Title

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Nguyen, Caitlin Elizabeth

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A Review of Laguerre-Gaussian Mode Laser Heater for Microbunching Instability Suppression in Free-Electron Lasers

Caitlin Nguyen*[1]

[1]*Electrical and Computer Engineering Department, University of California Los Angeles*
*caitguyen@g.ucla.edu

Abstract: This review goes over the proposal of using a laser heating to better the suppression of microbunching instability (MBI) in free-electron lasers (FEL) and explores alternative methods of generating a donut shaped beam, separate from the original paper.

INTRODUCTION

Tang *et al.* (2020) experimentally demonstrates that the tailoring of the transverse profile of a laser heater using a Laguerre-Gaussian 01 (LG_{01}) mode can significantly improve microbunching instability (MBI) suppression in free-electron lasers (FEL). The MBI is caused by collective beam effects such as longitudinal space charge and coherent synchrotron radiation, which then causes the amplification of small-scale density and energy modulations during magnetic compression. This degrades the gain of the FEL and introduces noise to the spectrum. The proposed LH is different from the conventional LHs. Typically, LH systems use a Gaussian transverse mode matched to the electron beam, but these have drawbacks due to constraints such as the need for much larger laser spot sizes to mitigate transverse jitter. However, those constraints also introduce other complications; the large laser spot leads to a suboptimal energy distribution that is described as a double-horn shape.

The LH proposed in the paper is modified to convert the infrared (IR) Gaussian beam into a LG_{01} mode with a spiral phase plate. With a spectrometer, the researchers verified that the LG_{01} mode LH produces a Gaussian slice energy distribution over a range of induced energy spreads. In comparison, the standard and conventional Gaussian mode reproduces the double-horn shape from medium to high heater powers. To quantify the difference between the LG_{01} and Gaussian results, the researchers compared the Gaussian R^2 fitting coefficients of the two modes, and after averaging over multiple shots, it is verified that the LG_{01} mode laser consistently produces the more Gaussian-shaped energy distribution.

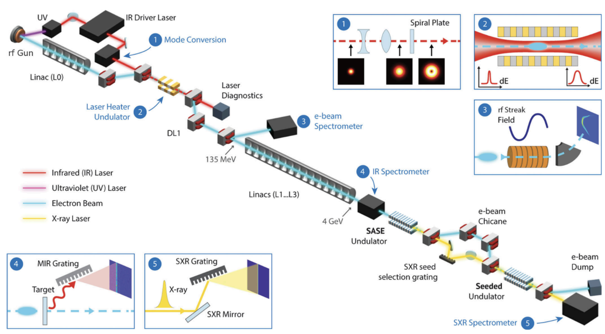

Fig. 1 Example figure from [1] showing the simplified experimental setup.

Further down the configuration, the researchers use a mid-infrared spectrometer to characterize the impact on microbunching, and those results also show that the LG_{01} mode laser better suppresses the MBI. They determine the impact by using the spectrometer to measure coherent emission corresponding to the longitudinal density modulations. When the coherent spectral signal is integrated across $k \in (3000, 5000)$ cm^{-1}, for an induced energy spread in the 20-30keV range, the LG_{01} mode reduces the coherent signal compared to the Gaussian; that indicates greater suppression of the MBI.

The paper further elaborates on how greater MBI suppression benefits one major application of FEL operational modes: soft X-ray self-seeded (SXRSS) FEL operation. The researchers' simulations show improved monochromaticity when using the LG_{01} heater by 20%.

To give an overarching view, the Tang paper demonstrates that the LG_{01} mode shaping of the laser provides an experimentally validated method to tailor the induced energy distribution such that the MBI is better suppressed and the FEL spectral performance is improved. This review paper proposes an alternative method to generating donut-shaped intensity beams while maintaining a fixed polarization for similar applications.

METHODS

To supplement Tang *et al.* (2020), we will determine an alternative method to generating donut-shaped intensity beams while maintaining a fixed polarization. Our goal is to compare methods that preserve the polarization and quantify how their transverse intensity profiles map into induced energy-modulation distributions for an electron beam near the beam center. We will look at the usage of a scalar axicon-based approach to create donut-shaped beams for a LH. We will then compare our results to the implementation of [1]. For this alternative method, the spiral-axicons should also create a Bessel-type beam with polarization preservation when implemented as isotropic scalar elements [2, 3]. We can approximate the radial phase delay for some wavelength as

$$\Phi_{axicon}(r) = k(n - 1)ar, \tag{1}$$

where $k = \frac{2\pi}{\lambda}$, from the equation $\Phi(r) = k(n - 1)r\, tan(a)$ using the small angle approximation to make $tan(a) \approx a$. This equation means that the axicon turns the incident plane or Gaussian wavefront into a conical one, and the radial phase is scalar (as in, independent of our polarization) when the material of our axicon is isotropic. This means that the input polarization is preserved [2].

A conical superposition of plane waves with our wavevector k_r will create a Bessel-type transverse field. The field can be modeled by a Bessel-Gauss form [2, 3] as

$$E_{BG}(r) = J_1(k_r r)exp(-\frac{r^2}{\omega_0^2}), \tag{2}$$

which comes from [4] with the paraxial approximation. We get intensity from this to be

$$I_{BG}(r) = J_1^2(k_r r)exp(-\frac{2r^2}{\omega_0^2}). \tag{3}$$

Where we have J_1 as the first-order Bessel function, $k_r = k\, sin\theta$ (θ is the conical angle from the axicon), and ω_0 is the Gaussian envelope waist (determined by the optics and the input beam [2, 5].

For our simulations, we will also use a Gaussian reference mode, much like how [1] used it as a baseline for the LH testing and quantification. This is defined as

$$E_G(r) = exp(-\frac{r^2}{\omega^2}) \tag{4}$$

with its intensity being defined as

$$I_G = |E_G|^2 \tag{5}$$

Finally, we can compare our Bessel-Gauss beam to the LG_{01} mode, which we can get from the explicit equation $E_{p,l}(r, \varphi, z)$ from [6] to become

$$E_{LG01}(r, \varphi) = \frac{\sqrt{2}r}{\omega} exp(-\frac{r^2}{\omega^2})e^{j\varphi} \tag{6}$$

and

$$I_{LG01}(r) = \frac{2r}{\omega} exp(-\frac{2r^2}{\omega^2}) \tag{7}$$

so that we can compare all three and determine if our Bessel-Gauss beam will create a donut-shaped beam as expected. The simulated intensity maps will be normalized for the three: the Gaussian mode reference beam, the LG_{01} beam described in [1], and the Bessel-Gaussian beam (spiral-axicon generated). If the Bessel-Gaussian matches the shape of the LG_{01} mode, then we will have determined an alternate method of generating donut-shaped beams: spiral axicon optical elements.

RESULTS AND INTERPRETATION

Using a Python script, we can input our equations to plot and generate Fig. 2, which does demonstrate that our Bessel-Gaussian beam from our theory does produce a donut-shaped beam. Fig. 3 shows the phase of the Bessel-Gaussian as well to demonstrate the OAM character of the spiral-axicon beam in order to visually prove that we have a donut shape.

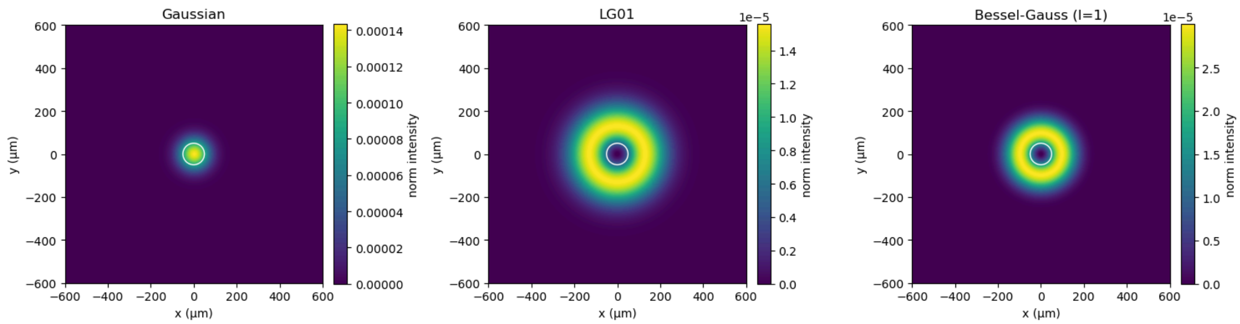

Fig. 2 Generated heatmaps for Gaussian, LG_{01} mode, and Bessel-Gaussian beams.

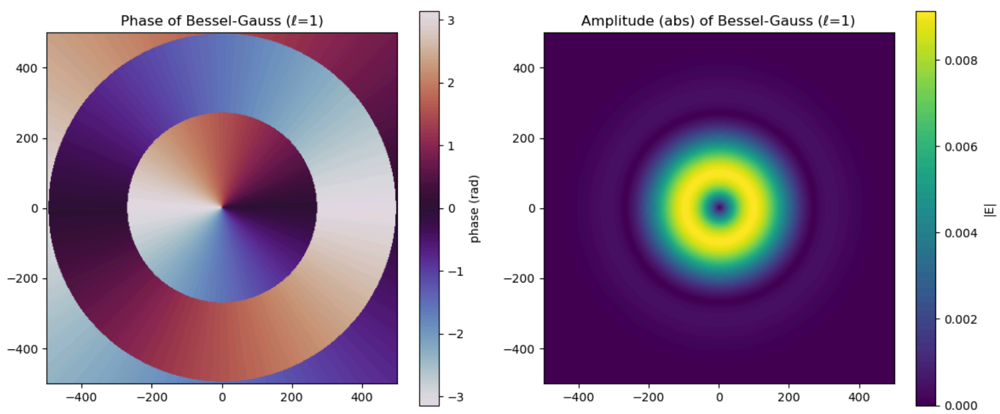

Fig. 3 Phase map and amplitude map of Bessel-Gaussian beam with $l = 1$.

In [1], Tang measures the R^2 to compare the LG_{01} mode to the Gaussian reference, and now we will compare the Bessel-Gaussian to the LG_{01} mode. We perform a sweep across values of k_r and ω to demonstrate that a Bessel-Gaussian can theoretically recreate the results of the LG_{01} mode LH. In Fig. 4, we can see the most optimal values and the Bessel-Gaussian's match to the LG_{01} mode. From Fig. 4, we can see that it is theoretically possible to recreate the results of a LG_{01} mode, with an R^2 value of 0.99921. This indicates that we have a near exact replication of the LG_{01} mode radially, as the spiral axicon creates a beam with a radial profile that closely matches that of the LG_{01} mode.

However, we must acknowledge that the simulations are run under the assumption that we have idealized conditions. Practical implementation of a spiral-axicon to obtain the Bessel-Gaussian beam depends on the fabrication of the optical component itself.

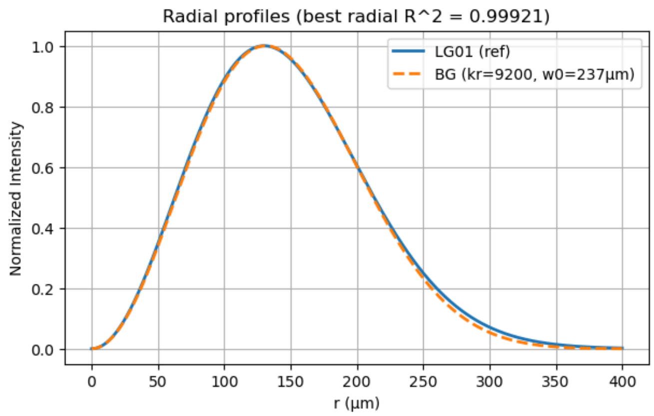

Fig. 4 Radial profile of the Bessel-Gaussian beam generated by the spiral-axicon compared to that of the LG_{01} mode, with parameters swept to determine the highest R^2 value.

CONCLUSIONS

This review examined the Laguerre–Gaussian LG_{01} laser heater proposed by Tang et al. (2020) and evaluated an alternative, axicon-based approach for generating donut-shaped transverse intensity profiles while preserving the polarization of the input beam. The LG_{01} heater demonstrated experimentally improved MBI suppression due to its ability to produce a more Gaussian slice energy distribution and reduce coherent spectral signatures associated with longitudinal density modulations. Based on these results, we investigated whether a scalar spiral-axicon could produce a comparably structured beam suitable for similar FEL LH applications.

Using analytical expressions from Gori, Tudor, and others, we modeled the Bessel-Gaussian beam formed by a spiral axicon and verified that its transverse intensity distribution exhibits the expected structure. Simulated radial profiles showed that, under idealized optical conditions, the Bessel-Gaussian mode can closely approximate the LG_{01} mode, achieving an R^2 value of 0.99921 relative to the LG_{01} radial intensity. This suggests that scalar axicon elements, due to their isotropic response and polarization preservation, represent an alternative for generating donut-shaped beams without requiring spiral phase plates.

However, this conclusion remains theoretical. Real-world implementation would depend on the fabrication tolerances, surface quality, and chromatic behavior of the axicon, all of which would influence the resulting Bessel-Gaussian profile and its stability within a LH system. Future work should therefore experimentally validate spiral-axicon-generated beams in a LH configuration, quantify their produced electron-beam energy spread, and compare their MBI suppression performance directly against the LG_{01} mode. If these results align with the simulations presented here, axicon-based beam shaping may offer a practical, polarization-preserving method for future FEL LH systems.

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