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RELATIVITY IN AN UNDERGRADUATE LABORATORY--MEASURING THE RELATIVISTIC MASS INCREASE

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ABSTRACT

A method is described for measuring the relativistic mass increase using simple, inexpensive equipment. The effect is large and easily seen.

Although the theory of relativity is one of the major developments of physics in this century, undergraduates (and many graduates) rarely if ever, see relativistic effects in their own laboratory courses.[1] This article describes a simple, inexpensive experiment to measure the relativistic mass increase. It uses a ^{90}Sr - ^{90}Y beta source, a rubber magnet, a small NaI scintillation counter and an oscilloscope. The beta momentum is measured with the magnet, and its kinetic energy is measured with the NaI counter which is placed at the 180 deg energy focus.

The kinetic energy of ^{90}Y electrons is typically several times their rest mass energy, and they can be easily produced, collimated, deflected, stopped, and detected. The kinetic energy predicted by the non-relativistic formula

$$K = p^2/2m \qquad (1)$$

can be over three times that predicted by the relativistic formula,

$$K = \sqrt{p^2c^2 + m^2c^4} - mc^2 \qquad (2)$$

and so the effect is large and easily measured.

It is a good idea to precede this experiment with some standard ones on atomic and quantum physics, both for their own value and to familiarize the students with the equipment. These could include studies of:

1. counting rates and Poisson statistics,
2. collimation of beta rays,
3. scattering by thin foils (e.g., ~ 0.1 mm Al)
4. particle range in matter,
5. peak pulse heights in the NaI scintillator with and without an absorber placed ahead,
6. bending in a magnetic field.

The range-pulse height data in particular is useful in establishing that the NaI scintillator is thick enough to completely stop the electrons and gives a signal proportional to the total energy absorbed.

Equipment

The experimental set-up is shown in Figs. 1 and 2. The source is in the middle of the lead cube. A modest amount of collimation, provided by the 3.8-cm long, 1.3-cm diameter hole, is used mainly to reduce the room radiation level. It is not directly needed for the experiment due to the 180 deg focusing.

The permanent magnet is made with Plastiform[2] poles, each 2.8 cm by 20 cm by 23 cm, has a return yoke of 1.2 cm thick cold-rolled steel, and a central field of 900 gauss. No machining is needed since the magnetic forces hold the pieces together, however the material is sold un-magnetized, so you will need some way to magnetize it. (We assembled the pieces, propped them apart with wood blocks, and placed them between the poles of a larger magnet which was then raised to 18 kG.)

The scintillator is a 2.5 cm diameter, 2.5 cm deep crystal of NaI with a 25 μ aluminum window.[3] The thin window is desirable for this experiment since the 0.81 mm window followed by 67 mg/cm^2 Al_2O_3 powder in the normal mounting absorbs about 1/2 MeV. It is essential for another experiment on the annihilation of anti-matter ($\leq$ .5 MeV positrons from ^{22}Na). An 8 cm long Lucite light pipe was used to keep the photomultiplier out of the magnetic field. The photomultiplier was an Amperex 56AVP and used a standard LRL base and magnetic shield system,[4] but most any reasonable combination will work as long as the noise rate is not too high.

We used a fairly hot source--about 100 μC. If a low intensity one is used (~ 1 - 10 μC), typical counting rates will range from 10 to 100 counts/sec, and the tube noise at comparable signal heights must be kept well below this level. Fortunately, the signals from NaI are quite large and should result in 10^3 or more photoelectrons collected, well above the noise level for most decent tubes.

Any triggerable, 1 MHz oscilloscope will do.[5] The one used here, a Tektronix 585, was actually too fast, so a 0.01 μF capacitor was placed across the input to smooth over the statistical fluctuations in the NaI output.

Procedure

With any preliminary experiments, such as those listed in the first section completed, four kinds of data are taken:

1. Run electrons directly into the NaI to determine the pulse height corresponding to the end-point of the 2.27 MeV beta spectrum. (See Fig. 3.) Allowance should be made for the fact that very few electrons are emitted near the exact end-point (about 1% for $0.9 \leq P/P_{max} \leq 1.0$).

2. Place the source and detector as in Figs. 1 and 2, but with the magnetic field reversed. The few pulses seen will come from tube noise, scattered electrons, and cosmic rays. Obvious tests can be performed to distinguish these sources if desired.

3. Pick up the magnet and turn it upside down. You should now see a clear signal. Vary the source-detector separation. You can see, almost immediately, that classical mechanics is in trouble. For instance, in Fig. 4, compare the pulse heights for source-detector separations of 8 cm and 16 cm. With a doubling of the radius of curvature and hence momentum (approximately) $p^2/2m$ increases by a factor of four. Nothing of that sort is seen. Smaller signals are seen from such sources as tube noise and Coulomb-scattered electrons but the

concentration of pulses near the maximum height makes it clear that most electrons do not suffer the latter fate. Saturation of the photomultiplier signal, of course, is a real danger, but it can always be checked (and eliminated) by lowering the supply voltage. To check all this quantitatively, next--

4. Map the magnetic field. I recommend a flip coil-ballistic galvanometer combination. It is simple, direct, does not easily go out of calibration and is not packaged in a black box. A 2-cm grid taken on the median plane over one magnet quadrant should be adequate. Typical orbits and (dotted) construction lines are shown in Fig. 2. Figure 5 shows the momentum-diameter relationship derived from such calculations.

Results

Figure 6 shows the kinetic energy determined from the NaI pulse height plotted against the momentum determined from magnetic bending. The beta end-point data from Fig. 3 was used to calibrate the pulse height scale. The errors indicated for the kinetic energy come mainly from errors in estimating the most probable pulse height. The momentum errors come about equally from possible systematic errors in the magnetic field measurement and from the error in estimating which orbit corresponds to the central momentum for any given source-detector separation. The disagreement with the non-relativistic formula, (1), and agreement with the relativistic one, (2), is clear. However, since the most accurate beta end-point values come from magnetic spectrometer measurements which rely on relativistic kinematics, the demonstration thus far is somewhat circular. To settle this point, the original values are multiplied by an arbitrary constant (2.25) chosen to fit the non-relativistic curve in the middle of its range, and are plotted as open circles in Fig. 6. The disagreement remains. With the relativistic relation established, the kinetic energy, K, can be interpreted as the relativistic mass increase

$$K = E_{total} - mc^2 = \sqrt{p^2c^2 + m^2c^4} - mc^2 = (m_{rel} - m)c^2 \qquad (3)$$

and so Fig. 6 is a plot of just that increase.

Acknowledgements

I would like to thank Peter Watson for his help in designing and assembling the magnet.

References

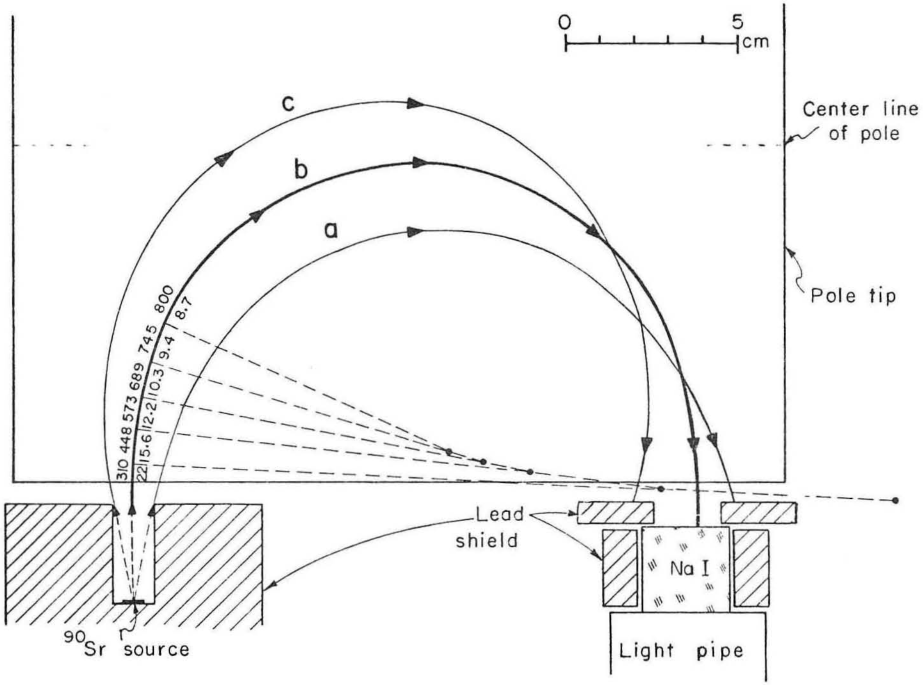
1. William Bertozzi, Am. J. Phys. 32, 551 (1964) describes an experiment using a 15 MeV linac which was the basis for the film, "The Ultimate Speed, an Exploration with High Energy Electrons." See also M. Steinberg, Am. J. Phys. 39, 582 (1971).

   J. R. Stevens and W. C. Winegrad, Am. J. Phys., 39, 34 (1971) describe a course featuring experiments with electrons, positrons and photons and films on relativity.
2. Plastiform, a rubber bonded, barium ferrite ($BaFe_{12}O_{19}$) composite material made by the 3M Company, Dielectric Materials and Systems Division, 4835 Para Drive, Cincinnati, Ohio 45237. Much useful design and background information is given in a 24-page booklet, "Plastiform Permanent Magnets" carrying the identification code DMS-MB (no date).
3. Available from the Harshaw Chemical Company, Crystal-Solid State Dept., 1945 East 97th Street, Cleveland, Ohio 44106. See also, the New York Times, Feb. 2, 1971, p. 19, col. 1.
4. S. Parker, W. Oliver and C. Rey, "A Photomultiplier Base and Shield System," Lawrence Radiation Laboratory Report UCRL-18031, Jan. 11, 1968 (unpublished).
5. Inexpensive, triggerable oscilloscopes with bandwidths in excess of 1 MHz are now sold by a number of companies; e.g., Telequipment, Pasco Scientific, and Heath.

Figure Captions

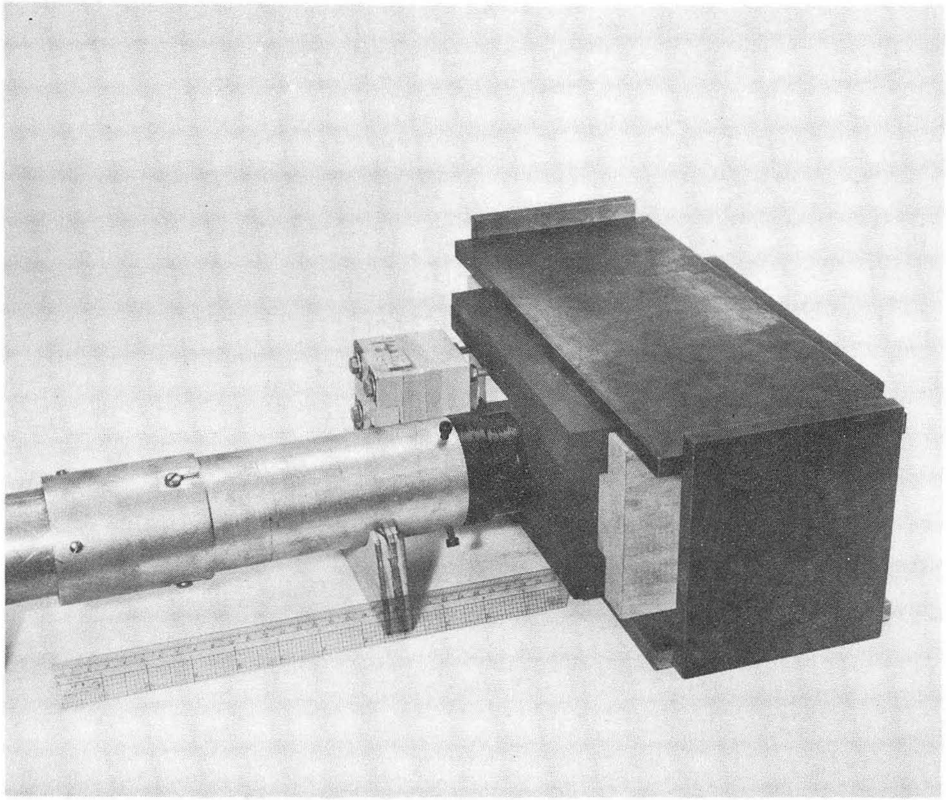
1. Experimental equipment--^{90}Sr source (rear), NaI scintillation counter (front), magnet (right).
2. Typical orbits for a 2.1 MeV/c electron. The numbers to the left of orbit are magnetic field values in gauss; those to the right are the corresponding radii of curvature in cm. The black dots indicate the centers of curvature used in calculating the orbit. A mild failure in 180 deg focusing is apparent.
3. (a) - (c) Signals from the ^{90}Sr beta source run directly into the NaI counter for 25, 50, and 100 traces.

   (d) Source-detector configuration as in Fig. 2, but with magnetic field reversed. An average of 27 traces would have been seen in an equally long exposure with the magnetic field set for electrons. Vertical scale - 0.1 V/cm, horizontal scale - 0.5 μsec/cm.
4. Signals from the ^{90}Sr beta source with a source-detector configuration as in Fig. 2 and for values of their separation, d, between 8 and 18.5 cm. Vertical scale - 0.1 V/cm, horizontal scale - 0.5 μsec/cm.
5. Momentum-diameter relationship derived from orbit tracing calculations. Curves labeled a, b, and c, calculated for orbits such as those shown in Fig. 2 with corresponding labels.
6. Electron kinetic energy determined from NaI pulse height plotted against momentum determined from magnetic bending. The solid circles indicate results derived using the beta end-point for calibration of the pulse height. The open circles are the result of further multiplication by an arbitrary constant (2.25) in an attempt to fit the non-relativistic curve.



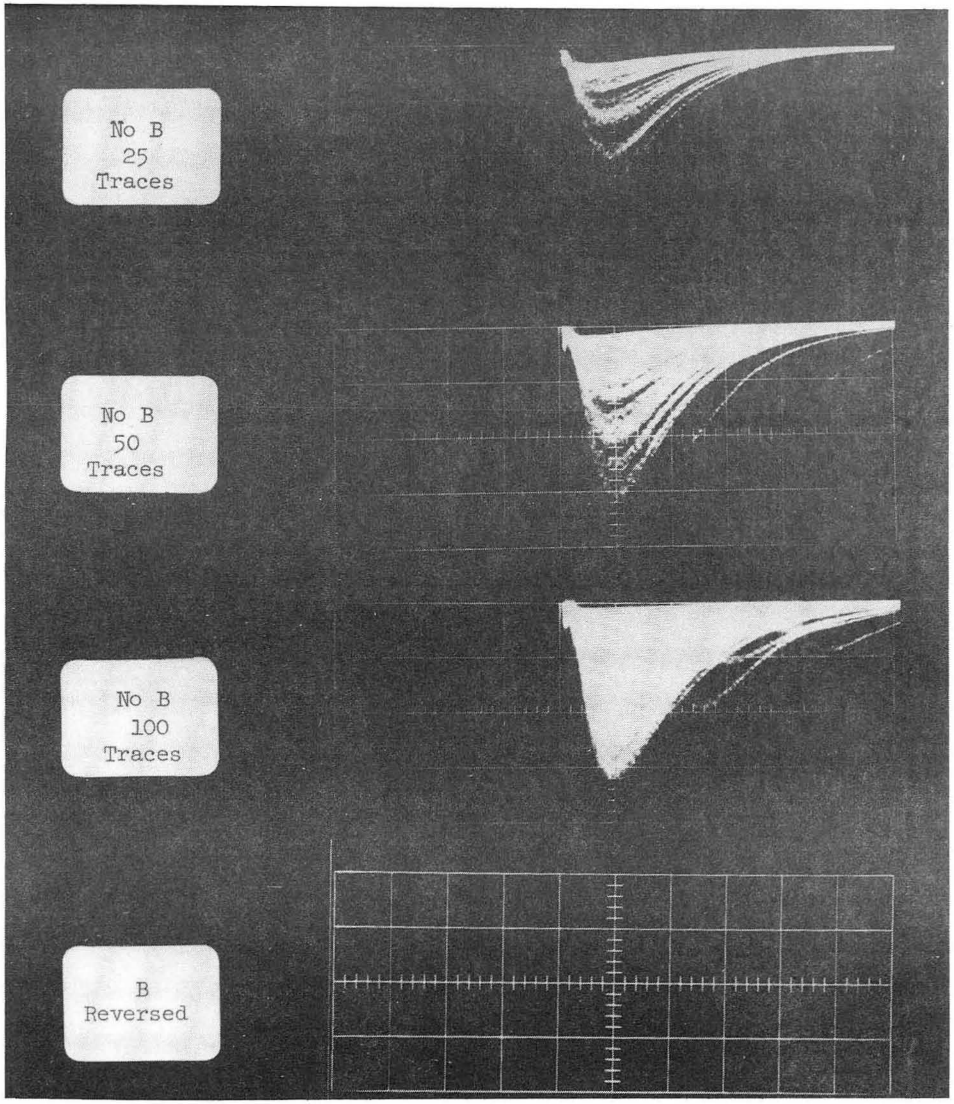
XBL 716-3672

Fig. 2



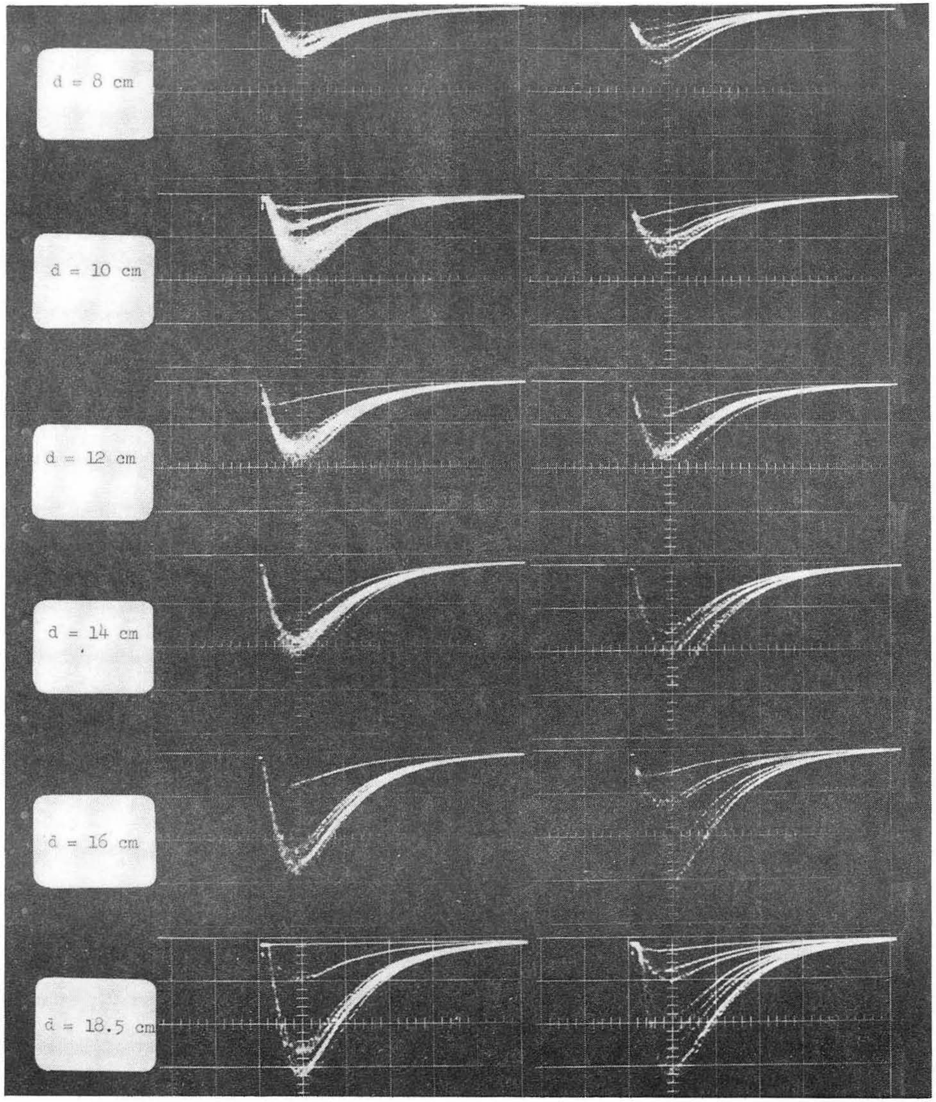
XBB 715-2234

Fig. 1



XBB 716-2362

Fig. 3



XBB 716-2363

Fig. 4

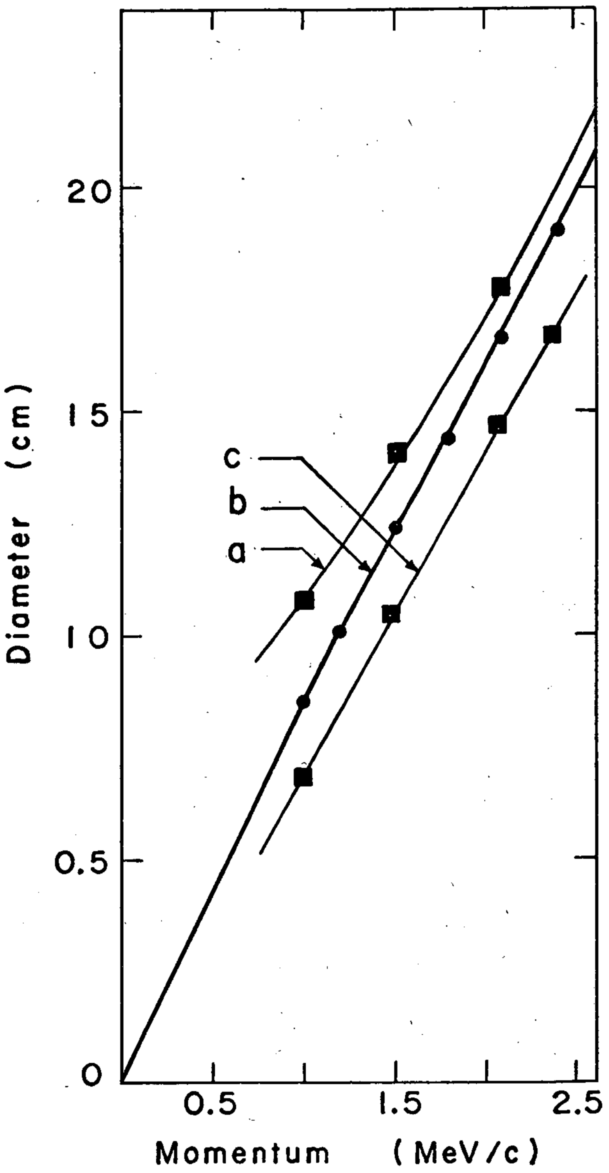


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Fig. 5

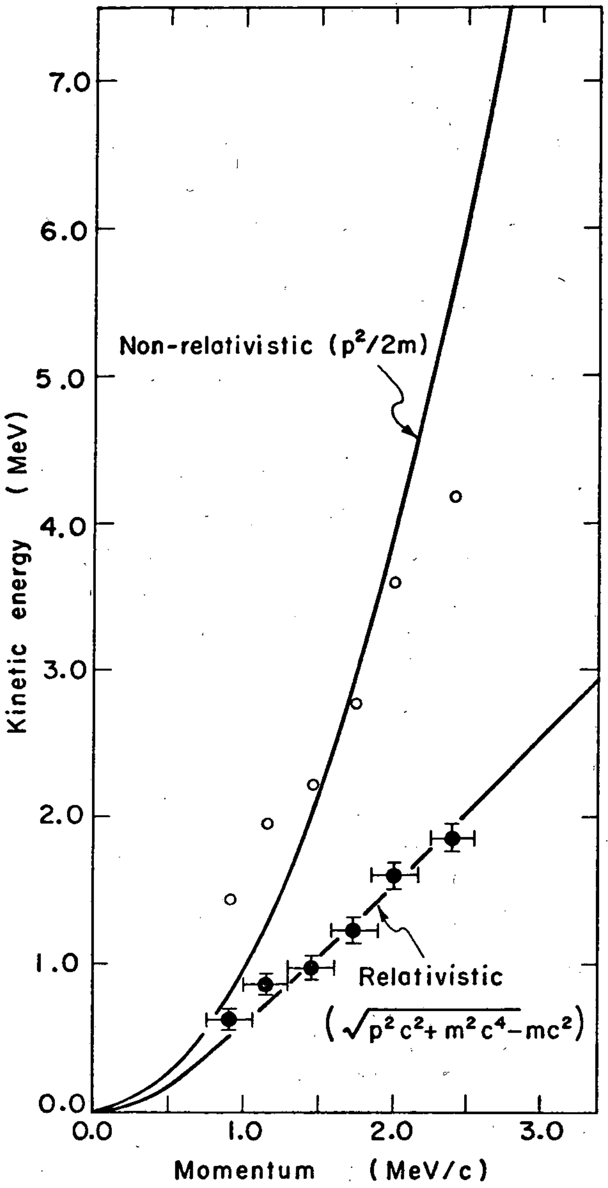

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Fig. 6

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