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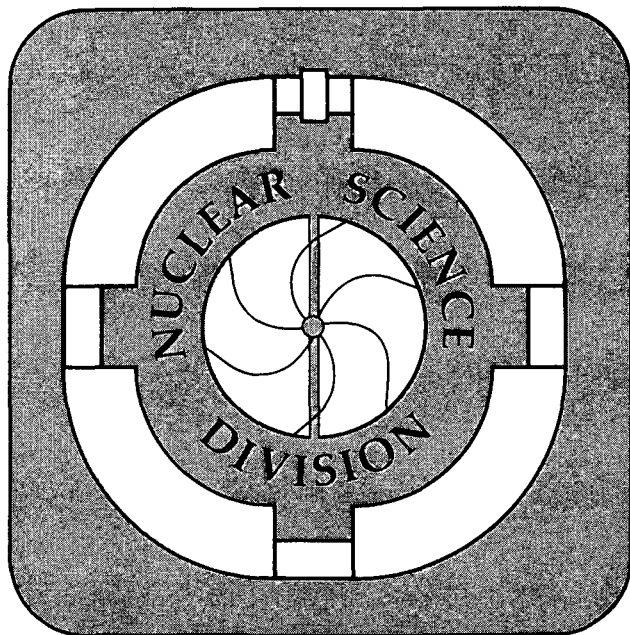
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# Neutron Stars in the Renormalized Chiral-Sigma Model<sup>†</sup>

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July 27, 1987

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<sup>†</sup>This work was supported by the Director, Office of Energy Research, Division of Nuclear Physics of the Office of High Energy and Nuclear Physics of the U.S. Department of Energy under Contract DE-AC03-76SF00098.

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## Abstract

Vacuum renormalization corrections are calculated for normal nuclear matter and neutron star matter in the chiral-sigma model. The theory is generalized to include hyperons in equilibrium with nucleons and leptons. It is shown that fully one half the mass of a neutron star at the limiting mass is composed of matter at less than twice nuclear density. Neutron star masses are therefore moderately sensitive to the properties of matter near saturation and to the domain of the hyperons, but dominated by neither. The predictions for a soft and stiff equation of state are compared with observed neutron star masses, and only the stiffer is compatible.

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# Neutron Stars in the Renormalized Chiral-Sigma Model

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In the last few years there has been great interest in relativistic nuclear field theory, both concerning the normal state of nuclear matter and nuclei and states of matter under extreme conditions.

In this paper we shall study the chiral-sigma model and take into account vacuum corrections [1], which are essential to obtain a normal saturation curve for nuclear matter.

Hyperons form an important component in neutron stars, and have been shown to soften the equation of state appreciably in the moderate to high baryon density domain [2,3]. As previously noted, this could be a critical factor in the first bounce mechanism for supernovae, since the time scales of star collapse are long (seconds) compared to the electroweak processes involved in relaxation of dense nucleon matter into hyperon matter [3]. The situation is opposite in high energy nuclear collisions which are fast compared to these processes so that net strangeness is not developed. Accordingly, for application to neutron stars we generalize the chiral-sigma model to include hyperons.

The Lagrangian for the chiral-sigma model is [4],

$$\begin{aligned}\mathcal{L}_\sigma &= \bar{\psi}_N[i\gamma_\mu\partial^\mu - g(\sigma + i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi})]\psi_N \\ &+ \frac{1}{2}(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi}) - \frac{1}{4}\lambda(\sigma^2 + \boldsymbol{\pi}\cdot\boldsymbol{\pi} - \sigma_0^2)^2\end{aligned}\quad (1)$$

to which we add the Lagrangians for the vector and vector-isovector mesons,  $\omega$  and  $\rho$ , which are coupled to the conserved baryon and isovector currents respectively. Thus,

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_\sigma + [-\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu] + [-\frac{1}{4}\rho_{\mu\nu}\cdot\rho^{\mu\nu} + \frac{1}{2}m_\rho^2\rho_\mu\cdot\rho^\mu] \\ &- \bar{\psi}_N[g_\omega\gamma_\mu\omega^\mu + \frac{1}{2}g_\rho\gamma_\mu\boldsymbol{\tau}\cdot\rho^\mu\cdots]\psi_N\end{aligned}\quad (2)$$

In this paper we shall be concerned only with the normal non-pion-condensed state of matter, so we take  $\boldsymbol{\pi} \equiv 0$ , and inconsequentially therefore, also  $m_\pi = 0$ .

The Dirac equation for the baryons is the Euler-Lagrange equation of  $\mathcal{L}$ , and is readily obtained as,

$$[\gamma_\mu(p^\mu - g_\omega\omega^\mu - \frac{1}{2}g_\rho\boldsymbol{\tau}\cdot\rho^\mu) - g\sigma]\psi_N = 0\quad (3)$$

This can be easily solved in the mean field approximation. The mass term in eq.(5) appears in the form  $g\sigma$ , which is referred to as the *effective* nucleon mass,  $m_N^* = g\sigma$ . to be compared with the vacuum mass,  $m_N = g\sigma_0$ . The energy eigenvalue is readily obtained as,

$$\epsilon_N(k) = g_\omega\omega_0 + g_\rho\rho_{03}I_{3N} + (k^2 + m_N^{*2})^{1/2}\quad (4)$$

where  $I_{3N}$  is the isospin of the baryon (nucleon)  $N$ .

The energy density can be found as the diagonal time-component of the stress-energy tensor and is found to be,

$$\begin{aligned} \epsilon = & -\frac{1}{2}m_\omega^2\omega_0^2 - \frac{1}{2}m_\rho^2\rho_{03}^2 + \frac{1}{4}\lambda(\sigma^2 - \sigma_0^2)^2 + \sum_N \frac{2J_N + 1}{2\pi^2} \int_0^{k_N} \epsilon_N(k)k^2 dk \\ & - \frac{m_N^4}{8\pi^2}F(\eta) + \frac{m_\sigma^4}{64\pi^2}F(\Delta) \end{aligned} \quad (5)$$

The last two terms have been separately added and represent the one-loop expressions for the renormalization of the energy arising from shifts in the spectrum of the filled Fermi sea of nucleons and the zero point energy of the  $\sigma$  field, in comparison with the vacuum [1].

It has not been found how to calculate the renormalization of the  $\rho$ -meson. In the uniform matter case, the only contribution of this meson is a term in the energy density that is quadratic in the isospin density. We may regard this as a phenomenological term, and determine the coupling by the empirical symmetry energy.

We have used the other definitions,

$$\eta = (\sigma/\sigma_0)^2 - 1 \quad (6)$$

$$\Delta = \frac{3}{2}\eta + \sum_N \frac{2J_N + 1}{2\pi^2} \frac{g^2}{m_\sigma^2} \int_0^{k_N} (k^2 + m_N^{*2})^{-3/2} k^4 dk \quad (7)$$

$$F(y) = (1+y)^2 \ln(1+y) - y - \frac{3}{2}y^2 \quad (8)$$

The field equation for the scalar, vector and isovector fields can be obtained as the condition that the energy is stationary at fixed baryon density.

The pressure can be found from  $p = \rho(d\epsilon/d\rho) - \epsilon$ .



The obvious generalization of the scalar coupling to hyperons is [5],

$$\sigma_0 = m_N/g = m_\Lambda/g_\Lambda = m_\Sigma/g_\Sigma \dots \quad (9)$$

The nucleon terms,  $N$ , in eq.(1,2,5) should be interpreted as sums over the *charge* states of  $N, \Lambda, \Sigma, \Xi, \dots$ . The pressure in this general case can be computed from  $p = \rho(d\epsilon/d\rho) - \epsilon$ . In addition, we add the energy and pressure of the leptons.

In fitting the theory to the empirical properties of nuclear matter, we shall use the three coupling constants,  $g, g_\omega, g_\rho$  and the scalar meson mass,  $m_\sigma$ . The vector meson masses are taken from experiment and are  $m_\omega = 783 \text{ MeV}$  and  $m_\rho = 770 \text{ MeV}$ . Since the value of the nuclear compression modulus is currently debated, we shall use two values for  $K$ , one that we refer to as soft,  $K = 200 \text{ MeV}$ , and one that is stiff,  $K = 300 \text{ MeV}$ . The other matter properties are  $\rho_0 = 0.151 \text{ fm}^{-3}$ ,  $B/A = -16.3 \text{ MeV}$ ,  $a_{sym} = 32.5$ ,  $m_N^*/m_N = 0.85$ , and the corresponding coupling constants are listed in Table 1. The ‘experimental’ values for the first three properties are taken from ref. [6] and the effective nucleon mass at saturation is taken from ref. [7].

The equation of state is shown in Fig. 1 for nuclear matter in the form of binding energy per nucleon,  $B/A$ , as a function of density. Also shown are the two contributions  $V_N$  and  $V_\sigma$  to the shift of the nucleon and  $\sigma$  vacua, whose sum is  $V$ , the two-body contribution  $E_2$  and the sum of the three- and four-body contributions,  $E_3 + E_4$ . The renormalization energy is seen to be strong and repulsive. It almost cancels the two-body attractive contribution in the region of nuclear saturation, but saturates at higher density. Recall that

Table 1: Parameters of the Theory

	$\frac{g^2}{4\pi}$	$\frac{g_\omega^2}{4\pi}$	$\frac{g_\rho^2}{4\pi}$	$m_\sigma$ (MeV)
soft	16.8	2.74	6.3	982
stiff	18.24	3.514	6.08	1071.5

the solution to eq.(15) in the absence of the renormalization terms yields a saturation curve that bears no resemblance to that of normal nuclear matter. Instead the normal state is bifurcated by an abnormal state and terminates.

The equilibrium admixture of nucleons, hyperons and leptons is shown in Fig. 2 for  $K = 300 \text{ MeV}$  for neutron star matter, which is charge neutral matter in equilibrium. The results are similar for  $K = 200 \text{ MeV}$ .

Below the hyperon thresholds, the electron chemical potential is a rapidly increasing function of density, which is reflected in the rapid increase in the lepton populations in the lower density domain. However when the hyperon threshold is reached, charge neutrality can be more economically maintained through the conversion of neutrons to  $\Lambda$ 's or protons to  $\Xi^-$ 's, etc., rather than neutron beta decay to proton and relativistic electron. The rapid increase in  $\mu_e$  is therefore arrested, and it never exceeds 280 MeV in the density domain of neutron stars. As observed before [2], this makes kaon condensation very unlikely.

The equation of state for neutron star matter corresponding to the stiff

equation of state is shown in Fig. 3. Three comparisons are made, pure neutron matter, matter in which neutrons and protons are in equilibrium with leptons, and the full generalized equilibrium consisting of nucleons, hyperons and leptons. The softening of pure neutron matter by beta decay of some neutrons to protons is evident by the shift to lower pressure, and the additional softening due to hyperons at their thresholds is clearly evident. All three lie below the causal limit,  $p = \epsilon$ , and reach it only asymptotically far beyond the domain of neutron star densities.

The limiting neutron star mass is especially interesting because an acceptable theory of matter must be able to account for neutron stars whose masses are known. The most accurately measured mass is for PSR1913+16 with  $M = 1.451 \pm 0.007M_{\odot}$  [8]. The largest measured mass is  $1.85^{+0.35}_{-0.30}$  for 4U0900-40. Until recently, measurements of neutron star masses were interpreted as though the stars belonged to a population all having the same mass. In this interpretation the common mass compatible with the existing measurements and their errors is  $1.4 \pm 0.2M_{\odot}$  [9]. It has been pointed out recently that the theoretical prejudice underlying this interpretation is no longer justified in view of recent developments [10]. In this case the constraint on theory must be taken provisionally as the largest mass that is apparently observed, namely  $1.85M_{\odot}$ .

In Fig. 4 we show the calculated masses corresponding to the stiff equation of state for pure neutron matter, beta stable neutron-proton matter, and the case of generalized beta equilibrium of nucleons, hyperons and leptons. The last is the one that provides the limiting mass of the corresponding

equation of state, the first two being shown to illustrate the magnitude of the effects of beta stability. The effects on the limiting mass are appreciable, neutron-proton stability and nucleon-hyperon stability amounting each to about  $1/4M_{\odot}$ . These effects are not as large as those found for the scalar-vector-isovector theory [2,3]. The soft equation of state cannot support a star of mass greater than  $1.3M_{\odot}$ . We conclude that  $K = 200 \text{ MeV}$  is marginally compatible with the limit  $1.4 \pm 0.2M_{\odot}$  in agreement with our earlier analysis of the constraints placed on  $K$  by neutron stars [11]. However it cannot account for the mass  $1.85M_{\odot}$ . In fact the “stiff” equation of state can only marginally account for the latter (ie. compatible within the lower error limit).

In fig. 5 the fraction of mass of the star,  $M(\rho)/M$ , that is composed of matter at baryon density greater than  $\rho$  is shown. From this figure we learn that about 85 percent of the star’s mass resides in matter that is at densities greater than nuclear ( $\rho = 0.153 \text{ fm}^{-3}$ ) but that half the mass is composed of matter that is at densities less than twice nuclear density! Thus the sensitivity to the compression modulus arises in two ways. First although  $K$  is a property at saturation density, by continuity of the theory and the causal constraint that the speed of sound in matter cannot exceed the speed of light, the stiffness or softness at saturation is reflected also in the high density equation of state. Secondly, because of the three dimensional geometry, much of the mass of the star is composed of matter at moderate density. The plateau region in Fig. 5 corresponds to the threshold for hyperons. The fact that only about a third of the star’s mass is contributed by matter at

Table 2: Properties of Neutron stars at the limiting mass, the central baryon density in units of normal nuclear density, radius, gravitational mass in solar mass units, total baryon number,  $A$ , and surface gravitational redshift  $z = \Delta\lambda/\lambda$ .

	$\rho_c/\rho_0$	R (km)	$M_{lim}/M_\odot$	A ( $10^{57}$ )	$z$
soft	6.22	12.0	1.30	1.73	0.215
stiff	5.90	12.3	1.65	2.23	0.286

densities above this threshold accounts for the moderate dependence of the limiting mass on the presence of hyperons as was noted above.

The properties of stars at the limiting mass for the two equations of state are summarized in Table 2.

To summarize, we have included vacuum renormalization corrections to the chiral-sigma model, and extended it to include the hyperons. The vacuum corrections are large. Because of the significant softening of the equation of state caused by hyperons, only the stiff one is compatible with the evidence on neutron star masses. This softening could play a crucial role in the first bounce mechanism of supernovae [3] and should be taken into account in supernova simulations. The effect on the limiting neutron star mass as compared to pure neutron matter is a reduction of  $1/2$  to  $3/4 M_\odot$  depending on whether the equation of state is otherwise stiff or soft. The central density of the limiting mass star is about six times the baryon density of normal nuclei, but fully one half of the mass of the star is contributed

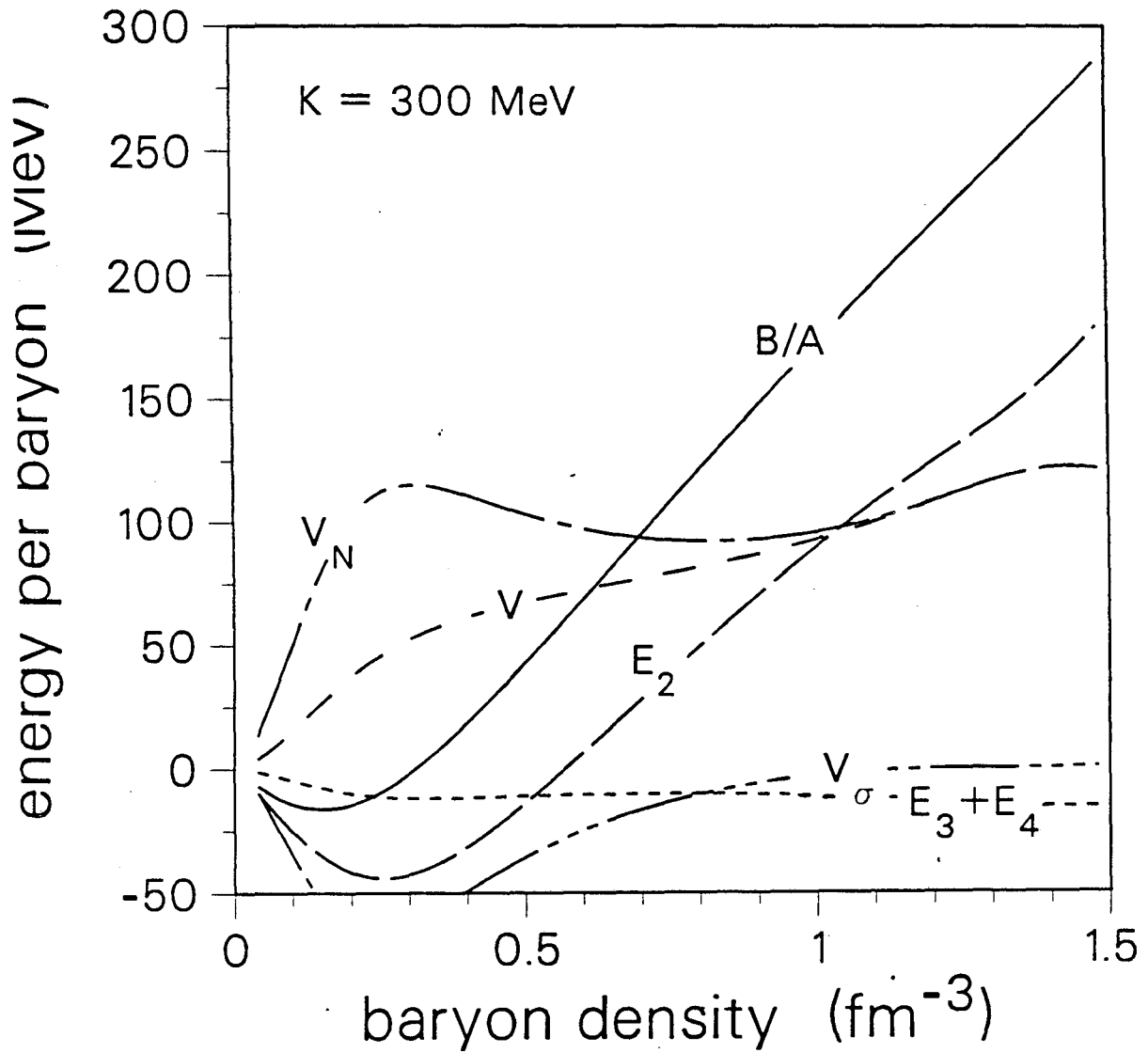
by matter at less than twice nuclear density in this model. Consequently neutron star properties depend on the properties of matter in the domain near saturation as well as the domain of the hyperons, and is dominated by neither.

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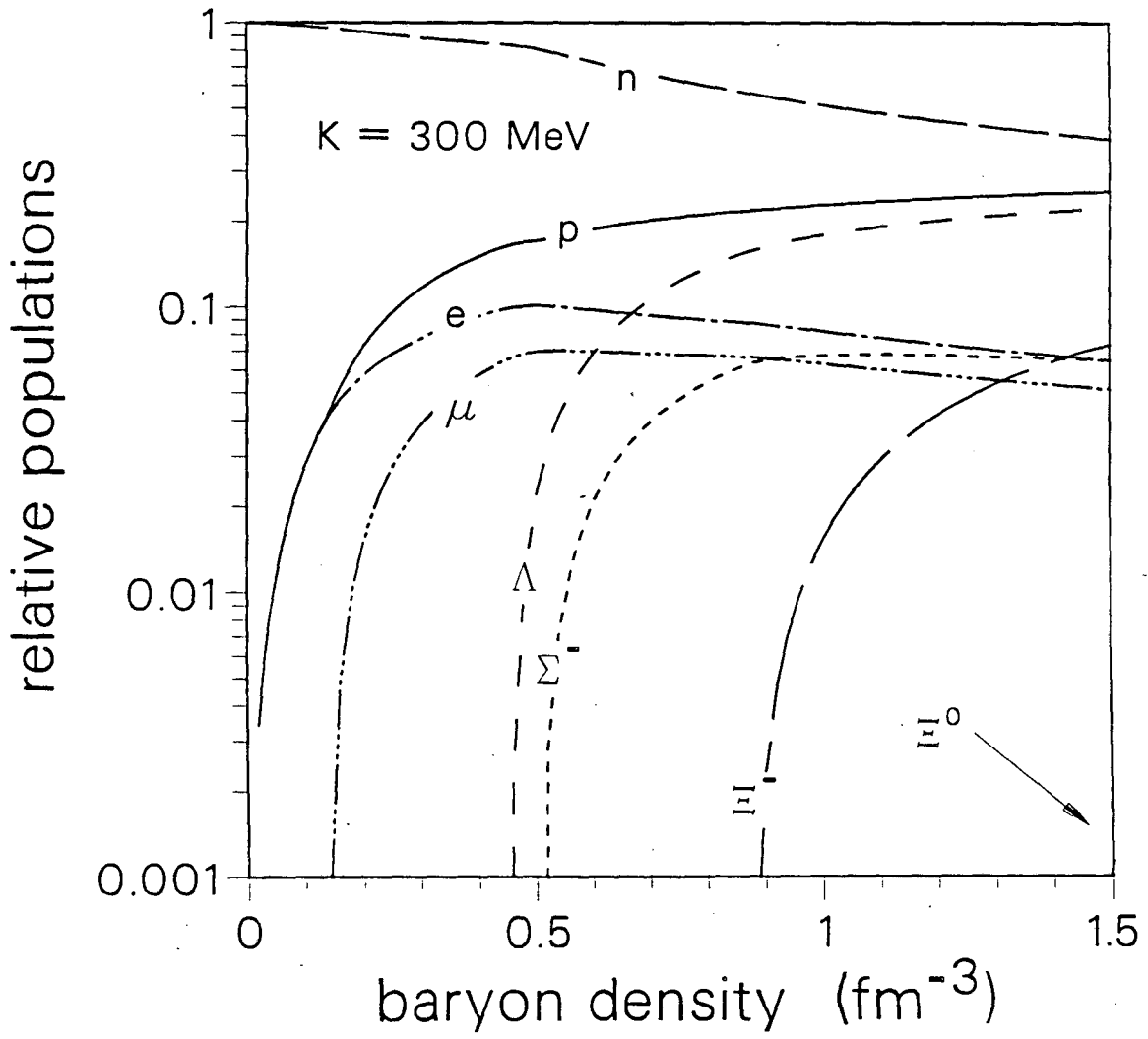
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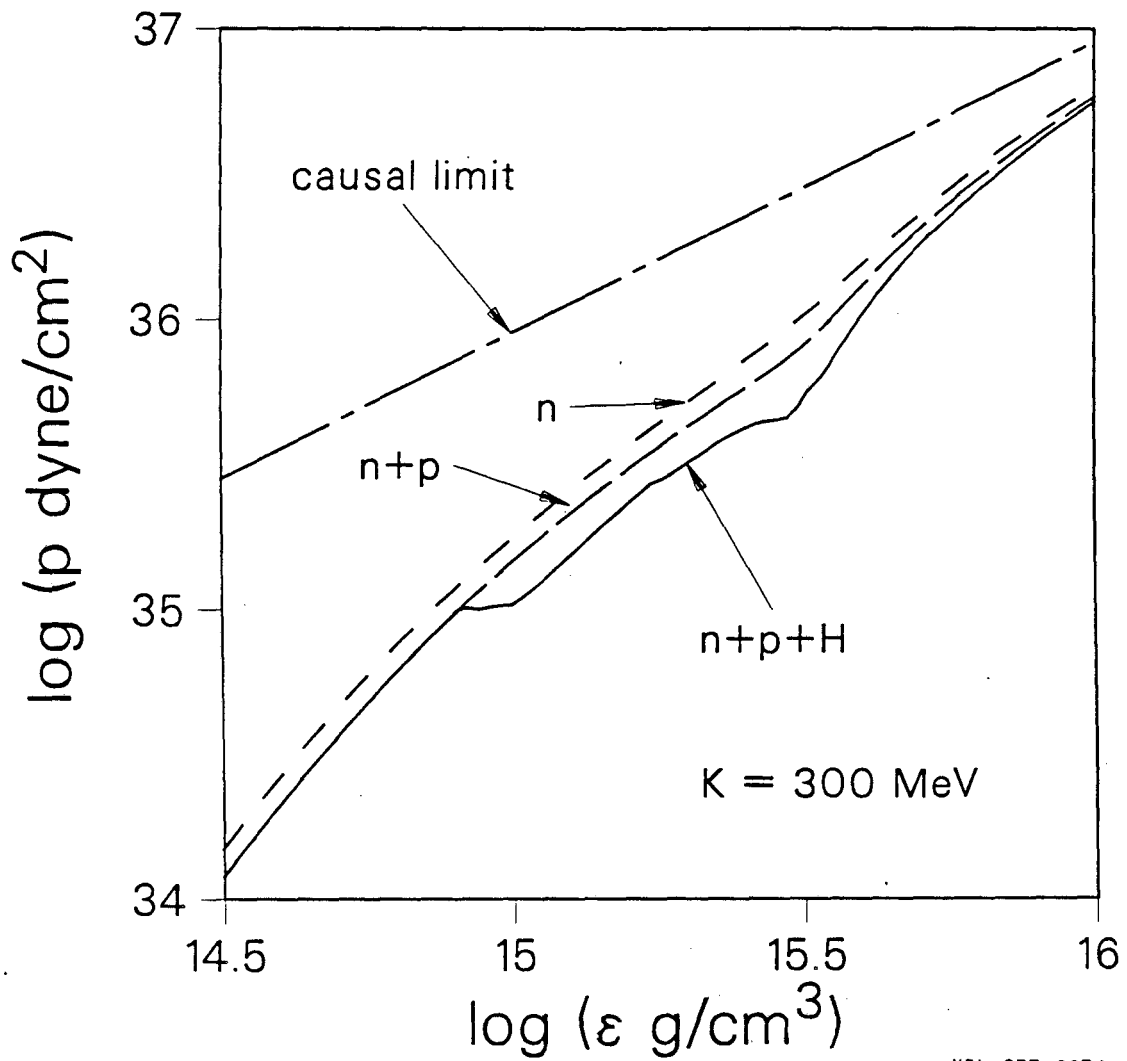
Fig. 1 For the 'soft' equation of state the binding per nucleon is shown for nuclear matter as a function of baryon density. Also shown are the contributions to it, the two, three and four-body parts, and the vacuum polarization energy of the nucleon ( $V_N$ ) and  $\sigma$  meson ( $V_\sigma$ ), and their sum, denoted by  $V$ .





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Fig. 2 Populations relative to total baryon density, in charge neutral beta stable neutron star matter as a function of total baryon density.



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Fig. 3 Equation of state,  $p$  vs  $\epsilon$ , in the case that  $K = 300$  MeV for nuclear matter. The curve marked 'n' is pure neutron matter, 'n+p' is neutrons and protons in equilibrium with electrons and muons, and 'n+p+H' has hyperons in addition. The causal limit is  $p = \epsilon$ .

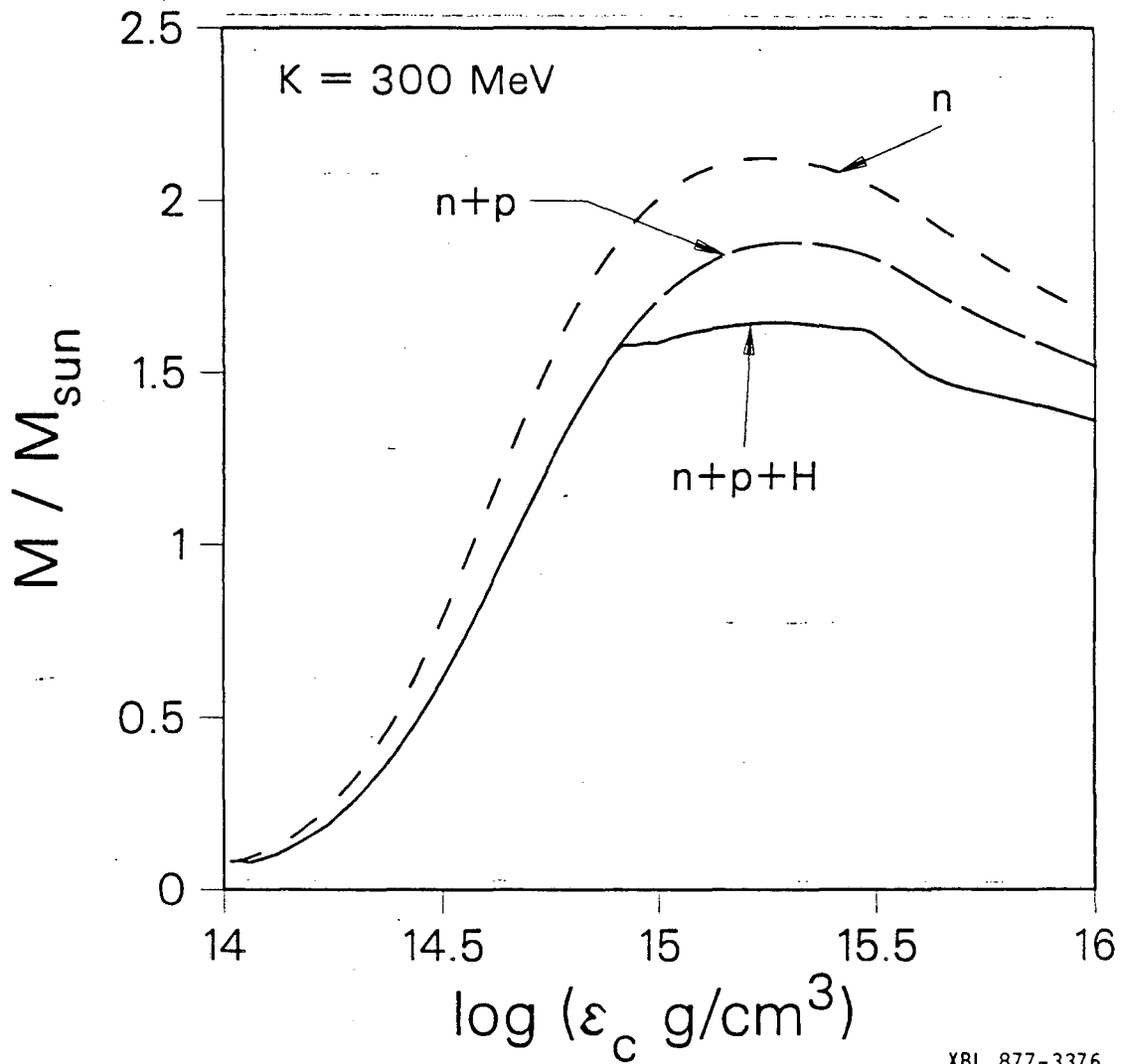
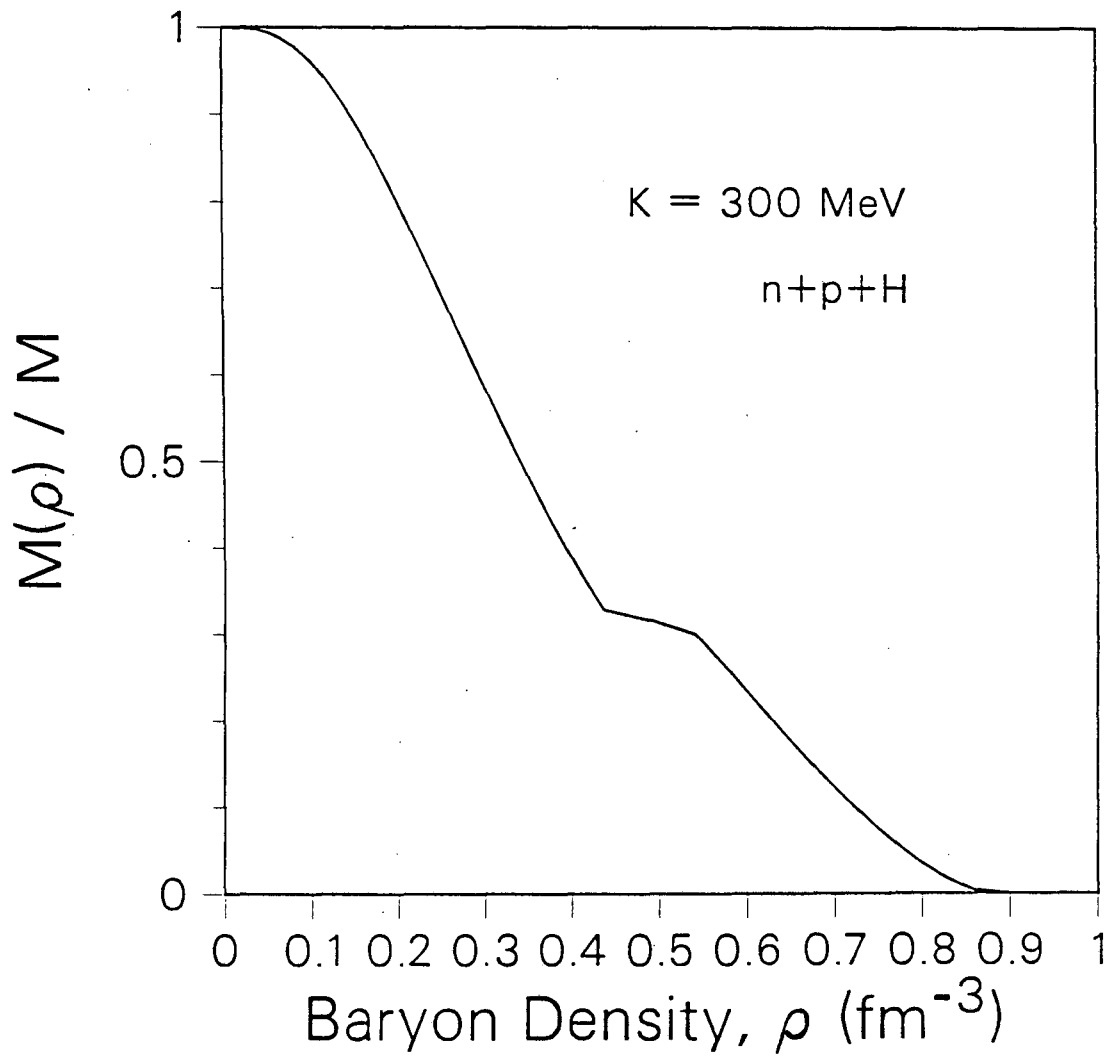


Fig. 4 Neutron star gravitational mass in solar mass units as a function of central energy density for the 'stiff' equation of state. The curve marked 'n' is pure neutron matter, 'n+p' is neutrons and protons in equilibrium with electrons and muons, and 'n+p+H' has hyperons in addition.



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Fig. 5 For the star at the limiting mass in the case of the 'stiff' equation of state, the fraction of the mass of the star  $M(\rho)/M$  contained at baryon density greater than  $\rho$  is shown as a function of  $\rho$ .

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