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Cell Loss and Output Process Analyses of a Finite-Buffer Discrete-Time ATM Queueing System with Correlated Arrivals*

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Abstract

This paper analyzes the performance of an ATM switching node considering cell arrival correlation. An ATM switching node is modeled as a discrete-time finite-buffer queue. Cell arrivals are assumed to follow a semi-Markovian process, where the number of cell arrivals in a slot depends on the states of the underlying (M-state) Markov chain in the current and previous slots. This paper presents analyses for various characteristics of the cell loss, as well as the distribution function of the cell output process from an ATM switching node. Obtained results include the cell loss probability, the consecutive loss probability, the distribution of loss period lengths, the joint distribution of successive cell interdeparture times, and the distributions of busy and idle periods. Through the numerical results, it is shown that both the correlation and the variation of cell arrivals significantly affect the cell loss and the output process characteristics.

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1 Introduction

ATM (Asynchronous Transfer Mode) is considered to be the most promising transfer technology for implementing B-ISDN [1], [5]. In ATM, information flow is organized into fixed-size cells which are transmitted through slotted channels. Cell switching in ATM is performed in hardware switching fabrics to match the speed of the channel.

ATM networks are expected to support diverse applications such as voice, video and data transfer. It is known that many of the traffic sources in ATM (such as a video source) exhibit a fair amount of correlation [15]. Correlations in cell arrivals are caused by, for instance, segmentation of large data frames or video signals into (small size) cells. It has been shown that neglecting correlations in cell arrivals may result in dramatic underestimation of various performance measures such as the loss probability and the transmission delay [9]. Thus, it is important to obtain performance considering the correlations in the ATM traffic.

This paper analyzes the performance of an ATM switching node considering the correlation in cell arrivals. In this paper, since ATM is a slot-based transfer technique and its cell size is constant, an ATM switching node is modeled as a discrete-time finite-buffer queueing system with a deterministic service time distribution. Cell arrivals to an ATM switching node are modeled as semi-Markovian correlated arrivals. Namely, the number of cell arrivals in a slot depends on the state of the underlying (M-state) Markov chain in the current slot and in the pervious slot. This model for the cell arrivals is fairly general and well describes the correlations found in many ATM traffic sources (such as video sources). Various characteristics of the cell loss at an ATM switching node are obtained, and the distribution function for the cell output process is derived.

Many researchers have studied discrete-time queueing systems with correlated arrivals (e.g., [7], [10], [13], [17], [22]). Most of the past work, however, assumes infinite buffers and focuses mainly on queue length and waiting time distributions, not on cell loss characteristics. Our paper investigates the cell loss of a finite buffer queueing system with correlated arrivals. There exist some papers which investigate cell loss, but most of them assume independent cell arrivals (e.g., [3], [6], [21]). Our paper investigates the cell loss assuming correlated arrivals. Important features of ATM are taken into consideration in our model.

The output process in a continuous-time finite-buffer queue with correlated arrivals is studied in [20], where the moment generating function for the sum of m successive interdeparture times is derived. In our paper, we analyze the discrete-time model, and our analytical results include the joint distribution function of m successive interdeparture times. The output process in a discrete-time queue is studied in [19] and [23], assuming infinite buffer. This paper analyzes the output process in a discrete-time queue, assuming finite buffer.

The remainder of this paper is organized as follows. In section 2, the queueing model is described. In section 3, we analyze the queueing model and present an efficient numerical method to calculate the stationary distribution of the number of cells in the system. Section 4 presents analysis for cell loss, and expressions are derived for various cell loss statistics such as the distribution of loss period lengths and the consecutive loss probability. Section 5 presents the output process analysis and derives expressions for various statistics of the output process, including the joint distribution of successive interdeparture times. In section 6, we show numerical results. Finally, concluding remarks are given in section 7.

2 Queueing Model

In this paper, an ATM switching node is modeled as a finite-buffer discrete-time single-server queueing system with correlated arrivals. Time is slotted, and the slot length is equal to a unit time. Cells arrive in batch, and a batch of cells potentially arrives immediately before a slot boundary. An arrival of a batch in the nth slot occurs immediately before the end of the nth slot. (See Fig.1.)

Cell arrivals are governed by an underlying M-state Markov chain. This Markov chain changes its state immediately after a slot boundary, and the transition matrix for this Markov chain is denoted by $\boldsymbol{P} = \{P_{ij}\}$ $(i, j = 1, \ldots, M)$. Without loss of generality, $\boldsymbol{P}$ is assumed to be irreducible and positive recurrent. Let P_n denote the state of the underlying Markov chain in the nth slot. Let $\boldsymbol{\pi} = (\pi_1, \ldots, \pi_M)$ denote the stationary state vector of this Markov chain, where $\pi_i = \lim_{n\to\infty} \Pr\{P_n = i\}$. Note that $\boldsymbol{\pi}$ satisfies $\boldsymbol{\pi} = \boldsymbol{\pi}\boldsymbol{P}$ and $\boldsymbol{\pi}\boldsymbol{e} = 1$, where $\boldsymbol{e}$ is an $M \times 1$ vector with all the components equal to one.

Let A_n denote the number of arriving cells in the nth slot (or the arrival batch size). We assume that A_{n+1} depends on both P_n and P_{n+1} (states of the underlying Markov chain in the nth and $(n+1)$st slots). We denote by $a_{ij}(k)$ the probability of having k cell arrivals in the current slot given that the underlying Markov chain was in state i in the previous slot and is in state j in the current slot:

$$a_{ij}(k) = \Pr\{A_{n+1} = k \mid P_n = i, P_{n+1} = j\}, \qquad i, j = 1, \ldots, M. \tag{1}$$

Note that we assume $a_{ij}(k)$ is time homogeneous and is independent of n.

Our queueing system has finite buffer which accommodates at most N cells including the one in service. Thus, when m $(m \geq N - k + 1)$ cells arrive to find k cells (including the one in service) in the system, only $N - k$ cells are accommodated in the system, and the remaining $m - N + k$ cells are discarded.

The service time of a cell is assumed to be constant and is equal to the unit time. The service of a cell (if there is at least one cell in the system) starts at the beginning of a slot and ends at the end of the slot (i.e., on slot boundaries). Cells depart from the system at slot boundaries. (See Fig.1.)

In the remainder of this section, we introduce some notations on the arrival process. These notations are used in the analysis in the following sections. Let $A_{ij}(k)$ denote the probability for the following event: k cells arrive in the $(n+1)$st slot, and the underlying Markov chain is in state j in the $(n+1)$th slot, given that the Markov chain was in state i in the nth slot. Namely,

$$A_{ij}(k) = \Pr\{A_{n+1} = k, P_{n+1} = j \mid P_n = i\} = a_{ij}(k)P_{ij}, \qquad i, j = 1, \ldots, M. \tag{2}$$

Let λ_i denote the mean number of cells arriving in a slot given that the underlying Markov chain is in state i in the previous slot. That is,

$$\lambda_i = \sum_{j=1}^{M} \sum_{k=1}^{\infty} kA_{ij}(k) = \sum_{j=1}^{M} \sum_{k=1}^{\infty} ka_{ij}(k)P_{ij}, \qquad i = 1, \ldots, M. \tag{3}$$

By noting that the service time for a cell is equal to a unit time, traffic intensity ρ is given by

$$\rho = \sum_{i=1}^{M} \pi_i \lambda_i \times 1. \tag{4}$$

3 Distribution of the number of cells in the system

Let X_n denote the number of cells in the system immediately after the end of the nth slot (i.e., immediately after the beginning of the $n+1$st slot). (See Fig.1.) Note that X_n includes cells arrived and accommodated into the system in the nth slot but excludes the cell receiving service in the nth slot (and therefore left the system at the end of the nth slot). The stochastic behavior of the number of cells in the system is completely described by the bivariate Markov chain $\{X_n, P_n\}$ $(n = 0, 1, \ldots)$. The transition probability for this bivariate Markov chain is given by

$$\begin{aligned}
&\Pr\{X_{n+1} = l, P_{n+1} = j \mid X_n = k, P_n = i\} \\
&= \begin{cases}
A_{ij}(l) & \text{if } k = 0 \text{ and } l \le N-1, \\
\sum_{m=N}^{\infty} A_{ij}(m) & \text{if } k = 0 \text{ and } l = N, \\
A_{ij}(l-k+1) & \text{if } k > 0 \text{ and } k-1 \le l \le N-2, \\
\sum_{m=N-k}^{\infty} A_{ij}(m) & \text{if } k > 0 \text{ and } l = N-1, \\
0 & \text{otherwise.}
\end{cases}
\end{aligned} \tag{5}$$

The above transition probability is obtained by noting that the number of cells in the system immediately after the $(n+1)$st slot, X_{n+1}, is given by the sum of the number of cells in the system immediately after the nth slot, X_n, and the number of cells arrived and accommodated in the system in the $(n+1)$st slot, minus one if there is a cell being served in the $(n+1)$st slot.

Let $\boldsymbol{A}_k$ and $\boldsymbol{B}_k$ denote an $M \times M$ matrix with its (i,j)th element given by $A_{ij}(k)$ and $\sum_{m=k}^{\infty} A_{ij}(m)$, respectively, where $k = 0, 1, \ldots$. Note that, by definition, $\boldsymbol{B}_0 = \boldsymbol{P}$, where $\boldsymbol{P}$ denotes the transition probability matrix of the underlying Markov chain (refer to section 2 for the definition of $\boldsymbol{P}$). Using $\boldsymbol{A}_k$, $\boldsymbol{B}_k$ and eq.(5), the transition probability matrix $\boldsymbol{T}$ for the bivariate Markov chain $\{X_n, P_n\}$ becomes

$$\boldsymbol{T} = \begin{bmatrix}
\boldsymbol{A}_0 & \boldsymbol{A}_1 & \boldsymbol{A}_2 & \cdots & \boldsymbol{A}_{N-2} & \boldsymbol{A}_{N-1} & \boldsymbol{B}_N \\
\boldsymbol{A}_0 & \boldsymbol{A}_1 & \boldsymbol{A}_2 & \cdots & \boldsymbol{A}_{N-2} & \boldsymbol{B}_{N-1} & \boldsymbol{0} \\
\boldsymbol{0} & \boldsymbol{A}_0 & \boldsymbol{A}_1 & \cdots & \boldsymbol{A}_{N-3} & \boldsymbol{B}_{N-2} & \boldsymbol{0} \\
\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_0 & \cdots & \boldsymbol{A}_{N-4} & \boldsymbol{B}_{N-3} & \boldsymbol{0} \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{A}_0 & \boldsymbol{B}_1 & \boldsymbol{0} \\
\boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{P} & \boldsymbol{0}
\end{bmatrix}. \tag{6}$$

Let $\boldsymbol{x} = (\boldsymbol{x}_0, \boldsymbol{x}_1, \ldots, \boldsymbol{x}_N)$ denote the stationary probability vector of the bivariate Markov chain $\{X_n, P_n\}$, where $\boldsymbol{x}_i$ is a $1 \times M$ vector whose jth element $x_{i,j}$ is given by

$$x_{i,j} = \lim_{n \to \infty} \Pr\{X_n = i, P_n = j\}. \tag{7}$$

Note that $x_{i,j}$ is the stationary joint probability that there are i cells in the system and the underlying Markov chain is in state j. Thus, $\boldsymbol{x}$ satisfies $\boldsymbol{x} = \boldsymbol{x}\boldsymbol{T}$, or equivalently,

$$\boldsymbol{x}_k = \boldsymbol{x}_0\boldsymbol{A}_k + \sum_{i=1}^{k+1} \boldsymbol{x}_i\boldsymbol{A}_{k+1-i}, \qquad 0 \le k \le N-2, \tag{8}$$

$$\boldsymbol{x}_{N-1} = \boldsymbol{x}_0\boldsymbol{A}_{N-1} + \sum_{i=1}^{N} \boldsymbol{x}_i\boldsymbol{B}_{N-i}, \tag{9}$$

$$\boldsymbol{x}_N = \boldsymbol{x}_0\boldsymbol{B}_N, \tag{10}$$

with the normalizing equation

$$\sum_{k=0}^{N} \boldsymbol{x}_k\boldsymbol{e} = 1. \tag{11}$$

In the remainder of this section, we consider a numerical method to obtain the stationary probabilities $\boldsymbol{x}_k$ $(k = 0, \ldots, N)$. Upon determining $\boldsymbol{x}_k$, the stationary queue length distribution can be derived from eq.(7). Note that the stochastic matrix given in eq.(6) takes a form of a block Hessenberg matrix. A numerical algorithm to calculate the stationary probability vector for this matrix has been studied in [11] and [12]. In the following, we modify the method presented in [12] and provide a recursive formula for computing $\boldsymbol{x}_k$.

From eq.(8) for $k = 0$, we have $\boldsymbol{x}_0 = \boldsymbol{x}_1\boldsymbol{R}_1$ where

$$\boldsymbol{R}_1 = \boldsymbol{A}_0(\boldsymbol{I} - \boldsymbol{A}_0)^{-1}. \tag{12}$$

From eq.(8) for $k = 1$ and $\boldsymbol{x}_0 = \boldsymbol{x}_1\boldsymbol{R}_1$, we have $\boldsymbol{x}_1 = \boldsymbol{x}_2\boldsymbol{R}_2$, where

$$\boldsymbol{R}_2 = \boldsymbol{A}_0(\boldsymbol{I} - \boldsymbol{A}_1 - \boldsymbol{R}_1\boldsymbol{A}_1)^{-1}. \tag{13}$$

In a similar manner, we obtain $\boldsymbol{x}_k = \boldsymbol{x}_{k+1}\boldsymbol{R}_{k+1}$ $(0 \le k \le N-2)$, where $\boldsymbol{R}_k$ $(3 \le k \le N-1)$ is given by

$$\boldsymbol{R}_k = \boldsymbol{A}_0\left(\boldsymbol{I} - \boldsymbol{A}_1 - \boldsymbol{R}_{k-1}\boldsymbol{R}_{k-2}\ldots\boldsymbol{R}_1\boldsymbol{A}_{k-1} - \sum_{i=2}^{k-1} \boldsymbol{R}_{k-1}\boldsymbol{R}_{k-2}\ldots\boldsymbol{R}_i\boldsymbol{A}_{k-i+1}\right)^{-1}. \tag{14}$$

Applying $\boldsymbol{x}_k = \boldsymbol{x}_{k+1}\boldsymbol{R}_{k+1}$ $(0 \le k \le N-2)$ recursively, we have

$$\boldsymbol{x}_k = \boldsymbol{x}_{N-1}\boldsymbol{R}_{N-1}\boldsymbol{R}_{N-2}\ldots\boldsymbol{R}_{k+1}, \qquad k = 0, \ldots, N-2. \tag{15}$$

It then follows from eqs.(9) and (15) that

$$\boldsymbol{x}_{N-1} = \boldsymbol{x}_{N-1}\boldsymbol{R}_{N-1}\ldots\boldsymbol{R}_1\boldsymbol{A}_{N-1} + \sum_{k=1}^{N-2}(\boldsymbol{x}_{N-1}\boldsymbol{R}_{N-1}\ldots\boldsymbol{R}_{k+1}\boldsymbol{B}_{N-k}) + \boldsymbol{x}_{N-1}\boldsymbol{B}_1 + \boldsymbol{x}_N\boldsymbol{P}. \tag{16}$$

Further, using eqs.(10) and (15), the last term of the right hand side of eq.(16), $\boldsymbol{x}_N\boldsymbol{P}$, becomes $\boldsymbol{x}_N\boldsymbol{P} = \boldsymbol{x}_{N-1}\boldsymbol{R}_{N-1}\ldots\boldsymbol{R}_1\boldsymbol{B}_N\boldsymbol{P}$. Substituting this into eq.(16), we obtain

$$\boldsymbol{x}_{N-1} = \boldsymbol{x}_{N-1}\boldsymbol{R}^*, \tag{17}$$

where

$$R^* = B_1 + R_{N-1} \dots R_1(A_{N-1} + B_N P) + \sum_{k=1}^{N-2} R_{N-1} \dots R_{k+1} B_{N-k}. \tag{18}$$

Note here that the $M \times M$ matrix R^* is stochastic. That is, R^* is a non-negative matrix whose row sums are all equal to one [12]. Therefore, eq.(17) has a unique solution up to a multiplicative constant, and this constant is determined by eq.(11).

At this point, we can calculate the joint stationary vector $x = (x_0, \dots, x_N)$ in the following manner:

1. Solve $x'_{N-1} = x'_{N-1} R^*$ (with a normalizing equation $x'_{N-1} e = 1$) with respect to a $1 \times M$ vector x'_{N-1}.

2. Compute $1 \times M$ vectors x'_k from $x'_k = x'_{k+1} R_{k+1}$ $(k = N-2, N-3, \dots, 1, 0)$.

3. Compute $1 \times M$ vector x'_N from $x'_N = x'_0 B_N$.

4. Compute the normalizing constant C from $C = \sum_{k=0}^{N} x'_k e$.

5. Compute the stationary joint vector x_k from $x_k = x'_k / C$ $(k = 0, 1, \dots, N)$.

Note that the stationary vector π for the state of the underlying Markov chain is given by $\sum_{k=0}^{N} x_k$. In the analyses presented in the following sections, we assume that the system is in equilibrium. In other words, we assume that the bivariate Markov chain $\{X_n, P_n\}$ has reached its steady state distribution $x = (x_0, x_1, \dots, x_N)$.

4 Loss Probability and Related Performance Measures

4.1 Cell loss probability

We define the cell loss probability P_{loss} as the fraction of the number of lost cells to the number of arriving cells. To obtain P_{loss}, we first consider the conditional cell loss probability $P_{loss}(i)$, the cell loss probability for a slot given that the underlying Markov chain was in state i in the previous slot. If we let L_n denote the number of lost cells in the nth slot, then, $P_{loss}(i)$ is defined as $P_{loss}(i) = \frac{E[L_{n+1} | P_n = i]}{E[A_{n+1} | P_n = i]}$. $P_{loss}(i)$ is obtained in the following manner.

Assume that $X_n = k$ $(k = 0, 1, \dots, N)$ given that $P_n = i$ in steady state. (In other words, assume that there are k cells in the system immediately after the end of the nth slot given that the underlying Markov chain was in state i in the nth slot.) This occurs with the probability $x_{k,i}/\pi_i$ (see eq.(7)). Further, assume that m $(m \geq N - k + 1)$ cells arrive to the system in the $(n+1)$st slot. This occurs with the probability $\sum_{j=1}^{M} A_{ij}(m)$. Then, $m - N + k$ cells among m arriving cells are lost. Therefore, $P_{loss}(i)$ becomes:

$$\begin{aligned} P_{loss}(i) &= \left[\sum_{k=0}^{N} \frac{x_{k,i}}{\pi_i} \sum_{m=N-k+1}^{\infty} (m - N + k) \sum_{j=1}^{M} A_{ij}(m) \right] / \lambda_i \\ &= 1 - \left[\sum_{k=0}^{N} x_{k,i} \left\{ N - k - \sum_{m=0}^{N-k} (N - k - m) \sum_{j=1}^{M} A_{ij}(m) \right\} \right] / (\pi_i \lambda_i), \end{aligned} \tag{19}$$

where λ_i is the mean number of cells arriving in a slot given that the underlying Markov chain was in state i in the previous slot.

From $P_{loss}(i)$, we obtain the cell loss probability $P_{loss} = \frac{E[L_{n+1}]}{E[A_{n+1}]}$. Note that $E[A_{n+1}]$ (the mean number of cell arrivals per slot) is equal to ρ (see eq.(4)) and that $E[L_{n+1}]$ is given by

$$E[L_{n+1}] = \sum_{i=1}^{M} E[L_{n+1} \mid P_n = i] \Pr\{P_n = i\} = \sum_{i=1}^{M} \pi_i \lambda_i P_{loss}(i). \tag{20}$$

Thus, the loss probability P_{loss} is given by

$$\begin{aligned} P_{loss} &= \sum_{i=1}^{M} \frac{\pi_i \lambda_i P_{loss}(i)}{\rho} = \left[\rho - \sum_{k=0}^{N}(N-k)\boldsymbol{x}_k\boldsymbol{e} + \sum_{k=0}^{N-1} \boldsymbol{x}_0 \boldsymbol{A}_k \boldsymbol{e} + \sum_{k=0}^{N-2}(N-1-k)\boldsymbol{x}_k\boldsymbol{e}\right] / \rho \\ &= \frac{\rho - (1 - \boldsymbol{x}_0\boldsymbol{e})}{\rho}. \end{aligned} \tag{21}$$

Eq.(21) can be intuitively explained as follows. From eq.(7), we have $\boldsymbol{x}_0\boldsymbol{e} = \sum_{i=1}^{M} x_{0,i}$ $= \Pr\{X_n = 0\}$. $\boldsymbol{x}_0\boldsymbol{e}$ thus represents the probability that the system is empty. Then, the utilization factor ρ' of the server is given by

$$\rho' = 1 - \boldsymbol{x}_0\boldsymbol{e}. \tag{22}$$

ρ' can also be interpreted as the mean number of accommodated cells (the mean number of arriving cells minus the mean number of lost cells) per slot, and ρ represents the mean number of cell arrivals in a slot (see eq.(4)). $\rho - \rho'$ then represents the mean number of lost cells per slot. Therefore, the loss probability P_{loss} is given by $(\rho - \rho')/\rho$.

4.2 Consecutive loss probability

Next we consider the consecutive loss probability, i.e., the conditional probability that the cell loss occurs in a slot given that the cell loss occurred in the previous slot. Let J_n denote the indicator function of the cell loss in the nth slot. Namely, $J_n = I_{\{L_n \geq 1\}}$, where L_n denotes the number of lost cells in the nth slot, and I_χ represents the indicator function of the event χ. $J_n = 1$, if the cell loss occurs in the nth slot, and $J_n = 0$, otherwise.

We will observe the system immediately after the end of the nth slot and consider a trivariate Markov chain $\{X_n, P_n, J_n\}$. (See Fig.1.) Assume that cell loss occurred in the nth slot. There are two possible cases: (1) a case where the system was empty at the beginning of the nth slot, and (2) a case where the system was not empty, and thus, there was a cell being served at the beginning of the nth slot. In both cases, cell arrivals at the end of the nth slot brought more cells than the system could accommodate, and thus, the system became full (i.e., there were N cells in the system) immediately after the arrivals. In the first case, no cell was served and departed at the end of the nth slot, and thus, there are still N cells in the system at the observation point (i.e., $X_n = N$). In the second case, since the cell receiving service at the beginning of the nth slot left the system at the end of the nth slot, the number of cells remaining in the system at the observation point becomes $N-1$ (i.e., $X_n = N-1$). As explained above, the number of cells in the system at the observation point (given that cell loss occurred in the nth slot) is either N or $N-1$. For the other cases, namely, for $0 \leq k \leq N-2$, we have

$$\Pr\{X_n = k, P_n = j, J_n = 1\} = 0, \qquad 1 \leq j \leq M. \tag{23}$$

In order to reduce the computational complexity in obtaining the distribution of the trivariate Markov chain $\{X_n, P_n, J_n\}$, we introduce the random variable $\widetilde{X}_n$ defined as

$$\widetilde{X}_n = \begin{cases} X_n & \text{if } J_n = 0, \\ N+1 & \text{if } X_n = N-1 \text{ and } J_n = 1, \\ N+2 & \text{if } X_n = N \text{ and } J_n = 1. \end{cases} \tag{24}$$

Since the trivariate Markov chain $\{X_n, P_n, J_n\}$ has a one to one mapping to the bivariate Markov chain $\{\widetilde{X}_n, P_n\}$, in the following we study $\{\widetilde{X}_n, P_n\}$.

The transition probability matrix $\boldsymbol{T}^*$ for the bivariate Markov chain $\{\widetilde{X}_n, P_n\}$ in block matrix form is given by

$$\boldsymbol{T}^* = \begin{bmatrix} \boldsymbol{A}_0 & \boldsymbol{A}_1 & \boldsymbol{A}_2 & \cdots & \boldsymbol{A}_{N-2} & \boldsymbol{A}_{N-1} & \boldsymbol{A}_N & \boldsymbol{0} & \boldsymbol{B}_{N+1} \\ \boldsymbol{A}_0 & \boldsymbol{A}_1 & \boldsymbol{A}_2 & \cdots & \boldsymbol{A}_{N-2} & \boldsymbol{A}_{N-1} & \boldsymbol{0} & \boldsymbol{B}_N & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{A}_0 & \boldsymbol{A}_1 & \cdots & \boldsymbol{A}_{N-3} & \boldsymbol{A}_{N-2} & \boldsymbol{0} & \boldsymbol{B}_{N-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_0 & \cdots & \boldsymbol{A}_{N-4} & \boldsymbol{A}_{N-3} & \boldsymbol{0} & \boldsymbol{B}_{N-2} & \boldsymbol{0} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{A}_0 & \boldsymbol{A}_1 & \boldsymbol{0} & \boldsymbol{B}_2 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{A}_0 & \boldsymbol{0} & \boldsymbol{B}_1 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{A}_0 & \boldsymbol{A}_1 & \boldsymbol{0} & \boldsymbol{B}_2 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{A}_0 & \boldsymbol{0} & \boldsymbol{B}_1 & \boldsymbol{0} \end{bmatrix}. \tag{25}$$

Let $(\boldsymbol{v}_0, \ldots, \boldsymbol{v}_N, \boldsymbol{u}_{N-1}, \boldsymbol{u}_N)$ denote the stationary vector for the Markov chain $\{\widetilde{X}_n, P_n\}$, where $\boldsymbol{v}_i$ and $\boldsymbol{u}_i$ are $1 \times M$ vectors whose jth elements $v_{i,j}$ and $u_{i,j}$ are the stationary joint probabilities of $\{\widetilde{X}_n = i, P_n = j\}$ for $0 \leq i \leq N$ and $\{\widetilde{X}_n = i+2, P_n = j\}$ for $N-1 \leq i \leq N$, respectively. That is,

$$v_{i,j} = \Pr\{\widetilde{X}_n = i, P_n = j\}, \qquad 0 \leq i \leq N, 1 \leq j \leq M, \tag{26}$$

$$u_{i,j} = \Pr\{\widetilde{X}_n = i+2, P_n = j\}, \qquad N-1 \leq i \leq N, 1 \leq j \leq M. \tag{27}$$

Note that $\{\widetilde{X}_n = i, P_n = j\}$ (which is equivalent to $\{X_n = i, P_n = j, J_n = 0\}$) is an event where there are i cells in the system, the underlying Markov chain is in state j, and cell loss did not occur. $\{\widetilde{X}_n = i+2, P_n = j\}$ (which is equivalent to $\{X_n = i, P_n = j, J_n = 1\}$) is an event where there are i cells in the system, the underlying Markov chain is in state j, and cell loss has occurred. The stationary probabilities $v_{i,j}$ and $u_{i,j}$ satisfy the following equation.

$$(\boldsymbol{v}_0, \ldots, \boldsymbol{v}_N, \boldsymbol{u}_{N-1}, \boldsymbol{u}_N) = (\boldsymbol{v}_0, \ldots, \boldsymbol{v}_N, \boldsymbol{u}_{N-1}, \boldsymbol{u}_N)\boldsymbol{T}^*. \tag{28}$$

Note that

$$\Pr\{X_n = i, P_n = j\} = \sum_{k=0}^{1} \Pr\{X_n = i, P_n = j, J_n = k\}. \tag{29}$$

It then follows from eqs.(23), (26) and (29) that

$$\boldsymbol{v}_i = \boldsymbol{x}_i, \qquad 0 \le i \le N-2. \tag{30}$$

Further, we have from eqs.(28) and (29)

$$\boldsymbol{v}_{N-1} = \boldsymbol{x}_0 \boldsymbol{A}_{N-1} + \textstyle\sum_{i=1}^{N} \boldsymbol{x}_i \boldsymbol{A}_{N-i}, \qquad \boldsymbol{v}_N = \boldsymbol{x}_0 \boldsymbol{A}_N, \tag{31}$$

$$\boldsymbol{u}_{N-1} = \textstyle\sum_{i=1}^{N} \boldsymbol{x}_i \boldsymbol{B}_{N+1-i} = \boldsymbol{x}_{N-1} - \boldsymbol{v}_{N-1}, \qquad \boldsymbol{u}_N = \boldsymbol{x}_0 \boldsymbol{B}_{N+1} = \boldsymbol{x}_N - \boldsymbol{v}_N. \tag{32}$$

The above eqs.(30), (31) and (32) represent the stationary probabilities of the trivariate Markov chain $\{X_n, P_n, J_n\}$ in terms of the $\boldsymbol{x}_n$, where $\boldsymbol{x}_n$ $(n = 0, 1, \ldots)$ have been already obtained in section 3. From these stationary probabilities, we obtain the consecutive loss probability C_{loss} in the following manner.

The consecutive loss probability C_{loss} is defined as

$$C_{loss} = \Pr\{J_{n+1} = 1 \mid J_n = 1\}. \tag{33}$$

By definition, we have

$$C_{loss} = \frac{\displaystyle\sum_{i=N-1}^{N} \sum_{j=N-1}^{N} \Pr\{X_{n+1} = j, J_{n+1} = 1 \mid X_n = i, J_n = 1\} \Pr\{X_n = i, J_n = 1\}}{\displaystyle\sum_{i=N-1}^{N} \Pr\{X_n = i, J_n = 1\}}. \tag{34}$$

It then follows from eqs.(25) and (34) that

$$C_{loss} = (\boldsymbol{u}_{N-1} \boldsymbol{B}_2 \boldsymbol{e} + \boldsymbol{u}_N \boldsymbol{B}_1 \boldsymbol{e}) / (\boldsymbol{u}_{N-1} + \boldsymbol{u}_N) \boldsymbol{e}. \tag{35}$$

4.3 Distribution of loss periods

Next we consider the distribution of loss period lengths. A loss period is defined as a time interval (measured in slots) during which cell loss occurs in every slot. Note that the event $\{J_{n-1} = 0, J_n = 1\}$ indicates a loss period starts at the nth slot and that the event $\{J_{m-1} = 1, J_m = 0\}$ indicates a loss period ends at the $(m-1)$st slot.

Let $q = \Pr\{J_{n-1} = 0, J_n = 1\}$ denote the stationary probability that a loss period starts. The probability q is obtained in the following manner:

$$\begin{aligned} q &= \sum_{i=0}^{N} \Pr\{J_n = 1 \mid X_{n-1} = i, J_{n-1} = 0\} \Pr\{X_{n-1} = i, J_{n-1} = 0\} \\ &= \sum_{i=1}^{N} \boldsymbol{v}_i \boldsymbol{B}_{N+1-i} + \boldsymbol{u}_N = \boldsymbol{u}_{N-1}(\boldsymbol{I} - \boldsymbol{B}_2)\boldsymbol{e} + \boldsymbol{u}_N(\boldsymbol{I} - \boldsymbol{B}_1)\boldsymbol{e} \\ &= (1 - C_{loss})(\boldsymbol{u}_{N-1} + \boldsymbol{u}_N)\boldsymbol{e}. \end{aligned} \tag{36}$$

The second equality follows from the transition probability given in eq.(25), the third equality follows from eqs.(31) and (32), and the last equality follows from eq.(35). The result of eq.(36)

($q = (1 - C_{loss})(\boldsymbol{u}_{N-1} + \boldsymbol{u}_N)\boldsymbol{e}$) is intuitively explained as follows. The term $(\boldsymbol{u}_{N-1} + \boldsymbol{u}_N)\boldsymbol{e}$ gives the probability that cell loss occurs in an arbitrary slot. $(1 - C_{loss})$ is the probability that there is no cell loss in an arbitrary slot given that the cell loss occurred in the previous slot. $(1 - C_{loss})(\boldsymbol{u}_{N-1} + \boldsymbol{u}_N)\boldsymbol{e}$, therefore, gives the joint probability that the cell loss occurred in the previous slot and there is no cell loss in the current slot, namely the probability that a loss period ends. Since the loss period that starts must eventually end, the probability q that a loss period starts is equal to the probability that a loss period ends $((1 - C_{loss})(\boldsymbol{u}_{N-1} + \boldsymbol{u}_N)\boldsymbol{e})$.

Note that the probability q is also represented as

$$q = \frac{1}{E[Y] + E[Z]}, \tag{37}$$

where $E[Y]$ and $E[Z]$ denote the mean length of a loss period and a non-loss period, respectively. From eqs.(36) and (37), we have

$$E[Y] + E[Z] = \frac{1}{\boldsymbol{u}_{N-1}(\boldsymbol{I} - \boldsymbol{B}_2)\boldsymbol{e} + \boldsymbol{u}_N(\boldsymbol{I} - \boldsymbol{B}_1)\boldsymbol{e}}. \tag{38}$$

Now we consider the state of the trivariate Markov chain $\{X_n, P_n, J_n\}$ in the slot that initiates a loss period. Let $\sigma_{i,j}$ be the stationary joint probability that there are i cells in the system and the underlying Markov chain is in state j in the first slot of a loss period. Namely,

$$\sigma_{i,j} = \Pr\{X_n = i, P_n = j \mid J_{n-1} = 0, J_n = 1\}. \tag{39}$$

Note that $\{J_{n-1} = 0, J_n = 1\}$ ensures that a loss period starts in the nth slot. By definition, we have

$$\begin{aligned} \sigma_{i,j} &= \sum_{i'=0}^{N}\sum_{j'=1}^{M} [\Pr\{X_n = i, P_n = j \mid X_{n-1} = i', P_{n-1} = j', J_{n-1} = 0, J_n = 1\} \\ &\quad \cdot \Pr\{X_{n-1} = i', P_{n-1} = j', J_{n-1} = 0, J_n = 1\}] \times \frac{1}{\Pr\{J_{n-1} = 0, J_n = 1\}}. \end{aligned} \tag{40}$$

By noting that $\Pr\{J_{n-1} = 0,\ J_n = 1\} = q$, and from the transition probability matrix in eq.(25) and its stationary vector (30)–(32), we obtain

$$\boldsymbol{\sigma}_{N-1} = \sum_{i=1}^{N} \boldsymbol{v}_i \boldsymbol{B}_{N+1-i}/q = (\boldsymbol{u}_{N-1}(\boldsymbol{I} - \boldsymbol{B}_2) - \boldsymbol{u}_N \boldsymbol{B}_1)/q, \tag{41}$$

$$\boldsymbol{\sigma}_N = \boldsymbol{v}_0 \boldsymbol{B}_{N+1}/q = \boldsymbol{u}_N/q. \tag{42}$$

Now we know $(\boldsymbol{\sigma}_{N-1}, \boldsymbol{\sigma}_N)$, the probability distribution of the system state in the first slot of a loss period. From this distribution, we derive the distribution of loss period lengths. Let Y be a random variable representing the length of a loss period (measured in slots). In the following, without loss of generality, we assume that the loss period starts in the 1st slot (i.e., slot 1) and that the system state in the 1st slot follows the distribution $(\boldsymbol{\sigma}_{N-1}, \boldsymbol{\sigma}_N)$. From these assumptions and by noting that with probability one, the loss period length becomes equal to or longer than one given that the loss period starts, we have

$$\begin{aligned} &\Pr\{Y \geq 1, X_1 = i, P_1 = j \mid J_0 = 0, J_1 = 1\} \\ &\quad = \Pr\{X_1 = i, P_1 = j \mid J_0 = 0, J_1 = 1\} = \sigma_{i,j}, \quad i = N-1, N,\ j = 1, \ldots, M. \end{aligned} \tag{43}$$

In obtaining the above equation, we used the following facts. If the system was empty at the beginning of the 1st slot which initiated a loss period, the number of cells in the system immediately after the end of the 1st slot is N (i.e., $X_1 = N$); on the other hand, if the system was not empty at the beginning of the 1st slot, X_1 is $N-1$, since the cell receiving service at the beginning of the 1st slot departs from the system at the end of the slot. (Refer to the discussion used to obtain eq.(23) in section 4.2.) Thus, $\Pr\{Y \geq 1, X_1 = i, P_1 = j \mid J_0 = 0, J_1 = 1\} = 0$ for $0 \leq i \leq N-2$.

For the probability that a loss period lasts longer than one slot, we have

$$\begin{aligned}\Pr\{Y \geq 2, P_2 = j \mid J_0 = 0, J_1 = 1\} &= \sum_{k=N-1}^{N} \sum_{m=1}^{M} \Pr\{Y \geq 2, P_2 = j \mid X_1 = k, P_1 = m\} \\ &\qquad \cdot \Pr\{X_1 = k, P_1 = m \mid J_0 = 0, J_1 = 1\} \\ &= \sum_{k=N-1}^{N} \sum_{m=1}^{M} \sigma_{k,m} \sum_{l=N+1-k}^{\infty} A_{m,j}(l). \end{aligned} \tag{44}$$

Note that in each slot in a loss period (except for the first slot), there is a cell departure. Thus, the number of cells in the system in the nth slot during a loss period is $N-1$ (i.e., $X_n = N-1$) for all n ($n \geq 2$). Thus, for $n \geq 2$,

$$\begin{aligned}&\Pr\{Y \geq n+1, X_{n+1} = N-1, P_{n+1} = j \mid J_0 = 0, J_1 = 1\} \\ &= \Pr\{Y \geq n+1, P_{n+1} = j \mid J_0 = 0, J_1 = 1\} \\ &= \sum_{m=1}^{M} \Pr\{Y \geq n+1, P_{n+1} = j \mid Y \geq n, P_n = m, J_0 = 0, J_1 = 1\} \\ &\qquad \cdot \Pr\{Y \geq n, P_n = m \mid J_0 = 0, J_1 = 1\}. \end{aligned} \tag{45}$$

Let $\hat{\boldsymbol{y}}_n$ denote a $1 \times M$ vector whose jth element $\hat{y}_{n,j}$ is $\Pr\{Y \geq n, P_n = j\}$, the joint probability that the length of a loss period is equal to or longer than n and the state of the underlying Markov chain in the nth slot is j. From eqs.(43), (44) and (45), we have

$$\hat{\boldsymbol{y}}_1 = \boldsymbol{\sigma}_{N-1} + \boldsymbol{\sigma}_N, \qquad \hat{\boldsymbol{y}}_2 = \boldsymbol{\sigma}_{N-1}\boldsymbol{B}_2 + \boldsymbol{\sigma}_N\boldsymbol{B}_1, \tag{46}$$

$$\hat{\boldsymbol{y}}_n = \hat{\boldsymbol{y}}_{n-1}\boldsymbol{B}_2 = (\boldsymbol{\sigma}_{N-1}\boldsymbol{B}_2 + \boldsymbol{\sigma}_N\boldsymbol{B}_1)(\boldsymbol{B}_2)^{n-2}, \qquad N \geq 3. \tag{47}$$

From eqs.(46) and (47), the distribution of loss period lengths is given by

$$\begin{aligned}\Pr\{Y = n\} &= \sum_{j=1}^{M} \Pr\{Y \geq n, P_n = j\} - \sum_{j=1}^{M} \Pr\{Y \geq n+1, P_{n+1} = j\} \\ &= \hat{\boldsymbol{y}}_n \boldsymbol{e} - \hat{\boldsymbol{y}}_{n+1} \boldsymbol{e} \\ &= \begin{cases} \boldsymbol{\sigma}_{N-1}(\boldsymbol{I} - \boldsymbol{B}_2)\boldsymbol{e} + \boldsymbol{\sigma}_N \boldsymbol{A}_0 \boldsymbol{e}, & n = 1, \\ (\boldsymbol{\sigma}_{N-1}\boldsymbol{B}_2 + \boldsymbol{\sigma}_N\boldsymbol{B}_1)(\boldsymbol{I} - \boldsymbol{B}_2)(\boldsymbol{B}_2)^{n-2}\boldsymbol{e}, & n \geq 2. \end{cases}\end{aligned} \tag{48}$$

Eq.(48) shows that the distribution of loss period lengths has the tail in the geometric form of $\boldsymbol{B}_2$.

From eq.(48), the mean loss period length is given as

$$\begin{aligned} E[Y] &= \sum_{n=1}^{\infty} n \Pr\{Y=n\} = 1 + \boldsymbol{\sigma}_N \boldsymbol{B}_1 \boldsymbol{e} + (\boldsymbol{\sigma}_{N-1} + \boldsymbol{\sigma}_N \boldsymbol{B}_1)\boldsymbol{B}_2[\boldsymbol{I} - \boldsymbol{B}_2]^{-1}\boldsymbol{e} \\ &= \frac{(\boldsymbol{u}_{N-1} + \boldsymbol{u}_N)\boldsymbol{e}}{\boldsymbol{u}_{N-1}(\boldsymbol{I} - \boldsymbol{B}_2)\boldsymbol{e} + \boldsymbol{u}_N(\boldsymbol{I} - \boldsymbol{B}_1)\boldsymbol{e}}, \end{aligned} \tag{49}$$

or equivalently,

$$E[Y] = \frac{1}{1 - C_{loss}}. \tag{50}$$

Note that Eq.(50) has the same form as that derived for the case of Poisson arrivals [2]. The mean length of a non-loss period $E[Z]$ is derived from eqs.(38) and (49).

5 Output Process and Related Performance Measures

In this section, we consider the output process of cells from the system. We obtain the distribution for the time intervals between two successive cell departures (cell interdeparture times). We also obtain the distribution of m successive interdeparture times. The cell departures from the system form a series of on and off periods where a cell departs on every slot during an on (or busy) period and no cell departs during an off (or idle) period. We also obtain the distributions of busy and idle periods.

5.1 Interdeparture time distribution

Let $\boldsymbol{d}_k$ denote a $1 \times M$ vector whose jth element $d_{k,j}$ is the stationary joint probability that there are k cells in the system and the state of the underlying Markov chain is j immediately after the departure of a cell. That is,

$$d_{k,j} = \Pr\{X_n = k, P_n = j \mid X_{n-1} \geq 1\}. \tag{51}$$

Note that $\{X_{n-1} \geq 1\}$ ensures that there is at least one cell that departs from the system at the end of the nth slot. We now rewrite eq.(51) as

$$d_{k,j} = \frac{\Pr\{X_{n-1} \geq 1, X_n = k, P_n = j\}}{\Pr\{X_{n-1} \geq 1\}} = \frac{\sum_{i=1}^{N} \Pr\{X_{n-1} = i, X_n = k, P_n = j\}}{1 - \Pr\{X_{n-1} = 0\}}. \tag{52}$$

Then, we have

$$\boldsymbol{d}_k = \sum_{i=1}^{k+1} \boldsymbol{x}_i \boldsymbol{A}_{k+1-i}/(1 - \boldsymbol{x}_0\boldsymbol{e}) = \frac{\boldsymbol{x}_k - \boldsymbol{x}_0 \boldsymbol{A}_k}{1 - \boldsymbol{x}_0 \boldsymbol{e}}, \qquad 0 \leq k \leq N-2, \tag{53}$$

$$\boldsymbol{d}_{N-1} = \sum_{i=1}^{N} \boldsymbol{x}_i \boldsymbol{B}_{N-i}/(1 - \boldsymbol{x}_0\boldsymbol{e}) = \frac{\boldsymbol{x}_{N-1} - \boldsymbol{x}_0 \boldsymbol{A}_{N-1}}{1 - \boldsymbol{x}_0 \boldsymbol{e}}. \tag{54}$$

We now consider the distribution of cell interdeparture times. Without loss of generality, we assume that a cell departure occurs at the end of the 0th slot and that the number of cells in the system and the state of the underlying Markov chain immediately after the departure follow the joint distribution $\boldsymbol{d}_k$. Let V_1 denote the length of the time interval between the first departure (at the end of the 0th slot) and the departure following this first departure. Let $\boldsymbol{\theta}_k^{(1)}(n)$ denote a $1 \times M$ vector whose jth element $\theta_{k,j}^{(1)}(n)$ represents the joint probability that the interdeparture time V_1 is of length n, the number of cells left behind by the second departure is k, and the state of the underlying Markov chain is j immediately after the second departure. That is

$$\theta_{k,j}^{(1)}(n) = \Pr\{V_1 = n, X_n = k, P_n = j\}. \tag{55}$$

Note that the upper subscript (1) indicates that it is for a single interdeparture time (as opposed to a series of m successive interdeparture times, in which case the upper subscript is (m) as seen in the next subsection).

We first consider the probability vector $\boldsymbol{\theta}_k^{(1)}(1)$. Suppose $X_0 = i$. If $i \geq 1$ (if there is at least one cell in the system immediately after the end of the 0th slot), the interdeparture time becomes 1, and X_1 (the number of cells in the system immediately after the 1st slot) is given as the sum of $i-1$ cells (i.e., cells left by the departure at the end of the 1st slot) and the cells arriving in the 1st slot and accommodated into the system. Thus, we have

$$\boldsymbol{\theta}_k^{(1)}(1) = \sum_{i=1}^{k+1} \boldsymbol{d}_i \boldsymbol{A}_{k+1-i} \quad (0 \leq k \leq N-1) \qquad \text{and} \qquad \boldsymbol{\theta}_{N-1}^{(1)}(1) = \sum_{i=1}^{N-1} \boldsymbol{d}_i \boldsymbol{B}_{N-i}. \tag{56}$$

Next we consider $\boldsymbol{\theta}_k^{(1)}(n)$ $(n \geq 2)$. Only when the first departure (i.e., the departure at the end of the 0th slot) leaves no cell behind, the interdeparture time becomes longer than one. In this case, if the first arrival occurs in the $(n-1)$st slot since the first departure, the interdeparture time becomes n. Thus, we have, for $n \geq 2$,

$$\boldsymbol{\theta}_k^{(1)}(n) = \boldsymbol{d}_0 \boldsymbol{A}_0^{n-2} \sum_{i=1}^{k+1} \boldsymbol{A}_i \boldsymbol{A}_{k+1-i}, \qquad 0 \leq k \leq N-2, \tag{57}$$

$$\boldsymbol{\theta}_{N-1}^{(1)}(n) = \boldsymbol{d}_0 \boldsymbol{A}_0^{n-2} \left(\sum_{i=1}^{N-1} \boldsymbol{A}_i \boldsymbol{B}_{N-i} + \boldsymbol{B}_N \boldsymbol{P} \right). \tag{58}$$

Let $\theta^{(1)}(n)$ denote the probability that the interdeparture time V_1 is of length n. From eqs.(56)–(58) (by summing over all the possible cases of X_n and P_n), we obtain the distribution of interdeparture times $(\theta^{(1)}(n))$ as follows:

$$\theta^{(1)}(1) = \sum_{k=0}^{N-1} \boldsymbol{\theta}_k^{(1)}(1) \boldsymbol{e} = 1 - \boldsymbol{d}_0 \boldsymbol{e}, \tag{59}$$

$$\theta^{(1)}(n) = \sum_{k=0}^{N-1} \boldsymbol{\theta}_k^{(1)}(n) e = \boldsymbol{d}_0 (\boldsymbol{I} - \boldsymbol{A}_0) \boldsymbol{A}_0^{n-2} \boldsymbol{e}, \qquad n \geq 2. \tag{60}$$

From the above equations, we observe that the interdeparture time distribution has a geometric decay of $\boldsymbol{A}_0^n$.

Let $\Theta^{(1)}(z)$ denote the PGF of the interdeparture time distribution. We have from eqs.(59) and (60)

$$\Theta^{(1)}(z) = \sum_{n=1}^{\infty} \theta^{(1)}(n) z^n = z \left\{ 1 - \boldsymbol{d}_0 \boldsymbol{e} + z \boldsymbol{d}_0 (\boldsymbol{I} - \boldsymbol{A}_0)(\boldsymbol{I} - z\boldsymbol{A}_0)^{-1} \boldsymbol{e} \right\}. \tag{61}$$

Let $V^{(m)}$ denote the mth factorial moment of the interdeparture time distribution, that is,

$$V^{(m)} = E[V(V-1)(V-2)\dots(V-m+1)]. \tag{62}$$

Taking the first derivative of $\Theta^{(1)}(z)$, we obtain the mean interdeparture time $V^{(1)}$ as

$$V^{(1)} = 1 + \boldsymbol{d}_0(\boldsymbol{I} - \boldsymbol{A}_0)^{-1}\boldsymbol{e} = 1/\rho'. \tag{63}$$

Here, ρ' is given in eq.(22). Eq.(63) is intuitively clear since exactly one cell is served in an interdeparture time, and thus the utilization factor ρ' becomes $1/V^{(1)}$. Further, taking the mth derivative of $\Theta^{(1)}(z)$, we obtain the mth factorial moment for the interdeparture time distribution as

$$V^{(m)} = \frac{m!\boldsymbol{x}_0(\boldsymbol{I} - \boldsymbol{A}_0)^{-1}\left\{(\boldsymbol{I} - \boldsymbol{A}_0)^{-1}\boldsymbol{A}_0\right\}^{m-2}\boldsymbol{e}}{1 - \boldsymbol{x}_0\boldsymbol{e}}, \qquad m \geq 2. \tag{64}$$

5.2 Joint distribution of successive interdeparture times

Next we consider the joint distribution of m successive interdeparture times. Note that successive interdeparture times are not independent. As in section 5.1, we assume that a cell departs from the system at the end of the 0th slot. Suppose that the number of cells in the system and the state of the underlying Markov chain immediately after this first departure follow the joint distribution $\boldsymbol{d}_k$. Let V_m denote the mth interdeparture time (i.e., the time interval between the mth departure and the $(m+1)$st departure).

Let $\boldsymbol{\theta}_k^{(2)}(n_1, n_2)$ denote a $1 \times M$ vector whose jth element $\theta_{k,j}^{(2)}(n_1, n_2)$ is the joint probability for the two successive interdeparture times, the number of cells in the system and the state of the underlying Markov chain at the end of the second interdeparture time. That is,

$$\theta_{k,j}^{(2)}(n_1, n_2) = \Pr\{V_1 = n_1, V_2 = n_2, X_{n_1+n_2} = k, P_{n_1+n_2} = j\}. \tag{65}$$

Following the same discussion used to derive eqs.(56)–(58), we have the following set of the equations, for $n_1 \geq 1$,

$$\boldsymbol{\theta}_k^{(2)}(n_1, 1) = \sum_{i=1}^{k+1} \boldsymbol{\theta}_i^{(1)}(n_1)\boldsymbol{A}_{k+1-i}, \qquad 0 \leq k \leq N-2, \tag{66}$$

$$\boldsymbol{\theta}_{N-1}^{(2)}(n_1, 1) = \sum_{i=1}^{N-1} \boldsymbol{\theta}_i^{(1)}(n_1)\boldsymbol{B}_{N-i}, \tag{67}$$

and, for $n_1 \geq 1$ and $n_2 \geq 2$,

$$\boldsymbol{\theta}_k^{(2)}(n_1, n_2) = \boldsymbol{\theta}_0^{(1)}(n_1)\boldsymbol{A}_0^{n_2-2}\sum_{i=1}^{k+1} \boldsymbol{A}_i\boldsymbol{A}_{k+1-i}, \qquad 0 \leq k \leq N-2, \tag{68}$$

$$\boldsymbol{\theta}_{N-1}^{(2)}(n_1, n_2) = \boldsymbol{\theta}_0^{(1)}(n_1)\boldsymbol{A}_0^{n_2-2}\left(\sum_{i=1}^{N-1} \boldsymbol{A}_i\boldsymbol{B}_{N-i} + \boldsymbol{B}_N\boldsymbol{P}\right). \tag{69}$$

The above results easily extends to a general case of m successive interdeparture times. Let $\boldsymbol{\theta}_k^{(m)}(n_1,\ldots,n_m)$ denote a $1 \times M$ vector whose jth element $\theta_{k,j}^{(m)}(n_1,\ldots,n_m)$ is the following joint probability:

$$\theta_{k,j}^{(m)}(n_1,\ldots,n_m) = \Pr\{V_1 = n_1,\ldots,V_m = n_m, X_{n^*} = k, P_{n^*} = j\}, \quad (70)$$

where $n^* = \sum_{i=1}^{m} n_i$ denotes the total length of the m successive interdeparture times. Using the same argument to obtain eqs.(66)–(69), we have for any $m \geq 2$,

$$\boldsymbol{\theta}_k^{(m)}(n_1,\ldots,n_{m-1},1) = \sum_{i=1}^{k+1} \boldsymbol{\theta}_i^{(m-1)}(n_1,\ldots,n_{m-1})\boldsymbol{A}_{k+1-i}, \quad 0 \leq k \leq N-2, \quad (71)$$

$$\boldsymbol{\theta}_{N-1}^{(m)}(n_1,\ldots,n_{m-1},1) = \sum_{i=1}^{N-1} \boldsymbol{\theta}_i^{(m-1)}(n_1,\ldots,n_{m-1})\boldsymbol{B}_{N-i}, \quad (72)$$

and for $n_m \geq 2$,

$$\boldsymbol{\theta}_k^{(m)}(n_1,\ldots,n_m) = \boldsymbol{\theta}_0^{(m-1)}(n_1,\ldots,n_{m-1})\boldsymbol{A}_0^{n_m-2}\sum_{i=1}^{k+1} \boldsymbol{A}_i\boldsymbol{A}_{k+1-i}, \quad 0 \leq k \leq N-2, \quad (73)$$

$$\boldsymbol{\theta}_{N-1}^{(m)}(n_1,\ldots,n_m) = \boldsymbol{\theta}_0^{(m)-1}(n_1,\ldots,n_{m-1})\boldsymbol{A}_0^{n_m-2}\left(\sum_{i=1}^{N-1} \boldsymbol{A}_i\boldsymbol{B}_{N-i} + \boldsymbol{B}_N\boldsymbol{P}\right). \quad (74)$$

Note that in obtaining $\theta_{k,j}^{(m)}(n_1,...,n_m)$, we assume that the number of cells in the system and the state of the underlying Markov chain immediately after the end of the mth interdeparture time are k and j, respectively. Thus, summing eqs.(71) and (72) (or eqs.(73) and (74)) over all the possible values of k and j, we have the following for the $\theta^{(m)}(n_1,\ldots,n_m)$ (the joint distribution of m successive interdeparture times):

$$\theta^{(m)}(n_1,\ldots,n_m) = \sum_{k=0}^{N-1} \boldsymbol{\theta}_k^{(m)}(n_1,\ldots,n_m)\boldsymbol{e}. \quad (75)$$

5.3 Distributions of the idle periods and the busy periods

We first consider the distribution of idle period lengths. Note that a cell departs in every slot during a busy period and that no cell departs during an idle period. Thus, when an interdeparture time (say, V_1) becomes longer than one, it indicates that a busy period has ended, and that an idle period has started. If V_1 is equal to $n+1$ ($n \geq 1$), the system is empty for the first n slots (during which there are no cell departures), and there is an arrival of at least one cell just before the end of the nth slot, creating a departure at the end of the $(n+1)$st slot. In this case, the idle period becomes of length n. From this, we obtain the distribution of the length Q of an idle period as follows:

$$\Pr\{Q = n\} = \Pr\{V_1 = n+1 \mid V_1 \geq 2\} = \frac{\boldsymbol{d}_0(\boldsymbol{I}-\boldsymbol{A}_0)\boldsymbol{A}_0^{n-1}\boldsymbol{e}}{\boldsymbol{d}_0\boldsymbol{e}}, \quad n \geq 1. \quad (76)$$

From the above equation, the mean idle period is given as

$$E[Q] = \frac{\boldsymbol{x}_0\boldsymbol{e}}{\boldsymbol{x}_0(\boldsymbol{I}-\boldsymbol{A}_0)\boldsymbol{e}}. \quad (77)$$

Next we consider the distribution of busy period lengths. We first obtain the mean busy period $E[S]$ as follows. Recall that the server is idle with probability $\boldsymbol{x}_0\boldsymbol{e}$. Since the probability of the server being idle is equal to the fraction of the time that the buffer is empty, we have $\boldsymbol{x}_0\boldsymbol{e} = E[Q]/(E[Q]+E[S])$. It then follows that

$$E[S] = \frac{1-\boldsymbol{x}_0\boldsymbol{e}}{\boldsymbol{x}_0(\boldsymbol{I}-\boldsymbol{A}_0)\boldsymbol{e}}. \tag{78}$$

We now proceed with the derivation of the distribution of busy period lengths. As we will see, the distribution of busy periods is closely related to the joint probability $\theta^{(m)}(1,1,\ldots,1)$ that m successive interdeparture times are all equal to 1. Recall that, in the analysis of the interdeparture time in section 5.2, we first selected an arbitrary departing cell, and then derived the joint distribution of m successive interdeparture times. Thus, the probability $\theta^m(1,...1)$ is the probability of a cell departing in m successive slots following the departure of an arbitrarily chosen cell, not necessarily the departure of a cell initiating a busy period. In other words, $\theta^m(1,...1)$ is the probability that the residual life of a busy period measured from the departure of an arbitrary cell is not less than m. Let $\hat{S}$ denote the residual busy period. From the above discussion, we have

$$\Pr\{\hat{S}\geq 0\} = 1, \qquad \Pr\{\hat{S}\geq m\} = \theta^{(m)}(1,\ldots,1), \qquad m\geq 1. \tag{79}$$

From eq.(79), we have

$$\Pr\{\hat{S}=0\} = 1-\theta^{(1)}(1), \tag{80}$$

$$\Pr\{\hat{S}=m\} = \theta^{(m)}(1,\ldots,1)-\theta^{(m+1)}(1,\ldots,1), \qquad m\geq 1. \tag{81}$$

On the other hand, the distributions of busy periods and residual busy periods are related by ([4])

$$\Pr\{\hat{S}=m\} = \Pr\{S\geq m+1\}/E[S]. \tag{82}$$

Thus, we have

$$\Pr\{S=m\} = \Pr\{S\geq m\}-\Pr\{S\geq m-1\} = E[S](\Pr\{\hat{S}=m-1\}-\Pr\{\hat{S}=m\}). \tag{83}$$

Therefore, the distribution of busy periods is found to be

$$\Pr\{S=1\} = E[S]\{1-2\theta^{(1)}(1)+\theta^{(2)}(1,1)\}, \tag{84}$$

$$\Pr\{S=m\} = E[S]\{\theta^{(m-1)}(1,\ldots,1)-2\theta^{(m)}(1,\ldots,1)+\theta^{(m+1)}(1,\ldots,1)\}, \quad m\geq 2. \tag{85}$$

5.4 Joint distribution of idle and busy periods

Next we consider the joint distribution of idle and busy periods. Without loss of generality, we assume that an idle period starts in the 1st slot. Let Q denote the length of this idle period, and let S denote the length of the busy period following this idle period. We obtain the joint probability $\Pr\{Q=n, S=m\}$ in the following.

Note that the event $\{V_1 \geq 2\}$ indicates that an idle period starts, and that the event $\{V_1 = n+1\}$ indicates $\{Q = n, S \geq 1\}$, as we saw in the above subsection. Therefore, we have, for $n \geq 1$,

$$\begin{aligned}\Pr\{Q = n, S \geq 1\} &= \Pr\{V_1 = n+1 \mid V_1 \geq 2\} = \Pr\{V_1 \geq n+1, V_1 \geq 2\} / \Pr\{V_1 \geq 2\} \\ &= E[S]\theta^{(1)}(n+1). \qquad (86)\end{aligned}$$

In the above equation we used $\Pr\{V_1 \geq 2\} = E[S]^{-1}$, which is derived from eqs.(53), (59) and (78).

Note that the event $\{V_1 = n+1, V_2 = 1\}$ is equivalent to the event $\{Q = n, S \geq 2\}$. In general, the event $\{V_1 = n+1, V_2 = 1, \ldots, V_m = 1\}$ is equivalent to the event $\{Q = n, S \geq m\}$ $(m \geq 2)$. Thus, we have

$$\begin{aligned}\Pr\{Q = n, S \geq m\} &= \Pr\{V_1 = n+1, V_2 = 1, \ldots, V_m = 1 \mid V_1 \geq 2\} \\ &= E[S]\theta^{(m)}(n+1, 1, \ldots, 1), \qquad m \geq 2. \qquad (87)\end{aligned}$$

From eqs.(86) and (87), we obtain the joint distribution of a pair of idle and busy periods as follows:

$$\begin{aligned}\Pr\{Q = n, S = 1\} &= E[S]\{\theta^{(1)}(n+1) - \theta^{(2)}(n+1, 1)\}, \qquad n \geq 1, \qquad (88) \\ \Pr\{Q = n, S = m\} &= E[S]\{\theta^{(m)}(n+1, 1, \ldots, 1) - \theta^{(m+1)}(n+1, 1, \ldots, 1)\}, \\ &\qquad n \geq 1, m \geq 2. \qquad (89)\end{aligned}$$

Note that the above argument extends easily to deriving the joint distribution of an arbitrarily long sequence of successive idle and busy periods.

6 Numerical Results

In this section, we investigate various cell loss statistics and characteristics of the output process through numerical examples. Throughout this section, we assume that the cell arrival process is modulated by a two-state Markov chain with states 1 and 2, where the state transition probabilities P_{ij} are given by $P_{11} = P_{22} = \alpha$ and $P_{12} = P_{21} = 1 - \alpha$ $(0 \leq \alpha < 1)$. The conditional probabilities $a_{1,j}(k)$ and $a_{2,j}(k)$ $(j = 1, 2)$ for the sizes of the cell arrival batches are given by

$$a_{1,j}(k) = e^{-(1+c)\rho}\{(1+c)\rho\}^k / k!, \qquad a_{2,j}(k) = e^{-(1-c)\rho}\{(1-c)\rho\}^k / k!. \qquad (90)$$

In other words, if the Markov chain was in state 1 (state 2) in the previous slot, the number of cells arriving in the current slot is Poisson distributed with the mean $(1+c)\rho$ $((1-c)\rho)$. Note that ρ denotes the overall traffic intensity, and c $(0 \leq c \leq 1)$ is a parameter.

Through numerical examples, we investigate the impact of the variation and the correlation in cell arrivals. For our cell arrival model, the squared coefficient of variation C_V^2 of the number of cells arriving in a slot is given as

$$C_V^2 = \rho^{-1} + c^2. \qquad (91)$$

For a fixed value of ρ, the squared coefficient of variation C_V^2 increases as the parameter c does. The correlation coefficient $C_C(n)$ of the number of arrivals at lag n for our cell arrival process becomes

$$C_C(n) = \frac{c^2\rho}{1+c^2\rho} \times (P_{11} + P_{22} - 1)^n = \frac{c^2\rho}{1+c^2\rho} \times (2\alpha - 1)^n. \quad (92)$$

Note that, by keeping ρ and c constant (which means keeping C_V^2 constant), the correlation coefficient $C_C(n)$ depends only on the term $2\alpha - 1$. Thus the term $2\alpha - 1$ is referred to as a correlation index in this paper. Note that by varying the correlation index from 0 to 1 (namely, by varying α from 0.5 to 1), we achieve varying degrees of non-negative correlations of arrivals.

In the following figures, traffic intensity ρ is kept constant as 0.75. Three levels of variations are considered: $c = 0.2$ (low variation), $c = 0.5$ (moderate variation), and $c = 0.8$ (high variation). Further, the buffer size is assumed to be 80, unless otherwise specified.

6.1 Cell Loss Statistics

Figs.2 through 5 show various cell loss statistics as a function of the correlation index $(2\alpha - 1)$. Fig.2 shows the cell loss probability P_{loss}. It is observed that both the variation and the correlation significantly affect the cell loss probability. Most of the past research focused on identifying the impact of the arrival correlations on the cell loss, and not much attention has been paid on the variation in the arrivals. This figure clearly shows that the variation of arrivals, as well as the correlation, affects the cell loss probability significantly.

Fig.3 shows the consecutive loss probability C_{loss} as a function of the correlation index. The consecutive loss probability curve for the low variation case is relatively flat and is not affected by the arrival correlation very much. However, as the arrival variation increases, the correlation affects the consecutive loss probability more significantly.

Fig.4 shows the mean length of the loss periods ($E[Y]$) as a function of the correlation index. A similar observation to that made in Fig.3 holds for this figure. When the arrival variation is low, the correlation does not affect $E[Y]$ much. When the variation becomes higher, the correlation affects $E[Y]$ more significantly. This is intuitively clear from eq.(50), which shows a simple relation between the mean loss period length and the consecutive loss probability.

Fig.5 shows the mean length of the non-loss periods ($E[Z]$) as a function of the correlation index for three different levels of the arrival variation. Contrary to the mean loss period (shown in Fig.4), the mean non-loss period strongly depends on the arrival correlation even when the arrival variation is low. From Figs.4 and 5, it is concluded that an increase in the correlation of arrivals creates more loss periods whose average lengths are approximately the same.

Fig.6 shows both the cell loss probability and the consecutive loss probability as a function of the buffer size. A parameter c for the arrival variation is assumed to be 0.5 in this figure (i.e., the moderate variation case). We observe that the cell loss probability reduces exponentially as the buffer size increases. On the other hand, the consecutive loss probability remains almost invariant to the buffer size. This suggests that increasing the buffer size creates longer non-loss periods, but it does not significantly affect the length of the loss periods.

6.2 Output Process Characteristics

Figs.7 through 12 show various statistics of the output process. Figs.7 and 8 show the mean idle and busy period lengths for the three different levels of the arrival variation (i.e., $c = 0.2$, 0.5, and 0.8). It is observed that except for the high-variation case, the mean idle and busy period lengths are not very sensitive to the correlation of arrivals. When the arrival variation is high, both the mean idle and busy period lengths become longer as the arrival correlation increases.

Fig.9 shows the coefficient of variation of the cell interdeparture times as a function of the correlation index $(2\alpha - 1)$ for different values of c. A similar observation to that made in Figs.7 and 8 holds for this figure. When the variation of arrivals is low ($c = 0.2$), the arrival correlation does not significantly affect the variation of the interdeparture time. On the other hand, when the variation is high, the correlation of arrivals has a significant influence on the variation of the interdeparture times.

Fig.10 shows the correlation coefficient of two successive cell interdeparture times for different levels of the arrival variation. Contrary to the coefficient of variation (in Fig.9), the correlation of the cell interdeparture times is insensitive to both the correlation and the variation of arrivals. Further, it is observed that the interdeparture time correlation becomes weaker as the arrival variation increases, although not significantly. Note that similar observations were also made for a continuous-time queue [20].

Fig.11 shows the coefficient of variation of the idle period lengths. We observe that the idle period variation is not significantly affected by either the correlation or the variation of arrivals.

Finally, Fig.12 shows the coefficient of variation of the busy period lengths. We observe that the variation in the busy period lengths increases as the correlation of arrivals increases.

From Figs.7 through 12, it is observed that the correlation of arrivals does not significantly affect the characteristics of the output process when the variation of arrivals is low. The correlation of arrivals becomes important, however, when the variation of arrivals is moderate or high. We thus conclude that both the variation and the correlation of arrivals play key roles in determining the characteristics of the output processes.

7 Concluding Remarks

This paper considered the discrete-time finite-buffer queue with correlated arrivals. We analytically obtained various cell loss statistics and output process statistics, and showed through numerical examples how these statistics are affected by the variation and correlation in the arrival process.

Our analysis presented in this paper applies, not only to a discrete-time queueing system, but also to a synchronous service queueing systems with continuous-time arrivals [8]. In such a queueing system, arrivals occur at any point in time, and service starts only at a slot boundary. We can apply the analysis in this paper to such a system by obtaining the number of arrivals in a unit time. For example, if cells arrive according to an M-state (continuous time) *MMPP* with an infinitesimal generator $\boldsymbol{R}$ for the underlying Markov process and a diagonal matrix $\boldsymbol{\Lambda}$

for arrival densities [18], the probability generating function for the number of cells arriving in a unit time (namely, $A_{ij}(z) = \sum_{k=0}^{\infty} A_{ij}(k)z^k$) is given by

$$A_{ij}(z) = [exp(\mathbf{R} - \mathbf{\Lambda} + z\mathbf{\Lambda})]_{ij}. \tag{93}$$

Once $A_{ij}(z)$ is obtained, the transition submatrices $\boldsymbol{A}_k$ are completely determined. The same discussion can be applied to more general (correlated) arrival processes such as the batch *MAP* [14]. The rest of the analysis presented in this paper, then, holds for the synchronous service queueing system with continuous-time arrivals.

Obtaining the waiting time distribution, although not presented in this paper, is somewhat straightforward. If we assume the FCFS service discipline, it is shown that the following relation holds between the waiting time distribution and the queue length distribution (see lemma 3.1 of [16]):

$$\Pr\{W = k\} = \boldsymbol{x}_{k+1}\boldsymbol{e}/\rho', \qquad k = 0, \ldots, N-1. \tag{94}$$

Thus, using the results in section 3, we obtain the waiting time distribution.

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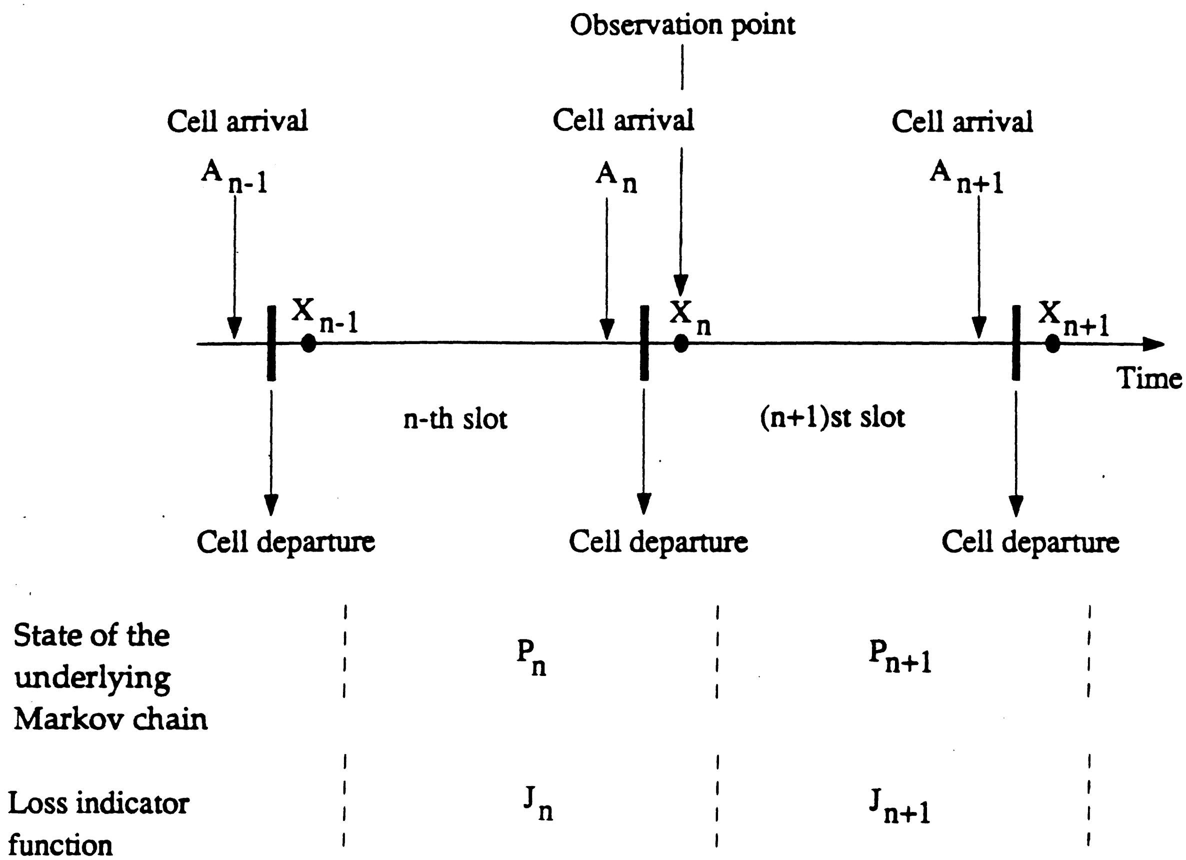

Fig.1 System Time Diagram

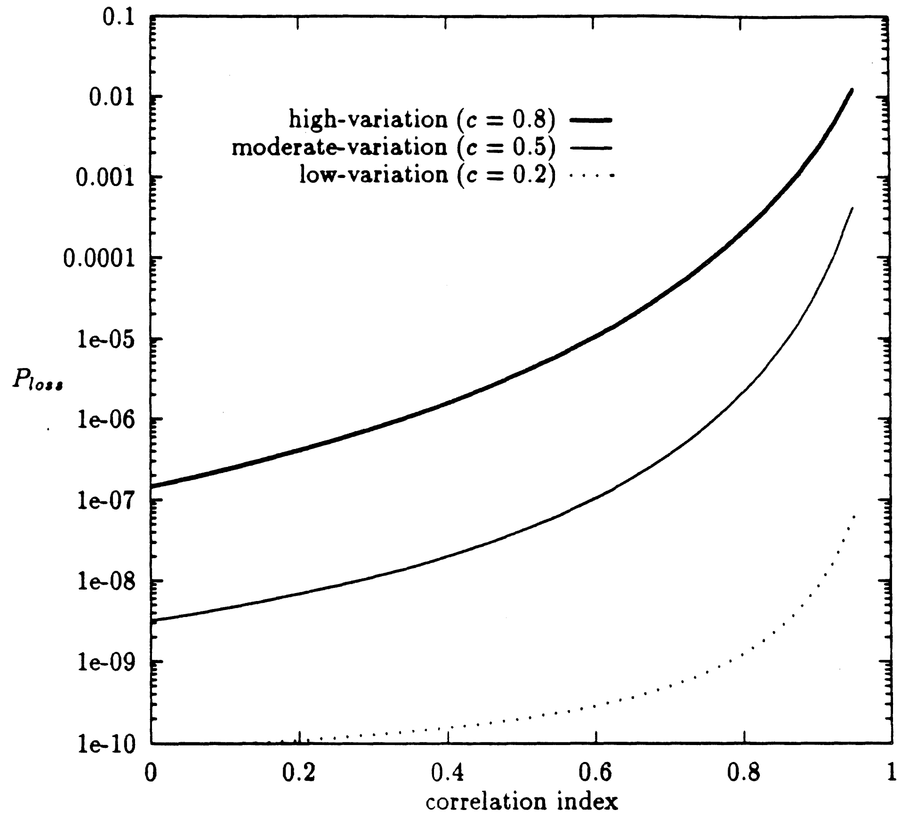

Fig.2 Cell Loss Probabilities ($N = 80$)

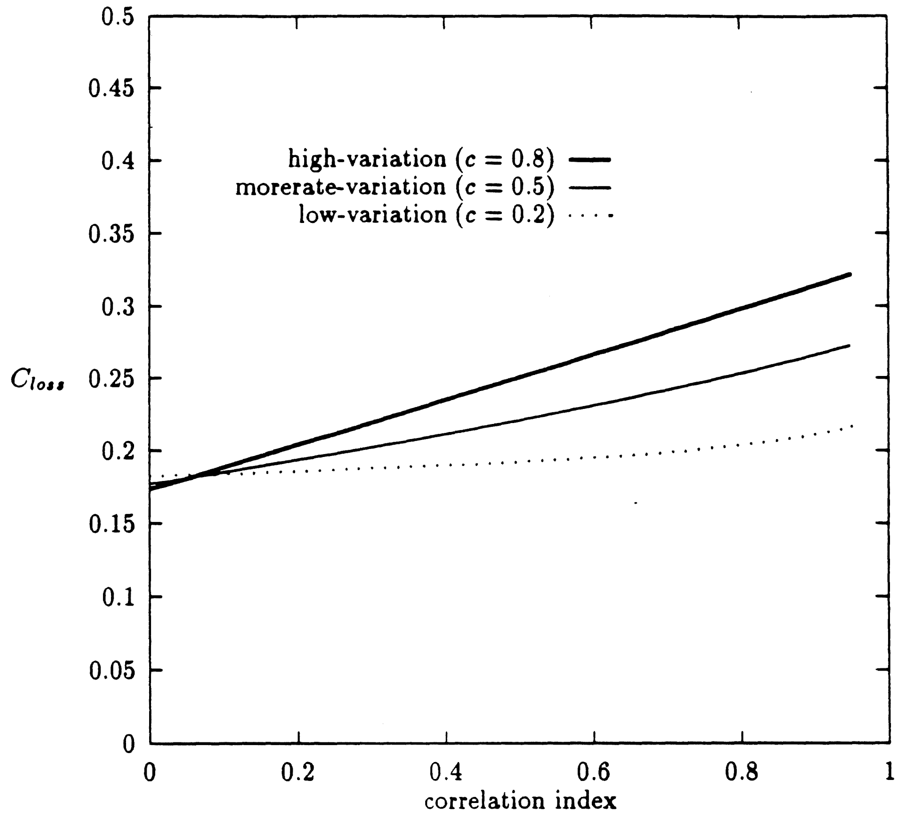

Fig.3 Consecutive Loss Probabilities ($N = 80$)

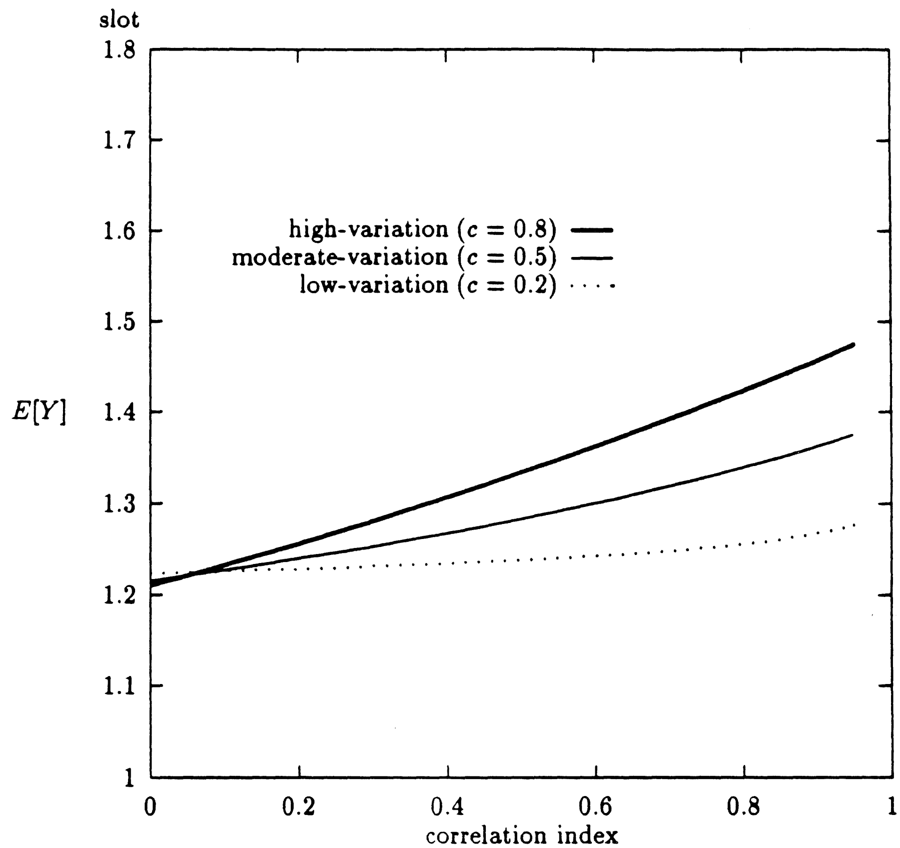

Fig.4 Mean Loss Period Lengths ($N = 80$)

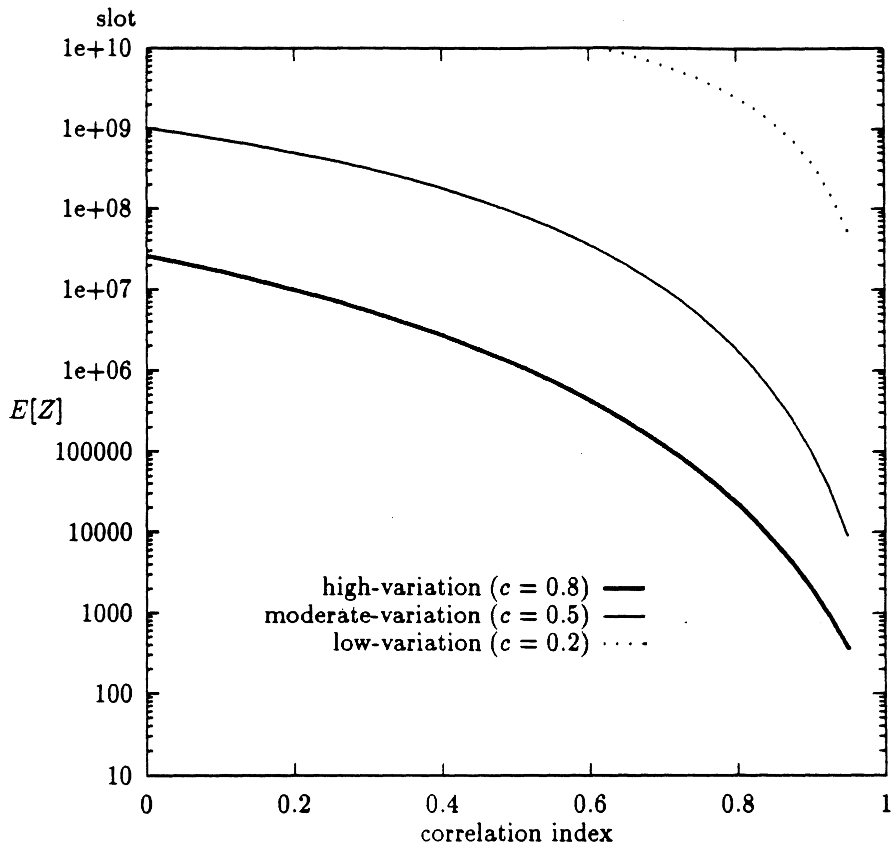

Fig.5 Mean Non-loss Period Lengths ($N = 80$)

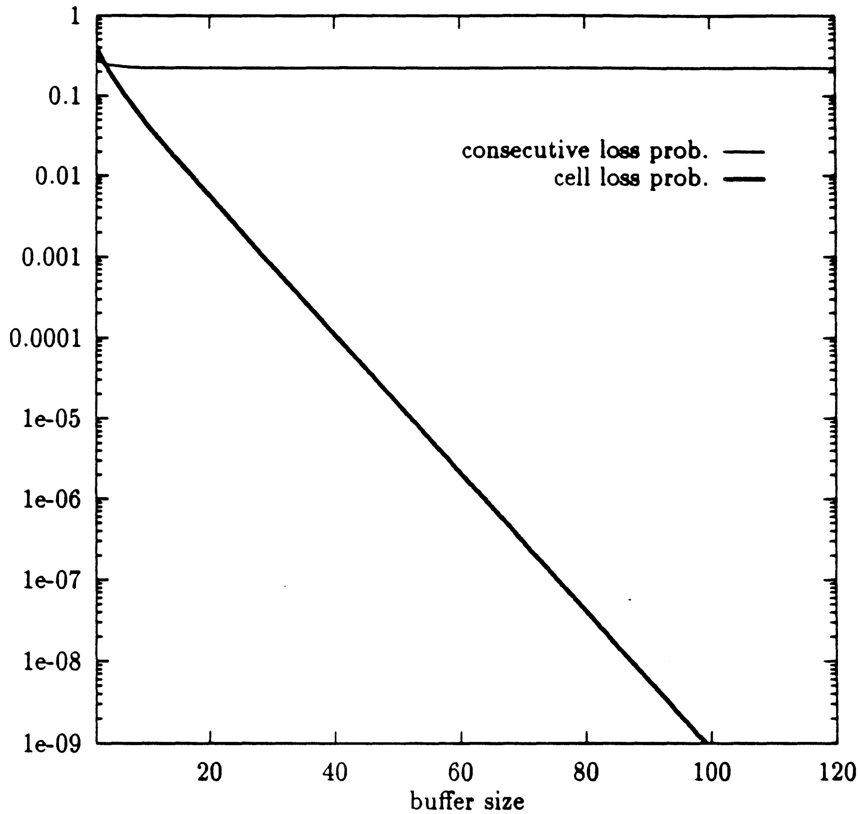

Fig.6 Loss Probabilities (moderate-variation)

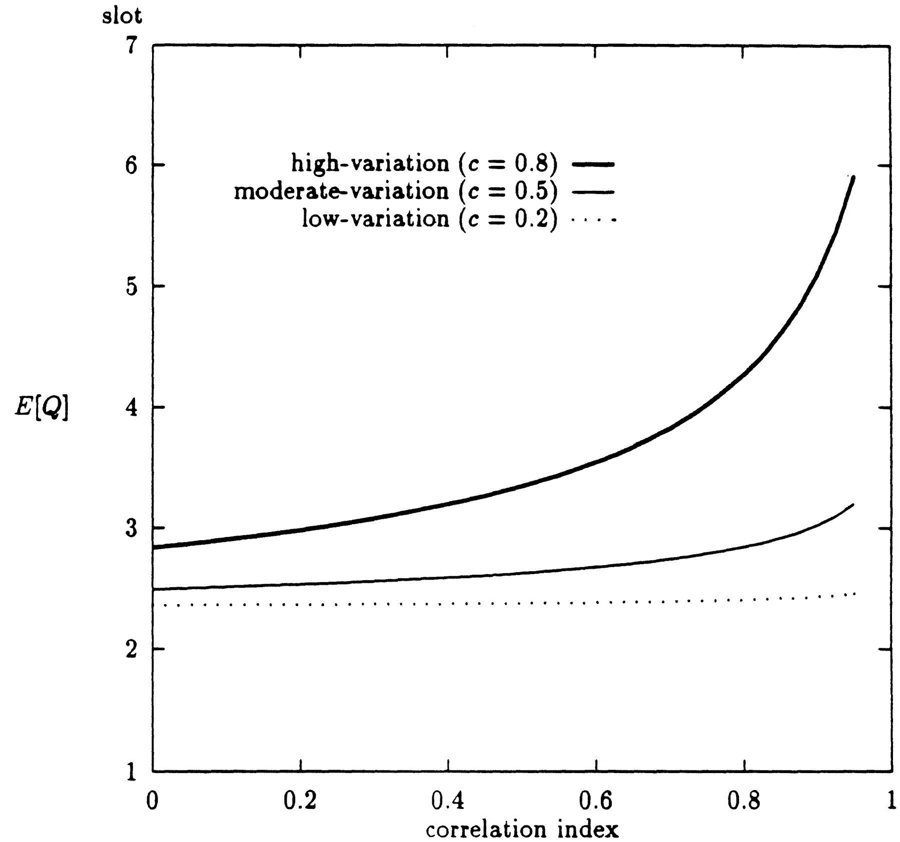

Fig.7 Mean Idle Periods ($N = 80$)

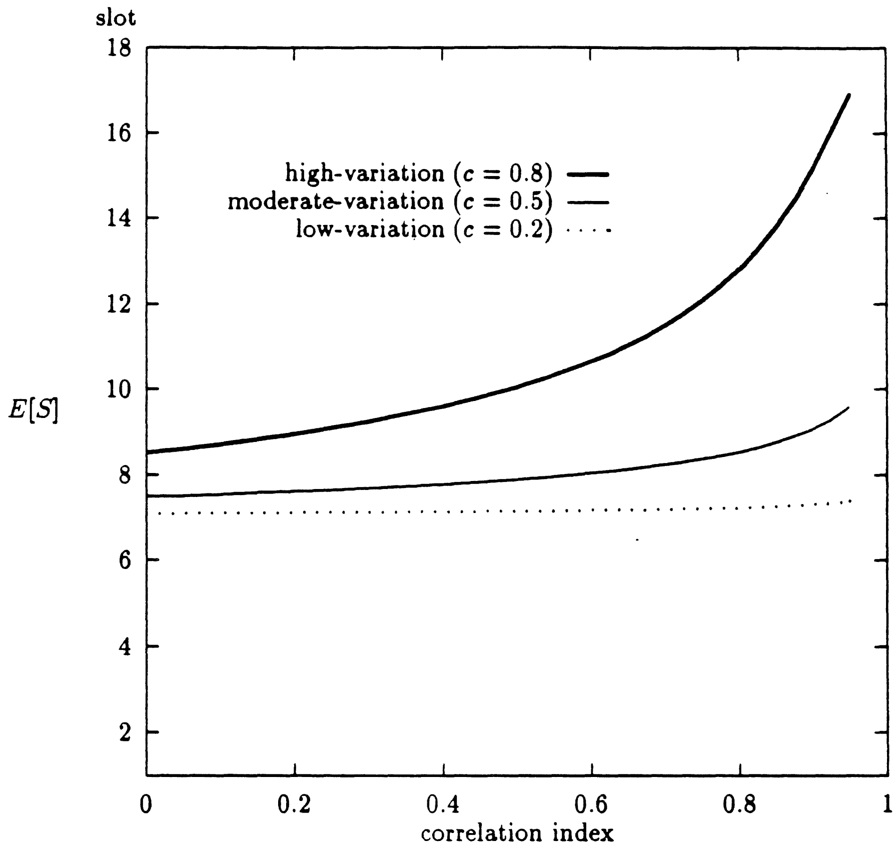

Fig.8 Mean Busy Periods ($N = 80$)

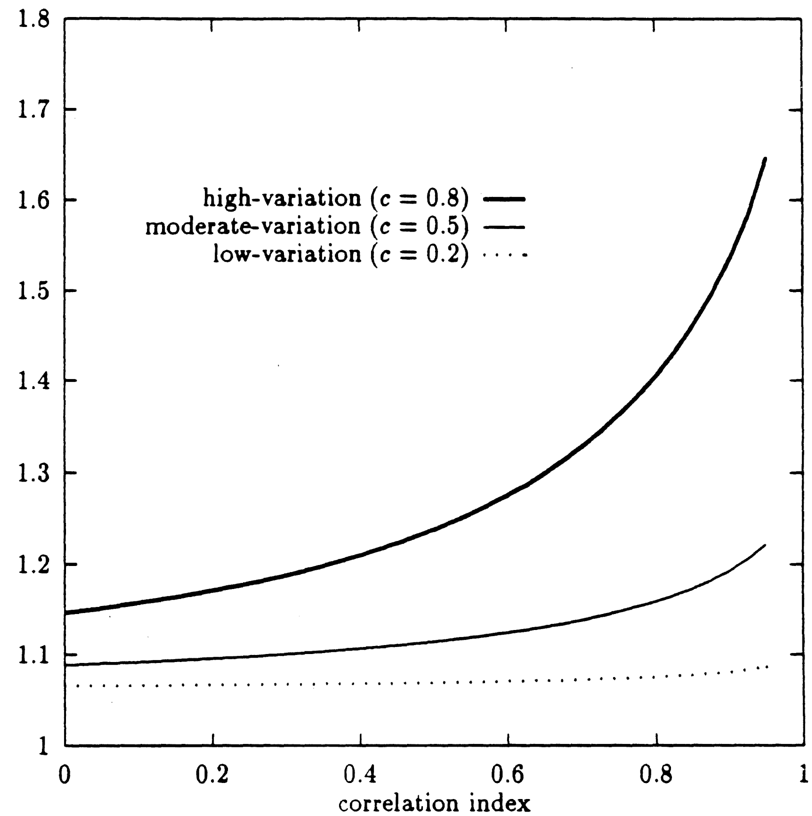

Fig.9 Coefficient of Variation of Interdeparture Times ($N = 80$)

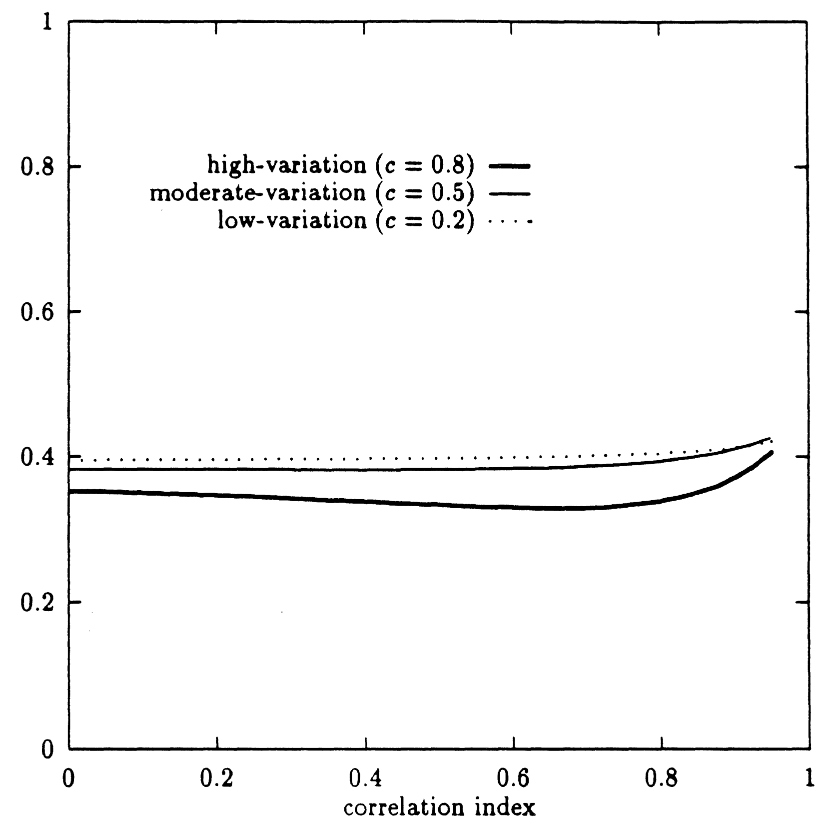

Fig.10 Correlation Coefficient **of Interdeparture Times** (N = 80)

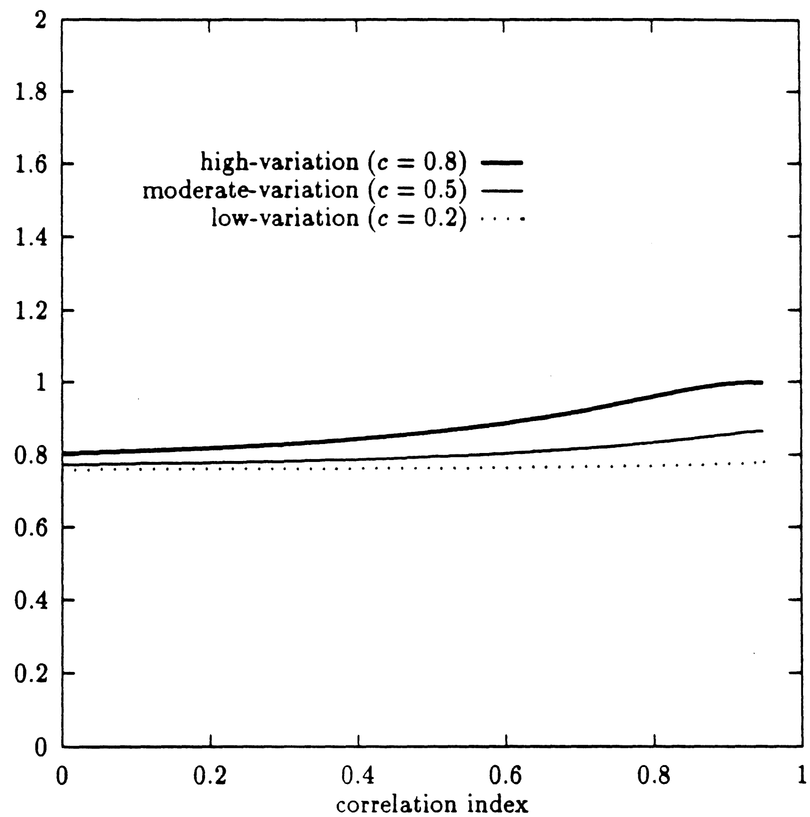

Fig.11 Coefficient of Variation of Idle Periods ($N = 80$)

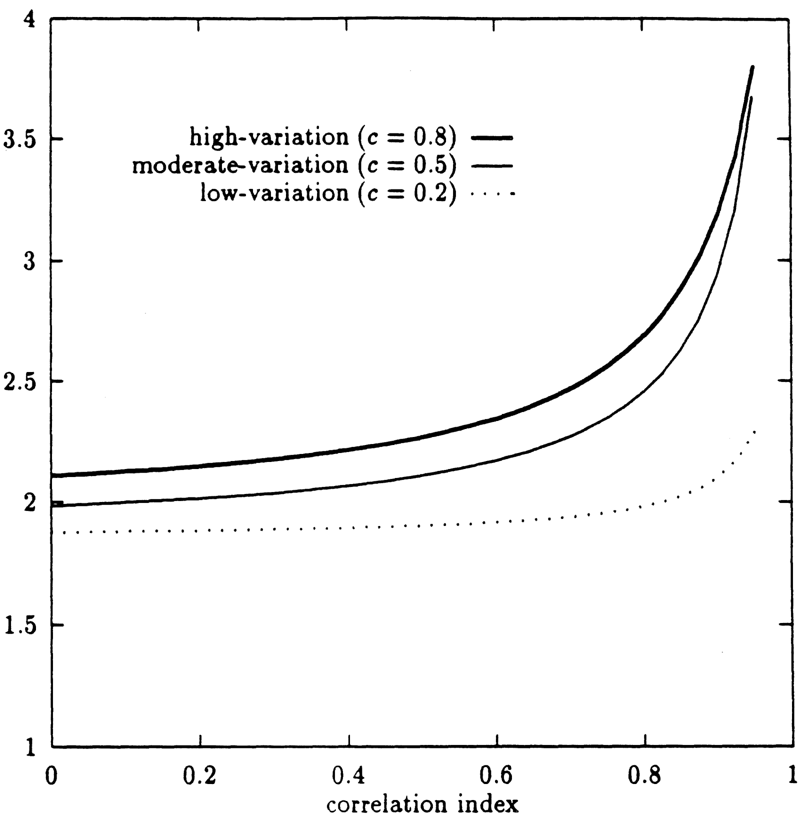

Fig.12 Coefficient of Variation of Busy Periods ($N = 80$)