}_t^R\right)}{\partial n_t^R} \right) \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} \\
& + \frac{1}{2a^R\sqrt{\bar{X}_t}} \frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^{IR}\right)} \frac{\partial^2 \left(\bar{\theta}_t^{IR}\right)}{\partial \left(n_t^R\right)^2} + \left(-\frac{1}{4a^R} \bar{X}_t^{-\frac{3}{2}} \frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^{IR}\right)} \frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^R\right)} + \frac{1}{2a^R\sqrt{\bar{X}_t}} \frac{\partial^2 \bar{X}_t}{\partial \left(\bar{\theta}_t^{IR}\right) \partial \left(\bar{\theta}_t^R\right)} \right) \frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} \frac{\partial \left(\bar{\theta}_t^R\right)}{\partial n_t^R} \\
& + \left(\sigma^R (1+\sigma^R)(\sigma^R - 1) \left(\bar{\theta}_t^R\right)^{\sigma^R - 2} - \frac{1}{4a^R} \bar{X}_t^{-\frac{3}{2}} \left(\frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^R\right)} \right)^2 + \frac{1}{2a^R\sqrt{\bar{X}_t}} \frac{\partial^2 \bar{X}_t}{\partial \left(\bar{\theta}_t^R\right)^2} \right) \left(\frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} \right)^2 \\
& + \left(\sigma^R (1+\sigma^R) \left(\bar{\theta}_t^R\right)^{\sigma^R - 1} + \frac{1}{2a^R\sqrt{\bar{X}_t}} \frac{\partial \bar{X}_t}{\partial \left(\bar{\theta}_t^R\right)} \right) \frac{\partial^2 \left(\bar{\theta}_t^R\right)}{\partial \left(n_t^R\right)^2} \Bigg] \\
& + n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} \left(\sigma^{IR} (1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR} - 1} \right)^2 \left(\frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} \right)^2 \\
& + n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} \left(\sigma^{IR} (1+\sigma^{IR})(\sigma^{IR} - 1) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR} - 2} \right) \left(\frac{\partial \left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} \right)^2 \\
& + n_t^{IR} e^{F^{IR} + (1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}} \left(\sigma^{IR} (1+\sigma^{IR}) \left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR} - 1} \right) \frac{\partial^2 \left(\bar{\theta}_t^{IR}\right)}{\partial \left(n_t^R\right)^2} + (\mathcal{C} - 1) \frac{\partial^2 \left(\bar{\theta}_t^R\right)}{\partial \left(n_t^R\right)^2},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 K(n_t^R, n_t^{IR})}{\partial\left(n_t^{IR}\right)^2} = {}& n_t^R \mathcal{C} e^{F^R+(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}-\frac{\eta^R}{(a^R)^2}+\frac{1}{a^R}\sqrt{\bar{X}_t}} \left(\frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)}\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}} + \left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1}\right.\right. \\
& \left.\left. + \frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)}\right)\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^{IR}}\right)^2 \\
& + n_t^R \mathcal{C} e^{F^R+(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}-\frac{\eta^R}{(a^R)^2}+\frac{1}{a^R}\sqrt{\bar{X}_t}} \\
& \times\left[-\frac{1}{4a^R}\bar{X}_t^{-\frac{3}{2}}\left(\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)}\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}} + \frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)}\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^{IR}}\right)\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)}\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}} + \frac{1}{2a^R\sqrt{\bar{X}_t}}\left(\frac{\partial^2 \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)^2}\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}} + \frac{\partial^2 \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)\partial\left(\bar{\theta}_t^R\right)}\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^{IR}}\right)\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}}\right. \\
& + \frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)}\frac{\partial^2\left(\bar{\theta}_t^{IR}\right)}{\partial\left(n_t^{IR}\right)^2} + \left(-\frac{1}{4a^R}\bar{X}_t^{-\frac{3}{2}}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)} + \frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial^2 \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)\partial\left(\bar{\theta}_t^R\right)}\right)\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}}\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^{IR}} \\
& + \left(\sigma^R(1+\sigma^R)(\sigma^R-1)\left(\bar{\theta}_t^R\right)^{\sigma^R-2} - \frac{1}{4a^R}\bar{X}_t^{-\frac{3}{2}}\left(\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)}\right)^2 + \frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial^2 \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)^2}\right)\left(\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^{IR}}\right)^2 \\
& \left. + \left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1} + \frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)}\right)\frac{\partial^2\left(\bar{\theta}_t^R\right)}{\partial\left(n_t^{IR}\right)^2}\right] \\
& + 2e^{F^{IR}+(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}}\left(\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\right)\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}} + n_t^{IR}e^{F^{IR}+(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}}\left(\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\right)^2\left(\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}}\right)^2 \\
& + n_t^{IR}e^{F^{IR}+(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}}\left(\sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1)\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-2}\right)\left(\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}}\right)^2 \\
& + n_t^{IR}e^{F^{IR}+(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}}\left(\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\right)\frac{\partial^2\left(\bar{\theta}_t^{IR}\right)}{\partial\left(n_t^{IR}\right)^2} + (\mathcal{C}-1)\frac{\partial^2\left(\bar{\theta}_t^R\right)}{\partial\left(n_t^{IR}\right)^2},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 K(n_t^R,n_t^{IR})}{\partial n_t^R \partial n_t^{IR}} = {} & \mathcal{C} e^{F^R+(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}-\frac{\eta^R}{(a^R)^2}+\frac{1}{a^R}\sqrt{\bar{X}_t}} \left(\frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^{IR}}\frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}} + \left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1} + \frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)}\right)\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^{IR}}\right) \\
& + n_t^R \mathcal{C} e^{F^R+(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}-\frac{\eta^R}{(a^R)^2}+\frac{1}{a^R}\sqrt{\bar{X}_t}} \left(\frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^{IR}}\frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{R}} + \left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1} + \frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)}\right)\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^{R}}\right) \\
& \times \left(\frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^{IR}}\frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}} + \left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1} + \frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)}\right)\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^{IR}}\right) \\
& + n_t^R \mathcal{C} e^{F^R+(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}-\frac{\eta^R}{(a^R)^2}+\frac{1}{a^R}\sqrt{\bar{X}_t}} \times \left[-\frac{1}{4a^R}\bar{X}_t^{-\frac{3}{2}}\left(\frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^{IR}}\frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{R}} + \frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)}\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^R}\right)\frac{\partial \bar{X}_t}{\partial \bar{\theta}_t^{IR}}\frac{\partial \bar{\theta}_t^{IR}}{\partial n_t^{IR}}\right. \\
& + \frac{1}{2a^R\sqrt{\bar{X}_t}}\left(\frac{\partial^2 \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)^2}\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} + \frac{\partial^2 \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)\partial\left(\bar{\theta}_t^R\right)}\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^R}\right)\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}} \\
& + \frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)}\frac{\partial^2\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R \partial n_t^{IR}} + \left(-\frac{1}{4a^R}\bar{X}_t^{-\frac{3}{2}}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)} + \frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial^2 \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)\partial\left(bar\theta_t^R\right)}\right)\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R}\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^{IR}} \\
& + \left(\sigma^R(1+\sigma^R)(\sigma^R-1)\left(\bar{\theta}_t^R\right)^{\sigma^R-2} - \frac{1}{4a^R}\bar{X}_t^{-\frac{3}{2}}\left(\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)}\right)^2 + \frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial^2 \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)^2}\right)\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^R}\frac{\partial\left(\bar{\theta}_t^R\right)}{\partial n_t^{IR}} \\
& \left. + \left(\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1} + \frac{1}{2a^R\sqrt{\bar{X}_t}}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)}\right)\frac{\partial^2\left(\bar{\theta}_t^R\right)}{\partial n_t^R \partial n_t^{IR}}\right] \\
& + e^{F^{IR}+(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}}\left(\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\right)\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R} + n_t^{IR} e^{F^{IR}+(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}}\left(\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\right)^2\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R}\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}} \\
& + n_t^{IR} e^{F^{IR}+(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}}\left(\sigma^{IR}(1+\sigma^{IR})\left(\sigma^{IR}-1\right)\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-2}\right)\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R}\frac{\partial\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^{IR}} \\
& + n_t^{IR} e^{F^{IR}+(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}}\left(\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1}\right)\frac{\partial^2\left(\bar{\theta}_t^{IR}\right)}{\partial n_t^R \partial n_t^{IR}} \\
& + (\mathcal{C}-1)\frac{\partial^2\left(bar\theta_t^R\right)}{\partial n_t^R \partial n_t^{IR}}.
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar{\theta}_t^{IR}\right)^2} = {} & \eta^{IR}\sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1)\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-2}\left(\left(bar\theta_t^{IR}\right)-\left(\bar{\theta}_t^R\right)\right) + \eta^{IR}\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-1} + 2\left(a^{IR}\right)^2(1+\sigma^{IR})\left(\sigma^{IR}\right)^3\left(\bar{\theta}_t^{IR}\right)^{2\sigma^{IR}-1} \\
& - \left(a^{IR}\right)^2\sigma^{IR}(1+\sigma^{IR})^2(2\sigma^{IR}-1)\left(\bar{\theta}_t^{IR}\right)^{2\sigma^{IR}-2}\left(\bar{\theta}_t^R\right) - \left(a^{IR}\right)^2\sigma^{IR}(1+\sigma^{IR})(\sigma^{IR}-1)\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}-2}\left(\bar{\theta}_t^R\right)^{\sigma^{IR}+1} \\
& - \frac{a^R}{4}\bar{X}_t^{-\frac{3}{2}}\sigma^R\left(\bar{\theta}_t^R\right)^{\sigma^R+1}\left(\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)}\right)^2 + \frac{a^R}{2\sqrt{\bar{X}_t}}\sigma^R\left(\bar{\theta}_t^R\right)^{\sigma^R+1}\frac{\partial^2 \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)^2} + \frac{1}{2}\left(\bar{\theta}_t^R\right)\frac{\partial^2 \bar{X}_t}{\partial\left(\bar{\theta}_t^{IR}\right)^2},
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar{\theta}_t^{R}\right)^2} = {} & \eta^{IR}\sigma^{IR}(1+\sigma^{IR})\left(\bar{\theta}_t^R\right)^{\sigma^{IR}-1} + 2\left(a^R\right)^2(1+\sigma^R)\left(\sigma^R\right)^3\left(\bar{\theta}_t^R\right)^{2\sigma^R-1} \\
& + \left(a^{IR}\right)^2\sigma^{IR}(1+\sigma^{IR})^2\left(\bar{\theta}_t^{IR}\right)^{\sigma^{IR}}\left(\bar{\theta}_t^R\right)^{\sigma^{IR}-1} - \left(a^{IR}\right)^2\sigma^{IR}(1+\sigma^{IR})^2\left(\bar{\theta}_t^R\right)^{2\sigma^{IR}-1} \\
& - \frac{a^R}{4}\bar{X}_t^{-\frac{3}{2}}\sigma^R\left(\bar{\theta}_t^R\right)^{\sigma^R+1}\left(\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)}\right)^2 + \frac{a^R}{2\sqrt{\bar{X}_t}}\sigma^R\left(\bar{\theta}_t^R\right)^{\sigma^R+1}\frac{\partial^2 \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)^2} + \frac{a^R}{\sqrt{\bar{X}_t}}\sigma^R(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R}\frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)} \\
& + a^R\left(\sigma^R\right)^2(1+\sigma^R)\left(\bar{\theta}_t^R\right)^{\sigma^R-1}\sqrt{\bar{X}_t} + \frac{\partial \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)} + \frac{1}{2}\left(\bar{\theta}_t^R\right)\frac{\partial^2 \bar{X}_t}{\partial\left(\bar{\theta}_t^R\right)^2},
\end{aligned}
$$

$$\frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^{IR}\right)\partial\bar\theta_t^R} = -\eta^{IR}\sigma^{IR}(1+\sigma^{IR})\left(\bar\theta_t^{IR}\right)^{\sigma^{IR}-1} - \left(a^{IR}\right)^2\sigma^{IR}(1+\sigma^{IR})^2\left(\bar\theta_t^{IR}\right)^{2\sigma^{IR}-1} + \left(a^{IR}\right)^2\sigma^{IR}(1+\sigma^{IR})^2\left(\bar\theta_t^{IR}\right)^{\sigma^{IR}-1}\left(\bar\theta_t^R\right)^{\sigma^{IR}}$$
$$-\frac{a^R}{4}\bar X_t^{-\frac{3}{2}}\sigma^R\left(\bar\theta_t^R\right)^{\sigma^R+1}\frac{\partial\bar X_t}{\partial\left(\bar\theta_t^{IR}\right)}\frac{\partial\bar X_t}{\partial\left(\bar\theta_t^R\right)} + \frac{a^R}{2\sqrt{\bar X_t}}\sigma^R(1+\sigma^R)\left(\bar\theta_t^R\right)^{\sigma^R}\frac{\partial\bar X_t}{\partial\left(\bar\theta_t^{IR}\right)} + \frac{a^R}{2\sqrt{\bar X_t}}\sigma^R\left(\bar\theta_t^R\right)^{\sigma^R+1}\frac{\partial^2\bar X_t}{\partial\left(\bar\theta_t^{IR}\right)\partial\left(\bar\theta_t^R\right)}$$
$$+\frac{1}{2}\frac{\partial\bar X_t}{\partial\left(\bar\theta_t^{IR}\right)} + \frac{1}{2}\left(\bar\theta_t^R\right)\frac{\partial^2\bar X_t}{\partial\left(\bar\theta_t^{IR}\right)\partial\left(\bar\theta_t^R\right)}.$$

Then,

$$\begin{aligned}
\frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(n_t^R\right)^2} =& \frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^{IR}\right)^2}\left(\frac{\partial\left(\bar\theta_t^{IR}\right)}{\partial n_t^R}\right)^2 + 2\frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^{IR}\right)\partial\left(\bar\theta_t^R\right)}\frac{\partial\left(\bar\theta_t^{IR}\right)}{\partial n_t^R}\frac{\partial\left(\bar\theta_t^R\right)}{\partial n_t^R}\\
&+\frac{\partial H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^{IR}\right)}\frac{\partial^2\left(\bar\theta_t^{IR}\right)}{\partial\left(n_t^R\right)^2} + \frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^R\right)^2}\left(\frac{\partial\left(\bar\theta_t^R\right)}{\partial n_t^R}\right)^2 + \frac{\partial H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^R\right)}\frac{\partial^2\left(\bar\theta_t^R\right)}{\partial\left(n_t^R\right)^2},\\
\frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(n_t^{IR}\right)^2} =& \frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^{IR}\right)^2}\left(\frac{\partial\left(\bar\theta_t^{IR}\right)}{\partial n_t^{IR}}\right)^2 + 2\frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^{IR}\right)\partial\left(\bar\theta_t^R\right)}\frac{\partial\left(\bar\theta_t^{IR}\right)}{\partial n_t^{IR}}\frac{\partial\left(\bar\theta_t^R\right)}{\partial n_t^{IR}}\\
&+\frac{\partial H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^{IR}\right)}\frac{\partial^2\left(\bar\theta_t^{IR}\right)}{\partial\left(n_t^{IR}\right)^2} + \frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^R\right)^2}\left(\frac{\partial\left(\bar\theta_t^R\right)}{\partial n_t^{IR}}\right)^2 + \frac{\partial H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^R\right)}\frac{\partial^2\left(\bar\theta_t^R\right)}{\partial\left(n_t^{IR}\right)^2},\\
\frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial n_t^R\partial n_t^{IR}} =& \frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^{IR}\right)^2}\frac{\partial\left(\bar\theta_t^{IR}\right)}{\partial n_t^R}\frac{\partial\left(\bar\theta_t^{IR}\right)}{\partial n_t^{IR}} + \frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^{IR}\right)\partial\left(\bar\theta_t^R\right)}\frac{\partial\left(\bar\theta_t^{IR}\right)}{\partial n_t^R}\frac{\partial\left(\bar\theta_t^R\right)}{\partial n_t^{IR}} + \frac{\partial H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^{IR}\right)}\frac{\partial^2\left(\bar\theta_t^{IR}\right)}{\partial n_t^R\partial n_t^{IR}}\\
&+\frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^R\right)^2}\frac{\partial\left(\bar\theta_t^R\right)}{\partial n_t^R}\frac{\partial\left(\bar\theta_t^R\right)}{\partial n_t^{IR}} + \frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^{IR}\right)\partial\left(\bar\theta_t^R\right)}\frac{\partial\left(\bar\theta_t^{IR}\right)}{\partial n_t^{IR}}\frac{\partial\left(\bar\theta_t^R\right)}{\partial n_t^R} + \frac{\partial H(n_t^R,n_t^{IR})}{\partial\left(\bar\theta_t^R\right)}\frac{\partial^2\left(\bar\theta_t^R\right)}{\partial n_t^R\partial n_t^{IR}}.
\end{aligned}$$

Fnally, the second-order derivatives of the disutility from supplying labor under more general case is

$$\frac{\partial^2 z(n_t^R,n_t^{IR})}{\partial\left(n_t^R\right)^2} = \frac{\frac{\partial^2 K(n_t^R,n_t^{IR})}{\partial\left(n_t^R\right)^2}K(n_t^R,n_t^{IR}) - \left(\frac{\partial K(n_t^R,n_t^{IR})}{\partial n_t^R}\right)^2}{K(n_t^R,n_t^{IR})^2} - \log\mathcal{C}\frac{\partial^2\left(\bar\theta_t^R\right)}{\partial\left(n_t^R\right)^2} - \frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(n_t^R\right)^2},$$
$$\frac{\partial^2 z(n_t^R,n_t^{IR})}{\partial\left(n_t^{IR}\right)^2} = \frac{\frac{\partial^2 K(n_t^R,n_t^{IR})}{\partial\left(n_t^{IR}\right)^2}K(n_t^R,n_t^{IR}) - \left(\frac{\partial K(n_t^R,n_t^{IR})}{\partial n_t^{IR}}\right)^2}{K(n_t^R,n_t^{IR})^2} - \log\mathcal{C}\frac{\partial^2\left(\bar\theta_t^R\right)}{\partial\left(n_t^{IR}\right)^2} - \frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial\left(n_t^{IR}\right)^2},$$
$$\frac{\partial^2 z(n_t^R,n_t^{IR})}{\partial n_t^R\partial n_t^{IR}} = \frac{\frac{\partial^2 K(n_t^R,n_t^{IR})}{\partial n_t^R\partial n_t^{IR}}K(n_t^R,n_t^{IR}) - \frac{\partial K(n_t^R,n_t^{IR})}{\partial n_t^R}\frac{\partial K(n_t^R,n_t^{IR})}{\partial n_t^{IR}}}{K(n_t^R,n_t^{IR})^2} - \log\mathcal{C}\frac{\partial^2\left(\bar\theta_t^R\right)}{\partial n_t^R\partial n_t^{IR}} - \frac{\partial^2 H(n_t^R,n_t^{IR})}{\partial n_t^R\partial n_t^{IR}}.$$

A.2.11 Model Extension

Because the indirect utility function of the representative family reduces to the standard functional form often used in the workhorse DSGE models, it is easy to extend the baseline model by embedding various other important frictions. First, it is easy to incorporate regular workers' wage rigidities into the model. Because regular workers tend to maintain longer-term relationships with firms, it is plausible that regular workers' wages exhibit more rigidities than those of irregular workers.[6] This makes regular workers much stickier than irregular workers in terms of adjustments. Second, the framework in the baseline model is flexible enough to incorporate habit formation in consumption. Lastly, it is easy to augment the baseline model to include a Bernanke, Gertler, and Gilchrist (1999) financial acceleration mechanism, which enables us to examine the interaction between financial frictions and labor adjustments.

A.2.11.1 Regular Workers' Wage Rigidities

In order to incorporate regular workers' nominal wage rigidities, I need to assume that a continuum of families, indexed by $h \in [0,1]$ supply *homogeneous* irregular labor input to intermediate-goods-producing firms directly, but supply *differentiated* regular labor input to a "labor packer." This labor packing firm then bundles the differentiated regular labor into a *homogeneous* regular labor input to intermediate-goods-producing firms for production.

The technology that this labor packer uses to bundle differentiated regular labor from families is:

$$n_t^R = \left(\int_0^1 n_t^R(h)^{\frac{\epsilon_w - 1}{\epsilon_w}} \mathrm{d}h \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \ \epsilon_w > 1, \tag{A.58}$$

where ϵ_w denotes the degree of substitutability between differentiated regular labor. Then the labor packer maximizes its profit by solving the following problem:

$$\max_{\{n_t^R(h)\}_h} W_t^R \left(\int_0^1 n_t^R(h)^{\frac{\epsilon_w - 1}{\epsilon_w}} \mathrm{d}h \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} - \int_0^1 W_t^R(h) n_t^R(h) \mathrm{d}h,$$

where W_t^R is the aggregate nominal wage for regular workers, and $W_t^R(h)$ is the nominal wage for regular labor supplied by a family, h.

The first order necessary condition of the above problem generates the following downward-sloping demand for each variety of regular-type labor:

$$n_t^R(h) = \left(\frac{W_t^R(h)}{W_t^R} \right)^{-\epsilon_w} n_t^R, \ \forall \ h \in [0,1]. \tag{A.59}$$

[6] Kudlyak (2010) shows the evidence that while wages of new hires exhibit high cyclicalities, those of existing workers do not.

where W_t^R is the aggregate wage index derived in a similar way to the aggregate price index as follows:

$$W_t^R = \left(\int_0^1 W_t^R(h)^{1-\epsilon_w} \mathrm{d}h \right)^{\frac{1}{1-\epsilon_w}}.$$

This downward-sloping demand curve for each variety of regular-type labor due to imperfect substitutabilities across varieties gives the family, h some wage-setting power. This modifies the family h's optimization problem slightly as below.

In addition to $\{C_t, n_t^R(h), n_t^{IR}, v_t, K_{t+1}, I_t, B_{t+1}\}$, a family h sets the wage for regular-type labor, $W_t^R(h)$, subject to regular workers' wage adjustment cost $C(W_t^R(h); W_{t-1}^R(h))$ and downward-sloping demand curve for regular-type labor for a family, h. Threfore, a family h's problem can now be written as follows:

$$\max_{\{C_t, n_t^R(h), W_t^R(h), n_t^{IR}, B_{t+1}, v_t, K_{t+1}, I_t\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \underbrace{u(C_t, n_t^R(h), n_t^{IR})}_{=\log C_t - z(n_t^R(h), n_t^{IR})} \tag{A.60}$$

subject to

$$\begin{aligned} P_t C_t + P_t I_t + B_{t+1} \leq & (1+i_{t-1})B_t + W_t^R(h) n_t^R(h) + W_t^{IR} n_t^{IR} + R_t K_t v_t \\ & - C(W_t^R(h); W_{t-1}^R(h)) + \text{Profits, Taxes, and Transfers}_t, \end{aligned}$$

$$K_{t+1} = Z_t \left[1 - \frac{\tau}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t + (1 - \delta(v_t)) K_t,$$

$$n_t^R(h) = \left(\frac{W_t^R(h)}{W_t^R} \right)^{-\epsilon_w} n_t^{R,d}, \ \forall \ h \in [0,1].$$

where the Rotemberg (1982)-type adjustment cost[7] for regular-type labor's nominal wage is defined as:

$$\mathcal{C}(W_t^R(h); W_{t-1}^R(h)) = \frac{\phi_w}{2} \left(\frac{1}{\Pi_t} \frac{W_t^R(h)}{W_{t-1}^R(h)} - 1 \right)^2 P_t Y_t. \tag{A.61}$$

Because the family h's indirect utility function features separability between consumption and leisure, families will be identical along the all margins except for the supply for regular-type of labor and its wages. Therefore, I suppress the dependence on h except for those two margins (see Erceg, Henderson, and Levin, 2000).

The necessary conditions for the above optimization problem are

$$\lambda_t \equiv P_t \Lambda_t = \frac{1}{C_t},$$

[7] Given complicated functional forms of $z(n_t^R(h), n_t^{IR})$, it is hard to represent $z(n_t^R(h), n_t^{IR})$ recursively, which makes using Calvo-type nominal wage rigidities difficult.

with Λ_t as a Lagrangian multiplier on a budget constraint,

$$\epsilon_w z_{n^R} \frac{n_t^R(h)}{W_t^R(h)} + \Lambda_t \Big\{ (1-\epsilon_w) n_t^R(h) - \phi_w \Big(\frac{1}{\Pi_t} \frac{W_t^R(h)}{W_{t-1}^R(h)} - 1 \Big) \frac{P_t Y_t}{\Pi_t W_{t-1}^R(h)} \Big\}$$
$$+ \beta \mathbb{E}_t \left[\Lambda_{t+1} \phi_w \left(\frac{1}{\Pi_{t+1}} \frac{W_{t+1}^R(h)}{W_t^R(h)} - 1 \right) \frac{W_{t+1}^R(h)}{W_t^R(h)^2} \frac{P_{t+1} Y_{t+1}}{\Pi_{t+1}} \right] = 0,$$

where z_{n^R} denotes $\partial z(n_t^R, n_t^{IR}) / \partial n_t^R$. With the symmetry of the equilibrium, hence suppressing the dependence on h, and if we define $\Pi_t^{R,w} \equiv W_t^R / W_{t-1}^R$, the above condition becomes

$$\epsilon_w z_{n^R} n_t^R + P_t \Lambda_t \left\{ (1-\epsilon_w) n_t^R \frac{W_t^R}{P_t} - \phi_w \left(\frac{\Pi_t^{R,w}}{\Pi_t} - 1 \right) \frac{\Pi_t^{R,w}}{\Pi_t} Y_t \right\}$$
$$+ \beta \mathbb{E}_t \left[P_{t+1} \Lambda_{t+1} \phi_w \left(\frac{\Pi_{t+1}^{R,w}}{\Pi_{t+1}} - 1 \right) \frac{\Pi_{t+1}^{R,w}}{\Pi_{t+1}} Y_{t+1} \right] = 0,$$

$$z_{n^{IR}}(n_t^R, n_t^{IR}) = \Lambda_t W_t^{IR},$$

where $z_{n^{IR}} \equiv \partial z(n_t^R, n_t^{IR}) / \partial n_t^{IR}$,

$$\Lambda_t R_t K_t = \mu_t \delta'(v_t) K_t,$$

with μ_t as a lagrangian multiplier on a capital accumulation process,

$$\Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} (1 + i_t),$$

$$\lambda_t = \mu_t Z_t \left(1 - \frac{\kappa}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \kappa \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right) + \beta \mathbb{E}_t \left[\mu_{t+1} Z_{t+1} \kappa \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right],$$

$$\mu_t = \beta \mathbb{E}_t \left[\Lambda_{t+1} R_{t+1} v_{t+1} + \mu_{t+1} (1 - \delta(v_{t+1})) \right].$$

A.2.11.2 Habit Formation in Consumption

As is the case of CTW, the baseline model is flexible enough to introduce habit formation of consumption, as long as consumption habit is governed by the family-wide consumption, C_{t-1}. If we assume that utility from consumption for each type of worker is given by $\log(c_t^m - bC_{t-1})$ for $m = R, IR$, and similarly for a non-worker's utility, $\log\left(c_t^{Um} - bC_{t-1}\right)$ for $m = R, IR$, then the indirect utility of a family h is now simplified as

$$u(C_t, n_t^R(h), n_t^{IR}) = \log(C_t - bC_{t-1}) - z(n_t^R(h), n_t^{IR}),$$

with the same functional form of $z(n_t^R(h), n_t^{IR})$ defined in Appendix A.2.3 and b denotes the degree of habit formation in consumption.

Note that in the case of utility with habit formation, only the necessary condition regarding the choice of C_t needs to be replaced with the one below:

$$P_t \Lambda_t = \frac{1}{C_t - bC_{t-1}} - \beta b \mathbb{E}_t \left[\frac{1}{C_{t+1} - bC_t} \right].$$

Appendix B

Appendix for Chapter 2

B.1 Additional Empirical Results

B.1.1 Alternative Cross-Sectional Specifications

The first type of robustness check we do is varying the horizon over which the cross-sectional regression is estimated, considering two natural alternative specifications: a two week horizon and a four week horizon. For the two week horizon specification, we consider cumulative initial claims between March 14 and March 28 regressed on SAH exposure over the same window; for the four week specification, the end date is April 11. We include the same set of controls as in our benchmark specification (Table 2.1, Column (5)).

Columns (1) and (2) of Table B.1 report the results from varying the horizon over which the model is estimated. Relative to our baseline result of 1.9%, estimating the model over just two weeks lowers the point estimate slightly to 1.83% (SE: 0.91%). Conversely, when the model is estimated over a four week horizon, the point estimate is 1.7% (SE: 0.59%).

In Column (3) of Table B.1 we estimate the effect of SAH exposure on UI claims, over the same three week horizon as in the benchmark case, weighting observations by state-level employment from the QCEW in 2018 (an approached advocated for by some papers in the local multiplier literature).[1] Again, we consider the same set of controls as in our benchmark specification. The point estimate from the WLS regression is elevated slightly: 2.10% (SE: 0.54%). Regardless, weighting delivers quantitatively similar estimates.

B.1.2 Influence of Specific States

One may also be concerned that individual states' responses, either in terms of rising unemployment claims or SAH orders, is driving our results. To understand whether this

[1]For arguments in either direction, see Ramey (2019) and Chodorow-Reich (Forthcoming), respectively. See also Solon, Haider, and Wooldridge (2015).

Table B.1: Effect of Stay-at-Home Orders on Cumulative Initial Weekly Claims Relative to State Employment: (i) 2-Week Horizon, (ii) 4-Week Horizon, (iii) Weighted Least Squares

	(1) Thru Mar. 28	(2) Thru Apr. 11	(3) WLS
SAH Exposure (varied horizons)	0.0183**	0.0166***	0.0209***
	(0.00908)	(0.00592)	(0.00541)
COVID-19 Cases per 1K	0.00197	0.000854	-0.00472
	(0.0109)	(0.00463)	(0.00306)
Excess Deaths per 1K	-0.0819	0.0691	0.214**
	(0.0959)	(0.0787)	(0.106)
Work at Home Index	-0.152	-0.587**	-0.486+
	(0.184)	(0.261)	(0.258)
Constant	0.111+	0.303***	0.242**
	(0.0649)	(0.0920)	(0.0921)
Adj. R-Square	0.0125	0.129	0.172
No. Obs.	51	51	51

This table reports results from estimating equation (2.4): $\frac{UI_{s,Mar.21,T}}{Emp_s} = \alpha + \beta_C \times SAH_{s,T} + X_s\Gamma + \epsilon_s$, where columns (1) and (2) estimate the model over horizon $T =$ March 28, 2020 and $T =$ April 11, 2020; column (3) estimates the model with $T =$April 4, 2020 by weighted least squares, weighting by state employment. In line with our benchmark specification (Table 2.1, Column (5)), in each column we specify a parsimonious model controlling for pandemic severity, political economy factors, and state sectoral composition. The dependent variable in all columns is our measure of cumulative new unemployment claims as a fraction of state employment, as calculated in Equation (2.3). The interpretation of the SAH Exposure coefficient ($\hat{\beta}_C$; top row) is the effect on normalized new UI claims of one additional week of state exposure to SAH. The Employment-Weighted exposure to SAH for a particular state is calculated by multiplying the number of weeks through T that each county in the state was subject to SAH with the 2018 QCEW average employment share of that county in the state, and summing over each states' counties.

Robust Standard Errors in Parentheses

$+\ p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

is the case, we replicate our benchmark specification (column (5) in Table 2.1) from above, dropping one state at a time. The resulting coefficient estimates for β_C are available in Figure B.1, along with 90 percent confidence intervals constructed from robust standard errors.

B.1.3 Pre-SAH Determinants of UI Claims

In this subsection, we broaden our analysis to adjust for determinants of state-level UI claims that may have been correlated with the timing of SAH implementation at the local level, as reported by the *New York Times*.

The first change that we make, relative to the results presented in Table 2.1, is to con-

Figure B.1: Benchmark Specification Estimated Dropping One State at a Time

This figure reports results from estimating equation (2.4): $\frac{UI_{s,Mar.21,Apr.4}}{Emp_s} = \alpha + \beta_C \times SAH_{s,Apr.4} + X_s\Gamma + \epsilon_s$, dropping one state at a time from the estimation. The set of controls, X_s, are those that appear in the benchmark specification (Table 2.1, Column (5))—a parsimonious model that controls for pandemic severity, political economy factors, and state sectoral composition. The dependent variable is our measure of cumulative new unemployment claims as a fraction of state employment, as calculated in Equation (2.3). The interpretation of the SAH Exposure coefficient ($\hat{\beta}_C$; top row) is the effect on normalized new UI claims of one additional week of state exposure to SAH. The Employment-Weighted exposure to SAH for a particular state is calculated by multiplying the number of weeks through April 4, 2020 that each county in the state was subject to SAH with the 2018 QCEW average employment share of that county in the state, and summing over each states' counties.

trol for the March 7 to March 14 change in consumer spending. Because consumption is a leading indicator, changes to consumer spending tend to precede changes to employment. Thus, this allows us to control for leading determinants—as manifested in changes to state-level consumer spending—of employment losses that may have also been correlated with the timing of the implementation of SAH orders.

To do so, we rely upon the newly available, daily consumer spending index constructed by Chetty et al. (2020). These high frequency indicators of state-level economic activity is constructed from proprietary private sector microdata and made publicly available at `https://trackthercovery.org`.

The second adjustment made in this subsection relates to the timing of state-level SAH implementation. In a few notable instances, the closure of non-essential businesses by state and local officials did not coincide with the broader SAH orders requiring all individuals to remain at home except for essential activities.[2] For example, on March 19 the governor of Pennsylvania issued a statewide executive order that required non-essential, in-person business activity to cease. This preceded by nearly a week the full statewide

[2]The closure of non-essential businesses is a prominent feature of most SAH orders.

SAH order that was put into effect on March 23. A similar discrepancy between SAH dates and non-essential business closure occurred in Nevada.

This is potentially important since both Pennsylvania and Nevada experienced larger cumulative increases in UI claims to employment than the rest of the country through April 4. If the discrepancy between non-essential business closure and SAH implementation (as reported by the *New York Times*) was systematically correlated with the severity of job losses, then our estimate of β_C may be biased. In particular, if the pattern for Pennsylvania and Nevada holds more generally—large UI claims increase and relatively early non-essential business closure—then our estimates of β_C in Table 2.1 will be biased downwards, leading us to understand both the relative employment effect of SAH orders and their implied aggregate effect.

We adjust for the discrepancy between SAH implementation as reported in the *New York Times* and non-essential business closures by constructing a combined SAH/business closure treatment variable:

$$SAHBIZ_{s,t} = \max\left\{SAH_{s,t}, BIZ_{s,t}\right\}, \tag{B.1}$$

where $BIZ_{s,t}$ is the number of weeks state s was subject to a non-essential business closure through date t.[3].

Table B.2 records the results after incorporating the March 7 to March 14 change in the consumer spending index and adjusting the treatment variable to handle discrepancies between reported SAH implementation dates and dates of non-essential business closures. This table is structured identically to Table 2.1 except for the aforementioned changes.

Both qualitatively and quantitatively the effect on unemployment of SAH orders is essentially unchanged relative to the benchmark specification. Consider Column (5): The point estimate of 1.9% (SE: 0.88%) implies that each additional week that a state was subject to a SAH order and/or non-essential business closures increased unemployment claims by 1.9% of the state's employment level.

While this point estimate is the same as our benchmark estimate, the relative-implied aggregate estimate of employment losses due to SAH orders through April 4, 2020 needs to be slightly adjusted. Incorporating non-essential business closure dates weakly increases each state's degree of SAH exposure. Recalculating equation (2.6) with the model estimated in Column (5) of Table B.2 yields an estimate of 4.6 million claims through April 4 attributable to SAH orders or approximately 27% of the overall increase in UI claims over the same period.[4]

[3]We use the state-level non-essential business closure dates compiled in Kong and Prinz (2020).

[4]The two controls we consider in this section each slightly alter the estimated coefficient for the specification analogous to our benchmark specification. Controlling only for the change in the consumer spending index attenuates the point estimate to 1.4% (SE: 0.80%). Only adjusting for the discrepancies between non-essential business closure dates and reported SAH dates amplifies the point estimate somewhat to 2.4% (SE: 0.68); however, this latter effect appears to be driven almost entirely by Pennsylvania and Nevada. Drop-

Table B.2: Effect of Stay-at-Home Orders on Cumulative Initial Weekly Claims Relative to State Employment for Weeks Ending March 21 thru April 4, 2020 After Accounting for Additional Pre-SAH Determinants of UI Claims.

	(1) Bivariate	(2) Covid	(3) Pol. Econ.	(4) Sectoral	(5) All
SAH/Business Closure Exposure	0.0214**	0.0218**	0.0215**	0.0224**	0.0191**
	(0.00855)	(0.00916)	(0.00972)	(0.00882)	(0.00884)
Mar. 7 to Mar. 14 Spending Change	-0.158	-0.183	-0.183	-0.310	-0.351
	(0.293)	(0.289)	(0.289)	(0.272)	(0.279)
COVID-19 Cases per 1K		-0.00295			0.00249
		(0.00579)			(0.00592)
Excess Deaths per 1K		0.0537			0.0637
		(0.120)			(0.109)
60+ Ratio to Total Population		0.308			
		(0.266)			
Avg. UI Replacement Rate			0.0740		0.0751
			(0.0764)		(0.0754)
2016 Trump Vote Share			0.00881		
			(0.0589)		
Work at Home Index				-0.500***	-0.563***
				(0.184)	(0.187)
Bartik-Predicted Job Loss				1.219	
				(7.388)	
Constant	0.0743***	0.0144	0.0372	0.259***	0.239***
	(0.0152)	(0.0517)	(0.0536)	(0.0793)	(0.0764)
Adj. R-Square	0.131	0.107	0.106	0.186	0.179
No. Obs.	51	51	51	51	51

This table reports results from estimating a variant of equation (2.4): $\frac{UI_{s,Mar.21,Apr.4}}{Emp_s} = \alpha + \beta_C \times SAHBIZ_{s,Apr.4} + X_s\Gamma + \epsilon_s$, where each column considers a different set of controls X_s. Column (5)—a parsimonious model controlling for pandemic severity, political economy factors, and state sectoral composition—is analogous to our benchmark specification. The dependent variable in all columns is our measure of cumulative new unemployment claims as a fraction of state employment, as calculated in Equation (2.3). The interpretation of the SAH Exposure coefficient ($\hat{\beta}_C$; top row) is the effect on normalized new UI claims of one additional week of state exposure to SAH, broadened to account for occasional discrepancy between non-essential business closure dates and reported SAH dates. The Employment-Weighted exposure to SAH for a particular state is calculated by multiplying the number of weeks through April 4, 2020 that each county in the state was subject to SAH with the 2018 QCEW average employment share of that county in the state, and summing over each states' counties.

Robust Standard Errors in Parentheses

$+ \ p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

ping these states from the estimation yields a point estimate of 1.9% (SE: 0.68). These results are available upon request.

B.1.4 County-Level Event Study Employment Specification

In Subsection 2.6.2 we use BLS-reported, month-to-month changes in county employment and unemployment to estimate the effect of SAH orders after controlling for state fixed effects. In what follows, we use county-level, high frequency employment indices to provide additional evidence that SAH orders had highly localized effects on county-level employment.[5]

Not only is the effect we estimate in this subsection consistent with our central finding, but by using high frequency, county-level data we are able to directly assess our assumption that the timing of local SAH implementation was uncorrelated with the relative severity of the local economic downturn. Consistent with the evidence presented in Subsection 2.4.2, we find no evidence of differential pre-trends in employment around the implementation of SAH orders.

For the subset of counties for which the high-frequency employment indices are available, we estimate the following event study specification:

$$EmpIDX_{c,t} = \alpha_c + \phi_{state(c),t} + \sum_{k=\underline{K}}^{\overline{K}} \beta_k SAH_{c,t+k} + X_{c,t} + \underline{D}_{c,t} + \overline{D}_{c,t} + \varepsilon_{c,t} \tag{B.2}$$

where $EmpIDX_{c,t}$ represents the county-level, employment index available at `https://tracktherecovery.org`, $SAH_{c,t}$ is a dummy variable equal to 1 on the day a county imposes SAH orders, and $\phi_{state(c),t}$ is a state-by-time fixed effect. As in Subsection 2.4.2, we set $\underline{K} = -17$ and $\overline{K} = 21$; the analysis thus examines three weeks prior and two and a half weeks following the imposition of SAH orders.[6] The event study is estimated over the period February 15th through April 24th, 2020. For this event study specification, we include no additional controls beyond county fixed effects and state-by-time fixed effects.

The results of this exercise are reported in Figure B.2. In the three weeks prior to the implementation of SAH orders, there is no statistically discernible pre-trend in employment.[7] However, there is a clear decline in employment after SAH orders were put into place. By one week following the SAH implementation, the employment index was down

[5]The county-level employment indices we use were constructed by Chetty et al. (2020) and are available at `https://tracktherecovery.org` The county-level employment statistics we use are built out from anonymized microdata from private companies. See Chetty et al. (2020) for a fuller description of the data construction and for evidence that these series tend to track lower-frequency, publicly available series constructed from representative surveys.

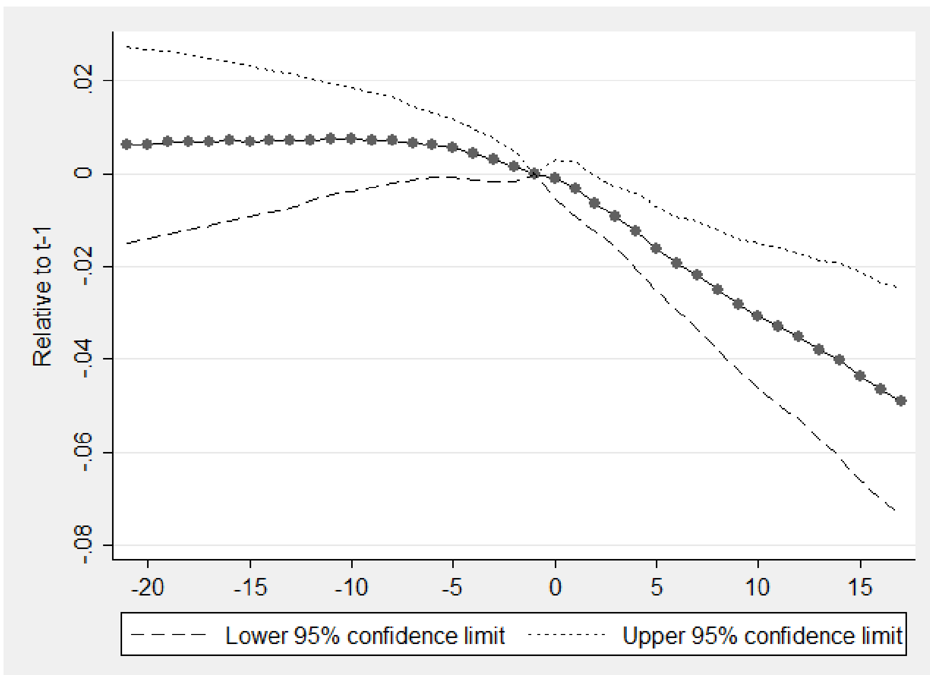
[6]Our sample is necessarily unbalanced in event time, so we include "long-run" dummy variables $\underline{D}_{c,t}$ and $\overline{D}_{c,t}$ which are equal to 1 if a county imposed a SAH order at least $\overline{K}$ days prior or will impose a SAH order at least $\underline{\text{K}}$ days in the future, respectively.

[7]While not statistically meaningful, there appears to be a slight inflection point approximately one week prior to SAH implementation. However, even this is likely a statistical artifact, since the county-level employment statistics we rely upon are primarily reliant upon weekly payroll data from the company Paychex. Chetty et al. (2020) write: We convert the weekly Paychex data to daily measures of employment by assuming that employment is constant within each week.

by 1.9% (SE: 0.5%). Two weeks following SAH implementation, the county-level index was down by by nearly twice as much.

For this analysis, we rely upon a subset of counties for which we have a high frequency measure of employment changes and for which there exist within-state variation. Nevertheless, despite relying upon a different subset of the variation for identification, the weekly effect on employment we estimate here is remarkably consistent with our state-level analysis, in terms of both magnitude and linearity of the effect. We view this as strongly corroborating our baseline finding and allaying concerns that the timing of SAH implementation was differentially correlated with the severity of each labor markets economic downturn.

Figure B.2: County Employment Event Study



This figure plots estimated coefficients from the county-level, event-study specification in equation (B.2), where coefficients have been normalized relative to one day prior to county-level SAH orders went into effect. The model includes as controls county fixed effects and state-by-time fixed effects. The outcome variable is the county-level employment index available at `https://trackthercovery.org`. This index is constructed using anonymized data from private companies; see Chetty et al. (2020) for additional details. The time unit is days.

Standard Errors: Two-Way Clustered by County and Day

Sources: `https://tracktherecovery.org`, the *New York Times*; Authors' Calculations

B.2 Local SAH Orders in a Currency Union Model

We develop a framework to help us interpret the "relative effect"—which we estimate in the data—as compared to the "aggregate effect" of stay-at-home orders. To that end, we use a simple version of Nakamura and Steinsson (2014) of a two-country monetary union model, albeit abstracting from government spending as that is not the focus of our paper.

Households

Consider a currency union comprised of two regions: a home region of size n, and a foreign region of size $1-n$. In each region, there are infinitely many households with *identical* preferences and initial wealth.

A household j in home region has the following preferences:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\delta_t \frac{\left(C_t^j\right)^{1-\sigma}}{1-\sigma} - \chi \frac{\left(N_t^j\right)^{1+\psi}}{1+\psi} \right]$$

where

$$C_t^j = \left[\phi_H^{\frac{1}{\eta}} \left(C_{H,t}^j\right)^{\frac{\eta-1}{\eta}} + \phi_F^{\frac{1}{\eta}} \left(C_{F,t}^j\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \text{ with } \phi_H + \phi_F = 1,$$

$$C_{H,t}^j = \left(\int_0^n \left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} c_{h,t}^j(i)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i \right)^{\frac{\epsilon}{\epsilon-1}}, \quad C_{F,t} = \left(\int_n^1 \left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} c_{f,t}^j(i)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}i \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Total consumption of a household j in a home region is a CES aggregator of a *bundle* of home goods, $C_{H,t}^j$ and a *bundle* of foreign goods, $C_{F,t}^j$. Here, ϕ_F denotes the steady state share of the foreign goods imported from by a household in the home region. When $\phi_H = 1 - \phi_F > n$, there is home bias.[8] η is the elasticity of substitution between home goods and imported goods from a foreign region, and ϵ denotes the elasticity of substitution across differentiated goods. β is discount factor and δ_t denotes consumption-preference shock in a home region, which evolves according to the following law of motion:

$$\log \delta_t = \rho^\delta \log \delta_{t-1} + \epsilon_t^\delta.$$

Then optimal allocations of expenditures (per household) are given by

$$C_{H,t}^j = \phi_H \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t^j, \quad C_{F,t} = \phi_F \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t^j,$$

$$c_{h,t}^j(i) = \left(\frac{p_{h,t}(i)}{P_{H,t}}\right)^{-\epsilon} C_{H,t}^j, \quad c_{f,t}^j(i) = \left(\frac{p_{f,t}(i)}{P_{F,t}}\right)^{-\epsilon} C_{F,t}^j,$$

[8] In the baseline calibration following Nakamura and Steinsson (2014), we calibrate $\phi_H = 0.69$ and $n = 0.1$, so that there is significant home bias.

with price indices defined as follows:

$$P_t = \left[\phi_H P_{H,t}^{1-\eta} + \phi_F P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}},$$
$$P_{H,t} = \left[\frac{1}{n}\int_0^n p_{h,t}(i)^{1-\epsilon}\mathrm{d}i\right]^{\frac{1}{1-\epsilon}},$$
$$P_{F,t} = \left[\frac{1}{1-n}\int_n^1 p_{f,t}(i)^{1-\epsilon}\mathrm{d}i\right]^{\frac{1}{1-\epsilon}}.$$

Here, P_t denotes consumper price index of a home region, and $P_{H,t}$ ($P_{F,t}$) is producer price index of home (foreign) goods.

In our baseline specification, we assume identical households in a given region with the same initial wealth and *complete* financial markets, which makes aggregation straightforward. Thus, we have

$$c_{h,t}(i) \equiv \int_0^n c^j_{h,t}(i)\mathrm{d}j = \left(\frac{p_{h,t}(i)}{P_{H,t}}\right)^{-\epsilon} C_{H,t}, \quad c_{f,t}(i) \equiv \int_0^n c^j_{f,t}(i)\mathrm{d}j = \left(\frac{p_{f,t}(i)}{P_{F,t}}\right)^{-\epsilon} C_{F,t}$$
$$C_{H,t} = \int_0^n C^j_{H,t}\mathrm{d}j = \phi_H\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t, \quad C_{F,t} = \int_n^1 C^j_{F,t}\mathrm{d}j = \phi_F\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t,$$
$$C_t = \int_0^n C^j_t\mathrm{d}j = nC^j_t,$$

where variables without j superscript are aggregate variables in a home region.

With the optimal allocations, we can write household j's budget constraint (in real terms with the home region's CPI as a numeraire) as follows:

$$C^j_t + \mathbb{E}_t\left[M_{t,t+1}B^j_{t+1}\right] \leq B^j_t + \frac{W_t}{P_t}N^j_t + \int_0^1 \frac{\Xi^j_{h,t}(i)}{P_t}\mathrm{d}i - \frac{T^j_t}{P_t}.$$

Note that W_t is home region's nominal wage, and N^j_t is a household j's labor supply. Here, we assume perfect immobility across the regions, meaning wages will be determined at the regional level. B^j_{t+1} is a household j's state-contingent asset holdings and note again that we assume complete financial markets. Here P_t denotes price index that gives the minimum price of one unit of consumption good, C_t. *i.e.* P_t is the Consumer Price Index (CPI) in the home region.

Optimality conditions for $j \in (0, n]$ are

$$\chi\left(N^j_t\right)^{\psi} = \delta_t\left(C^j_t\right)^{-\sigma}\frac{W_t}{P_t},$$
$$\delta_t\left(C^j_t\right)^{-\sigma} = \beta\mathbb{E}_t\left[\delta_{t+1}\left(C^j_{t+1}\right)^{-\sigma}\frac{1+i_t}{1+\pi_{t+1}}\right],$$

where i_t is one-period nominal spot interest rate which satisfies $\mathbb{E}_t[M_{t,t+1}] = 1/(1+i_t)$.

Households in the foreign region are symmetric relative to those in the home region, and we use $*$ to denote foreign variables. So we have

$$C_t^{*j} = \left[(\phi_H^*)^{\frac{1}{\eta}} \left(C_{H,t}^{*j}\right)^{\frac{\eta-1}{\eta}} + (\phi_F^*)^{\frac{1}{\eta}} \left(C_{F,t}^{*j}\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \text{ with } \phi_H^* + \phi_F^* = 1.$$

For *aggregate* optimal allocations in the foreign region, we have

$$c_{h,t}^*(i) \equiv \int_n^1 c_{h,t}^{*j}(i)\mathrm{d}j = \left(\frac{p_{h,t}^*(i)}{P_{H,t}^*}\right)^{-\epsilon} C_{H,t}^*, \quad c_{f,t}^*(i) \equiv \int_n^1 c_{f,t}^{*j}(i)\mathrm{d}j = \left(\frac{p_{f,t}^*(i)}{P_{F,t}^*}\right)^{-\epsilon} C_{F,t}^*$$

$$C_{H,t}^* = \int_n^1 C_{H,t}^{*j}\mathrm{d}j = \phi_H^* \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\eta} C_t^*, \quad C_{F,t}^* = \int_n^1 C_{F,t}^{*j}\mathrm{d}j = \phi_F^* \left(\frac{P_{F,t}^*}{P_t^*}\right)^{-\eta} C_t^*,$$

$$C_t^* = \int_n^1 C_t^{*j}\mathrm{d}j = (1-n)C_t^{*j}.$$

Optimality conditions for foreign households for $j \in [n, 1)$ are

$$\chi \left(N_t^{s,j*}\right)^{\psi} = \delta_t^* \left(C_t^{j*}\right)^{-\sigma} \frac{W_t^*}{P_t^*},$$

$$\delta_t^* \left(C_t^{j*}\right)^{-\sigma} = \beta \mathbb{E}_t \left[\delta_{t+1}^* \left(C_{t+1}^{j*}\right)^{-\sigma} \frac{1+i_t}{1+\pi_{t+1}^*}\right].$$

Terms of Trade, and Real Exchange Rate

Before moving on to firms in each region, let us define terms showing the relationships between various price measures. First, we define terms of trade, S_t as

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}}.$$

From this, we can write the relationship between CPI and Producer Price Index (PPI) in a home region as:

$$g(S_t) \equiv \frac{P_t}{P_{H,t}} = \left[\phi_H + \phi_F S_t^{1-\eta}\right]^{\frac{1}{1-\eta}}, \quad \frac{P_t}{P_{F,t}} = \frac{P_t}{P_{H,t}}\frac{P_{H,t}}{P_{F,t}} = \frac{g(S_t)}{S_t}.$$

For the case of the foreign region, we have

$$g^*(S_t) \equiv \frac{P_t^*}{P_{H,t}^*} = \left[\phi_H^* + \phi_F^* S_t^{1-\eta}\right]^{\frac{1}{1-\eta}}, \quad \frac{P_t^*}{P_{F,t}^*} = \frac{P_t^*}{P_{H,t}^*}\frac{P_{H,t}^*}{P_{F,t}^*} = \frac{g^*(S_t)}{S_t}.$$

Finally, we write the real exchange rate in terms of $g(S_t)$ and $g^*(S_t)$ as follows:

$$Q_t = \frac{P_t^*}{P_t} = \frac{g^*(S_t)}{g(S_t)}.$$

Firms

We assume that there is a continuum of intermediate-goods-producing firms in each region, producing differentiated intermediate goods by using labor as input. We assume a competitive labor market.

Production technologies of each intermediate-goods-producing firms are given by

$$y_{h,t}(i) = A_t N_{h,t}(i)^{\alpha}, \;\; \alpha < 1,$$
$$y_{f,t}(i) = A_t^* N_{f,t}^*(i)^{\alpha}, \;\; \alpha < 1,$$

where $y_{h,t}(i)$ $(y_{f,t}(i))$ is the production output of a firm i in the home (foreign) region, $N_{h,t}(i)$ $(N_{f,t}^*(i))$ is the amount of labor input hired by a firm i in the home (foreign) region, and A_t (A_t^*) is region-wide technology in the home (foreign) region. Both technology processes evolve according to the following laws of motion:

$$\log A_t = \rho^A \log A_{t-1} + \epsilon_t^A,$$
$$\log A_t^* = \rho^{A*} \log A_{t-1}^* + \epsilon_t^{A*}$$

This implies that region-wide labor demand can be written as

$$\begin{aligned}
N_t &= \int_0^n N_{h,t}(i)\mathrm{d}i = \int_0^n \left(\frac{y_{h,t}(i)}{A_t}\right)^{\frac{1}{\alpha}} \mathrm{d}i = \left(\frac{1}{A_t}\right)^{\frac{1}{\alpha}} \int_0^n y_{h,t}(i)^{\frac{1}{\alpha}}\mathrm{d}i \\
&= \left(\frac{Y_{H,t}}{A_t}\right)^{\frac{1}{\alpha}} \int_0^n \frac{1}{n}\left(\frac{p_{h,t}(i)}{P_{H,t}}\right)^{-\frac{\epsilon}{\alpha}} \mathrm{d}i = \left(\frac{Y_{H,t}}{A_t}\right)^{\frac{1}{\alpha}} \Delta_t^{\frac{1}{\alpha}}, \\
N_t^* &= \int_0^n N_{f,t}^*(i)\mathrm{d}i = \int_n^1 \left(\frac{y_{f,t}(i)}{A_t^*}\right)^{\frac{1}{\alpha}} \mathrm{d}i = \left(\frac{1}{A_t^*}\right)^{\frac{1}{\alpha}} \int_n^1 y_{f,t}(i)^{\frac{1}{\alpha}}\mathrm{d}i \\
&= \left(\frac{Y_{F,t}}{A_t^*}\right)^{\frac{1}{\alpha}} \int_n^1 \frac{1}{1-n}\left(\frac{p_{f,t}(i)}{P_{i,t}}\right)^{-\frac{\epsilon}{\alpha}} \mathrm{d}i = \left(\frac{Y_{F,t}}{A_t^*}\right)^{\frac{1}{\alpha}} (\Delta_t^*)^{\frac{1}{\alpha}},
\end{aligned}$$

by defining $\Delta_t \equiv \frac{1}{n}\int_0^n \left(\frac{p_{h,t}(i)}{P_t}\right)^{-\epsilon} \mathrm{d}i$, and $\Delta_t^* \equiv \frac{1}{1-n}\int_n^1 \left(\frac{p_{f,t}(i)}{P_t^*}\right)^{-\epsilon} \mathrm{d}i$ as price dispersion terms in each region.

Firms are subject to Calvo-type pricing frictions, so they solve the following problem:

$$\max_{p_{h,t}^{\#}(i)} \mathbb{E}_t \left[\sum_{k=0}^{\infty} Q_{t,t+k}\theta^k \left(p_{h,t}^{\#}(i) - MC_{h,t+k|t}(i)\right) y_{h,t+k|t}(i)\right]$$

subject to $y_{h,t+k|t}(i) = \left(\frac{p_{h,t}^{\#}(i)}{P_{H,t}}\right)^{-\epsilon} \left(C_{H,t} + C_{H,t}^*\right)$, and with $Q_{t,t+k} = \beta^k \frac{\delta_{t+k}u'(C_{t+k})}{\delta_t u'(C_t)}$. Note that here, $C_{H,t}^*$ denotes a composite index of foreign consumption of home goods, and $MC_{h,t+k|t}(i)$ is nominal marginal cost.

Then optimality conditions for pricing are given by

$$p^{\#}_{h,t}(i) = \frac{\epsilon}{\epsilon-1}\frac{\mathbb{E}_t\sum_{k=0}^{\infty}(\beta\theta)^k\delta_{t+k}u'(C_{t+k})mc_{h,t+k|t}(i)P^{\epsilon}_{H,t+k}\left(C_{H,t}+C^*_{H,t}\right)}{\mathbb{E}_t\sum_{k=0}^{\infty}(\beta\theta)^k\delta_{t+k}u'(C_{t+k})P^{\epsilon-1}_{H,t+k}\left(C_{H,t}+C^*_{H,t}\right)},$$

with $mc_{h,t+k|t}(i)$ is real marginal cost of a firm i in terms of PPI, $P_{H,t}$.

Aggregate real marginal cost with $\alpha < 1$ can be written as follows:

$$\begin{aligned}
mc_{h,t}(i) &= \frac{W_t/P_{H,t}}{\alpha A_t N_{h,t}(i)^{\alpha-1}} = \frac{w_t}{\alpha A_t}N_{h,t}(i)^{1-\alpha} \\
&= \frac{w_t}{\alpha A_t}\left(\frac{y_{h,t}(i)}{A_t}\right)^{\frac{1-\alpha}{\alpha}} = \frac{w_t}{\alpha A_t}\left(\frac{Y_{H,t}}{A_t}\right)^{\frac{1-\alpha}{\alpha}}\left(\frac{y_{h,t}(i)}{Y_{H,t}}\right)^{\frac{1-\alpha}{\alpha}} \\
&= mc_{H,t}\left(\frac{p_{h,t}(i)}{P_{H,t}}\right)^{-\frac{\epsilon(1-\alpha)}{\alpha}}, \\
mc_{H,t} &\equiv \frac{w_t}{\alpha A_t}\left(\frac{Y_{H,t}}{A_t}\right)^{\frac{1-\alpha}{\alpha}}.
\end{aligned}$$

with $w_t \equiv W_t/P_{H,t}$.

Combining this with the previous optimal pricing equation then generates

$$p^{\#}_{h,t}(i)^{1+\frac{\epsilon(1-\alpha)}{\alpha}} = \frac{\epsilon}{\epsilon-1}\frac{\mathbb{E}_t\sum_{k=0}^{\infty}(\beta\theta)^k u'(C_{t+k})mc_{H,t+k}P^{\epsilon/\alpha}_{H,t+k}Y_{H,t+k}}{\mathbb{E}_t\sum_{k=0}^{\infty}(\beta\theta)^k u'(C_{t+k})P^{\epsilon-1}_{H,t+k}Y_{H,t+k}}.$$

We have similar conditions for intermediate-goods-producing firms in the foreign region.

International Risk Sharing Condition and Market Clearing Conditions

Combining each region's Euler equation gives

$$\delta_t\left(\frac{1}{n}C_t\right)^{-\sigma} = \kappa\delta^*_t\left(\frac{1}{1-n}C^*_t\right)^{-\sigma}\frac{1}{\mathcal{Q}_t},$$

with complete markets and symmety of initial conditions, $\kappa = 1$, generating

$$\delta_t^{-\frac{1}{\sigma}}C_t = \frac{n}{1-n}\delta_t^{*-\frac{1}{\sigma}}C^*_t\mathcal{Q}_t^{\frac{1}{\sigma}},$$

with $\mathcal{Q}_t \equiv P^*_t/P_t$ for the real exchange rate.

Goods market clearing conditions in each region are:

$$Y_{H,t} = C_{H,t} + C^*_{H,t} = \phi_H \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \phi^*_H \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\eta} C^*_t,$$
$$Y_{F,t} = C_{F,t} + C^*_{F,t} = \phi_F \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t + \phi^*_F \left(\frac{P^*_{F,t}}{P^*_t}\right)^{-\eta} C^*_t.$$

Finally, we close the model by imposing the following monetary policy rule:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)(\phi_\pi \pi^{agg}_t + \phi_y \hat{y}^{agg}_t),$$

where π^{agg}_t is a union-wide inflation rate and $\hat{y}^{agg}_t$ is union-wide output gap.

Modelling Stay-at-Home Orders

We model the imposition of SAH orders in two ways: (i) as a local supply shock, and (ii) as a local demand shock. When we model the SAH as a local productivity shock, we introduce the negative productivity shock for intermediate-goods-producing firms by setting negative values for ϵ^A_t. Alternatively, we also model the imposition of SAH orders via a negative preference shock, since SAH orders may directly reduce consumption by limiting retail mobility, as discussed in Subsection 2.4.2. In this case, we introduce negative shocks to ϵ^δ_t.

B.3 Data Appendix

Table B.3 reports all sources used in this paper.

Table B.3: Data Sources

Variable	Source
Initial Unemployment Claims (Accessed 6/17/2020)	FRED (Mnemonic *ICLAIMS, where * indicates state abbreviation)
County Employment Data	BLS https://www.bls.gov/lau (Accessed 6/4/2020)
Stay-at-Home Orders (Accessed with *Internet Archive*)	*New York Times* https://www.nytimes.com/interactive/2020/us/coronavirus-stay-at-home-order.html.
Covid Confirmed Cases (Accessed 6/5/2020)	UsaFacts https://usafacts.org/visualizations/coronavirus-covid-19-spread-map/
State Excess Deaths (Accessed 6/4/2020)	CDC https://www.cdc.gov/nchs/nvss/vsrr/covid19/excess_deaths.htm
Share Age 60 (Accessed 6/16/2020)	Census Bureau https://www.census.gov/data/tables/time-series/demo/popest/2010s-state-detail.html
Average UI Replacement Rate (Accessed 6/16/2020)	Department of Labor's Employment and Training Administration https://oui.doleta.gov/unemploy/ui_replacement_rates.asp
2016 Trump Vote Share (Accessed 6/17/2020)	*New York Times* https://www.nytimes.com/elections/2016/results/president.
Work at Home Index	Dingel and Neiman (2020)
March Employment Losses for Bartik (Accessed 4/10/2020)	BLS https://download.bls.gov/pub/time.series/ce/ce.industry
Google Mobility Reports (Accessed 5/21/2020)	https://www.google.com/covid19/mobility/
Daily Consumer Spending and Employment	Track the Recovery https://tracktherecovery.org
State Non-Essential Business Closure Dates	Kong and Prinz (2020)

Appendix C

Appendix for Chapter 3

C.1 Proof of the Generality of Form

$$\underline{Y}_t = \underline{a} + \mathbf{A}_l Y_{t-1} + \mathbf{A}_f \mathbb{E}_t \left[\underline{Y}_{t+1} | \mathcal{I}_t^M \right] - \Gamma \underline{r}_t + \underline{u}_t$$

The goal is to turn Equation (3.1) (repeated above) into a structural form that better maps to the Jorda local projections we use to estimate impulse responses. The Jorda local projections involve running a series of K regressions of the following form:

$$Y_{i,t+k} = \alpha_i^k + \beta_i^k \underline{r}_t + \sum_{p=0}^{P} \delta_{i,p}^k Y_{i,t-p} + \gamma_{i,p}^k \underline{r}_{t-p} + \epsilon_t, \quad \forall k \in \{0, \ldots, K\} \; \forall i \in \{1, \ldots, N\}.$$

The collection $\{\beta_i\}_{k=1}^K$ represents the impulse response of the ith variable of $\underline{Y}$ to monetary policy changes. Naturally without an instrument (or exogenous variations of $\underline{r}_t$), the regression output will not be causal and the purpose of Appendix C.2.2 is to discuss why in the context of information transfer and so for this part of the Appendix, we derive a structural form representation for the above regression that we can then to discuss what goes wrong with OLS estimation.

To do this, for now we leave the interest rate rule unspecified and re-write the the system in Equation (3.1) in a VAR(1) form taking as given the expected future path of policy and future shocks:

$$\underline{Y}_t = \underline{c} + \mathbf{C}_l \underline{Y}_{t-1} + \sum_{s \geq 0} C_{\underline{r},s} \mathbb{E}_t \left[\underline{r}_{t+s} | \mathcal{I}_t^M \right] + \sum_{j \geq 0} C_{\underline{u},j} \mathbb{E}_t \left[\underline{u}_{t+j} | \mathcal{I}_t^M \right]. \tag{C.1}$$

Note that at this point no knowledge of the policy rule is needed to write the model in this form. As an example, for the 3-equation New Keynesian model this form is:

$$\pi = \beta \mathbb{E}_t \pi_{t+1} + \kappa y_t + u_t,$$
$$y_t = \mathbb{E}_t y_{t+1} - \sigma(r_t - \mathbb{E}_t \pi_{t+1}) + v_t,$$
$$\Leftrightarrow \begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \sum_{s\geq 0} \begin{pmatrix} \beta - \kappa\sigma & \kappa \\ \sigma & 1 \end{pmatrix}^s \left[\begin{pmatrix} 1 & \kappa \\ 0 & 1 \end{pmatrix} \begin{bmatrix} \mathbb{E}_t u_{t+s} \\ \mathbb{E}_t v_{t+s} \end{bmatrix} - \begin{pmatrix} \sigma\kappa \\ \sigma \end{pmatrix} \mathbb{E}_t r_{t+s} \right].$$

Here no knowledge of the policy rule was used to derive this form of the system but note that for it to have a unique solution (and for the system to be consistent with the imposition of transversality in writing the above Equation), there are restrictions on the set of allowable policy rules.

Next, note that we can iterate Equation (C.1) forward in time as follows:

$$\begin{aligned}
\underline{Y}_{t+1} = {} & \underline{c}(1 + \mathbf{C}_l) + \mathbf{C}_l^2 \underline{Y}_{t-1} + \sum_{s\geq 0} \mathbf{C}_l C_{\underline{r},s} \mathbb{E}_t \left[\underline{r}_{t+s} | \mathcal{I}_t^M \right] + \sum_{j\geq 0} \mathbf{C}_l C_{\underline{u},j} \mathbb{E}_t \left[\underline{u}_{t+j} | \mathcal{I}_t^M \right] \\
& + \sum_{s\geq 0} C_{\underline{r},s} \mathbb{E}_{t+1} \left[\underline{r}_{t+s+1} | \mathcal{I}_{t+1}^M \right] + \sum_{j\geq 0} C_{\underline{u},j} \mathbb{E}_{t+1} \left[\underline{u}_{t+j+1} | \mathcal{I}_{t+1}^M \right], \\
\Leftrightarrow \underline{Y}_{t+k} = {} & \underline{c} \left(\sum_{\tau=0}^{k} \mathbf{C}_l^\tau \right) + \mathbf{C}_l^{k+1} \underline{Y}_{t-1} + \sum_{s\geq 0} \mathbf{C}_l^k C_{\underline{r},s} \mathbb{E}_t \left[\underline{r}_{t+s} | \mathcal{I}_t^M \right] + \sum_{j\geq 0} \mathbf{C}_l^k C_{\underline{u},j} \mathbb{E}_t \left[\underline{u}_{t+j} | \mathcal{I}_t^M \right] \\
& + \sum_{\tau=1}^{k} \sum_{s\geq 0} \mathbf{C}_l^{k-\tau} C_{\underline{r},s} \mathbb{E}_{t+\tau} \left[\underline{r}_{t+s+\tau} | \mathcal{I}_{t+\tau}^M \right] + \sum_{\tau=1}^{k} \sum_{j\geq 0} \mathbf{C}_l^{k-\tau} C_{\underline{u},j} \mathbb{E}_{t+\tau} \left[\underline{u}_{t+j+\tau} | \mathcal{I}_{t+\tau}^M \right].
\end{aligned}$$

Without loss of generality we can write: $\mathbb{E}_{t+j} \left[x_{t+j+s} | \mathcal{I}_{t+j}^M \right] = a_{j,s} + \psi_{j,s} \mathbb{E}_t \left[x_{t+j+s} | \mathcal{I}_t^M \right] + v_{t+j}$ where $v_{s,t+j} \perp \mathbb{E}_t \left[x_{t+j+s} | \mathcal{I}_t^M \right]$. If these expectations are formed rationally, then $\psi_{j,s} = 1$ and $a_j = 0$. All other values of $(a_{j,s}, \psi_{j,s})$ allow for potentially non-rational expectations with arbitrary dependence on past forecasts. We can substitute these in to get:

$$\begin{aligned}
\underline{Y}_{t+k} = {} & \underline{c} \left(\sum_{\tau=0}^{k} \mathbf{C}_l^\tau \right) + \mathbf{C}_l^{k+1} \underline{Y}_{t-1} + \sum_{s\geq 0} \mathbf{C}_l^k C_{\underline{r},s} \mathbb{E}_t \left[\underline{r}_{t+s} | \mathcal{I}_t^M \right] + \sum_{j\geq 0} \mathbf{C}_l^k C_{\underline{u},j} \mathbb{E}_t \left[\underline{u}_{t+j} | \mathcal{I}_t^M \right] \\
& + \sum_{\tau=1}^{k} \sum_{s\geq 0} \mathbf{C}_l^{k-\tau} C_{\underline{r},s} \left(\alpha_{\tau,s} + \phi_{\tau,s} \mathbb{E}_t \left[\underline{r}_{t+s+\tau} | \mathcal{I}_t^M \right] + v_{\underline{r},s,t+\tau} \right) \\
& + \sum_{\tau=1}^{k} \sum_{j\geq 0} \mathbf{C}_l^{k-\tau} C_{\underline{u},j} \left(a_{\tau,j} + \psi_{\tau,j} \mathbb{E}_t \left[\underline{u}_{t+j+\tau} | \mathcal{I}_t^M \right] + v_{\underline{u},j,t+\tau} \right),
\end{aligned}$$

$$\Leftrightarrow \underline{Y}_{t+k} = \underbrace{\mathbf{C}_l^{k+1}}_{\equiv\Phi^k}\underline{Y}_{t-1} + \sum_{s\geq 0}\underbrace{\sum_{\tau=0}^{s}\mathbf{C}_l^{k-\tau}C_{\underline{r},s}\phi_{\tau,s}}_{\equiv-\Gamma_s^k}\mathbb{E}_t\left[\underline{r}_{t+s}|\mathcal{I}_t^M\right] + \sum_{j\geq 0}\underbrace{\sum_{\tau=0}^{j}\mathbf{C}_l^{k-\tau}C_{\underline{u},j}\psi_{\tau,j}}_{\equiv\Theta_j^k}\mathbb{E}_t\left[\underline{u}_{t+j}|\mathcal{I}_t^M\right]$$

$$+ \underbrace{\sum_{\tau=1}^{k}\sum_{s\geq 0}\mathbf{C}_l^{k-\tau}\left[C_{\underline{r},s}\phi_{\tau,s}v_{\underline{r},s,t+\tau} + C_{\underline{u},s}\psi_{\tau,s}v_{\underline{u},s,t+\tau}\right]}_{\equiv\tilde{\underline{u}}_{t+k}}$$

$$+ \underbrace{\sum_{\tau=1}^{k}\left(\sum_{s\geq 0}\mathbf{C}_l^{k-\tau}\left[\underline{c} + C_{\underline{r},s}\alpha_{\tau,s} + C_{\underline{r},s}a_{\tau,s}\right]\right)}_{\equiv\underline{c}^k},$$

$$\Leftrightarrow \underline{Y}_{t+k} = \underline{c}^k + \Phi^k\underline{Y}_{t-1} - \sum_{s\geq 0}\Gamma_s^k\mathbb{E}_t\left[\underline{r}_{t+s}|\mathcal{I}_t^M\right] + \sum_{j\geq 0}\Theta_j^k\mathbb{E}_t\left[\underline{u}_{t+j}|\mathcal{I}_t^M\right] + \tilde{\underline{u}}_{t+k},$$

where $\phi_{0,s} = \psi_{0,s} = 1\ \forall s \geq 0$ and by definition $\tilde{\underline{u}}_{t+k} \perp \{\mathbb{E}_t\left[\underline{r}_{t+s}|\mathcal{I}_t^M\right], \mathbb{E}_t\left[\underline{u}_{t+s}|\mathcal{I}_t^M\right]\}_{s\geq 0}$.

C.2 The Econometrics of Estimating the Effects of Monetary Policy

C.2.1 Bias of OLS

Even if the information effect is 0, estimating the effects of policy shocks is difficult because policy rates themselves depend on economic outcomes:

$$\begin{aligned} r_t &= r^* + \rho r_{t-1} + \sum_{l>0}\Psi_l\mathbb{E}_t^{CB}\underline{u}_{t+l} + \epsilon_t, \\ \mathbb{E}_t^M r_{t+s} &= \alpha_s + \beta_s r_t + e_{s,t}, \quad e_{s,t} \perp r_t, \end{aligned} \tag{C.2}$$

where we write the yield curve in a reduced form manner without any assumptions about the expectations formations process. For example, if the process were rational, then

$$\mathbb{E}_t^M r_{t+s} = \underbrace{\sum_{j=1}^{s}\rho^{s-j}r^* + \rho^s r_t + \sum_{j=1}^{s}\rho^{s-j}\left(\sum_{l>0}\Psi_l\mathbb{E}_t^M\mathbb{E}_{t+j}^{CB}\underline{u}_{t+j+l} + \mathbb{E}_t^M\epsilon_{t+j}\right)}_{\text{Expectations Hypothesis}} + \underbrace{\zeta_{t,t+s}}_{\substack{\text{Risk} \\ \text{Premium}}},$$

$$= \alpha_s + \beta_s r_t + e_{s,t},$$

$$\Rightarrow \beta_s \equiv \rho^s + \sum_{j=1}^{s}\rho^{s-j}\left(\sum_{l>0}\Psi_l\frac{Cov\left(r_t, \mathbb{E}_t^M\mathbb{E}_{t+j}^{CB}\underline{u}_{t+j+l}\right)}{Var(r_t)} + \frac{Cov\left(r_t, \mathbb{E}_t^M\epsilon_{t+j}\right)}{Var(r_t)}\right) + \frac{Cov\left(r_t, \zeta_{t,t+s}\right)}{Var(r_t)},$$

in order to deliver the result that $e_{s,t} \perp r_t$. Note also that $e_{s,t}$ is only orthogonal to r_t - it might still be correlated with other economic variables $\underline{Y}$ or shocks $\underline{u}$.

Next to model the signal extraction by the market, we can re-write Equation (3.2) as:

$$\underline{Y}_{t+k} = \underline{c}^k + \mathbf{\Phi}^k \underline{Y}_{t-1} - \underbrace{\sum_{s>0} \Gamma_s^k \mathbb{E}_t[\underline{r}_{t+s}|\mathcal{I}_t^M]}_{\text{Yield Curve}} + \sum_{j>0} \Theta_j^k \Big(\underbrace{\mathbb{E}_{t^-}\left[\underline{u}_{t+j}|\mathcal{I}_{t^-}^M\right]}_{\substack{\text{Market Beliefs}\\ \text{Before Policy Change}}} + \underbrace{\Upsilon_j(r_t - \mathbb{E}_{t^-}\left[r_t|\mathcal{I}_{t^-}^M\right]))}_{\substack{\text{Belief Update}\\ \text{From Policy Suprises}}} \Big) + \tilde{\underline{u}}_{t+k}. \tag{C.3}$$

In this expression we have decomposed the market expectations term as comprising a belief about future economic outcomes immediately prior to the interest rate announcement: $\{\mathbb{E}_{t^-}[\underline{u}_{t+j}]\}_{j>0}$ and a term that updates this belief once the interest rate r_t deviates from the level expected by the market $\mathbb{E}_{t^-}[r_t]$. Υ_j is a N x 1 vector describing how beliefs about each economic shock in $\mathbb{E}_{t^-}[\underline{u}|\mathcal{I}_{t^-}]$ is updated in response to the interest rate announcement. Note that if market's information set was the same as the central bank and they set their expectations rationally, then they would not update their beliefs based on interest rate changes and $\Upsilon_j = \underline{0}$, $\forall j > 0$.

Using more parsimonious matrix notation (and substituting in Equation (3.4)), we can write the system as:

$$\underline{Y}_{t+k} = \tilde{\underline{c}}^k + \mathbf{\Phi}^k \underline{Y}_{t-1} + \left(\mathbf{\Theta}\mathbf{Y}^k - \mathbf{\Gamma}^k \mathbf{B}\right) r_t + \mathbf{\Theta}^k \left(\mathbb{E}_{t^-}[\underline{u}_{t+j}] - \mathbf{Y}\mathbb{E}_{t^-}\left[r_t|\mathcal{I}_{t^-}^M\right]\right) + \hat{\underline{u}}_{t+k},$$

where $\mathbf{\Gamma}^k \equiv \begin{bmatrix}\Gamma_0^k & \Gamma_1^k & \dots\end{bmatrix}$, $\mathbf{B} \equiv \begin{bmatrix}1 & \beta_1 & \beta_2 & \dots\end{bmatrix}$, $\mathbf{\Theta}^k = \begin{bmatrix}\Theta_1^k & \Theta_2^k & \dots\end{bmatrix}$ and $\mathbf{Y} = \begin{bmatrix}\Upsilon_1 & \Upsilon_2 & \dots\end{bmatrix}$.

With this notation, the correct impulse responses are given by:

$$\begin{aligned} \Delta_{r_t}\underline{Y}_{t+k} &= \underbrace{-\sum_{s>0}\Gamma_s \frac{\partial \mathbb{E}_t^M r_{t+s}}{\partial r_t}}_{\substack{\text{Direct/ Yield curve effect}\\ \text{of monetary policy}}} + \underbrace{\sum_{j>1}\Theta_j \Delta_{r_t}\mathbb{E}_t^M \underline{u}_{t+j}}_{\substack{\text{Information Effect}\\ \text{of monetary policy}}} \ \forall k > 0, \\ &= \underbrace{-\mathbf{\Gamma}^k \mathbf{B}}_{\substack{\text{Direct/ Yield curve effect}\\ \text{of monetary policy}}} + \underbrace{\mathbf{\Theta}^k \mathbf{Y}}_{\substack{\text{Information Effect}\\ \text{of monetary policy}}} . \end{aligned}$$

$\mathbf{\Gamma}^k\mathbf{B}$ summarizes the effect of policy changes (both the short rate and yield curve changes in response to r_t) on economic outcomes and $\mathbf{\Theta}^k\mathbf{Y}$ summarizes the Information Effect.

Next section shows that estimating the following Jorda local projection regressions with OLS:

$$\underline{Y}_{t+k} = \underline{c}^k - \underline{\Lambda}^k r_t + \mathbf{\Phi}^k \underline{Y}_{t-1} + \xi_{t+k} \tag{C.4}$$

gives:

$$\underline{\hat{\Lambda}}^k_{OLS} = \underbrace{\mathbf{\Gamma}^k\mathbf{B} - \mathbf{\Theta}^k\mathbf{Y}}_{\text{Correct Impulse Response}} \underbrace{- \tilde{\mathbf{\Theta}}^k \frac{\mathbb{E}^M_{t+\tau}\underline{\mathbf{u}}_\mathbf{t}(1_k \otimes \mathbb{E}^{CB}_t \underline{\mathbf{u}}_t)'}{\mathbb{E}\left[(r^\perp_t)^2\right]}(1_k \otimes \mathbf{\Psi})'}_{\text{Central Bank is Forward Looking}} + \underbrace{\mathbf{\Theta}^k\mathbf{Y}\frac{\mathbb{E}\left[\mathbb{E}^M_{t^-}[r_t]^\perp r^\perp_t\right]}{\mathbb{E}\left[(r^\perp_t)^2\right]}}_{\substack{\text{Market already priced}\\ \text{in policy movement}}} \tag{C.5}$$

where $\underline{\mathbf{u}}'_t \equiv \begin{bmatrix}\underline{u}_t & \underline{u}_{t+1} & \ldots\end{bmatrix}$, $r^\perp_t$ is the interest rate vector with the lagged variables and a constant residualized away (using Frish-Waugh-Lovell), $\mathbb{E}^M_{t+\tau}\underline{\mathbf{u}}_\mathbf{t}$ is a matrix of market expectations of economic shocks (defined precisely in Appendix C.2.2); $\mathbf{\Theta}^k$, $\tilde{\mathbf{\Theta}}^k$ $\mathbf{\Psi}$ and $\mathbf{B}$ are matrices of parameters; and 1_k is a vector of ones of length k.

Essentially here OLS is biased for two reasons: central banks are forward looking and set policy anticipating future economic events. OLS then picks up an average of the direct effect of monetary policy on future outcomes and the ability of the central bank to forecast the future.

The second reason OLS is biased is that markets can forecast policy rate movements in advance and then react to it immediately. Interestingly, if markets were able to perfectly predict all policy movements then the market pricing effect would exactly cancel the learning effect in the correct impulse response. This suggests a method for future research to estimate the pure effect of monetary policy if signal extraction motives were absent.

C.2.2 Deriving bias of OLS

The original system is:

$$\begin{aligned}
\underline{Y}_{t+k} &= \underline{c}^k + \Phi^k \underline{Y}_{t-1} - \underbrace{\sum_{s>0} \Gamma^k_s \mathbb{E}_t[\underline{r}_{t+s}|\mathcal{I}^M_t]}_{\text{Yield Curve}} \\
&\quad + \underbrace{\sum_{j>0} \Theta^k_j \left(\mathbb{E}_{t^-}\left[\underline{u}_{t+j}|\mathcal{I}^M_{t^-}\right] + \Upsilon_j\left(r_t - \mathbb{E}_{t^-}[r_t|\mathcal{I}^M_{t^-}]\right)\right)}_{\substack{\text{Future Economic}\\ \text{Shocks}}} + \underline{\tilde{u}}_{t+k}, \\
r_t &= r^* + \rho r_{t-1} + \sum_{l>0} \Psi_l \mathbb{E}^{CB}_t \underline{u}_{t+l} + \epsilon_t, \\
\mathbb{E}^M_t r_{t+j} &= \alpha_j + \beta_j r_t + e_{j,t}.
\end{aligned}$$

For the bias calculations to be easier, let's expand out some of the terms in $\underline{\tilde{u}}_{t+k}$ to get:

$$
\begin{aligned}
\underline{Y}_{t+k} = \underline{c}^k + \Phi^k \underline{Y}_{t-1} - \underbrace{\sum_{s>0} \Gamma_s^k \mathbb{E}_t[\underline{r}_{t+s} | \mathcal{I}_t^M]}_{\text{Yield Curve}} \\
+ \underbrace{\sum_{j>0} \Theta_j^k \left(\mathbb{E}_{t^-} \left[\underline{u}_{t+j} | \mathcal{I}_{t^-}^M \right] + \Upsilon_j \left(r_t - \mathbb{E}_{t^-}[r_t | \mathcal{I}_{t^-}^M] \right) \right)}_{\substack{\text{Future Economic} \\ \text{Shocks}}} \\
+ \sum_{\tau=1}^{k} \sum_{j>0} \tilde{\Theta}_{\tau,j}^k \left(\mathbb{E}_{t+\tau} \left[\underline{u}_{t+j} | \mathcal{I}_{t^-}^M \right] - \psi_{\tau,j} \mathbb{E}_{t+\tau} \left[\underline{u}_{t+j} | \mathcal{I}_{t^-}^M \right] \right) + \xi_{t+k}. \\
r_t = r^* + \rho r_{t-1} + \sum_{l>0} \Psi_l \mathbb{E}_t^{CB} \underline{u}_{t+l} + \epsilon_t. \\
\mathbb{E}_t^M r_{t+j} = \alpha_j + \beta_j r_t + e_{j,t},
\end{aligned}
$$

where now there are additional terms of the effects of beliefs formed between t and $t+k$ to affect outcomes in $t+k$. Note that this includes actual shocks occurring at $t+k$ as well. The reason for including these terms is that when estimating with OLS, forward looking central banks may be able to predict these upcoming shocks (and then also market beliefs as the market learns about the shocks).

Let $Z_t = \begin{bmatrix} 1 & \underline{Y}_{t-1} & r_{t-1} \end{bmatrix}$, and first define $\underline{\mathbf{r}}_t' \equiv \begin{bmatrix} r_t & \mathbb{E}_t^M r_{t+1} & \ldots \end{bmatrix}$, $\underline{\mathbf{u}}_t' \equiv \begin{bmatrix} \underline{u}_t' & \underline{u}_{t+1}' & \ldots \end{bmatrix}$ and (with some abuse of notation) $\mathbb{E}_{t+\tau}^M \underline{\mathbf{u}}_{\mathbf{t}}' \equiv \begin{bmatrix} \underline{u}_t' & \underline{u}_{t+1}' & \mathbb{E}_{t+1}^M \underline{u}_{t+2}' & \ldots & \underline{u}_t' & \ldots & \mathbb{E}_{t+k}^M \underline{u}_{t+k+1}' & \ldots \end{bmatrix}$. Next, let's define $\mathbf{\Gamma}^k \equiv \begin{bmatrix} \Gamma_1^k & \Gamma_2^k & \ldots \end{bmatrix}$, $\mathbf{\Theta}^k \equiv \begin{bmatrix} \Theta_1^k & \Theta_2^k & \ldots \end{bmatrix}$, $\mathbf{\Psi} \equiv \begin{bmatrix} \Psi_1 & \Psi_2 & \ldots \end{bmatrix}$,$\mathbf{B} \equiv \begin{bmatrix} 1 & \beta_1 & \beta_2 & \ldots \end{bmatrix}$, $\tilde{\mathbf{\Theta}}^k \equiv \begin{bmatrix} \tilde{\Theta}_{1,1} & \tilde{\Theta}_{1,2} & \ldots & \tilde{\Theta}_{2,1} & \ldots \end{bmatrix}$, $\mathbf{ff} \equiv \begin{bmatrix} 0 & \alpha_1 & \alpha_2 & \ldots \end{bmatrix}$, and $\mathbf{e}_t \equiv \begin{bmatrix} 0 & e_{1,t} & e_{2,t} & \ldots \end{bmatrix}$.

We can now write the system as:

$$
\begin{aligned}
\underline{Y}_{t+k} &= \underline{c}^k + \Phi^k \underline{Y}_{t-1} - \mathbf{\Gamma}^k \underline{\mathbf{r}}_t + \mathbf{\Theta}^k \left(\mathbb{E}_{t^-}^M \underline{\mathbf{u}}_t + \mathbf{Y}(r_t - \mathbb{E}_{t^-}^M r_t) \right) + \tilde{\mathbf{\Theta}}^k \mathbb{E}_{t+\tau}^M \underline{\mathbf{u}}_{\mathbf{t}} + \xi_{t+k} \\
\underline{\mathbf{r}}_t &= \mathbf{ff} + \mathbf{B} \otimes (r^* + \rho r_{t-1} + \mathbf{\Psi} \mathbb{E}_t^{CB} \underline{\mathbf{u}}_t + \epsilon_t) + \mathbf{e}_t.
\end{aligned}
$$

We can then write this system in terms of r_t as:

$$
\underline{Y}_{t+k} = \underline{\gamma}^k + \mathbf{\Phi}^k \underline{Y}_{t-1} + \left(\mathbf{\Theta}^k \mathbf{Y} - \mathbf{\Gamma}^k \mathbf{B} \right) r_t + \mathbf{\Theta}^k \left(\mathbb{E}_{t^-}^M \underline{\mathbf{u}}_t - \mathbf{Y} \mathbb{E}_{t^-}^M [r_t] \right) + \tilde{\mathbf{\Theta}}^k \mathbb{E}_{t+\tau}^M \underline{\mathbf{u}}_{\mathbf{t}} + \xi_{t+k} - \mathbf{\Gamma}^{\mathbf{k}} \mathbf{e}_t,
$$

where $\underline{\gamma}^k = \underline{c}^k - \mathbf{\Gamma}^k \mathbf{ff}$. Note that the yield curve residuals are correlated with the individual elements of $\mathbf{u}_t$ but by construction are not correlated with r_t.

Next let's define the Frish-Waugh-Lovell residuals of x as $x^\perp \equiv (I - Z(Z'Z)^{-1}Z')x$, and consider the Jorda local projections regression run with OLS:

$$
\underline{Y}_{t+k} = \underline{c}^k - \mathbf{\Lambda}^k r_t + \phi^k \underline{Y}_{t-1} + \underline{\tilde{u}}_{t+k}.
$$

It is important to note that $\tilde{\underline{u}}_{t+k}$ contains terms that are correlated with r_t because in the derivation of this residual we made sure it was orthogonal to market expectations only. Naturally for $j < k$, the error term includes $\underline{u}_{t+j} - \phi_{k,j}\mathbb{E}_t^M \underline{u}_{t+j}$ which could be correlated with $\mathbb{E}^{CB}\underline{u}_{t+j}$.

We know that OLS:

$$
\begin{aligned}
\hat{\mathbf{\Lambda}}^k &= -\frac{\mathbb{E}(\underline{Y}_{t+k}^{\perp} r_t^{\perp})}{\mathbb{E}\left[(r_t^{\perp})^2\right])} \\
&= -\left(\mathbf{\Theta}^k\mathbf{Y} - \mathbf{\Gamma}^k\mathbf{B}\right)\frac{\mathbb{E}\left[(r_t^{\perp})^2\right]}{\mathbb{E}\left[(r_t^{\perp})^2\right]} - \mathbf{\Theta}^k\frac{\mathbb{E}\left[\mathbb{E}_{t^-}^M\underline{\mathbf{u}}_t\mathbb{E}_t^{CB}\underline{\mathbf{u}}_t'\right]}{\mathbb{E}\left[(r_t^{\perp})^2\right]}\mathbf{\Psi}' + \mathbf{\Theta}^k\mathbf{Y}\frac{\mathbb{E}\left[\mathbb{E}_{t^-}^M[r_t]r_t^{\perp}\right]}{\mathbb{E}\left[(r_t^{\perp})^2\right]} \\
&\quad - \tilde{\mathbf{\Theta}}^k\frac{\mathbb{E}_{t+\tau}^M\underline{\mathbf{u}}_{\mathbf{t}}(1_k\otimes\mathbb{E}_t^{CB}\underline{\mathbf{u}}_t)'}{E\left[(r_t^{\perp})^2\right]}(1_k\otimes\mathbf{\Psi})' \\
&= \mathbf{\Gamma}^k\mathbf{B} - \mathbf{\Theta}^k\mathbf{Y} - \mathbf{\Theta}^k\frac{\mathbb{E}\left[\mathbb{E}_{t^-}^M\underline{\mathbf{u}}_t\mathbb{E}_t^{CB}\underline{\mathbf{u}}_t'\right]}{\mathbb{E}\left[(r_t^{\perp})^2\right]}\mathbf{\Psi}' + \mathbf{\Theta}^k\mathbf{Y}\frac{\mathbb{E}\left[\mathbb{E}_{t^-}^M[r_t]r_t^{\perp}\right]}{\mathbb{E}\left[(r_t^{\perp})^2\right]} \\
&\quad - \tilde{\mathbf{\Theta}}^k\frac{\mathbb{E}_{\mathbf{t+\o}}^{\mathbf{M}}\underline{\mathbf{u}}_{\mathbf{t}}(1_k\otimes\mathbb{E}_t^{CB}\underline{\mathbf{u}}_t)'}{\mathbb{E}\left[(r_t^{\perp})^2\right]}(1_k\otimes\mathbf{\Psi})'.
\end{aligned}
$$

OLS is unbiased if $\mathbf{Y} = 0$ (i.e. there is no information effect) and any of:

- $\mathbf{\Psi} = \mathbf{0}$. In this case, monetary policy does not respond to future economic shocks. This is unlikely.
- $\underline{\mathbf{u}}_t, \mathbb{E}_t^M\underline{\mathbf{u}}_t \perp \mathbb{E}_t^{CB}\underline{\mathbf{u}}_t$. In this case, future shocks and central bank's forecasts of future shocks would be uncorrelated. This would be the case if the central bank's forecasts were just noise. This is unlikely.

A reasonable conclusion here is that OLS is likely biased!

C.2.3 Solutions from Previous Literature

In this framework, the ideal instrument to use for estimation would be the monetary policy shock (ϵ_t in Equation 3.4) if it could be observed. In this subsection we will briefly show how two popular monetary policy instruments approximate ϵ_t as an instrument.

Next covers solutions from previous literature quickly then adds in information with high-frequency instrument.

C.2.3.1 High Frequency Approach

The high frequency monetary policy approach pioneered by Kuttner (2001) and extended by Gürkaynak, Sack, and Swanson (2005) which use the fact that within a quarter, monetary policy is allowed to be changed only twice.[1]

With equations, we can represent this as follows:

$$\begin{aligned}
\underline{Y}_{t+k} &= \underline{\gamma}^k + \mathbf{\Phi}^k \underline{Y}_{t-1} + \left(\mathbf{\Theta}^k \mathbf{Y} - \mathbf{\Gamma}^k \mathbf{B}\right) r_t + + \mathbf{\Theta}^k \left(\mathbb{E}^M_{t^-} \underline{\mathbf{u}}_t - \mathbf{Y}\mathbb{E}^M_{t^-}[r_t]\right) + \tilde{u}_{t+k} - \mathbf{\Gamma^k e}_t, \\
r_t &= \mathbb{1}_{t \notin T^*} \times r_{t-1} + \mathbb{1}_{t \in T^*} \times \left[r^* + \rho r_{t-1} + \sum_{l>0} \Psi_l \mathbb{E}^{CB}_t \underline{u}_{t+l} + \epsilon_t \right], \\
r_{t+j} &= \mathbb{E}^M_t r_{t+j} + \tilde{e}_{j,t}, \;\; \tilde{e}_{j,t} \perp \mathbb{E}^M_t r_{t+j}, \\
\mathbb{E}^M_t r_{t+j} &= \alpha_j + \beta_j r_t + e_{j,t},
\end{aligned}$$

where $T^* \subset T = \{0, 1, \ldots, \}$ represents the subsets of time that are policy announcement dates and $\mathbf{e}_t = \begin{bmatrix} 0 \\ e_{1,t} \\ e_{2,t} \\ \vdots \end{bmatrix}$ - a vector of the yield curve terms that cannot be predicted with short rates r_t.

There are two changes here: Firstly, the short rate is not changed except during announcement dates $t \in T^*$. Secondly, short rate futures $\mathbb{E}^M_t[r_{t+j}]$ are rational forecasts in that forecast errors are not correlated with the market prediction.[2]

This approach essentially calculates $r_{t^*} - \mathbb{E}^M_{t^*-} r_{t^*}$ using very short horizon futures markets. In this framework, this evaluates to:

$$r_{t^*} - \mathbb{E}^M_{t^*-} r_{t^*} = \sum_{l>0} \Psi_l \left(\mathbb{E}^{CB}_{t^*} \underline{u}_{t^*+l} - \mathbb{E}^M_{t^*-} \mathbb{E}^{CB}_t \underline{u}_{t^*+l} \right) + (\epsilon_{t^*} - \mathbb{E}^M_{t^*-} \epsilon_{t^*})$$

Using similar steps to the bias derivation of the OLS estimator in Appendix C.2.2, it can be shown that using this high-frequency instrument to identify the effects of monetary policy gives:

[1]There are rare cases when central banks choose to make surprise announcements and change policy but by and large, monetary policy is adjusted 8 times a year on pre-scheduled days.

[2]To allow for the possibility of non-rational expectations we would need to write $r_{t+j} = a_j + \phi_j \mathbb{E}^M_t r_{t+j} + \tilde{e}_{j,t}$ to ensure that $\tilde{e}_{t+j} \perp \mathbb{E}^M_t r_{t+j}$. Setting $a_j = 0$ and $\phi_j = 1$ is imposing that the forecast is unbiased. Note also that without loss of generality, we can write this optimal forecast as being a function of a loading term β_j on the current short rate and a second term e_{jt} orthogonal to the short rate. See Appendix C.2.2 for more details

$$
\begin{aligned}
\underline{\hat{\Lambda}}_{HF}^{k} = & \underbrace{\boldsymbol{\Gamma}^k\mathbf{B}-\boldsymbol{\Theta}^k\mathbf{Y}}_{\text{Correct Impulse Response}} \quad \underbrace{-\tilde{\boldsymbol{\Theta}}^k\frac{\mathbb{E}\left[\mathbb{E}^M_{t^*+\tau}\underline{\mathbf{u}}_{\mathbf{t}^*}\left(1_k\otimes\left(\mathbb{E}^{CB}_{t^*}\underline{\mathbf{u}}_{t^*}-\mathbb{E}^M_{t^*-}\mathbb{E}^{CB}_{t^*}\underline{\mathbf{u}}_{t^*}\right)\right)'\right]}{\mathbb{E}\left[r^{\perp}_{t^*}(r_{t^*}-\mathbb{E}^M_{t^*-}r_{t^*})^{\perp}\right]}(1_k\otimes\boldsymbol{\Psi})'}_{\text{Mismatch of Market and Central Bank Information}} \\
& + \underbrace{\boldsymbol{\Theta}^k\mathbf{Y}\frac{\mathbb{E}\left[\mathbb{E}^M_{t^*-}[r_{t^*}](r_{t^*}-\mathbb{E}^M_{t^*-}r_{t^*})^{\perp}\right]}{\mathbb{E}\left[r^{\perp}_{t^*}(r_{t^*}-\mathbb{E}^M_{t^*}r_{t^*})^{\perp}\right]}}_{\text{Market Surprises are Predictable}}. \qquad \text{(C.6)}
\end{aligned}
$$

Identification in this setup is achieved if the following three conditions hold:

1. Private agent's information set is at least as large as the central bank: $\mathbb{E}^M_t\mathbb{E}^{CB}_t x_{t+j} = \mathbb{E}^{CB}_t x_{t+j}$.
2. The frequency is high enough that $\mathbb{E}_{t^*-}x_t = \mathbb{E}_{t^*}x_t$.
3. The market's prediction errors are not themselves predictable with their interest rate forecast.

If the first two of these conditions hold, $\mathbb{E}^{CB}_{t^*}\underline{u}_{t^*} - \mathbb{E}^M_{t^*-}E^{CB}_{t^*}\underline{u}_{t^*} = \underline{0}$, and the second term becomes 0. The intuition is that when the high-frequency instrument is only capturing exogenous monetary policy movements ϵ_t, then the forward looking parts of policy are ignored in the instrument.[3] When private agents have less or different information to the central bank, then an instrument that purges the market's policy rate forecast still doesn't purge all the information the central bank is using to set policy and therefore might still deliver biased estimates.

The final requirement is that market surprises are not themselves predictable with past interest rate forecasts. The problem is that markets may have responded to the current policy movement in advance. If the surprise component of the current policy movement is itself correlated with the part the market successfully predicted, then the amount of signal extraction done by markets cannot be identified.

C.2.3.2 Romer & Romer (2004) Approach

An alternative method pioneered by Romer and Romer (2004) involves using the central bank's forecasts to control for its information set. They use data of staff forecasts at the U.S. Federal Reserve (the "Greenbook" forecasts) and assume that these forecasts represent the entire information set the Federal Reserve uses to set policy. An appealing feature of this argument is that the forecasts themselves are forward looking and contain

[3] Because the market might know about some of the exogenous policy movements in advance - the instrument captures *unforecastable policy movements* $\epsilon_{t^*} - \mathbb{E}^M_{t^*-1}\epsilon_{t^*}$ not the total amount of exogenous policy movement ϵ_{t^*} - the first stage might be low even when there are considerable exogenous policy movements.

precisely those variables one might expect to be the only terms in a Taylor rule - GDP and inflation. The first step of their procedure involves linearly removing variation in the Fed Funds Rate that can be attributed to these forecasts. All that remains is an exogenous policy shock:

$$\hat{\epsilon}_t^{RR} = r_t - \mathbb{E}[r_t|\mathcal{F}_t] = r_t - \mathbb{E}_t^{RR}[r_t],$$

where $\mathcal{F}_t$ represents the information set contained in the Greenbook forecasts. One convenient feature of doing this for the US relative to other countries is that the Greenbook forecasts are released with a 5 year lag which means that the market is not informed of them prior to or even within the quarter of the interest rate adjustment.

Using similar steps to above, using $\hat{\epsilon}_t^{RR}$ as an instrument for monetary policy yields:

$$\begin{aligned}\underline{\hat{\Lambda}}_{RR}^{k} = {} & \underbrace{\boldsymbol{\Gamma}^k\mathbf{B} - \boldsymbol{\Theta}^k\mathbf{Y}}_{\text{Correct Impulse Response}} \underbrace{- \boldsymbol{\Theta}^k \frac{\mathbb{E}\left[\mathbb{E}_t^M \underline{\mathbf{u}}_t \left(\mathbb{E}_t^{CB}\underline{\mathbf{u}}_t - \mathbb{E}_t^{RR}\underline{\mathbf{u}}_t\right)'\right]}{\mathbb{E}\left[r_t^{\perp}(r_t - \mathbb{E}_t^{RR} r_t)^{\perp}\right]}\boldsymbol{\Psi}'}_{\text{Information set not complete}} + \underbrace{\boldsymbol{\Theta}^k\mathbf{Y}\frac{\mathbb{E}\left[\mathbb{E}_{t^-}^M[r_t](r_t - \mathbb{E}_t^{RR} r_t)^{\perp}\right]}{\mathbb{E}\left[r_t^{\perp}(r_t - \mathbb{E}_t^{RR} r_t)^{\perp}\right]}}_{\text{Market can predict these shocks}} \\ & \underbrace{- \tilde{\boldsymbol{\Theta}}^k \frac{\mathbb{E}\left[\mathbb{E}_{\mathbf{t+ø}}^{\mathbf{M}} \underline{\mathbf{u}}_{\mathbf{t}} \left(1_k \otimes \left(\mathbb{E}_t^{CB}\underline{\mathbf{u}}_t - \mathbb{E}_t^{RR}\underline{\mathbf{u}}_t\right)\right)'\right]}{\mathbb{E}\left[r_t^{\perp}(r_t - \mathbb{E}_t^{RR} r_t)^{\perp}\right]}\left(1_k \otimes \boldsymbol{\Psi}\right)'.}_{\text{Information set not complete}} \end{aligned} \tag{C.7}$$

The assumptions for identification are that:

- The information set used to compute $\mathbb{E}^{RR}[r_t|\mathcal{F}_t]$ contains the central bank's information set.
- The market cannot predict the Romer and Romer (2004) shocks in advance.

The first assumption is unlikely to be an issue given that central banks have very limited sets of variables they are directly concerned with and the Romer and Romer (2004) methodology uses forecasts of exactly these variables. The second assumption however is an issue as there is no reason not to think that the market is able to predict exogenous policy shocks in advance.[4]

A simple solution to the second condition would be to control for market's forecast of interest rates immediately prior to the policy announcement which is very similar to the hybrid approach taken by Miranda-Agrippino (2016) and Miranda-Agrippino and Ricco (2018).

[4]This in particular leads to an issue when there is signal extraction taking place in markets (as is visible here) but there is a more general external validity issue if one isn't careful as well. See Appendix C.3 for more details.

C.2.3.3 The Hybrid Instrument

Miranda-Agrippino (2016) and Miranda-Agrippino and Ricco (2018) both use a combination of the previous two instruments to try and identify the correct impulse response.

Their idea is to residualize *the market surprise series* with the central bank's forecasts as used by Romer and Romer (2004). This leads to the following instrument: $Z_t = r_t - E_{t^-}^{M,RR}\left[r_t|\mathcal{F}_t, \mathcal{I}_{t^-}^{M}\right]$. Using this as an instrument gives the following output:

$$\underline{\hat{\Lambda}}_{M,RR}^{k} = \underbrace{\mathbf{\Gamma}^k\mathbf{B} - \mathbf{\Theta}^k\mathbf{Y}}_{\text{Correct Impulse Response}} \underbrace{- \tilde{\mathbf{\Theta}}^k \frac{\mathbb{E}\left[\mathbb{E}_{t^*+\tau}^{M}\underline{\mathbf{u}}_{\mathbf{t}^*}\left(1_k \otimes \left(\mathbb{E}_{t^*}^{CB}\underline{\mathbf{u}}_{t^*} - \mathbb{E}_{t^*-1}^{M,RR}\mathbb{E}_{t^*}^{CB}\underline{\mathbf{u}}_{t^*}\right)\right)'\right]}{\mathbb{E}\left[r_{t^*}^{\perp}(r_{t^*} - \mathbb{E}_{t^*-1}^{M,RR}r_{t^*})^{\perp}\right]}(1_k \otimes \mathbf{\Psi})'}_{\text{Misses all Central Bank Information}}$$
$$\underbrace{+ \mathbf{\Theta}^k\mathbf{Y}\frac{\mathbb{E}\left[\mathbb{E}_{t^*-1}^{M}[r_{t^*}](r_{t^*} - \mathbb{E}_{t^*-1}^{M,RR}r_{t^*})^{\perp}\right]}{\mathbb{E}\left[r_{t^*}^{\perp}(r_{t^*} - \mathbb{E}_{t^*-1}^{M,RR}r_{t^*})^{\perp}\right]}}_{\text{Market Exogenous surprises are Predictable}}. \tag{C.8}$$

Here the identifications become:

- The combination of market and central bank forecasts used in the procedure capture the full information set the central bank uses to set policy.
- The "exogenous" policy surprises (i.e. the portion of the market surprises attributed as exogenous policy changes) are not predictable by the market.

The first seems fairly reasonable and the second still risks being a problem. However, Miranda-Agrippino and Ricco (2018) show that a lot of troubling past findings of the effects of monetary policy shocks are not as prevalent with their procedure.

C.3 Romer & Romer (2004) identification even with no learning

It was stated in Appendix C.2 that if the second assumption is not fulfilled there is the risk of an external validity issue. The purpose of this section is to discuss this issue.

The essence of the problem is that when we construct monetary policy shocks in macro models, we typically assume that 1. it is independent of the state of the economy (a movement ϵ_t in the framework) and 2. a surprise to markets: $\mathbb{E}_{t-s}^{M}\epsilon_t = 0 \; \forall s > 0$. If policy shocks in practice are forecastable by markets in advance, then the estimates produced by the Romer and Romer (2004) instrument will be identified in that they reflect the typical effect of monetary policy in the data, but they won't map correctly to the policy surprise concept used in our macro models.

To see the issue, let us adjust the structural DGP to the following:

$$\begin{aligned}
\underline{Y}_{t+k} &= \underline{c}^k + \Phi\underline{Y}_{t-1} - \mathbf{\Gamma B} r_t + \sum_{j>0} \Theta_j \mathbb{E}_t^M \underline{u}_{t+j} + \xi_t + \mathbf{\Gamma}\mathbb{E}_t^M \mathbf{e}_t - \sum_{p>0} v_p \mathbb{E}_{t-p}^M r_t, \\
r_t &= \mathbb{1}_{t \notin T^*} r_{t-1} + \mathbb{1}_{t \in T^*} \left[r^* + \rho r_{t-1} + \sum_{l>0} \Psi_l \mathbb{E}_t^{CB} \underline{u}_{t+l} + \epsilon_t \right], \\
r_{t+j} &= \mathbb{E}_t^M r_{t+j} + \tilde{e}_{j,t} \quad \tilde{e}_{j,t} \perp \mathbb{E}_t^M r_{t+j}, \\
\mathbb{E}_t^M r_{t+j} &= \alpha_j + \beta_j r_t + e_{j,t}.
\end{aligned}$$

Now current economic outcomes depend on a term that is related to past expectations of current policy rates - previous yield curves. Note that these yield curves could be included in the vector $\underline{Y}_{t-1}$ but variables in this vector are assumed to be controlled for. If however, we do not control for the past yield curve (which is very typical among papers using this instrument) then we estimate the following:

$$\hat{\mathbf{\Lambda}}_{RR}^k = \mathbf{\Gamma}^k \mathbf{B} - \mathbf{\Theta}^k \frac{\mathbb{E}\left[\mathbb{E}_t^M \underline{\mathbf{u}}_t \left(\mathbb{E}_t^{CB} \underline{\mathbf{u}}_t - \mathbb{E}_t^{RR} \underline{\mathbf{u}}_t\right)'\right]}{\mathbb{E}\left[((r_t - \mathbb{E}_t^{RR} r_t)^\perp)^2\right]} \mathbf{\Psi}' + \sum_{p>0} v_p \frac{\mathbb{E}\left[\mathbb{E}_{t-p}^M r_t^\perp (r_t - \mathbb{E}_t^{RR} r_t)^\perp\right]}{((r_t - \mathbb{E}_t^{RR} r_t)^\perp)^2}.$$

Now the identifying assumptions become:

- The information set used to compute $\mathbb{E}^{RR}[r_t|\mathcal{F}_t]$ contains the central bank's information set.
- The Romer and Romer (2004) policy shock is not predicted by the market in previous periods ($v_p = 0, \ \forall p$).

C.4 Other Results

C.4.1 Incorporating Crisis Periods

Our baseline results include the periods of Global Financial Crisis (GFC, hereafter) from December 2007 to June 2009. The results could be sensitive to the inclusion of this financial crisis periods. Therefore, we examine if the results we present in the main text are sensitive to this time periods. In particular, we consider the following to incorporate this GFC periods:

1. Include crisis dummies, $\mathbb{1}_{crisis_{t+h}}$. Here, crisis dummies are in terms of dependent variable, $Y_{i,t+h}$. If $t+h$ is in crisis periods or not. → Figure C.1 and Figure C.2.

2. Include the interaction of crisis dummies, monetary policy, and the dummy variables indicating whether Inflation Reports are released or not, , $\mathbb{1}_{crisis_{t+h}} \times r_t \times \mathbb{1}_{\text{Inflation Report}_t}$. Again, crisis dummies are in terms of dependent variable, $Y_{i,t+h}$. If $t+h$ is in crisis periods or not. → Figure C.3 and Figure C.4.

3. Include crisis dummies, $\mathbb{1}_{crisis_{t+h}}$, the interaction of crisis dummies with the monetary policy $\mathbb{1}_{crisis_{t+h}} \times r_t$, the interaction of crisis dummies, monetary policy, and the dummy variables indicating whether Inflation Reports are released or not, $\mathbb{1}_{crisis_{t+h}} \times r_t \times \mathbb{1}_{\text{Inflation Report}_t}$. Again, crisis dummies are in terms of dependent variable, $Y_{i,t+h}$. If $t+h$ is in crisis periods or not. → Figure C.5 and Figure C.6.

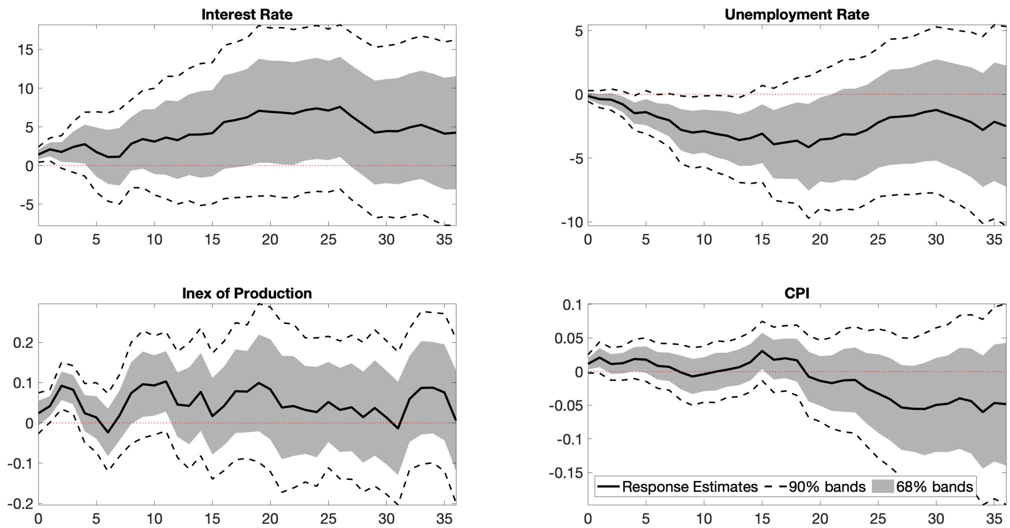
4. Include Crisis Dummies, $\mathbb{1}_{crisis_t}$. Here, crisis dummies are in terms of monetary policy, r_t. If t is during crisis periods or not. → Figure C.7 and Figure C.8.

5. Include the interaction of crisis dummies, monetary policy, and the dummy variables indicating whether Inflation Reports are released or not, , $\mathbb{1}_{crisis_t} \times r_t \times \mathbb{1}_{\text{Inflation Report}_t}$. Again, crisis dummies are in terms of monetary policy, $r_{i,t}$. If t is during crisis periods or not. → Figure C.9 and Figure C.10.

6. Excluding the crisis periods. → Figure C.11 and Figure C.12.

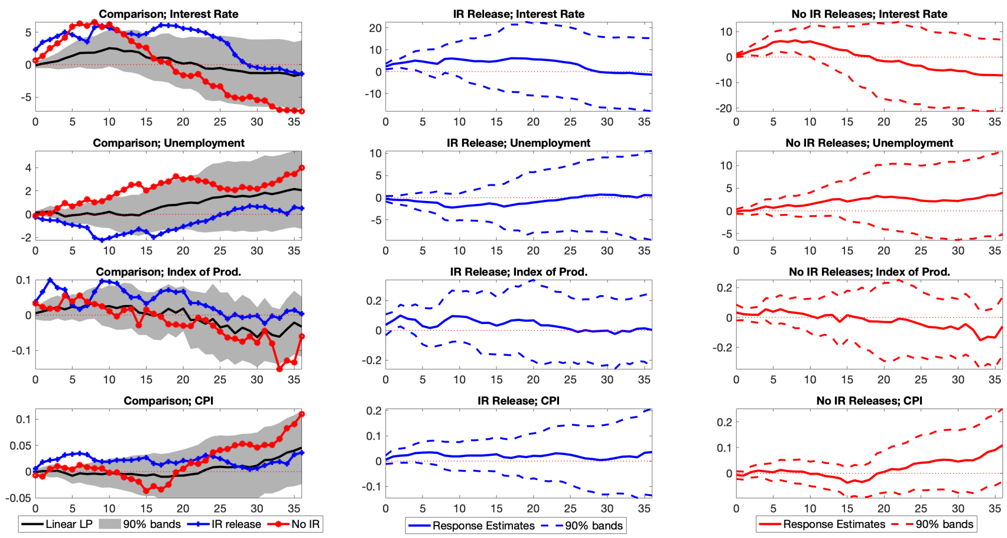
The results are essentially consistent across all these six ways of incorporating GFC periods.

Figure C.1: The Information Effect - 1st



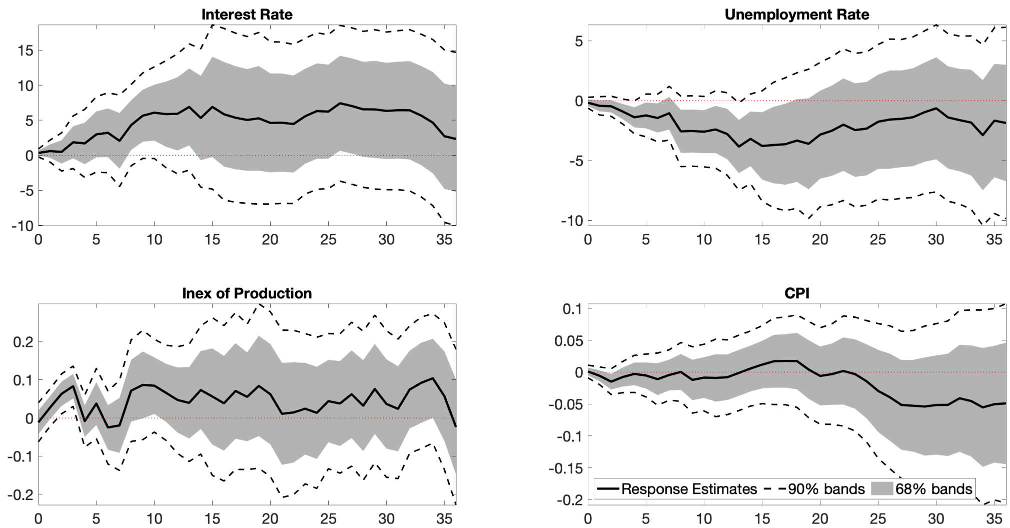
Notes: This figure shows the estimates, and 68% and 90% confidence bands from estimating Jordà (2005) Local Projections of the labelled outcome variable on high frequency monetary policy shock series and the shock series interacted with a time dummy for whether an Inflation Report was released, **with the first way of incorporating GFC periods**. These graphs all represent the *difference* between outcomes when an inflation report is released relative to when it isn't. Positive estimates indicate that releasing an inflation report makes the equilibrium response of this variable higher. The estimates are denoted as black solid line and 68% (90%) confidence bands are represented as grey shaded area (two black dotted lines).

Figure C.2: Estimates for Inflation Report and Non-Inflation Report Policy Changes



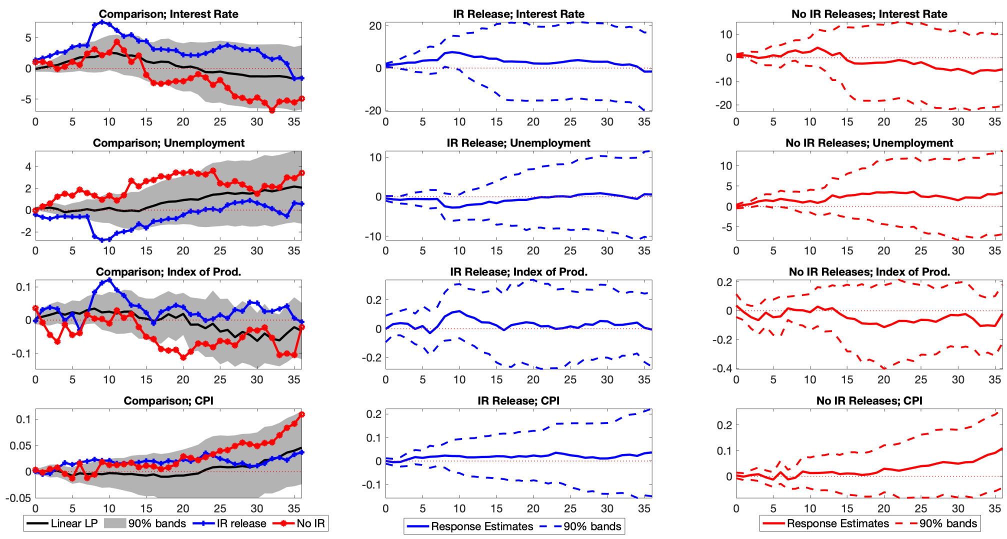
Notes: This figure shows the different effects of outcomes when there is no inflation report released (the red lines) and when there is an inflation report released (the blue lines), **with the first way of incorporating GFC periods**. The left-end columns show all the estimates of the effects of outcome. The black lines show the linear effect without considering the effect of inflation report releases along with grey area as 90% confidence bands. The blue line with plus signs (the red line with circles) show the effects when inflation report is released (not released). The middle column shows the estimates of the effects of outcome when inflation report is released with blue solid line along with 68% confidence bands denoted as sky blue shaded area and with 90 % bands with blue dashed lines. The right-end column shows the estimates of the effects of outcome when no inflation report is released with red solid line along with 68% confidence bands denoted as pink shaded area and with 90 % bands with red dashed lines.

Figure C.3: The Information Effect - 2nd



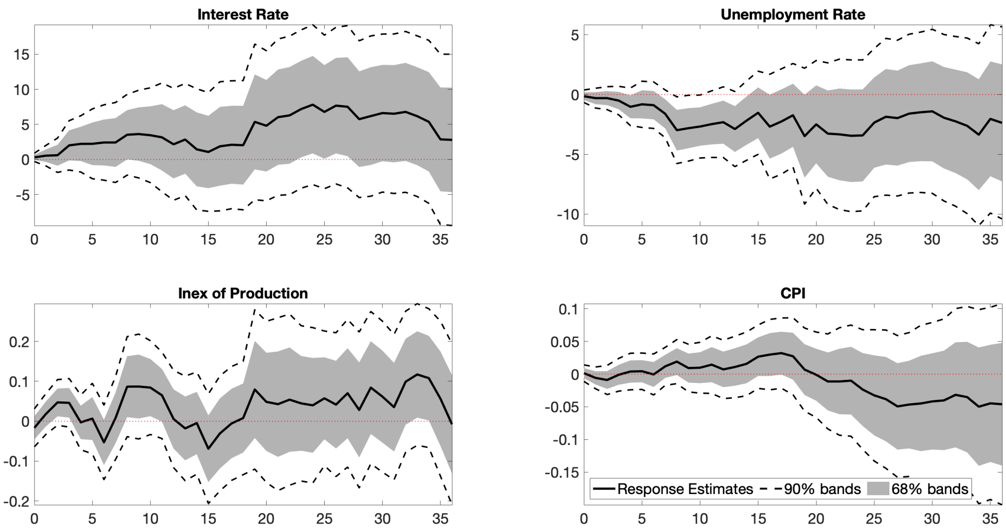
Notes: This figure shows the estimates, and 68% and 90% confidence bands from estimating Jordà (2005) Local Projections of the labelled outcome variable on high frequency monetary policy shock series and the shock series interacted with a time dummy for whether an Inflation Report was released, **with the second way of incorporating GFC periods**. These graphs all represent the *difference* between outcomes when an inflation report is released relative to when it isn't. Positive estimates indicate that releasing an inflation report makes the equilibrium response of this variable higher. The estimates are denoted as black solid line and 68% (90%) confidence bands are represented as grey shaded area (two black dotted lines).

Figure C.4: Estimates for Inflation Report and Non-Inflation Report Policy Changes



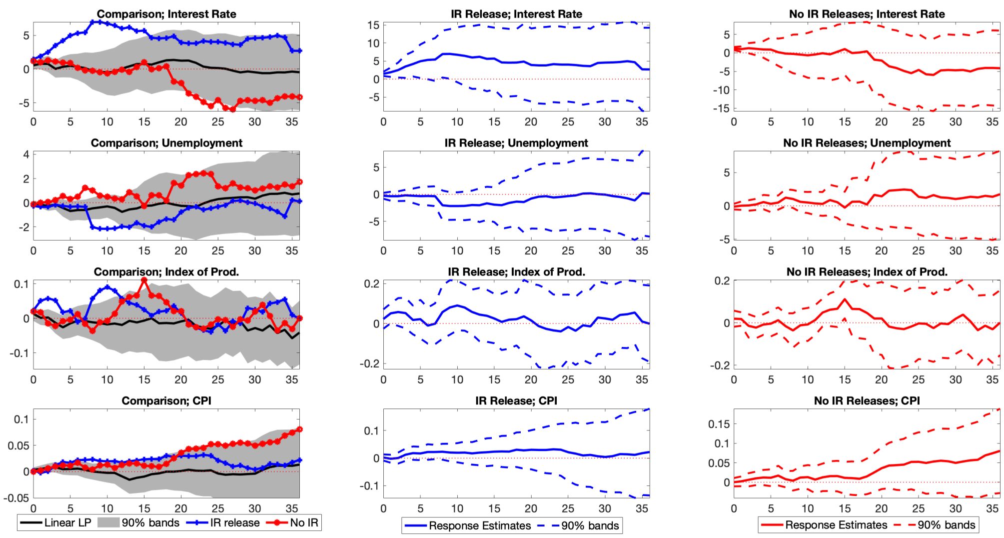
Notes: This figure shows the different effects of outcomes when there is no inflation report released (the red lines) and when there is an inflation report released (the blue lines), **with the second way of incorporating GFC periods**. The left-end columns show all the estimates of the effects of outcome. The black lines show the linear effect without considering the effect of inflation report releases along with grey area as 90% confidence bands. The blue line with plus signs (the red line with circles) show the effects when inflation report is released (not released). The middle column shows the estimates of the effects of outcome when inflation report is released with blue solid line along with 68% confidence bands denoted as sky blue shaded area and with 90 % bands with blue dashed lines. The right-end column shows the estimates of the effects of outcome when no inflation report is released with red solid line along with 68% confidence bands denoted as pink shaded area and with 90 % bands with red dashed lines.

Figure C.5: The Information Effect - 3rd



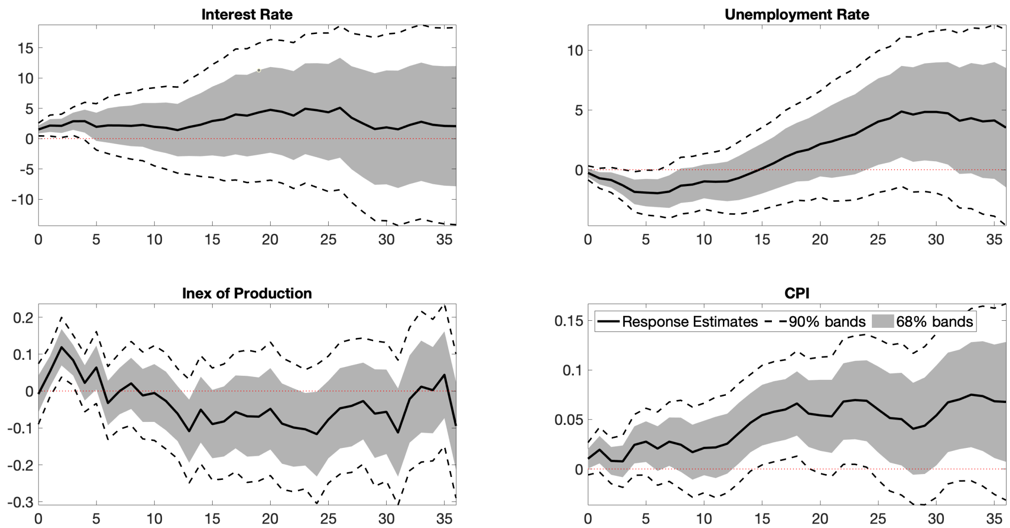
Notes: This figure shows the estimates, and 68% and 90% confidence bands from estimating Jordà (2005) Local Projections of the labelled outcome variable on high frequency monetary policy shock series and the shock series interacted with a time dummy for whether an Inflation Report was released, **with the third way of incorporating GFC periods**. These graphs all represent the *difference* between outcomes when an inflation report is released relative to when it isn't. Positive estimates indicate that releasing an inflation report makes the equilibrium response of this variable higher. The estimates are denoted as black solid line and 68% (90%) confidence bands are represented as grey shaded area (two black dotted lines).

Figure C.6: Estimates for Inflation Report and Non-Inflation Report Policy Changes



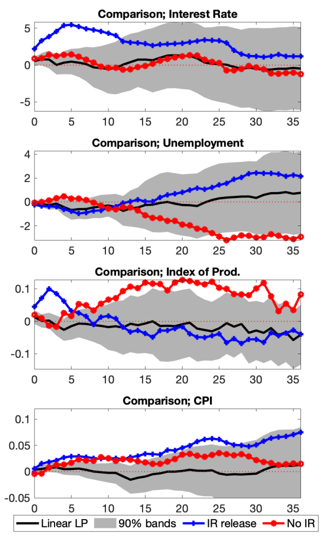
Notes: This figure shows the different effects of outcomes when there is no inflation report released (the red lines) and when there is an inflation report released (the blue lines), **with the third way of incorporating GFC periods**. The left-end columns show all the estimates of the effects of outcome. The black lines show the linear effect without considering the effect of inflation report releases along with grey area as 90% confidence bands. The blue line with plus signs (the red line with circles) show the effects when inflation report is released (not released). The middle column shows the estimates of the effects of outcome when inflation report is released with blue solid line along with 68% confidence bands denoted as sky blue shaded area and with 90 % bands with blue dashed lines. The right-end column shows the estimates of the effects of outcome when no inflation report is released with red solid line along with 68% confidence bands denoted as pink shaded area and with 90 % bands with red dashed lines.

Figure C.7: The Information Effect - fourth



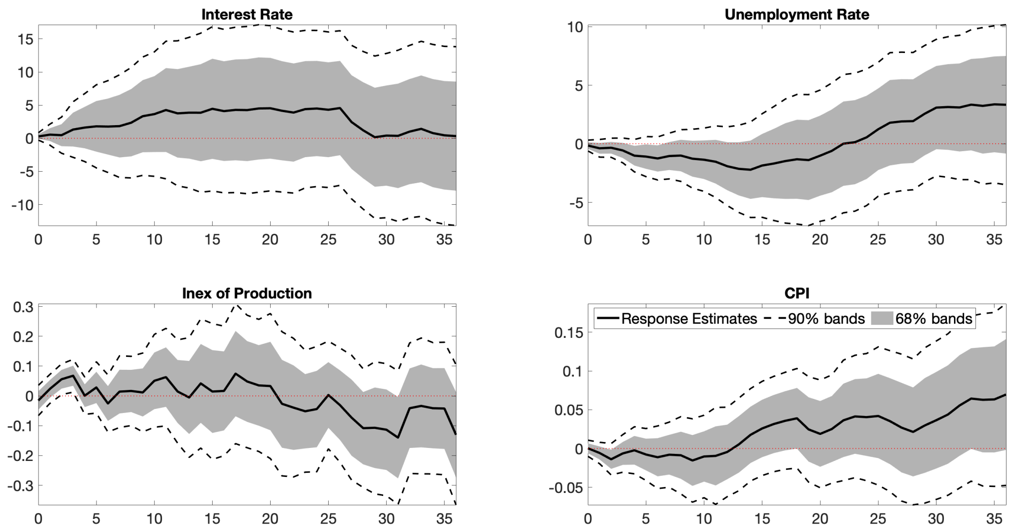
Notes: This figure shows the estimates, and 68% and 90% confidence bands from estimating Jordà (2005) Local Projections of the labelled outcome variable on high frequency monetary policy shock series and the shock series interacted with a time dummy for whether an Inflation Report was released, **with the fourth way of incorporating GFC periods**. These graphs all represent the *difference* between outcomes when an inflation report is released relative to when it isn't. Positive estimates indicate that releasing an inflation report makes the equilibrium response of this variable higher. The estimates are denoted as black solid line and 68% (90%) confidence bands are represented as grey shaded area (two black dotted lines).

Figure C.8: Estimates for Inflation Report and Non-Inflation Report Policy Changes



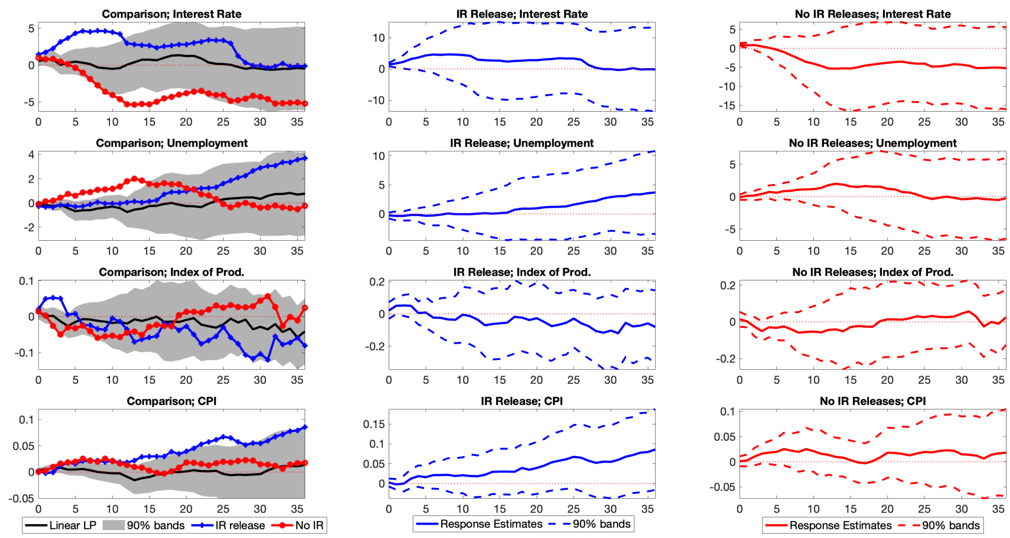
Notes: This figure shows the different effects of outcomes when there is no inflation report released (the red lines) and when there is an inflation report released (the blue lines), **with the fourth way of incorporating GFC periods**. The left-end columns show all the estimates of the effects of outcome. The black lines show the linear effect without considering the effect of inflation report releases along with grey area as 90% confidence bands. The blue line with plus signs (the red line with circles) show the effects when inflation report is released (not released). The middle column shows the estimates of the effects of outcome when inflation report is released with blue solid line along with 68% confidence bands denoted as sky blue shaded area and with 90 % bands with blue dashed lines. The right-end column shows the estimates of the effects of outcome when no inflation report is released with red solid line along with 68% confidence bands denoted as pink shaded area and with 90 % bands with red dashed lines.

Figure C.9: The Information Effect - 5th



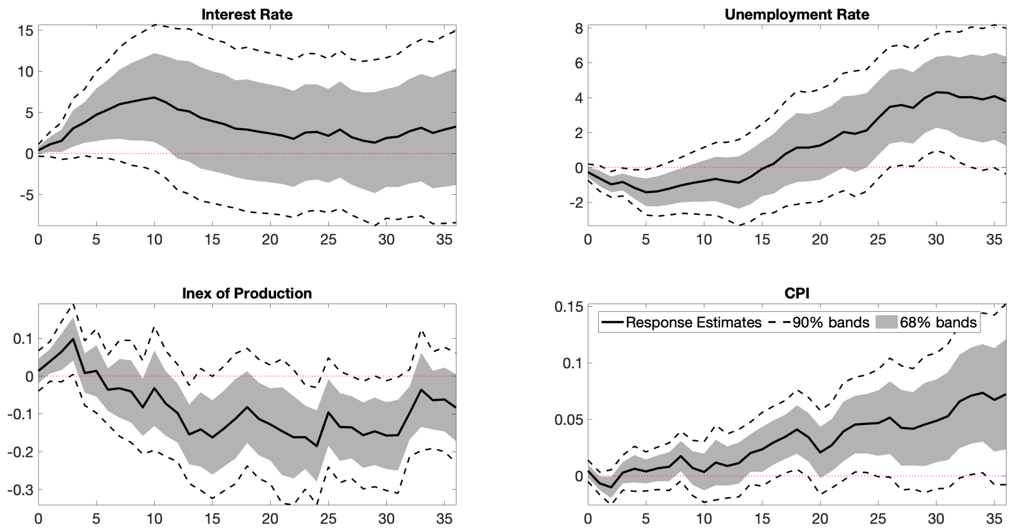
Notes: This figure shows the estimates, and 68% and 90% confidence bands from estimating Jordà (2005) Local Projections of the labelled outcome variable on high frequency monetary policy shock series and the shock series interacted with a time dummy for whether an Inflation Report was released, **with the fifth way of incorporating GFC periods**. These graphs all represent the *difference* between outcomes when an inflation report is released relative to when it isn't. Positive estimates indicate that releasing an inflation report makes the equilibrium response of this variable higher. The estimates are denoted as black solid line and 68% (90%) confidence bands are represented as grey shaded area (two black dotted lines).

Figure C.10: Estimates for Inflation Report and Non-Inflation Report Policy Changes



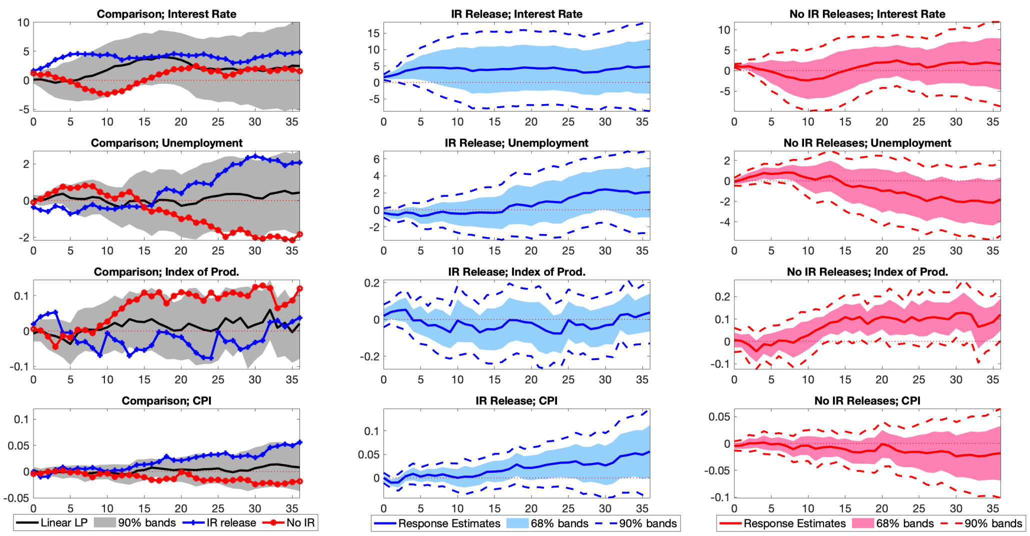
Notes: This figure shows the different effects of outcomes when there is no inflation report released (the red lines) and when there is an inflation report released (the blue lines), **with the fifth way of incorporating GFC periods**. The left-end columns show all the estimates of the effects of outcome. The black lines show the linear effect without considering the effect of inflation report releases along with grey area as 90% confidence bands. The blue line with plus signs (the red line with circles) show the effects when inflation report is released (not released). The middle column shows the estimates of the effects of outcome when inflation report is released with blue solid line along with 68% confidence bands denoted as sky blue shaded area and with 90 % bands with blue dashed lines. The right-end column shows the estimates of the effects of outcome when no inflation report is released with red solid line along with 68% confidence bands denoted as pink shaded area and with 90 % bands with red dashed lines.

Figure C.11: The Information Effect - 6th



Notes: This figure shows the estimates, and 68% and 90% confidence bands from estimating Jordà (2005) Local Projections of the labelled outcome variable on high frequency monetary policy shock series and the shock series interacted with a time dummy for whether an Inflation Report was released, **with the sixth way of incorporating GFC periods**. These graphs all represent the *difference* between outcomes when an inflation report is released relative to when it isn't. Positive estimates indicate that releasing an inflation report makes the equilibrium response of this variable higher. The estimates are denoted as black solid line and 68% (90%) confidence bands are represented as grey shaded area (two black dotted lines).

Figure C.12: Estimates for Inflation Report and Non-Inflation Report Policy Changes



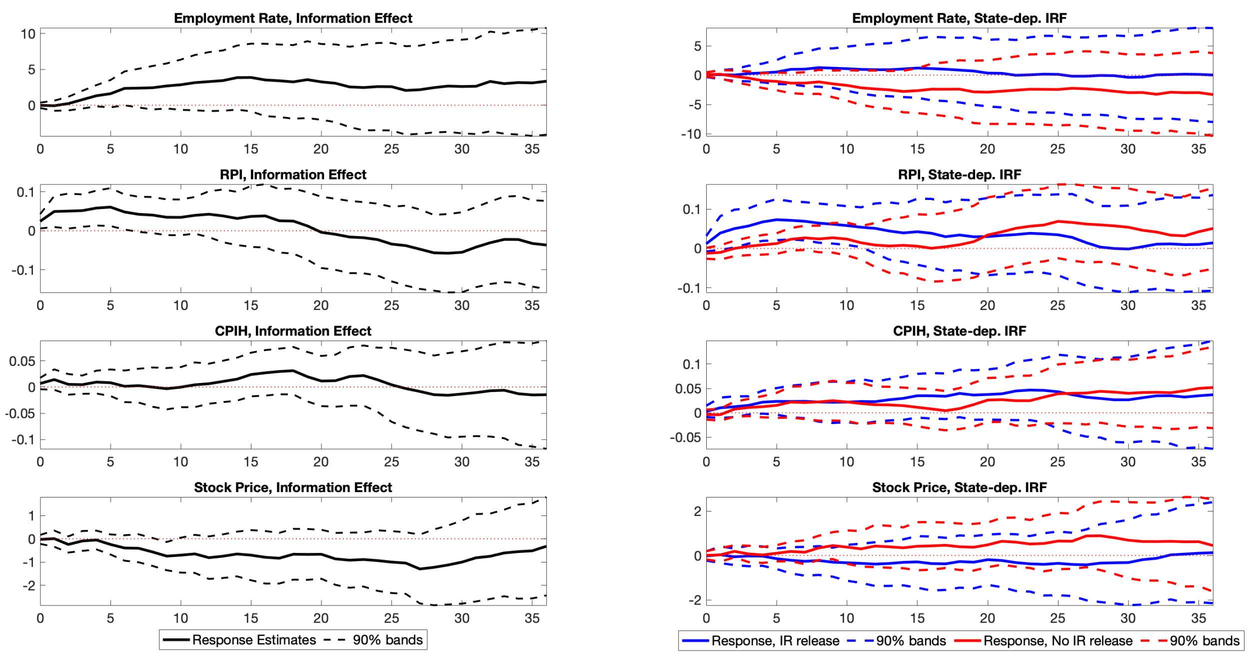
Notes: This figure shows the different effects of outcomes when there is no inflation report released (the red lines) and when there is an inflation report released (the blue lines), **with the sixth way of incorporating GFC periods**. The left-end columns show all the estimates of the effects of outcome. The black lines show the linear effect without considering the effect of inflation report releases along with grey area as 90% confidence bands. The blue line with plus signs (the red line with circles) show the effects when inflation report is released (not released). The middle column shows the estimates of the effects of outcome when inflation report is released with blue solid line along with 68% confidence bands denoted as sky blue shaded area and with 90 % bands with blue dashed lines. The right-end column shows the estimates of the effects of outcome when no inflation report is released with red solid line along with 68% confidence bands denoted as pink shaded area and with 90 % bands with red dashed lines.

C.4.2 Other Variables

In this section, we present results with other variables that we do not consider in the main text: employment rate, two other price indices, RPI (Retail Price Index), and CPIH (Consumer Price Index with Housing), and stock price index, FTSE-100.

Figure C.13 shows the results. Similar to the findings in the main text, employment rate and price indices tend to rise when Inflation Reports are released, and vice versa when no report is released to monetary surprises. In particular, unlike the results with CPI (consumer price index) in the main text where there the increase in CPI with the Inflation Reports releases was barely significant, here, we see clearly that RPI tends to significantly increase more with the release of Inflation Report here.

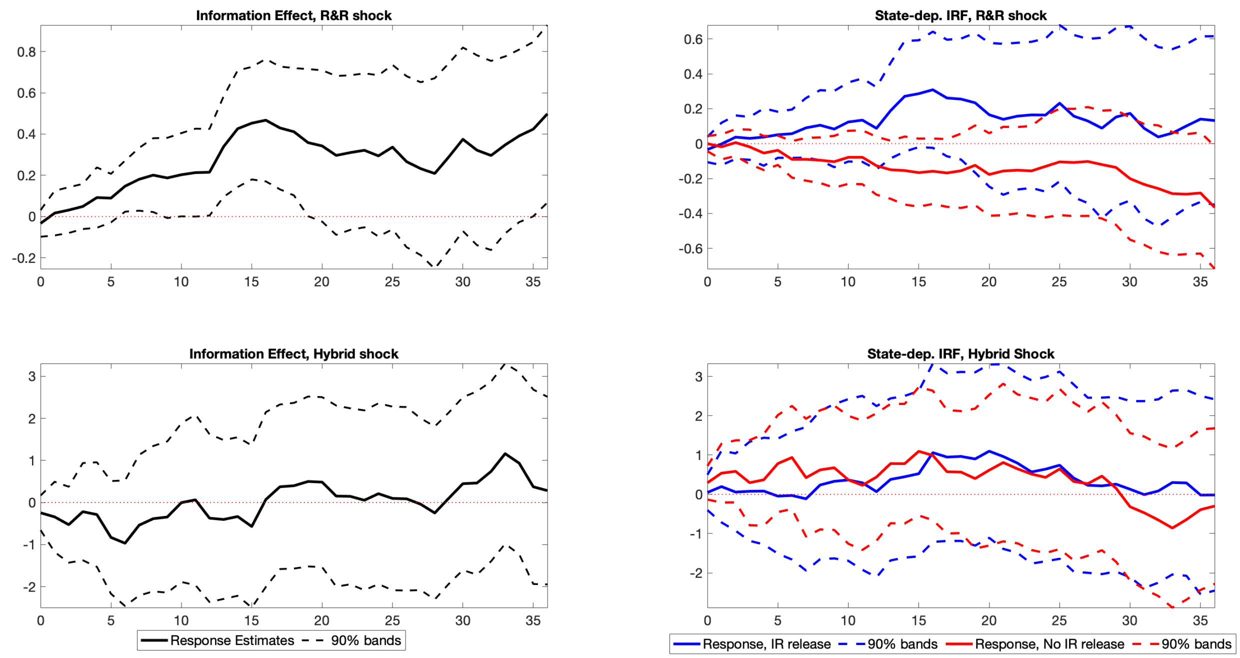
Figure C.13: Other Variables



Notes: This figure shows the effect of information on other variables: employment rate, two other price indices, RPI (Retail Price Index), CPIH (Consumer Price Index with Housing), and FTSE-100. The panels with black lines are showing the coefficient estimates of the difference between the impulse responses of month two when Inflation Report are released by the Bank of England and the other two months of that quarter with the two black lines for 90% confidence bands. Positive numbers mean that the responses from policy announcements in month 2 of each quarter exceed those from announcements in other months in each quarter. The second plot for each variable compares the level of the responses of month two surprise announcements to surprise announcements in other months. The blue solid lines are the impulse responses in the second months in each quarter with the two blue dashed lines for 90% confidence bands, and the red solid lines are the response coefficients in every other months in each quarter with the two red dashed lines for 90% confidence bands.

Unfortunately, however, stock price index, FTSE-100 does not show any clear difference between months when Inflation Reports are released and other months. Therefore, we implement similar exercise with two other monetary policy measures: Romer and Romer (2004)-type monetary policy shock series that we obtain from Cloyne and Hürtgen (2016), and hybrid shock series from Miranda-Agrippino (2016). Here, the results with Romer and Romer (2004)-type monetary policy shock series show promising result that stock price index tends to increase further when Inflation Reports are released compare to other months.

Figure C.14: Responses of FTSE-100 to Other Monetary Policy Measures



Notes: This figure shows the effect of information with on stock price index, FTSE-100. The first rows use Romer and Romer (2004)-type monetary policy shock seires that we obtain from Cloyne and Hürtgen (2016), and the second rows use hybrid policy measures that we obtain from Miranda-Agrippino (2016). The panels with black lines are showing the coefficient estimates of the difference between the impulse responses of month two when Inflation Report are released by the Bank of England and the other two months of that quarter with the two black lines for 90% confidence bands. Positive numbers mean that the responses from policy announcements in month two of each quarter exceed those from announcements in other months in each quarter. The second plot for each variable compares the level of the responses of month two surprise announcements to surprise announcements in other months. The blue solid lines are the impulse responses in the second months in each quarter with the two blue dashed lines for 90% confidence bands, and the red solid lines are the response coefficients in every other months in each quarter with the two red dashed lines for 90% confidence bands.

C.5 Sample Bank of England Press Release

The following is from the 8th of January 2009 - the middle of the financial crisis for the UK. Note that very little information is given about the wider economic situation and what the Bank of England believes will happen.

Figure C.15: Bank of England - Press Release 8th January 2009

BANK OF ENGLAND

News release

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London EC2R 8AH
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8 January 2009

Bank of England Reduces Bank Rate by 0.5 Percentage Points to 1.5%

The Bank of England's Monetary Policy Committee today voted to reduce the official Bank Rate paid on commercial bank reserves by 0.5 percentage points to 1.5%.

The world economy appears to be undergoing an unusually sharp and synchronised downturn. Measures of business and consumer confidence have fallen markedly. World trade growth this year is likely to be the weakest for some considerable time.

In the United Kingdom, business surveys suggest that the pace of contraction in activity increased during the fourth quarter of 2008 and that output is likely to continue to fall sharply during the first part of this year. Surveys of retailers and reports from the Bank's regional Agents imply that consumer spending has weakened. The outlook for business and residential investment has deteriorated. And the availability of credit to both households and businesses has tightened further, pointing to the need for further measures to increase the flow of lending to the non-financial sector. But the substantial depreciation in sterling over recent months may help to moderate the impact on UK net exports of the slowdown in global growth.

CPI inflation fell to 4.1% in November. Inflation is expected to fall further, reflecting waning contributions from retail energy and food prices and the direct impact of the temporary reduction in Value Added Tax. Measures of inflation expectations have come down. And pay growth remains subdued. But the depreciation in sterling will boost the cost of imports.

At its January meeting, the Committee noted that the recent easing in monetary and fiscal policy, the substantial fall in sterling and the prospective decline in inflation would together provide a considerable stimulus to activity as the year progressed. Nevertheless, the Committee judged that, looking through the volatility in inflation associated with the movements in Value Added Tax, there remained a significant risk of undershooting the 2% CPI inflation target in the medium term at the existing level of Bank Rate. Accordingly, the Committee concluded that a further reduction in Bank Rate of 0.5 percentage points to 1.5% was necessary to meet the target in the medium term.

The minutes of the meeting will be published at 9.30am on Wednesday 21 January.

All releases are available online at www.bankofengland.co.uk/publications/Pages/news/default.aspx

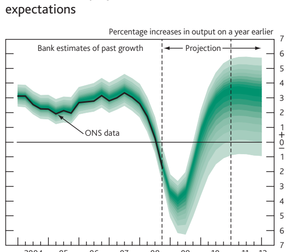
Next Figure C.16 shows the “Overview” page of the February 2009 Inflation Report.

Figure C.16: Bank of England - Inflation Report February 2009

5 Prospects for inflation

On the assumption that Bank Rate follows a path implied by market yields, the central projection is for GDP to contract sharply in the near term, and by more than assumed in the November *Report*. Further out, growth recovers, reflecting the substantial degree of stimulus from the easing in monetary and fiscal policy, the depreciation in sterling, past falls in commodity prices and actions by authorities at home and abroad to improve the availability of credit. CPI inflation falls well below the 2% target in the medium term, as the drag from the substantial margin of spare capacity more than outweighs the waning impact on import and consumer prices from the lower level of sterling. But the near-term path is uneven, reflecting sharp falls in energy prices, and the temporary reduction in VAT. The risks to growth are weighted heavily to the downside, reflecting in particular uncertainties over the pace at which the availability of credit improves and confidence returns. That also poses downside risks to inflation. But those risks are judged to be broadly matched by upside risks from the substantially lower level of sterling, leaving the overall balance of risks to inflation only modestly to the downside.

Chart 5.1 GDP projection based on market interest rate expectations



The fan chart depicts the probability of various outcomes for GDP growth. To the left of the first vertical dashed line, the distribution reflects the likelihood of revisions to the data over the past; to the right, it reflects uncertainty over the evolution of GDP growth in the future. If economic circumstances identical to today’s were to prevail on 100 occasions, the MPC’s best collective judgement is that the mature estimate of GDP growth would lie within the darkest central band on only 10 of those occasions. The fan chart is constructed so that outturns are also expected to lie within each pair of the lighter green areas on 10 occasions. Consequently, GDP growth is expected to lie somewhere within the entire fan on 90 out of 100 occasions. The bands widen as the time horizon is extended, indicating the increasing uncertainty about outcomes. See the box on page 39 of the November 2007 *Inflation Report* for a fuller description of the fan chart and what it represents. The second dashed line is drawn at the two-year point of the projection.

5.1 The projections for demand and inflation

The UK economy is undergoing a significant and sustained adjustment, as banks restructure their balance sheets, and the private sector cuts back on spending and increases saving. Monetary policy cannot — and should not — prevent necessary long-term adjustment: the challenge is to avoid excessive short-term movements in output and employment, while returning inflation to the 2% target.

Three forces shape the medium-term outlook for inflation: first, the pronounced deterioration in confidence, credit conditions and activity both at home and abroad, which threatens to pull inflation well below target. Second, the substantial stimulus from the greatly reduced levels of Bank Rate, sterling and commodity prices, expansionary fiscal policy and Government measures to support financial stability and lending, each of which will help to boost activity over time, but with varying scale and pace. And, third, the direct effects of the fall in sterling on import prices and CPI inflation. The projections presented here reflect the Committee’s judgement of the balance between those forces.

Chart 5.1 shows the outlook for GDP growth, on the assumption that Bank Rate follows a path implied by market yields — dipping down to ¾% in mid-2009 before rising gradually to 3% by the end of the forecast period (see the box on page 41). Despite the yield curve being materially lower than assumed in the November *Report*, the near-term

Available from https://www.bankofengland.co.uk/-/media/boe/files/inflation-report/2009/february-2009.pdf?la=en&hash=D8B10B7E69D515890540C18F0E095D69E2B67909