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Author
Green, Michael A.

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University of California

Ernest O. Lawrence Radiation Laboratory

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Michael A. Green

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Abstract

In 1965 Z. J. J. Stekly and J. L. Zar demonstrated that superconducting coils could be stabilized by the use of a substrate. The key to this stabilization technique is a local heat balance between heat generated in the substrate when the superconductor goes normal and heat removed.

The current density within a superconductor is a function of local magnetic field and local temperature. The local temperature is a function of heat transfer to the helium when the superconductor goes normal. The heat transfer from the superconductor to the fluid boundary is analyzed. The validity of basic assumptions often made while designing a stabilized system is discussed. The convective heat transfer to the liquid helium is analyzed. Included in this analysis is a discussion of single-phase pressurized helium cooling.

One important consideration that is often ignored by the designer of stabilized magnets is that a magnet may be stabilized when an additional heat load is being produced in the superconductor substrate combination. The external source of heat may be from interaction with an accelerated beam, natural radioactive decay, eddy currents generated by changing fields, or hysteresis-type losses. The additional heat loads must be removed from the superconductor substrate combination in order for stable superconducting operation to be maintained.

Several methods for improving the heat transfer from the superconductor to the helium are discussed. The development of ac magnets and improvements in substrate current density for large high-field dc magnets can result from substantial improvements in the heat transfer.

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The Basic Requirements for Stability

Since the publication of the Stekly reports on the stabilization of superconductors in 1965,[1,2] there has been a great deal of controversy associated with the subject of superconductor-substrate stabilization. Other authors have pointed out that the critical temperature of the superconductor does not influence the stability of the superconductor.[3] Brechna[4,5] has explained stability by the use of a steady wave-propagation model. This paper is an attempt to clear the controversy concerning the stabilization of superconductors.

The steady-state stability parameters for superconductor substrate combinations with outside heat loads presented here summarize the results of stability and heat-transfer studies at Lawrence Radiation Laboratory—Berkeley.[6,7] Stability parameters are presented for systems which have nucleate-boiling heat transfer and supercritical forced-convection heat transfer from the substrate to the bath. The effect of thermal resistance in the solid phase is discussed in detail.

The definition of stability that will be used in this paper is the short-sample current stability first described by Stekly. The stability parameter α is defined as the ratio of heat generated by normal currents to the maximum possible heat transfer to the bath. The basic stability equation given by Stekly for stability at a particular magnetic field H_0 and a bath temperature T_b (see Fig. 1) is

$$\alpha = \frac{I_b^2 R}{h_T A_T (T_{sc} - T_b)} ,$$

where T_{sc} is the superconductor critical temperature at a field H_0 (see Fig. 1), T_b is the bath temperature, R is the resistance (including magneto resistance) of the substrate per unit length, A_T is the area per unit length of heat transfer to the bath, h_T is an effective heat transfer coefficient from the superconductor to the bath, and I_b is the critical current carried by the superconductor at a field H_0 and a temperature T_b (see Fig. 1).

When $\alpha \leqslant 1$ the superconductor-substrate combination is stabilized at the superconductor short-sample current. When $\alpha > 1$ two regions can exist. When $I_\delta < I < I_b$ the superconductor substrate combination is not stable and normal regions propagate. When $I < I_\delta$ the superconductor is stabilized but at currents below the short-sample current. I_δ is the recovery current for the superconductor substrate combination. It is defined by

$$I_\delta = I_b / \sqrt{\alpha} \quad \text{for } \alpha > 1 ,$$

$$I_\delta = I_b \quad \text{for } \alpha \leqslant 1 .$$

When $I = I_\delta$ the propagation velocity is zero for $\alpha \geqslant 1$ and negative for $\alpha < 1$. It is important to consider the assumptions used to derive the basic Stekly equation. These assumptions are as follows: (a) All the normal current flows in the substrate. (b) Transient solutions are neglected. (c) The heat transfer to the bath follows a simple $hc\Delta T$ rule and all the temperature difference between T_{sc} and T_b is available for heat transfer. (d) There is no heating due to an external source (eddy currents in the substrate, interaction with charged particles, external heat leak, etc.). (e) The temperature dependency of normal-region resistivity and thermal conductivity is not considered. (f) The electrical and thermal resistance between the superconductor and the substrate is negligible.

This paper deals primarily with assumptions (c) and (d). The stability parameter given in the paper applies for nucleate-boiling heat transfer (where assumption (c) does not usually apply) and for forced-convection heat transfer (where assumption (c) does apply). The stability parameters presented here also apply when there is a continuous source of heat other than simple Joule heating due to normal currents. The outside heating may be due to (a) ac losses in the superconductor; (b) eddy currents; (c) interaction with an accelerated beam; or (d) external heat leaks through leads, etc.

The Basic Heat-Transfer Relations

Two basic types of heat transfer from the substrate-superconductor combination to the bath can be considered. They are nucleate-boiling heat transfer and single-phase forced-convection heat transfer. The heat flow from the fluid boundary to the fluid must be balanced by heat flow in the solid phase.

Today most superconducting systems are cooled by nucleate boiling of liquid helium. The heat transferred by boiling and the temperature drop from the wall to the bath are limited by the transition from nucleate to film boiling. In general nucleate boiling behaves according to

$$\Phi_b = M\,(T_w - T_b)^n\ ,$$

where Φ_b is the surface heat flux, T_w is the wall temperature, T_b is the bath temperature, and M is a boiling coefficient which is a function of pressure. A value $n = 2.5$ has been suggested by Kutateladze.[8] For this study I suggest $n = 3$ because it fits the experimental evidence better and will give larger values $T_w - T_b$. Nucleate-boiling heat flux is at its peak at about 0.8 atm ambient pressure. Table I gives values for T_b, the maximum nucleate boiling flux $\Phi_{b\,max}$, the maximum temperature difference $(T_w - T_b)_{max}$, and the boiling coefficient M as a function of ambient pressure.

Table I. Important nucleate boiling parameters for ambient pressures of 0.5 atm, 1.0 atm, and 1.5 atm; n = 3.

P_b (atm)	T_b (°K)	$\Phi_{b\,max}$ (W/cm^2)	$(T_w - T_b)_{max}$ (°K)	M [W/cm^2(°K)^3]
0.5	3.56	0.8	0.8	1.56
1.0	4.21	0.9	0.5	7.20
1.5	4.66	0.7	0.3	25.9

The nucleate-boiling relations given in Table I represent ideal conditions. The maximum nucleate-boiling heat flux decreases if the boiling takes place in confined channels.[3, 9] Boiling heat fluxes within coils will be of the order of 0.2 to 0.5 W/cm^2. Table II gives the temperature drop as a function of boiling heat flux. Table II will be useful for estimating boiling heat fluxes and temperature drops for the stabilizing parameters given in the next section. Boiling temperature drops exceeding the maximum values given in Tables I and II will result in a reduction of boiling heat flux until a minimum boiling flux of 0.18 to 0.2 W/cm^2 is reached, when $(T_w - T_b)$ is about 6.0 °K.[7, 8]

If liquid helium is pressurized above its critical pressure of 2.2 atm, the heat can be removed from the superconductor by single-phase forced convection.[10, 11] The surface heat flux is given by the following relations for fluid flow in tubes:[12]

$$\Phi_c = h_c\,(T_w - T_a)\ ,$$

$$h_c = \frac{k}{D}\,N_{U_D}\ ,$$

$$N_{U_D} = C\,Re_D^{0.8}\,Pr^{0.33}\ ,$$

where Φ_c is the convective heat flux, h_c is the convective heat transfer coefficient, k the thermal conductivity of the fluid, T_a the recovery temperature,

Table II. Boiling temperature drop (T_w - T_b) as a function of boiling heat flux for pressures of 0.5, 1.0 and 1.5 atm, n = 3.

Φ_b	Temperature drop (T_w - T_b)		
	0. atm	1.0 atm	1.5 atm
0.01	0.185	0.112	0.073
0.02	0.232	0.140	0.092
0.05	0.316	0.192	0.124
0.10	0.400	0.240	0.156
0.20	0.505	0.304	0.198
0.40	0.635	0.375	0.250
0.60	0.727	0.435	0.285
$\Phi_{b\ max}$	0.800	0.500	0.300

$T_a > T_b$ because of viscous dissipation, Nu_D is the nusults number based on the tube diameter D, Re_D is the Reynolds number based on D, and Pr is the Prandtl number. The Reynolds number and Prandtl number are based on the temperature T_a:

$$T_a = T_b + \sqrt{Pr}\ V_x^2/2C_p\ .$$

The $\sqrt{Pr}$ relation for recovery temperature given by Schlichting[13] is one of many proposed by a number of authors. V_x is the fluid velocity and C_p the fluid specific heat. $T_a - T_b$ is less than 0. 1°K for fluid velocities less than 25 meters per second, hence $T_a = T_b$ is assumed. The value of C measured by Brechna[5] is 0.04 greater than the 0.023 value suggested by most heat-transfer books.[7,12,13] Kolm[11] suggests that the helium be pressurized above 20 atm to avoid instabilities due to large changes in density of the helium with changes in temperature.

Heat-transfer coefficients of 2.0 W/cm°K should be easily obtainable if forced-convection heat transfer is used. The temperature drops of 10 °K could be obtained in Nb_3Sn superconducting systems. Local heat flux of the order of 50 W/cm^2 could be achieved in short high-pressure cooling circuits.

Heat transfer in the solid phase can be represented by the normal conduction relation and by an $h\Delta T$ relation,

$$\Phi_{con} = K_c \frac{\Delta T}{\Delta x} = h\Delta T\ ,$$

$$h = \frac{K_c}{\Delta x}\ .$$

For the substrate the heat transfer coefficient h_{s1} is

$$h_{s1} = \frac{2K_{s1}\,\beta_{s1}}{t_{s1}}$$ when heat is generated in the substrate,

$$h_{s1} = \frac{K_{s1}\,\beta_{s1}}{t_{s1}}$$ when heat is not generated in the substrate,

where K_{s1} is the thermal conductivity of the substrate, t_{s1} is the thickness of the substrate between the superconductor and the bath, and β_{s1} is a geometric factor; β_{s1} is unity for a flat slab or thin-walled tube. For copper K_{s1} = 3 W/cm °K at 4°K. A thin insulating layer between the substrate and the bath will have a heat-transfer coefficient of

$$h_{1w} = \frac{K_{1w}\,\beta_{1w}}{t_{1w}}\ .$$

The definitions of K, β, and t are the same except they apply to the insulation instead of the substrate. A typical value of K_{1w} for Mylar is 2.5×10^{-4} W/cm °K.

Even a small thickness of insulation, for which $\beta_{1w} = 1$, reduces the heat transfer coefficient h_{1w} by a large amount. The heat-transfer coefficient (h_{sw}) for the solid material between the superconductor and the bath can be expressed in terms of h_{s1} and h_{1w},

$$\frac{1}{h_{sw}} = \frac{1}{h_{s1}} + \frac{1}{h_{1w}} .$$

For a forced convection the heat transfer from the superconductor to the bath can be calculated by using the total heat transfer coefficient h_T suggested by Stekly,

$$\frac{1}{h_T} = \frac{1}{h_c} + \frac{1}{h_{sw}} .$$

<u>Stabilization of Superconductor with External Heat Loads</u>

Superconductors can be stabilized when outside heat loads are being deposited in them.[6] This term of stabilization can be applied to ac superconductors and to superconductors which receive heating from an accelerated beam. The outside heat load can be represented as an additional term in the basic Stekly stability equation, which assumes the form

$$\alpha = \frac{I_s^2 R + Q_{gen}}{h_T A_T (T_{sc} - T_b)} ,$$

for

$$T_b < T_s < T_{sc} \quad \text{and} \quad I_b > I_s > 0 .$$

Here I_s is the short-sample current for a superconductor at a field H_0 and a temperature T_s (see Fig. 1), T_s is the temperature of the superconductor when all the current flows in the superconductor and the heat transferred to the bath comes from the external source. We can define

$$T_s = T_b + \frac{Q_{gen}}{h_T A_T} ,$$

when

$$Q_{gen} \to 0, \quad T_s \to T_b, \quad I_s \to I_b ,$$

Q_{gen} is the external heat load per unit length of superconductor, h_T is the total heat transfer coefficient (a concept which applies only to forced flow), A_T is the heat transfer area per unit length to the bath, and R is the resistance per unit length of the substrate (including magneto resistance).

Stabilized operation at short-sample current occurs when $\alpha \leqslant 1$. All the current flows in the superconductor when $I \leqslant I_s$. When $I > I_s$ the current is shared between the superconductor and the substrate. When $\alpha > 1$ and $Q_{gen} \geqslant A_T h_T (T_{sc} - T_b)$ there is no superconducting operation. When $\alpha > 1$ and $Q_{gen} < A_T h_T (T_{sc} - T_b)$ two modes of operation can exist. The first mode occurs when $I_s > I > I_\delta$; this is an unstable region and normal regions propagate. The second mode of operation occurs when $I < I_\delta$; the superconductor is stabilized below its short-sample current. I_δ is a recovery current which is defined by

$$I_\delta = \left[\frac{A_T h_T (T_{sc} - T_s)}{R}\right]^{1/2} ,$$

as

$$Q_{gen} \to 0, \quad T_s \to T_b, \quad I_\delta \to \frac{I_b}{\sqrt{\alpha}} ,$$

The basic stability equation takes two forms when boiling heat transfer occurs at the fluid boundary. The heat transfer in one case is limited by the maximum boiling heat flux at the surface. In the other case the heat transfer is limited by heat flow in the solid. The dividing line between the two cases occurs when the effective heat transfer coefficient h_{sw} from the superconductor to the fluid wall is equal to h_{sw}^*. The value of h_{sw}^* is

$$h_{sw}^* = \frac{\Phi_{b\,max}}{(T_{sc} - T_b) - (T_w - T_b)} ,$$

$$T_w - T_b = \left[\frac{\Phi_{b\,max}}{M}\right]^{1/3} .$$

$\Phi_{b\,max}$ is specified at the maximum value that can occur for the conductor being stabilized. When $h_{sw} > h_{sw}^*$ boiling heat transfer controls and the stability parameter is defined by

$$\alpha = \frac{I_s^2 R + Q_{gen}}{A_T\ \Phi_{b\,max}} . \qquad (1)$$

Stability is independent of the superconductor critical temperature. As $Q_{gen} \rightarrow 0$ and therefore $I_s \rightarrow I_b$, the equation takes the form that has been suggested by Wheatstone[3] and others. When $h_{sw} < h_{sw}^*$ the heat transfer to the liquid helium is controlled by heat transfer in the solid phases. The stability parameter takes the form

$$\alpha = \frac{I_s^2 R + Q_{gen}}{A_T h_{sw}[(T_{sc} - T_b) - (T_w - T_b)]} . \qquad (2)$$

$T_w - T_b$ cannot be calculated directly because the boiling heat flux is an unknown. All that is known is the $\Phi_b < \Phi_{b\,max}$. An iterative technique can be used to calculate both Φ_b and $T_w - T_b$. The boiling heat flux at the surface equals the heat flow in the solid, hence

$$\Phi_b = h_{sw}[(T_{sc} - T_b) - (T_w - T_b)] .$$

For the first iteration assume $T_w - T_b = 0$. Using the calculated value of Φ_b, find $T_w - T_b$ from Table II or from the nucleate-boiling equation and substitute it into the equation above; both Φ_b and $T_w - T_b$ can be found in a couple of iterations. Stability is a function of the superconductor critical temperature in the second case. The addition of insulation to the heat-transfer surface can change the stability parameter from Eq. 1 to Eq. 2. The temperature T_s can be calculated by

$$T_s = T_b + \frac{Q_{gen}}{A_T h_{sw}} + (T_w - T_b) ;$$

here $T_w - T_b$ is based on the Q_{gen}/A_T heat flux at the boiling surface. As $Q_{gen} \rightarrow 0$ $(T_w - T_b) \rightarrow 0$, $T_s \rightarrow T_b$, and $I_s \rightarrow I_b$. The form that the stability equation takes for forced convection is the same as the basic equation

$$\alpha = \frac{I_s^2 R + Q_{gen}}{A_T h_T (T_{sc} - T_b)} . \qquad (3)$$

The full temperature range is available for heat transfer. The value of T_s is found from

$$T_s = \frac{Q_{gen}}{A_T h_T} + T_b .$$

The maximum current density achievable in a superconducting substrate combination will occur when $\alpha = 1$. When $\alpha = 1$ the following equations for maximum current density will apply:

$$\left(\frac{I_s}{A}\right)_{max} = \left[\frac{\Phi_b A_T - Q_{gen}}{\mathrm{Res}\ A}\right]^{1/2}$$ for boiling heat transfer to the liquid helium,

$\Phi_b = \Phi_{b\,max}$ when $h_{sw} \geq h_{sw}^*$. $\Phi_b < \Phi_{b\,max}$ when $h_{sw} < h_{sw}^*$,

$$\left(\frac{I_s}{A}\right)_{max} = \left[\frac{h_T A_T (T_{sc} - T_b) - Q_{gen}}{\mathrm{Res}\ A}\right]^{1/2}$$ for forced convection heat transfer to the liquid helium,

where A is the cross-sectional area of the substrate and Res is the resistivity of the substrate (including magneto resistance).

Table III shows the effect of insulation on heat flux to the bath. A flat slab model is used because the geometric factor $\beta = 1$. There is no external heat load and the Joule heating is generated uniformly in the substrate. The heat flow model is illustrated in Fig. 2.

Table III. Surface heat flux from an insulation-covered flat slab versus insulation thickness (see Fig. 2).

$T_{sc} = 12.0°K$, $T_b = 4.2°K$, $t_{s1} = 1.0$ cm, P = 1 atm,
$K_{s1} = 3$ W/cm°K, $K_{1w} = 2.5 \times 10^{-4}$ W/cm°K.

Mylar insulation thickness t_{1w} (mm)	Nucleate-boiling heat transfer $\Phi_{b\ max} = 0.5$	Forced convection heat transfer		
		$h_c = 0.5$	$h_c = 1.0$	$h_c = 2.0$
0.0	0.5	3.60	6.68	11.7
0.0025	0.5	2.46	3.59	4.68
0.005	0.5	1.87	2.46	2.92
0.01	0.5	1.26	1.51	1.67
0.025	0.5	0.641	0.698	0.731
0.05	0.360	0.352	0.369	0.377
0.10	0.186	0.184	0.189	0.192
0.25	0.077	0.076	0.077	0.078

Heat flux is in W/cm^2, h_c is in $W/cm^2°K$

Table III demonstrates dramatically that insulation at the liquid helium surface has a degrading effect on heat flow from the substrate to the bath. The surface heat flux for nucleate boiling is constant until $h_{sw} = h_{sw}^*$. The decrease in boiling heat flux for the larger insulation thicknesses indicates that $h_{sw} < h_{sw}^*$ and that the surface heat flux is controlled by the heat flow through the insulation. Table III illustrates why thin insulation layers don't affect the stabilization of some conductors in a boiling bath. Table III also illustrates large gains that can be achieved by using forced-convection heat transfer. It should be noted that for liquid helium flow in 5-mm tubes h_c's of 0.5 W/cm^2 °K, 1.0 W/cm^2 °K, and 2.0 W/cm^2 °K can be created when the helium flow velocities are: 1.9, 4.5, and 10.8 m/sec respectively.[5]

Hot-Spot Instability, Transient Effects, and Other Effects

Hot-spot instability can occur in nucleate-boiling systems when local heat generation due to flux jumping exceeds the maximum heat transfer rate to the bath. In some circumstances the instability will propagate. There are three ways of avoiding hot-spot instability: (a) use forced-convection heat transfer to cool the conductor; (b) treat the worst flux jump as a Q_{gen} term and stabilize accordingly; (c) stabilize the conductor on the basis of a local $\Phi_{b\ max} < 0.2$ W/cm^2.

The stability equations given in the preceding section assume that the heat transfer is steady. Thermal energy storage results in the slowing down of propagation of normal regions. The Brechna[4,5] propagation model shows that if the heat capacity of the substrate is high enough, large flux jumps can be absorbed by the substrate, which has a stabilizing effect on the superconductor. Unfortunately the substrate and the superconductor have extremely low specific heats at liquid helium temperatures.

The stability equations also assume that all the normal current flows in the substrate. A small portion of the normal current does flow in the superconductor. This current flow does not have a detrimental effect on stability unless the thermal bond between the superconductor and the substrate is poor. A poor thermal bond results in a degradation of T_{sc} in the stability equations.

Conclusions

Superconductor stability is greatly affected by heat transfer from the superconductor to the bath. Superconductors can be stabilized even when there is a steady flow of heat from an external source. Unless the heat generated by ac losses is removed, ac superconductors cannot exist.

The development of forced-convection heat-transfer systems can be an important key to solving a number of problems that superconducting magnet designers are facing. Forced convection will permit bath temperatures of 10°K to be used for an Nb_3Sn system, which could result in substantial savings in refrigeration. Forced-convection heat transfer will permit a high-strength high-resistance substrate to be used in high-field (> 70 kG) large bubble chamber magnets. This will permit an increase in substrate current density which is currently limited by stresses in the coil. Forced-convection heat transfer improves the feasibility of superconducting thin septum magnets.[14] I feel that forced-convection heat transfer may be the key to pulsed superconducting magnet development.

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List of Figures

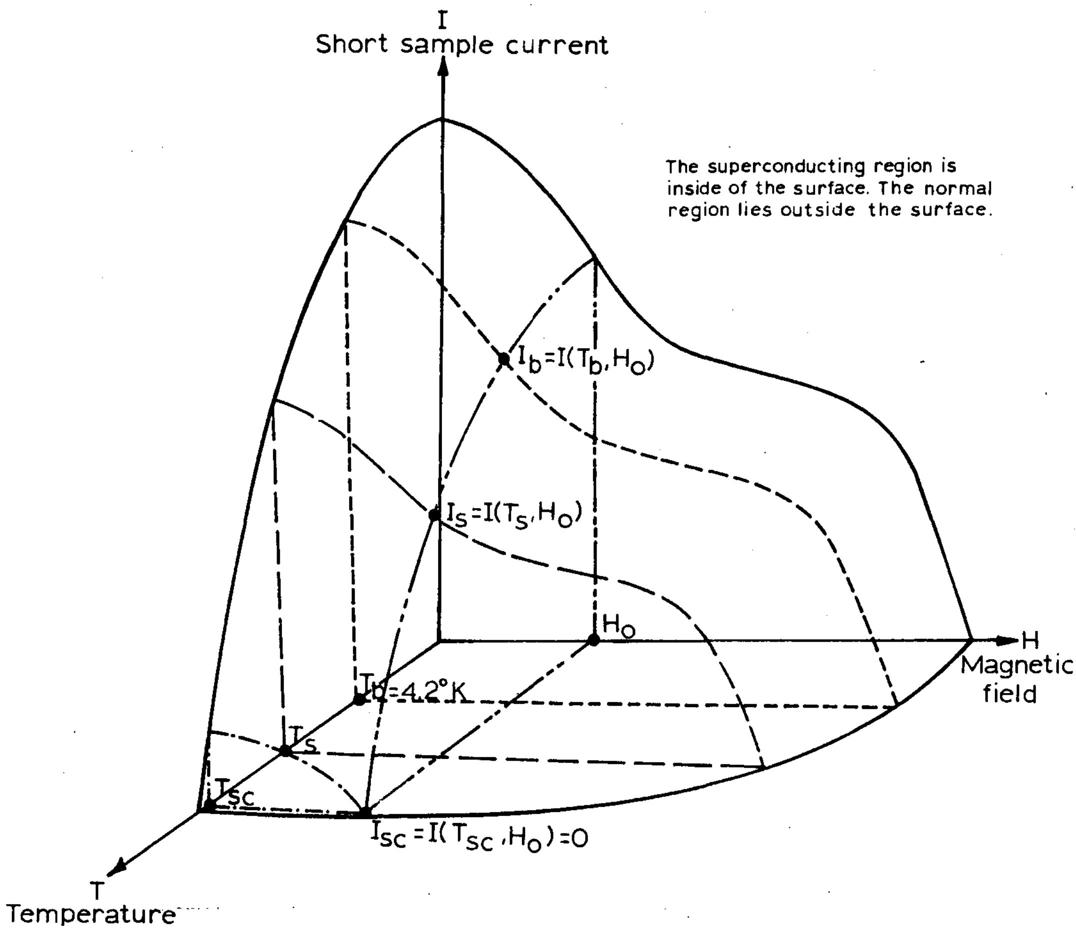

Fig. 1.

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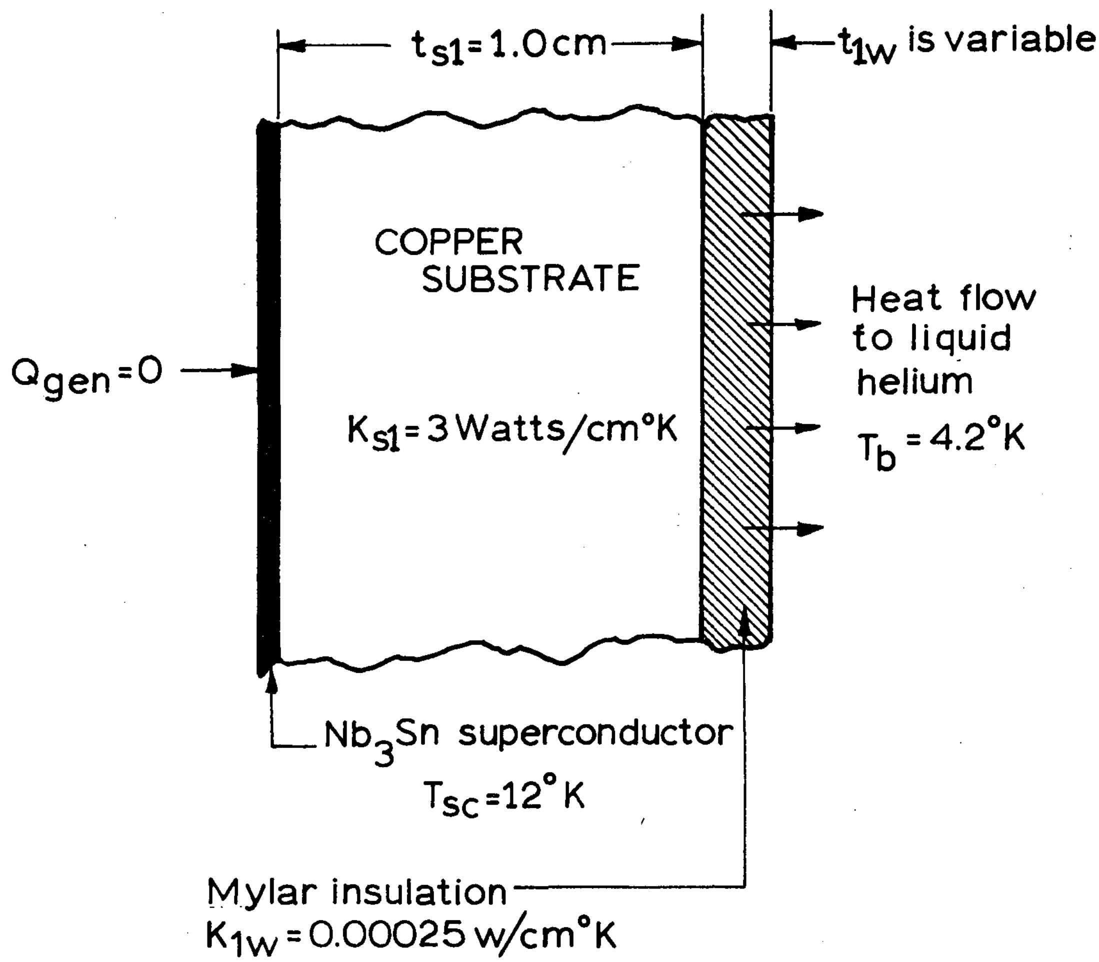

Fig. 2.

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