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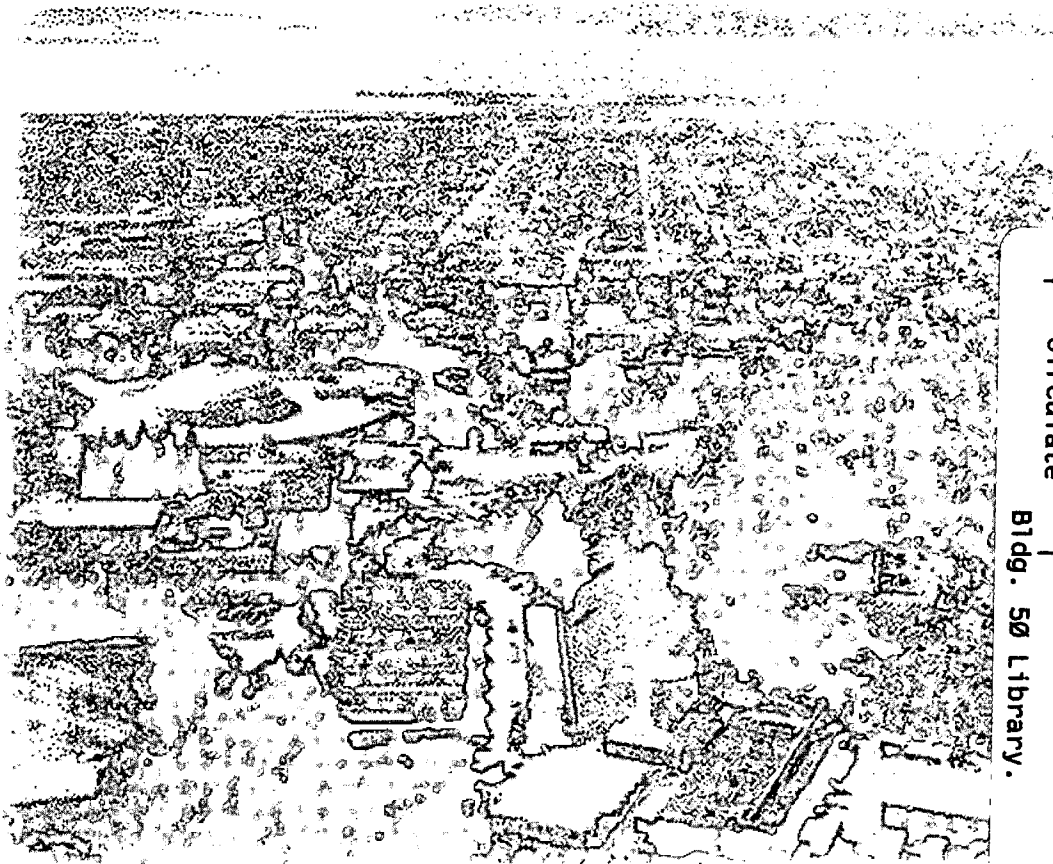**Publication Date**
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Dilaton Stabilization in the Context of Dynamical Supersymmetry Breaking through Gaugino Condensation

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Physics Division

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Dilaton Stabilization in the Context of Dynamical Supersymmetry Breaking through Gaugino Condensation [*]

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Abstract

We study gaugino condensation in the context of superstring effective theories using the linear multiplet formulation for the dilaton superfield. Including nonperturbative corrections to the Kähler potential for the dilaton may naturally achieve dilaton stabilization, with supersymmetry breaking and gaugino condensation; these three issues are interrelated in a very simple way. In a toy model with a single static condensate, a dilaton *vev* is found within a phenomenologically interesting range. The effective theory differs significantly from condensate models studied previously in the chiral formulation.

---

1 Introduction

Among the massless string modes, a real scalar (dilaton), an antisymmetric tensor field (the Kalb-Ramond field) and their supersymmetric partners can be described either by a chiral superfield S or by a linear multiplet L, which is known as the chiral-linear duality. By definition, the linear multiplet L is a vector superfield that satisfies the following constraints [1]:

$$-(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)L = 0,$$
$$-(\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - 8R^{\dagger})L = 0. \tag{1.1}$$

The lowest component of L is the dilaton field ℓ, and its *vev* is related to the gauge coupling constant as follows: $g^2(M_S) = 2\langle\ell\rangle$, where M_S is the string scale [2, 3]. Although the chiral-linear duality is obvious at tree level, it becomes obscure when quantum effects are included. Although scalar-2-form field strength duality, which is contained in chiral-linear duality, has been shown to be preserved in perturbation theory [4], the situation is less clear in the presence of nonperturbative effects, which are important in the study of gaugino condensation. It has recently been shown [5, 6] that gaugino condensation can be formulated directly using a linear multiplet for the dilaton. However, the content of the resulting chiral-linear duality transformation is in general very complicated. If there is an elegant description of gaugino condensates in the context of superstring effective theories, it may be simple in only one of these formulations, but not in both. Therefore, a pertinent issue is: which formulation is better?

In this paper we will construct the effective theory of gaugino condensation directly in the linear multiplet formulation without referring to the chiral formulation. There is reason to believe that the linear multiplet formulation is in fact more appropriate. The stringy reason for choosing the linear multiplet formulation is that the precise field content of the linear multiplet appears in the massless string spectrum, and $\langle L \rangle$ plays the role of string loop expansion parameter. Therefore, string information is more

naturally encoded in the linear multiplet formulation of string effective theory. In the context of gaugino condensation, it has been pointed out that the gaugino condensate U should be a constrained chiral superfield [5, 6, 17]; this constraint arises naturally in the linear multiplet formulation of gaugino condensation. Finally, in the linear formulation the symmetries of the underlying Yang-Mills theory in the weak coupling limit are automatically respected [7].

In the next section we describe the linear multiplet formulation of string effective Yang-Mills theory, whose effective theory below the condensation scale is constructed and analyzed in Sect. 3. It is then shown in Sect. 4 that supersymmetry is broken and the dilaton is stabilized in a large class of models of gaugino condensation. In this paper we use the Kähler superspace formulation [8], suitably extended to incorporate the linear multiplet [9].

2 The Linear Multiplet Formulation

2.1 Superstring Effective Yang-Mills Theory

In the realm of superstring effective Yang-Mills theory, there are two important ingredients, namely, the symmetry group of modular transformations and the linear multiplet. In order to make the discussion as explicit as possible, we consider here orbifolds with gauge group $E_8 \otimes E_6 \otimes U(1)^2$, which have been studied most extensively in the context of modular symmetries [2, 3, 10]. They contain three untwisted (1,1) moduli T^I, $I = 1, 2, 3$, which transform under SL(2,Z) as follows:

$$T^I \rightarrow \frac{aT^I - ib}{icT^I + d}, \quad ad - bc = 1, \quad a, b, c, d \in \mathbb{Z}. \qquad (2.1)$$

The corresponding Kähler potential is

$$G = \sum_I g^I + \sum_A \exp(\sum_I q_A^I g^I)|\Phi^A|^2 + O(\Phi^4), \qquad (2.2)$$

where $g^I = -\ln(T^I + \bar{T}^I)$, and the modular weights q_A^I depend on the particular matter field Φ^A as well as on the modulus T^I. However, it is well known that the effective theory obtained from the massless truncation of superstring is *not* invariant under the modular transformations (2.1) at one loop [11, 12]. Counterterms, that correspond to the result of integrating out massive modes, have to be added to the effective theory in order to restore modular invariance since string theory is known to be modular invariant to all orders of the loop expansion [13]. Two types of such counterterms have been discussed in the literature [2, 10, 12], the so-called f-type counterterm and the Green-Schwarz counterterm. The Green-Schwarz counterterm, which is analogous to the Green-Schwarz anomaly cancellation mechanism in D=10, is naturally implemented with the linear multiplet formulation [1]. Here we consider only those orbifolds for which the full modular anomaly is cancelled by the Green-Schwarz counterterm alone. This is the case unless the modulus T^I corresponds to an internal plane which is left invariant under some orbifold group transformations, which may happen only if an N=2 supersymmetric twisted sector is present [14]. Therefore, a large class of orbifolds, including the $\mathbb{Z}_3$ and $\mathbb{Z}_7$ orbifolds, is under consideration here.

The antisymmetric tensor field of superstring theories undergoes Yang-Mills gauge transformations. In the effective theory, it can be incorporated into a gauge invariant vector superfield L, the so-called modified linear multiplet, coupled to the Yang-Mills degrees of freedom as follows:

$$-(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)L = (\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)\Omega = \sum_a \mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)^a,$$

$$-(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^\dagger)L = (\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^\dagger)\Omega = \sum_a \mathrm{Tr}(\mathcal{W}_{\dot{\alpha}} \mathcal{W}^{\dot{\alpha}})^a, \quad (2.3)$$

where Ω is the Yang-Mills Chern-Simons superform. The summation extends over the indices a numbering simple subgroups of the full gauge group. The modified linear multiplet L contains the linear multiplet as well as the Chern-Simons superform, and its gauge invariance is ensured by imposing appropriate transformation properties for the linear multiplet. The generic

3

lagrangian describing the linear multiplet coupled to supergravity and matter in the presence of Yang-Mills Chern-Simons superform is [2]:

$$
\begin{aligned}
K &= k(L) + G, \\
\mathcal{L} &= -3 \int \mathrm{d}^4\theta\, E\, F(L) + \int \mathrm{d}^4\theta\, E\, \{\, bL \sum_I g^I \,\}, \qquad (2.4) \\
b &= \frac{C}{8\pi^2} = \frac{2}{3}b_0, \qquad (2.5)
\end{aligned}
$$

where L is the modified linear multiplet and $C = 30$ is the Casimir operator in the adjoint representation of E_8. b_0 is the E_8 one-loop β-function coefficient. The first term of $\mathcal{L}$ is the superspace integral which yields the kinetic actions for the linear multiplet, supergravity, matter and Yang-Mills fields. The second term in (2.4) is the Green-Schwarz counterterm, which is "minimal" in the sense of [2]. Furthermore, arbitrariness in the two functions $k(L)$ and $F(L)$ is reduced by the requirement that the Einstein term in $\mathcal{L}$ be canonical. Under this constraint, $k(L)$ and $F(L)$ are related to each other by the following first-order differential equation [9]:

$$
F - L\frac{\mathrm{d}F}{\mathrm{d}L} = 1 - \frac{1}{3}L\frac{\mathrm{d}k}{\mathrm{d}L}. \qquad (2.6)
$$

The complete component lagrangian of (2.4) with the tree-level Kähler potential (i.e., $k(L) = \ln L$ and $F(L) = \frac{2}{3}$) has been presented in [15] based on the Kähler superspace formalism. Similar studies have also been performed in the superconformal formalism of supergravity [16]. In the following sections, we are interested in the effective lagrangian of (2.4) below the condensation scale.

2.2 The Low-Energy Effective Degrees of Freedom

Below the condensation scale at which the gauge interaction becomes strong, the effective lagrangian of the Yang-Mills sector can be described by a composite chiral superfield U, which corresponds to the chiral superfield

$\mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)$ of the underlying theory. (We consider here gaugino condensation of a simple gauge group.) The scalar component of U is naturally interpreted as the gaugino condensate. It was pointed out only recently that the composite field U is actually a constrained chiral superfield [6]–[7],[17]. The constraint on U can be seen most clearly through the constrained superspace geometry of the underlying Yang-Mills theory. As a consequence of this constrained geometry, the chiral superfield $\mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)$ and its hermitian conjugate $\mathrm{Tr}(\mathcal{W}_{\dot\alpha} \mathcal{W}^{\dot\alpha})$ satisfy the following constraint:

$$(\mathcal{D}^\alpha \mathcal{D}_\alpha - 24R^\dagger)\mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) - (\mathcal{D}_{\dot\alpha} \mathcal{D}^{\dot\alpha} - 24R)\mathrm{Tr}(\mathcal{W}_{\dot\alpha} \mathcal{W}^{\dot\alpha}) = \text{total derivative.} \tag{2.7}$$

(2.7) has a natural interpretation in the context of a 3-form supermultiplet, and indeed $\mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)$ can be interpreted as the degrees of freedom of the 3-form field strength [18]. The explicit solution to the constraint (2.7) has been presented in [17], and it allows us to identify the constrained chiral superfield $\mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha)$ with the chiral projection of an unconstrained vector superfield L:

$$\begin{aligned}
\mathrm{Tr}(\mathcal{W}^\alpha \mathcal{W}_\alpha) &= -(\mathcal{D}_{\dot\alpha} \mathcal{D}^{\dot\alpha} - 8R)L, \\
\mathrm{Tr}(\mathcal{W}_{\dot\alpha} \mathcal{W}^{\dot\alpha}) &= -(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^\dagger)L.
\end{aligned} \tag{2.8}$$

Below the condensation scale, the constraint (2.7) is replaced by the following constraint on U and $\bar{U}$:

$$(\mathcal{D}^\alpha \mathcal{D}_\alpha - 24R^\dagger)U - (\mathcal{D}_{\dot\alpha} \mathcal{D}^{\dot\alpha} - 24R)\bar{U} = \text{total derivative.} \tag{2.9}$$

Similarly, the solution to (2.9) allows us to identify the constrained chiral superfield U with the chiral projection of an unconstrained vector superfield V:

$$\begin{aligned}
U &= -(\mathcal{D}_{\dot\alpha} \mathcal{D}^{\dot\alpha} - 8R)V, \\
\bar{U} &= -(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^\dagger)V.
\end{aligned} \tag{2.10}$$

(2.10) is the explicit constraint on U and $\bar{U}$.

In fact, the constraint on U and $\bar{U}$ enters the linear multiplet formulation of gaugino condensation very naturally. As described in Sect. 2.1, the linear multiplet formulation of supersymmetric Yang-Mills theory is described by a gauge-invariant vector superfield L which satisfies

$$
\begin{aligned}
-(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)L &= (\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)\Omega = \mathrm{Tr}(\mathcal{W}^{\alpha}\mathcal{W}_{\alpha}), \\
-(\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - 8R^{\dagger})L &= (\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - 8R^{\dagger})\Omega = \mathrm{Tr}(\mathcal{W}_{\dot{\alpha}}\mathcal{W}^{\dot{\alpha}}). \quad (2.11)
\end{aligned}
$$

For the linear multiplet formulation of the effective lagrangian below the condensation scale, (2.11) is replaced by

$$
\begin{aligned}
-(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)V &= U, \\
-(\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - 8R^{\dagger})V &= \bar{U}, \quad (2.12)
\end{aligned}
$$

where U is the gaugino condensate chiral superfield, and V contains the linear multiplet as well as the "fossil" Chern-Simons superform. In view of (2.12), it is clear that the constraint on U and $\bar{U}$ arises naturally in the linear multiplet formulation of gaugino condensation. Furthermore, the low-energy degrees of freedom (i.e., the linear multiplet and the gaugino condensate) are nicely merged into a single vector superfield V, and therefore the linear multiplet formulation of gaugino condensation can elegantly be described by V alone. The detailed construction of the effective lagrangian for the vector superfield V will be presented in the next section.

3 Gaugino Condensation in Superstring Effective Theory

3.1 A Simple Model

Constructing the linear multiplet formulation of gaugino condensation requires the specification of two functions of the vector superfield V, namely,

6

the superpotential and the Kähler potential. In the linear multiplet formulation, there is no classical superpotential [7], and the quantum superpotential originates from the nonperturbative effects of gaugino condensation. This nonperturbative superpotential, whose form was dictated by the anomaly structure of the underlying theory, was first obtained by Veneziano and Yankielowicz [19]. The details of its generalization to the case of matter coupled to $N=1$ supergravity in the Kähler superspace formalism has been presented in [20], and the superpotential term in the Lagrangian reads:

$$
\begin{aligned}
\int \mathrm{d}^4\theta \, \frac{E}{R} \, e^{K/2} W_{VY} &= \int \mathrm{d}^4\theta \, \frac{E}{R} \, \frac{1}{8} bU \ln(e^{-K/2} U/\mu^3), \\
\int \mathrm{d}^4\theta \, \frac{E}{R^\dagger} \, e^{K/2} \bar{W}_{VY} &= \int \mathrm{d}^4\theta \, \frac{E}{R^\dagger} \, \frac{1}{8} b\bar{U} \ln(e^{-K/2} \bar{U}/\mu^3),
\end{aligned} \qquad (3.1)
$$

where $U = -(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)V$ is the constrained gaugino condensate chiral superfield with Kähler weight 2, and μ is a constant with dimension of mass that is left undetermined by the method of anomaly matching.

As for the Kähler potential for V, there is little knowledge beyond tree level. The best we can do at present is to treat all physically reasonable Kähler potentials on the same footing and to look for possible general features and/or interesting special cases. Before discussing this general analysis, it is instructive to examine a simple linear multiplet model for gaugino condensation defined as follows [7]:

$$
\begin{aligned}
K &= \ln V + G, \\
\mathcal{L}_{eff} &= \int \mathrm{d}^4\theta \, E \left\{ -2 + bVG \right\} + \int \mathrm{d}^4\theta \, \frac{E}{R} \, e^{K/2} W_{VY} + \int \mathrm{d}^4\theta \, \frac{E}{R^\dagger} \, e^{K/2} \bar{W}_{VY}, \\
G &= -\sum_I \ln(T^I + \bar{T}^I).
\end{aligned} \qquad (3.2)
$$

This simple model describes the effective theory for (2.4) below the condensation scale, where the Kähler potential of V assumes its tree-level form. It is a "static" model in the sense that no kinetic term for U is included. From the viewpoint of the anomaly structure, static as well as nonstatic models

are interesting in their own right. In the chiral formulation of gaugino condensation, it can be shown that the static model corresponds to the effective theory of the nonstatic model after the gaugino condensate U is integrated out. Nonstatic models [5, 6] in the linear multiplet formulation have been studied less extensively. Here we will restrict our attention to the static case, since the points we wish to illustrate are not substantially altered by including a kinetic term for U. In Sect. 5 we will indicate how the model considered here can be generalized to the case of a dynamical condensate.

With $U = -(\mathcal{D}_{\dot{\alpha}}\mathcal{D}^{\dot{\alpha}} - 8R)V$ and $\bar{U} = -(\mathcal{D}^{\alpha}\mathcal{D}_{\alpha} - 8R^{\dagger})V$, we can rewrite the superpotential terms of $\mathcal{L}_{eff}$ as a single D-term, and therefore the simple model (3.2) can be rewritten as follows:

$$
\begin{aligned}
K &= \ln V + G, \\
\mathcal{L}_{eff} &= \int d^4\theta\, E\, \{ -2 + bVG + bV\ln(e^{-K}\bar{U}U/\mu^6) \}.
\end{aligned}
\tag{3.3}
$$

In (3.3), the modular anomaly cancellation by the Green-Schwarz counterterm is transparent [7]. The Green-Schwarz counterterm bVG and the superpotential D-term $bV\ln(e^{-K}\bar{U}U/\mu^6)$ are *not* modular invariant separately, but their sum is modular invariant, which ensures the modular invariance of the full theory. In fact, the Green-Schwarz counterterm cancels the T^I moduli-dependence of the superpotential completely. This is a unique feature of the linear multiplet formulation, and, as we will see later, has interesting implications for the moduli-dependence of physical quantities.

Throughout this paper only the bosonic and gravitino parts of the component lagrangian are presented, since we are interested in the vacuum configuration and the gravitino mass. In the following, we enumerate the definitions of bosonic component fields of the vector superfield V.

$$
\begin{aligned}
\ell &= V|_{\theta=\bar{\theta}=0}, \\
\sigma^m_{\alpha\dot{\alpha}}B_m &= \frac{1}{2}[\mathcal{D}_{\alpha},\mathcal{D}_{\dot{\alpha}}]V|_{\theta=\bar{\theta}=0} + \frac{2}{3}\ell\sigma^a_{\alpha\dot{\alpha}}b_a, \\
u &= U|_{\theta=\bar{\theta}=0} = -(\bar{\mathcal{D}}^2 - 8R)V|_{\theta=\bar{\theta}=0},
\end{aligned}
$$

8

$$\bar{u} = \bar{U}|_{\theta=\bar{\theta}=0} = -(\mathcal{D}^2 - 8R^\dagger)V|_{\theta=\bar{\theta}=0},$$

$$D = \frac{1}{8}\mathcal{D}^\beta(\bar{\mathcal{D}}^2 - 8R)\mathcal{D}_\beta V|_{\theta=\bar{\theta}=0}$$

$$= \frac{1}{8}\mathcal{D}_{\dot{\beta}}(\mathcal{D}^2 - 8R^\dagger)\mathcal{D}^{\dot{\beta}}V|_{\theta=\bar{\theta}=0}, \qquad (3.4)$$

where

$$-\frac{1}{6}M = R|_{\theta=\bar{\theta}=0}, \quad -\frac{1}{6}\bar{M} = R^\dagger|_{\theta=\bar{\theta}=0}, \quad -\frac{1}{3}b_a = G_a|_{\theta=\bar{\theta}=0} \qquad (3.5)$$

are the auxiliary components of supergravity multiplet. It is convenient to write the lowest components of $\mathcal{D}^2 U$ and $\bar{\mathcal{D}}^2\bar{U}$ as follows:

$$-4F_U = \mathcal{D}^2 U|_{\theta=\bar{\theta}=0}, \quad -4\bar{F}_{\bar{U}} = \bar{\mathcal{D}}^2\bar{U}|_{\theta=\bar{\theta}=0}. \qquad (3.6)$$

$(F_U - \bar{F}_{\bar{U}})$ can be explicitly expressed as follows:

$$(F_U - \bar{F}_{\bar{U}}) = 4i\nabla^m B_m + u\bar{M} - \bar{u}M. \qquad (3.7)$$

The expression for $(F_U + \bar{F}_{\bar{U}})$ contains the auxiliary field D. The bosonic components of T^I and $\bar{T}^I$ are

$$t^I = T^I|_{\theta=\bar{\theta}=0}, \quad -4F_T^I = \mathcal{D}^2 T^I|_{\theta=\bar{\theta}=0},$$

$$\bar{t}^I = \bar{T}^I|_{\theta=\bar{\theta}=0}, \quad -4\bar{F}_{\bar{T}}^I = \bar{\mathcal{D}}^2\bar{T}^I|_{\theta=\bar{\theta}=0}. \qquad (3.8)$$

We leave the details of constructing the component lagrangian for this simple model (in the Kähler superspace formalism) to Sect. 3.2, and present here only the scalar potential:

$$V_{pot} = \frac{1}{16e^2\ell}(1 + 2b\ell - 2b^2\ell^2)\mu^6 e^{-1/b\ell}. \qquad (3.9)$$

Eq.(3.9) agrees with the result obtained in [6], where the model defined by (3.2) was studied for the case of a single modulus using the superconformal formalism of supergravity.

However, this simple model is not viable. As expected, the weak-coupling limit $\ell = 0$ is always a minimum. As shown in Fig.1, the scalar potential

starts with $V_{pot} = 0$ at $\ell = 0$, first rises and then falls without limit as ℓ increases. Therefore, V_{pot} is unbounded from below, and this simple model has no well-defined vacuum. This may be somewhat surprising because the model defined by (3.2) superficially appears to be of the no-scale type: the Green-Schwarz counterterm, that destroys the no-scale property of chiral models and destabilizes the potential, is cancelled here by quantum effects that induce a potential for the condensate. However the resulting quantum contribution to the Lagrangian (3.3), $bV \ln(U\bar{U}/V)$, has an implicit T^I-dependence through the superfield U due to its nonvanishing Kähler weight: $w(U) = 2$. This implicit moduli-dependence is a consequence of the anomaly matching condition, and parallels the construction of the effective theory in the chiral formulation [19] which is also not of the no-scale form once the Green-Schwarz counterterm is included. By contrast, in [7] a no-scale model was constructed in the chiral formulation precisely through a cancellation of the Green-Schwarz counterterm. In the construction of that model, the point of view was adopted that a superpotential for the dilaton could arise only from nonperturbative effects on the string world sheet, and the anomaly matching condition was bypassed by directly writing an effective low energy theory that was exactly modular invariant. The relation between these approaches warrants further investigation.

If we take a closer look at (3.9), it is clear that the unboundedness of V_{pot} in the strong-coupling limit $\ell \to \infty$ is caused by a term of two-loop order: $-2b^2\ell^2$. This observation strongly suggests that the underlying reason for unboundedness is our poor control over the model in the strong-coupling regime. The form of the superpotential W_{VY} is completely fixed by the underlying anomaly structure. However the Kähler potential is much less constrained, and the choice (3.2) cannot be expected to be valid in the strong-coupling regime where the nonperturbative contributions should not be ignored. We conclude that the unboundedness shown in Fig. 1 simply simply reflects the importance of nonperturbative contributions [21, 22] to the Kähler potential. In the absence of a better knowledge of the exact Kähler potential, we will

consider models with generic Kähler potentials in the following sections.

3.2 General Static Model

In this section, we show how to construct the component lagrangian for generic linear multiplet models of gaugino condensation in the Kähler superspace formalism. Further computational details can be found in [8, 15]. Although our results can probably be rephrased in the chiral formulation, the equivalent chiral superfield formulation may be expected to be rather complicated because of the constraint on the condensate chiral superfield U. Quite generally we do not expect a simple ansatz in one formalism to appear simple in the other.

As suggested in Sect. 3.1, we extend the simple model in (3.2) to linear multiplet models of gaugino condensation with generic Kähler potentials defined as follows:

$$
\begin{aligned}
K &= \ln V + g(V) + G, \\
\mathcal{L}_{eff} &= \int d^4\theta \, E \, \{ \, (-2 + f(V)) + bVG + bV \ln(e^{-K}\bar{U}U/\mu^6) \, \}. \quad (3.10)
\end{aligned}
$$

For convenience, we also write $\ln V + g(V) \equiv k(V)$. $g(V)$ and $f(V)$ represent quantum corrections to the tree-level Kähler potential, and, according to (2.6), they are unambiguously related to each other by the following first-order differential equation:

$$
V\frac{dg(V)}{dV} = -V\frac{df(V)}{dV} + f, \quad (3.11)
$$

$$
g(V = 0) = 0 \quad \text{and} \quad f(V = 0) = 0. \quad (3.12)
$$

The boundary condition of $g(V)$ and $f(V)$ at $V = 0$ (the weak-coupling limit) is fixed by the tree-level Kähler potential. Before trying to specify $g(V)$ and $f(V)$, it is reasonable to assume for the present that $g(V)$ and $f(V)$ are arbitrary but bounded.

11

In the construction of the component field lagrangian, we use the chiral density multiplet method [8], which provides us with the locally supersymmetric generalization of the F-term construction in global supersymmetry. The chiral density multiplet r and its hermitian conjugate $\bar{r}$ for the generic model in (3.10) are:

$$
\begin{aligned}
r &= -\frac{1}{8}(\bar{\mathcal{D}}^2 - 8R)\{(-2 + f(V)) + bVG + bV\ln(e^{-K}\bar{U}U/\mu^6)\}, \\
\bar{r} &= -\frac{1}{8}(\mathcal{D}^2 - 8R^\dagger)\{(-2 + f(V)) + bVG + bV\ln(e^{-K}\bar{U}U/\mu^6)\}. \quad (3.13)
\end{aligned}
$$

In order to obtain the component lagrangian $\mathcal{L}_{eff}$, we need to work out the following expression

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}_{eff} &= -\frac{1}{4}\mathcal{D}^2 r|_{\theta=\bar{\theta}=0} + \frac{i}{2}(\bar{\psi}_m\bar{\sigma}^m)^\alpha \mathcal{D}_\alpha r|_{\theta=\bar{\theta}=0} \\
&\quad - (\bar{\psi}_m\bar{\sigma}^{mn}\bar{\psi}_n + \bar{M})r|_{\theta=\bar{\theta}=0} + \text{h.c.} \quad (3.14)
\end{aligned}
$$

An important point in the computation of (3.14) is the evaluation of the component field content of the Kähler supercovariant derivatives, a rather tricky process. The details of this computation have by now become general wisdom and we can to a large extent rely on the existing literature [23]. In particular, the Lorentz transformation and the Kähler transformation are incorporated in a very similar way in the Kähler superspace formalism, and the Lorentz connection as well as the so-called Kähler connection A_M are incorporated into the Kähler supercovariant derivatives in a concise and constructive way. The Kähler connection A_M is not an independent field but rather expressed in terms of the Kähler potential K as follows

$$
A_\alpha = \frac{1}{4}E_\alpha{}^M\partial_M K, \quad A_{\dot\alpha} = -\frac{1}{4}E_{\dot\alpha}{}^M\partial_M K, \quad (3.15)
$$

$$
\sigma^a_{\alpha\dot\alpha}A_a = \frac{3}{2}i\sigma^a_{\alpha\dot\alpha}G_a - \frac{1}{8}i[\mathcal{D}_\alpha, \mathcal{D}_{\dot\alpha}]K. \quad (3.16)
$$

In order to extract the explicit form of the various couplings, we choose to write out explicitly the vectorial part of the Kähler connection and keep

only the Lorentz connection in the definition of covariant derivatives when we present the component expressions. In the following, we give the lowest component of the vectorial part of the Kähler connection $A_m|_{\theta=\bar\theta=0}$ for our generic model.

$$A_m = e_m{}^a A_a + \frac{1}{2}\psi_m{}^\alpha A_\alpha + \frac{1}{2}\bar\psi_{m\dot\alpha}A^{\dot\alpha}. \tag{3.17}$$

$$A_m|_{\theta=\bar\theta=0} = -\frac{i}{4\ell}(\ell g_{(1)} + 1)B_m + \frac{i}{6}(\ell g_{(1)} - 2)e_m{}^a b_a$$
$$+ \sum_I \frac{1}{4(t^I + \bar t^I)}(\nabla_m \bar t^I - \nabla_m t^I). \tag{3.18}$$

$$g_{(m)} = g_{(m)}(\ell) = \frac{\mathrm{d}^m g(V)}{\mathrm{d}V^m}\Big|_{\theta=\bar\theta=0},$$
$$f_{(m)} = f_{(m)}(\ell) = \frac{\mathrm{d}^m f(V)}{\mathrm{d}V^m}\Big|_{\theta=\bar\theta=0}. \tag{3.19}$$

Another hallmark of the Kähler superspace formalism are the chiral superfield X_α and the antichiral superfield $\bar X^{\dot\alpha}$. They arise in complete analogy with usual supersymmetric abelian gauge theory except that now the corresponding vector superfield is replaced by the Kähler potential:

$$X_\alpha = -\frac{1}{8}(\mathcal{D}_{\dot\alpha}\mathcal{D}^{\dot\alpha} - 8R)\mathcal{D}_\alpha K,$$
$$\bar X^{\dot\alpha} = -\frac{1}{8}(\mathcal{D}^\alpha \mathcal{D}_\alpha - 8R^\dagger)\mathcal{D}^{\dot\alpha}K. \tag{3.20}$$

In the computation of (3.14), we need to decompose the lowest components of the following six superfields: X_α, $\bar X^{\dot\alpha}$, $\mathcal{D}_\alpha R$, $\mathcal{D}^{\dot\alpha}R^\dagger$, $(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}\bar X^{\dot\alpha})$ and $(\mathcal{D}^2 R + \bar{\mathcal{D}}^2 R^\dagger)$ into component fields. This is done by solving the following six simple algebraic equations:

$$(V\frac{\mathrm{d}g}{\mathrm{d}V} + 1)\mathcal{D}_\alpha R + X_\alpha = \Xi_\alpha, \tag{3.21}$$

$$3\mathcal{D}_\alpha R + X_\alpha = -2(\sigma^{cb}\epsilon)_{\alpha\varphi}T_{cb}{}^\varphi. \tag{3.22}$$

$$(V\frac{dg}{dV} + 1)\mathcal{D}^{\dot\alpha}R^\dagger + \bar{X}^{\dot\alpha} = \bar{\Xi}^{\dot\alpha}, \tag{3.23}$$

$$3\mathcal{D}^{\dot\alpha}R^\dagger + \bar{X}^{\dot\alpha} = -2(\bar\sigma^{cb}\epsilon)^{\dot\alpha\dot\varphi}T_{cb\dot\varphi}. \tag{3.24}$$

$$(V\frac{dg}{dV} + 1)(\mathcal{D}^2R + \bar{\mathcal{D}}^2R^\dagger) + (\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}\bar{X}^{\dot\alpha}) = \Delta, \tag{3.25}$$

$$3(\mathcal{D}^2R + \bar{\mathcal{D}}^2R^\dagger) + (\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}\bar{X}^{\dot\alpha}) = -2R_{ba}{}^{ba} + 12G^aG_a$$
$$+ 96RR^\dagger. \tag{3.26}$$

The identities (3.22), (3.24) and (3.26) arise solely from the structure of Kähler superspace. (3.22) and (3.24) involve the torsion superfields $T_{cb}{}^\varphi$ and $T_{cb\dot\varphi}$, which in their lowest components contain the curl of the Rarita-Schwinger field. The identities (3.21), (3.23) and (3.25) arise directly from the definitions of X_α, $\bar{X}^{\dot\alpha}$, $(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}\bar{X}^{\dot\alpha})$, and therefore they depend on the Kähler potential explicitly. Computing X_α, $\bar{X}^{\dot\alpha}$ and $(\mathcal{D}^\alpha X_\alpha + \mathcal{D}_{\dot\alpha}\bar{X}^{\dot\alpha})$ according to (3.20) defines the contents of Ξ_α, $\bar{\Xi}^{\dot\alpha}$ and Δ respectively. In the following, we present the component field expressions of the lowest components of Ξ_α, $\bar{\Xi}^{\dot\alpha}$ and Δ.

$$\frac{i}{2}(\bar\psi_m\bar\sigma^m)^\alpha\Xi_\alpha|_{\theta=\bar\theta=0} - \frac{i}{2}\bar{\Xi}_{\dot\alpha}(\bar\sigma^m\psi_m)^{\dot\alpha}|_{\theta=\bar\theta=0}$$
$$= -\frac{1}{8\ell}(\ell g_{(1)} + 1)(\bar{u} + \frac{4}{3}\ell\bar{M})(\psi_m\sigma^{mn}\psi_n)$$
$$-\frac{1}{8\ell}(\ell g_{(1)} + 1)(u + \frac{4}{3}\ell M)(\bar\psi_m\bar\sigma^{mn}\bar\psi_n)$$
$$+\frac{i}{4\ell}(\ell g_{(1)} + 1)(\eta^{mn}\eta^{pq} - \eta^{mq}\eta^{np})(\bar\psi_m\bar\sigma_n\psi_p)\nabla_q\ell$$
$$+\frac{i}{6}(\ell g_{(1)} + 1)\epsilon^{mnpq}(\bar\psi_m\bar\sigma_n\psi_p)e_q{}^a b_a$$
$$-\frac{i}{4\ell}(\ell g_{(1)} + 1)\epsilon^{mnpq}(\bar\psi_m\bar\sigma_n\psi_p)B_q$$
$$-\frac{1}{4}(\mathcal{D}^a\mathcal{D}^\alpha k)\psi_{a\alpha}|_{\theta=\bar\theta=0} - \frac{1}{4}\bar\psi_{a\dot\alpha}(\mathcal{D}^a\mathcal{D}^{\dot\alpha}k)|_{\theta=\bar\theta=0}. \tag{3.27}$$

The way $\Xi_\alpha|_{\theta=\bar\theta=0}$ and $\bar\Xi^{\dot\alpha}|_{\theta=\bar\theta=0}$ are presented in (3.27) will be useful for the computation of (3.14).

$$
\begin{aligned}
\Delta|_{\theta=\bar\theta=0} \\
= \;& -\frac{1}{\ell^2}(\ell^2 g_{(2)} - 1)\nabla^m\ell\,\nabla_m\ell + \frac{1}{\ell^2}(\ell^2 g_{(2)} - 1)B^m B_m \\
& + 4\sum_I \frac{1}{(t^I + \bar t^I)^2}\nabla^m\bar t^I\,\nabla_m t^I - \frac{4}{9}(\ell^2 g_{(2)} - \ell g_{(1)} - 2)\bar M M \\
& + \frac{4}{9}(\ell^2 g_{(2)} + 2\ell g_{(1)} + 1)b^a b_a - 4\sum_I \frac{1}{(t^I + \bar t^I)^2}\bar F^I_{\bar T} F^I_T \\
& - \frac{4}{3\ell}(\ell^2 g_{(2)} + \ell g_{(1)})B^m e_m{}^a b_a - \frac{1}{2\ell}(\ell g_{(1)} + 1)(F_U + \bar F_{\bar U}) \\
& - \frac{1}{6\ell}(2\ell^2 g_{(2)} - \ell g_{(1)} - 3)(u\bar M + \bar u M) - \frac{1}{4\ell^2}(\ell^2 g_{(2)} - 1)\bar u u \\
& + 2\nabla^m\nabla_m k - (\mathcal{D}^a\mathcal{D}^\alpha k)\psi_{a\alpha}|_{\theta=\bar\theta=0} - \bar\psi_{a\dot\alpha}(\mathcal{D}^a\mathcal{D}^{\dot\alpha} k)|_{\theta=\bar\theta=0}. \quad (3.28)
\end{aligned}
$$

It is unnecessary to decompose the last two terms in (3.27) and in (3.28) because they eventually cancel with one another.

Eqs.(3.15-28) describe the key steps involved in the computation of (3.14). The rest of it is standard and will not be detailed here. In the following, we present the component field expression of $\mathcal{L}_{eff}$ as the sum of the bosonic part $\mathcal{L}_B$ and the gravitino part $\mathcal{L}_{\bar G}$ as follows.[‡]

$$
\mathcal{L}_{eff} = \mathcal{L}_B + \mathcal{L}_{\bar G}. \quad (3.29)
$$

$$
\begin{aligned}
\frac{1}{e}\mathcal{L}_B = \;& -\frac{1}{2}\mathcal{R} - \frac{1}{4\ell^2}(\ell g_{(1)} + 1)\nabla^m\ell\,\nabla_m\ell \\
& + \frac{1}{4\ell^2}(\ell g_{(1)} + 1)B^m B_m - (1 + b\ell)\sum_I \frac{1}{(t^I + \bar t^I)^2}\nabla^m\bar t^I\,\nabla_m t^I \\
& + \frac{1}{9}(\ell g_{(1)} - 2)\bar M M - \frac{1}{9}(\ell g_{(1)} - 2)b^a b_a
\end{aligned}
$$

[‡]Only the bosonic and gravitino parts of the component field expressions are presented here.

15

$$+ (1 + b\ell) \sum_I \frac{1}{(t^I + \bar{t}^I)^2} \bar{F}_T^I F_T^I$$

$$+ \frac{1}{8\ell} \{ f + 1 + b\ell \ln(e^{-k} \bar{u} u / \mu^6) + 2b\ell \} (F_U + \bar{F}_{\bar{U}})$$

$$- \frac{1}{8\ell} \{ f + 1 + b\ell \ln(e^{-k} \bar{u} u / \mu^6) + \frac{2}{3} b\ell(\ell g_{(1)} + 1) \} (u \bar{M} + \bar{u} M)$$

$$- \frac{1}{16\ell^2} (1 + 2b\ell)(\ell g_{(1)} + 1) \bar{u} u$$

$$- \frac{i}{2} b \ln(\frac{\bar{u}}{u}) \nabla^m B_m - \frac{i}{2} b \sum_I \frac{1}{(t^I + \bar{t}^I)} (\nabla^m \bar{t}^I - \nabla^m t^I) B_m. \quad (3.30)$$

$$\begin{aligned}
\frac{1}{e} \mathcal{L}_{\bar{G}} &= \frac{1}{2} \epsilon^{mnpq} (\bar{\psi}_m \bar{\sigma}_n \nabla_p \psi_q - \psi_m \sigma_n \nabla_p \bar{\psi}_q) \\
&\quad - \frac{1}{8\ell} \{ f + 1 + b\ell \ln(e^{-k} \bar{u} u / \mu^6) \} \bar{u} (\psi_m \sigma^{mn} \psi_n) \\
&\quad - \frac{1}{8\ell} \{ f + 1 + b\ell \ln(e^{-k} \bar{u} u / \mu^6) \} u (\bar{\psi}_m \bar{\sigma}^{mn} \bar{\psi}_n) \\
&\quad - \frac{1}{4} (1 + b\ell) \sum_I \frac{1}{(t^I + \bar{t}^I)} \epsilon^{mnpq} (\bar{\psi}_m \bar{\sigma}_n \psi_p) (\nabla_q \bar{t}^I - \nabla_q t^I) \\
&\quad + \frac{i}{4\ell} (1 + b\ell)(\ell g_{(1)} + 1)(\eta^{mn} \eta^{pq} - \eta^{mq} \eta^{np})(\bar{\psi}_m \bar{\sigma}_n \psi_p) \nabla_q \ell \\
&\quad - \frac{i}{4} b\ell (\eta^{mn} \eta^{pq} - \eta^{mq} \eta^{np})(\bar{\psi}_m \bar{\sigma}_n \psi_p) \nabla_q \ln(\bar{u} u) \\
&\quad + \frac{1}{4} b\ell \, \epsilon^{mnpq} (\bar{\psi}_m \bar{\sigma}_n \psi_p) \nabla_q \ln(\frac{\bar{u}}{u}). \quad (3.31)
\end{aligned}$$

For completeness, we also give the definitions of covariant derivatives:

$$\begin{aligned}
\nabla_m \ell &= \partial_m \ell, \quad \nabla_m t^I = \partial_m t^I, \quad \nabla_m \bar{t}^I = \partial_m \bar{t}^I, \\
\nabla_m \psi_n{}^\alpha &= \partial_m \psi_n{}^\alpha + \psi_n{}^\beta \omega_{m\beta}{}^\alpha, \quad \nabla_m \bar{\psi}_{n\dot{\alpha}} = \partial_m \bar{\psi}_{n\dot{\alpha}} + \bar{\psi}_{n\dot{\beta}} \omega_m{}^{\dot{\beta}}{}_{\dot{\alpha}}. \quad (3.32)
\end{aligned}$$

To proceed further, we need to eliminate the auxiliary fields from $\mathcal{L}_{eff}$ through their equations of motion. The equation of motion of the auxiliary field $(F_U + \bar{F}_{\bar{U}})$ is

$$f + 1 + b\ell \ln(e^{-k} \bar{u} u / \mu^6) + 2b\ell = 0. \quad (3.33)$$

Eq. (3.33) implies that in static models the auxiliary field $\bar{u}u$ is expressed in terms of dilaton ℓ. The equations of motion of F_T^I, $\bar{F}_{\bar{T}}^I$ and the auxiliary fields b^a, M, $\bar{M}$ of the supergravity multiplet are (if $\ell g_{(1)} - 2 \neq 0$)

$$
\begin{aligned}
F_T^I &= 0, \quad \bar{F}_{\bar{T}}^I = 0, \\
b^a &= 0, \\
M &= \frac{3}{4}bu, \quad \bar{M} = \frac{3}{4}b\bar{u}.
\end{aligned}
\tag{3.34}
$$

Now we are left with only one auxiliary field to eliminate, where this auxiliary field can be either $i\ln(\bar{u}/u)$ or B_m. This corresponds to the fact that there are two ways to perform duality transformation. If we take $i\ln(\bar{u}/u)$ to be auxiliary, its equation of motion is

$$
\nabla_q\{ B^q - \frac{i}{2}\ell\,\epsilon^{mnpq}(\bar{\psi}_m\bar{\sigma}_n\psi_p)\} = 0,
\tag{3.35}
$$

which ensures that $\{B^q - \frac{i}{2}\ell\epsilon^{mnpq}(\bar{\psi}_m\bar{\sigma}_n\psi_p)\}$ is dual to the field strength of an antisymmetric tensor [6]. The term $B^m B_m$ in the lagrangian $\mathcal{L}_{eff}$ thus generates a kinetic term of this antisymmetric tensor field and its coupling to the gravitino. The other way to perform the duality transformation is to treat B_m as an auxiliary field by rewriting the term $-\frac{i}{2}b\ln(\bar{u}/u)\nabla^m B_m$ in $\mathcal{L}_{eff}$ as $\frac{i}{2}bB^m\nabla_m\ln(\bar{u}/u)$, and then to eliminate B_m from $\mathcal{L}_{eff}$ through its equation of motion as follows:

$$
\begin{aligned}
B_m &= -i\frac{b\ell^2}{(\ell g_{(1)} + 1)}\nabla_m\ln(\frac{\bar{u}}{u}) \\
&\quad + i\frac{b\ell^2}{(\ell g_{(1)} + 1)}\sum_I \frac{1}{(t^I + \bar{t}^I)}(\nabla_m\bar{t}^I - \nabla_m t^I).
\end{aligned}
\tag{3.36}
$$

The terms $B^m B_m$ and $\frac{i}{2}bB^m\nabla_m\ln(\bar{u}/u)$ in $\mathcal{L}_{eff}$ will generate a kinetic term for $i\ln(\bar{u}/u)$. It is clear that $i\ln(\bar{u}/u)$ plays the role of the pseudoscalar dual to B_m in the lagrangian obtained from the above after a duality transformation. With (3.33-36), it is then trivial to eliminate the auxiliary fields from $\mathcal{L}_{eff}$. The physics of $\mathcal{L}_{eff}$ will be investigated in the following sections.

3.3 Gaugino Condensate and the Gravitino Mass

Hidden-sector gaugino condensation in superstring effective theories is a very attractive scheme [24, 25] for supersymmetry breaking. However, before we can make any progress in phenomenology, two important questions must be answered: is supersymmetry broken, and is the dilaton stabilized? Past analyses have generally found that, in the absence of a second source of supersymmetry breaking, the dilaton is destabilized in the direction of vanishing gauge coupling (the so-called runaway dilaton problem) and supersymmetry is unbroken. To address the above questions in generic linear multiplet models of gaugino condensation, we first show how the three issues of supersymmetry breaking, gaugino condensation and dilaton stabilization are reformulated, and how they are interrelated, by examining the explicit expressions for the gravitino mass and the gaugino condensate. A detailed investigation of the vacuum will be presented in the following section.

The explicit expression for the gaugino condensate in terms of the dilaton ℓ is determined by (3.33):

$$\bar{u}u = \frac{1}{e^2}\ell\mu^6 e^{g-(f+1)/b\ell}. \tag{3.37}$$

With $g(\ell)=0$ and $f(\ell)=0$, we recover the result of the simple model (3.2) [6]. For generic models, the dilaton dependence of the gaugino condensate involves $g(\ell)$ and $f(\ell)$ which represent quantum corrections to the tree-level Kähler potential. According to our assumption of boundedness for $g(\ell)$ and $f(\ell)$ (especially at $\ell=0$ where following (3.12) we have the boundary conditions $g(\ell=0)=0$ and $f(\ell=0)=0$), $\ell=0$ is the only pole of $g-(f+1)/b\ell$. Therefore, we can draw a simple and clear relation between $\langle \bar{u}u \rangle$ and $\langle \ell \rangle$: gauginos condense (i.e., $\langle \bar{u}u \rangle \neq 0$) if and only if the dilaton is stabilized (i.e., $\langle \ell \rangle \neq 0$.)

Another physical quantity of interest is the gravitino mass $m_{\tilde{G}}$ which is the natural order parameter measuring supersymmetry breaking. The expression

18

for $m_{\tilde{G}}$ follows directly from $\mathcal{L}_{\tilde{G}}$.

$$m_{\tilde{G}} = \frac{1}{4}b\sqrt{\langle \bar{u}u \rangle},\qquad(3.38)$$

where we have used (3.33). This expression for the gravitino mass is simple and elegant even for generic linear multiplet models. From the viewpoint of superstring effective theories, an interesting feature of (3.38) is that the gravitino mass $m_{\tilde{G}}$ contains no dependence on the modulus T^I, which provides a direct relation between $m_{\tilde{G}}$ and $\langle \bar{u}u \rangle$. This feature can be traced to the fact that the Green-Schwarz counterterm cancels the T^I dependence of the superpotential completely, a unique feature of the linear multiplet formulation. We recall that, in the chiral formulations of gaugino condensation studied previously (with or without the Green-Schwarz cancellation mechanism), $m_{\tilde{G}}$ always involves a moduli-dependence, and therefore the relation between supersymmetry breaking (i.e., $m_{\tilde{G}} \neq 0$) and gaugino condensation (i.e., $\langle \bar{u}u \rangle \neq 0$) remains undetermined until the true vacuum can be found. By contrast, in generic linear multiplet models of gaugino condensation, there is a simple and direct relation, Eq.(3.38): supersymmetry is broken (i.e., $m_{\tilde{G}} \neq 0$) if and only if gaugino condensation occurs ($\langle \bar{u}u \rangle \neq 0$). We wish to emphasize that the above features of the linear multiplet model are unique in the sense that they are simple only in the linear multiplet model. This is related to the fact pointed out in Sect. 1 that, once the constraint (2.9) on the condensate field U is imposed, the chiral counterpart of the linear multiplet model is in general very complicated, and it is more natural to work in the linear formulation. Our conclusion of this section is best illustrated by the following diagram:

Supersymmetry Breaking	$\Longleftrightarrow$	Gaugino Condensation	$\Longleftrightarrow$	Stabilized Dilaton
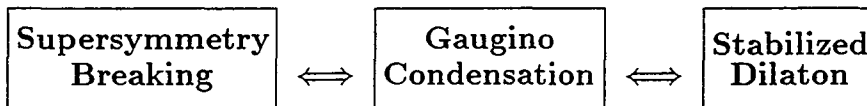
The equivalence among the above three issues is obvious. Therefore, in the following section, we only need to focus on one of the three issues in the

investigation of the vacuum, for example, the issue of dilaton stabilization.

4 Supersymmetry Breaking, Gaugino Condensation and the Stabilization of the Dilaton

As argued in Sect. 3.1, nonperturbative contributions to the Kähler potential should be introduced to cure the unboundedness problem of the simple model (3.2). In the context of the generic model (3.10), it is therefore interesting to address the question as to how the simple model should be modified in order to obtain a viable theory (i.e., with V_{pot} bounded from below). We start with the scalar potential V_{pot} arising from (3.30) after solving for the auxiliary fields (using (3.33), (3.34) and (3.37)). Recalling that (3.11) yields the identity $\ell g_{(1)} + 1 = 1 + f - \ell f_{(1)}$, we obtain

$$V_{pot} = \frac{1}{16e^2\ell}\{ (1 + f - \ell f_{(1)})(1 + b\ell)^2 - 3b^2\ell^2 \}\mu^6 e^{g - (f+1)/b\ell}, \qquad (4.1)$$

which depends only on the dilaton ℓ. The necessary and sufficient condition for V_{pot} to be bounded from below is

$$f - \ell f_{(1)} \geq -\mathrm{O}(\ell e^{1/b\ell}) \quad \text{for} \quad \ell \to 0, \qquad (4.2)$$

$$f - \ell f_{(1)} \geq 2 \qquad \text{for} \quad \ell \to \infty. \qquad (4.3)$$

It is clear that condition (4.2) is not at all restrictive, and therefore has no nontrivial implication. On the contrary, condition (4.3) is quite restrictive; in particular the simple model violates this condition. Condition (4.3) not only restricts the possible forms of the function f in the strong-coupling regime but also has important implications for dilaton stabilization and for supersymmetry breaking. To make the above statement more precise, let us revisit the unbounded potential of Fig.1, with the tree-level Kähler potential defined by $g(V) = f(V) = 0$. Adding physically reasonable corrections $g(V)$ and $f(V)$ (constrained by (4.2-3)) to this simple model should not

qualitatively alter its behavior in the weak-coupling regime. Therefore, as in Fig.1, the potential of the modified model in the weak-coupling regime starts with $V_{pot} = 0$ at $\ell = 0$, first rises and then falls as ℓ increases. On the other hand, adding $g(V)$ and $f(V)$ completely alters the strong-coupling behavior of the original simple model. As guaranteed by condition (4.3), the potential of the modified model in the strong-coupling regime is always bounded from below, and in most cases rises as ℓ increases. Joining the weak-coupling behavior of the modified model to its strong-coupling behavior therefore strongly suggests that its potential has a non-trivial minimum (at $\ell \neq 0$). Furthermore, if this non-trivial minimum is global, then the dilaton is stabilized. We conclude that not only does (4.2-3) tell us how to modify the theory, but a large class of theories so modified have naturally a stabilized dilaton (and therefore broken supersymmetry by the argument of Sect. 3.3). In view of the fact that there is currently little knowledge of the exact Kähler potential, the above conclusion, which applies to generic Kähler potentials subject to (4.2-3), is especially important to the search for supersymmetry breaking and dilaton stabilization. Though we are unable to study the exact Kähler potential at present, it is nevertheless interesting to study models with reasonable Kähler potentials for the purpose of illustrating the significance of condition (4.2-3) as well as displaying explicit examples with supersymmetry breaking. This will be done in the following example.

We start with the consideration of possible nonperturbative contributions to the Kähler potential. Aside from the Planck scale M_P, the only natural mass scale in the theory is the condensation scale Λ_c, that is, the scale at which the hidden-sector gauge interaction becomes strong. As is well known, it follows from the renormalization group equation for the running of the gauge coupling that Λ_c depends exponentially on the dilaton ℓ as $\Lambda_c \sim e^{-1/6b\ell}$, which is consistent with the results of the simple model in Sect. 3.1. Therefore, on dimensional grounds, the field-theoretical nonperturbative contribution to the Kähler potential has the generic form $V^{-m}e^{-n/6bV}/M_P^{n-2}$ ($M_P=1$ in our convention), where $n \geq 2$ and $m \geq 0$ [21].

21

In the following example, we consider the leading-order nonperturbative contribution ($n = 2$ and $m = 0$) to the Kähler potential:

$$f(V) = A_f e^{-1/3bV}, \tag{4.4}$$

where A_f is a constant to be determined by the nonperturbative dynamics. The regulation conditions (4.2-3) require $A_f \geq 2$. In Fig. 2, V_{pot} is plotted versus the dilaton ℓ, where $A_f = 6.92$ and $\mu=1$. Fig. 2 has two important features. First, V_{pot} of this modified theory is indeed bounded from below, and the dilaton is stabilized. Therefore, we obtain supersymmetry breaking, gaugino condensation and dilaton stabilization in this example. The gravitino mass is $m_{\tilde{G}} = 7.6 \times 10^{-5}$ in Planck units. Secondly, the *vev* of dilaton is stabilized at the phenomenologically interesting range ($\langle \ell \rangle = 0.45$ in Fig. 2). Furthermore, the above features involve no unnaturalness since they are insensitive to A_f. Fig. 2 is a nice realization of the argument in the preceding paragraph. It should be contrasted with the racetrack models where at least three gaugino condensates and large numerical coefficients are needed in order to achieve similar results. We can also consider possible stringy nonperturbative contributions to the Kähler potential suggested in [22]. It turns out that we obtain the same general features as those of Fig. 2. This is not surprising since, as argued in the preceding paragraph, the important features that we find in Fig. 2 are common to a large class of models.

Note that the value of the cosmological constant is irrelevant to the arguments presented here and in Sect. 3.3. In other words, the generic model (3.10) suffers from the usual cosmological constant problem, although we can find a fine-tuned subset of models whose cosmological constants vanish. For example, the cosmological constant of Fig. 2 vanishes by fine tuning A_f. It remains an open question as to whether or not the cosmological constant problem could be resolved within the context of the linear multiplet formulation of gaugino condensation if the exact Kähler potential were known.

5 Concluding Remarks

We have presented a concrete example of a solution to the infamous runaway dilaton problem, within the context of local supersymmetry and the linear multiplet formulation for the dilaton. We considered models for a static condensate that reflect the modular anomaly of the effective field theory while respecting the exact modular invariance of the underlying string theory. The simplest such model [6, 7] has a nontrivial potential that is, however, unbounded in the direction of strong coupling. Including nonperturbative corrections [21, 22] to the Kähler potential for the dilaton, the potential is stabilized, allowing a vacuum configuration in which condensation occurs and supersymmetry is broken. This is in contrast to previous analyses, based on the chiral formulation for the dilaton, in which supersymmetry breaking with a bounded vacuum energy was achieved only by introducing an additional source of supersymmetry breaking, such as a constant term in the superpotential [20, 25, 27].

In further contrast to most chiral models studied, supersymmetry breaking arises from a nonvanishing vacuum expectation value of the auxiliary field associated with the dilaton rather than the moduli: roughly speaking, in the dual chiral formulation, $\langle F_S \rangle \neq 0$ rather than $\langle F_T^I \rangle \neq 0$. As a consequence, gaugino masses and A-terms are generated at tree level. Although scalar masses are still protected at tree level by a Heisenberg symmetry [26], they will be generated at one loop by renormalizable interactions. For the model considered here, the hierarchy (about five orders of magnitude) between the Planck scale and the gravitino mass is insufficient to account for the observed scale of electroweak symmetry breaking. A possible avenue for improving this result is to consider multiple gaugino condensation; in realistic orbifold compactifications the hidden gauge group $\mathcal{G}$ is in general a product group: $\mathcal{G} = \Pi_a \mathcal{G}_a$. The generalization of our formalism to the multi-condensate case will be considered elsewhere.

The Kalb-Ramond field (or the axion, in the dual description) remains massless in the static models considered here, and therefore we still need to explain how the axion mass can be generated. It has recently been shown in the context of global supersymmetry [6] that a mass term for the axion is naturally generated if kinetic terms for U and $\bar{U}$ are included. It is therefore worth studying the extension of this paper to the nonstatic case. Consider the following generic linear multiplet model with a single dynamical condensate:

$$
\begin{aligned}
K &= \ln V + g(V, \bar{U}U) + G, \\
\mathcal{L}_{eff} &= \int d^4\theta \, E\{(-2 + f(V, \bar{U}U)) + bVG + bV\ln(e^{-K}\bar{U}U/\mu^6)\}. \quad (5.1)
\end{aligned}
$$

The model defined by (5.1) is a straightforward generalization of (3.10), where the quantum corrections to the Kähler potential, g and f, are now taken to be functions of $\bar{U}U$ as well as of V. The construction of the component lagrangian for the nonstatic model (5.1) is similar to that for the static model (3.10) presented in Sect. 3.2. For example, the condition for a canonical Einstein term for the generic nonstatic model turns out to be:

$$
(1 + Z\frac{\partial f}{\partial Z})(1 + V\frac{\partial g}{\partial V}) = (1 - Z\frac{\partial g}{\partial Z})(1 - V\frac{\partial f}{\partial V} + f), \quad (5.2)
$$

where $Z \equiv \bar{U}U$. It is clear that (3.11) is the static limit of (5.2), where g and f are independent of $\bar{U}U$. As suggested by terms that arise both from string corrections [28] at the classical level and from field-theoretical loop corrections [29], we have studied the nonstatic model with generic functions g and f that are s-duality invariant in the sense defined in [7]. That is, g and f are functions only of the s-duality invariant superfield variable $\bar{U}U/V^2$. It turns out that the scalar potential V_{pot} of the nonstatic model with s-duality invariance is always unbounded from below in the strong-coupling limit $\ell \to \infty$. The origin of this unboundedness problem is similar to that of the simple static model studied in Sect. 3.1, and again it reflects the absence of nonperturbative contributions to the Kähler potential. We expect that the unboundedness problem of the nonstatic model will be cured when

24

nonperturbative contributions to the Kähler potential are included. Studies along this line are in progress.

Acknowledgements

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FIGURE CAPTIONS

Fig.1: The scalar potential V_{pot} (in Planck units) is plotted versus the dilaton ℓ. $\mu=1$.

Fig.2: The scalar potential V_{pot} (in Planck units) is plotted versus the dilaton ℓ. $A_f = 6.92$ and $\mu=1$.
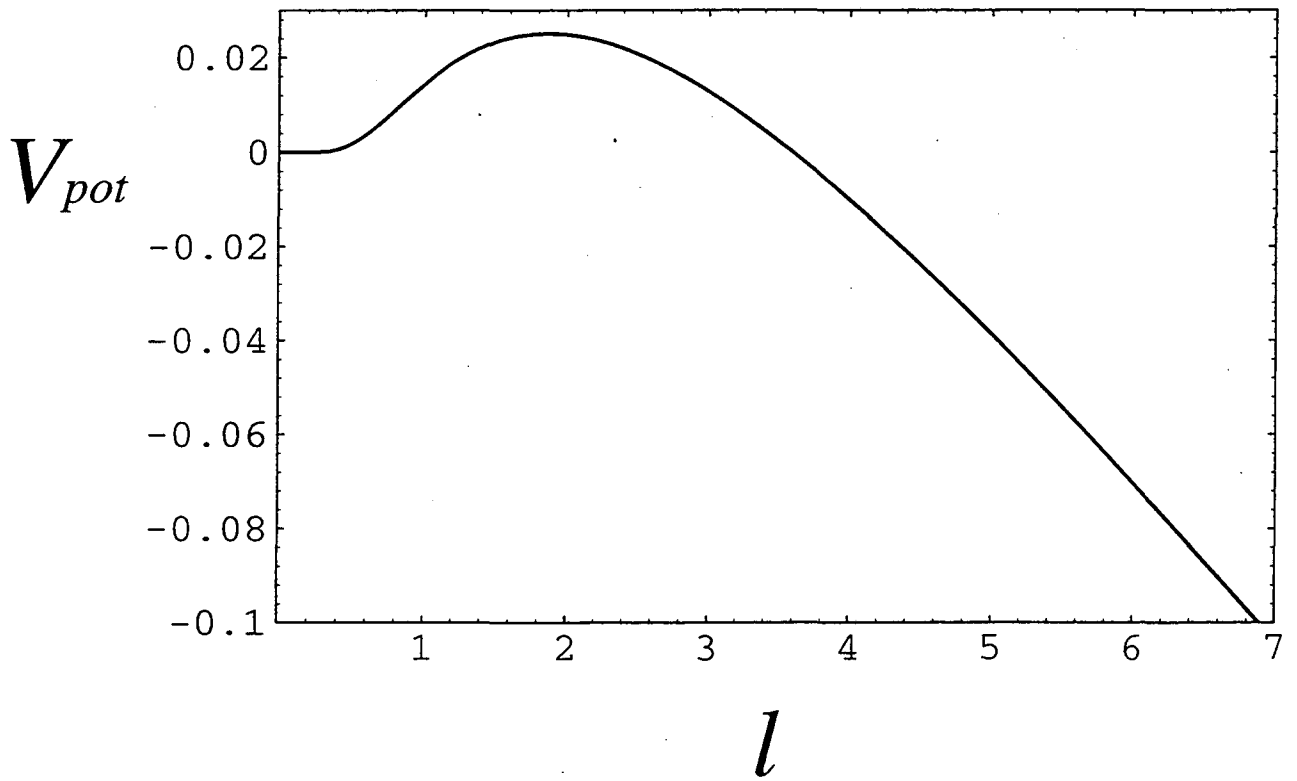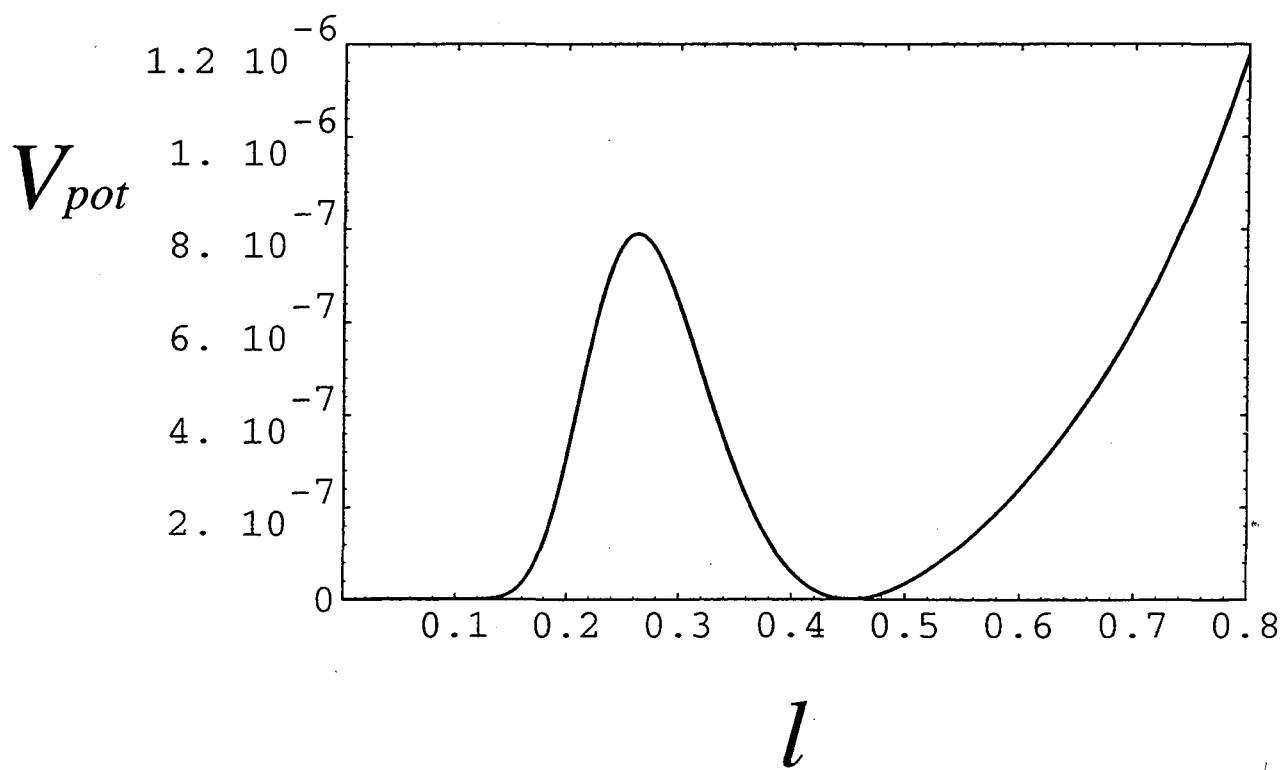
Fig.1

Fig.2