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Analytical study of tubular tee-joints

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ANALYTICAL STUDY OF TUBULAR TEE-JOINTS

BY

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Structures and Materials Research
Department of Civil Engineering
Division of Structural Engineering
and Structural Mechanics

ANALYTICAL STUDY
OF TUBULAR TEE-JOINTS

A Report of an Investigation

by

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INTRODUCTION

General Background

A tubular connection normally consists of a number of circular steel tube web members welded to a larger-diameter chord member. The critical influence on the ultimate strength of these connections is the radial flexibility of the chord tube wall. Several structural arrangements have been suggested to reduce the above influence to acceptable proportions (1)[1]. For joints under static loads the best structural solution consists of an overlapping and interwelding of the web branch members (2). However, for joints under alternating loads, fatigue failure can be postponed most effectively by increasing locally the radial stiffness of the chord wall through the insertion of a thicker-walled chord section and by separating simultaneously the branch members (3).

The above conclusions are all based on results obtained from extensive experimental studies. Because of the complexity of the problem, experimental research was and still is necessary. However, to optimize research efficiency in general and to aid designers, the need to analyze the stress distribution in these tubular joints has long been recognized. Unfortunately, no effective methods of analysis were available initially. However, through the development of general computer programs based on classical

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1. Numerals in parentheses refer to corresponding items in Appendix I--References.

shell theory, it has become possible to analyze simplified tube systems. This development is particularly significant in guiding researchers and engineers because it permits an evaluation of the basic influence of certain structural parameters by studying simplified tube-joint models. Although it is realized that such evaluations are not fully exact because of the simplified representation of the joint, the value of such studies is significant for the present developments in tubular joint design. Even for the most general methods of analysis which are presently under development the principal problem will be the accurate representation of the structural complexity of these joints in an analytical computer model. Hence, experimental research will remain a necessity.

The authors present the basic principles and theory used in developing several cylindrical shell programs which have been used to study the influence of two important geometric joint parameters, namely, the diameter-to-diameter ratio d/D of the web (d) and chord (D) members respectively, and the t/D ratio of the wall thickness (t) and diameter (D) of the chord section. The present paper discusses the accuracy of these programs and shows the validity of their application for simple parameter studies.

Previous Studies

A number of papers have been published on the analysis of circular cylindrical shells under localized loadings. A selected list of references in this subject is given at the end of this report, however, only a few of these will be mentioned below.

Roark (4) presented empirically derived formulas for stresses and deflections produced by a concentrated loading on a cylindrical shell. Yuan (5) (6) used Donnell's equation (7) and then Flugge's equation (8) to study the radial deflections of thin cylindrical shells subjected to concentrated, equal and opposite radial forces, acting at the ends of a vertical diameter. Hoff, Kempner, Nardo and Pohle (9) conducted a theoretical investigation of stresses in a pipeline branch connection under bending loads applied to the attached pipe. Later, Hoff, Kempner, Pohle and Sheng (10) (11) obtained the closed form expressions for the displacements and internal forces in cylindrical shells under sinusoidal line loads applied along a generator using Donnell's equation. Fourier series were than used to represent localized line loads including radial forces, axial moments and circumferential moments. Tabulated results were obtained.

Bijlaard (12) (13) (14) made extensive theoretical studies of the stresses produced in cylindrical pressure vessels by localized attachment loads. He used an analysis based on developing the loads and displacements into double Fourier series. The 8th order differential equation for radial deflection which he used differs somewhat from Donnell's equation, which has been used by other investigators. His equation attempts to overcome some of the known inaccuracies of Donnell's equation for long shells. For the case in which the loading is transferred by a rather rigid attachment such as a pipe, Bijlaard assumed the radial load to be uniformly distributed over a rectangular surface of the shell covered by the attachment. In order to take account of the rigidity of the attachment he recommended that for design purposes the values of the internal forces at the center of the loading surface, obtained from the analysis, be assumed to exist at the edge of the attachment. Extensive tables and curves for design purposes are included in Bijlaard's papers.

Cooper (15) studied the problem of localized line loading using shallow shell theory including the effect of shear deformation and found that his results for stresses were in good agreement with those found by Donnell's equation. Klein (16) presents results in diagrams and tables for stresses and displacements in shells subjected simultaneously to internal pressure and localized loadings. Morley (17) developed an improved shell equation, which is said to retain the simplicity of Donnell's equation, but which also has the desired property that its accuracy does not decrease for long shells.

Toprac and his colleagues (18) (19) (20) at the University of Texas have conducted a number of analytical and experimental studies on the behavior of large tubular connections. In his analytical studies he has used two approaches. One of these is based on Bijlaard's approach to determine the displacements and stresses in the vicinity of welded tubular joints. A second more recent approach, is based on a development by Dundrova (20) which permits the intensity of loading to be described at any point on the curve of intersection between the web and chord member.

In a recent investigation Greste and Clough (21) have successfully used a finite element method to study stresses in tubular joints including the effect of the flexibility of the web members as well as the chord member.

This method has great versatility since it can treat a variety of boundary conditions and it can be used in cases where several tubular members frame into the joint.

The approach to be used in the present investigation is based on the use of Donnell's equation for cylindrical shells. Detailed derivations of this equation and the necessary formulas for its application may be found in books by Jenkins (22), Gibson (23), or Billington (24). The treatment by Gibson was used by Scordelis and Lo (25) to develop a general computer program for the analysis of multiple cylindrical shell roofs. Extensions of this work to tubular shells under general loading and displacement conditions form the basis of the investigation reported herein.

Scope of Investigation

This investigation was concerned with the elastic analysis of tubular members subjected to specified loadings or displacements. The analytical model used throughout the investigation simulated a typical tee-type tubular connection, Fig. 1, which consists of a web tube interwelded and normal to a chord tube. The chord tube is assumed to be simply supported at its two ends, at which end diaphragms exist which are infinitely rigid in their own plane, but perfectly flexible normal to their own plane. Loads are applied to the chord tube by means of the web tube which is welded to it. The internal stresses and displacements in the chord tube of even this most elementary type of tubular joint have not as yet been solved completely by elastic theory. The type of loading, the radial flexibility of the chord tube and the relative diameters and thicknesses of the chord and web tubes have a great influence on the distribution of stresses and displacements in the chord tube.

In the present study, a general method of analysis is outlined by which a tubular cylindrical shell may be analyzed for any input loading or displacement pattern. Four computer programs, which have been developed, based on this method of analysis, will be described. Programs 1, 2, and 3 provide automatic solutions for the internal stresses and displacements in a circular cylindrical shell under a variety of loadings. Program 1 treats localized line loads applied along a single generator at the crown. Programs 2 and 3 extend the loadings to a number of generators around the circumference of the cylindrical shell. Program 4 provides a solution for the case in which either applied forces or displacements are imposed on the shell at specified points and the unknown corresponding displacements or forces are found. This program makes it possible to determine the load distribution to the chord tube, at the web tube-chord tube interconnection, Fig. 1, produced by a uniform vertical displacement of the chord tube along the line of interconnection. Once this load distribution is known, Program 3 can be used to determine stresses and displacements produced by the loading.

In order to check the accuracy of the computer programs, they are first used to analyze several cases which have been solved by other investigators and comparisons of results are made. The computer programs are then used to study typical tee connections in which a number of different assumptions are made as to the distribution of loading imposed on the chord tube by the web tube. The results are compared and discussed with respect to the validity of some of the present design assumptions being used. Finally, a computer analysis is made of a case studied experimentally by Toprac (18) and a comparison of the results is discussed.

METHOD OF ANALYSIS AND COMPUTER PROGRAMS

General Approach

The computer programs used in this investigation are based on the classical thin shell theory for the analysis of a simply supported circular cylindrical shell segment, Fig. 2. A short description of the steps which must be followed in analyzing a single shell is given below. For a detailed discussion the reader is referred to the book by Gibson (23).

1. Statics - The unknown internal forces shown in Fig. 3 are expressed in terms of the known surface loads. The number of unknown internal forces exceeds the number of equations of statics available; therefore, the system is statically indeterminate.

2. Geometry - The internal strains are expressed in terms of the displacements u, v, and w.

3. Properties of materials - The internal stresses are expressed in terms of the internal strains through Hooke's law and thence, by Step 2, in terms of u, v, and w.

4. Force-displacement equations - The internal forces are expressed in terms of the displacements u, v, and w by integrating the stresses, from Step 3, over the thickness of the shell.

5. Compatibility equation - The preceding steps will result in a set of partial differential equations equal in number to the number of unknown forces and displacements. By substitution and successive elimination this set of simultaneous equations can be reduced to a single equation with one unknown. This equation will be an 8th order partial differential equation and is called the compatibility equation. The compatibility equation used

in the computer programs was first proposed by Donnell (7) and subsequently derived independently by Jenkins (22). A full derivation of the equation is also given in Appendix II of the book by Gibson (23).

6. Solution of the compatibility equation - This consists of a solution in two parts: (1) the particular integral or a membrane solution which is dependent on the surface loading, but independent of the boundary conditions along the longitudinal edges; and (2) the complementary function or homogeneous solution containing eight constants of integration which are dependent on the boundary conditions along the two longitudinal edges of the circular shell.

7. Final internal forces and displacements - Once the compatibility equation is solved for the single unknown, the remaining internal forces and displacements may be found by back substitution into the previously derived equations.

The above steps for a single shell segment form the basis for the analysis of a closed cylindrical tube which is considered to be composed of two semicircular cylindrical shell segments joined along their two longitudinal edges to form a closed tube.

Analysis of Closed Cylindrical Shell Tube

The initial problem to be solved may be defined simply as follows: given the circular cylindrical shell shown in Fig. 4, which is subjected to a localized line load or displacement; find the resulting internal forces and displacements in the shell at specified points.

The closed cylindrical tube is made up of two semicircular shell elements or segments, ab and cd, interconnected at two longitudinal joints, 1 and 2, Figs. 5 and 6. At each longitudinal joint there are four degrees of freedom, either a known external force R or a known displacement r can exist in each of the four directions shown in Fig. 5. Any longitudinal distribution of these quantities can be replaced by the sum of the harmonic components of an appropriate Fourier series. For any single harmonic distribution of order n the forces will produce displacements of the same longitudinal distribution and vice-versa. Also for a typical harmonic, a single characteristic value may be used to describe any force or displacement pattern along a joint. This makes it possible to treat the entire longitudinal joint as a single nodal point and to operate with single forces and displacements instead of functions. If the conditions of static equilibrium and geometric compatibility are satisfied at this nodal point, they will be automatically maintained along the entire longitudinal joint.

Once the solution for a shell subjected to a force or displacement pattern varying as a typical harmonic of order n has been programmed for the digital computer, the results for any longitudinal variation can be readily obtained by a simple summation process using as many terms of the appropriate Fourier series as is deemed necessary for accuracy. In the computer programs to be described later, up to 100 non-zero terms may be used for this purpose. Only a typical case of a first harmonic force or displacement variation need thus be considered in the further discussion of the problem.

A direct stiffness solution in matrix form is used in the solution. The important steps will be briefly described.

Consider a semicircular shell element ab or cd taken as a free body, Fig. 6. These elements are subjected only to boundary forces S or boundary displacements v at their two longitudinal edges. These quantities are defined in an element coordinate system as shown. It is desired to determine the element stiffness matrix for each element relating the eight edge forces to the corresponding eight edge displacements, first in the element coordinate system shown in Fig. 6, and then by suitable transformations in the directions of the fixed coordinate system shown in Fig. 5. Note that since no surface loads exist in this problem, only the homogeneous solution of the compatibility equation described in the preceding section is necessary.

The geometry of a single shell element can be defined by its span L, radius R, thickness t, and central angle $\phi_k = 180^\circ$.

The following matrix relationships can be written:

$$\underset{8x1}{\{S\}} = \underset{8x8}{[B_s]} \underset{8x1}{\{A\}} \tag{1}$$

$$\underset{8x1}{\{v\}} = \underset{8x8}{[B_v]} \underset{8x1}{\{A\}} \tag{2}$$

The values S and v represent respectively the eight edge forces and eight corresponding edge displacements (in each case, four at each edge) shown in Fig. 6, which can be expressed in terms of eight arbitrary constants A premultiplied by coefficient matrices B_s or B_v. Formulas for the terms of the B_s and B_v matrices may be found in the book by Gibson (23). These coefficients are dependent on the geometry of the shell element together with its modulus of elasticity and Poisson's ratio. Thus in Eqs. (1) and (2) only S, v, and A are unknowns.

From Eq. (2):

$$\{A\} = [B_v]^{-1} \{v\} \tag{3}$$

Substituting into Eq. (1):

$$\underset{8x1}{\{S\}} = [B_s]\,[B_v]^{-1}\,\{v\}$$

$$= \underset{8x8}{[k]}\ \underset{8x1}{\{v\}} \qquad (4)$$

The matrix $k = B_s\ B_v^{-1}$ represents the element stiffness matrix in the element coordinate system of Fig. 6. The element stiffness matrix can be converted to the fixed coordinate system of Fig. 5, with any ordering of the forces and displacements desired, using simple transformation matrices.

$$\{v\} = [a]\ \{\bar{v}\} \qquad (5)$$

$$\{\bar{S}\} = [b]\ \{S\} = [a]^T\ \{S\} \qquad (6)$$

In Eqs. (5) and (6) the bar above S or v denotes that these quantities are referred to a fixed coordinate system. The values of a and b are respectively 8 x 8 displacement and force transformation matrices which can be easily calculated from the directions shown in Figs. 5 and 6. Note also that b equals the transpose of a. Now substituting Eq. (4) and then Eq. (5) into Eq. (6):

$$\begin{aligned} \underset{8x1}{\{\bar{S}\}} &= [b]\ \ [k]\ [a]\ \{\bar{v}\} \\ &= [a]^T\ [k]\ [a]\ \{\bar{v}\} \\ &= \underset{8x8}{[\bar{k}]}\ \underset{8x1}{\{\bar{v}\}} \end{aligned} \qquad (7)$$

The matrix $\bar{k}$ is the element stiffness matrix relating the shell element edge forces to its edge displacements, both in a fixed coordinate system. This process may be repeated for both shell elements ab and cd of Fig. 5, and Eq. (7) may be rewritten in terms of submatrices for each of these elements:

For shell element ab:

$$\underset{8x1}{\begin{Bmatrix} \bar{S}_a \\ \bar{S}_b \end{Bmatrix}} = \underset{8x8}{\begin{bmatrix} \bar{k}_{aa} & \bar{k}_{ab} \\ \bar{k}_{ba} & \bar{k}_{bb} \end{bmatrix}} \underset{8x1}{\begin{Bmatrix} \bar{v}_a \\ \bar{v}_b \end{Bmatrix}} \qquad (8)$$

For shell element cd :

$$\begin{Bmatrix} \bar{S}_d \\ \bar{S}_c \end{Bmatrix} = \begin{bmatrix} \bar{k}_{dd} & \bar{k}_{dc} \\ \bar{k}_{cd} & \bar{k}_{cc} \end{bmatrix} \begin{Bmatrix} \bar{v}_d \\ \bar{v}_c \end{Bmatrix} \tag{9}$$

$$\underset{8x1}{} \qquad \underset{8x8}{} \qquad \underset{8x1}{}$$

in which the 4 x 4 submatrices $\bar{k}_{ij}$ represent the forces at edge i produced by unit displacements at edge j .

Static equilibrium of joints 1 and 2 in Fig. 5 requires that the external joint forces R must be the sum of the edge forces $\bar{S}$ acting on the two shell elements which form the joint:

$$\begin{Bmatrix} R_1 \\ R_2 \end{Bmatrix} = \begin{Bmatrix} \bar{S}_a \\ \bar{S}_b \end{Bmatrix} + \begin{Bmatrix} \bar{S}_d \\ \bar{S}_c \end{Bmatrix} \tag{10}$$

Geometric compatibility of the joint requires that the external joint displacements r must be equal to the edge displacements $\bar{v}$ of the two shell elements which form the joint:

$$\begin{Bmatrix} r_1 \\ r_2 \end{Bmatrix} = \begin{Bmatrix} \bar{v}_a \\ \bar{v}_b \end{Bmatrix} = \begin{Bmatrix} \bar{v}_d \\ \bar{v}_c \end{Bmatrix} \tag{11}$$

Substituting Eqs. (8), (9), and (11) into Eq. (10) gives:

$$\begin{Bmatrix} R_1 \\ \cdots \\ R_2 \end{Bmatrix} = \left[\begin{array}{c:c} (\bar{k}_{aa} + \bar{k}_{dd}) & (\bar{k}_{ab} + \bar{k}_{dc}) \\ \hdashline (\bar{k}_{ba} + \bar{k}_{cd}) & (\bar{k}_{bb} + \bar{k}_{cc}) \end{array} \right] \begin{Bmatrix} r_1 \\ \cdots \\ r_2 \end{Bmatrix}$$

$$= \underset{8x8}{\left[\begin{array}{c:c} K_{11} & K_{12} \\ \hdashline K_{21} & K_{22} \end{array} \right]} \underset{8x1}{\begin{Bmatrix} r_1 \\ \cdots \\ r_2 \end{Bmatrix}} \tag{12}$$

or simply

$$\underset{8x1}{\{R\}} = \underset{8x8}{[K]} \; \underset{8x1}{\{r\}} \tag{13}$$

Equation (13) represents a set of eight simultaneous equations in which all of the elements in the structure stiffness matrix K are known. The joint forces R and the joint displacements r involve sixteen quantities, eight of which are specified as known applied joint forces or displacements and eight of which are corresponding unknown joint displacements or forces. Wherever a joint force is known the corresponding joint displacement is unknown and vice-versa. The eight unknown quantities may be found by solving the eight simultaneous equations.

After all of the joint displacements r have been found the eight arbitrary constants A for <u>each</u> shell element ab and cd can be determined using Eqs. (3) and (5).

$$\underset{8x1}{\{A\}} = [B_v]^{-1}\{v\} = \underset{8x8}{[B_v]^{-1}}\underset{8x8}{[a]}\underset{8x1}{\{r\}} \qquad (14)$$

With the constants known the internal forces and displacements at selected points in each shell element can be calculated using equations similar to Eqs. (1) and (2).

Types of Loading and Fourier Series Representation

The five different types of line loads which can be applied at a generator are shown in Fig. 7, and consist of:

(a) Type 1 - Radial load P

(b) Type 2 - Tangential load T

(c) Type 3 - Transverse moment load M_ϕ

(d) Type 4 - Londigitudinal moment load M_x

(e) Type 5 - Radial moment load M_z

Each load is defined by:

(a) Its total magnitude.

(b) Its uniformly distributed length along the generator δ .

(c) The distance from the left end of the shell to the center of the distributed load ξ .

The moment loads, M_x and M_z, are made up of statically equivalent sets of radial and tangential loads respectively as shown in Fig. 7.

The Fourier series expression for the radial load P, Fig. 7a, is as follows with the origin taken at midspan as indicated in Fig. 4.

$$P(x) = \sum_{n=1,3,\ldots}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=2,4,\ldots}^{\infty} b_n \sin\frac{n\pi x}{L}$$

$$= \sum_{n=1,3,\ldots}^{\infty} \frac{4P}{n\pi\delta} \sin\frac{n\pi\xi}{L} \sin\frac{n\pi\delta}{2L} (-1)^{\frac{n+3}{2}} \cos\frac{n\pi x}{L} \qquad (15)$$

$$+ \sum_{n=2,4,\ldots}^{\infty} \frac{4P}{n\pi\delta} \sin\frac{n\pi\xi}{L} \sin\frac{n\pi\delta}{2L} (-1)^{\frac{n}{2}} \sin\frac{n\pi x}{L}$$

Similar expressions may be written for the tangential load, Fig. 7b. and the transverse moment load, Fig. 7c, by replacing P by T and M_ϕ respectively in Eq. (15).

The Fourier series expression for the longitudinal moment M_x, Fig. 7d, is as follows with the origin at midspan as indicated in Fig. 4.

$$P(x) = \sum_{n=1,3,\ldots}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=2,4,\ldots}^{\infty} b_n \sin\frac{n\pi x}{L}$$

$$= \sum_{n=1,3,\ldots}^{\infty} \frac{16M_x}{n\pi\delta^2}\left(\cos\frac{n\pi\delta}{2L} - 1\right) \cos\frac{n\pi\xi}{L} (-1)^{\frac{n+3}{2}} \cos\frac{n\pi x}{L} \qquad (16)$$

$$+ \sum_{n=2,4,\ldots}^{\infty} \frac{16M_x}{n\pi\delta^2}\left(\cos\frac{n\pi\delta}{2L} - 1\right) \cos\frac{n\pi\xi}{L} (-1)^{\frac{n}{2}} \sin\frac{n\pi x}{L}$$

A similar expression may be written for the radial moment load, Fig. 7e, by replacing M_x by M_z in Eq. (16).

Description of Computer Programs 1, 2, and 3

These computer programs have been written for the IBM 7094 digital computer in FORTRAN language to perform analyses of cylindrical tubes based on the theory just described.

Programs 1, 2, and 3 permit solutions for internal forces, moments, and displacements, in a circular cylindrical shell subjected to variety of localized line loads. Program 1 treats line loads applied along a single x-generator at the crown. The program can treat simultaneously any combination of the five loading types shown in Fig. 7. Several loads of the same type applied along the crown can also be treated at one time. Program 2 is an extension of Program 1 so that the same types of loads can be applied at up to 20 different x-generators at the same time. This is accomplished internally in the computer by analyzing the shell for each loaded generator as in Program 1 and then superposing the output results for all loaded generators, after each harmonic, at the desired x, ϕ coordinates referenced to the crown of the actual shell. The location of each loaded generator is specified by its angle ϕ from the crown, Fig. 4. The loading directions are defined by the radial and tangential directions at each loaded generator. A maximum of 50 localized line loads, 10 of each of the types shown in Fig. 7, may be input at each loaded generator. Program 3 is a modification of Program 2. The changes are that the input loading directions at each generator are now specified with respect to vertical and horizontal directions and only loading types 1, 2, and 3 shown in Fig. 7 are permitted. In addition, the maximum permissible number of loaded generators is increased to 42.

The input and output for Programs 1, 2, and 3 are simmlar in nature and may be summarized as follows.

Input Data

1st card: Title of problem

2nd card: (1) Longitudinal span

(2) Radius of shell

(3) Shell thickness

(4) Modulus of elasticity

(5) Poisson's ratio

(6) Fourier series limit, therefore, the number of the highest term to be used to represent loading; maximum number is 100 for unsymmetrical loads with respect to midspan and 200 for symmetrical or antisymmetrical loads if non-contributing harmonics are skipped under (8).

(7) Indicators defining desired amount of output.

(8) Indicators defining whether or not odd or even harmonics terms are to be skipped; utilized for symmetrical and antisymmetrical loadings.

3rd card: Total number of loaded x-generators

4th card: Fourier series numbers after which results are to be printed; as many as desired up to the Fourier series limit

5th card: x-coordinates at which output results are desired

6th card: ϕ-coordinates at which output results are desired

Next cards: For each loaded x-generator three or more cards are required as follows.

(1) ϕ-angle defining location of loaded generator.

(2) Number of loads of each type, shown in Fig. 7, applied on this generator.

(3) One card for each load in sequence is required next which defines the magnitude, distributed length δ, and location from the left support ξ of the load Fig. 7.

Printed Output

(1) Input data is printed as a check

(2) The results for all internal forces, moments and displacements shown in Fig. 3 are given at the x, ϕ coordinates specified in the 3rd and 4th cards of the input

Description of Computer Program 4

In this program a set of specified points on the shell surface is considered. At each of these points either applied vertical or horizontal forces or displacements are input and the corresponding unknown displacements or forces are calculated and output. Each force at a specified point is defined as a line load of given length along a loaded generator. The corresponding displacement is at the specified point, which is the center of the loaded length.

In the program, the flexibility matrix F relating the forces and displacements at the considered points is first formed and the unknowns are then found by the following matrix operations:

$$r = FR \tag{17}$$

or after subdividing

$$\begin{Bmatrix} r \\ \cdots \\ \Delta \end{Bmatrix} = \left[\begin{array}{c:c} F_{11} & F_{12} \\ \hdashline F_{21} & F_{22} \end{array}\right] \begin{Bmatrix} X \\ \cdots \\ R \end{Bmatrix} \tag{18}$$

where r are the applied displacements and X are the corresponding unknown forces; R are the applied forces and Δ are the corresponding unknown displacements.

A maximum of 10 loading cases may be treated at one time in the computer program. The flexibility matrix F is determined using the method of analysis described for Programs 1, 2, and 3. Once the flexibility matrix is formed the unknowns are found using Eq. (18).

$$r = F_{11} X + F_{12} R \tag{19}$$

Solving for X in Eq. (19):

$$X = F_{11}^{-1} [r - F_{12} R]$$

After X is found:

$$\Delta = F_{21} X + F_{22} R \tag{20}$$

The maximum permissible number of considered points is 42 and at each point vertical and/or horizontal input forces or displacements may be included or if desired the restraint corresponding to either direction may be neglected. For sets of points and input forces and displacements which are symmetrical or antisymmetrical with respect to $x = 0$ and/or $\phi = 0$, Fig. 4, advantage may be taken of symmetry in the input. Thus for a case in which symmetry exists about both $x = 0$ and $\phi = 0$ a total of 42 points and the corresponding input may be considered on 1/4 of the whole shell.

The input and output for Program 4 may be summarized as follows:

Input Data

1st card: Title of problem

2nd card: Same as Programs 1, 2, and 3

3rd card: Fourier series numbers after which results are to be printed

4th card: (1) Number of considered points.

(2) Number of load cases.

(3) Indicator defining whether system is unsymmetrical, symmetrical, or antisymmetrical about $x = 0$.

(4) Indicator defining whether system is unsymmetrical, symmetrical or antisymmetrical about $\phi = 0$.

Next cards: One for each considered point

(1) ϕ-angle for location of point

(2) Distributed load length δ

(3) Location from left end ξ

(4) Indicators for both vertical and horizontal directions indicating whether an applied displacement or applied force exists or whether restraint is to be neglected.

Next cards: One set of cards for each load case specifying in sequence the magnitudes of the input applied displacements or forces.

Printed Output

(1) Input data is printed as a check.

(2) A complete list of the values of the input and the calculated forces and displacements at each considered point is given.

ACCURACY OF METHOD OF ANALYSIS

General

To evaluate the accuracy of the method of analysis, as presented in this paper, several different load conditions are considered and the results compared with other available solutions. First, the case of a horizontal tube under three concentrated load conditions, applied separately at the crown, was analyzed using Program 1. The significant results for particular locations are compared with those presented by Kempner (11). The second case study deals with a horizontal tube under a radially directed and uniformly distributed pad-load as solved previously by Bijlaard (12). The comparative results in that case are analyzed by using Program 2.

Tube Under Concentrated Loads

Three basic loads, namely a radially directed load P, a longitudinal-moment load M_x, and a transverse-moment load M_ϕ are considered in this first comparative study. These loads are applied separately at the mid-span point of the crown, as shown in Fig. 8. The shells are simply supported at both ends. The span-to-diameter ratio (L/D) in all cases is 0.5, and the wall thickness-to-diameter ratio (t/D) is 0.005. All loads are distributed over a finite length δ along the crown generator. The entire stress distribution in each case has been determined using Program 1. In each case the internal shell forces at a particular location are compared with those evaluated by Kempner (11). The results of both the Kempner and Program 1 solutions are presented in Table I.

TABLE I: COMPARATIVE STUDY OF DISPLACEMENTS

AND STRESS RESULTANTS FOR THREE LOAD CASES (FIG. 8)

LOAD CASE	A		B		C	
LOADING	$P = 1 \times 10^4$		$M_x = -1 \times 10^8$		$M_\phi = 1 \times 10^8$	
LOCATION	$x = 0$, $\phi = 0$		$x = 1.25$, $\phi = 0$		$x = 0$, $\phi = 0.0125$ rad	
METHOD	KEMPNER	PROGRAM 1	KEMPNER	PROGRAM 1	KEMPNER	PROGRAM 1
$-w$	$.111 \times 10^1$	$.1110 \times 10^1$	$.204 \times 10^3$	$.2039 \times 10^3$	$-.269 \times 10^3$	$-.268 \times 10^3$
M_ϕ	$.288 \times 10^4$	$.2876 \times 10^4$	$.675 \times 10^7$	$.6619 \times 10^7$	-1.00×10^7	$-.990 \times 10^7$
M_x	$.205 \times 10^4$	$.2034 \times 10^4$	$.667 \times 10^7$	$.6534 \times 10^7$	$-.300 \times 10^7$	$-.2971 \times 10^7$
$-N_\phi$	$.198 \times 10^4$	$.1981 \times 10^4$	$.114 \times 10^7$	$.1134 \times 10^7$	$-.399 \times 10^6$	$-.2358 \times 10^6$
$-N_x$	$.151 \times 10^4$	$.1509 \times 10^4$	$.279 \times 10^6$	$.2789 \times 10^6$	$-.464 \times 10^6$	$-.3129 \times 10^6$

The number of Fourier series terms used in each computer solution was 100 for load cases A , B and C . In each case the non-contributory terms of the Fourier series were skipped. The results due to the radial load P are in excellent agreement. This is particularly significant for the tubular joint problem under consideration. Also the different results for the two other load cases prove to be in very close agreement, except for the N_ϕ and N_x stress resultants for load case C .

Tube Under a Radial Uniformly Distributed Pad-Load

The second case study considers a loading which has been used in earlier pressure vessel studies as well as in initial studies of tube-to-tube connections. The load arrangement is shown in Fig. 9. The distributed load is directed radially and assumed to be acting uniformly along eleven generators each with a load length of $2c_2$. The internal stress resultants were evaluated for two different tubes and are compared at one location with results obtained by Bijlaard (12). The results of both methods, as presented in Table II, are generally in excellent agreement. The radial displacement w and the N_x forces as analysed by Program 2 show smaller values than the Bijlaard solution indicates, particularly for the longer shell. This phenomenon is typical for long shell solutions using Donnell equations but is not very critical for the study of tube-to-tube connections. Because of the significance of the radial stresses in such connections the accurate agreement of the contributing shell forces M_ϕ and N_ϕ is far more important.

TABLE II: COMPARATIVE STUDY OF DISPLACEMENTS AND STRESS RESULTANTS AT CENTER OF LOAD (FIG. 9)

SHELL	L/a = 3			L/a = 20		
METHOD	BIJLAARD	PROGRAM 2		BIJLAARD	PROGRAM 2	
FOURIER TERMS	m=41, n=61	n=100	n=120	m=41, n=61	n=161	n=201
$-w \,/\, \frac{P}{Ea}$	3645	3429	3429	12930	9544	9542
$M_\phi \,/\, P$	0.0863	0.0878	0.0879	0.1030	0.1032	0.1029
$M_x \,/\, P$	0.0559	0.0560	0.0562	0.0634	0.0631	0.0622
$-N_\phi \,/\, \frac{P}{a}$	6.4512	6.4312	6.4315	6.4336	6.4356	6.4090
$-N_x \,/\, \frac{P}{a}$	7.120	6.849	6.849	8.704	7.968	7.966

It should be noted that increasing the number of Fourier series terms does not improve the results in any significant fashion. Because of symmetry only odd numbered series terms were used in the Program solution.

In general it can be concluded from the comparative studies that the method of analysis presented gives reliable results which are generally in excellent agreement with results obtained through other solutions.

EFFECT OF THE LOAD TRANSFER IN A T-TYPE TUBE CONNECTION

To analyze the stress distribution in a horizontal tube due to an axially loaded vertical web member (see Fig. 1) engineers and researchers have made several assumptions regarding the load transfer between these two members. It is the purpose of this section to evaluate the accuracy of three assumptions by analyzing and comparing the internal tube forces, using the methods of analysis presented in this paper.

Considering the early availability of the Bijlaard solution the load transfer between the web member and the chord tube has been studied by others, assuming a uniformly distributed, radially directed pad-load. This assumption grossly neglects the actual geometric joint arrangement in which the load transfer takes place along the line of intersection between the two circular tubes. For joints with very small d/D ratios (less than 0.2) the discrepancies between the pad-load assumption and reality will be relatively small. However, for T-joints with larger d/D ratios the difference between the assumed pad-load and the actual loading will be substantial. Consequently, an unacceptable error in the predicted stress distribution will result. To improve the agreement a second assumption can be made in which the load transfer is represented by a uniformly distributed ring load along the circumference of the web tube. While such an arrangement reflects the spatial aspect of the load transfer, it still fails to recognize the basic stiffness characteristics of the horizontal chord tube.

Observing the chord wall geometry, it becomes obvious that the magnitude of a vertical concentrated force necessary to deflect the tube wall vertically over a unit distance will be quite different for a point located at the crown than for a point located further down along the wall of the tube. Considering furthermore on a relative basis, the almost-infinite longitudinal rigidity of the web member versus the low radial flexibility of the chord tube, one can assume readily that the displacement along the intersection between the two tubes will be virtually uniform. Consequently, the load distribution along the common intersection will not be uniform, but will show a concentrated force flow towards points of maximum stiffness. Hence, the load transfer will be small at the crown but will reach maximum values at the deepest point of the line of intersection between the two tubes. To reflect these considerations a third assumption for the load distribution between the members may be made based on a uniform displacement condition. All these load assumptions will be studied in this section.

Because the joint geometry, as reflected in the d/D ratio, affects the applicability of the load assumptions, two different joint arrangements with d/D ratios of 0.3 and 0.6 are selected for these studies. For both cases the L/d and the t/D ratios have been chosen the same, namely 5 and 0.03 respectively. Fig. 10 shows the three basic load assumptions, with, however, the pad-load directed vertically for reasons of comparison.

The load input for each of the load assumptions has been based on a total load of P = 1000 lb. for the d = 0.3 D joint and P = 2000 lb. for the connection with a d = 0.6 D web member. A reference chord tube diameter D = 1.0 ft. has been assumed for all cases. In the case of the joint with a d = 0.3 D the vertically directed pad-load has been represented by uniform loads along eleven generators evenly spaced in plan. Each line load, with a length d , carries a total load of 100 lb. except for the two outer generators which carry a load of 50 lb. each. For the joint with the larger diameter web member (d = 0.6 D) , twenty-three generators, evenly spaced in plan, were used to represent the pad-load. The total load on each of these generators is 2000/22 = 90.9090 lb. except for the two exterior ones which carry only half of this load. Fig. 11 shows the two load arrangements for the computer input.

The uniform ring load--assumption 2--has been distributed in plan over a number of generators with a total length equivalent to the circumference πd of the web tube. For the smaller diameter web member (d = 0.3 D) , a total of 20 generator sections, radially spaced in plan with equal intervals, were used. For the larger diameter web member (d = 0.6 D) the number of generator sections was increased to 44. The resulting load arrangement is shown in Fig. 12.

To develop the third load assumption the same 20 and 44 points respectively representing the centers of the circumferential segments, as shown in Fig. 12, were used. The primary condition imposed was the requirement of identical vertical displacements for each point in each case. From this condition the loads necessary to meet this requirement were obtained using Program 4. These were scaled to the desired total load for input in the second phase of the program in order to evaluate the internal stress distribution in the chord tube. With the same segmental lengths of 0.04714D and 0.04286D, as before, the segmental unit loads for the 0.3 D and 0.6 D web members are 50.0000 lb. and 45.4545 lb. respectively. The load concentration factors for each segment as analyzed from the primary condition of a uniform ring displacement are presented in Table III. It is of interest to note that the load at the lowest point of the intersection shows a concentration factor of about 2.5. Additional studies have indicated that the same general value is practically independent of the d/D and t/D ratios and it thus appears applicable for practically any tubular T-type joint.

Using the three previously defined load assumptions the internal stress resultants were analysed using Program 3. The results for Case I (P = 1000 lb. , D = 1.0 ft. , d/D = 0.3 , L/d = 5 , t/D = 0.03) are presented in Figures 13 and 14. Similarly, the results for Case II (P = 2000 lb. , D = 1.0 ft. , d/D = 0.6 , L/d = 5 , t/D = 0.03) are presented in Figures 15 and 16. Comparing the results for the three load assumptions it becomes immediately obvious that the pad-load is indeed basically in error with reality. While maximum stresses are expected to occur in the immediate vicinity of the intersection between the two tubular members the analysis shows invariably extreme stresses at the center of the branch pipe. The results of the ring-load assumption seem to reflect the basic behavior of the joint arrangement under study better, as illustrated by the obvious stress concentrations near the lower point of the line of intersection between the two tubes. However, the third load assumption, which is based on an assumed uniform ring displacement, does produce significantly higher stresses which are undoubtedly a better representation of the state of stress in an actual joint. It is realized that in the present method of analysis, the presence of the vertical web member has been omitted--although its behavior in transmitting load has been considered in evaluating the load distribution under assumption 3. However, while the results reflect the stress distribution only in an isolated cylindrical shell, it is believed that the results--particularly under assumption 3--describe the basic phenomenon of a tubular joint accurately enough to merit the use of this method to study the influence of the principal parameters of tube-to-tube connections, namely d/D and t/D ratios. Also the discrepancy in the assumption of an axially infinite rigidity of the web member versus the radial flexibility of the chord in evaluating the load transfer is recognized. For the most commonly used joint arrangements--d/D between 0.3 and 0.4--the resulting error will be relatively small. For joints with larger d/D ratios the error will of course increase. The above two discrepancies will invariably result in higher calculated peak stresses by this method than actually occur in a T-type joint.

In comparing the results it is of interest to note that in Case I, with $d = 0.3\ D$, the maximum values for M_ϕ, M_x, N_ϕ, and N_x are of the same order of magnitude for load assumptions 1 (pad load) and 3 (uniform displacement). The locations of these maximum values as noted before do not coincide. For Case II with $d = 0.6\ D$, the relative stress similarities no longer exist.

TABLE III: SEGMENTAL LOAD CONCENTRATION FACTORS* --LOAD ASSUMPTION 3

Case I (d = 0.3 D)	
segment	concentration factor
1	2.372
2	1.996
3	1.224
4	0.537
5	0.091
6	-0.066

Case II (d = 0.60 D)	
segment	concentration factor
1	2.677
2	2.488
3	2.036
4	1.489
5	1.056
6	0.745
7	0.558
8	0.411
9	0.324
10	0.324
11	0.219
12	0.204

*For numerical identification of segments see Fig. 12.

COMPARISON OF ANALYSIS VERSUS EXPERIMENTAL RESULTS

To test the applicability of the analysis, experimental results of a study by Toprac (18) were used to verify those predicted by the present method of analysis. The case study concerns a T-joint having a 12.5 in. O.D. horizontal chord member with a wall thickness of 0.250 in. In the center of the 48 in. long chord tube a vertical branch member with a 2.672 in. O.D. and a 0.203 in. wall thickness has been welded to the chord. The load consisted of an axial load applied to the branch tube producing a 1 ksi stress in this member.

The analysis is carried out by assuming a uniform vertical ring displacement along the line of intersection between branch and chord member. This condition is introduced at 40 points uniformly spaced in plan along the intersection. The loads necessary to meet this requirement are obtained in the first phase of the analysis. In order that the total load is identical in magnitude to the experimental value, the analytically determined loads were proportionally adjusted. These adjusted forces were introduced as uniformly distributed loads along 40 generators, the centers of which are spaced radially in plan at 9^{o} intervals. Each generator had a length of $(\pi d_m)/40$ in. where d_m = mean diameter = $(d_{inner} + d_{outer})/2$. The internal shell moments and forces (M_ϕ , M_x , and N_ϕ , N_x respectively) were evaluated by means of the computer and then the surface stresses σ_ϕ and σ_x were calculated. The stress results which are shown in Figures 17 and 18 are plotted versus ϕ for $x = 0$ and versus x for $\phi = 0$. On the same graph the experimental stress values according to Toprac are also presented. It is satisfactory to note that the important σ_ϕ stresses along the section

$x = 0$ (see Fig. 17) agree fairly closely with the experimental values. The agreement would undoubtedly improve if the outer diameter d instead of the mean diameter d_m of the branch tube would have been considered in the analysis. Also the normal build-up of the welds would further increase the effective diameter of the branch tube along the line of intersection between the branch and chord section. Such a large d value would result in a general shift towards the right (larger ϕ) of the maximum values of σ_ϕ and σ_x along the section $x = 0$ in both Figs. 17 and 18. Consequently the steep portions of these curves in that vicinity would also tend to move to the right and agree even better with the experimental values. The σ_ϕ and σ_x values along the crown ($\phi = 0$) have smaller maximum values and do not agree as well with the experimental results. The significant discrepancies in the σ_x stresses along the crown cannot be explained. However, in general, one can conclude that the comparison between analysis and experiment can be considered satisfactory, and that the method of analysis can be used to predict with reasonable accuracy the stress distribution in T-type joints built from circular tubes.

CONCLUSIONS

A solution for the determination of the state of stress in the chord-tube wall of a T-type tubular connection has been presented. The importance of correctly evaluating the basic load transfer from the branch tube to the chord member has been clearly illustrated. While the analytical model used in the procedure presented does not completely take account of the continuity created through the T-joint weld, it does include the predominant influence resulting from the differential stiffness of the radially flexible chord-tube wall and the axially rigid branch tube.

The method used can also be extended to investigate the structural integrity of tubular joints having branch-to-chord member arrangements other than the simple T-type joint treated in this paper.

ACKNOWLEDGEMENTS

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APPENDIX II - NOTATION

The following symbols are used in this paper:

a	= radius of the web tube
a_n, b_n	= coefficients in Fourier series expansion
$[a]$	= displacement transformation matrix
$\{A\}$	= matrix of arbitrary constants for shell
$[b]$	= force transformation matrix
$\{B_s\}$, $\{B_v\}$	= coefficient matrices
d	= diameter of the web tube
D	= diameter of the chord tube
E	= modulus of elasticity
$[F]$	= flexibility matrix
$[k]$, $[\bar{k}]$	= shell element stiffness matrix referred to an element system or fixed coordinate system respectively
$[\bar{k}_{ij}]$	= submatrix of element stiffness, represents the forces at edge i produced by unit displacements at edge j
$[K]$	= structure stiffness matrix
L	= span of the shell or chord tube
M_x	= external longitudinal moment load or internal longitudinal bending moment
M_z	= external radial moment load
M_ϕ	= external transverse moment load or internal transverse bending moment
N_x	= internal longitudinal membrane force
N_ϕ	= internal transverse membrane force
P	= external radial load
$\{r\}$	= external joint displacements
R	= radius of the shell or chord tube
$[R]$	= external forces

$\{S\}$, $\{\bar{S}\}$ = shell element edge forces referred to an element or a fixed coordinate system respectively

t = wall thickness of the shell or chord tube

T = external tangential load

x,y,z = fixed cartesian coordinates

u,v,w = displacements in x, y, and z directions respectively

$\{v\}$, $\{\bar{v}\}$ = shell element edge displacements referred to an element or a fixed coordinate system respectively

$\{X\}$ = unknown forces

δ = distributed load length along the generator

$\{\Delta\}$ = unknown displacements

ξ = distance from the left end of the shell to the center of the distributed load

ϕ = angle from the crown to a specified point

LIST OF FIGURE TITLES

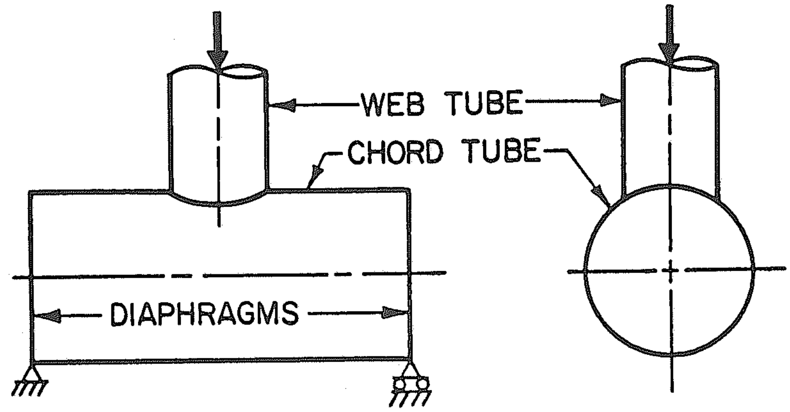


FIG. 1 TYPICAL TEE-TYPE TUBULAR CONNECTION.

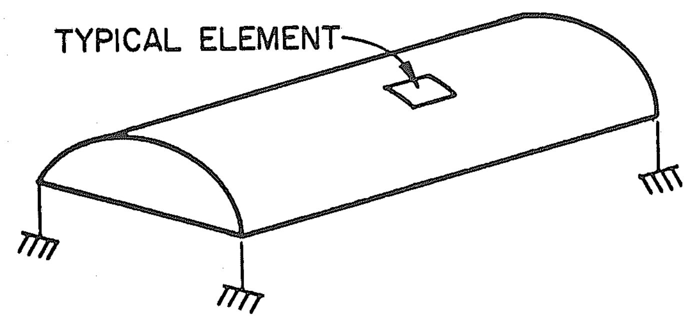


FIG. 2 TYPICAL SINGLE CYLINDRICAL SHELL.

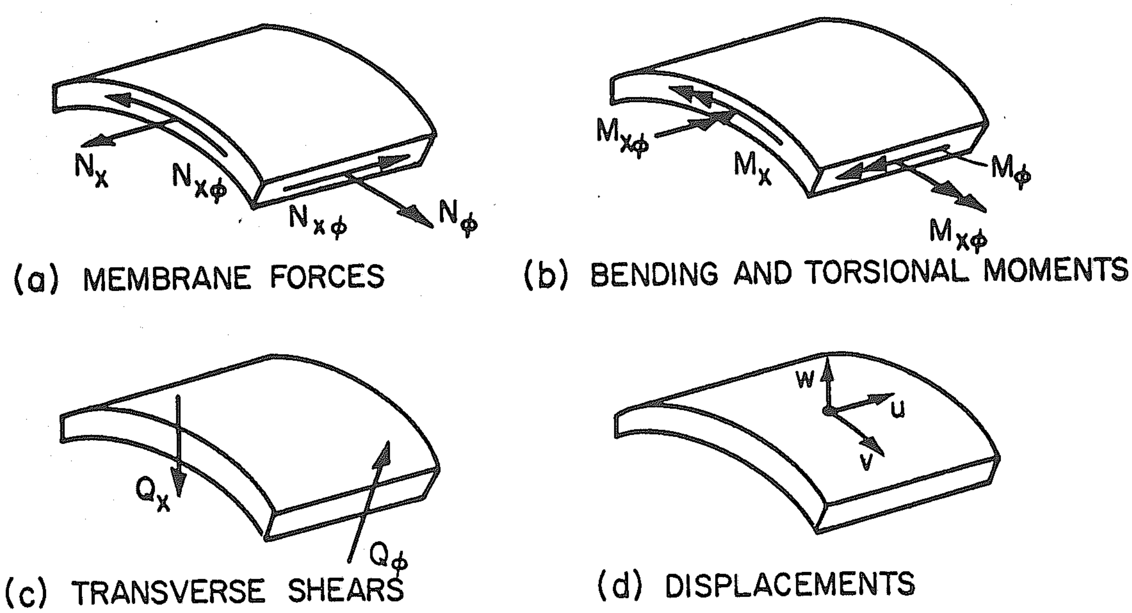


FIG. 3 INTERNAL FORCES AND DISPLACEMENTS FOR A TYPICAL ELEMENT.

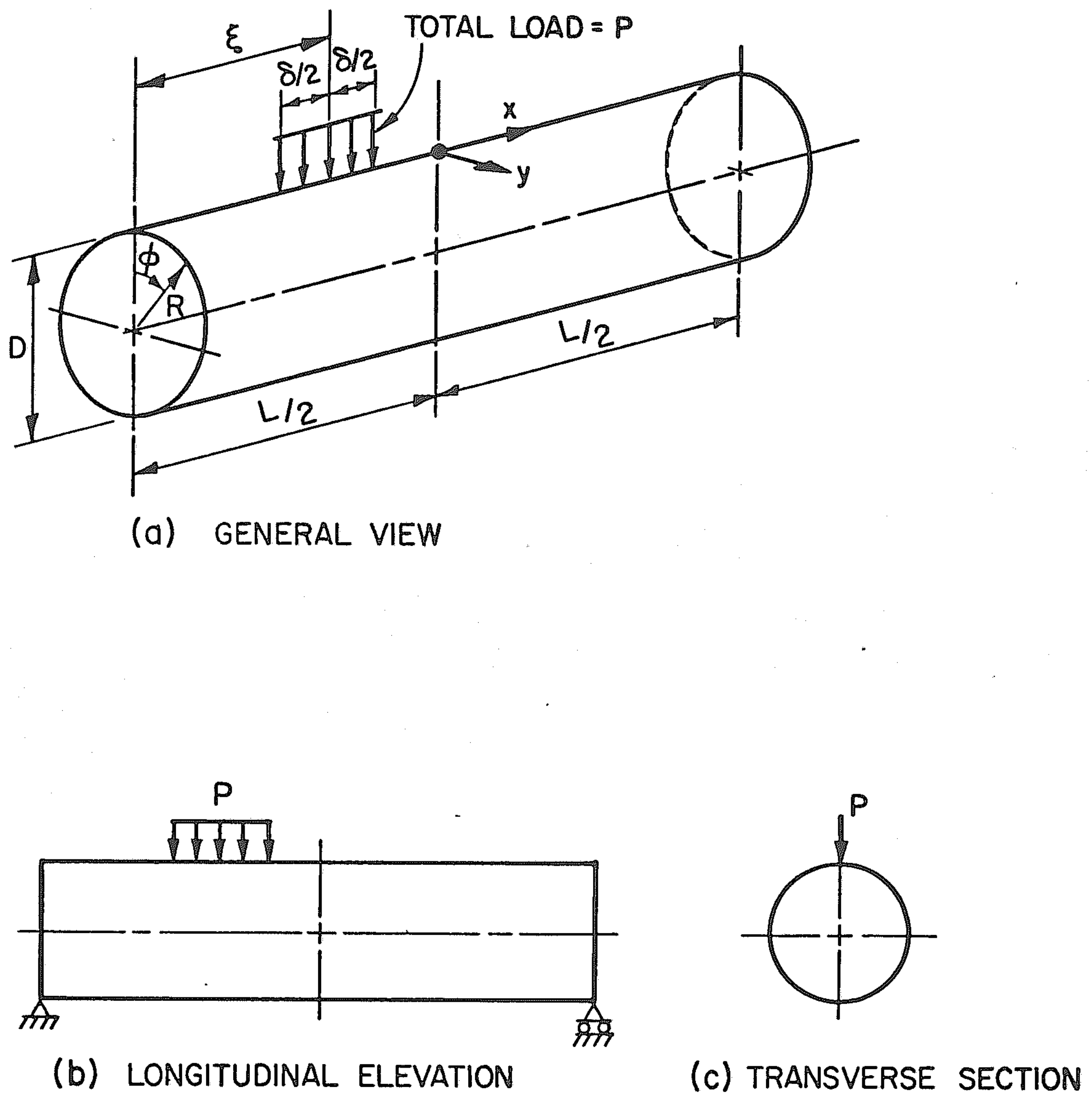


FIG. 4 CIRCULAR CYLINDRICAL TUBE UNDER LOCALIZED LINE LOAD

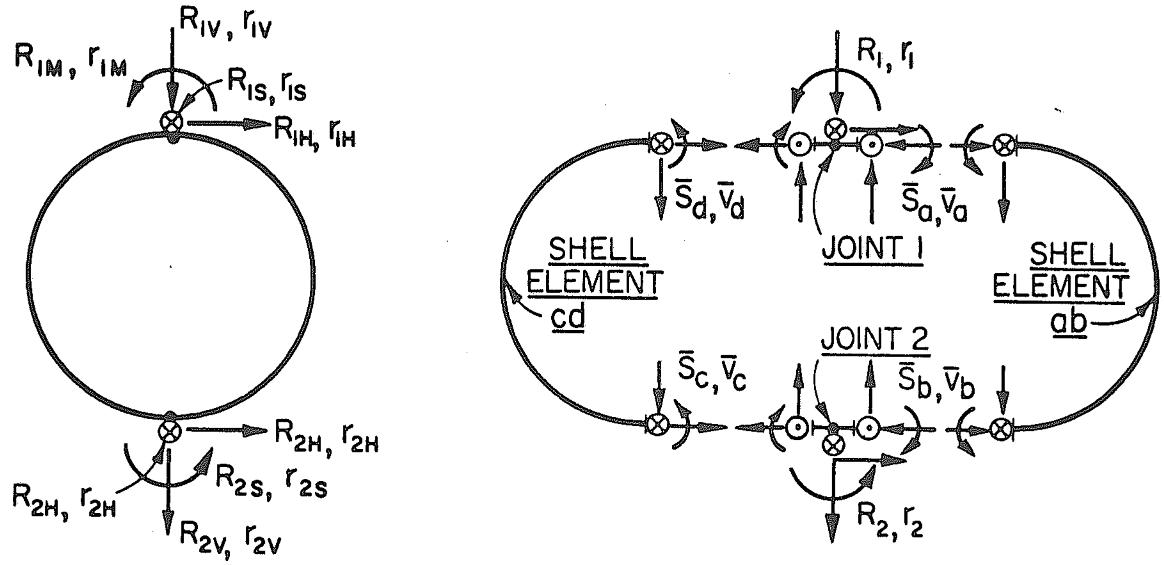


FIG. 5 EXTERNAL JOINT FORCES AND DISPLACEMENTS (R AND r) AND INTERNAL SHELL ELEMENT EDGE FORCES AND DISPLACEMENTS ($\bar{S}$ AND $\bar{v}$) IN THE FIXED (GLOBAL) COORDINATE SYSTEM.

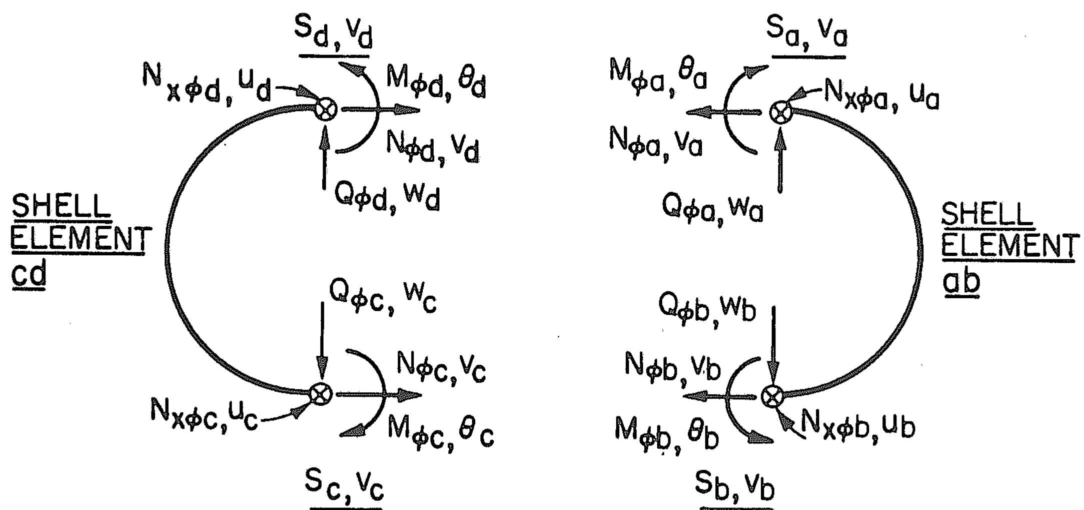


FIG. 6 INTERNAL SHELL ELEMENT EDGE FORCES AND DISPLACEMENT (S AND v) IN THE ELEMENT (LOCAL) COORDINATE SYSTEM.

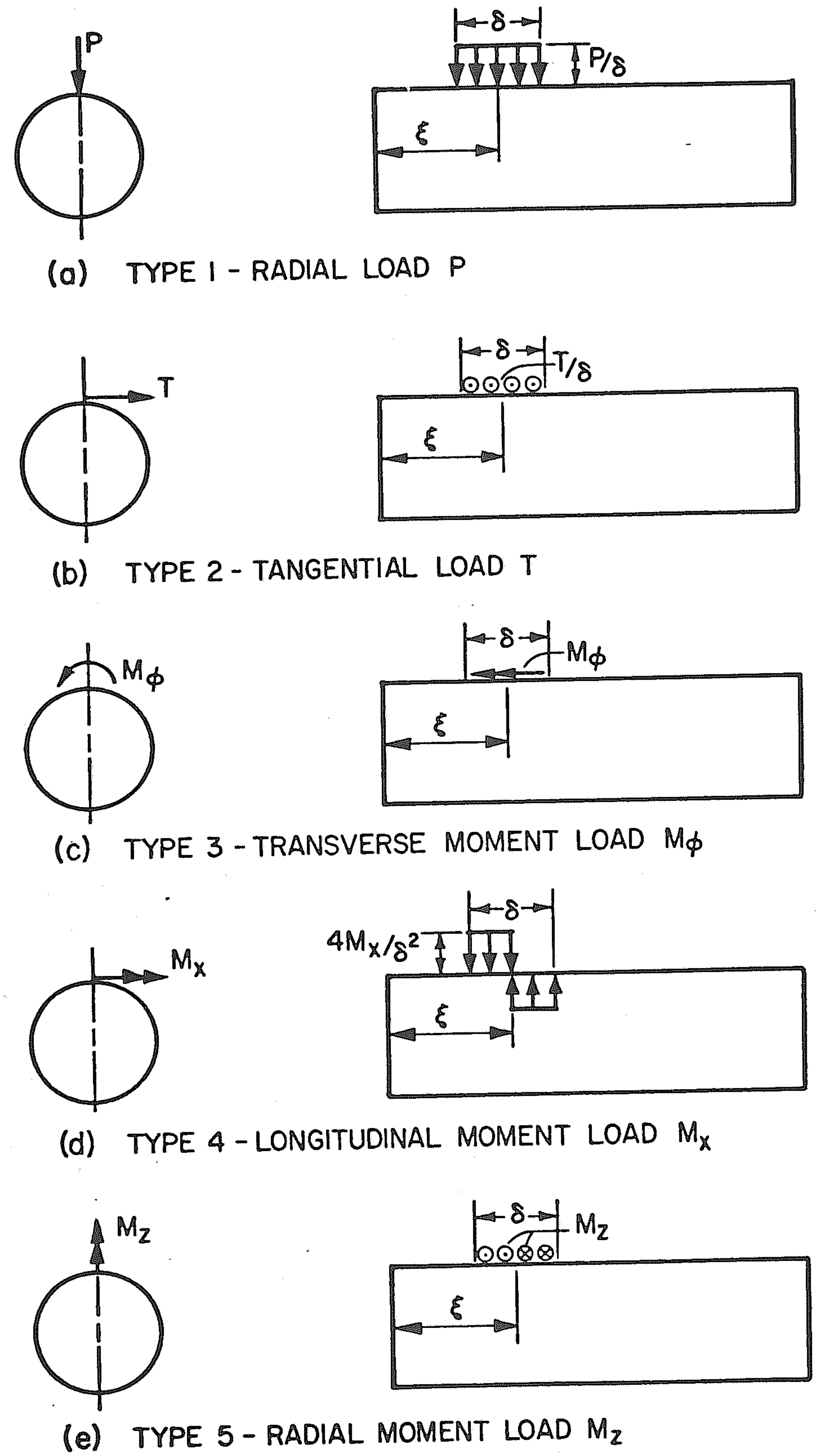
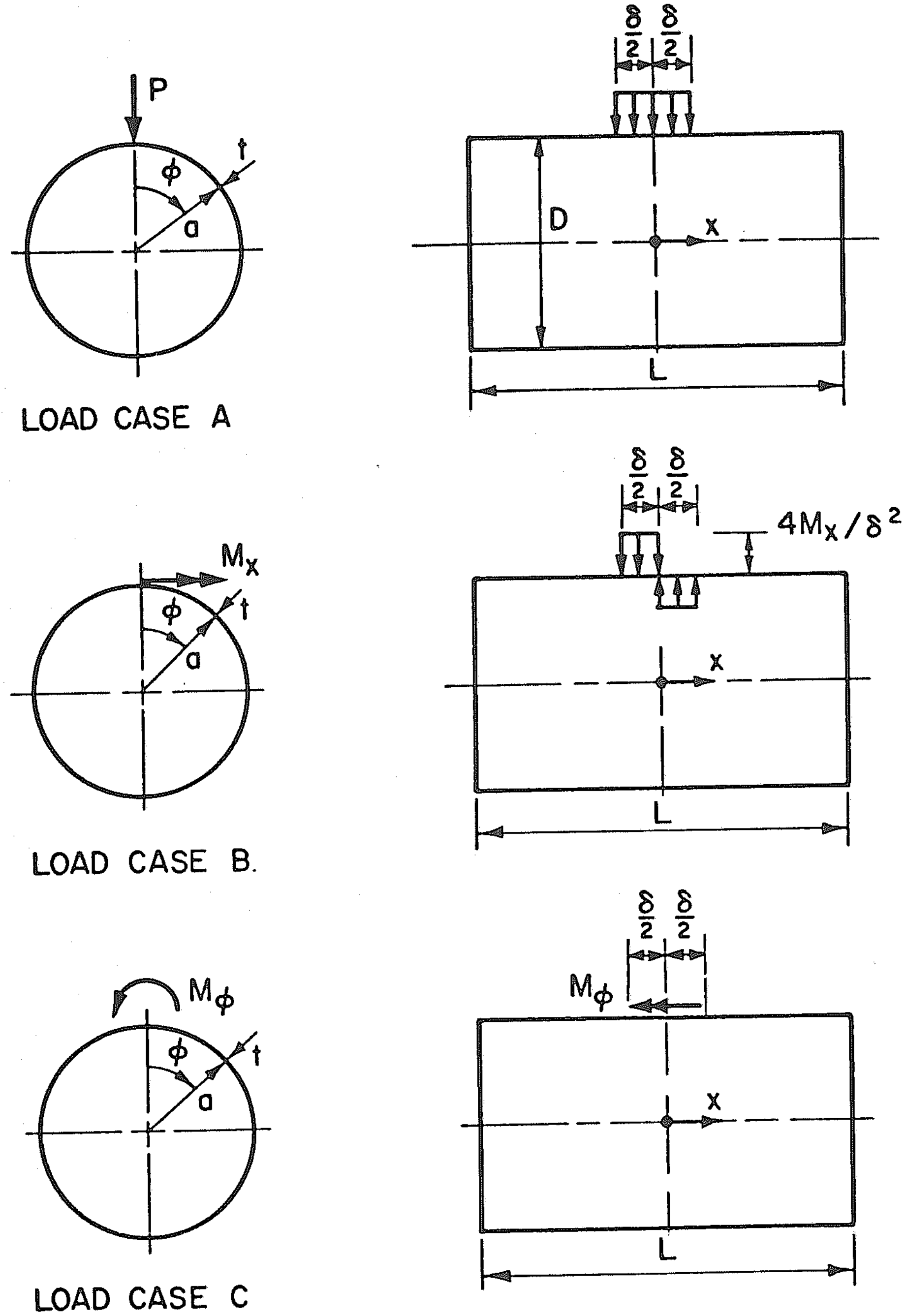


FIG. 7 TYPES OF LOADING



GENERAL: $L = 100$, $a = 100$
$t = 1.0$, $\delta/L = 0.05$ or $\delta = 5$
$E = 10^6$, $\nu = 0.3$

FIG. 8 COMPARATIVE STUDY I - THREE LOAD CASES

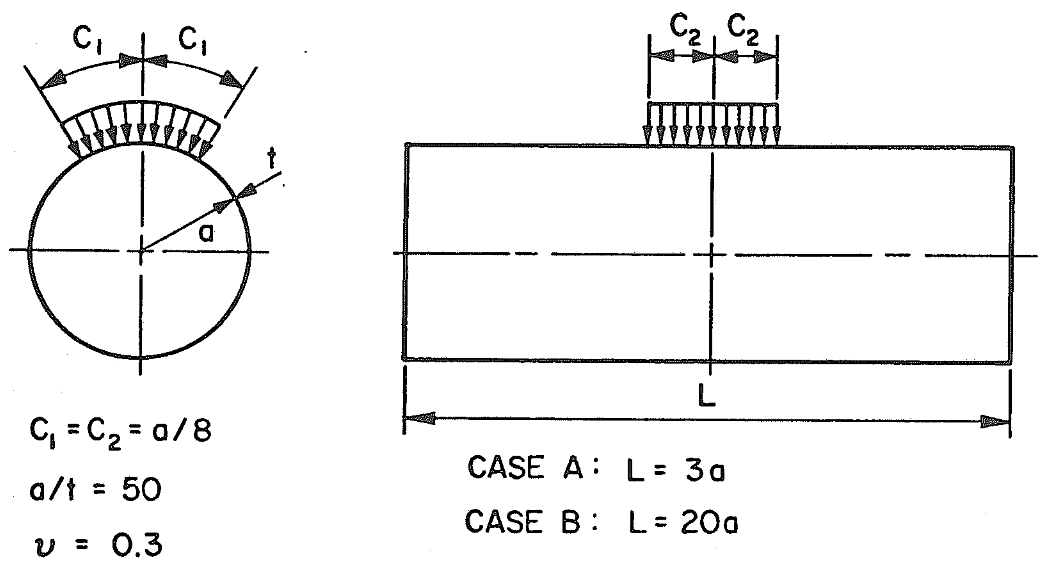


FIG. 9 COMPARATIVE STUDY 2 - PAD LOADING

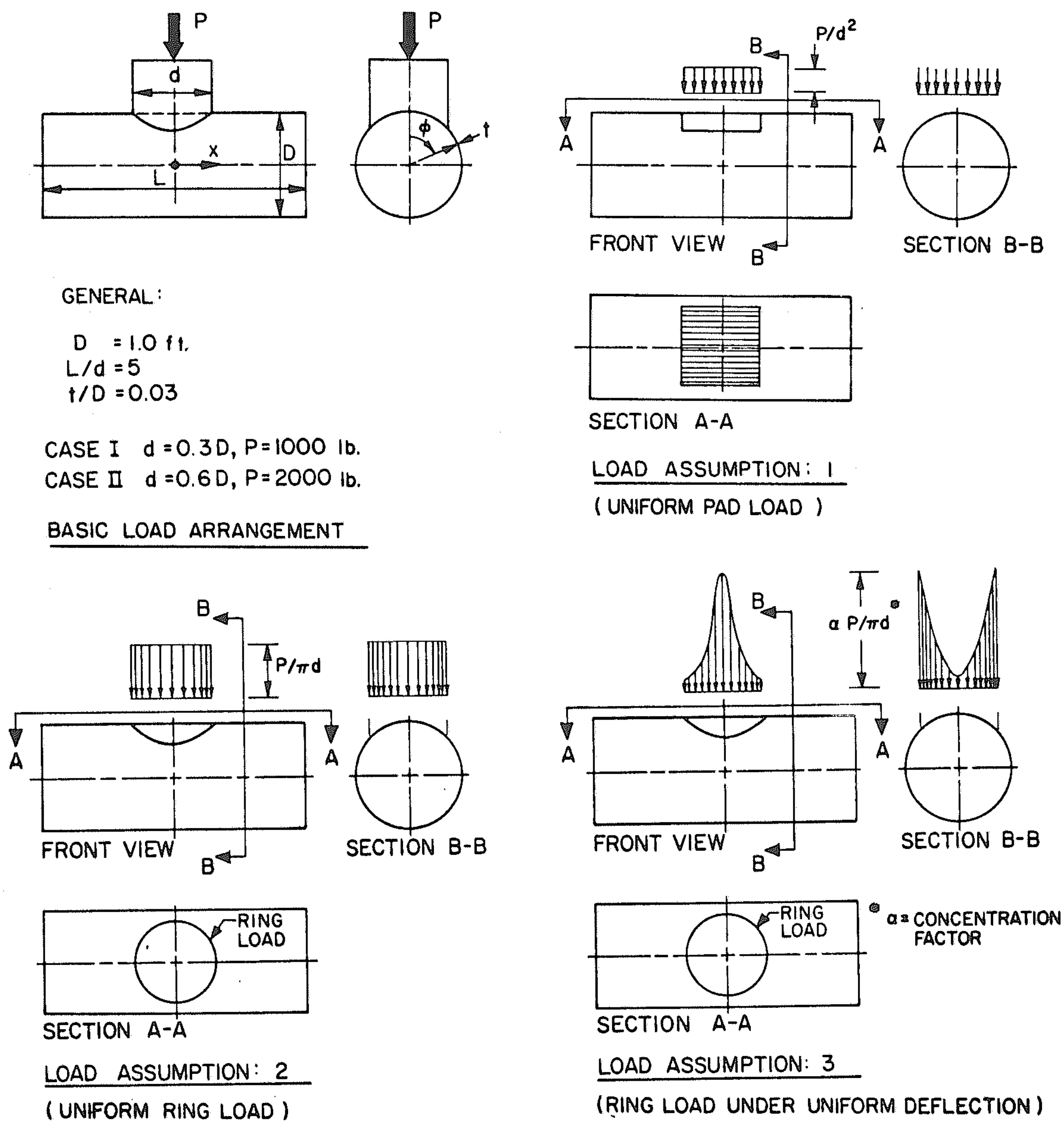


FIG. 10 LOAD ASSUMPTIONS FOR T-TYPE CONNECTION

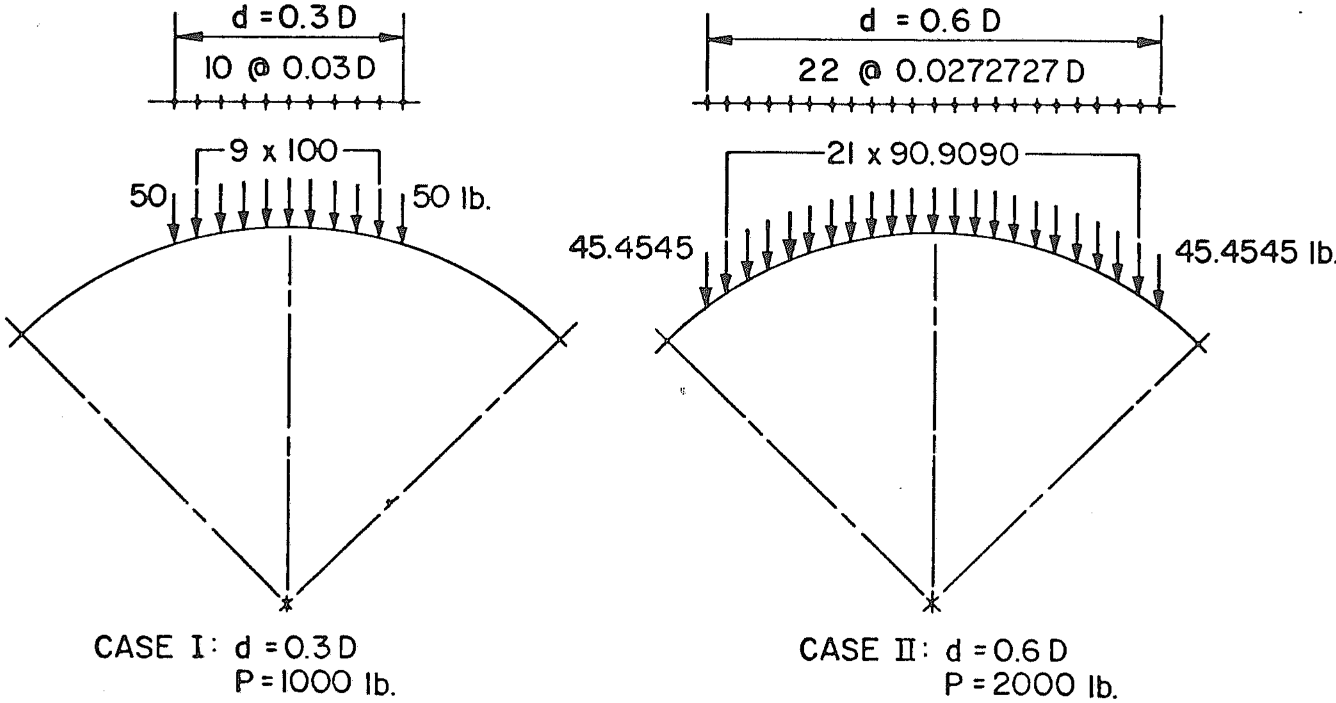
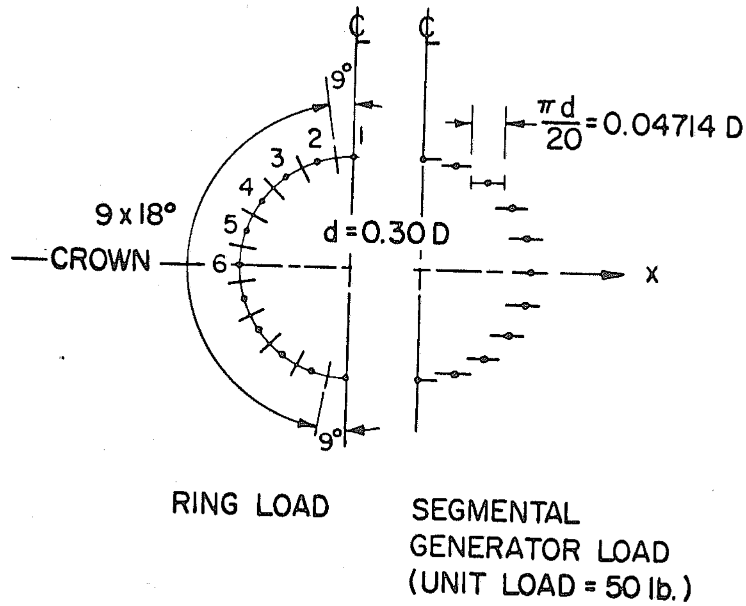
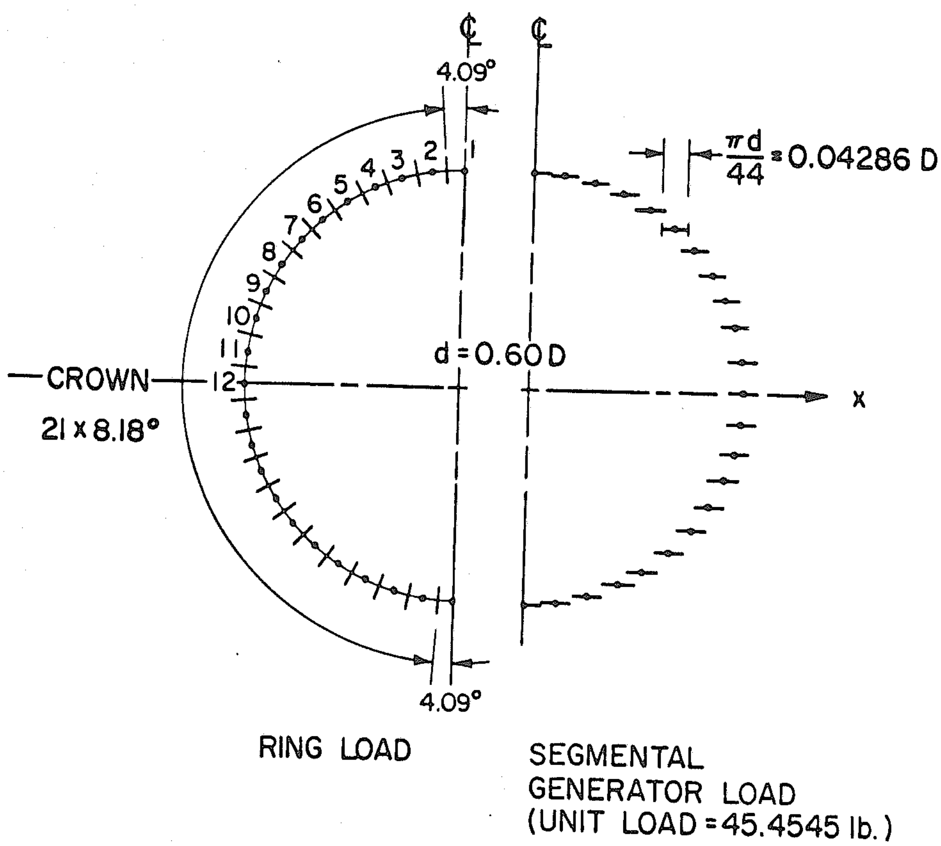


FIG. 11 GENERATOR LOAD DISTRIBUTION FOR LOAD ASSUMPTION I - PAD LOAD



CASE I: d = 0.3 D
P = 1000 lb.



CASE II: d = 0.60 D
P = 2000 lb.

FIG. 12 GENERATOR LOAD DISTRIBUTION FOR LOAD ASSUMPTION 2
UNIFORM RING LOAD

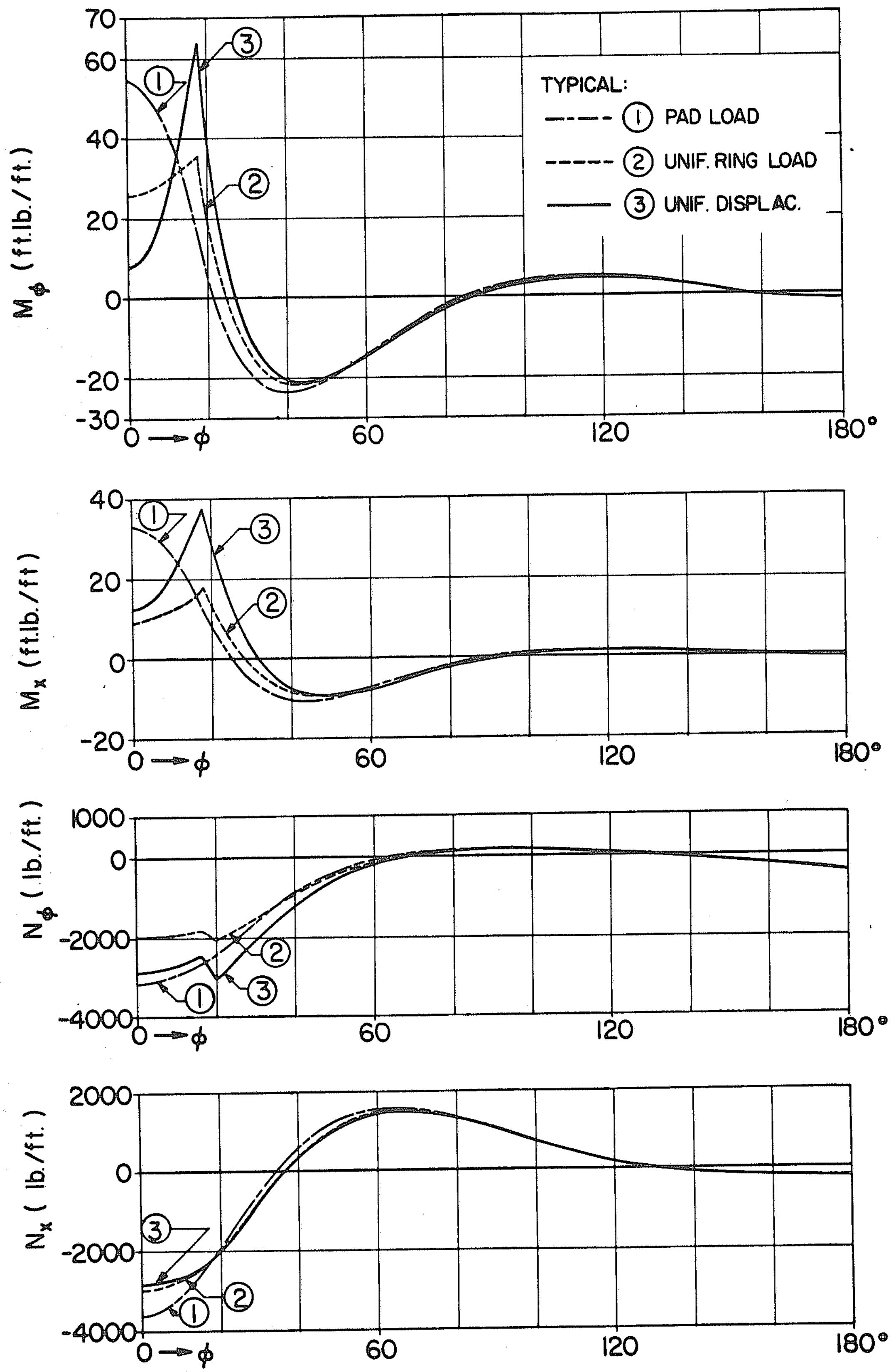


FIG. 13 M_ϕ, M_x, N_ϕ, N_x vs ϕ @ $x = 0$ FOR THREE DIFFERENT LOADS ($d = 0.3D$)

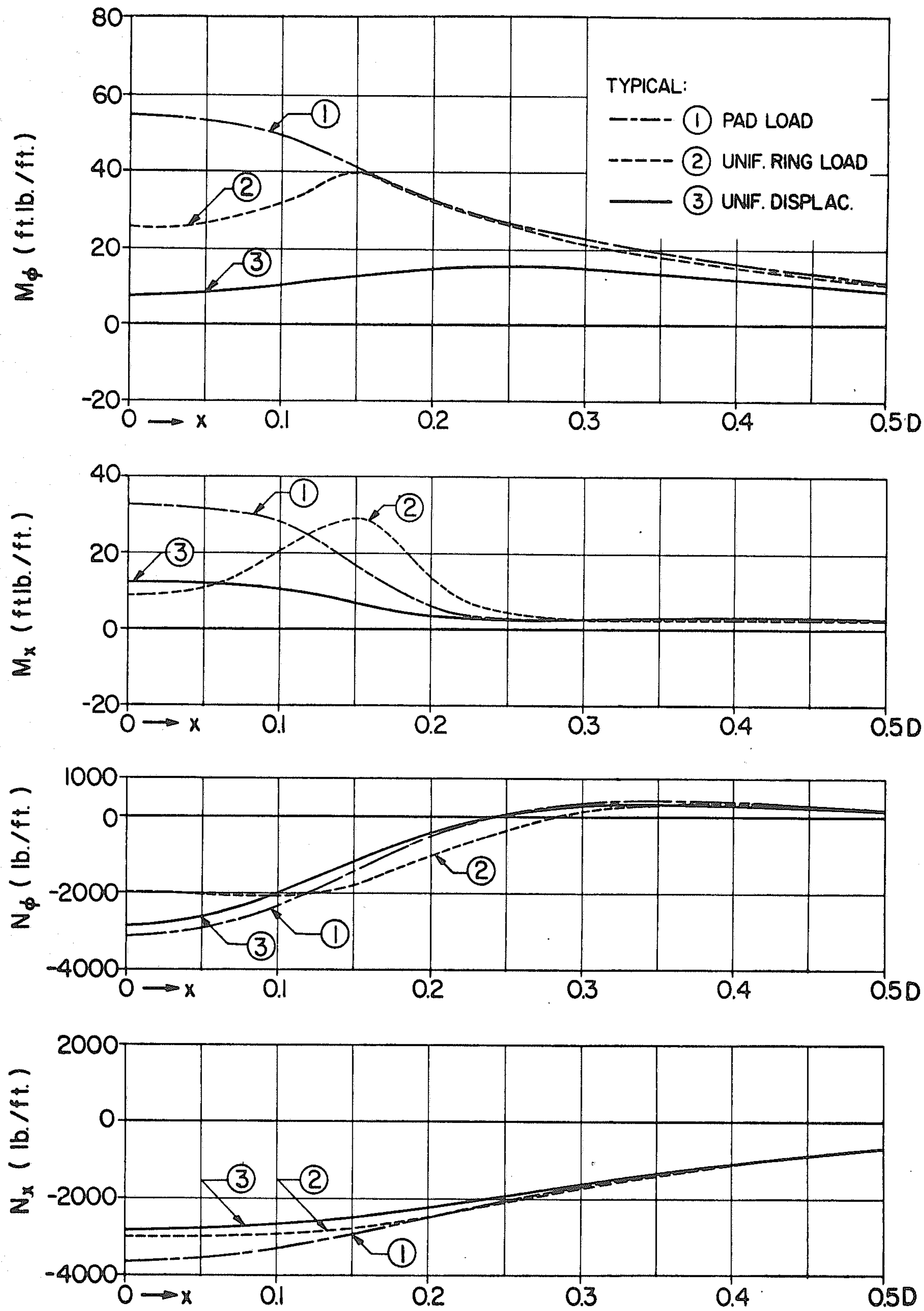


FIG. 14 M_ϕ, M_x, N_ϕ, N_x vs x @ ϕ=0 FOR THREE DIFFERENT LOADS (d = 0.3 D)

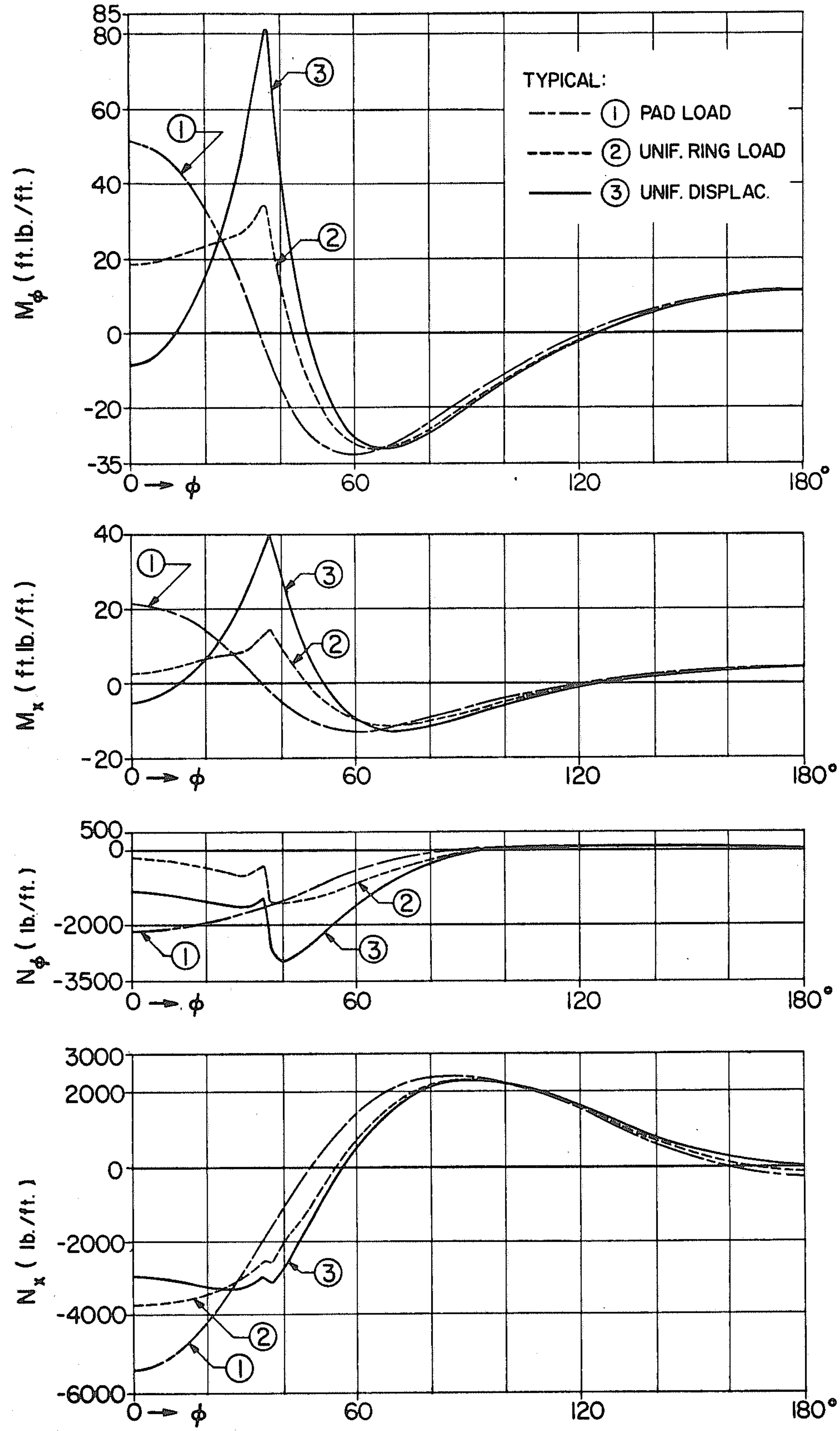


FIG. 15 M_ϕ, M_x, N_ϕ, N_x vs ϕ @ x=0 FOR THREE DIFFERENT LOADS (d=0.6D)

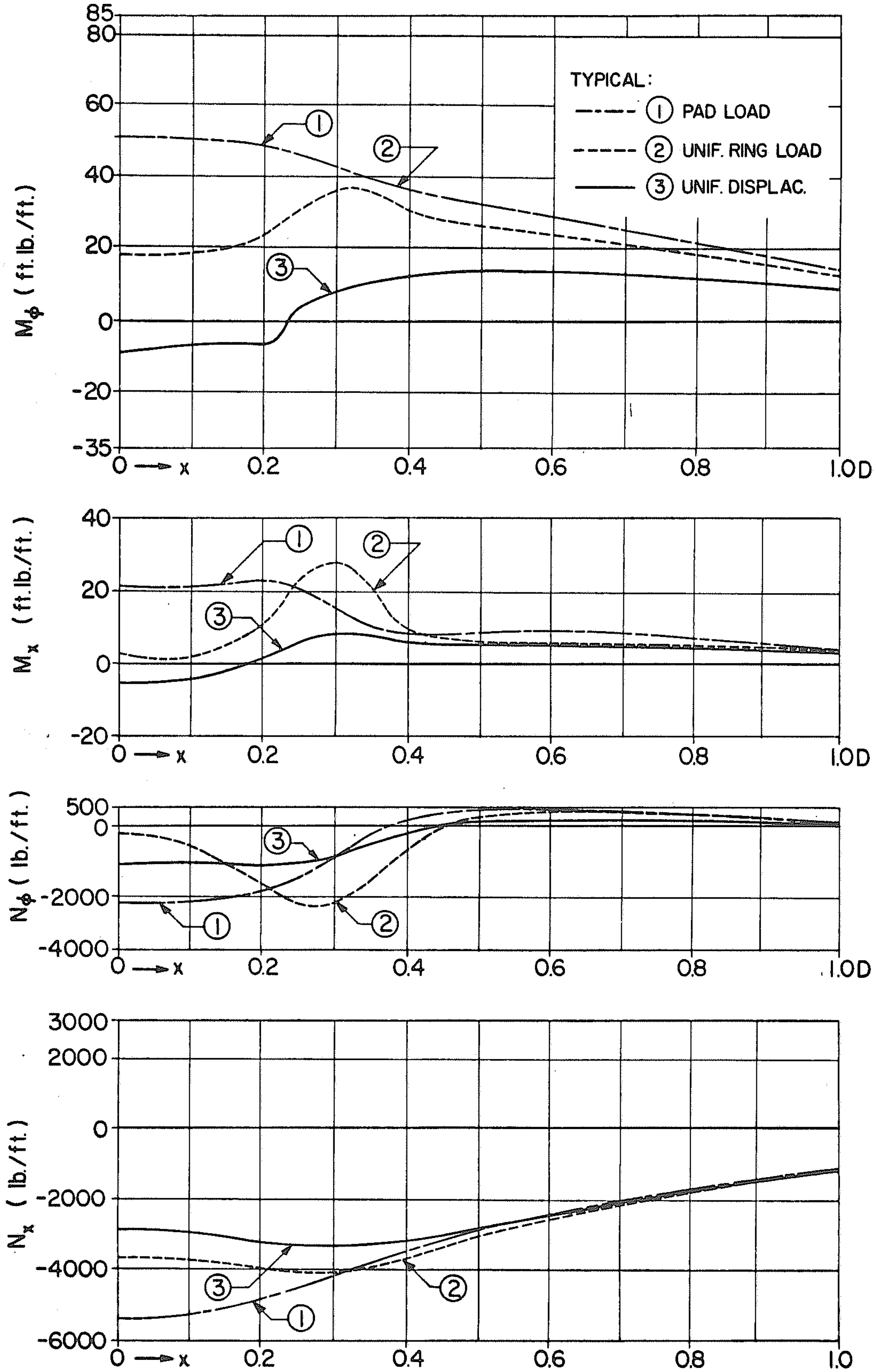


FIG. 16 M_ϕ, M_x, N_ϕ, N_x vs x @ $\phi = 0$ FOR THREE DIFFERENT LOADS (d = 0.6D)

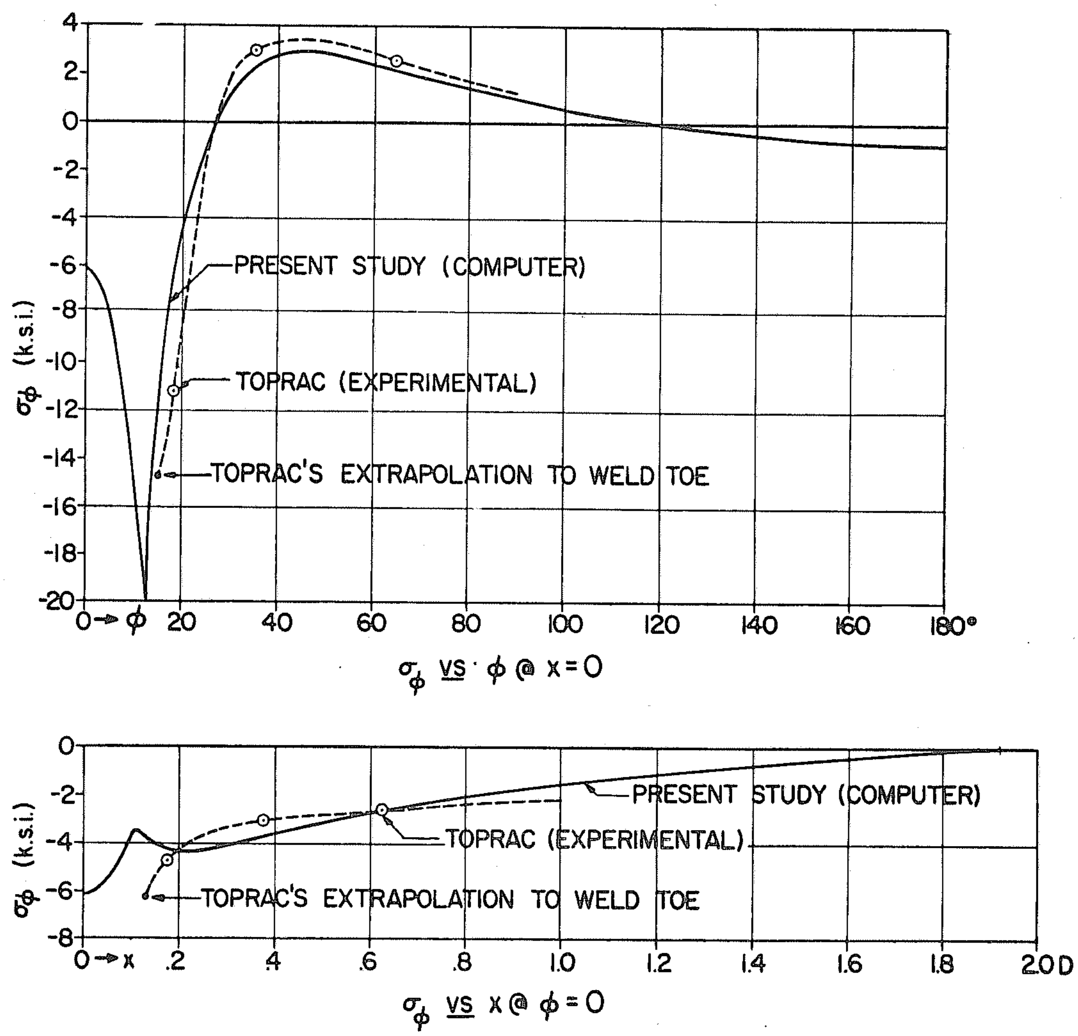
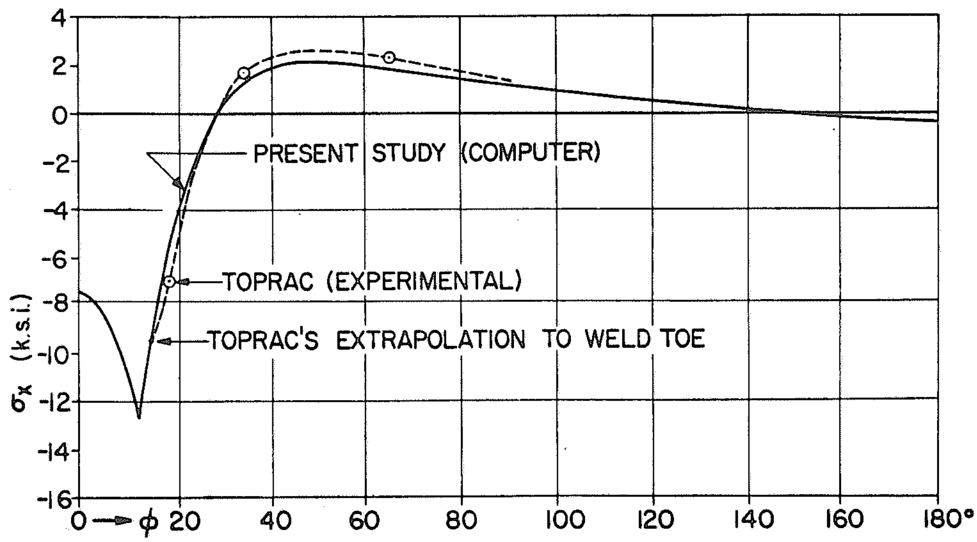
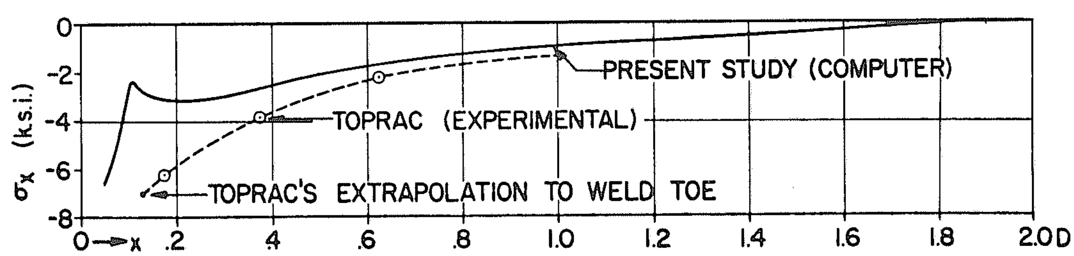


FIG. 17 CIRCUMFERENTIAL SURFACE STRESSES σ_ϕ



σ_x vs ϕ @ $x = 0$



σ_x vs x @ $\phi = 0$

FIG. 18 LONGITUDINAL SURFACE STRESSES σ_x

UNIVERSITY OF CALIFORNIA

Department of Civil Engineering
A. C. Scordelis and J. G. Bouwkamp

Notes on IBM 7094 Program 3 for Tubular Joint Analysis

1. IDENTIFICATION

CIR - 3: Solution of circular cylindrical shells with live loads applied along generators--Program 3--Nov. 1964 (vertical, horizontal and M(phi) loads only).

2. REFERENCES

a. "Analytical Study of Tubular Tee-Joints" by A. C. Scordelis and J. G. Bouwkamp.
b. "The Design of Cylindrical Shell Proofs" by J. E. Gibson, 2nd Ed. D. Van Nostrand Co., Inc., Princeton, N.J., 1961.

3. FORM OF INPUT

a. See attached FORTRAN II listing of program for detailed description of input.
b. Additional explanation of input is as follows:

(1) First Card Any 72 characters may be used for title.

(2) Second Card

Col. 51 - 54 Fourier series limit--the number of the highest harmonic term of the series to be used to represent the longitudinal distribution of loading; maximum number is 100 for unsymmetrical loads with respect to midspan and 200 for symmetrical or antisymmetrical loads if non-contributing harmonics are skipped in cols. 68 or 70.

Col. 55 - 58 Numpin--total number of times results are to be printed, one for each Fourier series number specified in fourth card. If Numpin is set at 0, results will be printed only after Fourier series limit has been reached.

Col. 59 - 62 and Col. 63 - 66 Number of cross-sections and number of angle sections refer to the total number of x and ϕ coordinates respectively at which output results are to be calculated.

Col. 68 for loadings antisymmetric about midspan, the odd Fourier terms can be skipped.

Col. 70 for loadings symmetric about midspan, the even Fourier terms can be skipped.

(3) Next Card The disturbance limit should be given a value of 1.0 E-16.

(4) <u>Next Card</u> If Numpin is set to zero on second card, skip this card and printout of results will be given only after Fourier series limit has been reached.

4. FORM OF OUTPUT

a. Input data is printed as a check.

b. Final internal forces and displacements are printed for each point (x,ϕ) designated in input. See references 2a and 2b for detailed description of following output quantities.

1) M_ϕ = Transverse moment per unit length
2) M_x = Longitudinal moment per unit length
3) Q_x = Normal shear on transverse section per unit length
4) N_ϕ = Transverse membrane force per unit length
5) N_x = Longitudinal membrane force per unit length
6) U = Longitudinal displacement
7) W = Radial displacement
8) Q_ϕ = Normal shear on longitudinal section
9) $Q'_\phi = Q_\phi + \partial M_{x\phi}/\partial x$ = Modified normal shear per unit length
10) $N_{x\phi}$ = Membrane shear per unit length
11) V = Transverse displacement
12) θ = Rotation about longitudinal axis
13) $M_{x\phi}$ = Torsional moment per unit length

```
*       DECKS
        LIST
        LABEL
*       FORTRAN
CCIR-3    (OLD 2A)
C
C  UNIVERSITY OF CALIFORNIA                        DEPT. OF CIVIL ENGINEERING
C
C     SOLUTION OF CIRCULAR CYLINDRICAL SHELLS WITH LINE LOADS APPLIED ALONG
C        GENERATORS --- PROGRAM 3 ---    NOV. 1964
C        VERT. HORIZ. AND M(PHI) LINE LOADS ONLY
C
C  PROGRAMMED BY..   K.S. LO
C  FACULTY INVESTIGATORS..   A.C. SCORDELIS AND J.G. BOUWKAMP
C
C     INPUT DATA
C
C     FIRST CARD - TITLE OF THE PROBLEM
C
C     SECOND CARD - COL.  1 TO 10 - SPAN (F10.0)
C                   COL. 11 TO 20 - RADIUS OF SHELL (F10.0)
C                   COL. 21 TO 30 - SHELL THICKNESS (F10.0)
C                   COL. 31 TO 40 - MODULUS OF ELASTICITY (F10.0)
C                   COL. 41 TO 50 - POISSON RATIO (F10.0)
C                   COL. 51 TO 54 - FOURIER SERIES LIMIT (I4)
C                   COL. 55 TO 58 - NUMPIN
C                                   NUMBER OF PRINT OF RESULTS, MAX. 100 (I4)
                    COL. 59 TO 62 - NUMBER OF CROSS-SECTIONS, MAX. 35 (I4)
                    COL. 63 TO 66 - NUMBER OF ANGLE SECTIONS, MAX. 35 (I4)
C                   COL.       68 - BLANK TO INCLUDE ODD FOURIER SERIES
C                                   1 TO SKIP ODD FOURIER SERIES
C                   COL.       70 - BLANK TO INCLUDE EVEN FOURIER SERIES
C                                   1 TO SKIP EVEN FOURIER SERIES
C
C     NEXT CARD - COL.  1 TO 10 - DISTURBANCE LIMIT (E10.3)
C                                 IF EXPF(-ALPHA*PHIK) IS LESS THAN THIS LIMIT
C                                 DISTURBANCE FROM OTHER JOINT IS NEGLECTED
C                 COL. 13 TO 14 - NUMBER OF GENERATORS ALONG WHICH LINE LOADS
C                                 ARE APPLIED, MAX. 42 (I2)
C
C     NEXT CARD - EXISTS ONLY IF 'NUMPIN' IS NOT ZERO
C                 FOURIER SERIES NUMBERS (18I4)
C                 RESULTS WILL BE PRINTED AFTER THESE SERIES NUMBERS
C                 USE NEXT CARDS IF NEEDED
C
C     NEXT CARD - X-COORDINATES AT WHICH RESULTS ARE DESIRED (9F8.0)
C                 USE NEXT CARDS IF NEEDED
C
C     NEXT CARD - PHI ANGLES (IN DEGREES) AT WHICH RESULTS ARE DESIRED
C                 (9F8.0), USE NEXT CARDS IF NEEDED
C
C     NEXT CARDS - LOADING DETAILS
C       FOR EACH LOADED GENERATOR, 3 OR MORE CARDS ARE REQUIRED
C          1 - COL.  1 TO 10 - PHI ANGLE (IN DEGREES) OF GENERATOR ALONG
                               WHICH LOADS ARE APPLIED (F10.0)
```

```
C
C          2 - COL.  1 TO  3 - NUMBER OF VERT. LOADS, MAX. 10 (I3)
C              COL.  4 TO  6 - NUMBER OF HORIZ. LOADS, MAX. 10 (I3)
C              COL.  7 TO  9 - NUMBER OF MOMENTS M(PHI), MAX. 10 (I3)
C
C          3 - COL.  1 TO 10 - MAGNITUDE (TOTAL FORCE OR MOMENT) (F10.0)
C              COL. 11 TO 20 - DISTRIBUTED WIDTH (F10.0)
C              COL. 21 TO 30 - LOCATION FROM LEFT END (F10.0)
C        ONE SUCH A CARD (NO.3) FOR EACH LOAD, INPUT ALL VERT. LOADS FIRST AND
C        THEN HORIZ. LOADS, AND MOMENTS M(PHI) IN THAT ORDER.
C      CARDS AS ABOVE ARE REPEATED FOR EACH LOADED GENERATOR
C
C      ALL ABOVE DATA CARDS ARE REPEATED FOR NEXT PROBLEM TO BE SOLVED
C
C      TWO BLANK CARDS ARE ADDED AT END OF THE DATA DECK
C
C
C      DIMENSION AND COMMON STATEMENT
C
      DIMENSION A1(35,35),A2(35,35),A3(35,35),A4(35,35),A5(35,35),
     1 A6(35,35),A7(35,35),A8(35,35),A9(35,35),A10(35,35),A11(35,35),
     2 A12(35,35),A13(35,35),XINT(35),YANG(35),M1(7),M2(7),TITLE(12),
     3 NPIN(102),YINT(42,35),IND(42,35),VL(42,10),HL(42,10),T3(42,10),
     4 DEL1(42,10),DEL2(42,10),DEL3(42,10),XI1(42,10),XI2(42,10),
     5 XI3(42,10),LL1(42),LL2(42),LL3(42),GANG(42),
     6                           DT(3),SERIES(35),SINKX(35),COSKX(35),
     7 SK(8,8),B(14,4),ADEL1(8,8),ADEL2(8,8),P(8),D(8),ZX(8),ZY(8),
     8 C1(8),C2(8),B1(13,8),B2(13,8),CE(8,35),AB(13,35)
      COMMON A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A12,A13,XINT,YANG,M1,M2
C
C      FORMAT STATEMENT
C
   11 FORMAT (12A6)
   12 FORMAT (5F10.0,4I4,2I2)
   13 FORMAT (18I4)
   14 FORMAT (9F8.0)
   15 FORMAT (2H1 )
   16 FORMAT (15H0SPAN LENGTH = F8.3/19H0RADIUS OF SHELL = F7.3/22H0THIC
     1KNESS OF SHELL = F9.6)
   17 FORMAT (25H0MODULUS OF ELASTICITY = E14.6/17H0POISSON RATIO = F6.4
     1/24H0FOURIER SERIES LIMIT = I3)
   18 FORMAT (39H0PRINT RESULTS AFTER FOURIER SERIES NO.)
   19 FORMAT (46H0PRINT RESULTS AT CROSS-SECTIONS OF X EQUAL TO)
   20 FORMAT (50H0PRINT RESULTS AT PHI (IN DEGREES) ANGLES EQUAL TO )
   21 FORMAT (10F12.4)
   22 FORMAT (//41H0CALCULATIONS SKIP ALL ODD FOURIER SERIES)
   23 FORMAT (//42H0CALCULATIONS SKIP ALL EVEN FOURIER SERIES)
   24 FORMAT (5I3)
   25 FORMAT (3F10.0)
   26 FORMAT (3E15.6)
   27 FORMAT (//54H0INPUT LOADS AT GENERATOR OF PHI ANGLE (IN DEGREES) =
     1  F12.6)
   28 FORMAT (14H0VERTICAL LOAD)
   29 FORMAT (59H      MAGNITUDE            WIDTH     LOCATION (FROM LEFT
     1END))
```

```
   30 FORMAT (16HOHORIZONTAL LOAD)
   31 FORMAT (14HOMOMENT M(PHI))
   50 FORMAT (1E10.3,I4)
   51 FORMAT (21HODISTURBANCE LIMIT = E10.3//)
   52 FORMAT (67H1DISTURBANCE FROM OTHER JOINT IS NEGLECTED AFTER FOURIE
     1R SERIES NO. I4)
   53 FORMAT (22HOAT FOURIER SERIES NO. I4/24HO   EXPF(-ALPHA1*PHIK) =
     1 E15.6/24HO   EXPF(-ALPHA3*PHIK) = E15.6)
C
C     READ, PRINT AND MODIFY INPUT DATA
C
  101 READ 11, (TITLE(I),I=1,12)
      READ 12, SPAN,R,TH,E,FNU,MAXSER,NUMPIN,NUMX,NUMY,NODD,NEVEN
      IF (SPAN) 999,999,102
  102 PRINT 15
      PRINT 11, (TITLE(I),I=1,12)
      PRINT 16, SPAN,R,TH
      PRINT 17, E,FNU,MAXSER
      READ  50, DT(1),NGEN
      PRINT 51, DT(1)
      IF (NUMPIN) 104,104,103
  103 READ 13, (NPIN(I),I=1,NUMPIN)
      PRINT 18
      PRINT 13, (NPIN(I),I=1,NUMPIN)
  104 J=NUMPIN+1
      NPIN(J)=MAXSER
      READ 14, (XINT(I),I=1,NUMX)
      READ 14, (YANG(I),I=1,NUMY)
      PRINT 19
      PRINT 21, (XINT(I),I=1,NUMX)
      PRINT 20
      PRINT 21, (YANG(I),I=1,NUMY)
      IF (NODD) 106,106,105
  105 PRINT 22
  106 IF (NEVEN) 108,108,107
  107 PRINT 23
  108 PI=3.141592654
      PIL=PI/SPAN
      PRINT 15
      DO 125 J=1,NGEN
      READ 25, GANG(J)
      PRINT 27, GANG(J)
      READ 24, L1,L2,L3
      LL1(J)=L1
      LL2(J)=L2
      LL3(J)=L3
      IF (L1) 113,113,111
  111 READ  25, (VL(J,I),DEL1(J,I),XI1(J,I),I=1,L1)
      PRINT 28
      PRINT 29
      PRINT 26, (VL(J,I),DEL1(J,I),XI1(J,I),I=1,L1)
      DO 112 I=1,L1
      VL(J,I)=4.*VL(J,I)/(PI*DEL1(J,I))
      DEL1(J,I)=0.5*DEL1(J,I)*PIL
  112 XI1(J,I)=XI1(J,I)*PIL
```

```
  113 IF (L2) 116,116,114
  114 READ  25, (HL(J,I),DEL2(J,I),XI2(J,I),I=1,L2)
      PRINT 30
      PRINT 29
      PRINT 26, (HL(J,I),DEL2(J,I),XI2(J,I),I=1,L2)
      DO 115 I=1,L2
      HL(J,I)=4.*HL(J,I)/(PI*DEL2(J,I))
      DEL2(J,I)=0.5*DEL2(J,I)*PIL
  115 XI2(J,I)=XI2(J,I)*PIL
  116 IF (L3) 125,125,117
  117 READ  25, (T3(J,I),DEL3(J,I),XI3(J,I),I=1,L3)
      PRINT 31
      PRINT 29
      PRINT 26, (T3(J,I),DEL3(J,I),XI3(J,I),I=1,L3)
      DO 118 I=1,L3
      T3(J,I)=4.*T3(J,I)/(PI*DEL3(J,I))
      DEL3(J,I)=0.5*DEL3(J,I)*PIL
  118 XI3(J,I)=XI3(J,I)*PIL
  125 CONTINUE
      DO 110 J=1,NGEN
      DO 110 I=1,NUMY
      YNG=YANG(I)-GANG(J)
      IF (YNG) 305,301,301
  301 X=YNG-180.
      IF (X) 302,302,303
  302 YINT(J,I)=PI*(0.5-YNG/180.)
      IND(J,I)=1
      GO TO 110
  303 YINT(J,I)=PI*(0.5-X/180.)
      IND(J,I)=0
      GO TO 110
  305 X=YNG+180.
      IF (X) 307,306,306
  306 GO TO 303
  307 X=-X-90.
      YINT(J,I)=PI*(X/180.)
      IND(J,I)=1
  110 CONTINUE
      DO 308 I=1,NGEN
  308 GANG(I)=PI*(GANG(I)/180.)
C
C     OUTPUT IS CLEARED
C
      DO 131 I=1,NUMY
      DO 131 J=1,NUMX
      A1(I,J)=0.
      A2(I,J)=0.
      A3(I,J)=0.
      A4(I,J)=0.
      A5(I,J)=0.
      A6(I,J)=0.
      A7(I,J)=0.
      A8(I,J)=0.
      A9(I,J)=0.
      A10(I,J)=0.
```

```
      A11(I,J)=0.
      A12(I,J)=0.
  131 A13(I,J)=0.
      DO 132 I=1,7
      M2(I)=I*6
  132 M1(I)=M2(I)-5
      I=1
  133 IF (M2(I)-NUMX) 134,135,135
  134 I=I+1
      GO TO 133
  135 M2(I)=NUMX
      MM=I
C
C     COEFFICIENTS PI*X/L ARE COMPUTED
C
      DO 136 I=1,NUMX
  136 SERIES(I)=PIL*XINT(I)
C
C     CYCLE FOR EACH HARMONIC IS INITIATED
C
      LDT = 0
      LL=1
      LPIN=NPIN(1)
      DO 700 NN=1,MAXSER
      FQ=(-1.)**(NN+1)
      IF (FQ) 137,137,138
  137 IF (NEVEN) 139,139,650
  138 IF (NODD) 139,139,650
C
C     SPAN AND K ARE GENERALIZED
C
  139 FN=NN
      FK=FN*PIL
C
C     STIFFNESS MATRIX IS COMPUTED BY SUBROUTINE STIMAT
C
      CALL STIMAT (FK,FQ,R,TH,E,FNU,SK,B,ADEL1,ADEL2,LDT,DT)
      AL1=B(14,1)
      AL3=B(14,2)
      BE1=B(14,3)
      BE3=B(14,4)
      CALL INVERT (SK,8,8,ZX,ZY)
C
C     CALCULATE SINKX AND COSKX MATRICES
C
      IF (FQ) 160,160,165
  160 N=NN/2
      DO 162 I=1,NUMX
      X=FN*SERIES(I)
      SINKX(I)=COSF(X)
  162 COSKX(I)=SINF(X)
      GO TO 170
  165 N=(NN+3)/2
      DO 167 I=1,NUMX
      X=FN*SERIES(I)
```

```
      SINKX(I)=SINF(X)
  167 COSKX(I)=COSF(X)
  170 X=(-1.)**N
C
C     CALCULATE INPUT LOADS FOR EACH LOADED GENERATOR
C
      DO 200 J=1,NGEN
      L1=LL1(J)
      L2=LL2(J)
      L3=LL3(J)
      TS=SINF(GANG(J))
      TC=COSF(GANG(J))
      DO 140 I=1,8
  140 P(I)=0.
      IF (L1) 143,143,141
  141 DO 142 I=1,L1
      SD=SINF(FN*DEL1(J,I))
      SX=SINF(FN*XI1(J,I))
      T4=(VL(J,I)/FN)*SD*SX
      P(1)=P(1)+T4*TS
  142 P(2)=P(2)+T4*TC
  143 IF (L2) 146,146,144
  144 DO 145 I=1,L2
      SD=SINF(FN*DEL2(J,I))
      SX=SINF(FN*XI2(J,I))
      T4=(HL(J,I)/FN)*SD*SX
      P(2)=P(2)-T4*TS
  145 P(1)=P(1)+T4*TC
  146 IF (L3) 155,155,147
  147 DO 148 I=1,L3
      SD=SINF(FN*DEL3(J,I))
      SX=SINF(FN*XI3(J,I))
  148 P(3)=P(3)+(T3(J,I)/FN)*SD*SX
  155 CONTINUE
      DO 156 I=1,3
  156 P(I)=P(I)*X
C
C     UNKNOWN JOINT DISPLACEMENTS ARE COMPUTED
C
      DO 171 I=1,8
      D(I)=0.
      DO 171 K=1,3
  171 D(I)=D(I)+SK(I,K)*P(K)
C
C     CALCULATE ARBITRARY CONSTANTS FOR SHELLS
C
      DO 185 I=1,8
      C1(I)=0.
      C2(I)=0.
      DO 185 K=1,8
      C1(I)=C1(I)+ADEL1(I,K)*D(K)
  185 C2(I)=C2(I)+ADEL2(I,K)*D(K)
C
C     TO FIND MAX. INTERNAL FORCES AT DIFFERENT PHI ANGLES
C
```

```
      DO 190 I=1,13
      B1(I,1)=C1(1)*B(I,1)-C1(2)*B(I,2)
      B1(I,2)=C1(1)*B(I,2)+C1(2)*B(I,1)
      B1(I,3)=C1(3)*B(I,3)-C1(4)*B(I,4)
      B1(I,4)=C1(3)*B(I,4)+C1(4)*B(I,3)
      B1(I,5)=C1(5)*B(I,1)-C1(6)*B(I,2)
      B1(I,6)=C1(5)*B(I,2)+C1(6)*B(I,1)
      B1(I,7)=C1(7)*B(I,3)-C1(8)*B(I,4)
      B1(I,8)=C1(7)*B(I,4)+C1(8)*B(I,3)
      B2(I,1)=C2(1)*B(I,1)-C2(2)*B(I,2)
      B2(I,2)=C2(1)*B(I,2)+C2(2)*B(I,1)
      B2(I,3)=C2(3)*B(I,3)-C2(4)*B(I,4)
      B2(I,4)=C2(3)*B(I,4)+C2(4)*B(I,3)
      B2(I,5)=C2(5)*B(I,1)-C2(6)*B(I,2)
      B2(I,6)=C2(5)*B(I,2)+C2(6)*B(I,1)
      B2(I,7)=C2(7)*B(I,3)-C2(8)*B(I,4)
  190 B2(I,8)=C2(7)*B(I,4)+C2(8)*B(I,3)
      DO 191 I=8,13
      DO 191 K=5,8
      B1(I,K)=-B1(I,K)
  191 B2(I,K)=-B2(I,K)
      DO 192 I=1,NUMY
      PHI=YINT(J,I)
      CC=COSF(BE1*PHI)
      C3=COSF(BE3*PHI)
      S1=SINF(BE1*PHI)
      S3=SINF(BE3*PHI)
      IF (LDT-1) 351,351,350
  350 PHJ= PHI-1.570796327
      E1=EXPF(AL1*PHJ)
      E2=EXPF(AL3*PHJ)
      PHJ=-PHI-1.570796327
      E3=EXPF(AL1*PHJ)
      E4=EXPF(AL3*PHJ)
      GO TO 352
  351 E1=EXPF(AL1*PHI)
      E2=EXPF(AL3*PHI)
      E3=EXPF(-AL1*PHI)
      E4=EXPF(-AL3*PHI)
  352 CE(1,I)= CC*E1
      CE(2,I)=-S1*E1
      CE(3,I)= C3*E2
      CE(4,I)=-S3*E2
      CE(5,I)= CC*E3
      CE(6,I)= S1*E3
      CE(7,I)= C3*E4
  192 CE(8,I)= S3*E4
      DO 197 IJ=1,NUMY
      IF (IND(J,IJ)) 195,195,193
  193 DO 194 I=1,13
      AB(I,IJ)=0.
      DO 194 K=1,8
  194 AB(I,IJ)=AB(I,IJ)+B1(I,K)*CE(K,IJ)
      GO TO 197
  195 DO 196 I=1,13
```

```
      AB(I,IJ)=0.
      DO 196 K=1,8
  196 AB(I,IJ)=AB(I,IJ)+B2(I,K)*CE(K,IJ)
  197 CONTINUE
C
C      INTERNAL FORCES ARE COMPUTED AND ADDED
C
      DO 200 I=1,NUMY
      DO 200 K=1,NUMX
      A1(I,K)=A1(I,K)+AB(1,I)*COSKX(K)
      A2(I,K)=A2(I,K)+AB(2,I)*COSKX(K)
      A3(I,K)=A3(I,K)+AB(3,I)*SINKX(K)
      A4(I,K)=A4(I,K)+AB(4,I)*COSKX(K)
      A5(I,K)=A5(I,K)+AB(5,I)*COSKX(K)
      A6(I,K)=A6(I,K)+AB(6,I)*SINKX(K)
      A7(I,K)=A7(I,K)+AB(7,I)*COSKX(K)
      A8(I,K)=A8(I,K)+AB(8,I)*COSKX(K)
      A9(I,K)=A9(I,K)+AB(9,I)*COSKX(K)
      A10(I,K)=A10(I,K)+AB(10,I)*SINKX(K)
      A11(I,K)=A11(I,K)+AB(11,I)*COSKX(K)
      A12(I,K)=A12(I,K)+AB(12,I)*COSKX(K)
      A13(I,K)=A13(I,K)+AB(13,I)*SINKX(K)
  200 CONTINUE
C
C     PRINT CHECK AND PRINT INTERNAL FORCES AND DISPLACEMENTS
C
  650 IF (NN-LPIN) 652,651,652
  651 CALL PINFOR (LPIN,NUMY,MM)
      LL=LL+1
      LPIN=NPIN(LL)
  652 IF (LDT-1) 700,653,700
  653 LDT=2
      PRINT 52,NN
      PRINT 53, NN,DT(2),DT(3)
  700 CONTINUE
      GO TO 101
  999 CALL EXIT
       END


*      LIST
*      LABEL
*      FORTRAN
CPIN-2-
      SUBROUTINE PINFOR (NP,NY,MN)
      DIMENSION A1(35,35),A2(35,35),A3(35,35),A4(35,35),A5(35,35),
     1 A6(35,35),A7(35,35),A8(35,35),A9(35,35),A10(35,35),A11(35,35),
     2 A12(35,35),A13(35,35),XINT(35),YANG(35),M1(7),M2(7)
      COMMON A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A12,A13,XINT,YANG,M1,M2
  100 FORMAT (60H1INTERNAL FORCES AND DISPLACEMENTS AFTER FOURIER SERIES
     1 NO. I4///)
  101 FORMAT (16H0         M(PHI)  )
  102 FORMAT (16H0         M(X)    )
  103 FORMAT (16H0         Q(X)    )
```

```
  104 FORMAT (16H0        N(PHI)   )
  105 FORMAT (16H0        N(X)     )
  106 FORMAT (16H0        U        )
  107 FORMAT (16H0        W        )
  108 FORMAT (16H0        Q(PHI)   )
  109 FORMAT (16H0        Q'(PHI)  )
  110 FORMAT (16H0        N(X-PHI))
  111 FORMAT (16H0        V        )
  112 FORMAT (16H0        THETA    )
  113 FORMAT (16H0        M(X-PHI))
C
      NUMY=NY
      MM=MN
C
C     PRINT INTERNAL FORCES AND DISPLACEMENTS BY SUBROUTINE FORPIN
C
      PRINT 100, NP
      PRINT 101
      CALL FORPIN (A1,XINT,YANG,M1,M2,NUMY,MM)
      PRINT 102
      CALL FORPIN (A2,XINT,YANG,M1,M2,NUMY,MM)
      PRINT 103
      CALL FORPIN (A3,XINT,YANG,M1,M2,NUMY,MM)
      PRINT 104
      CALL FORPIN (A4,XINT,YANG,M1,M2,NUMY,MM)
      PRINT 105
      CALL FORPIN (A5,XINT,YANG,M1,M2,NUMY,MM)
      PRINT 106
      CALL FORPIN (A6,XINT,YANG,M1,M2,NUMY,MM)
      PRINT 107
      CALL FORPIN (A7,XINT,YANG,M1,M2,NUMY,MM)
      PRINT 108
      CALL FORPIN (A8,XINT,YANG,M1,M2,NUMY,MM)
      PRINT 109
      CALL FORPIN (A9,XINT,YANG,M1,M2,NUMY,MM)
      PRINT 110
      CALL FORPIN (A10,XINT,YANG,M1,M2,NUMY,MM)
      PRINT 111
      CALL FORPIN (A11,XINT,YANG,M1,M2,NUMY,MM)
      PRINT 112
      CALL FORPIN (A12,XINT,YANG,M1,M2,NUMY,MM)
      PRINT 113
      CALL FORPIN (A13,XINT,YANG,M1,M2,NUMY,MM)
      CONTINUE
      RETURN
       END


*      LIST
*      LABEL
*      FORTRAN
CFOR-2-
      SUBROUTINE FORPIN (A,X,Y,M1,M2,NUMY,MM)
      DIMENSION A(35,35),X(35),Y(35),M1(7),M2(7)
```

```
   10 FORMAT (10H0      (PHI)2X,6(7H     X =F11.4))
   11 FORMAT (F10.4,2X,6E18.8)
      NY=NUMY
      DO 30 M=1,MM
      N1=M1(M)
      N2=M2(M)
      PRINT 10, (X(I),I=N1,N2)
      DO 20 K=1,NY
   20 PRINT 11, (Y(K),(A(K,I),I=N1,N2))
   30 CONTINUE
      RETURN
       END


*     LIST
*      LABEL
*      FORTRAN
CSTIMAT
      SUBROUTINE STIMAT (VK,SL,VR,VA,VE,FNU,SMALLK,BMAT,ADEL1,ADEL2,
     1 LDT,DT)
      DIMENSION SMALLK(8,8),B(13,4),RBAR(13),BMAT(14,4),BQ(8,8),
     1 BDEL(8,8),CE(4,8),ADEL1(8,8),ADEL2(8,8),X(8,8),Y(8,8),Z(8,8),
     2 ZX(8),ZY(8),DT(3)
      R =VR
      A = VA
      E = VE
      FK = VK
      FMU = FNU
      EA = E*A
      FKK = FK*FK
      RR = R*R
      FKR = FK*R
      FKR2 = FKR**2
      PHIK=1.570796327
      XDT=DT(1)
C
C     CALCULATE B MATRIX
C
      G = 1.-FMU**2
      FI = A**3/12.
      PD = (3.*G)**0.125*(R/A)**0.25*(R*FK)**0.5
      GAMMA = R*FK*(A/R)**0.5/(3.*G)**0.25
   17 FM1 = ((1.+GAMMA)**2+1.)**0.5
      FN1 = ((FM1-1.-GAMMA)/2.)**0.5
      FM1 = ((FM1+1.+GAMMA)/2.)**0.5
      FM2 = ((1.-GAMMA)**2+1.)**0.5
      FN2 = ((FM2+1.-GAMMA)/2.)**0.5
      FM2 = ((FM2-1.+GAMMA)/2.)**0.5
      ALPHA1 = PD*FM1
      ALPHA3 = PD*FM2
      BETA1 = PD*FN1
      BETA3 = PD*FN2
      BETA2 = PD*PD
      BETA4 = BETA2
```

```
      ALPHA2 = BETA2*(1.+GAMMA)
      ALPHA4 = BETA2*(GAMMA-1.)
      DO 20 I=3,7
      DO 20 J=1,4
   20 B(I,J) = 1.
      B(1,1) = ALPHA2-FMU*FKR2
      B(1,2) = BETA2
      B(1,3) = ALPHA4-FMU*FKR2
      B(1,4) = BETA4
      B(2,1) = 1.-FMU*ALPHA2/FKR2
      B(2,2) = -FMU*BETA2/FKR2
      B(2,3) = 1.-FMU*ALPHA4/FKR2
      B(2,4) = -FMU*BETA4/FKR2
      B(3,3) = -1.
      B(4,1) = 0.
      B(4,3) = 0.
      B(4,4) = -1.
      B(5,1) = -1.
      B(5,2) = 1.+GAMMA
      B(5,4) = 1.-GAMMA
      B(6,1) = -1.
      B(6,2) = 1.+GAMMA*(1.+FMU)
      B(6,4) = 1.-GAMMA*(1.+FMU)
      B(7,2) = 0.
      B(7,4) = 0.
   21 B(8,1) = FM1-FN1
      B(8,2) = FM1+FN1
      B(8,3) = -FM2-FN2
      B(8,4) = FM2-FN2
      GAMMA1 = GAMMA*(1.-FMU)
      B(9,1) = FM1*(1.-GAMMA1)-FN1
      B(9,2) = FN1*(1.-GAMMA1)+FM1
      B(9,3) = -FM2*(1.+GAMMA1)-FN2
      B(9,4) = -FN2*(1.+GAMMA1)+FM2
      B(10,1)= -FN1
      B(10,2)= FM1
      B(10,3)= FN2
      B(10,4)= -FM2
      GAMMA2 = GAMMA*(1.+FMU)
      B(11,1)= FM1+FN1*(1.-GAMMA2)
      B(11,2)= FN1-FM1*(1.-GAMMA2)
      B(11,3)= -FM2+FN2*(1.+GAMMA2)
      B(11,4)= -FN2-FM2*(1.+GAMMA2)
      DO 22 I=1,4
   22 B(12,I) = 0.
      B(13,1) = ALPHA1
      B(13,2) = BETA1
      B(13,3) = ALPHA3
      B(13,4) = BETA3
   23 EIG = E*FI/G
      RBAR(1) = 2.*EIG/RR
      RBAR(2) = -2.*EIG*FKK
      RBAR(3) = RBAR(2)*FK/GAMMA
      RBAR(4) = RBAR(3)*2.*FKR/GAMMA
      RBAR(5) =-RBAR(4)/GAMMA
```

```
      RBAR(6)  =  4.*FI*FKR*FKK/(G*A*GAMMA**3)
      RBAR(7)  =  2.
      RBAR(8)  =  -RBAR(3)/GAMMA**0.5
      RBAR(9)  =  RBAR(8)
      RBAR(10)=(RBAR(8)*2.*FKR/GAMMA)*SL
      RBAR(11)=  -RBAR(6)/GAMMA**0.5
      RBAR(12)=  1.
      RBAR(13)=(-2.*E*FI*FK/((1.+FMU)*R))*SL
      RBAR(3)=RBAR(3)*SL
      RBAR(6)=RBAR(6)*SL
      DO 24  I=1,13
      DO 24  J=1,4
   24 B(I,J)  =  B(I,J)*RBAR(I)
      B(12,1)  =  (B(11,1)-2.*ALPHA1)/R
      B(12,2)  =  (B(11,2)-2.*BETA1)/R
      B(12,3)  =  (B(11,3)-2.*ALPHA3)/R
      B(12,4)  =  (B(11,4)-2.*BETA3)/R
      DO 25  I  =  1,13
      DO 25  J  =  1,4
   25 BMAT(I,J)  =  B(I,J)
      BMAT(14,1)  =  ALPHA1
      BMAT(14,2)  =  ALPHA3
      BMAT(14,3)  =  BETA1
      BMAT(14,4)  =  BETA3
C
C     SET UP BQ AND BDEL MATRICES
C
      C1  =  COSF(BETA1*PHIK)
      C3  =  COSF(BETA3*PHIK)
      S1  =  SINF(BETA1*PHIK)
      S3  =  SINF(BETA3*PHIK)
      IF  (LDT)  28,28,27
   27 E1=1.
      E2=1.
      E3=0.
      E4=0.
      GO TO 29
   28 E1  =  EXPF(ALPHA1*PHIK)
      E2  =  EXPF(ALPHA3*PHIK)
      E3  =  EXPF(-ALPHA1*PHIK)
      E4  =  EXPF(-ALPHA3*PHIK)
      IF  (E3-XDT)  61,61,60
   60 IF  (E4-XDT)  61,61,29
   61 LDT=1
      DT(2)=E3
      DT(3)=E4
   29 DO 30  I=1,4
      DO 30  J=1,8
   30 CE(I,J)  =  0.
      CE(1,1)  =  C1*E1
      CE(1,2)  =-S1*E1
      CE(2,1)  =  CE(1,2)
      CE(2,2)  =-CE(1,1)
      CE(3,3)  =  C3*E2
      CE(3,4)  =-S3*E2
```

```
      CE(4,3) = CE(3,4)
      CE(4,4) =-CE(3,3)
      CE(1,5) = C1*E3
      CE(1,6) = S1*E3
      CE(2,5) = CE(1,6)
      CE(2,6) =-CE(1,5)
      CE(3,7) = C3*E4
      CE(3,8) = S3*E4
      CE(4,7) = CE(3,8)
      CE(4,8) =-CE(3,7)
   31 CALL MPYBCE    (B,1,CE,BQ,4)
      CALL MPYBCE    (B,4,CE,BQ,2)
      CALL MPYBCE    (B,6,CE,BDEL,1)
      CALL MPYBCE    (B,7,CE,BDEL,3)
      DO 32 I=1,4
      DO 32 J=5,8
   32 CE(I,J) = -CE(I,J)
      CALL MPYBCE    (B,9,CE,BQ,3)
      CALL MPYBCE    (B,10,CE,BQ,1)
      CALL MPYBCE    (B,11,CE,BDEL,2)
      CALL MPYBCE    (B,12,CE,BDEL,4)
      DO 33 I=1,4
      DO 33 J=1,4
      BQ(I+4,J) = BQ(I,J+4)*(-1.)**I
      BQ(I+4,J+4) = BQ(I,J)*(-1.)**I
      BDEL(I+4,J) = BDEL(I,J+4)*(-1.)**(I+1)
   33 BDEL(I+4,J+4) = BDEL(I,J)*(-1.)**(I+1)
C
C     INVERT BDEL MATRIX
C
      CALL INVERT (BDEL,8,8,ZX,ZY)
C
C     CALCULATE ADEL1,ADEL2, AND SMALLK MATRICES
C
      DO 35 I=1,8
      DO 35 J=1,8
      Z(I,J)=0.
      DO 35 K=1,8
   35 Z(I,J)=Z(I,J)+BQ(I,K)*BDEL(K,J)
      DO 40 I=1,8
      ADEL1(I,1) =-BDEL(I,2)
      ADEL1(I,2) =-BDEL(I,3)
      ADEL1(I,3) = BDEL(I,4)
      ADEL1(I,4) = BDEL(I,1)
      ADEL1(I,5) = BDEL(I,6)
      ADEL1(I,6) = BDEL(I,7)
      ADEL1(I,7) = BDEL(I,8)
      ADEL1(I,8) = BDEL(I,5)
      ADEL2(I,1) =-BDEL(I,6)
      ADEL2(I,2) =-BDEL(I,7)
      ADEL2(I,3) = BDEL(I,8)
      ADEL2(I,4) = BDEL(I,5)
      ADEL2(I,5) = BDEL(I,2)
      ADEL2(I,6) = BDEL(I,3)
      ADEL2(I,7) = BDEL(I,4)
```

```
      ADEL2(I,8) = BDEL(I,1)
      X(I,1) =-Z(I,2)
      X(I,2) =-Z(I,3)
      X(I,3) = Z(I,4)
      X(I,4) = Z(I,1)
      X(I,5) = Z(I,6)
      X(I,6) = Z(I,7)
      X(I,7) = Z(I,8)
      X(I,8) = Z(I,5)
      Y(I,1) =-Z(I,6)
      Y(I,2) =-Z(I,7)
      Y(I,3) = Z(I,8)
      Y(I,4) = Z(I,5)
      Y(I,5) = Z(I,2)
      Y(I,6) = Z(I,3)
      Y(I,7) = Z(I,4)
   40 Y(I,8) = Z(I,1)
      DO 50 I=1,8
      SMALLK(1,I) =-X(2,I)+Y(6,I)
      SMALLK(2,I) = X(3,I)-Y(7,I)
      SMALLK(3,I) =-X(4,I)+Y(8,I)
      SMALLK(4,I) = X(1,I)-Y(5,I)
      SMALLK(5,I) =-X(6,I)+Y(2,I)
      SMALLK(6,I) = X(7,I)-Y(3,I)
      SMALLK(7,I) = X(8,I)-Y(4,I)
   50 SMALLK(8,I) =-X(5,I)+Y(1,I)
      RETURN
      END


*     LIST
*     LABEL
*     FORTRAN
CMPYBCE
      SUBROUTINE MPYBCE (B,I,CE,BQD,J)
C
C     TO MULTIPLY B BY CE MATRIX AND STORE IN BQ OR BDEL
C
      DIMENSION B(13,4),CE(4,8),BQD(8,8)
      DO 1 K=1,8
      BQD(J,K) = 0.
      DO 1 L=1,4
    1 BQD(J,K) = BQD(J,K)+B(I,L)*CE(L,K)
      RETURN
       END


*     LIST
*     FORTRAN
*     LABEL
C          GENERAL MATRIX INVERSION SUBROUTINE
C
      SUBROUTINE INVERT(A,NN,N,M,C)
```

```
C
      DIMENSION A(1),M(1),C(1)

      IF (NN-1) 80,70,80
   70 A(1)=1./A(1)
      GO TO 300
   80 DO 90 I=1,NN
   90 M(I)=-I
C
      DO 140 I=1,NN
C
C     LOCATE LARGEST ELEMENT
C
      D=0.0
      DO 112 L=1,NN
      IF (M(L)) 100,100,112
  100 J=L
      DO 110 K=1,NN
      IF (M(K)) 103,103,108
  103 IF (ABSF(D)-ABSF(A(J))) 105,105,108
  105 LD=L
      KD=K
      D=A(J)
  108 J=J+N
  110 CONTINUE
  112 CONTINUE
C
C     INTERCHANGE ROWS
C
      TEMP=-M(LD)
      M(LD)=M(KD)
      M(KD)=TEMP
      L=LD
      K=KD
      DO 114 J=1,NN
      C(J)=A(L)
      A(L)=A(K)
      A(K)=C(J)
      L=L+N
  114 K=K+N
C
C     DIVIDE COLUNM BY LARGEST ELEMENT
C
      NR=(KD-1)*N+1
      NH=NR+N-1
      DO 115 K=NR,NH
  115 A(K)=A(K)/D
C
C     REDUCE REMAINING ROWS AND COLUMNS
C
      L=1
      DO 135 J=1,NN
      IF (J-KD) 130,125,130
  125 L=L+N
      GO TO 135
```

```
  130 DO 134 K=NR,NH
      A(L)=A(L)-C(J)*A(K)
  134 L=L+1
  135 CONTINUE
C
C     REDUCE ROW
C
      C(KD)=-1.0
      J=KD
      DO 140 K=1,NN
      A(J)=-C(K)/D
  140 J=J+N
C
C     INTERCHANGE COLUMNS
C
      DO 200 I=1,NN
      L=0
  150 L=L+1
      IF(M(L)-I) 150,160,150
  160 K=(L-1)*N+1
      J=(I-1)*N+1
      M(L)=M(I)
      M(I)=I
      DO 200 L=1,NN
      TEMP=A(K)
      A(K)=A(J)
      A(J)=TEMP
      J=J+1
  200 K=K+1
C
  300 RETURN
C
      END
```

UNIVERSITY OF CALIFORNIA

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Notes on IBM 7094 Program 4 for Tubular Joint Analysis

1. IDENTIFICATION

CIR - 4: Solution of circular cylindrical shells with live loads applied along generators--Program 4--Oct. 1964 (relation between forces and displacements at surface of circular tube).

2. REFERENCES

a. "Analytical Study of Tubular Tee-Joints," by A. C. Scordelis and J. G. Bouwkamp

3. FORM OF INPUT

a. See attached FORTRAN II listing of program for detailed description of input.

b. Additional explanation of input is as follows:

(1) First Card Any 72 characters may be used for title.

(2) Second Card

Col. 51 - 54 Fourier series limit-- the number of the highest harmonic term of the series to be used to represent the longitudinal distribution of loading; maximum number is 100 for unsymmetrical loads with respect to midspan and 200 for symmetrical or anti-symmetrical loads if non-contributing harmonics are skipped in cols. 60 or 62.

Col. 55 - 58 Numpin--total number of times results are to be printed, one for each Fourier series number specified in fourth card. If Numpin is set at 0 results will be printed only after Fourier series limit has been reached.

Col. 60 for loadings antisymmetric about midspan, the odd Fourier series terms can be skipped.

Col. 62 for loadings symmetric about midspan, the even Fourier series terms can be skipped.

(3) Next Card The disturbance limit should be given a value of 1.0 E-16.

(4) Next Card If Numpin is set to zero on second card, skip this card and printout of results will be given only after Fourier series limit has been reached.

(5) Next Card — Note that for a case with symmetry or antisymmetry about both $x = 0$ and $\phi = 0$, a total of 42 points may be considered on 1/4 of the whole circular tube.

(6) Next Card — Note that either or both vertical and horizontal displacement restraints can be assumed to exist.

(7) Next Card — Input action is either total force magnitude or total displacement at point depending on tag specification on preceding card.

4. FORM OF OUTPUT

a. Input data is printed as a check.
b. Final total forces and corresponding displacements at each specified point. One set of answers is printed sequentially after each Fourier series number specified for printout.

```
*       DECKS
*       LIST
        LABEL
*       FORTRAN
CCIR-4    (OLD 3)
C
C   UNIVERSITY OF CALIFORNIA                              DEPT. OF CIVIL ENGINEERING
C
C       SOLUTION OF CIRCULAR CYLINDRICAL SHELLS WITH LINE LOADS APPLIED ALONG
C          GENERATORS --- PROGRAM 4 ---  OCT.1964
C          RELATION BETWEEN FORCES AND DISPLACEMENTS AT SURFACE OF CIRCULAR TUBE
C
C   PROGRAMMED BY..  K.S. LO
C   FACULTY INVESTIGATORS..  A.C. SCORDELIS AND J.G. BOUWKAMP
C
C       INPUT DATA
C
C       FIRST CARD - TITLE OF THE PROBLEM
C
C       SECOND CARD - COL.  1 TO 10 - SPAN (F10.0)
C                     COL. 11 TO 20 - RADIUS OF SHELL (F10.0)
C                     COL. 21 TO 30 - SHELL THICKNESS (F10.0)
C                     COL. 31 TO 40 - MODULUS OF ELASTICITY (F10.0)
C                     COL. 41 TO 50 - POISSON RATIO (F10.0)
C                     COL. 51 TO 54 - FOURIER SERIES LIMIT (I4)
C                     COL. 55 TO 58 - NUMPIN
C                                     NUMBER OF PRINT OF RESULTS, MAX. 100 (I4)
.                     COL.        60 - BLANK TO INCLUDE ODD FOURIER SERIES
.                                     1 TO SKIP ODD FOURIER SERIES
C                     COL.        62 - BLANK TO INCLUDE EVEN FOURIER SERIES
C                                     1 TO SKIP EVEN FOURIER SERIES
C
C      NEXT CARD - COL.  1 TO 10 - DISTURBANCE LIMIT (E10.3)
C                                 IF EXPF(-ALPHA*PHIK) IS LESS THAN THIS LIMIT
C                                 DISTURBANCE FROM OTHER JOINT IS NEGLECTED
C
C      NEXT CARD - EXISTS ONLY IF 'NUMPIN' IS NOT ZERO
C                  FOURIER SERIES NUMBERS (18I4)
C                  RESULTS WILL BE PRINTED AFTER THESE SERIES NUMBERS
C                  USE NEXT CARDS IF NEEDED
C
C      NEXT CARD - COL. 1 TO  3 - NUMBER OF CONSIDERED POINTS, MAX. 42 (I3)
C                  COL. 4 TO  6 - NUMBER OF LOAD CASES, MAX. 10 (I3)
C                  COL.       8 - INDICATOR FOR SYMMETRICAL PROPERTY ABOUT X=0
C                                   0 - UNSYM.  1 - SYM.  2 - ANTISYM.
C                  COL.      10 - INDICATOR FOR SYMMETRICAL PROPERTY ABOUT PHI=0
C                                   0 - UNSYM.  1 - SYM.  2 - ANTISYM.
C
C      NEXT CARDS - BOUNDARY CONDITIONS FOR THE POINTS, ONE CARD FOR EACH POINT
C                   COL.  1 TO 10 - PHI ANGLE (IN DEGREES) OF THE POINT (F10.0)
C                   COL. 11 TO 20 - DISTRIBUTED LOAD WIDTH (F10.0)
C                   COL. 21 TO 30 - LOCATION FROM LEFT END (F10.0)
C                   COL.       32 - INDEX FOR VERT. FORCE OR DISPL. AT THAT PT.
C                                    1-GIVEN DISPL.  2-GIVEN FORCE  0-NEGLECTED
.                   COL.       34 - INDEX FOR HORIZ. FORCE OR DISPL. AT THAT PT.
```

```
C                                      1-GIVEN DISPL.  2-GIVEN FORCE  0-NEGLECTE
C
C     NEXT CARDS - FORCE MAGNITUDE OR DISPLACEMENT FOR THE POINTS
C                  ONE SET OF CARDS FOR EACH LOAD CASE
C                  INPUT THE VERT. ACTION FIRST AND THEN HORIZ. ACTION OF THE
C                    SAME POINT
C                  INPUT THE ACTION(S) OF THE 1ST POINT FIRST AND THEN THE
C                    ACTION(S) OF THE 2ND POINT AND SO ON
C                  (7F10.0) USE NEXT CARD IF NEEDED
C                  START WITH A NEW CARD FOR EACH LOAD CASE
C
C     ALL ABOVE DATA CARDS ARE REPEATED FOR NEXT PROBLEM TO BE SOLVED
C
C     TWO BLANK CARDS ARE ADDED AT END OF THE DATA DECK
C
C
C     DIMENSION AND COMMON STATEMENTS
C
      DIMENSION TITLE(12),DT(3),NPIN(102),YANG(42),DEL(42),IVH(42,2),
     1 SINY(42),COSY(42),VH(42),XI1(42),SYMP(2),YINT(42,42),IND(42,42),
     2 YINT1(42,42),IND1(42,42),IKM(84),L1(84),MAXR(3),GR(84,10),XI(42),
     3 A1(84,84),SERIES(42),SK(8,8),B(3,4),ADEL1(8,8),ADEL2(8,8),
     4 ZX(8),ZY(8),COSKX(42),P(2),D(8),C1(8),C2(8),B1(2,8),B2(2,8),
     5 CE(8),AB(2,42)
      COMMON YINT,IND,YINT1,IND1
C
C     FORMAT STATEMENT
C
   11 FORMAT (12A6)
   12 FORMAT (5F10.0,2I4,2I2)
   13 FORMAT (18I4)
   14 FORMAT (2I3,2I2)
   15 FORMAT (2H1 )
   16 FORMAT (15H0SPAN LENGTH = F8.3/19H0RADIUS OF SHELL = F7.3/22H0THIC
     1KNESS OF SHELL = F9.6)
   17 FORMAT (25H0MODULUS OF ELASTICITY = E14.6/17H0POISSON RATIO = F6.4
     1/24H0FOURIER SERIES LIMIT = I3)
   18 FORMAT (39H0PRINT RESULTS AFTER FOURIER SERIES NO.)
   19 FORMAT (7F10.0)
   22 FORMAT (//41H0CALCULATIONS SKIP ALL ODD FOURIER SERIES)
   23 FORMAT (//42H0CALCULATIONS SKIP ALL EVEN FOURIER SERIES)
   25 FORMAT (3F10.0,2I2)
   50 FORMAT (1E10.3)
   51 FORMAT (21H0DISTURBANCE LIMIT = E10.3//)
   52 FORMAT (67H1DISTURBANCE FROM OTHER JOINT IS NEGLECTED AFTER FOURIE
     1R SERIES NO. I4)
   53 FORMAT (22H0AT FOURIER SERIES NO. I4/24H0    EXPF(-ALPHA1*PHIK) =
     1 E15.6/24H0    EXPF(-ALPHA3*PHIK) = E15.6)
   60 FORMAT (27H1NUMBER OF LOADING POINTS = I3)
   61 FORMAT (27H0NUMBER OF LOADING CASES  = I3)
   62 FORMAT (33H0SYSTEM IS SYMMETRICAL W.R.T. X=0 )
   63 FORMAT (37H0SYSTEM IS ANTISYMMETRICAL W.R.T. X=0 )
   64 FORMAT (35H0SYSTEM IS SYMMETRICAL W.R.T. PHI=0 )
   65 FORMAT (39H0SYSTEM IS ANTISYMMETRICAL W.R.T. PHI=0 )
   66 FORMAT (76H0POINT     PHI(DEGREES)          LOAD WIDTH   LOCATION FROM
```

```
     1 LEFT END     IV     IH )
  67 FORMAT (I4,3E18.6,9X,I1,5X,I1)
  68 FORMAT (59H0IV =   1-GIVEN V DISPL.   2-GIVEN V FORCE   0-NEGLECT V C
     1OMP. )
  69 FORMAT (59H0IH =   1-GIVEN H DISPL.   2-GIVEN H FORCE   0-NEGLECT H C
     1OMP. )
  70 FORMAT (42H0INPUT FORCES OR DISPL. FOR LOAD CASE NO. I2)
  71 FORMAT (8F15.6)
C
C     READ, PRINT AND MODIFY INPUT DATA
C
 101 READ 11, (TITLE(I),I=1,12)
     READ 12, SPAN,R,TH,E,FNU,MAXSER,NUMPIN,NODD,NEVEN
     IF (SPAN) 999,999,102
 102 PRINT 15
     PRINT 11, (TITLE(I),I=1,12)
     PRINT 16, SPAN,R,TH
     PRINT 17, E,FNU,MAXSER
     READ  50, DT(1)
     PRINT 51, DT(1)
     IF (NUMPIN) 104,104,103
 103 READ 13, (NPIN(I),I=1,NUMPIN)
     PRINT 18
     PRINT 13, (NPIN(I),I=1,NUMPIN)
 104 J=NUMPIN+1
     NPIN(J)=MAXSER
     READ 14, NPT,NLD,LSYMX,LSYMP
     IF (NODD) 106,106,105
 105 PRINT 22
 106 IF (NEVEN) 108,108,107
 107 PRINT 23
 108 PI=3.141592654
     PIL=PI/SPAN
     PRINT 60, NPT
     PRINT 61, NLD
     PRINT 66
     DO 115 J=1,NPT
     READ 25, (YANG(J),DEL(J),XI(J),IVH(J,1),IVH(J,2))
     PRINT 67, (J,YANG(J),DEL(J),XI(J),IVH(J,1),IVH(J,2))
     YRAD=(YANG(J)/180.)*PI
     SINY(J)=SINF(YRAD)
     COSY(J)=COSF(YRAD)
     VH(J)=4./(PI*DEL(J))
     DEL(J)=0.5*DEL(J)*PIL
 115 XI(J)=XI(J)*PIL
     PRINT 68
     PRINT 69
     IF (LSYMX-1) 120,116,117
 116 SYMX=1.
     PRINT 62
     GO TO 118
 117 SYMX=-1.
     PRINT 63
 118 DO 119 J=1,NPT
 119 XI1(J)=PI-XI(J)
```

```
      GO TO 121
  120 SYMX=0.
  121 IF (LSYMP-1) 124,122,123
  122 SYMP(1)=1.
      SYMP(2)=-1.
      PRINT 64
      GO TO 124
  123 SYMP(1)=-1.
      SYMP(2)=1.
      PRINT 65
  124 CONTINUE
      DO 308 J=1,NPT
      DO 308 I=1,NPT
      YNG=YANG(I)-YANG(J)
      IF (YNG) 305,301,301
  301 X=YNG-180.
      IF (X) 302,302,303
  302 YINT(J,I)=PI*(0.5-YNG/180.)
      IND(J,I)=1
      GO TO 308
  303 YINT(J,I)=PI*(0.5-X/180.)
      IND(J,I)=0
      GO TO 308
  305 X=YNG+180.
      IF (X) 307,303,303
  307 X=-X-90.
      YINT(J,I)=PI*(X/180.)
      IND(J,I)=1
  308 CONTINUE
      IF (LSYMP) 320,320,310
  310 DO 318 J=1,NPT
      DO 318 I=1,NPT
      YNG=YANG(I)+YANG(J)
      IF (YNG) 315,311,311
  311 X=YNG-180.
      IF (X) 312,312,313
  312 YINT1(J,I)=PI*(0.5-YNG/180.)
      IND1(J,I)=1
      GO TO 318
  313 YINT1(J,I)=PI*(0.5-X/180.)
      IND1(J,I)=0
      GO TO 318
  315 X=YNG+180.
      IF (X) 317,313,313
  317 X=-X-90.
      YINT1(J,I)=PI*(X/180.)
      IND1(J,I)=1
  318 CONTINUE
  320 CONTINUE
      K=0
      KK=0
      KKK=0
      DO 323 J=1,NPT
      DO 323 I=1,2
      IF (IVH(J,I)-1) 323,321,322
```

```
  321 K=K+1
      KK=KK+1
      IKM(KK)=K
      GO TO 323
  322 K=K+1
      KKK=KKK+1
      L1(KKK)=K
  323 CONTINUE
      MAXR(1)=K
      MAXR(2)=KK
      MAXR(3)=KKK
      DO 324 I=1,KKK
      J=KK+I
  324 IKM(J)=L1(I)
      DO 326 I=1,NLD
      READ 19, (GR(J,I),J=1,K)
  326 CONTINUE
      DO 328 I=1,NLD
      PRINT 70, I
  328 PRINT 71, (GR(J,I),J=1,K)
C
C     OUTPUT IS CLEARED
C
      DO 327 I=1,K
      DO 327 J=1,K
  327 A1(I,J)=0.
C
C     COEFFICIENTS PI*X/L ARE COMPUTED
C
      PI2=0.5*PI
      DO 136 I=1,NPT
  136 SERIES(I)=XI(I)-PI2
C
C     CYCLE FOR EACH HARMONIC IS INITIATED
C
      LDT = 0
      LL=1
      LPIN=NPIN(1)
      DO 700 NN=1,MAXSER
      FQ=(-1.)**(NN+1)
      IF (FQ) 137,137,138
  137 IF (NEVEN) 139,139,650
  138 IF (NODD) 139,139,650
C
C     SPAN AND K ARE GENERALIZED
C
  139 FN=NN
      FK=FN*PIL
C
C     STIFFNESS MATRIX IS COMPUTED BY SUBROUTINE SIMSTI
C
      CALL SIMSTI (FK,FQ,R,TH,E,FNU,SK,B,ADEL1,ADEL2,LDT,DT)
      AL1=B(3,1)
      AL3=B(3,2)
      BE1=B(3,3)
```

```
  321 K=K+1
      KK=KK+1
      IKM(KK)=K
      GO TO 323
  322 K=K+1
      KKK=KKK+1
      L1(KKK)=K
  323 CONTINUE
      MAXR(1)=K
      MAXR(2)=KK
      MAXR(3)=KKK
      DO 324 I=1,KKK
      J=KK+I
  324 IKM(J)=L1(I)
      DO 326 I=1,NLD
      READ 19, (GR(J,I),J=1,K)
  326 CONTINUE
      DO 328 I=1,NLD
      PRINT 70, I
  328 PRINT 71, (GR(J,I),J=1,K)
C
C     OUTPUT IS CLEARED
C
      DO 327 I=1,K
      DO 327 J=1,K
  327 A1(I,J)=0.
C
C     COEFFICIENTS PI*X/L ARE COMPUTED
C
      PI2=0.5*PI
      DO 136 I=1,NPT
  136 SERIES(I)=XI(I)-PI2
C
C     CYCLE FOR EACH HARMONIC IS INITIATED
C
      LDT = 0
      LL=1
      LPIN=NPIN(1)
      DO 700 NN=1,MAXSER
      FQ=(-1.)**(NN+1)
      IF (FQ) 137,137,138
  137 IF (NEVEN) 139,139,650
  138 IF (NODD) 139,139,650
C
C     SPAN AND K ARE GENERALIZED
C
  139 FN=NN
      FK=FN*PIL
C
C     STIFFNESS MATRIX IS COMPUTED BY SUBROUTINE SIMSTI
C
      CALL SIMSTI (FK,FQ,R,TH,E,FNU,SK,B,ADEL1,ADEL2,LDT,DT)
      AL1=B(3,1)
      AL3=B(3,2)
      BE1=B(3,3)
```

```
  321 K=K+1
      KK=KK+1
      IKM(KK)=K
      GO TO 323
  322 K=K+1
      KKK=KKK+1
      L1(KKK)=K
  323 CONTINUE
      MAXR(1)=K
      MAXR(2)=KK
      MAXR(3)=KKK
      DO 324 I=1,KKK
      J=KK+I
  324 IKM(J)=L1(I)
      DO 326 I=1,NLD
      READ 19, (GR(J,I),J=1,K)
  326 CONTINUE
      DO 328 I=1,NLD
      PRINT 70, I
  328 PRINT 71, (GR(J,I),J=1,K)
C
C     OUTPUT IS CLEARED
C
      DO 327 I=1,K
      DO 327 J=1,K
  327 A1(I,J)=0.
C
C     COEFFICIENTS PI*X/L ARE COMPUTED
C
      PI2=0.5*PI
      DO 136 I=1,NPT
  136 SERIES(I)=XI(I)-PI2
C
C     CYCLE FOR EACH HARMONIC IS INITIATED
C
      LDT = 0
      LL=1
      LPIN=NPIN(1)
      DO 700 NN=1,MAXSER
      FQ=(-1.)**(NN+1)
      IF (FQ) 137,137,138
  137 IF (NEVEN) 139,139,650
  138 IF (NODD) 139,139,650
C
C     SPAN AND K ARE GENERALIZED
C
  139 FN=NN
      FK=FN*PIL
C
C     STIFFNESS MATRIX IS COMPUTED BY SUBROUTINE SIMSTI
C
      CALL SIMSTI (FK,FQ,R,TH,E,FNU,SK,B,ADEL1,ADEL2,LDT,DT)
      AL1=B(3,1)
      AL3=B(3,2)
      BE1=B(3,3)
```

```
      BE3=B(3,4)
      CALL INVERT (SK,8,8,ZX,ZY)
C
C     CALCULATE SINKX AND COSKX MATRICES
C
      IF (FQ) 160,160,165
  160 N=NN/2
      DO 162 I=1,NPT
      X=FN*SERIES(I)
  162 COSKX(I)=SINF(X)
      GO TO 170
  165 N=(NN+3)/2
      DO 167 I=1,NPT
      X=FN*SERIES(I)
  167 COSKX(I)=COSF(X)
  170 X=(-1.)**N
C
C     CALCULATE INPUT LOADS FOR EACH LOADED GENERATOR
C
      KK=0
      DO 200 J=1,NPT
      SD=SINF(FN*DEL(J))
      SX=SINF(FN*XI(J))+SINF(FN*XI1(J))*SYMX
      VHJ=(VH(J)/FN)*SD*SX
      DO 200 JJ=1,2
      IF (IVH(J,JJ)) 200,200,141
  141 KK=KK+1
      IF (JJ-1) 142,142,143
  142 P(2)=VHJ*COSY(J)
      P(1)=VHJ*SINY(J)
      GO TO 144
  143 P(2)=-VHJ*SINY(J)
      P(1)=VHJ*COSY(J)
  144 DO 199 JJJ=1,2
      JJJ1=JJJ-1
      IF (JJJ1) 155,155,145
  145 IF (LSYMP) 199,199,146
  146 P(2)=P(2)*SYMP(1)
      P(1)=P(1)*SYMP(2)
      GO TO 157
  155 CONTINUE
      DO 156 I=1,2
  156 P(I)=P(I)*X
  157 CONTINUE
C
C     UNKNOWN JOINT DISPLACEMENTS ARE COMPUTED
C
      DO 171 I=1,8
      D(I)=0.
      DO 171 K=1,2
  171 D(I)=D(I)+SK(I,K)*P(K)
C
C     CALCULATE ARBITRARY CONSTANTS FOR SHELLS
C
      DO 185 I=1,8
```

```
      C1(I)=0.
      C2(I)=0.
      DO 185 K=1,8
      C1(I)=C1(I)+ADEL1(I,K)*D(K)
  185 C2(I)=C2(I)+ADEL2(I,K)*D(K)
C
C     TO FIND MAX. INTERNAL FORCES AT DIFFERENT PHI ANGLES
C
      DO 190 I=1,2
      B1(I,1)=C1(1)*B(I,1)-C1(2)*B(I,2)
      B1(I,2)=C1(1)*B(I,2)+C1(2)*B(I,1)
      B1(I,3)=C1(3)*B(I,3)-C1(4)*B(I,4)
      B1(I,4)=C1(3)*B(I,4)+C1(4)*B(I,3)
      B1(I,5)=C1(5)*B(I,1)-C1(6)*B(I,2)
      B1(I,6)=C1(5)*B(I,2)+C1(6)*B(I,1)
      B1(I,7)=C1(7)*B(I,3)-C1(8)*B(I,4)
      B1(I,8)=C1(7)*B(I,4)+C1(8)*B(I,3)
      B2(I,1)=C2(1)*B(I,1)-C2(2)*B(I,2)
      B2(I,2)=C2(1)*B(I,2)+C2(2)*B(I,1)
      B2(I,3)=C2(3)*B(I,3)-C2(4)*B(I,4)
      B2(I,4)=C2(3)*B(I,4)+C2(4)*B(I,3)
      B2(I,5)=C2(5)*B(I,1)-C2(6)*B(I,2)
      B2(I,6)=C2(5)*B(I,2)+C2(6)*B(I,1)
      B2(I,7)=C2(7)*B(I,3)-C2(8)*B(I,4)
  190 B2(I,8)=C2(7)*B(I,4)+C2(8)*B(I,3)
      DO 191 K=5,8
      B1(2,K)=-B1(2,K)
  191 B2(2,K)=-B2(2,K)
      DO 197 IJ=1,NPT
      IF (JJJ1) 345,345,346
  345 PHI=YINT(J,IJ)
      IK=IND(J,IJ)
      GO TO 347
  346 PHI=YINT1(J,IJ)
      IK=IND1(J,IJ)
  347 CC=COSF(BE1*PHI)
      C3=COSF(BE3*PHI)
      S1=SINF(BE1*PHI)
      S3=SINF(BE3*PHI)
      IF (LDT-1) 351,351,350
  350 PHJ= PHI-1.570796327
      E1=EXPF(AL1*PHJ)
      E2=EXPF(AL3*PHJ)
      PHJ=-PHI-1.570796327
      E3=EXPF(AL1*PHJ)
      E4=EXPF(AL3*PHJ)
      GO TO 352
  351 E1=EXPF(AL1*PHI)
      E2=EXPF(AL3*PHI)
      E3=EXPF(-AL1*PHI)
      E4=EXPF(-AL3*PHI)
  352 CE(1)= CC*E1
      CE(2)=-S1*E1
      CE(3)= C3*E2
      CE(4)=-S3*E2
```

```
      CE(5)= CC*E3
      CE(6)= S1*E3
      CE(7)= C3*E4
      CE(8)= S3*E4
      IF (IK) 195,195,193
  193 DO 194 I=1,2
      AB(I,IJ)=0.
      DO 194 K=1,8
  194 AB(I,IJ)=AB(I,IJ)+B1(I,K)*CE(K)
      GO TO 197
  195 DO 196 I=1,2
      AB(I,IJ)=0.
      DO 196 K=1,8
  196 AB(I,IJ)=AB(I,IJ)+B2(I,K)*CE(K)
  197 CONTINUE
C
C     V AND H DISPLACEMENTS ARE COMPUTED AND ADDED
C
      KKK=0
      DO 198 I=1,NPT
      IF (IVH(I,1)) 354,354,353
  353 KKK=KKK+1
      A1(KKK,KK)=A1(KKK,KK)-(AB(2,I)*SINY(I)+AB(1,I)*COSY(I))*COSKX(I)
  354 IF (IVH(I,2)) 198,198,355
  355 KKK=KKK+1
      A1(KKK,KK)=A1(KKK,KK)+(AB(1,I)*SINY(I)-AB(2,I)*COSY(I))*COSKX(I)
  198 CONTINUE
  199 CONTINUE
  200 CONTINUE
C
C     PRINT CHECK AND PRINT FORCES AND DISPLACEMENTS
C
  650 IF (NN-LPIN) 652,651,652
  651 CALL SOLA (A1,MAXR,IKM,GR,NLD)
      LL=LL+1
      LPIN=NPIN(LL)
  652 IF (LDT-1) 700,653,700
  653 LDT=2
      PRINT 52,NN
      PRINT 53, NN,DT(2),DT(3)
  700 CONTINUE
      GO TO 101
  999 CALL EXIT
       END


*      LIST
*      LABEL
*      FORTRAN
CSOLA
      SUBROUTINE SOLA (A1,MAXR,IKM,GR,NLD)
      DIMENSION A1(84,84),IKM(84),GR(84,10),MAXR(3),A(84,84),D(84),
     1 R(84),X(84),RR(84)
      MM=MAXR(1)
```

```
      MD=MAXR(2)
      MR=MAXR(3)
      MD1=MD+1
      IF (MD) 5,10,5
    5 IF (MR) 30,20,30
   10 DO 15 I=1,NLD
      DO 11 J=1,MM
   11 R(J)=GR(J,I)
      DO 12 K=1,MM
      D(K)=0.
      DO 12 J=1,MM
   12 D(K)=D(K)+A1(K,J)*R(J)
      CALL PINRD (D,R,MM,I)
   15 CONTINUE
      GO TO 100
   20 DO 21 I=1,MM
      DO 21 J=1,MM
   21 A(I,J)=A1(I,J)
      CALL INVERT (A,MM,84,R,D)
      DO 25 I=1,NLD
      DO 22 J=1,MM
   22 D(J)=GR(J,I)
      DO 23 K=1,MM
      R(K)=0.
      DO 23 J=1,MM
   23 R(K)=R(K)+A(K,J)*D(J)
      CALL PINRD (D,R,MM,I)
   25 CONTINUE
      GO TO 100
   30 DO 31 I=1,MM
      II=IKM(I)
      DO 31 J=1,MM
      JJ=IKM(J)
   31 A(I,J)=A1(II,JJ)
      CALL INVERT (A,MD,84,R,D)
      DO 45 I=1,NLD
      DO 35 J=1,MM
      JJ=IKM(J)
   35 R(J)=GR(JJ,I)
      DO 36 J=1,MD
      D(J)=0.
      DO 36 K=MD1,MM
   36 D(J)=D(J)+A(J,K)*R(K)
      DO 37 J=1,MD
      X(J)=0.
      DO 37 K=1,MD
   37 X(J)=X(J)+A(J,K)*(R(K)-D(K))
      DO 39 J=MD1,MM
      X(J)=0.
      DO 38 K=1,MD
   38 X(J)=X(J)+A(J,K)*X(K)
      DO 39 K=MD1,MM
   39 X(J)=X(J)+A(J,K)*R(K)
      DO 40 J=1,MD
      JJ=IKM(J)
```

```
      D(JJ)=R(J)
   40 RR(JJ)=X(J)
      DO 41 J=MD1,MM
      JJ=IKM(J)
      D(JJ)=X(J)
   41 RR(JJ)=R(J)
      CALL PINRD (D,RR,MM,I)
   45 CONTINUE
  100 RETURN
      END


*      LIST
*      LABEL
*      FORTRAN
CPINRD
      SUBROUTINE PINRD (D,R,MM,I)
      DIMENSION D(84),R(84)
    1 FORMAT (52H1FINAL FORCES AND DISPLACEMENTS FOR LOADING CASE NO.I3)
    2 FORMAT (43H0              FORCES          DISPLACEMENTS )
    3 FORMAT (I4,2E20.8)
      PRINT 1, I
      PRINT 2
      PRINT 3, (J,R(J),D(J),J=1,MM)
      RETURN
      END


*      LIST
*      LABEL
*      FORTRAN
CSIMSTI
      SUBROUTINE SIMSTI (VK,SL,VR,VA,VE,FNU,SMALLK,BMAT,ADEL1,ADEL2,
     1 LDT,DT)
      DIMENSION SMALLK(8,8),B(13,4),RBAR(13),BMAT(3,4),BQ(8,8),
     1 BDEL(8,8),CE(4,8),ADEL1(8,8),ADEL2(8,8),X(8,8),Y(8,8),Z(8,8),
     2 ZX(8),ZY(8),DT(3)
      R =VR
      A = VA
      E = VE
      FK = VK
      FMU = FNU
      EA = E*A
      FKK = FK*FK
      RR = R*R
      FKR = FK*R
      FKR2 = FKR**2
      PHIK=1.570796327
      XDT=DT(1)
C
C     CALCULATE B MATRIX
C
      G = 1.-FMU**2
```

```
      FI = A**3/12.
      PD = (3.*G)**0.125*(R/A)**0.25*(R*FK)**0.5
      GAMMA = R*FK*(A/R)**0.5/(3.*G)**0.25
   17 FM1 = ((1.+GAMMA)**2+1.)**0.5
      FN1 = ((FM1-1.-GAMMA)/2.)**0.5
      FM1 = ((FM1+1.+GAMMA)/2.)**0.5
      FM2 = ((1.-GAMMA)**2+1.)**0.5
      FN2 = ((FM2+1.-GAMMA)/2.)**0.5
      FM2 = ((FM2-1.+GAMMA)/2.)**0.5
      ALPHA1 = PD*FM1
      ALPHA3 = PD*FM2
      BETA1 = PD*FN1
      BETA3 = PD*FN2
      BETA2 = PD*PD
      BETA4 = BETA2
      ALPHA2 = BETA2*(1.+GAMMA)
      ALPHA4 = BETA2*(GAMMA-1.)
      DO 20 I=4,7
      DO 20 J=1,4
   20 B(I,J) = 1.
      B(1,1) = ALPHA2-FMU*FKR2
      B(1,2) = BETA2
      B(1,3) = ALPHA4-FMU*FKR2
      B(1,4) = BETA4
      B(4,1) = 0.
      B(4,3) = 0.
      B(4,4) = -1.
      B(6,1) = -1.
      B(6,2) = 1.+GAMMA*(1.+FMU)
      B(6,4) = 1.-GAMMA*(1.+FMU)
      B(7,2) = 0.
      B(7,4) = 0.
      GAMMA1 = GAMMA*(1.-FMU)
      B(9,1) = FM1*(1.-GAMMA1)-FN1
      B(9,2) = FN1*(1.-GAMMA1)+FM1
      B(9,3) = -FM2*(1.+GAMMA1)-FN2
      B(9,4) = -FN2*(1.+GAMMA1)+FM2
      B(10,1)= -FN1
      B(10,2)= FM1
      B(10,3)= FN2
      B(10,4)= -FM2
      GAMMA2 = GAMMA*(1.+FMU)
      B(11,1)= FM1+FN1*(1.-GAMMA2)
      B(11,2)= FN1-FM1*(1.-GAMMA2)
      B(11,3)= -FM2+FN2*(1.+GAMMA2)
      B(11,4)= -FN2-FM2*(1.+GAMMA2)
   23 EIG = E*FI/G
      RBAR(1) = 2.*EIG/RR
      RBAR(2) = -2.*EIG*FKK
      RBAR(3) = RBAR(2)*FK/GAMMA
      RBAR(4) = RBAR(3)*2.*FKR/GAMMA
      RBAR(6) = 4.*FI*FKR*FKK/(G*A*GAMMA**3)
      RBAR(7) = 2.
      RBAR(8) = -RBAR(3)/GAMMA**0.5
      RBAR(9) = RBAR(8)
```

```
      RBAR(10)=(RBAR(8)*2.*FKR/GAMMA)*SL
      RBAR(11)= -RBAR(6)/GAMMA**0.5
      RBAR(12)= 1.
      RBAR(6)=RBAR(6)*SL
      DO 24 I=1,11
      DO 24 J=1,4
   24 B(I,J) = B(I,J)*RBAR(I)
      B(12,1) = (B(11,1)-2.*ALPHA1)/R
      B(12,2) = (B(11,2)-2.*BETA1)/R
      B(12,3) = (B(11,3)-2.*ALPHA3)/R
      B(12,4) = (B(11,4)-2.*BETA3)/R
      DO 25 J = 1,4
      BMAT(1,J) = B(7,J)
   25 BMAT(2,J) = B(11,J)
      BMAT(3,1) = ALPHA1
      BMAT(3,2) = ALPHA3
      BMAT(3,3) = BETA1
      BMAT(3,4) = BETA3
C
C     SET UP BQ AND BDEL MATRICES
C
      C1 = COSF(BETA1*PHIK)
      C3 = COSF(BETA3*PHIK)
      S1 = SINF(BETA1*PHIK)
      S3 = SINF(BETA3*PHIK)
      IF (LDT) 28,28,27
   27 E1=1.
      E2=1.
      E3=0.
      E4=0.
      GO TO 29
   28 E1 = EXPF(ALPHA1*PHIK)
      E2 = EXPF(ALPHA3*PHIK)
      E3 = EXPF(-ALPHA1*PHIK)
      E4 = EXPF(-ALPHA3*PHIK)
      IF (E3-XDT) 61,61,60
   60 IF (E4-XDT) 61,61,29
   61 LDT=1
      DT(2)=E3
      DT(3)=E4
   29 DO 30 I=1,4
      DO 30 J=1,8
   30 CE(I,J) = 0.
      CE(1,1) = C1*E1
      CE(1,2) =-S1*E1
      CE(2,1) = CE(1,2)
      CE(2,2) =-CE(1,1)
      CE(3,3) = C3*E2
      CE(3,4) =-S3*E2
      CE(4,3) = CE(3,4)
      CE(4,4) =-CE(3,3)
      CE(1,5) = C1*E3
      CE(1,6) = S1*E3
      CE(2,5) = CE(1,6)
      CE(2,6) =-CE(1,5)
```

```
      CE(3,7) = C3*E4
      CE(3,8) = S3*E4
      CE(4,7) = CE(3,8)
      CE(4,8) =-CE(3,7)
   31 CALL MPYBCE    (B,1,CE,BQ,4)
      CALL MPYBCE    (B,4,CE,BQ,2)
      CALL MPYBCE    (B,6,CE,BDEL,1)
      CALL MPYBCE    (B,7,CE,BDEL,3)
      DO 32 I=1,4
      DO 32 J=5,8
   32 CE(I,J) = -CE(I,J)
      CALL MPYBCE    (B,9,CE,BQ,3)
      CALL MPYBCE    (B,10,CE,BQ,1)
      CALL MPYBCE    (B,11,CE,BDEL,2)
      CALL MPYBCE    (B,12,CE,BDEL,4)
      DO 33 I=1,4
      DO 33 J=1,4
      BQ(I+4,J) = BQ(I,J+4)*(-1.)**I
      BQ(I+4,J+4) = BQ(I,J)*(-1.)**I
      BDEL(I+4,J) = BDEL(I,J+4)*(-1.)**(I+1)
   33 BDEL(I+4,J+4) = BDEL(I,J)*(-1.)**(I+1)
C
C     INVERT BDEL MATRIX
C
      CALL INVERT (BDEL,8,8,ZX,ZY)
C
C     CALCULATE ADEL1,ADEL2, AND SMALLK MATRICES

      DO 35 I=1,8
      DO 35 J=1,8
      Z(I,J)=0.
      DO 35 K=1,8
   35 Z(I,J)=Z(I,J)+BQ(I,K)*BDEL(K,J)
      DO 40 I=1,8
      ADEL1(I,1) =-BDEL(I,2)
      ADEL1(I,2) =-BDEL(I,3)
      ADEL1(I,3) = BDEL(I,4)
      ADEL1(I,4) = BDEL(I,1)
      ADEL1(I,5) = BDEL(I,6)
      ADEL1(I,6) = BDEL(I,7)
      ADEL1(I,7) = BDEL(I,8)
      ADEL1(I,8) = BDEL(I,5)
      ADEL2(I,1) =-BDEL(I,6)
      ADEL2(I,2) =-BDEL(I,7)
      ADEL2(I,3) = BDEL(I,8)
      ADEL2(I,4) = BDEL(I,5)
      ADEL2(I,5) = BDEL(I,2)
      ADEL2(I,6) = BDEL(I,3)
      ADEL2(I,7) = BDEL(I,4)
      ADEL2(I,8) = BDEL(I,1)
      X(I,1) =-Z(I,2)
      X(I,2) =-Z(I,3)
      X(I,3) = Z(I,4)
      X(I,4) = Z(I,1)
      X(I,5) = Z(I,6)
```

```
      X(I,6) = Z(I,7)
      X(I,7) = Z(I,8)
      X(I,8) = Z(I,5)
      Y(I,1) =-Z(I,6)
      Y(I,2) =-Z(I,7)
      Y(I,3) = Z(I,8)
      Y(I,4) = Z(I,5)
      Y(I,5) = Z(I,2)
      Y(I,6) = Z(I,3)
      Y(I,7) = Z(I,4)
   40 Y(I,8) = Z(I,1)
      DO 50 I=1,8
      SMALLK(1,I) =-X(2,I)+Y(6,I)
      SMALLK(2,I) = X(3,I)-Y(7,I)
      SMALLK(3,I) =-X(4,I)+Y(8,I)
      SMALLK(4,I) = X(1,I)-Y(5,I)
      SMALLK(5,I) =-X(6,I)+Y(2,I)
      SMALLK(6,I) = X(7,I)-Y(3,I)
      SMALLK(7,I) = X(8,I)-Y(4,I)
   50 SMALLK(8,I) =-X(5,I)+Y(1,I)
      RETURN
      END


*     LIST
*     LABEL
*     FORTRAN
CMPYBCE
      SUBROUTINE MPYBCE (B,I,CE,BQD,J)
C
C     TO MULTIPLY B BY CE MATRIX AND STORE IN BQ OR BDEL
C
      DIMENSION B(13,4),CE(4,8),BQD(8,8)
      DO 1 K=1,8
      BQD(J,K) = 0.
      DO 1 L=1,4
    1 BQD(J,K) = BQD(J,K)+B(I,L)*CE(L,K)
      RETURN
       END


*     LIST
*     FORTRAN
*     LABEL
C          GENERAL MATRIX INVERSION SUBROUTINE
C
      SUBROUTINE INVERT(A,NN,N,M,C)
C
      DIMENSION A(1),M(1),C(1)
C
      IF (NN-1) 80,70,80
   70 A(1)=1./A(1)
      GO TO 300
```

```
C
      C(KD)=-1.0
      J=KD
      DO 140 K=1,NN
      A(J)=-C(K)/D
  140 J=J+N
C
C     INTERCHANGE COLUMNS
C
      DO 200 I=1,NN
      L=0
  150 L=L+1
      IF(M(L)-I) 150,160,150
  160 K=(L-1)*N+1
      J=(I-1)*N+1
      M(L)=M(I)
      M(I)=I
      DO 200 L=1,NN
      TEMP=A(K)
      A(K)=A(J)
      A(J)=TEMP
      J=J+1
  200 K=K+1
C
  300 RETURN
C
      END
```