Persevering, Alone or Together? Mathematics Teachers Navigating Individualist Narratives About Perseverance

By

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Abstract

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The construction of equitable, ambitious mathematics classrooms in which students learn together hinges on teachers representing students' mathematical struggle as supported and constrained by interactions they have with peers and teachers. However, commonsense narratives about "perseverance" locate the responsibility for struggling productively in the individual, diminishing or ignoring the role that interpersonal interactions play in students' perseverance. Individualist orientations diminish teachers' agency by placing responsibility for productive mathematical struggle with students. In contrast, group-oriented perspectives towards perseverance have the potential to broaden how teachers conceive of productive student thinking and empower teachers to work towards ambitious mathematics teaching practices. However, supporting teachers to locate perseverance in the collective is complex work. It is complicated by challenges professional developers face in organizing teachers for collaborative inquiry and by how entrenched individualist ideologies are in education—both traditional and reform—and society at large.

This dissertation follows four high school mathematics teachers over a semester as they navigated tensions between commonsense individualist narratives of perseverance and ambitious visions of mathematics instruction. It explores how features of the professional development (PD) environment in which the teachers worked supported or constrained their collaborative inquiry. Interactions among the teachers and PD facilitator were observed and videoed. As summarized below, close examination of the work of these PD participants yields findings with implications for the research and practice of practice-based PD for mathematics teachers.

Analysis examines how the teachers *represented* perseverance as they engaged in PD activities. How the teachers represented perseverance is situated in two broad but distinct discourses of mathematics teaching and learning, "traditional" and "ambitious." Those discourses have ties to broader discourses and ideologies used in society at large, which tend to privilege individualist narratives. Focusing on representations of perseverance illuminates the explicit or tacit systems of categorization that the teachers used to organize people, phenomena, and ideas, which the teachers in turn used to make teaching decisions.

The teachers were found to use both individualist and group-oriented representations of perseverance throughout the PD, with group-oriented representations gaining in explanatory power and prevalence as the PD progressed. Over time, the teachers increasingly used group representations of perseverance to explain successful perseverance and inquired into how they could support perseverance by encouraging students to build on each other's mathematical thinking. Nonetheless, individualist representations of perseverance continued to hold significant explanatory power for the teachers, especially as first resort and to explain failed perseverance. Facilitation that problematized individualist representations of perseverance, frameworks that drew teachers' attention to specific ways in which collaboration can support productive mathematical struggle, and iterative, collaborative classroom-video watching followed by collaborative planning were found to support inquiry that broadened the teachers' horizons of observation around perseverance to include more group-oriented perspective. These findings strongly suggest that PD should be grounded in explicit theoretical frameworks, demand reasoning grounded in examples rather than justified by vague principles, and be guided by facilitation that explicitly problematizes individualist reasoning about students.

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Chapter 1 Introduction

"How do we build student perseverance?" Introducing the themes of the dissertation

How mathematics teachers make sense together of their students' perseverance, and how professional development supports or constrains their sense-making, are the central themes of this dissertation. As the dissertation shows, these issues are decidedly non-trivial. The question that the teachers whose collaborative work this dissertation unpacks pursued together, "How do we build student perseverance?" encompasses a host of thorny yet crucial issues in education broadly and mathematics education in particular. Issues of equity, individualism, and merit, and to what extent teachers have agency in building classroom conditions to support resilience, are implicated in this question. How the teachers traversed these thorny issues and how professional development did and did not afford discourse among the teachers towards ambitious and equitable conceptions of those issues are the topics explored in this dissertation.

The question, "How do we build student perseverance?" is really two questions: *What* is perseverance—as in, to what can we attribute students' persistence in mathematical learning situations? And, *how* can classroom mathematical learning environments be arranged so as to be conducive to perseverance? These questions are interrelated. Where teachers locate perseverance has implications for the teaching decisions they make on a daily basis about how to support their students' perseverance. But these questions also extend beyond individual assessments and decisions made in the course of teaching. How teachers answer each of those questions evokes different discourses common in mathematics education communities and society at large about where to locate the foundations of resilience. As this dissertation discusses, traditional discourses of mathematics instruction, and commonsense discourses of learning and achievement in society at large, locate perseverance in the *individual*. Ambitious discourses of mathematics instruction situate perseverance in the environment, in which *community* and *the collective* are central (Apple, 2006; Martin, 2013; Sengupta-Irving & Agarwal, 2017; Valero, 2017).

How teachers use these discourses to make sense of students, mathematics, and teaching has meaningful implications for the learning environments that teachers cultivate and the opportunities that students have to succeed in them (Bannister, 2015; Horn & Kane, 2015; Nasir et al., 2014). As this dissertation will show, locating perseverance in the individual is consistent with building classroom environments in which students struggle alone. Individual students' success or failure is attributed to innate traits, leaving teachers with little agency to build classroom environments that support productive struggle. Locating perseverance in the collective, however, is consistent with building classroom environments in which students build mathematical ideas together. Students' success or failure is attributed to the affordances of classroom environment, giving teachers ample agency to change their pedagogical practice to encourage richer and more equitable learning. Scholars of mathematics learning have long found that pedagogical practices consistent with group-oriented conceptions of perseverance, practices

that support perseverance by building opportunities for students to collaborate around challenging and worthwhile mathematics tasks, lead to richer and more equitable learning (Engle, 2012; Nasir et al., 2014; Schoenfeld, 2014). They have also found that pedagogical practices consistent with individualist conceptions of perseverance perpetuate inequitable learning disparities and exclusive narratives about who can and should succeed in mathematics (Boaler & Greeno, 2000; Schoenfeld, 2014; Sengupta-Irving & Vossoughi, 2019). But encouraging teachers to move towards ambitious and equitable pedagogies goes hand in hand with shifting their collaborative discourse (Bannister, 2015; Horn, 2007; Horn & Kane, 2015; Nasir et al., 2014), in this case away from individualist and deficit-oriented narratives about perseverance and towards situated narratives that represent perseverance as a group accomplishment. And shifting teachers' discourse is no easy task, especially around concepts such as perseverance that have deeply embedded, problematic representations that are commonsense in society at large but run against ambitious instruction discourses and pedagogies that provide the richest learning for students.

This dissertation focuses on the discursive side of the shifting discourse – shifting pedagogical practice work involved in learning for teachers. It provides a case study of a group of high school math teachers who, supported by a professional development program aimed at ambitious mathematics instruction, inquired into how to build their students' perseverance. As hinted in the previous paragraphs and further unpacked in this dissertation, this case study is situated at the intersection of several robust bodies of educational research: research into teachers' learning around problems of practice (Little, 2002), teachers' sense-making about traditional and ambitious mathematics instruction discourses (Bannister, 2015; Horn, 2007; Louie, 2016), professional development conditions that support generative collaborative discourse for teachers (Lefstein et al., 2020), and conceptual change about deeply embedded ideological issues (diSessa & Sherin, 1998; Philip, 2011; Vosniadou, 2009).

The next sections of this introduction outline the key findings and contributions of this dissertation to the domains of research in which it is situated. First, however, I share a short classroom vignette of perseverance-in-action. To illustrate the significant ideological and practical differences between the two perspectives on perseverance outlined in the previous paragraphs, I unpack this vignette from those perspectives: perseverance as *located in the individual*, and perseverance as a *group accomplishment*. I provide this short vignette and analysis to ground the reader in the stances taken in this dissertation. I also provide this vignette to highlight for the reader how commonsensical and deeply-embedded individualist representations of perseverance are in traditional discourses of mathematics instruction, indicating the challenging work undertaken by the teachers whose learning is examined in this dissertation.

Locating perseverance in the individual or the community

In the following sections I provide a brief vignette of students struggling with challenging mathematics in a high school classroom. I follow the vignette with short analysis from the two

perspectives on perseverance central to this dissertation: perseverance as individual capacity or trait, or perseverance as a group accomplishment. This vignette and analysis encapsulate the key issues with which the teachers in this dissertation grappled. I encourage the reader to answer the following questions themselves before reading my analysis:

1. In this vignette, who persevered, and with what did they persevere?
2. To what do you attribute their perseverance or lack thereof?
3. How could a teacher support perseverance for all students in this situation?

Vignette

Four students sit around a table in a high school math classroom, working on a task. One of the students makes a bid to discuss the task with another student. She suggests one possible answer, then another, leaving room for conversation. But her partner continues working in silence.

Then, a third student announces his answer. It is one of the two answers suggested by the first student. Without question, the second student—who had ignored the first student's bid for discussion—acknowledges the third student's answer and writes it down. The fourth student follows suit. Alternately frustrated and affirmed, the first student writes down the answer and asserts that she had said the same thing. No one responds.

They move onto the next question. Quickly, the third student announces his answer. It happens to be wrong. Nonetheless, all four students write it down without discussion. Task seemingly complete, the third student calls to the teacher to check their work.

Analysis

The vignette recalled above came from a real math classroom. Readers familiar with math classrooms may notice that it reflects a relatively routine interaction. Given the dynamics of the vignette, it is relevant to ask the three questions about perseverance presented before the vignette. Whenever students struggle, perseverance, persistence, resilience, and other related ideas can be invoked in analysis of that struggle. However, there are distinctly different ways of representing those ideas, ways that are derived from distinct discourses about resilience, success, and society. How a teacher represents those ideas in the course of making sense of the students and their interactions has implications for how the teacher might plan to support perseverance or address its lack.

Here is one way of making sense of *who* persevered, and with *what*, and *to what that perseverance can be attributed*. Note how this line of reasoning problematizes how the students positioned one another with respect to their mathematical capabilities and status in the group and highlights the problematic implications of that positioning for *each* of the students' perseverance and productive struggle and the perseverance of the *group:*

> The first student struggled both with a challenging math task and to engage her peers with her ideas. She met with resistance from her peers, who did not respond to her bids for collaboration. Because her peers did not engage with her, her productive mathematical struggle was curtailed. The third student solved the challenging task, although without obviously struggling with it. His solution was immediately taken up by the other students. The disparate reactions of the students to Student 1 and Student 3 positioned Student 1 as less mathematically capable than Student 3, and Student 3 as unequivocally correct. Disheartened by this student's leveraging of his high status to silence his peers, the other three students declined to engage in mathematical reasoning about the next part of the task, leading the group's mathematical learning to suffer and each student to have reduced opportunities to struggle productively, and persevere, with mathematics.

This framing derives from discourses of ambitious mathematics instruction (Lampert, 2003; Schoenfeld, 2014). Such discourses attend to how students' productive struggle with challenging mathematics tasks is co-constructed, potentially supported or constrained by interactional dynamics that affect students' abilities to collaborate and build knowledge together (Engle, 2012; Sengupta-Irving & Agarwal, 2017). The summary provided here draws on these discourses and how they represent an important idea in mathematics teaching and learning: *perseverance.* In discourses of ambitious mathematics instruction, perseverance is a group accomplishment. Addressing how students do or do not support each other to struggle together with mathematics, and how the learning environment supports or constrains their efforts at collaborative struggle, is essential to reasoning about perseverance in ways consistent with ambitious mathematics instruction discourses.

Notably, this line of reasoning opens many avenues for a teacher seeking to support these students' perseverance. In pointing to problematic positioning and status among the students and its cascading effects on their collaborative mathematical discourse as a cause of their lack of perseverance, together and separately, this reasoning invites a teacher to entertain pedagogical strategies that intentionally disrupt status among students and compel students to push each other for mathematical justifications (Cohen et al., 1999).

However, this is not the only possible way to make sense of the interaction recalled above. Here is another approach. Note how this line of reasoning does not problematize issues of status and positioning among the students. Instead, this line of reasoning uses trait- and character-based logic to determine who persevered and to what that perseverance or lack thereof could be attributed:

> The first student struggled with a challenging task. She was not able to offer a correct answer. The third student successfully completed the task. His mathematical skills and confidence resulted, unsurprisingly, in his peers taking up

> his correct answer. Then, the less mathematically-able and confident peers awaited his solution to the next task. They could join in the work, but they chose not to. Left to work alone, but persevering nonetheless, the second student attempted the next task. Unfortunately, he got it wrong. His peers should have pitched in to help him.

While the summary inspired by discourses of ambitious mathematics instruction attended to how the four students in the group co-constructed the perseverance (or lack thereof) that unfolded in the anecdote, the latter summary located perseverance and its lack in the individual students. It attended to individual students' *character* and *choices*, considered out of context. This summary drew on discourses of traditional mathematical instruction. It named character traits—confidence, innate mathematical skill—and individual students' decisions to work or not work as it made sense of what unfolded in the anecdote. This summary dwelled on some students' deficits while lionizing the student who most obviously worked. In doing so, it left a teacher seeking to support all of the students in this group to persevere with few options to intervene. If perseverance is part of character that some students lack or choices that some students inevitably make, regardless of context, what is to be done?

The latter summary reflects predominant and commonsense orientations to perseverance. In traditional mathematics instruction discourses, perseverance is represented as part of *individual students' character* (Sengupta-Irving & Agarwal, 2017). Individualist approaches to perseverance extend beyond mathematics instruction discourses. They are commonplace in broader discourses and ideologies that help explain why some people succeed and others fail in society (Valero, 2017). They perpetuate status-quo, inequitable learning environments wherever they are used (Ledgerwood et al., 2011). And because they are so pervasive, offering explanatory power consistent with broader discourses and ideologies, they are difficult to disrupt (Horn, 2007; Louie, 2016).

I encourage the reader to reflect on their own experience making sense of the students' perseverance in the vignette. What work was required to see perseverance through the two lenses offered here? The reader may have felt that asking about the "perseverance" in this vignette invited individualist interpretations. This is a reasonable reaction given the pervasiveness of individualist representations of perseverance in society. Perhaps asking about something related, such as "agency," would more readily produce reasoning aligned with discourses of ambitious mathematics instruction. Agency is a term commonly adopted by discourses of ambitious mathematics instruction to convey something akin to perseverance (Boaler & Greeno, 2000; Schoenfeld, 2014; Sengupta-Irving, 2015): students' sense that continuing to struggle with mathematics is worthwhile and will be fruitful. For better or worse, however, the teachers whose collaborative work is analyzed in this dissertation chose to frame their problem of practice in terms of *perseverance*. With that word came the hard work of disrupting traditional discourses of mathematics instruction in teachers' collaborative conversation and encouraging ambitious discourses.

This dissertation explores the collaborative work of four high school mathematics teachers as they attempted to make sense of and support their students' perseverance. The preceding vignette and short analyses frame the challenging nature of the work done by the teachers and PD facilitator analyzed in this dissertation. In this dissertation, I examine teachers' uses of different representations of perseverance over time as they inquired together into how to support their students' perseverance in mathematics, supported by a PD program. I employ discourse analysis (Lefstein et al., 2020; Russ et al., 2016) to investigate how teachers' representations changed or stayed the same, reflected discursive traditions in mathematics education and beyond, and positioned the teachers to talk about students and teaching in ways conducive to building powerful mathematical learning environments in their classrooms. I also investigate how PD workgroup conditions, including PD activities, frameworks, and facilitation, supported or constrained discourse generative of collaborative inquiry. I take a grounded, rather than evaluative, perspective, seeking to *understand*, not *assess*, the relationship between PD workgroup conditions, features of teachers' collaborative discourse, and how the teachers leveraged traditional or ambitious instruction discourses to make sense of their work.

The next section of this chapter provides a brief overview of this dissertation's theoretical and methodological approach and its findings.

Overview of key approaches and findings

This dissertation is divided into seven chapters, listed below:

Chapter 1: Introduction

Chapter 2: Theoretical Frameworks

Chapter 3: The Study and Methods

Chapter 4: Working on the Problem of Perseverance

Chapter 5: Towards Students Persevering "Together," Through PD

Chapter 6: Individual Representations of Perseverance Persist

Chapter 7: Conclusion

This first chapter introduces the dissertation. The second chapter outlines two theoretical frameworks central to this analysis: the framework of ambitious mathematics instruction discourse, and the framework describing how PD workgroup conditions may support or constrain generative collaborative discourse among teachers that can lead to learning. This second chapter situates this dissertation's work in a rich body of research on teachers' learning towards discourses of ambitious mathematics instruction. Drawing on this body of work, this chapter frames ambitious instruction discourses as supportive of content-rich, equitable mathematics learning for students (Schoenfeld, 2014). Ambitious mathematics instruction

discourses are also generative, positioning teachers to reason about students and learning in more robust ways and collectively plan for instructional change (Horn & Little, 2010; Kazemi & Franke, 2004). Chapter 2 also explores how teachers' learning towards ambitious mathematics instruction is slow and piecemeal, given the resilience of traditional mathematics instruction discourses (Bannister, 2015; Horn, 2007; Louie, 2016; Philip, 2011). Chapter 2 situates this dissertation in prior research into how PD workgroup conditions may support or constrain generative collaborative discourse. Chapter 2 reviews literature on how facilitation, video-watching activities, and PD frameworks support or constrain generative discourse (Beisiegel et al., 2018; van Es et al., 2014; van Es & Sherin, 2010). Finally, Chapter 2 describes this dissertation's divergence from typical evaluation research exploring PD workgroup conditions.

The third chapter describes the study, PD program, and methods used in this dissertation. It provides an overview of the Teaching for Robust Understanding (TRU) of Mathematics Framework (Schoenfeld & the Teaching for Robust Understanding Project, 2016), Lesson Study (Murata, 2011), and the PD program combining the two in which the teachers in this dissertation participated, TRU-LS (Schoenfeld et al., 2019). It also introduces the Benton High School teachers and facilitator whose discourse this dissertation analyzes. Finally, Chapter 3 describes the methods for longitudinal and case-based discourse analysis used in this dissertation. It introduces the two representations of perseverance that take center stage in this dissertation, perseverance as individual students' capacity or choice to work and perseverance as a group accomplishment.

Chapters 4, 5, and 6 dive into analysis. Chapter 4 explores patterns in how the teachers leveraged individual and group representations of perseverance over time. It draws on quantitative evidence to uncover a pattern of the teachers using more group representations of perseverance as the PD progressed. It also shows that individual representations of perseverance persisted in the teachers' discourse. In several cases, Chapter 4 highlights how the teachers drew on the two representations in different phases of their collaborative work. Chapter 4 shows that teachers can take up ambitious mathematics instruction discourses, and that those discourses afford richer and more complex reasoning about students and collaborative planning for pedagogical change.

Chapter 5 investigates how PD workgroup conditions supported and constrained collaborative discourse among the participants that led to the patterns uncovered in Chapter 4. Using quantitative evidence and a series of cases, Chapter 5 shows that the combined activities of watching classroom video together and collaboratively planning interventions broadened the teachers' horizons of observation about perseverance (Hutchins, 1996), supporting them to represent perseverance in more complex, group-oriented ways.

Chapter 6 returns to the patterns identified in Chapter 4 to further investigate when and how the teachers returned to individual representations of perseverance, even as group representations became more prevalent in their conversations. Chapter 6 situates its analysis in literature highlighting the widespread and deeply-embedded nature of individualist representations of perseverance in mathematics education discourses and beyond. Chapter 6

uncovers patterns in the teachers' discourse that indicate that individual representations retained significant explanatory power for the teachers, especially as representations of first resort, to explain students' deficits, and to emphasize the importance of independence.

Finally, Chapter 7 offers key conclusions and implications of this dissertation for future research and practice. Chapter 7 argues that this dissertation's findings motivate further research into how teachers represent perseverance in their collaborative discourse. It also provides specific recommendations useful to PD designers and scholars of teachers' learning.

In summary, this dissertation takes on two questions important to understanding of teachers' learning: How do teachers represent perseverance as they inquire together into a problem of practice? And, how do PD workgroup conditions support or constrain teachers' learning towards discourses of ambitious mathematics instruction? The next chapter frames these questions, setting up the analysis that follows in subsequent chapters.

Chapter 2 Theoretical Frameworks

This dissertation examines the learning of a group of high school mathematics teachers as they inquired into a problem of practice together. The teachers were supported by a practice-based professional development program (PD) (Ball & Cohen, 1999; Kazemi & Hubbard, 2008), a program of inquiry with activities grounded in teachers' classroom work and supported by a vision of ambitious mathematics instruction.

To examine these teachers' learning, I followed their inquiry with the facilitator and researchers in the PD workgroup over several months. I examined how participants represented core teaching ideas and patterns in those representations over time, with particular focus on how engagement with one another and the PD positioned teachers to take up more ambitious representations of teaching ideas and mobilize for work in their classrooms. As described in the introduction to this dissertation, I seek to answer these research questions:

1. How did the participants represent core teaching ideas as they inquired into their problem of practice, over time?
2. How did engagement with facilitation, PD activities, and frameworks support the participants to come to see and plan to act on in their classrooms more robust, ambitious representations of core teaching ideas?

Broadly speaking, this dissertation engages in discourse analysis of conversations among participants in a teacher learning community or workgroup. Discourse analysis uncovered the patterns of and pivotal moments in teachers' inquiry into an important problem of teaching practice as they collaborated. Analytic focus on *representations of core ideas* and *how engagement within PD supported new patterns* in those representations positions this research in a rich and diverse field of research into teachers' collaborative discourse (Lefstein et al., 2020; Russ et al., 2016). This chapter situates the research presented in this dissertation within this diverse field. It also highlights how this dissertation extends prior research.

The chapter is divided into two parts, corresponding to the two frameworks that shape the analysis in this dissertation. The first part describes the framework used in this dissertation to analyze how the teachers in this study represented core teaching ideas over time. It draws on dominant discourses in research on teachers' collaborative work to define the relationship between how teachers represent core ideas of teaching and their collaborative learning around problems of practice. It situates this dissertation in relation to prior research on teachers' uses of discourses of ambitious and traditional mathematics instruction in collaborative workgroups.

The second part draws on a rich body of research into the conditions that support teachers' collaborative learning around problems of practice to outline a framework for the conditions that can support teachers to come to see ambitious representations of core teaching ideas and plan to act on them in their classrooms. This part situates this dissertation in relation to prior research on the relationship between teachers' patterns of participation in collaborative workgroups and PD and their learning around problems of practice.

Ambitious and traditional discourses of teaching in teachers' collaborative talk

For this dissertation, learning *for teachers* is conceptualized as a process of defining, studying, and attempting to resolve problems of practice in and across their workgroup and classrooms. Problems of practice are more significant than the routine issues that teachers experience and collaborate around daily. "Should I assign this homework problem?" and, "How do I pace the curriculum?" are routine teaching problems that do not rise to the level of problems of practice (Bannister, 2015; Horn & Kane, 2015; Horn & Little, 2010; Louie, 2016). Problems of practice can surface from routine teaching issues such as these, given supportive conditions; or they can surface from exceptional teaching events that highlight significant problems precisely because of their exceptional nature. Regardless of how they emerge, problems of practice are "located at the level of the 'instructional triangle'—or the relationships among teachers, students, and content" (Horn & Little, 2010; p. 189). In other words, teaching problems, whether routine or exceptional, can be read as problems concerning students, teaching, curriculum, or content, each in isolation: e.g., some students are "not trying"; the reform-mandated curriculum is "too challenging"; teacher so-and-so is "ineffective". Conceptualizing problems of practice involves surfacing the conflicts that arise from relationships among communities that meet in classrooms that lead to these problems: e.g., we see these students as "not trying" because we envision content learning as skills-mastery, which blinds us to the strengths students are bringing and leaves us unable to work with the curriculum to create adequate supports for productive struggle. Problems of practice center fundamental issues of teaching and learning, experienced in practice (Ball & Cohen, 1999; Levine, 2010; Philip et al., 2018). Working on them requires challenging taken-for-granted conceptions of teaching, learning, students, and content.

Teachers' collaborative learning around problems of practice is illuminated by close examination of participants' "Discourses", meaning the types of communication, consisting of shared symbols, meanings, representations, metaphors, and interactional and communicational norms and routines, both linguistic and non-linguistic, used by communities (Gee, 2004; Sfard, 2008). How teachers' talk reflects different Discourses—how teachers represent teaching, learning, mathematics, and students, communicate with one another about their work and problem, and plan to change classroom instruction—provides a window into how teachers make sense of the goals of mathematics teaching and learning. Teachers' discourse bounds how they look at and comprehend their work and the phenomena of learning mathematics that they encounter daily (Goodwin, 1994; Sfard, 2008). How teachers make sense of the goals of mathematics instruction and what discursive resources they bring to collaborative work directly influences the mathematics learning experiences they provide for students (Schoenfeld, 2010b). Closely examining teachers' discourses as they collaborate together around problems of practice showcases how teachers see core ideas of teaching and learning and whether and how they can collaboratively mobilize to transform their teaching practice (Horn et al., 2016). This dissertation builds on other work examining teachers' discourse in collaborative workgroups to highlight how the ways in which teachers leverage different discourses of mathematics teaching and

learning influences what they see, how they interpret what they see, how they frame problems, and whether and how they mobilize to transform learning environments—in other words, their collaborative inquiry into problems of practice. While this dissertation builds on this prior research, it also takes it in new directions, examining teachers' discourse about an understudied but core idea of teaching and learning in mathematics—perseverance—and how teachers' discourses about perseverance interact with broader discourses of individualism, merit, and power in society (Lefstein et al., 2020).

Prior research on teachers' collaborative discourse has examined how teachers *represent* core ideas of teaching and learning. In discourse analysis, representations are claims, explanations, and reifications participants develop about key ideas of teaching and learning (Vygotsky, 1986; Wenger, 1998). Representations can function as explicit or tacit systems of categorization, organizing objects, people, phenomena, and ideas into systems of meaning. Within discourse communities, representations, categories, and classifications facilitate communication. They also have power to influence decision-making. And when representations guide decision-making in the social sphere, such as in education, they have moral consequences (Bowker & Star, 2000). For example, Horn (2007) illustrated how teachers' uses of explicit and fixed-meaning category systems that sift students into "fast" and "slow" hampered the teachers from taking up ambitious, equitable pedagogies with potential to support their students' mathematical learning. The categories "fast" and "slow" relieved teachers of the burden of changing their practice towards equitable and ambitious instruction. Representations need not have fixed meanings and are not always so explicit in teachers' talk; but their fluid or tacit nature does not diminish their power to influence teachers' collaborative approaches to problems of practice. For example, Bannister (2015) described how a group of teachers shifted how they framed "struggling students" over a year of collaborative work. While the teachers continued to use language of and similar to "struggling," over time they shifted from describing "struggling students" as having limited fixed characteristics and laying the burden of change on the "struggling students" to describing these students with complexity and taking it upon themselves as teachers to support them. As in Horn's (2007) work, Bannister showed the moral consequences different representations of "struggling students": representing "struggling students" in narrow, fixed ways shifted the moral burden from teacher to student, while representing "struggling students" in complex, changeable ways empowered teachers to act. This research has highlighted the complex but pivotal relationship between teachers' representations of core ideas of teaching and learning and their collaborative learning around problems of practice. This dissertation adds to the field's understanding of that complex relationship.

How teachers represent core ideas can shift as they collaboratively work on problems of practice. Importantly, teachers can move towards discourses associated with ambitious mathematics instruction (Bannister, 2015; Kazemi & Franke, 2004; Nasir et al., 2014), an umbrella term that encompasses mathematically rich, cognitively demanding, equitable, responsive approaches to teaching that help students become powerful thinkers and doers with mathematics (Boaler, 2002a; Schoenfeld, 2002; Schoenfeld & Floden, 2014). Ambitious

mathematics instruction and its associated discourses contrasts with what is sometimes called "traditional" mathematics instruction and its associated discourses.

Discourses of ambitious and traditional mathematics instruction differ in two essential ways. First, ambitious mathematics instruction builds from the premise that students can and should engage with rich and important mathematical concepts and representations. Ambitious mathematics instruction discourses represent complex mathematics as sensible to all students, given time and robust learning environments. Traditional mathematics instruction, on the other hand, tends to be skills-oriented and procedural. Its discourses organize mathematical knowledge into a ladder of content to be mastered and treat complex understanding as the reward of climbing the ladder (Schoenfeld, 2002, 2014).

Second, discourses of ambitious mathematics teaching build from the premise that instruction should position *all* students to engage with rich mathematics content and become powerful mathematical actors (Lampert, 2003; Schoenfeld, 2014). As a result of this commitment to equity, discourses of ambitious mathematics teaching take a student-centered approach to building mathematics learning environments. Discourses of ambitious mathematics instruction repeatedly ask, "How are students experiencing mathematics instruction in this classroom today?" (Schoenfeld, 2014) Ambitious mathematics instruction seeks to remedy inequitable participation and/or outcomes by inquiring into students' historical experiences of mathematics instruction and designing better learning environments accordingly (broadly speaking; I expand on specifics and how discourses of ambitious instruction differ on this front later in this chapter). As a result, students' ideas, voices, and histories tend to play pivotal roles in how learning unfolds in classrooms that follow ambitious mathematics instruction. Ambitious mathematics instruction elicits and builds on students' mathematical thinking and socio-emotional responses to learning environments to set an appropriate level of mathematical challenge, support collaboration, and allow students' ideas to drive instructional decision-making. Discourses of traditional mathematics instruction, however, tend to frame inequitable learning outcomes as the result of students' individual deficits, rather than of impoverished learning environments and historical, society-wide inequities (Martin, 2013; Schoenfeld, 2002). They tend to center the teacher's or other non-student actor's (such as the curriculum or an important test) perspectives when designing learning environments and evaluating students' participation and achievement in them (Horn, 2007; Louie, 2016).

The Teaching for Robust Understanding of Mathematics (TRU) framework (Schoenfeld, 2014; Schoenfeld & Floden, 2014) was the articulation of ambitious mathematics instruction that guided the PD in which participants studied in the dissertation engaged. I elaborate on the dimensions of the TRU framework in Chapter 3 as I describe my methodology and the design of the PD in which the teachers in this dissertation participated. Essential to this chapter, however, is the way in which the TRU framework theorizes powerful mathematics instruction as engaging students with rich mathematics in student-centered, equitable, and responsive ways. The TRU framework guides teachers to examine their classrooms through the eyes of their students, focusing on five essential dimensions of mathematics classrooms: the Mathematics; Cognitive

Demand; Equitable Access to Content; Agency, Ownership, and Identity; and Formative Assessment. Those dimensions and how the TRU framework encourages teachers to inquire into them through the eyes of their students are represented in Figure 2-1.

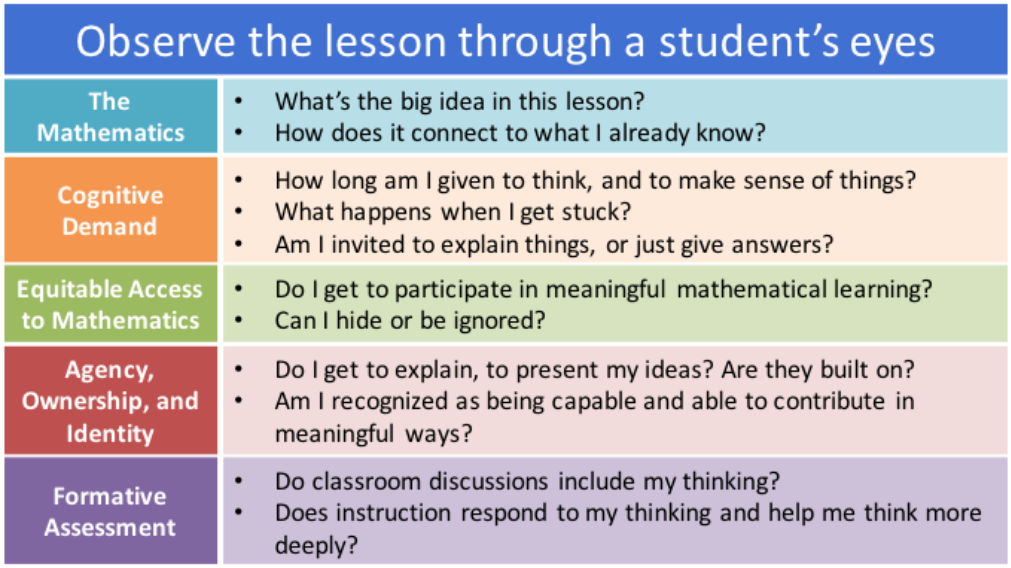


Figure 2-1: The five dimensions of the TRU framework, through students' eyes.

The five dimensions of the TRU framework represent ambitious mathematics instruction's focus on rich, connected mathematics content and attention to students' experiences.

As the content of the TRU framework indicates, ambitious mathematics instruction is characterized more by a set of shared goals for and representations of the learning environment than by particular pedagogical strategies or curricula employed. The TRU framework does not prescribe particular teaching methods. Rather, it takes a student-centered and mathematically-robust stance towards mathematics instruction and guides teachers towards that stance through descriptions, reflection questions, and other tools (further described in Chapter 3). In other words, the TRU framework specifies a *discourse* of mathematics instruction. Of course, certain sets of pedagogical strategies are more likely than others to support the kinds of environments valued by ambitious instruction (Cohen et al., 1999; Kavanagh et al., 2019; McDonald et al., 2013). For example, whole class discussions in which students share and build on each other's thinking to collaboratively make sense of important mathematical concepts support the aims of ambitious instruction. Such whole class discussions provide opportunities to build a community of mathematicians in the classroom by supporting students to share, listen to, and engage with each other's ideas (Boerst et al., 2011). However, it is difficult to prescribe teaching practices outside of context. Teaching is fundamentally contextual and relational work (Gutiérrez, 2012; Philip et al., 2018). Teachers must continually learn about their students, the contexts in which they live and learn, and how pedagogical strategies interact with students in contexts. And then

they must adapt instruction accordingly. Continuing the example, strategies that support whole class discussions in one context may not work as well in another.

Furthermore, the same pedagogical strategies take on different meanings within different discourses around teaching and learning (Horn, 2007; Louie, 2016; Ma & Singer-Gabella, 2011). For example, leading a whole class discussion is part of both ambitious mathematics instruction and traditional instruction. However, in ambitious or reform-oriented discourse "whole class discussion" means students and teachers collectively building mathematical meaning. In more traditional discourses on teaching it means students sharing mathematical ideas and teachers evaluating them for correctness. Ambitious or reform-oriented discourses of mathematics education frame mathematics as sensible, all students as capable, and the work of teaching as collaborative. These discourses contrast with those that frame mathematics as about skills, some students as more capable than others, and the work of teaching as private (Horn, 2007; Little, 1982; Lortie, 1975). The same actions take on different meaning in the different discourses.

That the practice of teaching is context-dependent and ever-evolving increases the importance of teachers' discourses. Teachers make sense of students, content, curricula, and pedagogies with the representations made available in different discourses. As previously discussed, discourses of ambitious mathematics instruction offer teachers powerful representations through which to make sense of their students as smart, content as complex, and pedagogy as responsive. Research has shown that collaboration among teachers can support inquiry into problems of practice, inquiry that in turn supports and is supported by shifts in teachers' collaborative discourse from tending to leverage discourses of traditional mathematics instruction towards discourses of ambitious mathematics instruction. I review such research that is most relevant to this dissertation in the following section.

Lessons learned from prior research on teachers' collaborative discourse

Research on teachers' collaborative discourse most relevant to this dissertation is that which examines how teachers represent their students' capacities to learn complex mathematics and the kinds of learning environments that teachers should create to best support robust mathematics learning (Bannister, 2015; Horn, 2007; Horn & Little, 2010; Kazemi & Franke, 2004; Louie, 2016; Nasir et al., 2014). Some of this research highlights examples of success in shifting teachers discourse towards that of ambitious mathematics instruction through collaborative inquiry into problems of practice. For example, Bannister (2015) and Horn (2007) found that through collaboratively inquiring into a problem of practice, groups of teachers changed how they represented "struggling" (Bannister, 2015) or "slow" (Horn, 2007) students, taking up more contextually-nuanced and strengths-oriented representations to make sense of their students' mathematical challenges. Kazemi and Franke (2004) found that collaborative inquiry into a problem of practice in a PD program supported teachers to expand their visions of what elementary school students were capable of doing with mathematics, leading them to be surprised by their students' ingenuity and flexibility. The rich body of research situated at the

Railside School provides a longitudinal look at how shifts in teachers' collaborative discourse towards ambitious mathematics instruction as teachers worked together on problems of practice compounds over time, with significant affordances for robust and equitable learning for students (Nasir et al., 2014). Together, these studies show that such shifts are possible.

Research also highlights examples in which success was mixed. This research showcases the challenges in moving teachers' collaborative discourse towards ambitious instruction. Some of this research points to the hold that traditional discourses have in teacher communities. Louie (2016) found that even in math teacher communities explicitly committed to taking up ambitious mathematics instruction language, meanings, and practices, teachers struggled to shake traditional mathematics discourses. Teachers were comfortable toggling between traditional and ambitious discourses, often doing so seemingly without noticing. Louie's work points to how deeply embedded and commonsensical traditional mathematics instruction discourses are in mathematics teacher communities, raising significant challenges for learning.

Research that showcases mixed results in change in teachers' collaborative discourse also highlights the complexity in the relationship between teachers' ambitious instruction discourses and collaborative learning around problems of practice. The connection between teachers organizing to work on problems of practice and shifts in discourse towards ambitious mathematics instruction is not straightforward. While collaborative work around problems of practice can support shifts in teachers' discourse towards ambitious instruction, teachers' capacity to collaboratively work on problems of practice is supported and constrained by how they represent core ideas of teaching and learning. Horn (2007) and Horn and Little's (2010) look at two cases of collaborative learning among teachers in two different departments at two different schools points to this feedback relationship between teachers' collective work on problems of practice and collaborative discourse. In these two studies, Horn and Little compared the discourse of a group of teachers who collaboratively shifted their discourse towards ambitious instruction to the discourse of a group of teachers who struggled to move away from traditional discourses. For the group that accomplished a shift, ambitious instruction discourses were *generative*. In other words, new ways of representing students, content, curricula, and pedagogies corresponding with ambitious instruction discourses led to further shifts in discourse as the teachers addressed problems of practice together and mobilized for change in instruction. Horn and Little's findings about the generativity of ambitious instruction discourses aligns with similar findings from Bannister (2015) and Kazemi and Franke (2004). Kazemi and Franke (2004) in particular demonstrated how new teaching practices that teachers adopted because of the shifts in discourse and their resulting new desire to see more of their students' mathematical thinking led to further shifts in discourse.

However, Horn and Little found that for the group that did not accomplish a shift, traditional discourses inhibited their collaboration and inquiry. Horn (2007) documented teachers' work around a problem of practice she called the "Mismatch Problem." The Mismatch Problem was a tension experienced by teachers between reform initiatives, teachers' readings of their students' mathematical abilities and achievements, and their students' reactions to reform-

based curricula. Reform initiatives called upon teachers to shift instruction towards problem-solving, mathematical communication, and reasoning. However teachers, reflecting on their knowledge of their students' experiences of newly challenging mathematics curricula, saw students' mathematical abilities and achievements "as incommensurate with a rigorous mathematics curriculum (p.42)". Horn described how the ways that teachers made sense of mathematics curricula reflected a common representation of mathematics as sequential. Similarly, how the teachers' read their students' abilities reflected a common representation of students seen as "low-achieving" as unable to learn math through inquiry and collaborative sense-making. These representations were part of the cultural-historical traditions of mathematics teaching in which the teachers in Horn's work were steeped. Students themselves saw the new curricula as overly challenging, in part reinforcing their teachers' representations. Hence, the Mismatch Problem—in which teachers saw reform curricula as inappropriate for the needs of their students—was a conflict between discourses. Representing students, curriculum, and teaching as incommensurate, drawing on traditional discourses of mathematics instruction, hindered the teachers from organizing to address the problems they noticed in their routine practice (Horn, 2007).

Horn's work shows how traditional discourses of mathematics instruction can put teachers in a bind, making them powerless to address problems because of how they represent core ideas. Research by Horn and Little (2010) and Louie (2016) illustrated further how teachers' uses of representations central to discourses of traditional mathematics instruction correlates with interaction routines that hamper teachers' collaborative inquiry, including glossing over details of classroom experience and sharing ideas in a "round robin" format in lieu of building on or problematizing each other's ideas. As Horn, Little, and Louie show, teachers' uses of traditional instruction discourses correlates with teacher workgroup interaction that treats teaching as private and idiosyncratic. Such interaction is itself consistent with traditional views of the work of teaching (Little, 1993; Lortie, 1975; McLaughlin & Talbert, 2001).

Teachers' collaborative learning around problems of practice and use of discourses are inseparable. The feedback loop between them is part of what makes collaborative learning for teachers so slow and non-straightforward. The commonsensical nature of traditional instruction discourses also contributes to the challenge of shifting teachers' collaborative discourse towards ambitious instruction. Even in cases of successful shift in collaborative discourse, progress was slow and piecemeal. The cases profiled here of successful shift in collaborative discourse towards ambitious instruction happened over months and, in the notable case of the teachers at Railside School, years (Bannister, 2015; Horn, 2007; Kazemi & Franke, 2004; Nasir et al., 2014). However, the generative relationship between collaboration around problems of practice and movement towards discourses of ambitious instruction gives cause for optimism. Given appropriate conditions, if teachers can collaboratively engage with problems of practice and begin to take up representations of core teaching ideas consistent with ambitious instruction discourses, potentially even in small ways, their learning can build over time, with each new representation and practice precipitating further changes.

This dissertation explores teachers' learning around problems of practice supporting teachers to draw on representations from discourses of ambitious instruction. It provides another case of the generativity of discourses of ambitious instruction and the constraints of discourses of traditional instruction. Furthermore, it highlights how discourses of ambitious and traditional instruction can exist simultaneously in teachers' talk. This dissertation adds further complexity to the field's understanding of how teachers' collaborative learning around problems of practice supports and is supported by their collaborative discourse.

New directions

This dissertation centers another challenge to shifting teachers' collaborative discourse towards discourses of ambitious mathematics instruction, one not prominently highlighted in literature on teachers' collaborative discourse: that representations of certain ideas key to ambitious instruction discourses may be underspecified in those discourses. Much research on teachers' collaborative discourse tends to treat discourses of "ambitious" and "traditional" instruction as well-specified. In other words, much research does not consider how representations of key ideas in either set of discourses may be neither well-defined nor incontrovertible. This is particularly the case in research on teachers' collaborative discourse in structured PD. The vision of ambitious instruction used in analysis of teachers' discourse in structured PD is often that which also organized the PD, such as the Mathematics Quality of Instruction framework (Beisiegel et al., 2018), the Cognitively Guided Instruction framework (Kazemi & Franke, 2004), or the vision of Complex Instruction (Jilk, 2016). What constitutes a discourse of ambitious instruction is more loosely defined in studies of teachers' learning in informal workgroups. Scholars often focus on teachers' talk about a particular feature of ambitious and traditional instruction discourses, such as how teachers represent students' struggle (Bannister, 2015; Horn, 2007), reducing the need to define discourse of ambitious instruction at large. When they do not focus on a particular feature, scholars rely on the shared knowledge of their readers of these discourses (Louie, 2016). Regardless of the specificity of definition, however, scholars represent discourses of ambitious and traditional instruction as well-defined and not in conflict.

However, what is called "ambitious" or "reform-oriented" discourses of math education contain concepts whose representations are debated in communities that make use of those discourses. There is significant diversity within discourses of ambitious math instruction, and this diversity influences teachers' learning. The under-specification of representations of key ideas in ambitious instruction discourses is highlighted by existing literature, albeit often tacitly. The implications of this under-specification for teachers' collaborative learning has not been addressed, however.

For example, what constitutes *equitable* ambitious mathematics instruction varies in different ambitious mathematics instruction discourses. The difference between how ambitious instruction discourses represent equity are not as stark as the differences between ambitious and

traditional instruction. Traditional mathematics instruction tends to be comfortable with colorblind ideologies that center individualism and glorify merit while ignoring ways in which these ideologies perpetuate racial and ethnic injustices (Battey & Leyva, 2016; Gutiérrez, 2017; Martin, 2009; Nasir et al., 2012). Ambitious mathematics instruction frames students as members of cultural-historical communities and calls for mathematics instruction to help overturn histories of injustices against marginalized racial, ethnic, and socioeconomic groups (R. Gutiérrez, 2012; Gutstein, 2012; Ladson-Billings, 1995; Martin, 2009). However, there are diversity and even conflict within discourses of ambitious mathematics instruction around what it means to work towards equity in mathematics instruction. Sometimes the diversity lies in what is made explicit and left implicit. For example, the TRU framework does not routinely explicitly center racial justice in mathematics instruction, while discourses of culturally responsive teaching do (Ladson-Billings, 1995; Schoenfeld, 2014). This does not mean that the TRU framework is incompatible with culturally responsive teaching's emphasis on racial justice through mathematics education. On the contrary, resources associated with the TRU framework cite scholars of culturally responsive pedagogy (TRU-LS Team, n.d.). Nonetheless, explicit attention to race and racial justice in mathematics education differentiates discourses about equity in ambitious mathematics instruction (Martin, 2009, 2013).

Outright conflict about equity has emerged among scholars of ambitious math instruction around whether or not the field can and should define a set of "core" or "high leverage" teaching practices that support ambitious instruction (Kavanagh et al., 2019; Philip et al., 2018). Both sides frame their arguments in terms of equity, but draw different conclusions. Proponents of core practices argue that equitable instruction for all students relies on teacher educators instilling in new teachers a core set of pedagogical moves that position teachers to teach responsively (Forzani, 2014; McDonald et al., 2013). Advocates of core practices frame their rationale as one of equity. They argue in part that teachers working in traditionally hard-to-staff schools that typically serve students of color and from working-poor families, who also tend to have less teaching experience, need "high leverage" practices to provide robust mathematics education to their students and engage in professional learning (*TeachingWorks*, n.d.). However, critics of the core practices movement in teacher education argue that delineating a set of pedagogical moves runs the risk of privileging the experiences of white students and erasing the histories of students of color. They also pinpoint problematic affiliations between organizations supporting core or high leverage practices and organizations advocating for alternative paths to teacher preparation and charter schools (Philip et al., 2018). Philip et al. (2018) show cross-fertilization between ambitious, reform-oriented math instruction communities and communities that also advocate for reform but with neo-liberal and neo-conservative tendencies. Thus how discourses of ambitious mathematics instruction treat equity differ in both emphasis and fundamental definition, especially in how they attend to racial injustice.

That scholars working within ambitious math instruction discourses can disagree over the meaning of equity illustrates how fragmented discourses around ambitious math instruction are. These debates highlight two features of discourses on ambitious mathematics instruction: first,

that there are *discourses* of ambitious mathematics instruction, plural intentional; and second, that there are core ideas of teaching and learning—including equity—that are *under-specified across discourses of ambitious instruction*. However, representations of these key ideas can become both fixed and operationalized within communities of teachers, with direct consequences for students. Continuing with the example of equity, research by Horn (2007) overviewed in the previous section showed that how teachers represent and categorize students, content, curricula, and pedagogies, derived from key ideas of equity central to discourses of ambitious or traditional instruction, guides how teachers navigate important problems together. The representations and categories of "fast" and "slow" students had well-defined-enough meaning for teachers to influence the equity-related decisions that teachers made. How might the under-specification of key ideas in ambitious mathematics instruction affect teachers working to shift their discourse away from commonsensical traditional instruction discourses and the at least locally well-defined representations and category systems that have historically guided their pedagogical problem-solving? This dissertation takes up that important question. This dissertation examines a core idea of teaching and learning, that of students' perseverance, and illustrates how it, like equity, is also underspecified across discourses of ambitious instruction, with consequences for teachers' learning around it. In these ways, this dissertation extends prior research on teachers' collaborative discourse and learning.

This dissertation also extends prior research by contextualizing patterns in how teachers represented key ideas of teaching and learning in discourses beyond those typically treated as directly relevant to mathematics instruction. As Lefstein et al., (2020) note in their recent review of literature on teachers' collaborative discourse, attention to broader cultural historical contexts is rare in studies of teachers' learning in professional settings. Discourses of ambitious and traditional mathematics instruction exist within broader discourses of education and society at large. The representations of key ideas of teaching and learning that PD participants collaboratively build, revise, and/or reject in professional learning are seeded by meanings that themselves are often commonsense in discourse communities that reach beyond the collaborative workgroup (Martin, 2013; Philip, 2011). Understanding what representations teachers find salient and how and why those representations come to be salient in a collaborative workgroup requires looking beyond the workgroup for cultural-historical context. Scholars who have taken this approach overwhelmingly examined teachers' discourse about equity and/or race in education. For example, Philip (2011) examined ideologies of race and racism in teachers' discourse as teachers participated in PD aimed at supporting them to incorporate social justice into their mathematics teaching. This dissertation examines teachers' collaborative discourse around a parallel topic, perseverance. Thus, this dissertation's approach of situating the discourses that emerged in the workgroup in broader discourses is rare. Its examination of teachers' discourse around perseverance, especially contextualized by broader discourse communities, is unique.

Generative collaborative discourse for teachers and conditions that support it

The previous section of this chapter described a main focus of this dissertation: examining patterns in representations in teachers' collaborative discourse. It situated this dissertation's analysis of representations in teachers' discourse in prior research on discourses of traditional and ambitious mathematics instruction in teachers' collaborative workgroups. This section outlines a second main focus of this dissertation: examining some of the workgroup and PD conditions that support discourse among teachers that is generative of learning. This dissertation builds in particular on research into the workgroup and PD conditions that support collaborative discourse generative of movement towards discourses of ambitious mathematics instruction. While the first section of this chapter situated my analysis of "what" was learned in prior research, this section situates my analysis of "how" that learning happened in prior research.

Teachers' collaborative learning in workgroups is advanced through "generative" discourse (Kazemi & Franke, 2004; Lefstein et al., 2020). Generative workgroup discourse is discourse that can support teachers to move towards discourses of ambitious mathematics instruction. Lefstein et al. (2020) outlined four features of discourse that are considered generative of learning for teachers in a workgroup:

- Discourse *reveals* and *probes* problems of practice. Discourse also orients to problems encountered during teaching as problems of practice.
- Discourse is *dialogic* and positions participants in a *dialogic stance*. Participants orient to their collaborative work as exploring each other's different perspectives and engage in discourse practices that support dialogic interaction such as sharing ideas, building on each other's thinking, and offering new perspectives.
- Discourse involves participants *making claims* and *supporting claims with evidence and reasoning*.
- Discourse engages with key *concepts* of teaching and learning, connecting classroom events and observations to larger ideas.

Research on teachers' learning in collaborative workgroups points to generative discourse as creating opportunities for teachers to learn, or the conditions that support teachers to define, study, and collaboratively address important problems of practice (Horn et al., 2016). Previous work has attempted to systematically track the variability of these features in teachers' collaborative discourses and show how that variability supports or constrains opportunities to learn (Bannister, 2015; Horn et al., 2016; Horn & Kane, 2015; Horn & Little, 2010; Little, 2003; Louie, 2016). For example, Horn and Little (2010) used a comparative case study to examine the extent to which teachers in two workgroups deliberately worked on problems of practice with transparent evidence from their classrooms and, as a result, were afforded opportunities to shift their practice. Bannister (2015) looked longitudinally at discourse among a group of teachers to trace change in how the teachers engaged with each other's ideas, correlating this change with

the opening of opportunities to shift classroom practice over a school year. Horn and colleagues (Horn et al., 2016) examined discourse among twenty-four different teacher workgroups to characterize teachers' engagement with key concepts. This body of research provides a foundation for this dissertation's analysis of PD and workgroup conditions that support movement towards discourses of ambitious instruction. While this dissertation does not systematically track features of generative discourse in teachers' talk, either across multiple cases or over time, it builds on this research that has established a connection between features of generative discourse and opportunities for teachers to learn.

Given that these features of teachers' collaborative discourse are generative of shifts towards ambitious instruction, what PD and workgroup conditions support their development? This question is a main focus of this dissertation, captured by the second research question. Figure 2-2 outlines the relationship between PD and workgroup conditions, features of generative collaborative discourse, and teachers' learning around discourses of ambitious mathematics instruction.

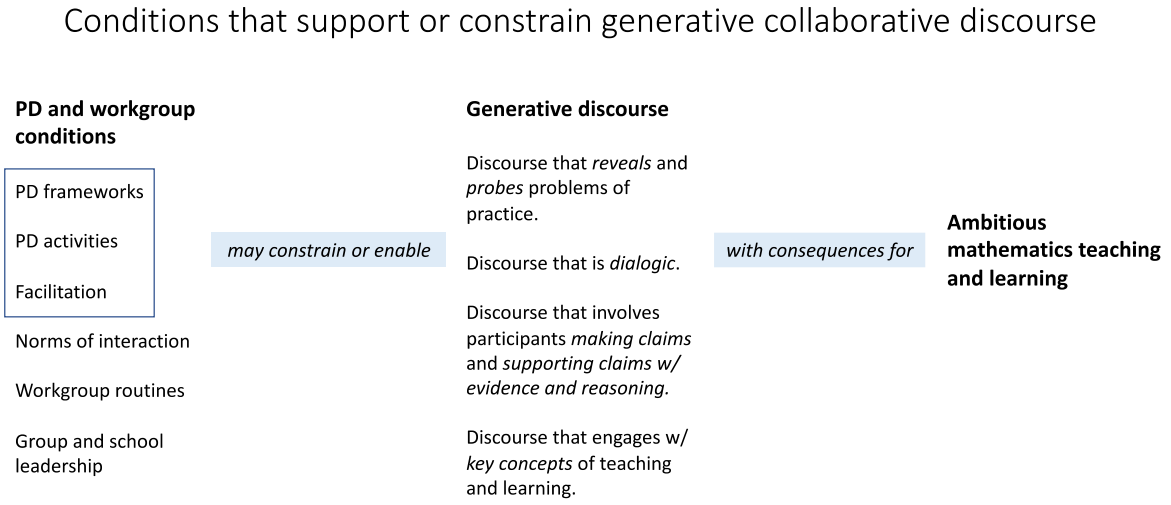


Figure 2-2: Framework for the relationship between PD and workgroup conditions, generative discourse, and shifts in discourse towards ambitious instruction.

As shown in Figure 2-2, an array of PD and workgroup conditions may support or constrain teachers' generative collaborative discourse, with consequences for how teachers discourse takes up ambitious mathematics instruction. This dissertation investigates the relationship between certain PD/workgroup conditions and patterns in teachers' uses of discourses of ambitious and traditional mathematics instruction, with how PD and workgroup conditions constrain or enable generative discourse as the link between these two points.

In particular, as shown in Figure 2-2, this dissertation builds on prior research by examining how three PD and workgroup conditions constrain or enable generative collaborative discourse. These are: (1) use of PD frameworks, (2) teachers' engagement with PD activities, and (3) facilitation choices. Literature shows that these are not the only three PD and workgroup conditions that influence teachers' generative discourse. For example, research has also focused on the relationship between norms and routines for interaction in the workgroup and generative discourse (Horn & Little, 2010); and the influence of school and district structural and political context on workgroup cohesiveness, focus, and longevity (Coburn & Russell, 2008; Nasir et al., 2014). While facilitator expertise and school and district context undoubtedly contributed to variability in teachers' generative discourse and learning around ambitious mathematics instruction in this dissertation, examining how is outside the scope of current analysis. Analysis in this dissertation builds on prior work examining participants' interaction with two structural aspects of the PD environment, namely PD frameworks and activities, and facilitation choices. It also diverges from prior work in several key ways, outlined in the following sections.

Frameworks and activities

PD programs are typically distinguished by their "trademark" activities and frameworks. For example, video clubs (van Es & Sherin, 2010) engage teachers in collaborative conversation about excerpts of video from their own classrooms of students' working on mathematics. Lesson study (Fernandez & Yoshida, 2004; Murata, 2011) engages teachers in iterative cycles of inquiry around a research question, grounded in an individual lesson that teachers collaboratively plan, teach, and reflect on. VIDEO-LM (Viewing, Investigating, and Discussing Environments of Learning Mathematics) (Karsenty & Arcavi, 2017) engages teachers in a program of watching and reflecting on videos of other teachers' practice, supported by an analytic framework. PD programs engage participants in designated activities, often supported by frameworks, with the goal of effecting specific learning outcomes. For example, video clubs aim to build teachers' ability to notice and interpret students' mathematical thinking (van Es, 2011; van Es et al., 2014; van Es & Sherin, 2010). The Problem Solving Cycle (Borko et al., 2015) has a similar aim, with the addition of supporting teachers to notice effective pedagogical strategies and build planning routines.

Literature on PD programs typically takes one of two forms. First, the literature offers *descriptions* of PD programs (Borko et al., 2011; Karsenty & Arcavi, 2017; Lewis, 2009). Literature of this form details the structure and desired engagement with the PD program, often illustrated with examples from practice. It largely outlines *theoretical* connections between engagement with PD/workgroup activities and frameworks, aspirational teacher collaborative discourse, and goals for learning around ambitious mathematics instruction. For example, Borko and colleagues (Borko et al., 2011) compared the activities and frameworks used in two PD programs, the Problem Solving Cycle and Learning and Teaching Geometry. In the article referenced here, the authors motivated the different activity and framework choices of the two

PD programs by drawing on literature about teachers' generative collaborative discourse and learning towards ambitious mathematics instruction. They then compared the programs using selected examples of teachers' engagement with them. These examples illustrate goals but do not constitute detailed analysis of teachers' learning. Such literature is valuable for PD designers looking to compare activity and framework choices and goals, or for practitioners choosing among an array of PD programs. I contributed to a paper of this type describing the PD program analyzed in this dissertation (Schoenfeld et al., 2019). This dissertation moves beyond theoretical connections and brief illustrations to provide a detailed analysis of the relationship between PD/workgroup conditions, generative discourse variability, and shifts towards ambitious mathematics instruction.

Second, the literature offers *evaluation* of PD programs, focusing on well-defined learning goals specified by the PD design. Literature of this form details participant engagement that contributed to particular outcomes and compares those outcomes to the specified PD goals. This literature tends to provide connections between teachers' engagement with PD/workgroup activities and frameworks, variability in generative discourse, and learning around a *pre-determined slice* of ambitious mathematics instruction. Focus on evaluating teachers' learning of a pre-determined slice of ambitious mathematics instruction has methodological and interpretative limitations for this body of research, limitations that this dissertation aims to reach beyond.

For example, van Es and Sherin (2010) evaluated teachers' learning to notice and interpret certain aspects of classroom interactions in the context of a video club. The learning outcome, noticing and interpreting students' thinking about mathematics, was specified by the PD program at the outset. As a result, van Es and Sherin's analysis leveraged a pre-established set of codes, in essence analyzing for the extent to which teachers' collaborative discourse, supported by the specific PD activity of the video club, hit a set of targets for learning around noticing and interpreting students' mathematical thinking. These codes were based on findings from literature on teachers' generative collaborative discourse. For example, van Es and Sherin coded for "stance" in teachers' discourse, whether teachers reported, evaluated, or interpreted classroom events. Research on teachers' generative collaborative discourse has found that discourse that takes an inquiry-stance, or makes, supports, and reasons about claims grounded in detailed classroom observations, affords opportunities for teachers to learn together (Horn et al., 2016; Little, 2003). Using these pre-determined codes, van Es and Sherin analyzed teachers' workgroup discourse and found that over time, the teachers came to notice and interpret students' mathematical thinking when discussing classroom videos.

Similarly, Beisiegel and colleagues (Beisiegel et al., 2018) evaluated teachers' learning in four variations on a video club, each supported by a framework for teaching mathematics called the Mathematics Quality of Instruction (MQI) framework. Their study compared teachers' learning in MQI-enhanced video clubs with four different types of activity, in which participants (a) were led by either an expert facilitator or teacher, and (b) watched stock video or videos from their own classrooms. As in van Es and Sherin's analysis, Beisiegel and colleagues specified the

learning goals at the outset, namely that teachers would come to analyze videos in ways consistent with the goals presented by the MQI framework. Beisiegel and colleagues evaluated teachers' learning against these goals using pre-set codes and correlated their findings with treatment condition. They found that teachers' learning met their goals more often in the teacher-led, own-video treatment.

PD evaluation research such as that in the two studies profiled here (Beisiegel et al., 2018; van Es & Sherin, 2010) is sufficient to answer the question, "Does this PD work?" That is, it is sufficient so long as whether or not the PD "works" or, in other words, to what extent teachers learned, is narrowly defined by pre-specified learning goals. However, as described in the first part of this chapter, teachers' learning around discourses of ambitious instruction is a wide and complex space. How teachers notice and interpret students' mathematical thinking is an important dimension of ambitious mathematics instruction. Discourses of ambitious mathematics instruction include rich representations of students' mathematical thinking, much but not all of which is captured by the MQI framework (Schoenfeld et al., 2018). How teachers notice and interpret student thinking has implications for the pedagogical choices that teachers make, especially in their attempts to broaden access to robust mathematical learning for all students (Jilk, 2016; van Es et al., 2017). However, investigating whether teachers learned a particular slice of ambitious mathematics instruction, such as noticing students' mathematical thinking, or a slice from a particular angle, such as that specified by the MQI, does not shed light on teachers' learning of the complex and diverse space that is ambitious mathematics instruction. This dissertation moves beyond evaluative research on PD activities and frameworks by taking a grounded, ethnographic approach to teachers' learning of ambitious mathematics instruction discourses. While this dissertation specifies the broad space covered by discourses of ambitious mathematics instruction as the learning goal for teachers, it does not specify a particular outcome, practice, or perspective on ambitious mathematics instruction. Rather than asking whether the PD "worked," this dissertation inquires into teachers' collaborative sense-making of discourses of ambitious mathematics instruction writ large.

Furthermore, PD evaluation research does not do a sufficient job of answering the question, "*How* does this PD work?" at the level of drawing connections between PD activities and frameworks and variability in teachers' generative collaborative discourse. Such research does not provide a detailed look at teachers' interactions within PD activities and with frameworks. Sherin, van Es, and Beisiegel and colleagues' work, for example, provides illustrative cases of teacher's conversation within each PD program and activity, but does not examine the intricate connections between PD activity and framework support and variation in generative discourse (Beisiegel et al., 2018; van Es & Sherin, 2010). This dissertation moves beyond illustrations provided in PD evaluation research to detail some of the complexity in interactions between participants and PD activities and frameworks.

In a notable variation from typical evaluation analysis, Babichenko, Asterhan, and Lefstein (Babichenko et al., 2020) compared how three different types of PD activity— peer consultation, planning, and video analysis—supported and constrained teachers' generative

collaborative discourse. They identified features of generative collaborative discourse, coded for those features, and correlated their relative presence or absence with activity-type. This dissertation offers complementary analysis of a similar question about similar types of PD activity. As will be unpacked in Chapter 3, my analysis examined patterns in how participants represented important ideas first, followed by case-based analysis of patterns in collaborative discourse. In contrast, Babichenko and colleagues' analysis did not take into account *what* teachers were talking about, focusing exclusively on *how* they were talking. This approach was necessary given the breadth of their study, looking at conversations from 60 different teams of teachers. In this dissertation, the topic of the teachers' collaborative inquiry, perseverance, is of central importance to the analysis.

Facilitation choices

While research that focuses on PD activities and frameworks as a condition for teachers' generative discourse and learning is plentiful, research on the influence of PD facilitation on generative discourse and learning is in early stages (van Es & Sherin, 2017). Scholars focusing on PD facilitation as a condition leading to variation in generative discourse in teacher workgroups have highlighted the complexity of a facilitator's work. That facilitating PD for math teachers is complex work should come as no surprise to those who already see teaching children mathematics as complex work. Much like math teachers who work with children, PD facilitators must manage an intricate relationship between themselves, their students (the teachers), and content (the PD "curriculum" and discourses of ambitious instruction).

Faced with this complexity, much research on facilitation of PD has sought to contribute to frameworks for the expertise needed to successfully facilitate PD (Kintz et al., 2015; Tekkumru-Kisa & Stein, 2017; van Es et al., 2014; van Es & Sherin, 2017). These frameworks tend to draw on analyses of "successful" facilitation. For example, van Es and colleagues (2014) analyzed the facilitation of two video club workgroups to outline how "expert" facilitators structure teachers' collaborative discourse in PD. Others look closely at novice facilitators' struggles and successes with facilitating PD, often attending to fidelity as well as the generativity of teacher discourse (Beisiegel et al., 2018; Borko et al., 2014). A handful of studies have shown correlations between facilitator "expertise" and teachers' learning in PD, although Beisiegel and colleagues notably found that teacher-facilitated PD outperformed PD facilitated by experts (Beisiegel et al., 2018; Kintz et al., 2015).

This dissertation takes a step back from this work. Instead, this dissertation takes a grounded look at the interactions between facilitator, teachers, and content as participants engaged with each other in the context of the PD. Rather than trying to characterize facilitator expertise or classify facilitator moves as effective or missing-the-mark, this dissertation analyzes the interactions among participants and PD activities and resources that unfolded, with the facilitator as a uniquely-positioned participant. In so doing, this dissertation draws on prominent approaches to analyzing the role of a teacher in creating opportunities for children to learn mathematics. For example, the Teaching for Robust Understanding of Mathematics (TRU)

Framework guides those analyzing the role of the teacher in classroom learning to take a student perspective: How do the teacher's actions and choices influence students' experiences of instruction, content, and other students (Schoenfeld, 2014; Schoenfeld et al., 2018)? This dissertation asks a similar question of facilitation: How do the facilitator's actions and choices influence teachers' experiences of PD activities and frameworks, discourses of ambitious mathematics instruction, and other participants? While such analysis may in fact contribute to the development of frameworks for facilitation expertise, it will also help fill in gaps in the field's knowledge of the role of facilitation in variability in teachers' generative discourse and learning.

Conclusion

This chapter elaborated on the two theoretical frameworks used in this dissertation: a framework for the discourses of traditional and ambitious mathematics instruction and their connection to teachers' workgroup and classroom practice, and a framework for the relationship between PD and workgroup conditions, generative teacher discourse, and learning. Articulation of these frameworks situated this dissertation in rich body of work that describes what ambitious mathematics instruction is, how teachers learn it together, what conditions support and constrain that learning. This chapter also showed how this dissertation diverges from typical research into teachers' learning around ambitious mathematics instruction in PD contexts and what it aims to contribute. Namely, this dissertation examines teachers' collaborative discourse around *perseverance*, a topic that has not yet been studied but builds on prior research into teachers' collaborative discourse around equity. This dissertation also situates analysis of teachers' collaborative discourse in larger social conversations, something that is not typically done in research on teachers' discourse. Finally, this dissertation seeks to move beyond typical PD evaluation research in articulating the relationships between PD conditions, teachers' generative discourse, and learning outcomes, rather than drawing arrows directly from PD conditions to learning outcomes as is typical.

In the next chapter, I detail how the theoretical approaches outlined in this chapter translate to research methodologies. The theoretical frameworks adopted here inform my analytic approach. I also describe the study in which this dissertation was situated.

Chapter 3 The Study and Methods

My research examines the learning of four high school math teachers as they participated in a professional development program called TRU-Lesson Study. In this chapter, I elaborate on the methodology underlying this study. I begin by describing the design of the professional development program in which the teachers participated. Next, I describe the setting, participants, and their school context. I provide an overview of the Benton teachers' work together over their three years in the professional development program, part of one of which this dissertation analyzes. I then describe data collection procedures used throughout the study. Finally, I describe the methods used to reduce and analyze the data to investigate teacher learning over time and the relationship between the teachers' learning and the activities in which they engaged. I also describe how my methods derive from the theoretical approaches outlined in Chapter 2.

TRU-Lesson Study Professional Development

The Benton teachers were participating in a professional development program designed as part of a multi-year design-based research-practice partnership project (Confrey, 2005; Penuel et al., 2015; Schoenfeld, 2010a) called, "TRUMath and Lesson Study: Supporting fundamental and sustainable improvement in high school mathematics teaching" (NSF Grant 1503454, PIs Suzanne Donovan, Catherine Lewis, and Alan Schoenfeld). I refer to the project as TRU-LS and its PD program as TRU-LS PD, or simply "the PD." TRU-LS investigated how school-based professional development for high school math teachers can cultivate the teaching expertise necessary to create ambitious mathematics learning environments. The project PIs partnered with the coaching staff of a large urban school district in California to design and implement the professional development at seven schools in the district.

TRU-LS PD took place at Benton for the latter three years of the four-year project. In their first year, the Benton teachers did just the TRU part of TRU-LS PD. In years two and three, they participated in the full TRU-LS PD.

TRU-LS PD combines the inquiry structure of Lesson Study (Huang et al., 2019; Lewis et al., 2019; Murata, 2011) and the vision of ambitious mathematics instruction described by the Teaching for Robust Understanding (TRU) framework (Schoenfeld, 2014, 2018; Schoenfeld & Floden, 2014). TRU-LS PD can be classified as a practice-based professional development program (Ball & Cohen, 1999; Kazemi & Hubbard, 2008). This means that TRU-LS PD engages teachers in planned PD activities around their own problems of practice, supported by artifacts from the classroom. Chapter 2 provided a theoretical model of how teachers learn around problems of practice, with examples from teachers engaged in practice-based professional development programs. This section of this chapter elaborates on how TRU-LS PD supports teachers' inquiry into problems of practice. I focus on two main features of TRU-LS PD that afford inquiry into problems of practice: a robust vision of ambitious instruction that

problematizes students' experiences of instruction, and a structure for collaborative inquiry in and across the workgroup and classroom.

I begin this section by describing the components of TRU-LS PD separately—the TRU Framework and lesson study. Then, I describe the TRU-LS PD that evolved over the 2017-18 school year as the TRU-LS team designed and implemented it.

The Teaching for Robust Understanding (TRU) Framework

The TRU Framework provided the vision of ambitious mathematics instruction that grounded TRU-LS PD. Schoenfeld and colleagues described the three criteria for a vision of mathematics instruction that supports teachers' collaborative inquiry into problems of practice (Schoenfeld et al., 2019):

1. A useful vision of ambitious math instruction should *focus on what matters for instruction*, highlighting essential aspects of practice for inquiry.
2. A useful vision of ambitious math instruction should *focus on the ways that students engage with mathematics and classroom*.
3. A useful vision of ambitious math instruction should help teachers *problematize instruction*.

The TRU Framework meets these three criteria.

The TRU Framework deconstructs what matters for robust, ambitious mathematics instruction into five dimensions (Figure 3-1): the Mathematics (or Content, in versions used outside of math); Cognitive Demand; Equitable Access to Content; Agency, Ownership, and Identity; and Formative Assessment (Schoenfeld & Floden, 2014). The TRU Framework was developed over several years by a team of mathematics education researchers who collected and synthesized literature on features important to students' learning of mathematics. The team found an overwhelmingly long list of important features. Seeking to capture all of these features in a framework that was manageable but also comprehensive, the team categorized the features into the five dimensions of the framework (Schoenfeld, 2018). The five dimensions span the space of what matters for mathematics instruction. They are not the only way to span that space, but they do so in a way that describes the student experience of math instruction more powerfully than other available frameworks do (Schoenfeld, 2018; Schoenfeld et al., 2018). The five dimensions fulfill the first requirement of visions of ambitious math instruction that afford learning: focusing on what matters.

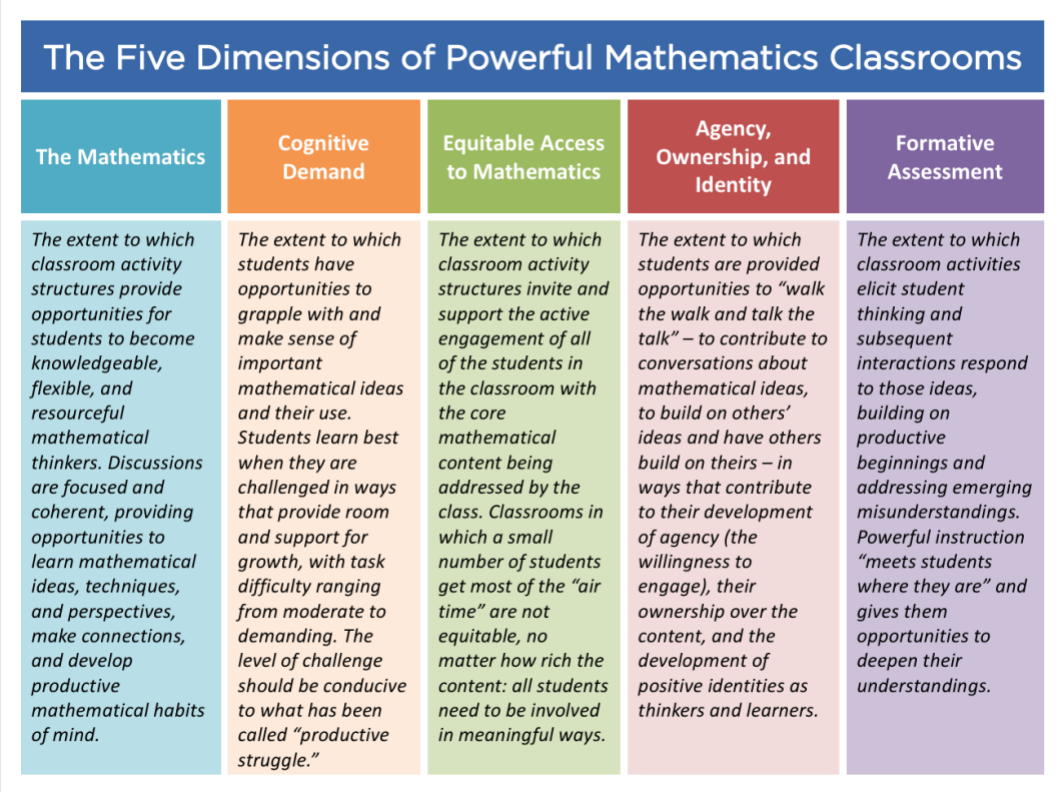


Figure 3-1: The TRU Framework.

As the text of the dimensions in Figure 3-1 shows, each dimension attends to how *students experience* mathematics and classroom activity, meeting the second criterion for visions of math instruction that afford inquiry and learning. For example, the framework asks teachers to examine "the extent to which students have opportunities to grapple with and make sense of important mathematical ideas and their use" when thinking about cognitive demand. This language frames cognitive demand from the students' perspective. Cognitive demand is not inherent in tasks, and productive struggle is not something that teachers can expect from students without careful attention to the learning environment. Rather, cognitive demand has to do with how students experience mathematical challenge. To evaluate the cognitive demand present in a mathematics classroom, teachers need to pay attention to how their students are approaching tasks. Armed with this information, teachers can then think about whether tasks and pedagogy are supporting cognitive demand. In this way, the TRU Framework encourages teachers to problematize tasks, learning environments, and what they notice from interactions with and among students.

The affordances of the TRU Framework for problematizing instruction, the third criterion, are most apparent in the following alternative way of representing the dimensions, shown here in Figure 3-2:

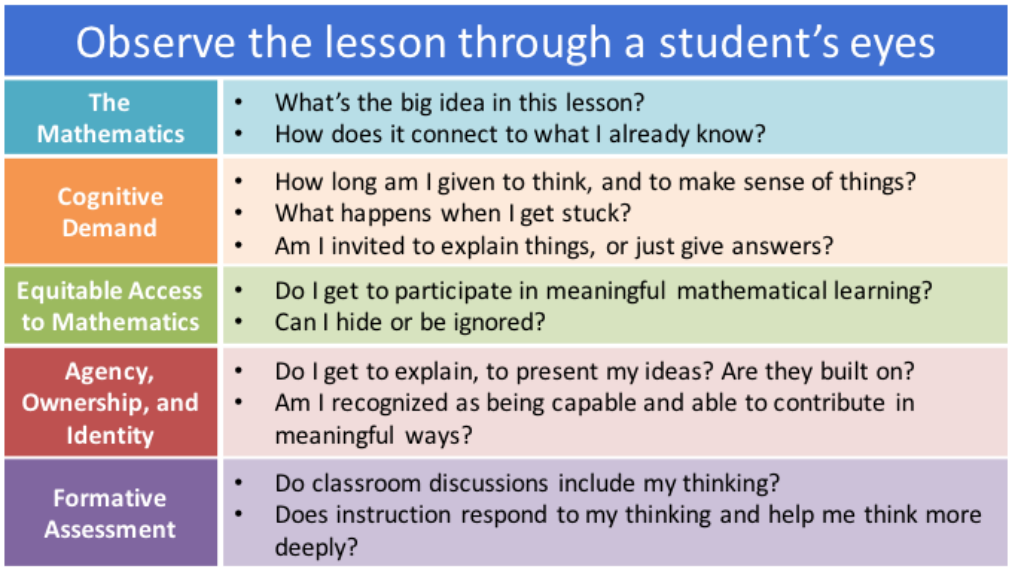


Figure 3-2: The TRU Framework through the students' eyes

As shown in Figure 3-2, the TRU Framework encourages teachers to ask questions about how their students experience instruction. Importantly, the questions it encourages are not agnostic as to how students should experience instruction. The TRU Framework does not say that anything goes. The dimensions are grounded in research into what matters for students learning mathematics. Nonetheless, the TRU Framework problematizes instruction in a way that is sensitive to context and encourages teachers to learn more about their students, rather than imposing values and practices.

The TRU Framework and its accompanying tools, such as the table of questions for observing through the students' eyes, the TRU Conversation Guide (Baldinger & Louie, n.d.), and the TRU Observation Guide (Schoenfeld & the Teaching for Robust Understanding Project, 2018), can support teachers' collaborative inquiry into problems of practice. They can be used with a variety of practice-based professional education activities intended to support inquiry into problems of practice, including collaboratively watching video of instruction (Schoenfeld, 2018) and lesson study (Schoenfeld et al., 2019). TRU-LS PD layered the TRU Framework and its accompanying tools with lesson study.

Lesson Study

Lesson study provided the inquiry structure for teachers' learning in TRU-LS PD. Lesson study is a practice-based form of professional development in which teachers collaborate to inquire into a problem of practice (Huang et al., 2019; Lewis et al., 2009; Murata, 2011). Lesson study originated in China and Japan, but has now spread around the world (Fernandez & Yoshida, 2004; Lewis & Lee, 2017). In lesson study, teams of teachers—sometimes grade-level teams or departments but, ideally, entire schools (Takahashi & McDougal, 2016)— build and

share knowledge around a research theme by engaging in a series of inquiry cycles (Lewis, 2009; Lewis et al., 2019; Murata, 2011). Lesson study can be done by teachers in any domain.

Each inquiry cycle has four stages, as shown in Figure 3-3: study, plan, teach, and reflect. In the study phase, participants establish the learning community, set a learning goal called the research theme, and learn about the research theme, often by reading research related to the research theme. In the plan phase, the participants plan a lesson that they hope will help them learn more about their research theme. As they plan, they think horizontally and vertically across the curriculum about how their lesson highlights content standards across units and courses. Participants also build a theory of action, a hypothesis that articulates how their lesson will support their research theme. Next, participants teach their lesson and observe each other's teaching. Teaching the lesson tests their hypothesis, as described in the theory of action, and builds solidarity as teachers engage in what is typically a solitary practice as a community. Finally, participants reflect on the cycle thus far. They describe what they learned from teaching and observing the lesson, connect what they learned to their teaching practice outside of the lesson study cycle, and commit to engaging in the cycle again.

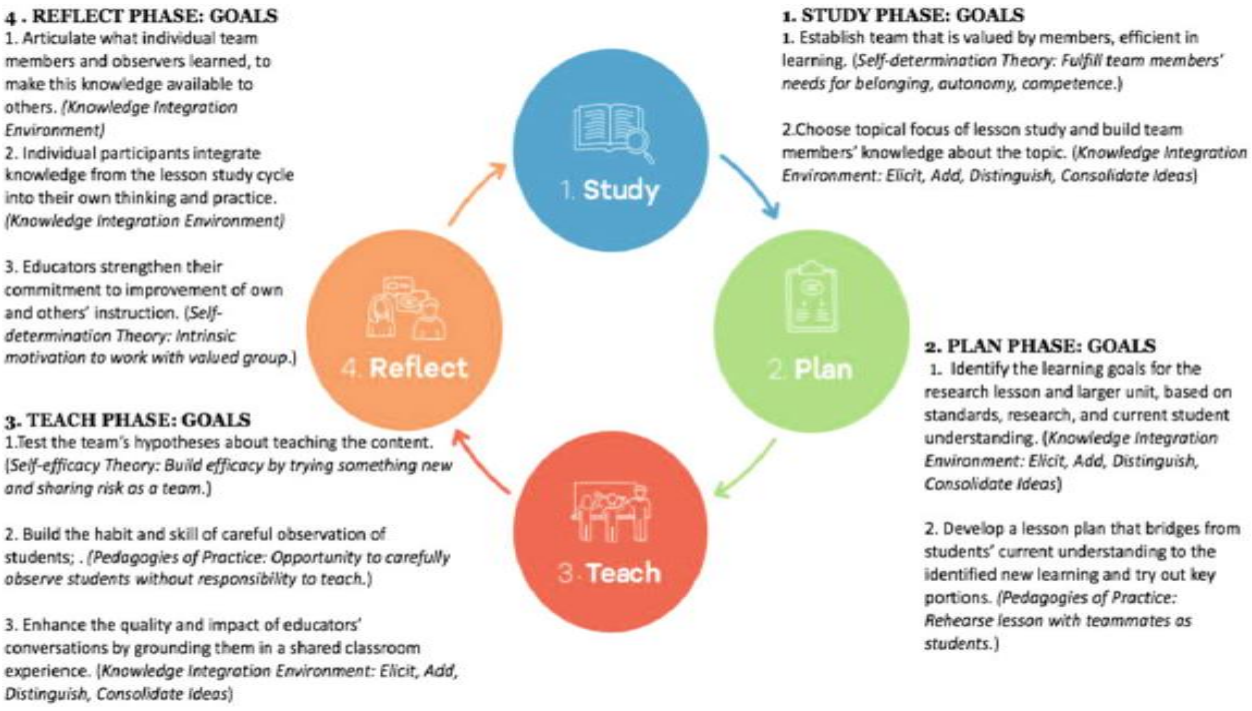


Figure 3-3: The Lesson Study Cycle (Lewis et al., 2019, pg. 19)

Lesson study aims to bring about transformational, cross-context learning (C. Foster et al., 2013; Lewis, 2009; Lewis et al., 2019; Wake et al., 2015). Figure 3-4 is a model of learning in lesson study.

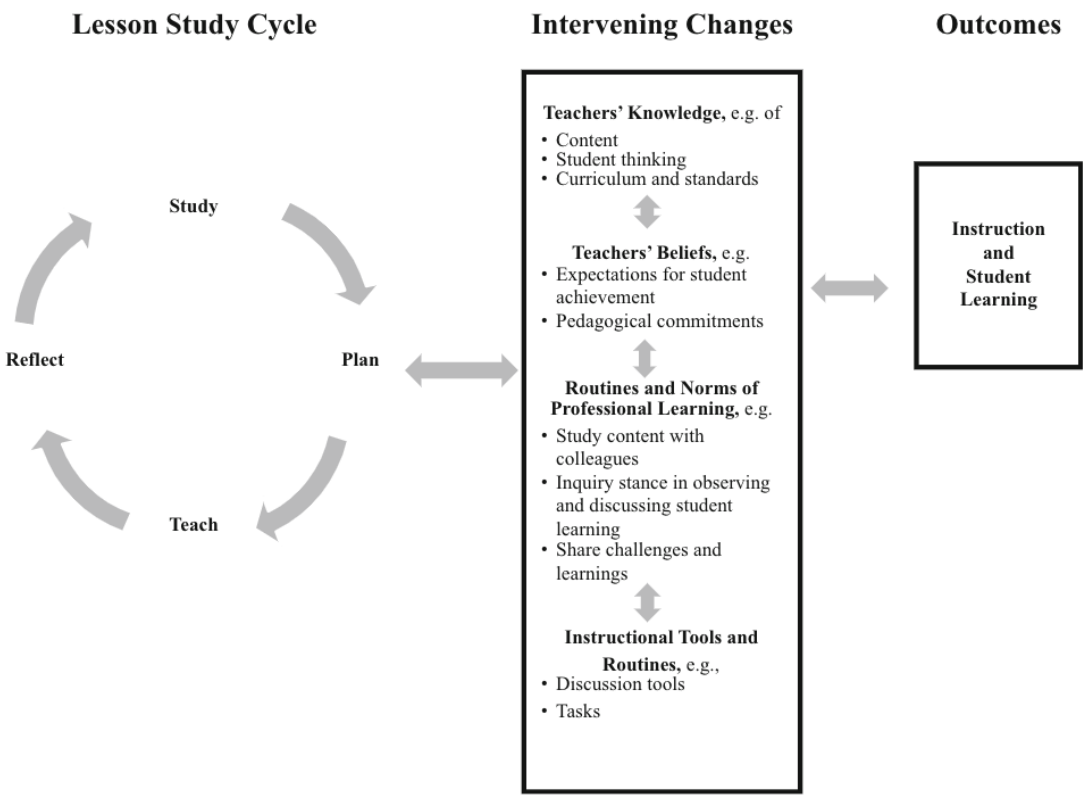


Figure 3-4: The theoretical model of learning in lesson study (Lewis et al., 2019, pg. 15)

The model of lesson study shown in Figure 3-4 posits that activity in the workgroup ultimately changes teaching practice and improves student learning. Along the way, lesson study changes how teachers think and talk about their work ("Teachers' Beliefs"), teachers' explicit knowledge of content and students ("Teachers' Knowledge"), and the structure of teachers' professional and classroom environments ("Routines and Norms of Professional Learning" and "Instructional Tools and Routines") (Lewis et al., 2019). All of this is achieved by collaboratively inquiring into genuine problems of practice, working across the workgroup and classroom contexts.

TRU-LS Professional Development

In this section, I describe the structure of TRU-LS PD. For further information about TRU-LS PD and narrative cases of its implementation at sites beyond Benton, see Schoenfeld et al., 2019.

TRU-LS PD combines the inquiry structure of lesson study with the vision of ambitious instruction and supporting tools of the TRU Framework. In TRU-LS PD, mathematics departments led by a district facilitator or teacher-leader engage in a series of nested inquiry cycles. Over a single school year, the department does two or three lesson study cycles around at least one research theme chosen by the department. Within those lesson study cycles, teachers

engaged in inquiry cycles that last two workgroup sessions aimed at learning more about their research theme through planning interventions, trying them, and reflecting on them with classroom-based artifacts.

Teachers generate potential research themes in an early workgroup session. To brainstorm research themes, they reflect together on their successes and challenges in the classroom and connect them to the TRU Dimensions. They also reflect on their goals for their students' learning. Over the first few sessions, they craft a shared research theme from their list of successes, challenges, and goals, grounding it in at least one TRU Dimension. The research theme that the teachers choose is a problem of practice that is relevant to them and stated in their own words. The TRU Dimensions help them frame their research theme as a question about how students experience the learning environment and situate their research question in some combination of the five features that matter for learning environments.

Then teachers begin inquiry into their research theme, starting the Study phase of lesson study. The teachers engage in short cycles of inquiry, over two workgroup sessions, in which they learn about their research theme by moving between teaching experiments in their classrooms and reflection and planning activities grounded in the TRU Framework in the workgroup. A single inquiry cycle is represented by the cycle in Figure 3-5.

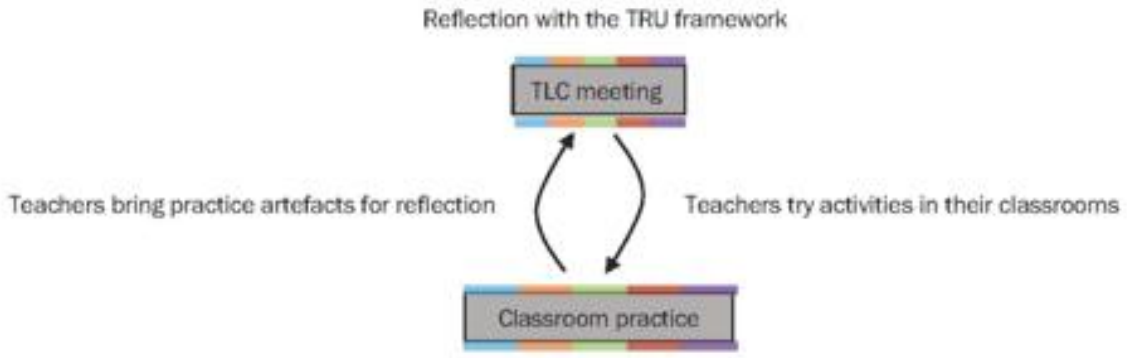


Figure 3-5: One inquiry cycle in TRU-LS (Schoenfeld et al., 2019)

How the two-session-long inquiry cycles represented in Figure 3-5 are nested within a lesson study cycle is represented in Figure 3-6.

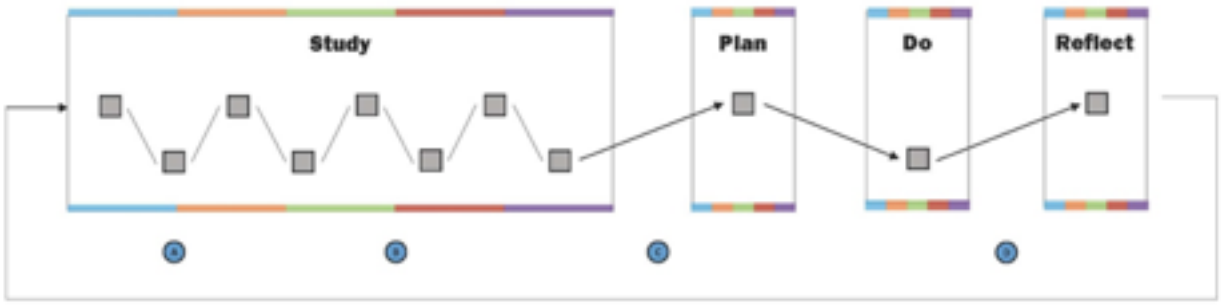


Figure 3-6: The TRU-LS cycle (Schoenfeld et al., 2019)

As shown in Figure 3-6, the inquiry cycles compose the Study phase of lesson study.

As part of these inquiry cycles, the teachers engage in three activities in the workgroup: planning together, sharing classroom events, and watching video together. Table 3-1 describes each activity and what classroom artifacts and TRU resources are used in each.

Table 3-1: The three workgroup activities that make up an inquiry cycle.

Activity	Description	Classroom artifacts	TRU Resources
Plan together	Teachers select instructional strategies from the list provided by the facilitator that they think will help them learn more about their research theme. They come to consensus around one strategy and agree to try it in their classrooms. They may modify the strategy from what is described in the list.	None	Instructional strategies list developed by the TRU-LS planning team.
Share classroom events	Teachers take turns recounting what happened when they tried the instructional strategy. They are encouraged by the facilitator and researcher to use *replays*, detailed descriptions of classroom events that include evidence and reasoning. They may make and develop claims about the efficacy of the strategy.	Teachers often rely on recollections of classroom events but sometimes are supported by: - Notes - Student work samples - Tasks	TRU Dimensions descriptions
Watch video together	Teachers watch together and discuss a video clip recorded in one of their classrooms, typically during the two weeks between workgroup sessions. They typically observe for a short list of student behaviors, selected from the TRU Observation Guide Student Look Fors. In the discussion that follows video-watching, the teachers make claims about what happened in the video and are encouraged to justify their claims with evidence from the video.	Video from the teachers' own classrooms, recorded and clipped by the facilitator and/or researcher.	TRU Observation Guide Student Look Fors TRU Conversation Guide

The three activities described in Table 3-1—planning, sharing classroom events, and watching video together—were designed to allow the teachers to share their individual experiences with their problem of practice in their classrooms and collaboratively define, study, and explore possible resolutions for their problem of practice. Each workgroup session that was part of the Study phase of the two lesson study cycles typically included all three activities. The session began with sharing classroom events from the teachers' experiences trying the intervention they had chosen in the previous session. As described in Table 3-1, sometimes teachers brought classroom artifacts to share. Then the teachers watched video of one of them and/or their students engaging with the intervention. They discussed the video, which led into

planning the next inquiry cycle's intervention. The teachers either modified the intervention or chose a new intervention as they juxtaposed their and their students' experiences with the goals and issues captured in their research theme.

Table 3-2 maps the activities in each session of the first two lesson study cycles in which the Benton teachers participated. Table 3-2 also shows when the Benton teachers engaged in other aspects of lesson study during PD time, including choosing a research theme, planning their research lessons, teaching the research lessons, and reflecting.

Table 3-2: Benton teachers' first two lesson study cycles and session activities.

Sept.		**Oct.**		**Nov.**	**Dec.**		**Jan.**		**Feb.**
1	2	3	4	5	6		7	8	
Choose research theme. Plan together.	Share class events. Watch video. Plan together.	Share class events. Watch video. Plan together.	Share class events. Watch video. Plan together.	Watch video. Plan together.	Share class events. Do math of the research lesson.	**RL 1**	Reflect on research lesson.	Share class events. Watch video. Plan together.	**RL 2**
Cycle 1							**Cycle 2**		

As Table 3-2 shows, a typical session of TRU-LS PD at Benton followed a routine of the teachers sharing classroom events, watching video together, and then planning together. Chapter 4 examines how the activities and the ways in which they built on each other supported the teachers' inquiry.

As Table 3-2 also shows, the Benton teachers did not spend extensive PD time planning their research lessons. During Session 6, the teachers did the math activity that students would be doing during the first research lesson. Otherwise, however, they did not plan the research lesson during PD time. Because of limited PD time and the Benton teachers' willingness to use other departmental time and time outside of working hours to plan the research lesson, the facilitator chose to focus PD time on the facilitated Study activities and have the teachers plan their research lessons largely unfacilitated and on their own time. Therefore, this dissertation does not delve significantly into the teachers' work during the Plan phase. Nonetheless, I describe that phase here.

During the Plan phase, the teachers use what they learned from the inquiry cycles to plan the research lesson. They choose an upcoming lesson that deals with content important across courses by studying their curricula and course sequences. The facilitator encourages them to think vertically and horizontally across the curriculum. One teacher or a pair of teachers commit to teaching the research lesson. Then, members of the planning team (typically the teacher(s) who will teach the lesson and other teachers that teach the same course, but sometimes the entire department) revises the lesson plan provided by the curriculum to incorporate their hypotheses and instructional strategies. Final planning for the research lesson takes into account all of the

learning that teachers have done around their problem of practice in the small inquiry cycles leading up to the lesson. They generate a list of anticipated challenges that students will encounter during the lesson, reflect on those challenges using the TRU Dimensions, and plan possible teacher responses. They also make a list of data that they want observers to collect. Immediately before the research lesson, the whole department meets to reflect on what they have learned so far and what they hope to learn when they teach the lesson.

Finally, the teachers teach their research lesson. One or two teachers teach, observed by the rest of the department and a few invited visitors. Research Lesson Day is an opportunity for the department to include other important members of the school community in their learning. Visitors often include administration and teacher-leaders from other departments, as well as members of the design and research team from the overarching research project. The teachers also invite a knowledgeable outsider, who will provide final commentary on their lesson. Observers attend to the questions posed by the lesson planning team and collect data by taking notes.

After the lesson, the teachers reflect with the help of their invited visitors. They review the data and revisit their research theme. The TRU Dimensions help them keep students' experiences at the center of their reflections and remind them of why they chose their research theme. The knowledgeable outsider gives final commentary, often structured by the TRU Dimensions. At the end of their reflection, the department recommits to engage in lesson study and the cycle begins again.

TRU-LS and lesson study share a similar structure, focus on problems of practice, and emphasis on mathematical content that is important in the curriculum. Unlike typical lesson study, TRU-LS incorporates small cycles of inquiry into the study phase of the lesson study cycle. Teachers reflect on their inquiry via videos of their own classrooms.

Research Setting, Participants, and Context

Benton High School

Benton High School is one of several dozen schools in a large, urban California school district. It serves grades 9-12 and is one of the smaller comprehensive high schools in the district, with around 800 students. The city in which Benton is located is economically and racially segregated, with wealthier, white families living in the "highlands" and poorer families of color living in the "lowlands." Benton is a neighborhood school in the "lowlands." It is located in a neighborhood with high poverty and large migrant and refugee populations, mainly from Latin America and the Middle East. In 2017-18, 98% of students at Benton were from minoritized backgrounds and 94% qualified for free- or reduced-price lunch. Half of Benton students were designated as English language learners. Benton is one of a handful of high schools in the district with a program designated for students who have recently arrived in the US and are learning English. This district uses the term "newcomer" to designate the program and the students in it.

This program for recent US arrivals operates separately from the "mainstream" program. Even in mainstream classes, however, students can frequently be heard speaking Spanish and Arabic in addition to English.

In any given year, Benton high school has between six and eight math teachers, the size of the department varying with district budget booms and busts. Two or three of those math teachers teach in newcomer program, while four or five teach in the mainstream program.

Participants

In the 2017-18 school year, the year of data taken up by this dissertation, the four mainstream math teachers participated in TRU-LS PD. Their PD was facilitated by a district coach. Two researchers from the project team attended most to all of their PD sessions. The Benton principal sat in on half of the first session but did not return. Table 3-3 shows the participants, the courses they taught, and their years of teaching or coaching experience as of the end of the 2017-18 school year. Years of experience at Benton is specified when a teacher also taught at another school.

Table 3-3: Participants, courses, and years of experience (where applicable).

	Participant	**Courses**	**Years of experience**
Benton teachers	Leo Lin (department chair)	Geometry and Math Analysis	15
	Cindy Guo	Algebra 1 and 2	15 (13 at Benton)
	Jamie Rossi	Algebra 1 and 2	3
	Soledad Castillo	Geometry and Math Analysis	2
Coach	Elizabeth Davies	N/A	3 (as a coach) 8 (as a teacher, high school math)
Research/ Design	Anna Weltman	N/A	7 (as a teacher; middle and high school math)
	Matt Ellinger	N/A	14 (as a teacher; elementary all subjects)

In this section, I provide backgrounds of the four Benton teachers, focusing on their previous and contemporary experiences with professional development and what is known about other professional interactions they had with their colleagues during the study year.

Leo Lin was serving as department chair and had taught at Benton for 15 years. This was his second year as department chair. His previous experiences with PD were PD provided by the district and the school. Several years previously, he had participated in monthly PD about the district's mathematics curriculum. He described this PD as unhelpful because it was infrequent, mandatory, and mainly devoted to identifying ways in which the different school math departments were not in sync with the curriculum. Leo co-planned with Soledad. He described his collaboration with Soledad as primarily "troubleshooting" Soledad's teaching challenges. As department chair, Leo visited his colleagues' classrooms to observe and debriefed with them afterwards. He visited Jamie and Soledad's classrooms up to once a month. I do not know how often he visited Cindy's classroom. When Elizabeth left after their second research lesson for maternity leave, Leo assumed the role of facilitator.

Cindy Guo had taught at Benton for 13 years and had 15 years of teaching experience. She had served as the department chair before Leo. Cindy's previous PD experiences were also PD provided by the district and school. She had participated in PD for department chairs and teacher leaders and in the curriculum PD that Leo also described. She appreciated the community built by the teacher leader PD, but otherwise did not describe these PD experiences as useful. Cindy co-planned with Jamie. They met multiple times per week to discuss lesson plans, pedagogical strategies, and pacing goals. Cindy described her professional relationship with Jamie as important to her. She described learning from Jamie about pedagogical practices around groupwork that were new to her but which Jamie had learned in his credential program and in PD he participated in outside of the district.

Jamie Rossi had taught at Benton for three years. He received his teaching credentials and masters degree from a locally well-regarded teacher preparation program known for its emphasis on Complex Instruction. Jamie participated in PD offered by the district and other local institutions. During the 2017-18 school year, Jamie was part of a multi-year fellowship and PD program offered by a nearby university for early-career high school teachers who taught in schools that served students from low-income backgrounds. The program emphasized deepening beginning teachers' instructional practices for supporting equitable learning environments. In addition to regular co-planning meetings with Cindy, Jamie occasionally met one-on-one with Soledad to troubleshoot issues with students and discuss pedagogy. Jamie described his professional relationship with Cindy as important to him. He described valuing Cindy's extensive teaching experience, and he benefited from being able to collaborate in planning. Jamie was an active member in the district teachers' union.

Soledad Castillo was in her second year of teaching at Benton. She was concurrently earning her masters and credential from a local university. Soledad had changed careers from working in biotech. She attended classes nights and weekends and had regular visits from a university observer. One of the classes she took during the 2017-18 school year included at least one session about the TRU Framework. During the 2017-18 school year, Soledad attended a regional conference for mathematics teachers. During PD meetings and in interviews, Soledad routinely expressed gratitude for the generosity of her colleagues and the support they offered

her as a new teacher. Soledad described the strong network she experienced among the Benton math teachers as one of the primary reasons she did not leave the teaching profession after her first year.

Elizabeth Davies was the facilitator. She worked as a secondary mathematics coach for the district. In that role, she facilitated TRU-LS PD at several other schools and coordinated other district PD efforts for secondary math teachers. She was in her third year in this role. Previous to becoming a district math coach, Elizabeth taught secondary mathematics in a large urban school district in a different region of the country. Elizabeth received a teaching credential and Master's degree in Education through an alternative-pathway credential and Master's program popular in the region in which she worked. She later received a Master's degree in Educational Policy from a well-regarded school of education. Elizabeth began working as a district coach in 2015. In addition to designing and running TRU-LS PD at several school sites, Elizabeth co-planned and taught district-wide professional learning courses for math teachers focused on building content and curriculum knowledge. She also coached beginning teachers in the California Teacher Induction program. Finally, she worked with department chairs to help them build professional learning plans for teachers in their departments.

Researcher positionality

In this section, I describe my role in TRU-LS PD at Benton and the relationships I developed with the teachers and facilitator. I also describe my background because I consider myself a participant in the TRU-LS PD at Benton.

I came to this project as a former math teacher. My experience as a math teacher was similar to and unlike the experiences of the Benton teachers in important ways. Before I began work on the TRU-LS project, I taught elementary and secondary math at a private school in Brooklyn, New York. While working on the TRU-LS project, I taught middle school math at a private school and then at a public school in a district near the district in which the Benton teachers taught. I was not teaching during the 2017-18 school year.

The majority of my teaching experience was in schools unlike Benton in student demographics, teachers' and students' access to resources, and demands on teachers' time outside of the classroom. At both private schools, students were admitted after a vetting process that included interviews and testing. More than half of students paid full tuition, which was at least $20,000 a year. Up to a quarter of students at both schools attended with full scholarships and would have qualified for free- or reduced-price lunch had they attended a public school. Neither school admitted English language learners. The public school in the city near Benton in which I taught was a neighborhood elementary and middle school in a gentrifying community traditionally home to working class families of color. Half of students at this school qualified for free or reduced-price lunch and a quarter were English language learners.

While teaching at both private schools, I was not expected to participate in professional development. During my time teaching in Brooklyn, I voluntarily participated in professional

development through professional organizations, but mostly without my colleagues. As a public school teacher, I participated in mandatory school professional development, some of which was lesson study. I also participated in optional professional development on Complex Instruction offered at the district level.

Before my work on the TRU-LS project, I had not worked as a coach or professional developer but had mentored new teachers and led workshops for math teachers at conferences. During the 2017-18 school year, while working with the Benton teachers, I co-planned and helped facilitate a different professional development program for math teachers in a nearby district.

Before the 2017-18 school year, I met with Elizabeth and the other district coaches who worked on the TRU-LS project up to once a month to advise them on designing and facilitating the PD. In 2017-2018, I worked more closely with Elizabeth. In that year, Elizabeth became the only district coach assigned to the TRU-LS project. Knowing that she no longer had as much district support but was expected to lead PD at the same number of schools, I offered to collaborate with Elizabeth. Elizabeth and I met as often as once a week to plan and debrief PD sessions at Benton and other schools. We also created year- and semester-long learning plans for the Benton teachers. I typically collected and curated the video-based representations of practice that we used in workgroup meetings. When I collected the video, I always offered to help the teacher plan ahead of time and debrief afterwards. Soledad and Jamie regularly took up those offers; Cindy and Leo did not.

After the 2017-2018 school year, I remained in touch with Elizabeth and the Benton teachers, although my formal planning and facilitation support ended. I now continue to consult with Jamie, who currently leads their professional development, and I served as the final commentator for one of their research lessons.

Three years of TRU-LS at Benton: An overview

As previously stated, the Benton teachers participated in TRU-LS for three years. This dissertation focuses on their first two lesson study cycles in the second year in the project, which was also their first year engaging with TRU-LS PD. In this section, I provide overviews of their work in TRU-LS over the three years. I present reasoning behind selection for analysis of the first two lesson study cycles of Year 2 and characterize their work during the years not analyzed.

Table 3-4 provides an overview of the Benton teachers' work in TRU-LS over the three years.

Table 3-4: Three years of TRU-LS at Benton High School

Year	PD type	Participants and facilitator	Description
2016-17	**TRU only**	***Facilitators:*** Elizabeth and Swati *(second district facilitator)* ***Teachers:*** *(Mainstream math department)* Leo, Cindy, Jamie, Soledad, and Olivia	Bi-monthly meetings; introduced participants to the TRU Dimensions and engaged them in two "inquiry cycles."
2017-18	**TRU-LS**	***Facilitators:*** Elizabeth (until February); Leo (February to end of year) ***Teachers:*** *(Mainstream math department)* Leo, Cindy, Jamie, and Soledad	Bi-monthly meetings; engaged teachers in three TRU-LS cycles.
2018-19	**TRU-LS**	***Facilitator:*** Leo ***Teachers:*** *(Mainstream math department)* Leo, Cindy, Jamie, Soledad, and Angela; *(Newcomer math department)* Patricia and Carmen	Bi-monthly meetings; engaged teachers in three TRU-LS cycles.

As shown in Table 3-4, the Benton teachers' first year in TRU-LS introduced them to the TRU Framework and established them as a professional workgroup. Their second and third years followed the TRU-LS PD program described in the previous section.

Year 1, 2016-17: Building community with TRU

In 2016-17, TRU-LS was supported by three district coaches and two mathematics curriculum and instruction leaders at the district level. Two of those coaches facilitated the PD at Benton. They were often supported by one of the math leaders. At Benton, TRU-LS PD took place every two weeks during a common planning period. The newcomer math teachers did not share that common planning period so they could not participate in TRU-LS PD. That year, the participants were as new to working with each other as they were to the TRU Framework. Teachers Soledad and Olivia were new to the school; Jamie was beginning his second year. Facilitator Elizabeth was beginning her second year in the district and her first working with the Benton teachers. While facilitator Swati had several years of experience as a district coach, she did not have strong relationships with the Benton teachers before the 2016-17 school year. Because of the nascent and unstable relationships among the Benton teachers and district facilitators, Swati and Elizabeth were wary of inviting project team members to the TRU-LS PD sessions. Anna attended a handful of sessions that year, but her irregular presence and newness of the project and PD resulted in spotty video records of PD sessions.

Swati and Elizabeth launched the PD by supporting the teachers to reflect on classroom events during workgroup sessions and engaging the teachers in workshops focused on each TRU Dimension. Swati and Elizabeth progressively worked towards the teachers bringing artifacts

from their classrooms to the sessions, culminating in several of the teachers agreeing to share video of their classrooms. Swati and Elizabeth also worked towards having the teachers select one TRU Dimension and a problem of practice to launch an inquiry cycle in which the teachers learned more about how to support that dimension in their classrooms.

During the spring semester of 2017, the teachers chose to inquire into the Agency, Ownership, and Identity dimension. They investigated their own questioning practices with the aim of building routines that encouraged students to share their thinking. Towards the end of the school year, the teachers named perseverance as a character trait they wanted to build in their students. The teachers would return to perseverance in the 2017-18 school year.

In interviews at the end of the 2016-17 school year, all teachers but Olivia reflected positively on their experiences in TRU-LS. Leo, Cindy, Jamie, and Soledad described the PD as useful and expressed eagerness to participate in the PD the following year. Olivia did not describe the PD as useful. Thus, during the 2016-17 school year, the four Benton teachers that would return in 2017-18 largely developed into a professional learning community, became familiar with the TRU Dimensions, and began to articulate and investigate a problem of practice that would focus their inquiry in the following year.

Year 2, 2017-18: District and school-site reorganization and beginning TRU-LS

Between Years 1 and 2 of TRU-LS, the district underwent a budget crisis that forced staffing cuts in schools and at the district level. District policies implemented to deal with the budget crisis changed the professional environment for the Benton teachers. Full examination of the influence of these district policies on the Benton teachers and their work with TRU-LS is for another paper. I will provide a rough characterization, however. The position at Benton held by Olivia was eliminated going into the 2017-18 school year, leaving the remaining teachers with larger class sizes. Despite the increased workload for teachers, school leaders scheduled the Benton math teachers' classes so that they still had a common planning period. Leo's leverage as department chair and a long-time Benton teacher proved pivotal as the teachers advocated for common planning time. The four remaining mainstream teachers were able to use the common planning period for department meetings, co-planning of courses they shared, and, every two weeks, TRU-LS PD. However, the Newcomer math teachers still did not share the same planning period, so they were excluded from TRU-LS PD despite wanting to attend.

The second district facilitator in 2016-17, Swati, left the district in the turmoil over the budget shortfall, as did several other members of the TRU-LS project team in the district office. Only Elizabeth remained in the district office; a third coach took a teaching job in the district in a school that continued to do TRU-LS PD, although he no longer facilitated. In 2017-18, Elizabeth was the only district-level coach assigned to the TRU-LS project. Her continued presence at Benton, along with the return of four of the five teachers who had participated in 2016-17, resulted in a sense of stability at Benton, despite the turmoil at the district level.

As will be shown in forthcoming analysis, in 2017-18 the teachers returned to the problem of practice around student perseverance that they had first named during the spring of 2017. Their first two lesson study cycles inquired into how to support student perseverance. The teachers followed the TRU-LS PD program described in the previous section of this chapter and outlined in Table 3.

In the beginning of 2018, Elizabeth, Leo, and Anna began to prepare for Elizabeth's family leave, which was to start at the end of February. Leo officially assumed the role of TRU-LS facilitator at Benton following their second research lesson. While Leo was supported by Elizabeth until her leave began and by Anna throughout, and was encouraged to follow the PD structure developed by Elizabeth and Anna, the Benton teachers' work shifted with Leo as facilitator. They chose a new research theme: "How can teachers support students to use prior knowledge to access new content?" Between the end of February and mid-May, the Benton teachers met for TRU-LS eight times and did one research lesson. The activities that the teachers did during these sessions resembled typical department meetings more than they resembled TRU-LS PD sessions. In three of those eight sessions, the teachers collaboratively planned the third research lesson, something which they had not done during PD time in the first two lesson study cycles and had instead done during routine department meetings. The teachers also did not watch video together of each other's classrooms and did not commit to or spend time planning around a shared pedagogical strategy. Thus, while the teachers continued to meet and completed a third lesson study cycle after Elizabeth's departure, there was a shift in their activity and inquiry focus.

In interviews at the end of the 2017-18 school year, the four teachers again reflected positively on the PD. All teachers described feeling closer to their colleagues, holding shared ideas about teaching, and having learned from their work together. The teachers looked forward to continuing the PD the following year with Leo facilitating.

Year 3, 2018-19: Continuing with TRU-LS, now with everyone

Before the 2018-19 school year, Leo negotiated with Benton administration to provide a common planning period for all math teachers at Benton, mainstream and newcomer alike. The teachers in the school's newcomer program had expressed interest in joining the TRU-LS PD since the PD began in 2016. A common planning period was arranged, and newcomer math teachers Patricia and Carmen joined Leo, Cindy, Jamie, Soledad, and new mainstream teacher Angela for their bi-monthly TRU-LS PD during the 2018-19 school year. That year, the teachers again completed three lesson study cycles. Leo continued as primary facilitator, although Jamie took on an informal leadership role in their PD work. They were informally supported by Elizabeth and, at times, Anna. The 2018-19 school year was the final year of the TRU-LS project, and Elizabeth and the project team encouraged the Benton teachers to continue the PD with as little district and project support as possible. Data collection during TRU-LS PD sessions was incomplete because of Elizabeth and Anna's irregular attendance.

Interviews at the end of the school year pointed to several challenges faced by the Benton teachers in their third year of the PD. The 2018-19 school year was challenging for the entire district, and those challenges made their way into the Benton teachers' PD efforts. Significant school disruptions occurred across California in the fall of 2018 because of massive wildfires that displaced residents in nearby cities, cut power across the region, and made the air unhealthy for many residents. School was closed for several days because of the fires and TRU-LS PD was disrupted. Early in 2019, the Benton teachers and others in the district went on strike. These disruptions pushed the Benton teachers' second research lesson into the late spring, with the third following closely.

Introducing the two newcomer math teachers to the community already established among the mainstream math teachers also introduced challenges. In interviews, the newcomer teachers described feeling like outsiders and as if they had been expected to join the work already in progress, rather than the work shifting to incorporate their needs and interests. The mainstream teachers described feeling less aligned with their colleagues' goals and views on teaching than they had at the end of the previous year. Jamie and Cindy in particular expressed some frustration with the slow pace of change across the department. Despite these challenges, however, all teachers committed to continuing TRU-LS PD the following year, even without the formal support of the TRU-LS project. The four teachers with less than five years' experience at Benton—Jamie, Soledad, Angela, and Patricia—cited the support and professional community provided by TRU-LS PD in their decisions to remain teachers at Benton.

Anecdotal evidence showed the Benton teachers continuing to meet during the 2019-20 school year. Jamie served as facilitator. Elizabeth and Anna joined for a research lesson in the fall of 2019. Of course, the 2019-20 school year was also disrupted, this time by the COVID-19 pandemic. Time will tell whether the Benton teachers continue to meet regularly for self-run professional development resembling TRU-LS PD.

Selection of two lesson study cycles from 2017-18 for dissertation analysis

As the previous section indicates, the TRU-LS project generated a rich, diverse set of data at Benton over three years. Videos of workgroup sessions, interviews, notes from classroom visits, and written notes describing district and school-site dynamics paint a robust picture of the environment in which the Benton math teachers worked and learned. Many stories could be told from these data.

For this dissertation, I chose to focus on Leo, Cindy, Jamie, and Soledad's work together during the first half of the 2017-18 school year, while TRU-LS PD was facilitated by Elizabeth and they inquired into the research theme, "How do we build student perseverance and capacity to struggle productively, together?" This selection incorporates two lesson study cycles and eight workgroup sessions. These sessions and research lessons and the data collected around them were best suited to my research questions: How did the teachers represent core teaching ideas as they inquired into their problem of practice, over time? And, how did engagement with PD

activities support the teachers to take up more robust, ambitious representations of core teaching ideas? Over their first two lesson study cycles in 2017-18, the teachers inquired into the same problem of practice, which anchors my analysis for the first research question. They also engaged in regular PD activities with stable facilitation, which anchor my analysis for the second research question.

Data collection

Data for this dissertation were collected as part of the TRU-LS project. TRU-LS collected a wide array of data types from a variety of sources over four years, as part of the project's aim of making sense of change at the district, school site, and classroom levels. This dissertation primarily utilizes video of workgroup sessions and pre- and post-research lesson discussions collected during the fall of 2017 and winter and early spring of 2018. I collected all data analyzed in this dissertation. Table 3-5 represents the dates and lengths of the sessions analyzed in this dissertation, attendance at each session, and the type and quality of video in each session.

Table 3-5: Data table for session videos.

Session	Date *Length*	Attendance	Video type and quality
1	09/13/17 *59:50*	Elizabeth, Anna Leo, Cindy, Jamie, Soledad	Wide-angle lens video. Video captured all participants. Complete audio.
2	09/27/17 *01:00:15*	Elizabeth, Anna, Matt Leo, Cindy, Soledad	Regular lens video. Video captured up to four participants at a time. Camera was pivoted towards speaker. Video ended after about 30 minutes. Complete audio.
3	10/13/17 *01:16:31*	Elizabeth, Anna, Matt Leo, Cindy, Jamie, Soledad	Regular lens video. Video captured up to four participants at a time. Camera was pivoted towards speaker. Complete audio.
4	10/25/17 *01:00:05*	Elizabeth, Anna, Matt Leo, Cindy, Jamie, Soledad	Regular lens video. Video captured up to four participants at a time. Camera was pivoted towards speaker. Complete audio.
5	11/08/17 *01:03:04*	Elizabeth, Anna Leo, Cindy, Jamie	Wide-angle lens video. Video captured all participants. Complete audio.
6	12/06/17 *01:07:42*	Elizabeth, Anna, Matt Leo, Cindy, Jamie, Soledad	Wide-angle lens video. Video captured all participants. Complete audio.
RL 1 pre	12/20/17 *58:40*	Elizabeth, Anna, Matt, Catherine, Lara Leo, Cindy, Jamie, Soledad Lucy	Wide-angle lens video. Video captured up to six participants at a time. Camera was inconsistently pivoted towards speaker. Complete audio.
RL 1 post	12/20/17 *01:09:11*	Elizabeth, Anna, Matt, Catherine, Lara Leo, Cindy, Jamie, Soledad Lucy	Wide-angle lens video. Video captured up to six participants at a time. Camera was inconsistently pivoted towards speaker. Complete audio.
7	01/07/18 *55:01*	Elizabeth, Anna, Matt Leo, Cindy, Jamie, Soledad	Wide-angle lens video. Video captured all participants. Complete audio.
8	01/31/18 *01:02:02*	Elizabeth, Anna, Matt Leo, Cindy, Jamie, Soledad	Wide-angle lens video. Video captured all participants. Complete audio.
RL 2 pre	02/07/18 *58:22*	Elizabeth, Anna, Matt, Catherine, Alan, Suzanne Leo, Cindy, Jamie, Soledad Lucy, Stig	Wide-angle lens video. Video captured up to six participants at a time. Camera was inconsistently pivoted towards speaker. Complete audio.
RL 2 post	02/07/18 *01:56:34*	Elizabeth, Anna, Matt, Catherine, Alan, Suzanne Leo, Cindy, Jamie, Soledad Lucy, Stig	Wide-angle lens video. Video captured up to six participants at a time. Camera was inconsistently pivoted towards speaker. Complete audio.

As shown in Table 3-5, videos of eight workgroup sessions and two research lessons were collected. Sessions were typically one hour long, while pre- and post-lesson discussion ranged from one to two hours. Complete video and audio were available for all sessions with the exception of Session 2. For Session 2, video was available for the first thirty minutes of the session; audio was available for the entire session.

As Table 3-5 indicates, video did not consistently show the faces and bodies of all participants. A wide-angle lens camera was used in Session 1 and from Session 5 onwards. With the wide-angle lens, video captured all participants at all times during the regular workgroup sessions. Without the wide-angle lens and in pre- and post-lesson discussion sessions, video did not capture all participants at all times but an attempt was made to pivot the camera towards the speaker when possible.

To triangulate conclusions about what happened during sessions, video data were supplemented by: session agendas; commentaries and reflections written by Elizabeth and Anna during and after each workgroup session and research lesson; and material resources used and/or created during sessions. Agendas, reflections, and material resources were collected for all workgroup sessions and pre- and post-lesson discussions. To triangulate conclusions about teachers' relationships and reactions to the PD, video data of sessions were supplemented by video and transcript of end-of-year interviews with each teacher.

Data reduction and analysis methods

Analysis methods follow from my research questions and theoretical framework. The questions I undertook to answer in this dissertation were:

1. How did the teachers represent core teaching ideas as they inquired into their problem of practice, over time?
2. How did engagement with PD activities and frameworks support the teachers to take up more robust, ambitious representations of core teaching ideas?

The theoretical framework was described in Chapter 2. I conceptualize learning as an iterative process of teachers defining, studying, and attempting to resolve problems of practice in and across their workgroup and classrooms. As teachers engage in inquiry, they iteratively build, hybridize, and transform problems of practice, representations of key ideas, and norms and patterns of participation in and across contexts, including but not limited to the workgroup itself, their classrooms, their schools, and reform-oriented networks and organizations (Akkerman & Bakker, 2011; Bannister, 2018; Horn et al., 2016; Little, 2003; Rogoff, 1994).

My research questions operate at two temporal levels. They call for *longitudinal analysis* across the entire dataset, and for examination of *pivotal cases* at a grain-size that surfaces interactions. Longitudinal analysis results in descriptions of patterns and trajectories*:* What about

the teachers' representations of core teaching ideas and participation changed and stayed the same over time? How did teachers' engagement with PD activities unfold over time? Case analyses identify interactions that were pivotal to patterns identified in longitudinal analysis: What happened in moments when a change in representation or participation took place or when a representation or practice surfaced or stabilized? How did surfacing, change, or stabilization of representations and practices come about through interactions among participants and with PD activities, norms, and materials?

Following from the two temporal levels of my research questions, my analysis unfolded in two phases. First, I worked across the entire dataset to identify themes in how the teachers represented ideas core to their problem of practice. I also tracked the development of pedagogical practices that the teachers planned together and used in their classrooms between workgroup sessions. Then, I selected, transcribed, and closely analyzed pivotal cases to describe moments of emergence, change, or stabilization and the interactions that brought them about. These two phases of analysis and the analysis activities that constituted them are represented in Table 3-6.

Table 3-6: Phases and steps of analysis.

	Step	Outcome
Phase 1	(1) Segmenting and making logs (with a second coder)	How participants represented ideas about teaching central to their problem of practice, over time. What pedagogical practices the participants planned and used in their classrooms, over time.
	(2) Open-coding, and memoing	
	(3) Collapsing and categorizing codes	
	(4) Flagging counterexamples and revising codes	
	(5) Applying final codes to entire dataset	
	(6) Representing coded data to identify surfacing, changing, and stabilizing representations	
Phase 2	(7) (Concurrent with above) Flagging pivotal cases	How representations and pedagogical practices identified in Phase 1 emerged through interactions among participants and PD activities, norms, and materials.
	(8) Transcribing cases and building out triangulating data	
	(9) Writing the cases	

As shown in Table 3-6, data were segmented, coded, selected as cases, and eventually transcribed in nine stages: (1) segmenting and making logs; (2) memoing and open-coding; (3) collapsing and categorizing codes; (4) flagging counterexamples and revising codes based on

them; (5) applying the finalized codes to all of the videos; (6) representing the coded data to identify trajectories including surfacing, changing, and stabilizing representation of core ideas; (7) flagging pivotal cases for transcription and close analysis, in conversation with my research questions; (8) transcribing selected cases and building out the data used to triangulate video findings; and, finally, (9) writing the cases and engaging in close interaction analysis through writing.

I elaborate on these analysis steps in the following sections.

Phase 1: Longitudinal analysis and preparing to analyze pivotal cases

In the first phase of my analysis, I endeavored to describe how the participants represented ideas central to their research theme and what pedagogical interventions they planned and used across the eight workgroup sessions and two research lessons. As described in Chapter 2, how teachers represent core ideas of teaching and learning is indicative of different discourses of teaching and learning. The descriptions of how the Benton teachers represented core ideas of teaching and learning built in this phase of analysis drew on the frameworks of ambitious and traditional mathematics instruction elaborated in Chapter 2. However, drawing on a theoretical stance on these discourses that they are broad, diverse, and only roughly bounded (also elaborated in Chapter 2), coding began as open coding, only engaging the literature on ambitious and traditional mathematics instruction in later stages. This coding process is elaborated in the following sections. Phase 1 analysis consisted of preparing the video data for coding, building and applying codes and labels, memoing, and representing the coded data to afford description of trajectories. I describe these analysis steps in the following sections.

Segmenting and building activity logs

I began by segmenting the data and building activity logs for each workgroup session. The goals for segmenting were to divide the session videos into units of analysis that: (1) captured topically coherent conversation, to afford thematic coding, and (2) contained consistent activity and participation structure, to afford interaction analysis.

When making activity logs, I drew on the session agendas and video records to divide each session into its separate activity components. For example, in Session 2, the participants were first welcomed to the session; then they shared what happened in their classrooms when they tried a pedagogical strategy selected in the previous session; then they watched and discussed video from one of their classrooms; next they planned around a pedagogical strategy; and finally they closed the session. The activity logs captured these distinct activities along with short descriptions of what happened in each. The descriptions made note of when participants reached consensus around a pedagogical strategy that they would use in their classrooms and described the strategy.

After building activity logs, two undergraduate research assistants and I segmented each video. My unit of analysis is a segment of conversation called *micro-activity*. A micro-activity is defined by topic and participation structure. Micro-activities change when either the topic or the participation structure changes. The segmenting process that resulted in the micro-activity unit was built semi-inductively by a team of coders, including myself, who were working with data from across the multi-site project. We began by defining criteria for our unit of analysis. For example, we wanted our unit of analysis to be topically coherent, signal changes in participation structure, apply to workgroup sessions at different school sites, and allow us to look exclusively at facilitation if we chose. We reviewed session agendas and fieldnotes from several school sites to collect a set of possible participation structures. Then, we individually inductively segmented sample sessions, compared our results, and refined our segmentation process. After developing a segmentation process that we could reliably apply to unproblematic sessions, we reviewed our fieldnotes to find unconventional sessions—sessions with atypical activities, different facilitators, little written record, etc.—and segmented them, refining the process as we went. We built a training manual that documented the segmentation process.

Table 3-7 defines the participation structures used in segmenting.

Table 3-7: Participation structures and their descriptions.

Participation structure	Description
FRAME	The facilitator moves participants through the agenda. Micro-activities labelled FRAME include the facilitator describing session goals, launching a discussion, wrapping up a discussion, or giving instructions for a group activity. Questions asked by the facilitator in the course of discussion are not given their own micro-activity unless they are explicitly named in the agenda. In FRAME, the facilitator is typically the only speaker. Other participants may ask procedural questions.
SHARE	Participants take turns sharing responses to a prompt. The SHARE participation structure is always set by the facilitator in a preceding FRAME micro-activity. In SHARE, each sharing participant gets his or her own micro-activity. Typically only one participant speaks. Other participants may ask clarifying questions, or the facilitator may restate or refine the prompt.
DISCUSS	Participants freely converse with one another about a single topic. The facilitator may mediate conversation.
WVIDEO	Participants watch video together.
READ/WRITE	Participants read or write individually. Typically no one speaks, except to ask procedural questions.
GRP(#a, b, c, etc.)	Participants work in groups. The GRP label is following by a number indicating the number of people in the group(s) that can be heard in the video, and then a letter indicating which group(s) can be heard. For example, GRP3ab indicates that groups a and b can be heard, and each has three members.
CHAT	Some subset of participants talk off-topic. CHAT is the only participation structure that can be used in conjunction with the other participation structures. For example, if participants are writing individually but two participants have an off-topic conversation about what they did over the weekend when they finish the writing activity, the micro-activity is labelled READ/WRITE/CHAT. CHAT can be its own micro-activity if a micro-activity is interrupted by more than 20 seconds of off-topic conversation.

Most micro-activity transitions resulted from changes in participation structure. This was in part because the sessions were facilitated and each planned question asked by a facilitator earned its own micro-activity. Some micro-activity transitions resulted from a change in topic. These typically occurred during discussions. The resulting segmentation has several DISCUSS micro-activities back-to-back, each with a different title and description. To identify a change in topic not tied to a planned facilitator move, we relied on language cues and a holistic evaluation process. Topic changes often occurred when the facilitator asked an unplanned question or another participant asked a non-procedural question. Sometimes they occurred when a participant suggested a new interpretation of the event, artifact, or idea being discussed and the participants embarked on a discussion of that new interpretation. Therefore, questions and comments that indicate that the speaker is suggesting an alternative served as language cues to a

change in topic. Rarely, a segmenter felt that a new micro-activity was needed because of a topic change but could not identify a language cue. In these cases, the segmenters reflected together on whether a participant in the room would feel that something different was happening.

The finalized segmentation process consisted of these steps:

1. Two segmenters complete segmentation individually. They:
    a. Find all written documentation of the session. Documentation includes the session agenda, public displays created for or during the session, material resources used during the session, and notes or reflections written by the facilitator and/or attending researcher.
    b. Create a *macro-activity* segment for each activity written in the session agenda. For example, if the agenda specified a video-watching activity, the segmenters would create a macro-activity called, "Watch video."
    c. Begin a first pass of watching the video. Each time there is a change in participation structure and/or a change in topic, stop the video and note the time of the potential *micro-activity*. Also roughly describe the participation structure and what happened in the segment of video.
    d. Watch the video a second time. On this second pass, refine the start/stop times of micro-activities. Also label each micro-activity with its participation structure, create a one-sentence title for the micro-activity that captures topic, and, if appropriate, describe the micro-activity in more detail separately from the title. Finally, describe and, if possible, link to material resources used.
2. The two segmenters compare their segmentation. They discuss discrepancies and arrive at a consensus segmentation.
3. A third expert segmenter who is already familiar with the data watches the video with the consensus segmentation. She revises the consensus segmentation as needed. For example, she might edit the title for clarity.

Each video in my data corpus was segmented by two research apprentices. I served as the expert segmenter.

The segmented videos served as robust activity logs for each session. As described in the segmentation process above, each segment was annotated with participation structure, a descriptive title, and a several sentence description of what the participants did and said during that segment.

After segmentation was completed, I moved to building codes to capture core ideas and how participants represented them.

Building codes

Coding began with open coding and memoing (Charmaz, 2006). My coding described how the participants represented the ideas central to their research theme: "How do we build student perseverance and capacity to struggle productively, together?" Drawing on the research theme, I began open coding teachers' representations of these three ideas: perseverance, productive struggle, and the notion of students doing these things "together." I inductively built codes from the data and memoed about my progress. I also coded for explicit references to the TRU Dimensions.

While this dissertation focuses on the eight workgroup sessions and two research lessons in which the teachers engaged in the first half of the 2017-18 school year, I open-coded and memoed all sixteen workgroup sessions and three research lessons for the whole year. I began the coding process in Excel and migrated to Angles[1] once it was appropriate to begin building codes in the "markup window." Figure 3-7 shows an example of the markup window during the initial open coding and memoing stage.

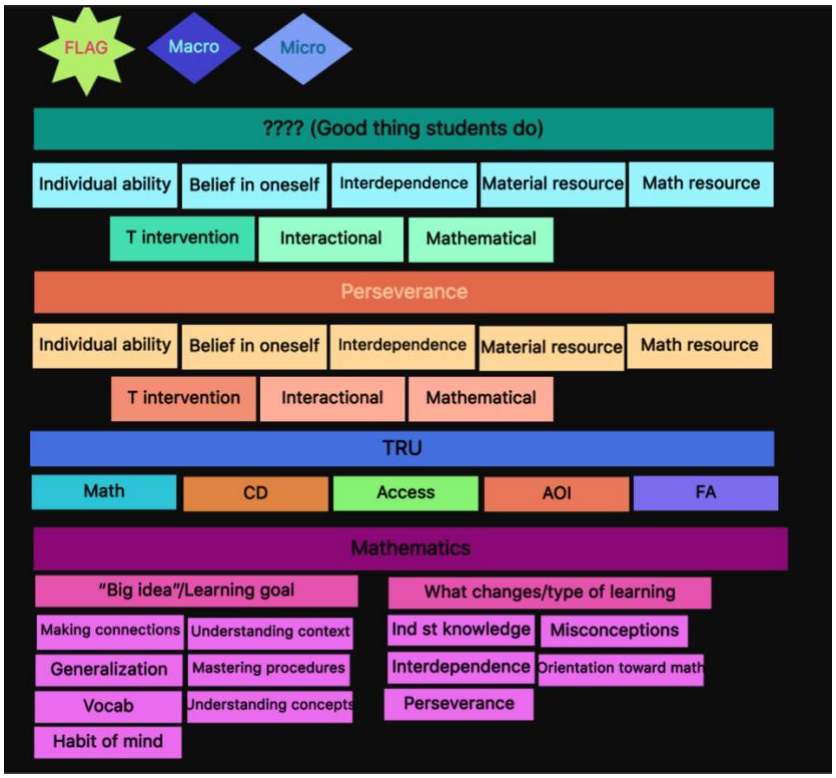


Figure.3-7: An example of the markup window during open coding. The markup window changed often; this is an image at one moment in time. Open coding resulted in several broad codes and an array of more fine-grained codes that were later categorized and collapsed.

[1] Angles is a video analysis software. It supports layered segmenting, coding, and memoing of videos. A single file contains segmented and coded video, captured as a "timeline," and any memos made about video segments. Angles also supports generating quantitative data about coded videos. The software can produce counts of coded segments and segments coded with a combination of codes, using logical operators. It also supports iterative code development. "Markup windows" can be revised and saved, producing a record of changes in coding over time.

As shown in Figure 3-7, open coding produced a set of codes that captured the core teaching ideas participants discussed and how they represented those ideas. Two main codes emerged, reflecting two core teaching ideas salient to the teachers' discussions, "perseverance" and "mathematical struggle." I applied "perseverance" to segments in which the participants discussed their students' experiences of perseverance. I applied "mathematical struggle" to segments in which the participants discussed their students' experiences of mathematical struggle. While these codes were often applied to the same segments—perseverance and mathematical struggle are overlapping ideas—they also applied to separate segments.

An array of sub-codes also emerged that captured how participants represented their students' perseverance or mathematical struggle. In particular, these sub-codes represented how the participants talked about students persevering alone or together; whether perseverance was an individual trait or emerged from the classroom environment; and whether or not mathematical struggle was productive for learning. For example, I developed the code "belief in oneself" for segments in which participants described perseverance as an individually-held feeling of self-efficacy.

I then collapsed, categorized, and added sub-codes, toggling between the data and my research questions. I documented my reflections in memos. This second stage of coding confirmed the two broad codes "perseverance" and "mathematical struggle." I collapsed the descriptive sub-codes into a set of labels that I could apply to coded segments. The labels captured different ways that participants described the central concepts of their research themes: perseverance as an *individual's ability or inclination to work*; perseverance as a *group accomplishment*; mathematical struggle as *productive for learning*; and mathematical struggle as *unproductive for learning*. These labels will be described in detail in the next section of this chapter.

In the third stage of coding, I revisited the data and re-coded as needed. I also flagged challenging cases, segments that did not clearly fall into the categories. Finally, in the fourth stage of coding, I returned to the flagged cases. I finalized coding them and documented my decisions, filling in the details of my coding scheme. Figure 3-8 shows the finalized markup window.

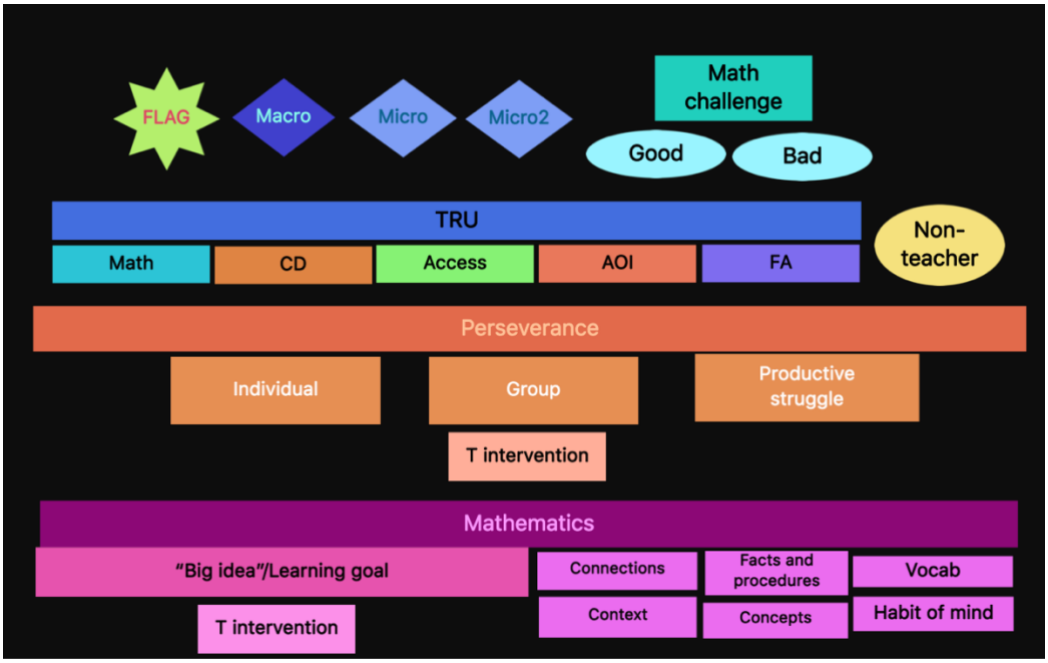


Figure 3-8: The finalized markup window.

In the next sections, I elaborate on the code used for analysis in this dissertation, perseverance, and its associated labels. The mathematical struggle code produced a line of analysis that is not reported in this dissertation.

Codes and labels: Perseverance

For segments coded "perseverance," I built two labels that capture the two primary ways in which participants tended to represent perseverance. These labels are: perseverance as an *individual's choice or ability to work* and perseverance as a *group accomplishment.* The labels subsumed the sub-codes I built during open coding and expanded to include concepts not captured during open coding but noted in memos. *Individual choice or ability to work* applies to segments in which participants reify descriptions of students' perseverance as a capacity or choice to work, or result of other individual trait, possession, or action. *Group accomplishment* applies to segments in which participants reify descriptions of students' perseverance as the result of interaction or, more robustly, as constituting students' collaborative engagement.

Table 3-8 provides examples of each label. I do not provide transcript of an entire micro-activity, even though the whole micro-activity received the label. I provide a sample of conversation that is indicative of the label. I selected examples from Session 3.

Table 3-8 also provides an example of unlabeled talk about perseverance. Sometimes participants mentioned perseverance but did not describe it in enough depth to warrant a label. This often occurred when the facilitator launched a discussion about perseverance, especially in the first four sessions, as the example in Table 3-8 indicates.

Table 3-8: Codebook for Perseverance code and its labels.

Code and label	Description	Example
Perseverance	Participants' joint work is oriented towards building their students' perseverance. Participants mention or describe perseverance or one of its synonyms or antonyms, such as persistence, engagement, productive struggle, "giving up," "getting stuck," or successfully or unsuccessfully working through challenge. The segment remains unlabeled if there is not enough information to determine how the speaker(s) is/are treating perseverance.	When Elizabeth reads their research theme, as in Session 3 Micro 2: *"First I want to just remind us of our draft research theme, here? We're working on…, 'Building student perseverance, capacity to struggle productively, together.' And that's our draft. We can always come back to it and tweak it as we go through this process."*
As individual's choice or ability to work	Participants represent perseverance as the work of individual students. They may attribute perseverance or lack thereof to individuals' "capacity," personality, mathematical ability, personal beliefs, or choices. They may describe students persevering or not persevering alone or in one-on-one interactions with teachers, or describe students working together but attribute perseverance or lack thereof to individuals.	When Cindy reacts to Jamie's story about a student refusing to explain his thinking and Jamie responds, Session 3 Micro 4: *Cindy: When I heard that comment from my student, I jumped to the conclusion, he's being lazy about it. Like, ok.* *Jamie: He—I mean—definitely not the kind of student who's like—he was of course the only person when I said, 'Ok, now try and make some matches and color code,' he was the only one who started doing it.*
As group accomplishment	Participants represent perseverance as the collaborative work of students. They may attribute perseverance or lack thereof to interactional aspects of the learning environment, such as successful or failed collaboration, relationships, or groups of students' use or non-use of interactional resources, including group roles.	When Elizabeth responds to Cindy's description of how she used a pedagogical strategy with students working in groups: *"Which I think also speaks to that piece in your research theme about 'together'? Like, 'building the capacity to struggle,' like, 'together'? So they're using, like, all the—everyone's pieces to come together, to persevere."*

These two categories are analytically separate, both conceptually and in the data. That is, they represent meaningfully different interpretations of perseverance and participants often invoked only one of them when talking about perseverance. They are also associated with different prominent sets of discourse of mathematics instruction, "traditional" and "ambitious," as described in Chapter 2. While these labels were induced through open coding, their alignment with these two sets of discourses is not surprising given the orientation of the PD and the prevalence of traditional instruction discourses. Nonetheless, sometimes individual speakers invoked more than one of them in a single talk-turn, often when offering multifaceted explanations of students' perseverance or lack thereof. Also, interactions among multiple speakers about the same topic often included references to more than one of these categories. Participants implicitly and explicitly discussed these categories. For my research questions, this adds to the analytic salience of these categories.

Table 3-9 provides an example of conversation from a segment that earned more than one label. Again, I do not provide transcript of the entire micro-activity, even though the micro-activity as a whole earned the labels. This examples was also a challenging case that required multiple passes to code. Table 3-9 also explains how it was challenging and why I chose the labels I did.

Table 3-9: A challenging case with more than one label.

Sample transcript from a micro-activity with **two labels, multiple speakers in conversation:**
The teachers discussed how they wanted to use the pedagogical strategy "Three Things" going forward. With Three Things, teachers ask students who are stuck for three things they know and three things they want to know. Session 3, Micro-activity 23: *Soledad: I was thinking that it's a bridge so they move on. So that, you know, when they're stuck and you're like [mimes pushing slowly with one hand] 'Oh, I don't know what to do, so. I'm gonna…' [Pretends to play with her phone under the desk.] You know? Um. That instead of doing that, that the—that the asking of the three things become—will become more automatic for them, that they might start asking themselves? And that they move forward. Autonomously. Tomorrow. Starting on Monday. [Laughs]* *Jamie: Like, immediately. [Laughs]* *Elizabeth [to Cindy, Jamie, and Leo]: Is that how you guys were thinking about it?* *Leo: Yes, but. Even if they can't move forward, if they move to a place where they know what they should be, like, what should come next. Like, I play video games, and I make plans—like, when you're playing, you're like, 'I need to go here, I need to go there, I need to go there.' But, I just don't stop and don't do anything. I [moves his hand away from his head, as though miming using his brain to move forward].*

Soledad: Yeah.

Leo: So. Even if you don't know content-wise what you're supposed to do, you could be like, 'If I ask Lin [Leo's last name] this, and I find this out, then I'll be able to do this.' Or if I ask somebody else this, then I'll be able to do this. Um. Instead of just getting stuck. So, that's sorta why I brought up the directions, because once they don't—some, some kids really just don't know the directions, so if they knew the directions they would try something.

Elizabeth: Cindy, I saw you shaking your head earlier.

Cindy: Yeah, because it has started to evolve for me. Uh. I keep bringing up, like, I want to engage the unengaged. So. Like, I'm starting to see this not as a one-on-one strategy? Like, me and another student. But, like, something public. Because it's—I want—I want to be able to publicly acknowledge that somebody is successful. Like, they get, 'Yeah, that's something that we could see. And point out. You're right.' Like, 'You know something here.' So that's why I'm like—I really like [pointing to Leo] that idea, of the doing it in a group. Because then maybe it's not three things, maybe it's four things or customize it to how—whatever the size of the team even is. So it's [mimes pointing to students], 'What's something you know, something you know, what's something you know.'

Jamie: I like that.

Cindy: So that everybody gets a say and everybody's like, 'Got it.' And so hopefully what that does for the person who's unengaged is that, 'Oh, ok, I do know something.'

Labelled *individual* and *group.*

Rationale: Soledad described her desire to use Three Things to get individual students back on task. She described and mimed individual students choosing to disengage. She hoped that individual students would internalize the Three Things strategy so that they would no longer rely on teachers. Her turn is an example of attributing perseverance to individual choices and skills. Thus, her turn earned the segment the label *individual.*

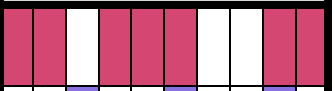
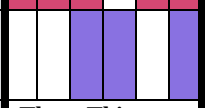
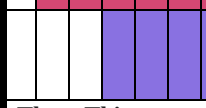
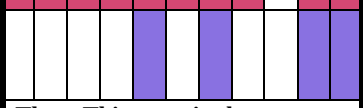
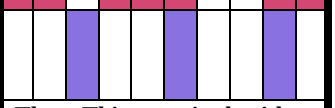
Leo described using Three Things to show students that if they are stuck, they can seek help from the teacher or other students. His turn is a challenging case because his replay showed him engaging in productive struggle while playing video games. However, in the end of his turn, he described students moving forward in problem-solving by collaborating, not by planning or making incremental steps as described in his video gaming replay. Therefore, Leo's turn earned the segment the label *group.*

Cindy described using Three Things with groups of students. She hoped that Three Things would help "unengaged" students see how they can and do contribute to their group's work. Labeling certain students as "unengaged" is a move typical of treating perseverance as an individual endeavor. Cindy described the power of collaboration in helping "unengaged" students build confidence. Had Cindy's turn been the only turn in the micro-activity, it would have earned both labels *individual* and *group.*

Representing the coded data to identify patterns

After coding and labeling was completed, I represented the coded data to identify patterns in participants' representations of perseverance. I triangulated the coded data with activity logs embedded in the segmentation and reflective notes written after each session by myself and the facilitator to identify when particular representations *surfaced, faded, stabilized,* or *changed.* I represented coded data in a timeline, tagged with the pedagogical interventions that the participants planned and used. Representations of these data provided a picture of when and how participants used the two representations of perseverance and three representations of mathematical struggle over the eight sessions and two research lessons. These representations also correlated the participants' use of different representations of key ideas with planning different interventions. An example of a timeline is shown in Table 3-10.

Table.3-10: Sample timeline. The first row represents each micro-activity coded perseverance as labeled individual; second row represents each micro-activity coded perseverance and labeled group. The third row describes the intervention planned in each session. Coding and interventions represented in this timeline indicates that individual and group representations of perseverance were present in Session 1 and individual representations stabilized through Sessions 2 and 3.

	Session 1	*Session 2*	*Session 3*	*Session 4*
Ind				
Grp				
Intervention	**Three Things:** "If students are stuck on a problem, ask them to state three things they know about the problem and three things they are wondering about. This can help students realize that they are making progress and have information that they can potentially draw on to make more progress." (Emailed on 9/15/17)	**Three Things revised:** "…we agreed to continue trying the Three Things strategy, but modifying if needed, using Leo's suggestions of prompting students for three things they know with questions like 'what have you done so far?' and 'what has someone told you?'" (Emailed on 10/04/17 and 10/10/17)	**Three Things revised:** "Everyone agreed to continue to try Three Things, but this time using it in a public setting (either in front of the whole class or in front of a small group) by asking 'What do you know?' to three different students. Our hypothesis is that by doing this in a public setting and hearing from a variety of students, students will be able to work through productive struggle by using their own and each other's ideas." (Emailed on 10/16/17)	**Three Things revised, with unspecified new strategy:** "We also agreed to continue our investigation on Three Things, collecting evidence/reflections on how that strategy is impacting student perseverance. We did notice, however, that the strategy gave students more access to content, but not to individual perseverance (as we provided the scaffold that we hope they would be doing on their own). In addition to continuing with Three Things, we agreed to brainstorm other strategies that would get a building *individual* perseverance, for discussion on 11/8." (Emailed on 10/26/17)

Creating data representations such as the timeline in Table 3-10 and pattern narratives culminated Phase 1 of analysis.

Phase 2: Analysis of pivotal cases

Phase 1 of data analysis produced patterns in the participants' planned interventions and representations of ideas central to their research theme, "How do we build student perseverance/ capacity to struggle productively, together?" Descriptions of these trajectories positioned me to move into Phase 2 of analysis, analyzing pivotal cases to determine how those patterns came about and the role of PD activities and frameworks and facilitation in teachers' inquiry. I elaborate on the stages of Phase 2 analysis in the following sections.

Case selection, transcription, and writing

Timelines and trajectory narratives built in Phase 1 pointed towards cases worthy of interaction analysis. To narrow the collection of potential cases, I flagged micro-activities surrounding moments when representations of perseverance surfaced, changed, faded, or stabilized. I also flagged micro-activities surrounding moments when participants created new, revised, or hybridized pedagogical interventions.

Flagging resulted in around 30 micro-activities with the potential to be analyzed as cases. Many of these micro-activities clumped together, spanning much of a planned workgroup activity. For example, often a representation surfaced in a session as the teachers discussed classroom video and was followed by a planning activity in which teachers developed a new intervention using that representation. Clumping the flagged micro-activities according to workgroup session activity illuminated connections between the trajectories identified in Phase 1 and the PD.

Once flagged micro-activities were grouped into robust, coherent cases, I transcribed them. Transcription attempted to represent words, phrases, and slang used by participants. Transcription was also annotated when possible with gestures made by the speaker. Video limitations did not make full gesture transcription possible. Transcript was also annotated with artifacts used by participants and how those artifacts were indicated by the speaker. However, I did not attempt to embed the transcript with all gestures, uses of artifacts, and indications of interactions among the participants. When analyzing interactions among participants, artifacts, and PD activities in each case, I moved between the transcript, video, and descriptions of segments captured in activity logs. Full analysis of each case unfolded through writing as I attempted to create rich descriptions of the interactions among participants, PD activities, and frameworks present in each case and their connections to the representations and pedagogical interventions developed.

Conclusion

This chapter described the study in which this dissertation was situated, including the design of the PD and participants' histories. It also motivated the selection of the eight workgroup sessions and two research lessons that made up the first half of the 2017-18 school

year for analysis. This chapter then described the analytic methods employed to answer the two research questions:

1. How did the teachers represent core teaching ideas as they inquired into their problem of practice, over time?
2. How did engagement with PD activities and frameworks support the teachers to take up more robust, ambitious representations of core teaching ideas?

The next three chapters show the fruits of that analysis. In them, I describe the patterns in how the teachers represented core teaching ideas that unfolded as the participants engaged with their problem of practice. I also look for connections between those patterns and teachers' engagement with one another in the PD activities, supported by frameworks of ambitious mathematics instruction. In so doing, I present a detailed picture of four teachers' learning around an important problem of practice in a collaborative workgroup.

Chapter 4 Working on the Problem of Perseverance

This chapter begins the analysis of the teachers' learning in two lesson study cycles, during which they inquired into building their students' perseverance. As elaborated in Chapter 2, I conceptualize learning as an iterative process of teachers defining, studying, and attempting to resolve problems of practice in and across their workgroup and classrooms (Ball & Cohen, 1999). Through this process, teachers may construct or move away from representations of important teaching ideas. I consider learning as situated in both immediate settings and interactions and in larger cultural-historical contexts (K. D. Gutiérrez & Rogoff, 2003; Little, 2002; Russ et al., 2016). I examine shifts in collaborative discourse within the workgroup and consider the roles that interactions between and among participants, resources, and PD activities play in the learning process. These theoretical stances inform my analysis of learning.

In their first two lesson study cycles, the teachers inquired into the problem of practice captured by their research theme: "How do we build student perseverance and capacity to struggle productively, together?" Through their inquiry, the teachers defined, refined, and sometimes rejected representations of key ideas captured by their research theme. These ideas included perseverance, productive struggle, and the notion of students doing these things "together." How the teachers took up the last of these issues, the role of interaction and collaboration in students' perseverance, is the focus of Chapters 4, 5, and 6. Chapter 7 investigates teachers' sense-making around the role of productive struggle in perseverance. This chapter engages with the question, *How did the teachers represent the role of interaction in perseverance as they inquired into their problem of practice, over time?* It examines patterns in teachers' representations of the role of interaction in perseverance. The next chapter, Chapter 5, engages with the question, *How did engagement with PD activities support teachers to take up different representations of the role of interaction in perseverance?* Chapter 5 examines how interactions facilitated by PD activities shaped teachers' uses of representations of perseverance over time. Chapter 6 engages with a question raised by analysis of patterns in representations in this chapter, *How and in what contexts did individual representations of perseverance remain salient to the teachers as they explained their students' behavior?* Chapter 6 takes a broader approach, considering teachers' use of individual representations of perseverance in the context of prevalent ideologies that make individual, merit- and character-based interpretations of students' behavior common-sense.

In my analysis of the teachers' learning in their first two lesson study cycles, I find that two ways of representing perseverance were predominant in the teachers' conversations: perseverance as *individual students' capacity or choice to work*, and perseverance as *a group accomplishment*. Uses of these representations varied across the two lesson study cycles. Over time, teachers increasingly took up representations of perseverance as a group accomplishment. However, representations of perseverance as individual students' capacity or choice to work remained salient throughout. Using representations of perseverance as a group accomplishment over time, the teachers were able to reason together about how students and teachers co-construct

opportunities for all students to do challenging mathematics and learn. This reasoning supported the teachers to see perseverance the result of collaboration, rather than character, and to plan interventions that would support students to engage together with mathematics. Thus, analysis in this chapter serves as an existence proof that teachers collaborating around a meaningful problem of practice, supported by PD, can move towards discourses of ambitious mathematics instruction, in ways that have potential to create more powerful learning environments for their students. However, analysis in this chapter is not solely an existence proof. It also illuminates the complexity of the teachers' interactions with each other around perseverance and sets up for analysis in the next two chapters of PD conditions that supported and constrained learning and the persistence of individual representations of perseverance.

Analysis of the teachers' learning around perseverance in this chapter is divided into two parts. In Part 1, I discuss what it means to represent perseverance as individual students' capacity or choice to work or as a group accomplishment. I connect those representations to discourses of ambitious and traditional mathematics instruction and contextualize the importance and challenge of shifting teachers' collaborative discourse towards ambitious mathematics instruction. In Part 2, I examine changes over time in how the teachers represented perseverance. Part 2 looks at quantitative evidence to describe longitudinal patterns in participants' representations. Part 3 illustrates the patterns described in Part 2 with four cases. These cases also attend to the role of the facilitator and TRU PD tools in the patterns that unfolded.

Individual and group representations of perseverance in discourses of mathematics instruction

Perseverance is central to discourses of mathematics instruction—so much so that it is one of the Common Core State Standards for mathematical practice (*Standards for Mathematical Practice | Common Core State Standards Initiative*). In the Standards for Mathematical Practice, "perseverance" is represented as successful problem-solving. "Perseverant" students are "mathematically proficient." The "perseverance" standard is shown in Figure 4-1.

Mathematical Practice 1: Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

*Figure 4-1: CCSS Mathematical Practice Standard 1 (*Standards for Mathematical Practice | Common Core State Standards Initiative, *n.d.)*

As shown in Figure 4-1, then, "perseverance" in the CCSS for Mathematical Practice is, essentially, successful problem-solving. Students who persevere are those who proficiently solve mathematics problems. The standard describes the mathematical work that students do when they persevere in problem-solving. But the wording of this standard leaves open the questions of what conditions support perseverance. Are students either capable or incapable of persevering, as a matter of character or choice, or does perseverance emerge through interaction? Can students' problem-solving be unsuccessful, partially successful, or alternately successful and unsuccessful, as students engage in the fraught process of learning, and students still be considered perseverant? These questions about the relationship between perseverance, interaction with peers, and productive struggle are not answered by the perseverance standard. And yet how teachers answer them is essential to the sense teachers make of students' real struggles with mathematics and to the opportunities teachers provide for mathematical learning. With this lack of definition, how are teachers to make sense of why students do and do not persevere and how to support perseverance?

In discourses of ambitious mathematics instruction, supporting students' perseverance in mathematics is equivalent to supporting students to struggle productively, together, with important mathematics and come to see themselves and their peers as mathematically powerful. Supports for perseverance come from many aspects of a classroom environment. Appropriate task cognitive demand and the availability and usefulness of material resources are both important factors. Central to this chapter and the next are how the teachers made sense of the ways in which interactions among students and teachers support perseverance.

How teachers make sense of the role of interpersonal interaction and collaboration in perseverance is an important question. This is because as discussed in Chapter 2, teaching in robust and ambitious ways relies on seeing students as learning mathematics *together*, in classroom contexts that allow them to struggle productively. Discourses of ambitious instruction

represent the opportunities students have to struggle productively and learn in mathematics classrooms as collaboratively constructed. Looking to interactions among students and teachers, rather than to individual students' character or choices outside of classroom contexts, to learn about why students may or may not struggle productively with mathematics in certain situations is consistent with discourses of ambitious instruction (Schoenfeld, 2014). And yet predominant, taken-for-granted discourses of education represent perseverance and lack thereof as the result of individual abilities and choices examined outside of context (Horn, 2007; Sengupta-Irving & Agarwal, 2017). In other words, dominant discourses of education tend to represent perseverance as part of an individual student's character, while discourses of ambitious instruction tend to represent perseverance as a group accomplishment. Teachers' uses of these representations, as they are part of different discourses of mathematics teaching and learning, have consequences for the sense teachers make of observations of students' behavior and for the kinds of classroom environments teachers cultivate (Goodwin, 1994; Schoenfeld, 2014).

In the context of ambitious instruction discourses, perseverance is a *group accomplishment*. Perseverance, or identity- and agency-building productive struggle and meaning-making, is built by students engaging with each other and teachers in their classroom environment (Schoenfeld, 2014). One can attend to the perseverance of an individual student situated within a group of students and/or classroom environment or the perseverance of a group of students as a collective (Sengupta-Irving & Agarwal, 2017). Essential to either perspective is attention to how a student's or a group of students' perseverance is constructed through interaction with peers. Interaction with peers in the classroom places students in "collaborative zones of proximal development" (Goos et al., 2002), changing the ways they experience mathematical struggle and what they can learn from it. Empowering collaboration among students can richen the cognitive demand that students experience and expand students' opportunities to struggle productively (Boaler & Greeno, 2000; Cohen et al., 1999). Learning environments that provide opportunities for students to collaborate around cognitively demanding tasks can support students' engagement with, enjoyment of, and feelings of agency and power with mathematics (Engle, 2012; Gresalfi et al., 2008; Sengupta-Irving, 2015). To support students' perseverance, then, teachers should attend to the ways in which students interact with each other around challenging tasks. As described in Chapter 3, TRU-LS PD aimed to encourage the Benton teachers to inquire into the relationship between interaction with peers, collaboration, cognitive demand, and perseverance.

Notably, however, discourses of ambitious mathematics instruction do not often explicitly discuss perseverance. For example, the TRU Framework (Schoenfeld, 2014), the vision of ambitious mathematics instruction used by the PD analyzed in this dissertation, does not use the word "perseverance." Instead, the TRU Framework uses the word "agency" to describe the sense that individual students or students working in groups can have that continuing to work on challenging math problems is worthwhile and possible. The TRU Framework argues that the richest mathematical learning happens when students struggle productively with appropriately-challenging mathematics and have opportunities to "build on

others' ideas and have others build on theirs" (Schoenfeld & the Teaching for Robust Understanding Project, 2016). Similarly, Complex Instruction (CI) (Cohen, 1998; Cohen et al., 1999; *Home | Complex Instruction*, n.d.), an ambitious instruction pedagogy which the teachers in this dissertation frequently referenced, does not deal with "perseverance." Instead, CI specifies the need to engage students "at a high intellectual level" (*Home | Complex Instruction*, n.d.). It specifically names group work as a mechanism to combat pervasive and problematic status issues in education that prevent some students from engaging at that high level. Thus, the TRU Framework and CI pedagogy, both of which teachers in this dissertation leveraged in their collaborative inquiry, represent perseverance as a group accomplishment, although they do so implicitly. They emphasize collaboration differently, however. Chapter 6 takes up some of the notable differences in how these two important discourses of ambitious mathematics instruction represent perseverance as a group accomplishment, in preparation for a closer look at manifestations of predominant discourses and ideologies of perseverance in the teachers' talk.

The pull of locating perseverance within the individual strong. Traditional, commonplace discourses of mathematics instruction represent perseverance as part of an individual student's character or the result of an individual student's choices, disregarding group context (Bannister, 2015; Horn, 2007). Individual representations of perseverance are consistent with predominant learning theories that take up individual and trait-based perspectives on the relationship between students' learning, culture, and context (Gresalfi, 2009; Gutiérrez & Rogoff, 2003). Research on students' learning conducted from a situative perspective has shown that in contrast to this predominant perspective, students' productive struggle and persistence with mathematics is collaboratively constructed, not the result of character traits or acontextual choices (Horn, 2008; Sengupta-Irving & Agarwal, 2017; Sengupta-Irving & Vossoughi, 2019). However, individual representations of perseverance persist in discourses of mathematics education in part because they derive from broader ideologies that elevate individual achievement, shame individual failure, and frame education—and math education in particular— as preparing students to compete as individuals rather than to participate in a mathematically rich community (Gutstein, 2012; Valero, 2017). Individualist representations of perseverance also derive from ideologies that tend to represent minoritized students as character-deficient and, consequently, in particular need of schooling that builds character traits associated with success in a meritocratic society (Adiredja, 2019; Leonard & Martin, 2013; Valero, 2017). Individualist representations of perseverance are central to the popular concept of "grit," a character trait that education scholars and reformers have trumpeted as behind some students' resilience to adversity (Duckworth, 2016). The similarly popular concept of "growth and fixed mindsets" also draws on individualist representations of perseverance, attributing some students' capacity to learn from failure to individually-held inclinations and beliefs (Boaler, 2015; Dweck, 2006).

It is worth noting that the strong individualist connotations of the word "perseverance" and the close relationship between "grit," a concept that many adherents to ambitious mathematics instruction find problematic, and "perseverance" may explain why prominent ambitious mathematics instruction frameworks and pedagogies such as the TRU Framework and

CI shy away from using the word "perseverance" in their supporting documents. The strong individualist connotations behind the word "perseverance" described in this section of this chapter make "perseverance" a potentially loaded word for teachers to engage with in collaborative workgroups that aim to support movement towards ambitious mathematics instruction. However, as the first case analyzed in this chapter will show, the Benton teachers themselves chose to focus on supporting their students' *perseverance*. For better or worse, they did not phrase their problem of practice using words taken from the TRU Framework, the framework used in the PD. To them, perseverance and all of its complex connotations best represented the problem of practice that they wanted to tackle together.

The use of individual representations of perseverance has consequences for teachers' sense-making about students' behavior and pedagogical decisions, with further consequences for students' mathematical learning. Teachers' discourse can bound or broaden teachers' professional vision (Goodwin, 1994; Hutchins, 1996), constraining or opening up what teachers notice and what they can do. Teachers' uses of discourses of math instruction that represent mathematical sense-making as unfolding through interaction are consistent with their use of ambitious mathematics instruction pedagogies (Horn & Kane, 2015). Group representations of perseverance broaden teachers' horizons of observation to include the role of interaction in student perseverance. They have the potential to support teachers' attempts to restructure their classroom environment because of how they include important aspects of classroom context in teachers' field of vision. However, individual representations of perseverance bound teachers' horizons of observation by hindering teachers' ability to see the role of interactions in perseverance. Individual representations of perseverance also have the potential to confound teachers' attempts to restructure their classroom environments to support students' perseverance because of the how they blame individual students for a classroom-constructed lack of opportunity for collaborative productive struggle. Representing perseverance or lack thereof as individual ability or choice is consistent with deficit-oriented positioning of students (Bannister, 2016; Horn, 2008; Jilk, 2016). Deficit-oriented pedagogical practices reduce the opportunities that students read as "lacking"—in this case, lacking perseverance—have to engage in rich mathematical learning (Cohen et al., 1999).

Thus, supporting teachers to move away from locating perseverance within the individual and towards representations of perseverance as a group accomplishment has promise for teachers' ambitious pedagogical noticing, reasoning, and decision-making. However, as framed in Chapter 2, such a shift in discourse from traditional, dominant discourses and towards ambitious, reform-oriented discourses is challenging for teachers, even in PD contexts intentionally designed to support the shift (Bannister, 2015; Horn, 2007; Kazemi & Franke, 2004). Chapters 4, 5, and 6 trace the Benton teachers' journey as they engaged in the challenging and important work of expanding and enriching their discourse around the role of interaction in students' mathematical perseverance.

There are three parts to this story. In this chapter, I examine the variability in how the teachers represented perseverance and trace their slow, piecemeal movement towards discourses

of ambitious mathematics instruction. In Chapter 5, I examine how the PD supported the teachers to take up new representations of perseverance and increasingly consider students' learning as situated in a learning environment in which interactions with peers could afford or hinder perseverance. This chapter and the next show that teachers can move towards representations of perseverance as a group accomplishment, in the context of professional development. In Chapter 6, I examine the persistence of individual representations of perseverance in the teachers' conversations and conjecture why individual representations were so persistent. Chapter 6 shows that moving towards group representations of perseverance is a slow and complex process, unavoidably shaped by the larger culture in which teachers work.

Quantitative evidence of individual and group representations of perseverance in teachers' talk, over time

This part looks at quantitative evidence of how the teachers represented perseverance over time. Quantitative evidence provides an opportunity to look for broad-strokes patterns in how the teachers represented the role of interpersonal interaction in perseverance, over time in the PD context. In the next part, I provide detailed discourse analysis of four cases, each from a pivotal time in the teachers' collaborative work. Case analysis contextualized by quantitatively-derived longitudinal patterns provides an opportunity to observe relationships between patterns in representation and teachers' collaborative reasoning, uncover variability within broad-strokes patterns, and point to potential mechanisms for change, which will be further examined in Chapter 5.

Participants drew on individual and group representations of perseverance throughout the eight workgroup sessions and two research lessons that made up their first two lesson study cycles. Nonetheless, over time, they increasingly used group representations of perseverance. These patterns are highlighted in Figure 4-2. Figure 4-2 shows a frequency count of the number of topically- and participant-structure-coherent units of conversation in each workgroup session in which the participants (teachers and non-teachers) used individual or group representations of perseverance.

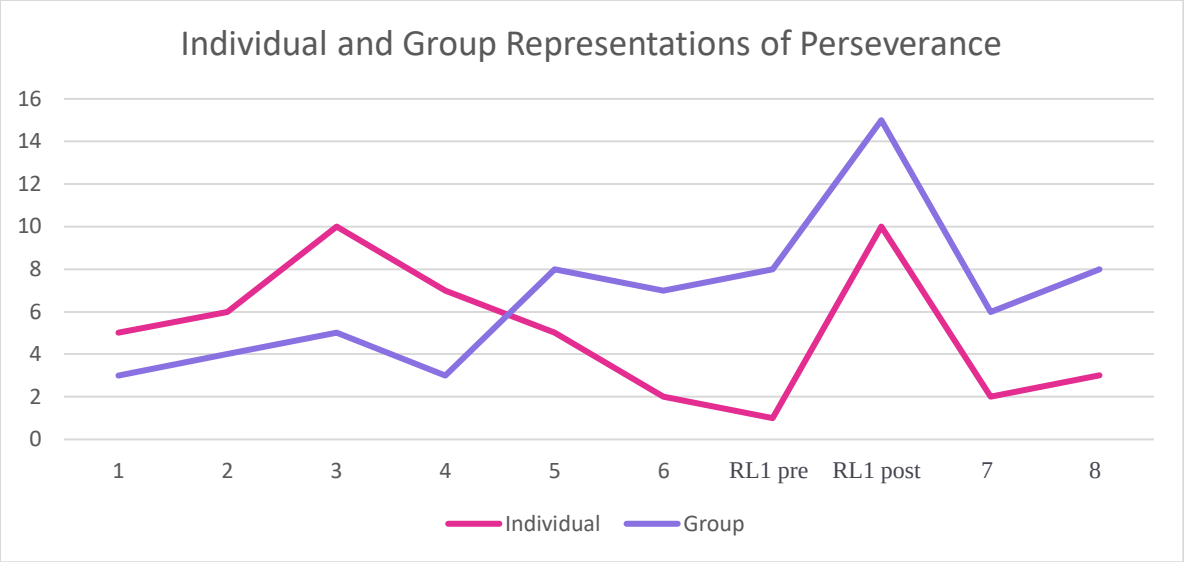


Figure 4-2: Graph of instances of individual and group representations of perseverance across the eight sessions and Research Lesson 1. Counts include teacher and non-teacher talk.

The graph in Figure 4-2 characterizes the ways in which participants represented perseverance as individual choice or capacity to work or as a group accomplishment in conversation, over time. As described in Chapter 3, the codes "perseverance as individual choice or capacity to work" and "perseverance as group accomplishment" were applied to units of conversation called micro-activities (topically- and participation-structure-coherent units of conversation). A code was applied to a micro-activity when at least one participant elaborated on a representation of perseverance in the manner captured by the code. As described in Chapter 3, a micro-activity could receive more than one code, but received each code only once.

Figure 4-2 shows that participants' conversation included both representations throughout the two lesson study cycles. It also shows that over time, more of the participants' conversation included group representations of perseverance than individual representations of perseverance. As shown in Figure 4-2, before Session 5 individual representations of perseverance dominated in teachers' discussions. Group representations also surfaced but were less common. From Session 5 onwards, group representations dominated but individual representations of perseverance persisted and were sometimes common, notably in the post-lesson discussion following Research Lesson 1.

Figure 4-2 supports analysis of patterns in participants' talk at the group level. Notably, it represents counts at the level of micro-activity. As a unit of analysis, coding discourse at the micro-activity level supports holistic analysis of group conversation. It allows us to see patterns in the shape of conversation among the collective, including non-teacher participants as well as teacher participants. But it does not support analysis of individual teachers' talk. Figure 4-3, shown below, supplements Figure 4-2 by disaggregating use of representations by individual teachers. The four teachers drew on individual and group representations differently. Nonetheless, among all four teachers, uses of individual representations fell and uses of group representations grew over time. Figure 4-3 disaggregates the counts presented in Figure 4-2 by

teacher to show how often each teacher contributed to individual or group representations of perseverance that were raised in micro-activities.

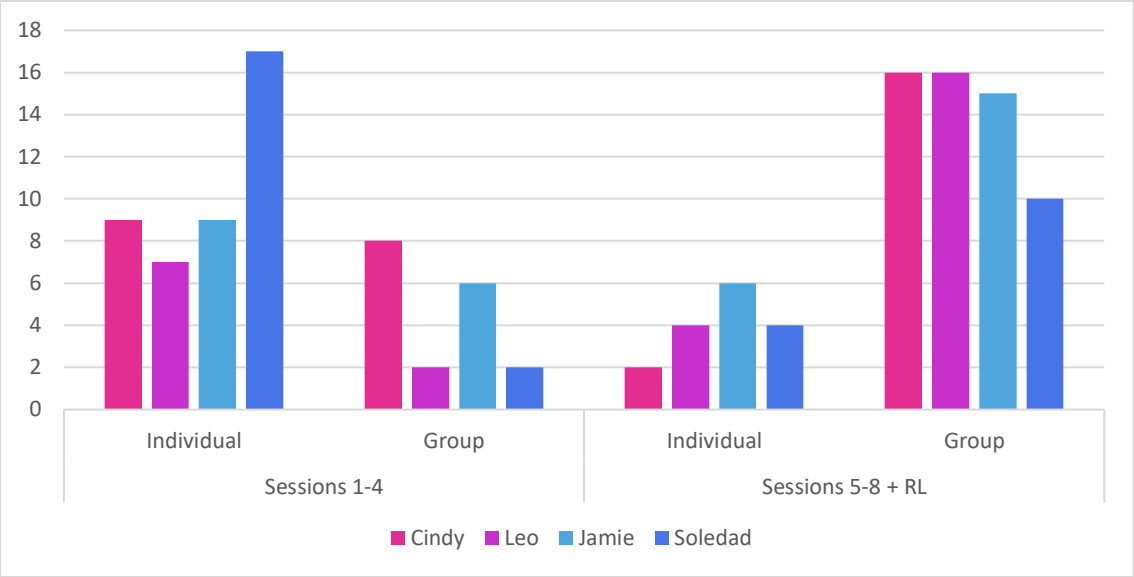


Figure 4-3: Each teachers' use of individual and group representations of perseverance over time.

As shown in Figure 4-3, the teachers used individual and group representations of perseverance differently. Uses of individual and group representations of perseverance were more variable among the teachers in Sessions 1-4. In particular, Soledad drew on individual representations almost twice as often as her colleagues. Soledad and Leo drew on group representations only twice across the first four sessions, while Cindy and Jamie drew on group representations eight and six times, respectively. Nonetheless, all four teachers used fewer individual and more group representations of perseverance after Session 4. Thus, over time discourse around the relationship between interaction and perseverance shifted for the collective and for individual teachers. The two patterns identified through analysis of quantitative evidence—that individual and group representations were used throughout but group representations were used more often over time—hold for the collective and for individual teachers.

Figure 4-4 provides quantitative evidence of the use of individual and group representations of perseverance by another central participant, the facilitator. Figure 4-4 shows the number of times that Elizabeth made a contribution to the conversation that explicitly drew on an individual or group representation of perseverance. Figure 4-4 also shows the number of times that Elizabeth talked about perseverance but did not explicitly reference an individual or a group representation. Counts in Figure 4-4 are broken down as in Figure 4-3, focusing on Sessions 1-4 and Sessions 5-8 + RL separately.

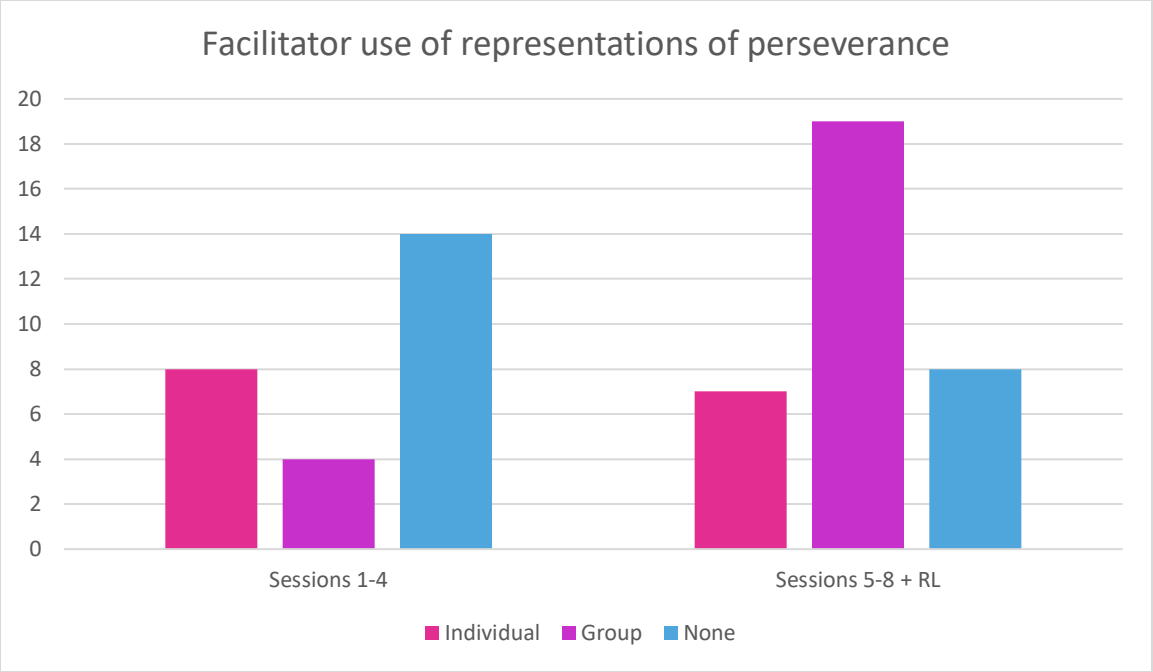


Figure 4-4: Counts of the facilitator's use of individual and group representations of perseverance in the two phases, and counts of the facilitator's references to perseverance without explicit representation.

As shown in Figure 4-4, Elizabeth also tended to use individual representations more often than group representations in Sessions 1-4, and group representations more often than individual representations in Sessions 5-8 and the research lesson. Furthermore, Figure 4-4 shows that in Sessions 1-4, Elizabeth most often talked about perseverance without explicitly drawing on either representation. 14 of Elizabeth's 26 contributions that dealt with perseverance in Sessions 1-4 did not explicitly draw on either representation of the role of interpersonal interaction in perseverance. However, in the later sessions, Elizabeth overwhelmingly used group representations of perseverance, 19 times out of 34 total contributions. Thus, as Figure 4-4 indicates, Elizabeth's participation largely mirrored the participation of the teachers. Furthermore, in the early sessions, Elizabeth seemingly did not push representations of perseverance as a group accomplishment. Elizabeth often did not play a role of spokesperson for discourses of ambitious mathematics instruction in these early sessions. Her language largely did not contain representations indicative of either traditional or ambitious discourses; but when it did, individual representations drawn from traditional discourses were more prevalent.

Patterns highlighted by the relationship between quantitative evidence in Figures 4-3 and 4-4 indicate that the low use of group representations of perseverance in Sessions 1-4 was driven by *all* participants, not just particular teachers or even the teachers as a block. Similarly, increased use of group representations of perseverance in Sessions 5-8 and the research lesson was driven by all participants.

To capture the switch in which representations were more plentiful—individual or group—I refer to Sessions 1 through 4 as "Phase 1" and Sessions 5 through 8, including Research Lesson 1, as "Phase 2," shown in Figure 4-5.

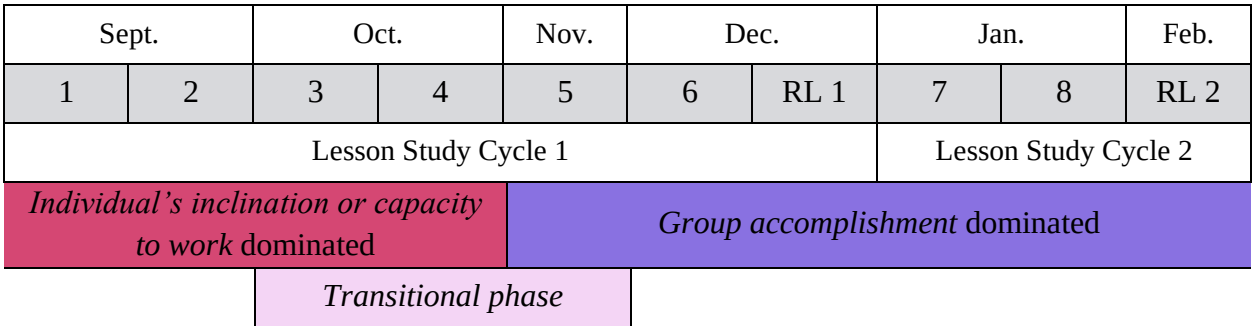


Figure 4-5: Phase 1 and Phase 2 in the first two lesson study cycles.

The language of Phase 1 and Phase 2 highlights that over time, the participants took up new representations of perseverance, and many of these represented perseverance as a group accomplishment. Sessions 3-5 mark a transitional phase, as participants' talk moved towards more group representations. However, it is important to remember that teachers' uses of individual and group representations of perseverance were variable within and across the phases. While group representations overtook individual representations in Phase 2, both were used in both phases. Chapter 6 examines the persistence of representations of perseverance as individual students' capacity or choice to work.

Four cases of individual and group representations of perseverance in teachers' collaborative discourse

The four cases presented here illustrate the broad-strokes longitudinal patterns presented in the preceding section. They characterize how the participants, including teachers and facilitator, leveraged individual and group representations in the different "phases" of their collaborative work around the problem of practice, "How do we build student perseverance and capacity to struggle productively, together?" They also point to the variability in how the teachers represented perseverance, even as they took up more group representations in later sessions. Finally, they begin to show how interactions among participants and with PD activities and resources facilitated a shift towards group representations of perseverance, a line of analysis pursued extensively in Chapter 5. Analysis of these cases pays particular attention to the role of the facilitator and TRU PD resources, including the TRU Dimensions and other TRU tools, in the patterns characterized in Part 2. The TRU Dimensions and accompanying tools had the potential to serve as representatives of discourses of ambitious mathematics instruction, available for the teachers to leverage as they made sense of their students' behavior and planned for changes in classroom practice. The facilitator also had the potential to serve a similar role. However, longitudinal analysis from Part 2 indicated that she may not have served this role throughout. Analysis in this part unpacks her participation across several cases.

Four findings emerged from analysis of the teachers' uses of individual and group representations over time:

1. In the beginning of their collaborative work, the teachers used individual and group representations interchangeably, without explicit discussion of the tension between those representations. Over time, however, the teachers reasoned away from individual representations in favor of group representations.
2. Over time, group representations of perseverance became salient to explain cases of successful student perseverance.
3. Over time, the teachers came to conclude that the classroom activity of students' building on each other's thinking was important to perseverance, and that teachers should work to support that activity.
4. While facilitator actions and TRU tools at times supported the teachers to use group representations of perseverance, they did not do so consistently. But the teachers' collaborative discourse incorporated more group representations of perseverance over time nonetheless.

The four cases analyzed in this section address these four findings. Figure 4-6 maps the four cases across the sessions analyzed.

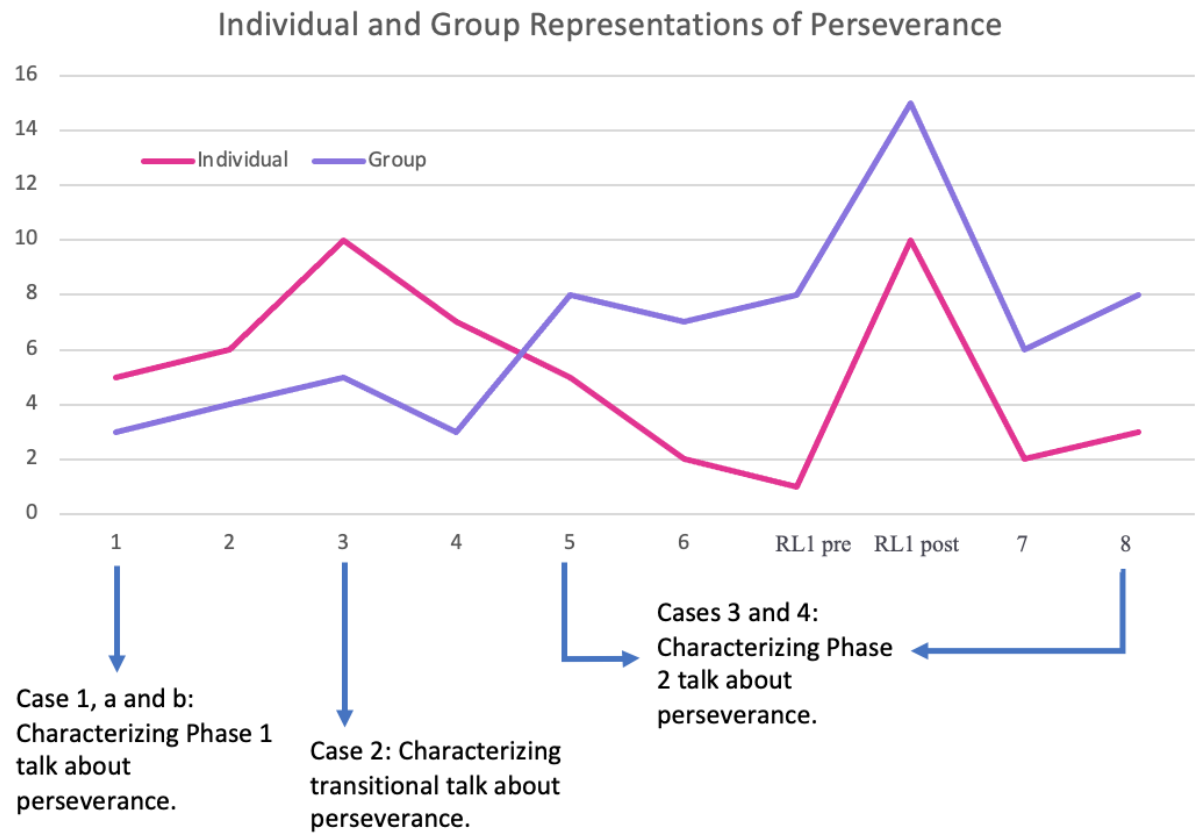


Figure 4-6: Distribution of the four cases.

As shown in Figure 4-6, the four cases provided in this part of this chapter are distributed across the eight workgroup sessions and research lesson. Considered individually, they

characterize Phase 1, transitional, and Phase 2 talk about perseverance, as well as the variability in how teachers represented perseverance. As a set, they highlight the slow, difficult work done by the participants to shift their talk about perseverance towards representations characteristic of discourses of ambitious mathematics instruction, even as traditional discourses retained a significant hold.

Case 1, a and b: Leaning on individual representations even as group representations appear

In Phase 1, the teachers leaned heavily on individual representations of perseverance but also used group representations at times (as shown in Figure 4-7). Case 1, which is divided into parts a and b, from Session 1 highlights this.

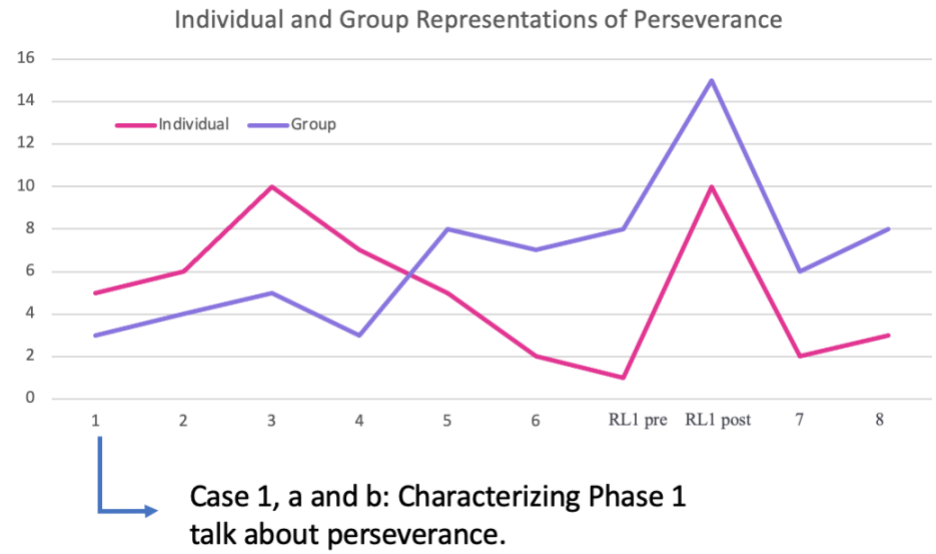


Figure 4-7: Situating Case 1 in the longitudinal trend

Case 1 comes from the first session, in which the participants worked together to articulate the problem of practice that they would pursue over two research lessons, "How do we build student perseverance and capacity to struggle productively, together?" As such, it provides insight into how perseverance and the role of interpersonal interaction in perseverance first surfaced in the participants' talk. As discussed in Chapter 3, perseverance had grown into an important topic in the Benton teachers' collaborative work at the end of the previous school year. Going into this first session of the 2017-18 school year, then, perseverance was on the minds of the participants, but it was not a given that the teachers would return to perseverance as the focus of their collaborative work.

Case 1a draws particular attention to how the teachers tended to use individual representations to make sense of students whose behavior they felt did not show perseverance. This pattern in the teachers' use of individual representations to explain "deficit" behavior is

further analyzed in Chapter 6. Case 1b shows how the teachers used individual and group representations without attending to the tension inherent in these disparate ways of seeing students.

Case 1a: Cindy and Soledad recall challenges from the first weeks of school in Session 1

All four teachers, Elizabeth, and Anna were present for Session 1. Elizabeth launched the first session by asking the teachers to share two successes and one challenge or question from their first few weeks of school. She asked the teachers to make connections between their successes and challenges and the TRU Dimensions by naming a dimension associated with their success or challenge. The teachers had not yet defined perseverance as the focus of their lesson study work. Elizabeth did not mention perseverance in her launch. Nonetheless, perseverance surfaced, in the form of engagement, during this recall-based activity (Horn, 2005) when the teachers shared challenges they were facing in their classrooms. In the early sessions in particular, the participants used the words "perseverance" and "engagement" interchangeably.

Cindy was the first teacher to share. After sharing her two successes (one, that she felt she had a better understanding of the curriculum this year, and two, that she was proud of an activity she had planned) she shared a question:

Cindy: So! One challenge. Um. How can I reengage students who are quickly disengaging? Particularly those who don't like math or those who are repeating the math class? So that's AOI right there. Agency, Ownership, and Identity.

Cindy framed her challenge as one of students who "don't like math or… who are repeating the math class" opting out of engaging with classwork. Cindy's challenge represented lack of engagement as an individual student's choice. She implicitly tied the choice to disengage to a lack of mathematical ability via her mention of students who were repeating the class because they had failed it the previous year. She named the AOI Dimension of the TRU Framework, but did not elaborate further on this challenge or its connection to AOI. Nor was she pressed to do so.

Soledad shared next. Again after sharing two successes (one, that her students had remembered the uses of a geometric compass after she taught them a chant, and two, that a class of students had successfully evaluated each other's work during a gallery walk), she shared a challenge. She explicitly framed her challenge as building on Cindy's. Her challenge also leveraged individual representations of perseverance.

Soledad: Um. A question, a challenge is I think mine's similar to um Cindy's is um I have some repeat students. And some of them are like, "Pshhh, I am not gonna get a D and F again." So they're just like self-motivated in that sense. But there are some who are not. And I don't know. I'm doing it a lot different than last year and some

> of them didn't have me last year for first semester. But I guess, you know, had not passed it the first time with any other Geo teacher. Um. But yeah, how to keep repeat students engaged when they feel like, oh I kind of done material like this before.

Soledad elaborated on Cindy's reference to "repeat students" as potentially choosing not to engage in math class. She added reasoning that "repeat students" might disengage because they felt that they had already learned the mathematics in the class that they were taking for a second time. Tied to that reasoning, Soledad described feeling an obligation to do something to keep these students engaged. Soledad's challenge did not represent "repeat students" disengagement as connected to a lack of mathematical ability, as Cindy's implicitly had. However, Soledad described some "repeat students" as "self-motivated," and some as not. In this way, Soledad explicitly represented successful engagement as the result of individual character and implicitly represented disengagement similarly.

When Elizabeth asked Soledad to connect her challenge to the TRU Dimensions, she added:

> Soledad: I mean I guess it could be any of them [the TRU Dimensions] because if I give them a different way to access mathematical content—access to content, um, it might engage them. Right? Um. Umm. Uh but. I think like I could really quickly see it fall under Agency and Ownership? Um. I'm still trying to see which of my students identify, how they how they-- I kind of think it's probably most likely in AOI.

Soledad reached beyond individual representations of perseverance when she suggested that how she presented the course material might help engage her unmotivated repeat students. However, she backtracked when she suggested that disengagement related more to "how they [the repeat students]" "identify" than to her attempts to broaden access to the course content. In her representation, how students identify comes across as an individual choice.

Discussion of Case 1a

In these recalls, at the very beginning of the first session of the teachers' collaborative work, Cindy and Soledad represented the lack of perseverance as the result of individual attributes of students. Cindy's representation lacked complexity, largely because she did not elaborate and was not pressed to do so. Soledad's elaboration on Cindy's challenge added complexity to the representation. Soledad suggested that she as a teacher was responsible for attempting to engage "repeat students" who might be inclined to disengage, adding that how she gave these students "access to content" could support them to engage productively. However, Soledad returned to representations of perseverance as individual ability and choice at the end of her final turn. Both teachers indicated that they were at a loss for what to do to help students who

they explicitly and implicitly represented as lacking individual ability to persevere or as making choices to not persevere as individuals, without reference to classroom context.

Cindy and Soledad's reasoning in this case indicated that individual representations of perseverance were not supporting them to individually or collaboratively build changes their instruction and classroom environments to better support students to persevere. The teachers were not supposed to brainstorm interventions, as the conversation was framed as one in which teachers could share "challenges." However, Cindy and Soledad's focus on "repeat students" individual deficits and framing of their struggles with repeat students as challenges for which they could not yet think of solutions suggests a relationship between Cindy and Soledad's representation of the problem as lodged in individual students and lack of instructional, classroom-based solution. These students who may not have been well-served by the curriculum and classroom environment in previous years would seemingly continue to encounter learning environments unsupportive of their productive struggle with mathematics.

Both Cindy and Soledad drew on the Agency, Ownership, and Identity dimension of the TRU-Framework to situate their individual representations of perseverance. There was not enough detail in their references to the AOI dimension to justify robust claims about their understanding of the dimension. However, their references to AOI seem to be misunderstandings of the dimension. As discussed in Chapter 3, the TRU Dimensions do not attend to features of individual students. Rather, all of the TRU Dimensions, including AOI, ask what opportunities students are given to have robust mathematical experiences. The AOI dimension focuses on opportunities students are given to build agency, ownership, and positive mathematical identities. Neither the facilitator nor PD materials available to the participants pushed against Cindy and Soledad's non-normative uses of the TRU Dimensions or the individualist, character-based claims about perseverance that the teachers supported with the dimensions. This case points to a pattern of the teachers' making their own meanings with the TRU Dimensions, albeit non-normatively, to situate and bolster individual representations of perseverance. The teachers also used the TRU Dimensions and accompanying resources to situate group-oriented representations of perseverance, as Case 2 shows. However, teachers' uses of the TRU Dimensions to make individual representations of perseverance are notable. This pattern is taken up further in Chapter 6.

Case 1b: Adding "together" to their research theme at the end of Session 1

The previous case demonstrated how the teachers drew on representations of perseverance as individual ability or choice to work and how those representations inadequately supported the teachers to collaborative shift instruction to better support "repeat students." It came from the first session, during Phase 1, when the teachers drew on individual representations of perseverance more often than they drew on group representations. Nonetheless, by the close of their first session, the teachers had articulated their research theme, "How do we build student perseverance and capacity to struggle productively, *together*?" Their research theme explicitly

connected perseverance and student collaboration around mathematics. This next case demonstrates how the word "together" came to be part of their research theme in Session 1.

After the teachers formulated the research theme without the word "together," and discussion waned, Jamie signaled that he was not ready to join the tacit consensus that arose in the group about it. After saying, "I'm still thinking," and waiting for several seconds, Jamie jumped in with a suggestion. Jamie suggested that they add something to the theme about the importance of collaboration. He recalled that earlier in the conversation, the teachers had discussed the importance of collaboration in their teaching goals. But, he noted, their research theme did not say anything about the relationship between perseverance and collaboration.

Jamie:	Oh, I was just going to say, for the second one, "Building student perseverance," um. I feel like what's missing there that we brought up here is around, is something around, like, promoting that collaborative work.
Anna:	Mmm.
Elizabeth:	Mmm.
Soledad:	[Nods]
Jamie:	So it's there, it's just not explicit?
Anna:	Mmhmm.
Jamie:	Um. But I feel like each of us in one facet or another mentioned how we can get students to work together better or, um, what it, what it would require or take to promote that kind of collaborative work so that students are hopefully taking ownership as much as possible. So there's, so there's something there that's missing for me that wasn't, that wasn't a big part of our discussion last year but that is already surfacing this year.

In these turns, Jamie claimed that collaborative classwork could support students to take "ownership" over their mathematical learning. He did not elaborate on how working collaboratively supported ownership. Nonetheless, Jamie articulated a connection between collaboration, students' feelings of ownership over mathematics and mathematical sense-making, and perseverance. Jamie implicitly drew on the AOI Dimension of the TRU Framework to make this claim. Jamie's claim was consistent with representing perseverance as a group accomplishment. When students make sense of mathematics together, they can develop agency and ownership over their learning (Engle, 2012; Sengupta-Irving, 2015).

Elizabeth then oriented the group towards working something about collaboration into the statement of their research theme. Elizabeth's suggested phrasing positioned collaboration or group work as the goal of students' perseverance. Elizabeth and Anna then prompted the teachers to discuss why students might persevere.

Elizabeth:	Maybe something like, "Building student perseverance and the capacity to struggle productively, to engage in group work?" Or, "to…" [Shrugs]

Jamie: That's a good question.
Anna: Yeah, I'm hearing something like a goal, or like a why.
Elizabeth: Mmhmm.
Anna: Like a, why are they doing that? Or.
Jamie: Why are they doing what?
Anna: Why are they persevering? Or like.
Elizabeth: Persevering to do what?
Anna: Mmhmm!
Jamie: Hmm.
Anna: Or. I don't know.
Jamie: It's not formed yet. [Laughs]
Anna: Yeah, that's fine.
Elizabeth: Maybe we just add something around working together.

In this exchange, Elizabeth attempted to incorporate "group work" into the research theme as the goal of perseverance and Anna prompted the teachers to consider why students are persevering. However, the teachers did not respond to their questions or take up Elizabeth's suggested phrasing. Jamie's response indicated that he did not know how to answer their questions.

Soledad suggested a way to add collaboration to the research theme that implicitly defined perseverance as students' capacity to collaboratively struggle productively. Her suggestion was taken up by the group.

Soledad: Maybe it's like the capacity to struggle productively and collaborate. Is that what it is?
Cindy: Struggle collaboratively. [Laughs]
Elizabeth: Collaboratively productively. [Laughter] Productively collaboratively.
Cindy: Struggle together, maybe. Or.
Anna: [Adds "together" to the end of the provisional research theme written on the board, "How do we build student perseverance and capacity to struggle productively?"]

Soledad's suggestion, "Maybe it's like the capacity to struggle productively and collaborate," implicitly defined perseverance as collaborative productive struggle. Her wording explicitly represented perseverance as a group accomplishment. Anna's addition of Cindy's concise and compelling wording, "struggle together," to their research theme closed the conversation. With three of the four teachers contributing to the discussion of whether and how to add wording around collaboration to the research theme, the group reached near-consensus around the importance of group work and collaboration to students' perseverance.

Discussion of Case 1b

Coming at the end of Session 1, the explicit addition of wording about collaboration and group work to their research theme indicated that perseverance as a group accomplishment was a part of the teachers' discourse around perseverance and mathematics instruction. The teachers did not discuss the connection between collaboration and perseverance in much detail. Adding the word "together" to their research theme was primarily a token of their commitment to providing opportunities for students to work together, only slightly tethered to their perseverance goals. Nonetheless, it signaled from the very first session the teachers' engagement with both individual and group representations of perseverance. That these representations were in tension did not surface in this session. Notably, both Cindy and Soledad contributed to adding the word "together" to the research theme, even though they had leveraged individual representations earlier in the session.

Case 1b provides a contrast with Case 1a not only for the teachers' use of group representations of perseverance but for the role of the facilitator and PD resources in how group representations were developed and shared. Jamie leveraged a normative interpretation of the AOI Dimension to support his reasoning about the relationship between collaborative work, students' feelings of ownership, and perseverance. Jamie introduced collaboration into the conversation without prompting from the facilitator. However, once collaboration was part of the teachers' discourse around perseverance, Elizabeth encouraged it. She facilitated a group discussion about how to add language about collaboration into the group's research theme. Elizabeth's participation in this case built on the teachers' own reasoning about perseverance, encouraging them to move towards discourses of ambitious mathematics instruction but only once they showed inclination. Considered alongside the quantitative evidence presented in Part 2 of Elizabeth's use of the two representations, this case highlights Elizabeth's stance of mirroring the teachers' discourse rather than pointedly introducing new discourse.

Discussion of Case 1

Case 1 characterized the participants' uses of individual and group representations of perseverance in what has been called Phase 1, the series of sessions in which individual representations predominated. Two findings emerge from this analysis.

The teachers were comfortable toggling between seemingly contradictory representations of perseverance.

As Case 1a highlighted, participants leveraged individual representations to make sense of students' behavior, particularly non-perseverant behavior, and represented it as reflective of individual students' character deficits. Individual representations did not support participants to collaboratively move towards changes in classroom practice. At the same time, Case 1 pointed to the variability in participants' representations. In Case 1b, the teachers added the word

"together" to their research theme, signaling a commitment to building students' collaborative perseverance even though no one pointed to the tension between adding "together" to the research theme and conversation earlier in the session. Elizabeth facilitated this work, even as quantitative evidence in Part 2 showed that she tended to use individual representations over group during Phase 1, when she used an explicit representation at all. Teachers drew on the TRU Dimensions when using individual and group representations. Neither the PD materials themselves nor facilitation pushed back against non-normative uses of the TRU Dimensions.

Thus, Case 1 shows that while conversation in Phase 1 was characterized by individual representations, the teachers' collaborative work was complex and variable. Teachers comfortably shifted between inconsistent representations of perseverance. Facilitation and PD materials provided mixed affordances for the teachers to move towards discourses of ambitious mathematics instruction.

Case 2: Coming to use group representations of perseverance to make sense of students' behavior and plan classroom interventions in Session 3

In Sessions 3-5, the transitional phases, the teachers increasingly used more group representations of perseverance and fewer individual representations of perseverance. Case 2 from Session 3 (positioned in the quantitative evidence in Figure 4-8) shows a slice of this transition.

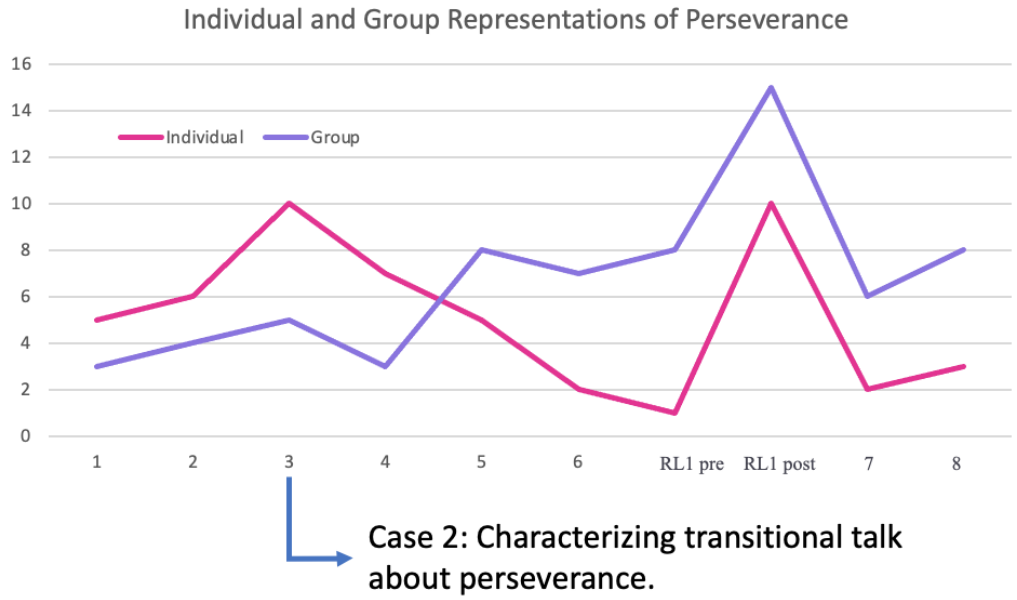


Figure 4-8: Situating Case 2 in the longitudinal analysis.

As shown in Figure 4-8, Case 2 comes from the third session, when the teachers were deep into their collaborative work around supporting students' perseverance. By Session 3, the

teachers had completed two cycles of trying a shared intervention aimed at supporting students' perseverance in their classrooms. They had also watched video together from one of their classrooms and used the TRU Dimensions to support their sense-making of students' perseverance in the video. Thus by Session 3, the teachers had experience with the activities of TRU-LS PD and discussing perseverance with each other.

Case 2 highlights how during the three sessions characterized as the "transitional phase," the teachers explicitly attended to how individual and group representations of perseverance support different interpretations of students' behavior, the former supporting character-based assessments and the latter supporting interactional assessments. Case 2 also draws attention to how frameworks for ambitious mathematics instruction used in the PD, namely the TRU Framework and the perspective of CI pedagogy, supported the teachers to build and use new, group-oriented representations of perseverance.

Case 2: Collaboratively making sense of video from Leo's classroom in Session 3

This case comes from the middle of Session 3, when the teachers collaboratively watched a segment of video from Leo's Math Analysis class. All four teachers, Elizabeth, Anna, and Matt were present.

Overview of Session 3 discussion before the selected case

Before watching Leo's video together, the teachers had shared what happened in their classrooms over the previous two weeks when they tried an intervention they hoped would support their students' perseverance. Conversation during this sharing activity was characterized by heavy use of individual representations of perseverance. For example, at the end of the sharing activity, Jamie had presented two samples of student work from two different classes, one of which he claimed reflected perseverance and the other of which he claimed reflected lack of perseverance. The "perseverant" work consisted of a completed assignment, while the contrasting work consisted of a mostly blank poster. In describing the classroom events that led to the production of these two student work samples, Jamie commented that groups with "an academically strong student" made more progress, attributing some groups' perseverance to individual students' talents. He also commented that the students who had produced the largely blank poster had explicitly refused to engage with him when he attempted to use the planned intervention. The students had pushed back on his questioning, telling him, as he paraphrased, "Mr. Rossi, we're simple people. We don't need this kind of thing." Thus in these replays, Jamie used individual representations of perseverance to reason about students' perseverance or lack thereof. Neither Jamie's colleagues nor the facilitator pushed on Jamie's representations.

Video-watching followed this sharing activity.

Elizabeth launches watching Leo's video

Elizabeth launched watching video by reminding the teachers of a video-watching tool they had used during Session 2. This tool was the TRU Observation Guide Student Look-Fors (called the Look-Fors; see Appendix C for the complete tool). The Look-Fors is a strengths-based list of behaviors teachers might see if students are engaged in mathematical reasoning with peers, organized by TRU Dimension. In Session 2, the teachers had chosen three Look-Fors that they thought best described the behavior of "engaged" students. The three Look-Fors they chose are shown in Table 4-1.

Table 4-1: The three Look-Fors used by the teachers in Session 2.

Look-For	TRU Dimension
Explains their thinking, even if somewhat preliminary.	Formative Assessment
Builds on the contributions of others and helps other see or make connections.	Agency, Ownership, Identity
Explains their reasoning processes as well as their answers.	Mathematics

As shown in Table 4-1, the three Look-Fors nominated by the teachers in Session 2 represented engagement, and implicitly perseverance, as interactive. The first and third Look-Fors they selected, from the Formative Assessment and Mathematics dimensions, describe how engaged students explain their thinking. These Look-Fors do not make it explicit to whom students explain their thinking, whether to the teacher or to other students. The second Look-For, however, explicitly represents engagement as involving student-to-student interaction. It also explicitly represents mathematical knowledge as being co-constructed by students as they build on the contributions of others and help other students make important mathematical connections.

In Session 2, the teachers did not explain why they chose these Look-Fors, nor were they pressed to do so. In her launch of video-watching in Session 3, Elizabeth reminded the teachers of these Look-Fors and encouraged them to reflect on them while watching the video from Leo's classroom.

Leo's video was short, only two-and-a-half minutes long. In it, Leo helped a student, Eden, with a problem about using the vertex formula to find the vertex of a parabola. As the video progressed, Eden voiced her confusion but struggled to explain her thinking. By asking Eden questions, Leo gained a clear enough idea of the source of her confusion—not knowing how to use the formula—to walk her through a relevant example problem. Eden continued to ask questions as he explained until she let him know that she understood. (A full vignette of the video is provided in Appendix A.)

After a few minutes of small-group discussion, Elizabeth launched a whole-group discussion about the video. Elizabeth's launch was simple, with no explicit reference to perseverance, let alone individual or group representations: "I'm interested to see which each group, like, focused your conversations on." Thus Elizabeth's launch invited any interpretations of the video and did not orient the teachers towards particular themes or representations. Her launch also did not remind the teachers of the Look-Fors.

Jamie responds to Elizabeth's launch with a group-oriented representation

Jamie shared first for himself, Cindy, and Leo. He shared that he and his groupmates had noticed Eden building on Leo's ideas, an example of the AOI Look-For, "Builds on the contributions of others and helps others see or make connections."

Jamie: Uh, so in our conversation, Cindy pointed out that it felt like Eden in the video was really building on Leo as a thought partner? So this, so there's an— there's a moment here for AOI in which he's uh she's building on the contributions of others, right? So like every question he's asking is prompting her to continue to build her explanation?

Cindy's claim, as shared by Jamie, represented Eden and Leo's interaction as collaborative. To make this representation, Cindy drew on a non-normative use of the Look-For, "Builds on the contributions of others and helps others see or make connections." The notion of building on the contributions of others is intended to describe the affordances of student-student interactions. Cindy expanded it to include students and teachers building on each other's ideas. In this way, Cindy and Jamie created an opportunity to represent what might otherwise be seen as a student's individual perseverance as a group accomplishment between Eden and Leo.

Soledad offers another interpretation

However, Soledad had another interpretation of Eden's interaction with Leo. Speaking for herself, Kyle, and Anna, she wondered whether Eden actually knew the answer but lacked the confidence to share it. Soledad wondered whether Eden had asked for Leo's help to get him to confirm her answer. She expanded on her claim when prompted by Elizabeth.

Soledad: We— we came up with a question on, does Eden actually know it, but she wanted affirmation?
Jamie: Mmm.
Soledad: And is that why she was asking Leo and then waiting til she was getting bits from Leo to go on with the next step. Was it like a worry about being vulnerable about her, about what she held? You know?
Jamie: Mmhmm.
Soledad: Or was it that, was it that her, like, thinking there was still very partial. And so she, um, she like was really wanting or needing, uh, the bits and pushes from Leo.
Elizabeth: What made you think that that might be a possibility?

Soledad: Because I think that lots of our students, um, lack confidence in their math ability? Um. Or, maybe, and maybe some don't lack confidence but some are for some reason, like, reticent to share? Um. I mean it could be either way, but for some reason there's something that keeps them from, like, sharing completely what's like, what they're what they're thinking and what they have? Um. But some I do think, like, have, have some ideas and are still putting together the puzzle pieces.

Jamie: Mmhmm.

Soledad drew on individual representations of perseverance in her alternative interpretation of the video. She suggested that Eden actually knew how to solve the problem, but lacked the confidence to share her answer. In Soledad's telling, Eden's lack of confidence made her "reticent to share." This latter part of Soledad's claim was drawn out in response to prompting by Elizabeth for further explanation. In this representation, Soledad attributed what she saw as Eden's lack of perseverance—choosing not to share her thinking—to Eden's lack of confidence, a character trait. Soledad framed her claim a challenge to Jamie and Cindy's, asking (emphasis mine), "does Eden *actually* know it…?"

Elizabeth then opened the reasoning that Soledad had begun to the rest of the group, asking, "What do you guys think?"

Leo and Jamie respond to Soledad's claim with claims about Eden's character

Both Leo and Jamie indicated that they would like to respond. Jamie stepped back to make room for Leo. Leo, and then Jamie, rejected Soledad's hypothesis. However, Leo and Jamie drew on their personal knowledge of Eden's character to argue against Soledad's claim.

Leo: Well, I personally think that Eden has no issue with being wrong. She, like, voices her opinions. And I think she was just really confused.

Elizabeth: Mmhmm.

Jamie: Although I will say knowing Eden, I was surprised she wasn't willing to at least formulate something. Because she usually in my class last year was like, even even at the most, like, "I-duh-know [shrugs], this is way off, I'm confused as hell," she was always willing to offer something. And eager. Um. When focused. So, I was surprised that it didn't seem like she was explaining her reasoning process or explaining her thinking? Um. And saying, like, "I can't— I can't say it, I can show it, I can't say it, I can show it."

In this exchange, Jamie and Leo used evidence that only they were privy to in order to build a character-based argument against Soledad's claim that Eden was reticent to share her

thinking. Jamie and Leo's claims were meant to push back against Soledad's, but they perpetuated the individual representations of perseverance that Soledad used.

Cindy responds to Soledad's claim with a group representation of perseverance

Cindy followed up on Leo and Jamie's claims. She drew on evidence of Eden and Leo's interaction that she observed when watching the video to continue to counter Soledad's claim.

Cindy: Um, from that again, like, like I haven't taught her, I just know her from passing periods and other places. But, like, I e-even I would side with folks who've taught her, like, I don't, I don't think she was try— she knows it and trying to get affirmation because if she really wasn't able to complete any of her sentences in the beginning? And yet, like, after some prompting or some questioning, like, I— I was just saying in this group, some students would be like, "Oh, ok," and then they'd move on to their work. And she actually kept him there a little longer, and like, "So…" And then she either questioned, give some questions to get confirmation or give some statements to get confirmation on what she just discovered?

In her turn, Cindy leveraged representations of perseverance as a group accomplishment to explicitly counter Soledad's individualist interpretation of Eden's perseverance. Cindy drew out a contrast between the interaction she observed in the video, in which Eden engaged with Leo's explanations, and her own experience of students listening to her explanations without interacting with them. In doing so, Cindy highlighted the role of interaction, rather than individual character, in what the teachers' characterized as Eden's perseverance. Cindy framed her reasoning as building on Leo and Jamie's. However, Cindy's was of a different character. Where Leo and Jamie drew on individualist orientations to argue against Soledad, Cindy used interactional evidence derived from the video.

Soledad offers a summary

Soledad took the next turn. To confirm that she understood Cindy's argument, Soledad suggested a new Look-For from the AOI Dimension that might better describe Eden's behavior.

Soledad: So I think she was, "Asking questions and making suggestions to support analyzing, evaluating, applying and synthesizing mathematical ideas"?

Soledad's suggestion continued to highlight the role of Eden and Leo's interaction in the perseverance that the teachers observed. In sharing this, Soledad indicated that she was convinced by Cindy's argument and had come to see the evidence from the video as Cindy represented it.

Discussion of Case 2

The discussion in this case was characteristic of the transitional phase in the teachers' collaborative work, Sessions 3-5 in which their discourse shifted from largely drawing on individual representations of perseverance to drawing more on group representations of perseverance. Two findings stand out from this analysis.

In contrast with Case 1, in this case the teachers attended to the difference between individual and group representations.

In this case, unlike in Case 1, the teachers explicitly and pivotally attended to the different interpretational power afforded by individual and group representations of perseverance. When Soledad offered her individualist interpretation of Eden's behavior, she framed it as different from Jamie's initial group-oriented interpretation by posing it as a question to the group. In asking her question, Soledad opened up space for an intentional conversation about whether aspects of Eden's character, namely her lack of confidence, were behind the perseverance or lack thereof that the teachers had witnessed in the video—or whether Cindy's interpretation as relayed by Jamie was more appropriate. The intentional conversation led to growth for Soledad. By the end of the conversation, Soledad explicitly rejected her initial character-based interpretation of the video and took up Cindy's interpretation of Eden and Leo persevering together.

TRU tools supported the teachers' reasoning.

In this case, TRU tools provided support for the teachers' reasoning about perseverance as a group accomplishment. Cindy, Jamie, and Soledad drew on the descriptions of interpersonal interactions captured by the Look-Fors to support their claims about Eden and Leo persevering together. The facilitator did not explicitly orient the teachers to discourses of ambitious mathematics instruction beyond prompting them to use the Look-Fors. In the discussion analyzed here, she did not make any explicit references to perseverance or either representation of it. However, her moves pushing the teachers to elaborate on their claims and opening the discussion to differing views afforded deeper reasoning and intentional conversation about different perspectives on perseverance.

Cases 3 and 4: Leveraging group representations of perseverance to organize for change in classroom practice

In Phase 2, the teachers used group representations of perseverance more often than individual representations of perseverance. Even individual teachers who had drawn heavily on

individual representations during Phase 1 frequently turned to group representations. Cases 3 and 4, drawn from Sessions 5 and 8 (represented in Figure 4-9), illustrate this.

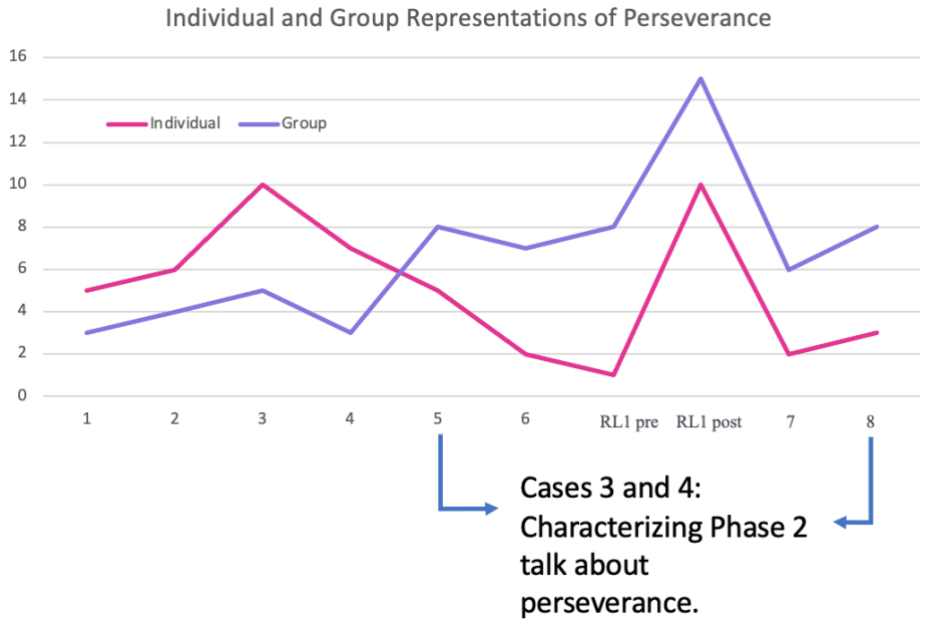


Figure 4-9: Situating Cases 3 and 4 in longitudinal analysis.

Case 3, from Session 5, falls at the end of the transitional period. Case 4, from Session 8, was the teachers' last session before their second research lesson, which marked the end of the sessions analyzed in this dissertation. Cases 3 and 4 in particular demonstrate how the teachers used group representations of perseverance to make sense of students' behavior and then leveraged those interpretations to plan group-oriented teaching interventions. Together, these cases point to how the teachers' increasing uses of group representations afforded potentially powerful changes in their teaching practice.

Case 3: Deepening collective discourse about what it means to persevere "together" while discussing Jamie's video in Session 5

This case comes from the video-watching activity in Session 5. Leo, Cindy, Jamie, Elizabeth, and Anna were present.

Elizabeth launches watching Jamie's video with an emphasis on individual perseverance

Elizabeth launched watching video from Jamie's classroom with explicit attention to perseverance as an individual's ability or choice to work. Her launch referenced the teachers' work in Session 4, which will be analyzed in Chapter 6. In Session 4, the teachers had reached a tentative consensus around trying to support students to persevere as individuals. Their Session 4

conclusions contrasted with the progress they had made towards perseverance as a group accomplishment in Session 3. Elizabeth launched video-watching in Session 5 by explicitly building on the individualist orientations taken up in Session 4. She acknowledged that the students in the video they were about to watch were working as a group, but primarily encouraged the teachers to treat the students in the video as individuals with individual reasons for persevering (or not).

> Elizabeth: And as we're watching this video, we've— we've been working on the Three Things strategy, but we've also recognized that it comes with a lot of — or it doesn't work for everything, right? Like, Three Things we've been working on when the teacher is present, but we're also interested in what happens when kids are individually persevering or indi—like, persevering as a group. So I think that's what we want to be looking at in this video, is like how are— how does each student, each of the four students we're going to see, persevere to get through this problem, or how do they not persevere. And what are the different reasons behind, like what are the root causes of those issues that come up.

Elizabeth represented the situation in the video as "each of the four students… persever(ing) to get through this problem." She encouraged the teachers to interpret the students' actions individually, rather than in interaction. Elizabeth's launch of video-watching in this session was markedly different from her launch in Session 3, as shown in Case 2. She did not direct the teachers to use the TRU Look-Fors, which had supported discussion around perseverance as a group accomplishment in Session 3, and she explicitly prompted the teachers to attend to students' perseverance as individuals.

Jamie's video

Jamie's video was seven minutes long. A full description of the video is provided in Appendix A and a longer description in Chapter 5. In brief, the video shows a group of four students working on a problem about simple, or linearly-growing, interest. The task asked the students to choose from among three graphs one that best represented the interest growth described in a situation about a bank account. Then, the students were asked to write a formula for the growth of the money as described in the situation.

In the beginning of the video, one student, Zakira, asked questions of another student, Mohammed, to try to encourage him to work with her on the graphing question, while the two other students, Omar and Khalid, conversed in Arabic. Zakira tentatively voiced two potential answers. Failing to grab Mohammed's attention, Zakira turned to the videographer, Elizabeth, who redirected Zakira to her groupmates. Hearing Elizabeth's prompt, Omar announced to the whole group his answer to the graphs question. Mohammed immediately agreed with Omar, despite not agreeing with Zakira when she tentatively shared the same answer. Omar then

provided reasoning, which Zakira also agreed with, albeit while complaining that she had had the same idea. All four students wrote down the answer. Then, Omar offered an equation. His equation was incorrect: he provided an exponential equation, when a linear equation was required. However, the other students did not press him on his answer and all wrote it down.

Omar then called to Jamie to check their work. Upon approaching the group, Jamie noticed that the equation was incorrect. Positioning himself near Omar and visually directing his questions at Omar, Jamie asked a series of questions that led Omar to conclude that his graph was incorrect. Mohammed offered Omar a sheet of notes to help him formulate a correct equation. Omar developed the correct equation and Jamie left the group, encouraging Omar to explain his work to his peers. Zakira shared her distrust in Omar's new equation, given that his first attempt had been incorrect. However, Omar explained his work, and eventually all four students wrote it down.

The teachers begin discussing Jamie's video

After watching the clip, the teachers read and organized their notes. Then Elizabeth launched their discussion of the clip:

Elizabeth: What do we notice about each student's level of participation and how they persevered or did not persevere?

Elizabeth's launch again framed their inquiry as focused on individual students. The teachers responded in kind, by describing how individual students participated. They noted that they could not tell how Khalid was participating because he spoke in Arabic throughout the clip. They also noted which students were writing on their worksheets.

Jamie describes a "cute moment"

Finally, they described the moment when Omar switched from talking to Khalid in Arabic to talking with Zakira and Elizabeth in English about the graphing problem. Leo provided a positive assessment of the interaction between Zakira and Omar, without evidence: "He [Omar] provides a good balance to her [Zakira]." Jamie agreed, also without evidence.

Elizabeth then jumped in with a replay of Zakira's attempts early in the video clip at engaging her peers with her reasoning. Although she began her replay by saying, "Yeah," as if she was also agreeing with Leo, Elizabeth's replay tacitly problematized the claim that Omar provided "a good balance" to Zakira.

Elizabeth: Yeah, there's a moment when she's eliminating and I was like, 'Come on, ask her some question about that!' Like, 'Ask her why she's eliminating A!'
Anna: Yeah.

Elizabeth: And, um. But they were just both kind of like so in.

Elizabeth tacitly problematized the claim that Omar was a productive collaborator with Zakira by pointing out that none of the students—Omar included—engaged with Zakira as she attempted to work on the graphing question. However, her problematizing was undercut by her explicit vocal agreement with Leo and Jamie (beginning her replay by saying, "Yeah") and unsubstantiated concluding comment that Omar and Zakira "were just both kind of like so in."

Seemingly not noticing Elizabeth's implicit problematizing, Jamie then offered evidence of Omar and Zakira's "balanced" relationship through a replay of the moment when Omar made his first English-language contribution. In describing that moment, Jamie attended to Zakira's frustration that Omar seemingly restated her idea without giving her credit. However, he did not attend to the problematic nature of the interaction for the students' collaborative perseverance.

Jamie: So Zakira is like, 'Well it's adding the same.' No, Zakira said something like it was going up by the same amount, which is 20. And then, Omar, 'Yeah it's adding the same number every time.' There's that interesting moment where it's like not clear if they're actually listening to each other? And Zakira is like, 'Yeah, I said that.' Like [laughs] 'I already said that.' Sort of like, 'You're not listening to me.' Um. Yeah, and Omar I think was just providing a little more specificity in the explanation? But.

Elizabeth: Mmm.

Jamie: That was kind of a cute moment. Where it's not clear whether they're listening to each other. Or whether Zakira is saying, 'You are not listening to me.'

Jamie interpreted the interaction between Zakira and Omar about the answer to the graphing problem as relatively benign. He called the interaction "cute." Jamie's replay of and claims about the video did not attend to the problematic positioning that unfolded as the students affirmed Omar's answer without probing when they had not afforded the same status to Zakira's identical answer.

Elizabeth pivots the discussion towards perseverance as a group accomplishment with explicit problematizing

However, Elizabeth then drew attention to the problematic nature of this exchange: Zakira asserted the correct answer; no one attended; Omar asserted the correct answer; Mohammed and Zakira immediately agreed.

Elizabeth: Yeah. I was really interested in that moment where, you know, Zakira was like, 'It's B, I think it's B.' Omar said, 'It's B.' And everyone was like

[pounds table], 'Yeah, it's B.' What do we— what do we think around, um, that moment? How students were acting.

In highlighting how the students only agreed with Zakira's answer when Omar said it, Elizabeth tacitly problematized the interaction. While she did not explicitly name perseverance, in the context of the conversation Elizabeth's replay implied that the interactions among the students had implications for the students' perseverance. Elizabeth asked the teachers for their thoughts.

In his reply, Jamie admitted that he had not seen the interaction in that way. Cindy did, however. She responded to Elizabeth's question by characterizing Omar as the "leader" of the group and then problematizing that characterization.

Jamie: I'm not sure I caught that as clearly.

Cindy: Well, I, uh, saw when you [Jamie] were next to them, and, you know, Zakira was like, 'Oh, I'm writing in pen, so you're not sure of this equation? You don't know it?' Like, so, and then, it's like, and then while he [Omar] was talking to you— you [to Elizabeth] you were panning your camera to other students who were writing stuff down, and so, you know, you could see, Omar's kind of the lead of the whole group.

Jamie: Mmhmm.

Cindy: And they, they're all, like, waiting to see what he does

Jamie: Right.

Cindy: or what he writes. And so, um, I see two things— I mean, one is that, yeah, a team needs a leader? I guess. But, also, like, um, what can we do about it? Because right now, we're— we're taking notes and we have two people that we know a lot about, and two who we don't. So is that a spot to interrupt? Or to encourage, like, just I think that's a moment where we could do something.

In this exchange, Cindy explicitly attended to the ways in which the interactions among the students had positioned Omar as having greater mathematical authority than the others. She provided evidence from the video for her claim. Cindy was of two minds about the influence of Omar's outsize authority on the students' collaborative work. She claimed both that "a team needs a leader," although she did not specify why, and that Omar's high status had precluded participation and, implicitly, perseverance by other students. She seemed to come down on the side of Omar's status being problematic, as she called for the teachers to "interrupt" interactions such as those captured in the video. Cindy's use of the word "interrupt" was an implicit nod to CI pedagogy's attention to status among students in group work and the teacher's role in disrupting problematic patterns.

The teachers organize for pedagogical change

In his response to Cindy's replays, claims, and call for action, Jamie laid the burden of "breaking up that status" on himself. He recalled that when he approached the students, he talked mainly to Omar and did not direct his questions to the other students in the group.

> Jamie: Yeah, I noticed that I did not do a great job of breaking that status up there. [Laughs] There was like a conversation with me and Omar. Um. I missed an opportunity to ask any of the others, like, directly.

"Status" is an explicit reference to CI pedagogy. In the conversation that followed, Jamie, Leo, and Cindy, supported by Elizabeth, brainstormed together how a teacher could "break up" problematic status dynamics that undermine students' perseverance as a group. Their conversation about what Jamie could have done in the interaction captured by the video they watched transitioned into the planning activity, in which they agreed to try an intervention intentionally directed at supporting students to listen to and build on each other's thinking, thereby persevering together (analyzed in Chapter 5).

Discussion of Case 3

The conversation in this case marked movement from the transitional phase to Phase 2, in which the participants primarily leveraged group representations of perseverance to make sense of students' behavior. Three findings stand out in this analysis.

Elizabeth played a pivotal role in problematizing the teachers' claims.

One of the most notable features of this case is the variability in Elizabeth's facilitation. Elizabeth launched the video discussion by explicitly prompting the teachers to look for reasons why students did or did not persevere as individuals. Elizabeth's launch built on the teachers' conclusions from Session 4, which will be analyzed in Chapter 6. As such, Elizabeth's facilitation at the beginning of this case can be characterized as responsive: she was mirroring and building on the teachers' own representations of perseverance, despite their inconsistency with the values and goals of the PD. Elizabeth's facilitation at the beginning of this case was consistent with her Phase 1 participation, in which she drew on individual representations more often than group, largely tracking the teachers' participation, as shown in the reproduction of Figure 4-4 below. Elizabeth's launch did not provide opportunities for the teachers to deepen their reasoning about perseverance as a group accomplishment. In this launch, Elizabeth potentially missed an opportunity to problematize the teachers' conclusions from Session 4 and direct them towards ways of noticing student participation in the video that would have deepened their collaborative inquiry.

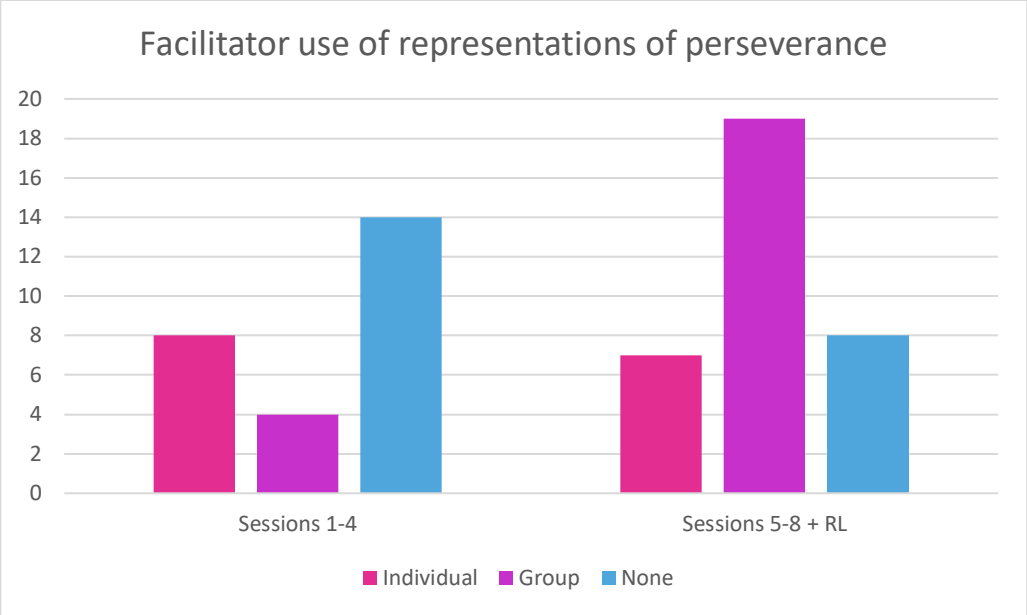


Figure 4-10/4-4: Elizabeth's use of individual and group representations of perseverance.

However, at a pivotal point in the teachers' discussion of Jamie's video, Elizabeth posed a question that drew the teachers' attention to the role of problematic status dynamics in students' collaborative perseverance. Elizabeth's question constituted a new contribution. Prior to her question, the teachers had not problematized the status-laden interaction among Zakira, Omar, and Mohammed that had led Zakira's ideas to be side-lined and Omar's ideas to be taken up by the group without probing. Elizabeth made a subtle first attempt at problematizing Leo and Jamie's as-of-yet-unsubstantiated claim that Omar and Zakira were productive collaborators, but her first attempt went unnoticed by the teachers. Then, she explicitly problematized the interactions among the students by asking a targeted question. In problematizing these interactions, Elizabeth offered a new interpretation of the students' perseverance, one supported by an implicit representation of perseverance as a group accomplishment. Offering new contributions supported by representations from ambitious instruction discourses was atypical participation for Elizabeth, as shown by the contrast between this move and Elizabeth's participation at the beginning of this case and in Cases 1 and 2. Nonetheless, in this case, her participation as a voice of ambitious mathematics instruction proved pivotal. Jamie and then Cindy revisited their interpretations of the interactions in the video in light of Elizabeth's new representation. Building on this new way of seeing the video, the teachers moved into planning an intervention with potential to support their students to persevere together by building on each other's thinking. In Phase 2, as shown in Figure 4, Elizabeth leaned heavily on group representations of perseverance. Her change in discourse from Phase 1 to Phase 2 thus tracked the teachers' discourse. But, as shown in this case, it also encouraged the teachers to take up group representations of perseverance and use them to make sense of students' behavior.

TRU discourse took a backseat to Complex Instruction.

TRU tools and the TRU Dimensions did not play much of a role in the conversation that unfolded in this case. While TRU tools and the Dimensions supported the teachers to use new, group-oriented representations of perseverance in Case 2, from Session 3, here TRU tools were not used and the dimensions were not mentioned. In fact, across all of Session 5, the TRU Dimensions were only briefly mentioned in one micro-activity, when Elizabeth reminded the teachers of their conclusions from Session 4 before they watched Jamie's video. Instead, the teachers leaned on discourse from CI, which is consistent with and complementary to discourse from the TRU Framework. The roles played in the teachers' collaborative discourse by explicit uses of discourse from the TRU Dimensions and CI will be examined further in Chapter 5.

Jamie and Cindy, two teachers who drew on group representations from the beginning, dominated this conversation.

Case 3 shows two teachers, Jamie and Cindy, using group representations of perseverance to deepen their inquiry into how to support their students to persevere together. Case 3 shows the power of those representations to support teachers to organize to make important changes in their instructional practice. As discussed in Part 2 of this chapter, however, Jamie and Cindy drew on group representations frequently from the beginning of the PD. In contrast, Leo and Soledad did not. However, all four teachers came to increasingly use group representations. See the reproduction of Figure 4-3, below.

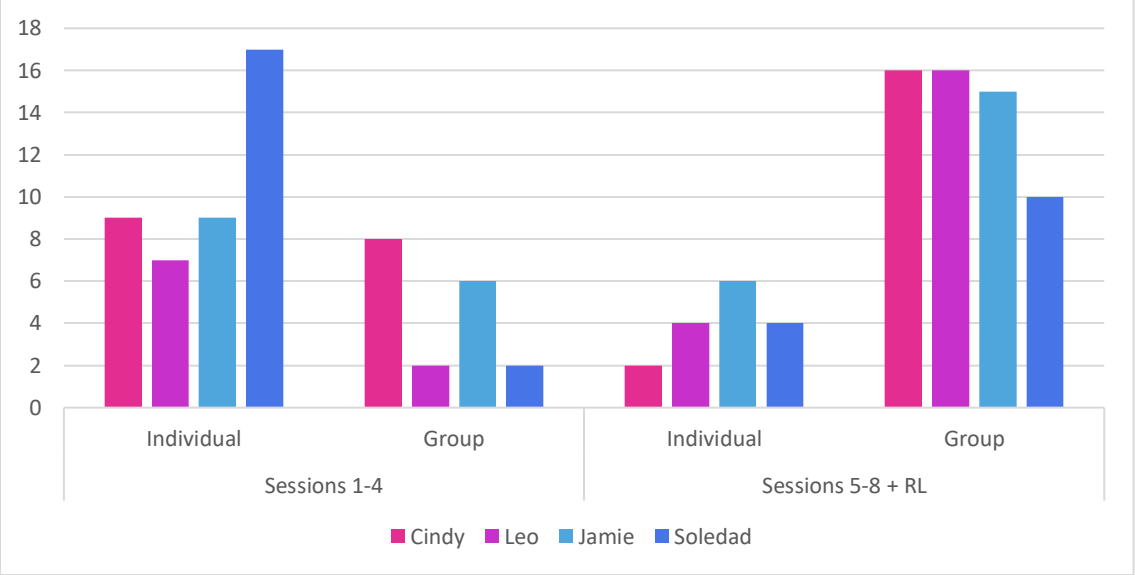


Figure 4-11/4-3: Uses of individual and group representations of perseverance by teacher.

Case 3 does not illustrate Leo or Soledad's use of group representations over time. Case 4 does, however, with a focus on Leo.

Case 4: Leo plans sentence stems to help students persevere by sharing and discussing ideas in Session 8

This case is from the planning activity that closed Session 8. As in many sessions, the planning in Session 8 followed video-watching.

Overview of Session 8 discussion before this case

Immediately preceding this discussion, the teachers had watched and discussed a video from Cindy's class of a group of students working on a mathematics task. In the video, the students had reached a moment in the task when none of them knew how to proceed. The students ventured tentative suggestions until one student noticed a salient piece of information that they were able to use to make more progress. In their discussion of the video, the teachers attended to the tentative nature of the students' interactions. They also attended to the ways in which the structure of the task had supported the students to collaboratively assemble the information they eventually used to continue to make progress.

Leo suggests sentence stems to facilitate confident group conversation

Elizabeth launched the planning activity by encouraging the teachers to reflect on what they saw in the video as they thought of questions to ask students that would help them persevere. Leo made the first suggestion: sentence stems to facilitate more confident and mathematically useful conversation among the students when they were making tentative claims. He used observations from the video they had just watched to justify his suggestion.

Leo: They [the students in the video] were very confident during instruction. There were complete sentences, "This, this, this, this, this." But then when they were making claims, it was sort of like, this little bit, this little bit, waiting for a response. So, not a question so much, but like a stem. Like, "I think blank because blank blank blank blank blank." Or, "This should look like blank because blank blank blank blank blank." Um. So to give— so so that they sound more confident, and to give other students a way to be like, "No, I disagree because this blank says whatever." You know. Just to structure the conversation so that maybe it's fine. And so.

In his replay of the video, Leo noted that when the students were making claims about how to proceed, claims that at times were not correct and could have benefited from discussion among the students, the students did not use complete sentences. Instead, they offered "this little

bit, this little bit" and waited for a response from their peers. Leo's replay attended to interactions among the students, although it did not yet explicitly connect the nature of those interactions to the students' perseverance with the task. Following the replay, Leo suggested giving the students sentence stems to facilitate sharing claims in confident and complete ways. He argued that confident-sounding claims would support other students to build on or disagree with those claims.

Leo elaborates on the importance of confidence for students to build on each other's thinking

Elizabeth then asked Leo to elaborate on how he imagined the sentence stem helping students. Leo elaborated on the importance of sentence stems for helping students build on each other's thinking.

Leo:	It would give them a structure to respond.
Matt:	Sounds like you're concerned that they didn't have the context— these comments were without context, so they weren't--
Leo:	That's part of it. That's part of it. Um. But, like when I think you're wrong, or I think you're right, I don't know how to respond to that. Like especially if I think you're wrong. Cuz then kids won't say that. Well, a lot of kids won't say that. "You're wrong." But if they can say, "Oh, I disagree because the notes say this." Or, "But what about this thing, that contradicts that?" I think they're more likely to respond.

Leo's elaboration tied together his representation of perseverance as a group accomplishment. In this exchange with Elizabeth and Matt, Leo argued that students disagreeing with each other during problem-solving was important for their mathematical sense-making, especially in challenging mathematical situations such as the one the teachers had observed in the video. In Leo's telling, justified with a rehearsal, students struggle productively with challenging mathematics together, in part by building on and challenging each other's tentative thinking. To facilitate the difficult work of sharing and building on tentative thinking, Leo suggested offering the students sentence stems.

Following Leo's elaboration, the teachers verbally agreed to his suggestion and moved on to other potential interventions.

Discussion of Case 4

In this brief planning episode, Leo imagined supporting students to persevere by helping them share and respond to ideas. In doing so, he represented perseverance as a group accomplishment. Two findings stand out in this analysis.

Leo represented both successful and unsuccessful perseverance as a group accomplishment.

In suggesting that the stems could support perseverance by helping students voice disagreement, Leo indicated that the students in the video may have struggled to persevere because their communication did not include enough constructive disagreement. In this way, Leo represented both successful and failed perseverance as group accomplishments. Perseverance failed because students did not constructively respond to each other's potentially flawed thinking. Perseverance could succeed if the students were assisted with these challenging interactions. Leo's representation of failed perseverance as related to unsupported student interactions was a change from how Cindy and Soledad had represented failed perseverance in Case 1. In that case, the teachers had turned to individual representations of perseverance to make sense of students' disengagement. Here, however, Leo looked to group dynamics to explain lack of perseverance, in this case video record of mathematical struggle that the teachers represented as unnecessary and unproductive. Leo also leveraged group representations of perseverance to brainstorm a potential intervention aimed at supporting students to persevere, together. Representing the perseverance of the students in the video as a group accomplishment supported Leo to develop an intervention that had the potential to help students' build on each other's thinking in challenging problem-solving moments.

Notably, Leo's use of group representations of perseverance to make sense of *lack of* perseverance diverged from a pattern that will be examined in Chapter 6, that the teachers tended to use individual representations to explain lack of perseverance throughout their collaborative work.

Leo's discourse in this case represents a shift for him towards perseverance as a group accomplishment.

Leo's rich and generative use of group representations of perseverance in this case showcases how the teachers increasingly turned to group representations of perseverance over time. As shown in Figure 4-12 below, Leo rarely represented perseverance as a group accomplishment in Phase 1, Sessions 1-4.

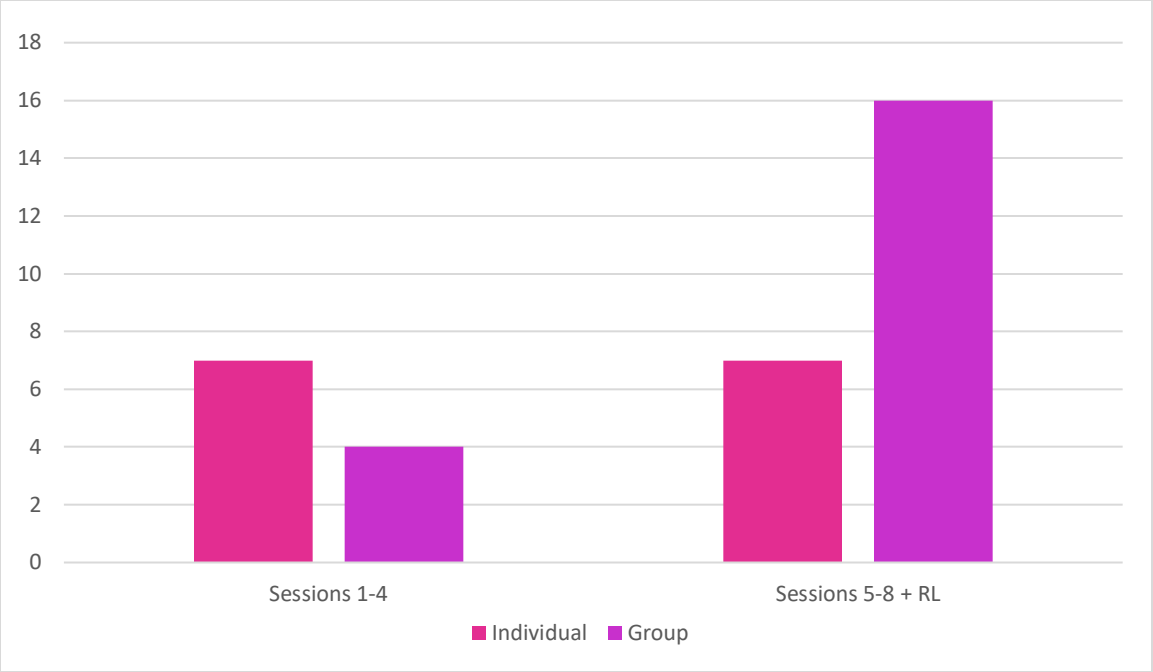


Figure 4-12: Leo's uses of individual and group representations of perseverance across the two phases.

In fact, as shown here in Figure 4-12, he represented perseverance as a group accomplishment only twice across those four sessions, in contrast to Cindy and Jamie's respective eight and six group representations of perseverance in those same four sessions. However, in this case from Session 8, Leo powerfully used representations of perseverance as a group accomplishment to make sense of students' behavior in a video and devise a potentially productive intervention. Leo's increased uptake of group representations of perseverance was part of a pattern among all four teachers and in their collaborative discourse towards using more group representations of perseverance. The generative discourse from Leo displayed in this analysis from Session 8 was co-constructed by the participants over time, as they interacted with one another in the PD.

Discussion of the four cases

The four cases analyzed here, ranging from the very first session to the last, illustrate the patterns in the teachers' uses of individual and group representations over time demonstrated quantitatively in Part 2. Quantitative evidence showed that the teachers increasingly drew on group representations of perseverance in their collaborative discourse. However, quantitative evidence did not, and cannot, show *how* the teachers used group representations, the difficulty of shifting to group representations, or the *power* of those representations to deepen the teachers' inquiry and help them mobilize to make important changes to their classroom instruction. These four cases provide that illustration. The following sections outline key findings from this analysis and their implications.

Teachers can move towards group representations of perseverance, and group representations offer power for their collaborative reasoning about students and pedagogical planning.

Together, these four cases show that over time and with significant work, teachers can increasingly use discourses of ambitious mathematics instruction in their collaborative discourse, and that those discourses offer affordances for their reasoning and planning to change practice towards more powerful mathematics instruction. Case 1a shows how the teachers began their work, tending to use individual representations of perseverance to make sense of their students' behavior, in particular their non-perseverant behavior, with focus on individual students' character deficits. Part of Case 2 similarly shows how the teachers were drawn to individual representations, with particular attention to students' perceived characters, in this case to determine whether or not behavior was perseverant and explain it. In contrast, Cases 2, 3, and 4 show how over time and after problematizing, conflict, and personal vulnerability, the teachers came to use group representations of perseverance to make sense of perseverance. In Cases 2 and 3, the teachers together reasoned about students' behavior with group representations only after a teacher or the facilitator explicitly pointed to conflict or problematized individual representations. Building group representations as a collective and individuals, in the face of pervasive and commonsensical individual representations, thus took work for the participants.

Cases 3 and 4 also show the power afforded to the teachers by group representations. In Case 3, group representations supported Cindy and Jamie to collaboratively reason about the effects of problematic status arrangements among students on their perseverance with math tasks as a group. In both Cases 3 and 4, representing perseverance, or lack thereof, as a group accomplishment empowered the teachers to imagine potential changes they could make to their classroom instruction that would better support students to persevere in a constructive, collaborative learning environment. Thus group representations of perseverance afforded collaborative discourse among the teachers that broadened and complexified what they noticed about students' mathematical learning and opened constructive pathways for collaborative pedagogical change.

But individual representations persisted, with potential consequences for teachers' collaborative inquiry.

These cases also illustrate another pattern highlighted by the quantitative evidence, that individual representations persisted. Sometimes, as in Case 1, the teachers used both individual and group representations without seeming to notice the contradiction between them. In Case 1, the teachers used the TRU Dimensions in non-normative ways to build individual representations without noticing the misfit. Other times, as in Case 2, different interpretations of students' behavior afforded by individual and group representations were explicitly discussed. In Cases 1 and 2, individual representations had the feel of commonsense for the teachers in that they seemed self-evident, especially in contrast with the work required to incorporate group representations into conversation in Cases 2 and 3. Cases 3 and 4 do not illustrate how individual

representations of perseverance persisted in the teachers' discourse beyond what has been called Phase 1. Quantitative evidence showed that they did persist, however. Chapter 6 examines how individual representations continued to hold explanatory power for the teachers.

Nonetheless, with the quantitative evidence analyzed in Part 2, the four cases analyzed in this chapter provide an illustration of how the teachers' collaborative discourse took up group representations of perseverance over time, even as individual representations endured. They also show what that partial but still notable shift in discourse afforded: deeper collective reasoning among the teachers about what supports and inhibits students' perseverance and opportunities for the teachers to plan together to change their classroom practice.

Well-timed problematizing from the facilitator can pivot teachers' collaborative discourse, even amidst other facilitation moves with mixed affordances.

These four cases also illustrate the facilitator's participation. Quantitative evidence showed a pattern of the facilitator mirroring the teachers' discourse about perseverance, drawing more heavily on individual representations in earlier sessions and more heavily on group representations in later sessions. These cases illustrate that mirroring for what has been called Phase 1, with Cases 1, 2, and 3 demonstrating how Elizabeth followed the teachers' lead when she posed questions about perseverance. In those cases, Elizabeth tended to use individual and group representations when the teachers did.

However, Case 3 showed that what in quantitative evidence might look like more mirroring during Phase 2 was actually the facilitator driving the conversation towards group representations of perseverance. In Case 3, Elizabeth offered her own group-oriented interpretation of Jamie's video by problematizing status issues that unfolded in the group of students captured by the video *and* the teachers' initial claims about the video, after the teachers had made arguments that downplayed those issues. Case 3 shows how the facilitator was able to introduce group representations of perseverance into the teachers' discourse through problematizing and how the teachers productively took up her introduction. This case demonstrates the power of the facilitator to act as a representative of discourses of ambitious mathematics instruction by problematizing teachers' traditional-discourse-oriented claims about students, teaching, and learning.

That Elizabeth's problematizing move in Case 3 proved productive for collaborative discourse does not imply that more problematizing from facilitators is always better, however. Facilitators face a delicate decision when choosing how, or whether or not, to problematize teachers' reasoning. Problematizing is delicate work. Done with a rough hand, it runs the risk of undercutting hard-won trust among PD participants. Trust among PD participants is essential to building an environment in which collaborative inquiry into teaching practice is possible. Sharing classroom challenges, reasoning about each other's teaching practice, and committing to collective goals runs against typical professional practice in schools, where idiosyncratic, individualistic teaching is the norm (Little, 1982; Lortie, 1975). For this reason, problematizing by the facilitator would have been exceptionally challenging in Case 1a, when the teachers drew

on individual and deficit-oriented representations of perseverance. Case 1a was the first opportunity during the 2017-18 school year that the teachers had to share in the PD what was challenging for them in their classrooms. In that case, Cindy, Soledad, and the other teachers who shared but whose talk was not analyzed in this chapter made themselves vulnerable by sharing challenging classroom experiences. The teachers drew on representations of perseverance that are problematic and would later be less relevant for the teachers as they deepened their collective inquiry. They also relied on non-normative uses of the TRU Dimensions. Nonetheless, explicit problematizing by the facilitator at that early stage could have weakened the teachers' trust by making them feel that their vulnerability was being abused. The fallout from ill-timed or unsubtle problematizing in those moments could have limited Elizabeth's opportunities to productively problematize later.

Elizabeth's individual-oriented launch at the beginning of Case 3 reads as more ill-advised than her lack of problematizing in Case 1a. In Case 3, Elizabeth prompted the teachers to make sense of students' perseverance as shown in Jamie's video as individual accomplishments. Elizabeth's launch built on the teachers' conclusions from the previous session, in which they had named perseverance as something done by individual students. However, Elizabeth eventually productively problematized the individualist stance through her participation in the video discussion that unfolded. Case 3 thus points to the affordances even of imperfect facilitation, facilitation that does not take full advantage of opportunities to problematize and introduce discourses of ambitious mathematics instruction but provides a push in a necessary moment.

Growth-oriented, precise PD tools can facilitate teachers' noticing consistent with ambitious instruction discourses, while vaguely-worded tools have potential to be coopted.

These cases further illustrate how participants used the TRU Dimensions and TRU tools as they reasoned together about their students' perseverance. How participants drew on two discourses of ambitious mathematics instruction prominent in their conversation, discourse associated with the TRU Framework and with Complex Instruction, will be further unpacked in Chapter 5. Case analysis in this chapter, however, showed that the participants sometimes used TRU tools and discourse associated with the TRU Framework and CI to support noticing of perseverance as a group accomplishment. Notably in Case 2, Cindy and Jamie drew on the TRU Look-Fors as they interpreted interaction between Leo and his student, Eden, as them persevering together. Soledad then drew on another TRU Look-For as she made sense of perseverance as a group accomplishment, a pivot for her away from perseverance as part of a student's character. The TRU Look-Fors' descriptions of specific productive and collaborative student behaviors supported the teachers to notice students persevering together and reason about claims collectively. In Case 3, language around status derived from CI supported the teachers' reasoning about the problematic dynamics that unfolded in the video. CI teachers' commitment to actively disrupting status issues in groups of students afforded Jamie's recognition that he could and should have done more to disrupt the dynamics that impeded perseverance among the

students in the video, a recognition that spurred the teachers to collaboratively plan for pedagogical change.

However, this chapter also showed that the teachers sometimes used TRU tools and discourses non-normatively as part of reasoning that represented perseverance as individual capacity and choice. In Case 1a, Cindy and Soledad named TRU Dimensions as they reasoned about claims that "repeat" students' lack of perseverance was based in character and context-independent choices. The teachers' uses of the TRU Dimensions in this case were non-normative. As discussed in Part 1 of this chapter, the TRU Dimensions represent perseverance as a group accomplishment, albeit with less direct emphasis on the importance of student collaboration than other discourses of ambitious mathematics instruction, including CI. Nonetheless, Cindy and Soledad drew on the TRU Dimensions as they built individual representations of perseverance, and none of the other participants pushed back on them.

How teachers drew on the TRU Dimensions as they continued to use individual representations of perseverance will be taken up further in Chapter 6. In this chapter, however, the teachers' normative use of the TRU Look-Fors and non-normative use of the TRU Dimensions is notable. While the Look-Fors provide detailed descriptions of positive student behaviors, the text of the TRU Dimensions is broad and leaves room for interpretation. Neither the Look-Fors nor the TRU Dimensions explicitly discuss perseverance. Analysis of Case 1a in this chapter points to the consequences for teachers' collaborative learning around ambitious mathematics instruction of the lack of specificity around perseverance in discourses of ambitious mathematics instruction. In Case 1a, the teachers took advantage of the broad wording to make their own meaning. That meaning was not aligned with the values underlying the TRU Dimensions, but the TRU tools provided and norms of participation in place in the workgroup at the time did not afford pushing back on the contradiction. As previously noted, the teachers' non-normative uses of the TRU Dimensions to build individual representations of perseverance and lack of pushback described in Case 1a was indicative of more widespread participation. This analysis is a preview of further analysis in Chapter 6.

Conclusion

This chapter drew on quantitative evidence and analysis of cases to describe patterns in how the Benton teachers and PD facilitator used individual and group representations of perseverance over time. It also used case analysis to illustrate the affordances that increased uptake of group representations of perseverance provided for the teachers' collaborative sense-making and planning around how to better support their students to persevere. Analysis in this chapter showed that over time, the teachers drew on representations of perseverance both as individual students' capacity or inclination to work and as a group accomplishment. While teachers used both representations throughout, individual representations were more prevalent in the first four meetings, which I call Phase 1. Over time, the teachers took up new representations of perseverance. In Phase 2, teachers looked more often to interactions among students for the

reasons behind student perseverance. The teachers also planned interventions that could help students work together and rely on each other to build perseverance.

The teachers were variably supported by the facilitator and TRU tools. While the facilitator's participation largely mirrored the teachers, drawing more heavily on individual representations in Phase 1 and using more group representations in Phase 2, case analysis illustrated how mirroring by the facilitator may have built trust among participants while well-timed if infrequent problematizing supported deeper reasoning and new perspectives. While TRU tools at times helped the teachers to notice classroom examples of group perseverance, at other times the teachers assigned their own non-normative meanings to the TRU Dimensions as they constructed individual representations of perseverance to support claims about students' character and choices. Chapters 5 and 6 provide deeper analysis of the role of the PD, including facilitation, TRU tools, and frameworks, in the patterns of representation described in this chapter.

Taking up group representations of perseverance was a meaningful and difficult shift for the Benton teachers. Moving away from representations of perseverance that center students' characters and individual choices, without regard for context, is challenging work. As discussed in Part 1, individual representations of perseverance are part of traditional, taken-for-granted discourses of mathematics instruction. The cases analyzed here show how teachers drew on individual representations from the beginning of their work together and throughout, even going so far as to build non-normative meanings around the TRU Dimensions to continuing using individual representations in a PD environment that did not support them. The teachers moved to group representations through explicit problematizing and vulnerable, challenging discussion. But despite the challenge, over time the teachers' understanding of perseverance shifted towards the understanding supported by discourses of ambitious mathematics instruction. When teachers reason together about students, content, and teaching in ways that problematize status and other problematic group dynamics that lead to inequitable and disempowering learning environments, they have opportunities to collaboratively organize to shift their practice to support more powerful and equitable learning for students. The cases analyzed in this chapter show the Benton teachers doing precisely that—incompletely as traditional discourses continued to pervade their talk, but doing it nonetheless, collaboratively over time. In taking up perseverance as a group accomplishment, the teachers began to shift responsibility for persevering from individual students to the affordances for collaborative work provided by interactions among students and teachers, supported by classroom interactional routines. This redistribution of responsibility supported the teachers to devise and use interventions to support students to persevere, supporting further inquiry into perseverance to and how to encourage it.

Notably, the group-oriented interpretations of students' perseverance that the teachers constructed together attended primarily to the relational dimensions of the classroom environment. The teachers focused on the role of interactions in supporting and inhibiting students' perseverance. They largely did not attend to the role of the task, for instance, or opportunities for productive struggle afforded by students' interactions with each other, teachers,

and tasks in students' perseverance. Furthermore, the teachers returned to individual representations of perseverance even in the midst of developing and using group representations of perseverance throughout their work together. Individual representations retained significant explanatory power for the teachers. In light of the pervasiveness and self-evident-nature of individual representations of perseverance in traditional discourses of mathematics instruction, the persistence of individual representations in these teachers' discourse is not surprising. But it also warrants closer examination. I explore the teachers' return to individual representations further in Chapter 6.

A limitation of this analysis is lack of access to evidence of the teachers' actual change in classroom practice. Quantitative evidence points to a change in collaborative discourse towards ambitious mathematics instruction, and analysis of cases shows that the teachers *planned* together and *committed to* changes to their classroom practice aimed at supporting students to persevere together. However, this workgroup-based evidence does not demonstrate what actually happened in teachers' classrooms. The missing evidence linking workgroup learning to classroom practice and student learning is a limitation of this research. Nonetheless, change in collaborative discourse around students, teaching, and learning, and then planning and collectively committing to changes in classroom practice, are pivotal first steps towards change in classroom practice and richer student experiences.

As shown in this chapter, representations of perseverance as a group accomplishment supported the teachers to inquire into how to support their students to persevere. The shift towards taking up group representations of perseverance made available ways of noticing students' behavior that emphasized the role of interactions in perseverance and empowered the teachers to begin to make changes in their classrooms that had the potential to improve students' opportunities to learn. The teachers' uptake of group representations of perseverance and the affordances of those representations for the teachers' collaborative inquiry and learning remain significant, despite the problematizing to come in Chapter 6.

Chapter 5 Towards Students Persevering "Together," Through PD

The previous chapter examined patterns in how the Benton teachers represented the role of interaction in their students' perseverance in math class. It engaged with the question, *How did the teachers represent the role of interaction in perseverance as they inquired into their problem of practice, over time?* As the teachers inquired into the problem of practice captured by their research theme, "How do we build student perseverance and capacity to struggle productively, together?" they explored the relationship between students' perseverance, or lack thereof, and students' interactions with each other and teachers. Analysis showed that over time, the teachers increasingly used representations of perseverance as a group accomplishment to make sense of observations of student behavior and while planning potential classroom interventions to support perseverance. Representations of perseverance as individual students' ability or choice to work remained salient to the teachers throughout. Nonetheless, individually and as a collective, the teachers increasingly turned to group representations of perseverance to support their inquiry into their problem of practice. The previous chapter gave a window into the power of group representations of perseverance for supporting the teachers' inquiry through four cases. In those cases, teachers could be seen leveraging group representations of perseverance to expand what they noticed in videos of students working on mathematics and to begin to organize around interventions with the potential support perseverance through collaboration and interaction. This tentative yet distinct pattern in the teachers' collaborative discourse towards group representations is notable because group representations of perseverance are consistent with discourses of ambitious mathematics instruction known to support teachers to teach in more mathematically robust and powerful ways (Schoenfeld, 2014).

In this chapter I examine how the teachers' representations of perseverance were shaped by engagement with the PD. I explore the question, *How did engagement with PD activities support teachers to take up different representations of the role of interaction in perseverance, moving towards ambitious mathematics instruction?* Analysis in this chapter considers the roles that interactions between and among participants, activities, and resources played in the ways teachers represented perseverance identified in Chapter 4. I find that iterative cycles of collaborative video-watching and planning interventions, supported by activity structures that encouraged teachers to problematize, build on, and revise their own and each other's ideas, supported the teachers to explore new and more complex representations of perseverance. Collaborative discourse around classroom videos and interventions supported the teachers' to broaden their horizons of observation (Hutchins, 1996), seeing the relationship between interaction and perseverance in new ways that supported them to mobilize for change in their classroom teaching.

This chapter begins by revisiting the theoretical framework for the relationship between PD and workgroup conditions, generative discourse, and teachers' learning outlined in Chapter 2. It then turns to data analysis, first examining how different ways of representing perseverance corresponded with the PD activities in which the teachers engaged, including recalling classroom

events, watching video, and planning interventions. Then, I examine how the PD activities of watching video and planning interventions jointly encouraged the teachers to discuss complex representations of perseverance, increasingly represent perseverance as a group accomplishment, and plan interventions that supported student collaboration. I use a sequence of cases of teachers discussing video and planning interventions to show how interactions among participants and between participants, video representations, and PD resources in these activities broadened the teachers' horizons of observation (Hutchins, 1996) and facilitated the uptake of group representations of perseverance.

Tracing the relationship between PD activities and frameworks, generative discourse, and learning in teacher workgroups

As discussed in Chapter 4, over time the Benton teachers shifted from using individual representations of perseverance more often than group representations of perseverance to the opposite, using group representations more often than individual. This pattern is highlighted by Figure 5-1. Figure 5-1 shows a frequency count of the number of micro-activities (see Chapter 3 for a description of this unit of analysis) in which the participants used individual or group representations of perseverance.

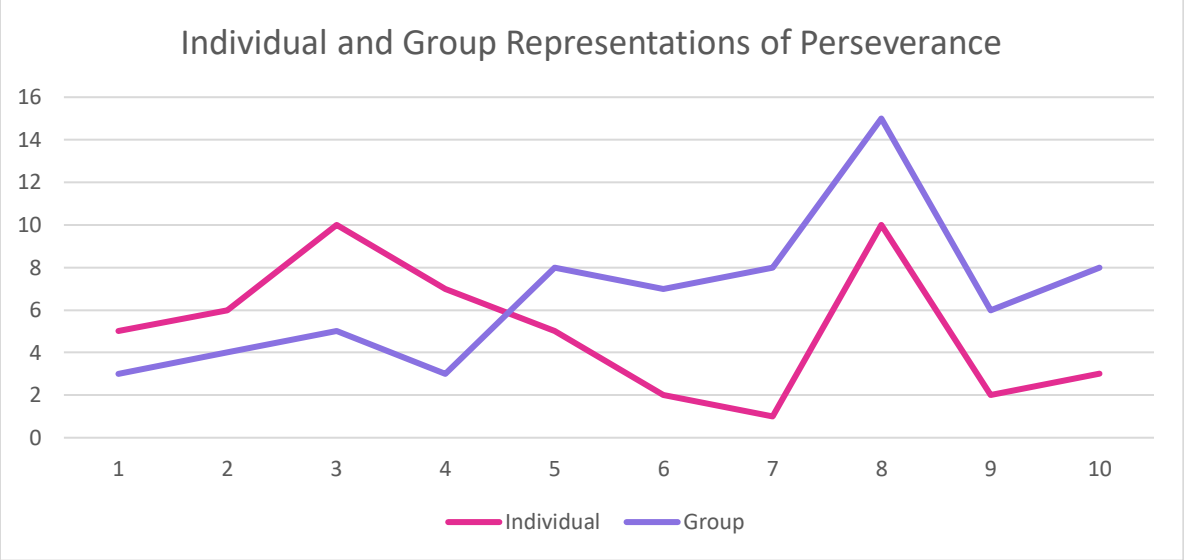


Figure 5-1: Graph of instances of individual and group representations of perseverance across the eight sessions and Research Lesson 1.

As shown in Figure 5-1, before Session 5 the participants drew on individual representations more often than group representations. Session 5 is a pivotal session in the teachers' collaborative discourse. From Session 5 onwards, the teachers drew on group representations of perseverance more often than individual. Chapter 4 presented this pattern and provided cases to characterize the teachers' uses of both representations over time.

This chapter seeks to characterize the relationship between the patterns identified in Chapter 4 and the teachers' engagement with one another during TRU-LS PD. As discussed in Chapter 2, an array of PD and workgroup conditions may constrain or enable generative discourse among teachers in a workgroup. Some of those conditions are shown in Figure 5-2.

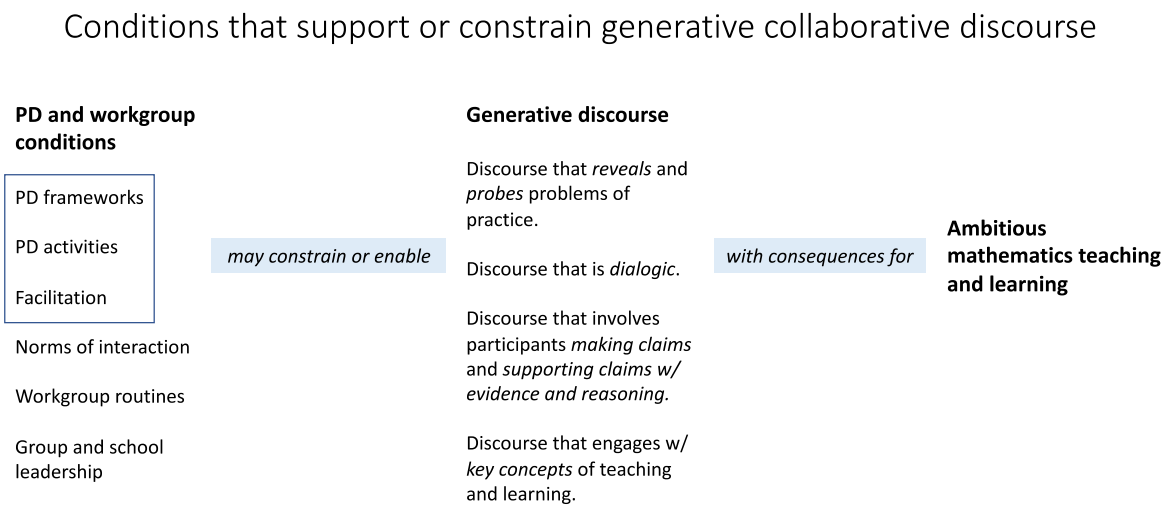


Figure 5-2: Framework for the relationship between PD and workgroup conditions, generative discourse, and shifts in discourse towards ambitious instruction.

Chapter 4 addressed the influence of facilitation on the discourse that unfolded among the Benton teachers over time. It also provided a preview of analysis on how participants' uses of PD frameworks, namely the TRU Dimensions (Schoenfeld & the Teaching for Robust Understanding Project, 2016) and the framework of Complex Instruction (Cohen et al., 1999), constrained and enabled generative collaborative discourse. This chapter examines how participants' engagement with the activities and frameworks of TRU-LS PD constrained and enabled generative discourse. It also examines the relationship between the discourse that unfolded and the Benton teachers' learning. This chapter primarily focuses on teachers' engagement with TRU-LS PD activities in Sessions 1-5, the sessions leading up to and including the most notable shift in collaborative discourse around perseverance. Chapter 4 indicated that something about the teachers' engagement with one another in the context of TRU-LS PD, especially in Sessions 1-5, was generative of a shift towards group representations of perseverance. This chapter explores the teachers' interactions within and across the different activities of TRU-LS PD to uncover PD conditions that supported and constrained generative discourse that in turn supported that shift.

Analytic stance taken in this chapter

Analysis in this chapter diverges from typical research into teachers' learning in PD. As discussed in Chapter 2, research into teachers' learning in PD typically takes either a descriptive or evaluative approach. Namely, research either describes a PD program and its learning goals without detailed analysis, or evaluates whether teachers' learning in PD program met a set of pre-established, specific learning targets. This chapter takes neither of these typical approaches. Instead, it builds on the grounded analysis of the teachers' discourse described in Chapter 4 to uncover the relationship between teachers' interactions with and within the PD activities and their *collaborative inquiry process*. As described in Chapter 2, this dissertation conceptualizes teachers' learning as a process of defining, studying, and attempting to resolve problems of practice in and across their workgroup and classrooms. That learning is supported by generative discourse of the kind described in Figure 2: discourse that reveals and probes problems of practice, is dialogic, involves participants making claims and supporting them with evidence and reasoning, and engages with key concepts of teaching and learning. In typical evaluative research into teachers' learning in PD, however, "learning" is defined more narrowly, as teachers adopting particular pedagogical practices, such as noticing students' mathematical thinking (van Es & Sherin, 2010), or discourse patterns, such as talk about students, pedagogy, and content aligned with a framework for mathematics teaching (Beisiegel et al., 2018). In other words, typical evaluative research into teachers' learning in PD overlooks the link between PD conditions and generative discourse, skipping straight to how PD conditions precipitated learning outcomes, and describes those learning outcomes narrowly.

As described in Chapter 2, this dissertation takes a broader approach to ambitious mathematics instruction than is typical in studies of teachers' learning in PD because teaching mathematics ambitiously is a complex activity that cannot be narrowed to a set of pre-determined practices (Philip et al., 2018; Schoenfeld, 2014). It also focuses on learning as a discursive and collaborative process, rather than as achievement of a narrow set of outcomes, because learning to teach mathematics ambitiously happens slowly as teachers inquire into important problems of practice together (Bannister, 2018; Little, 2002). This chapter builds on a broader tradition of research characteristic of ethnographic studies of teachers' learning in everyday work (Horn, 2005; Little, 2002) by bringing it to bear on analysis of how conditions for learning established by PD activities and frameworks enabled or constrained generative discourse among teachers in an organized PD context. The remainder of this chapter consists of that analysis.

Outline of the rest of the chapter

Part 2 looks at quantitative evidence to correlate patterns in the teachers' use of individual and group representations of perseverance with teachers' engagement in each of the three TRU-LS PD activities. As described in Chapter 3, TRU-LS PD primarily consisted of three types of activity, roughly organized around three different representations of practice and PD resources: planning interventions, video-watching, and recollecting and reflecting on classroom

events. Part 2 identifies PD activities that were generative of discourse that indicated teachers were exploring more complex representations of the relationship between perseverance and interaction.

Part 3 looks closely at the teachers' collaborative discourse in two PD activities—video-watching and planning— in Sessions 3 and 5, two pivotal sessions in their work around interaction and perseverance. Analysis in Part 3 highlights patterns in discourse that were generative of the teachers broadening their horizons of observation (Hutchins, 1996) around interpersonal interaction and perseverance. Part 3 addresses the question of how the PD activities and discourse frameworks used in those pivotal sessions enabled generative discourse among the teachers.

Representations of perseverance and the three types of PD activity

In this part of this chapter, I examine patterns in in the teachers' use of individual and group representations of perseverance at the level of a TRU-LS PD activity. To explore their problem of practice and the key ideas in it, the teachers engaged in the activities of TRU-LS PD, described in Chapter 3. TRU-LS PD activities can be roughly grouped into three categories: planning interventions, video-watching, and recollecting and reflecting on classroom events. As described in Chapter 3, these three types of activities were integrated into the study phase of lesson study (Murata, 2011). They were integrated as inquiry cycles that mirrored the larger lesson study inquiry cycle but which the teachers completed over a two-week period, bookended by their PD sessions (Schoenfeld et al., 2019). Each two-week inquiry cycle began with the teachers planning a pedagogical strategy that they hoped would support their students to persevere. In the following PD session, the teachers recalled what happened when they tried the strategy. Then they watched video of one of them using the strategy with their students and, sometimes, of their students working before and after the teacher used the strategy. Finally, they returned to planning, often revising the strategy they had used in the preceding two weeks.

Different kinds of collaborative discourse were characteristic of these different activities. Chapter 3 described the design of these activities. Looking at the level of activity sheds light on how specific PD activity conditions enabled or constrained generative discourse that encouraged learning as deepening inquiry into a problem of practice (Ball & Cohen, 1999; Lefstein et al., 2020; Little, 2003).

Distribution of individual and group representations of perseverance across the three types of PD activity

Even in the beginning of their collaborative work, when the teachers predominantly turned to individual representations of perseverance to make sense of student behavior, individual and group representations of perseverance were not used evenly in the three different types of activity. Table 5-1 shows the distribution of the two representations of perseverance among the three activity types in the first four sessions of the Benton teachers' collaborative

work. In Chapter 4, I called these first four sessions Phase 1 because during these sessions, the teachers used individual representations of perseverance more often than group representations of perseverance. In Phase 2, the teachers used group representations of perseverance more often than individual representations of perseverance.

Table 5-1: Individual and group representations of perseverance in the three types of PD activities, in Phase 1.

	Phase 1	
Activity	**Individual**	**Group**
Share classroom events	14	4
Watch video together	6	5
Plan together	6	4

As shown in Table 5-1, in Phase 1, individual representations of perseverance were more common than group representations across all activities. However, representations of perseverance were more evenly divided among individual and group when teachers watched video and planned together. Teachers drew on individual representations heavily when sharing classroom events. Examining the teachers' uses of individual and group representations of perseverance in the three different PD activities during Phase 1 indicates that watching video together and planning together supported collaborative discourse generative of greater diversity and complexity in representations of perseverance.

Furthermore, in both phases, video-watching in which the teachers attended to perseverance as a group accomplishment, and did so at least as often as they attended to perseverance as individual choice or ability, was followed by planning in which the teachers devised interventions to help students work together. This is represented in Table 5-2.

Table 5-2: Individual and group representations of perseverance during video-watching, followed by planned interventions.

Session	Representations of perseverance in video-watching		Intervention planned after watching video
	Individual	Group	
Session 2 *Jamie's video*	4	3	Individual-oriented
Session 3 *Leo's video*	2	2	Group-oriented
Session 4 *Soledad's video*	0	0	Individual-oriented
Session 5 *Jamie's video*	3	4	Group-oriented

As Table 5-2 also shows, video-watching in which the teachers did not attend to perseverance as a group accomplishment or attended to perseverance as individual choice or ability more often was not followed by planning to support students to work together. Both research lessons also included pedagogical strategies aimed at supporting students to persevere by collaborating. They followed sessions in which the teachers used group representations of perseverance when discussing video. Video-watching in which group representations of perseverance were salient supported the teachers to plan groupwork and collaborative interventions—in other words, it was generative of learning examined in this dissertation, teachers defining, studying, and attempting to resolve problems of practice.

Table 5-2 also highlights that watching video did not always lead to generative conversations about perseverance. Notably, in Session 4, the teachers used neither representation of perseverance when talking about the video they watched together. Video-watching in Session 4 was not generative of conversation in which the teachers explored their problem of practice. Table 5-2 indicates features of the video-watching activity as enacted in Sessions 2, 3, and 5 were conducive to inquiry, whereas features of the video-watching activity as enacted in Session 4 were not. I explore these findings further in the next section of this part. Session 4 is also analyzed in greater depth in the next chapter, Chapter 6, as part of a discussion of when and how the teachers continued to draw on individual representations of perseverance even as they took up more group representations.

Table 5-2 also points to Sessions 3 and 5 as sessions in which teachers explored more group representations of perseverance, with the video-watching and planning in Sessions 3 and 5 playing important roles. The next section examines Sessions 3 and 5 in more detail.

Zooming in on Sessions 3 and 5

Between Session 3 and 5, the teachers transitioned from mostly using individual representations of perseverance to mostly using group representations of perseverance. To

investigate this further, Table 5-3 focuses on individual and group representations of perseverance in the three PD activities in Sessions 3 and 5 only.

Table 5-3: Individual and group representations of perseverance across activities in Sessions 3 and 5.

	Session 3		Session 5	
Activity	**Individual**	**Group**	**Individual**	**Group**
Total	**10**	**5**	**5**	**8**
Share classroom events	5	1	N/A	N/A
Watch video together	2	2	3	4
Plan together	3	2	2	4
Intervention	Group-oriented		Group-oriented	

Table 5-3 shows the distribution of the two representations of perseverance across the three activities in Sessions 3 and 5, along with the total number of times each representation was used in each session. As Table 5-3 shows, in Session 3 as a whole, the teachers used individual representations of perseverance twice as often as they used group representations. Session 3 was thus squarely in Phase 1. Teachers also heavily leaned on individual representations when recalling classroom events. However, in Session 3 the teachers used individual and group representations of perseverance equally when discussing video and they planned a group-oriented intervention. Thus in Session 3, teachers' discourse around perseverance in video-watching and planning activities was complex, entertaining both types of representation. By Session 5, the teachers had shifted into Phase 2. They often used group representations of perseverance while watching video and planned a group-oriented intervention. In subsequent sessions, the teachers continued to use predominantly group representations of perseverance and plan group-oriented interventions, including their two research lessons.

Thus, analysis of quantitative evidence shows that video-watching and planning were pivotal in the teachers' taking up more complex representations of perseverance and, eventually, using more group representations of perseverance. Video-watching and planning did not always support the teachers to explore group representations of perseverance, however. Video-watching and planning were more effective in certain sessions than in others—notably, in Sessions 3 and 5.

In Part 3, I pursue how participants represented perseverance during video-watching and planning activities in the pivotal sessions, Sessions 3 and 5. I examine the affordances of these activities for supporting teachers to deepen their inquiry into perseverance. I also examine how

teachers drew on the framework designed to be used in the PD, the TRU Framework, and a second framework, that of Complex Instruction, which the teachers introduced on their own. As analysis of these activities in these sessions shows, the teachers used more group representations of perseverance to illustrate increasingly more complex reasoning about students' perseverance as they engaged in PD activities and explored the frameworks.

Affordances of video-watching for exploring group representations of perseverance

Before moving into Part 3 for a close examination of teachers' collaborative discourse and the PD conditions that supported it in Sessions 3 and 5, I look more closely at the nature of the videos that the teachers watched together in all sessions.

Motivation for a closer look at the features of the videos that the Benton teachers' watched

Prior research into teachers' collaborative learning supported by watching classroom videos has sought to characterize the kinds of videos that best support generative workgroup discourse. Research has explored whether it is preferable for teachers to watch videos of their own practice or stock video (Beisiegel et al., 2018; Borko et al., 2008; Karsenty & Arcavi, 2017). An argument in favor of stock video is that stock videos can be intentionally chosen by facilitators to highlight particular aspects of instruction (Beisiegel et al., 2018). Own-video does not have this affordance, as facilitators simply have to take what they can get. The Benton teachers exclusively watched video of their own practice, and video-watching proved generative of collaborative discourse. But because the teachers watched "own video," the videos did not always show classroom interaction with all of the features considered desirable for discourse by the PD designers and facilitators. It is thus worth exploring whether there is any correlation between the kinds of classroom interactions that the videos showed and generative collaborative discourse.

How the videos were selected

The teachers often explored group representations of perseverance while talking about videos they watched together. They watched video in five of the eight sessions. As described in Chapter 3, the videos they watched were videos of their own practice, all but one taken during the 2017-18 school year. The exception was the video they watched in Session 2. This video was recorded in Jamie's classroom during the 2016-17 school year. Each video was recorded by either the facilitator, Elizabeth, or the researcher, Anna. Elizabeth and Anna selected a clip from the lesson video that they thought would provoke productive discussion amongst the teachers. Clips were typically between three and eight minutes long.

Elizabeth and Anna tried to choose clips that included mathematical conversation among students, as opposed to whole-class instruction with no student talk. They hypothesized that

watching such clips would be most likely to lead to conversation among teachers highlighting the role of student-student interaction in perseverance. They also tried to choose clips that showed the teacher using the planned intervention and showed students engaging in mathematical struggle. Elizabeth and Anna selected these three features for videos watched in the PD because of their relationship to the teachers' research theme, "How do we build student perseverance and capacity to struggle productively, together?" They wanted to provide teachers with clips that would show students struggling productively with mathematics through interaction with other students, ideally with the planned intervention playing a role in what unfolded.

Not all clips met these criteria. Nonetheless, the teachers often had generative discussions about videos that did not meet all of the criteria. The next section demonstrates this through analysis correlating the features of the videos watched with quantitative evidence pointing towards the generativity of discussion.

Correlating video features with generativity of collaborative video discussion

None of the clips that the teachers watched met all three criteria set by Elizabeth and Anna. Table 5-4 provides brief summaries the clips watched during Sessions 2-5, the sessions examined in this chapter, highlighting which of the three criteria were met.

Table 5-4: Characterizations of the videos watched in Sessions 2, 3, 4, and 5.

Session	Summary of clip	Clip contents (as determined by facilitators)		
		Math conversation among students?	Teacher using planned intervention?	Mathematical struggle?
2	*Jamie's Alg 2 class:* Four students worked together on a problem calculating a waitress's hourly wage after 12 months of compound growth. They attempted to solve the problem explicitly without already knowing how to write an explicit function. But their approach did not work. The short-cut was based on cognitively demanding and correct reasoning, albeit incorrectly executed. They realized their mistake and reverted to the longer and less cognitively demanding recursive method for solving the problem. One student in the group said nothing throughout the clip. A second student participated in generating the short cut, but was discouraged by its failure.	Yes	No	Yes
3	*Leo's Math Analysis class:* Leo and a student talked together after the student called him over to ask for help. The student had misunderstood how to use the formula for finding the vertex of a parabola. Leo asked her questions to expose what she did and did not knew and then worked on an example problem with her.	No	No	Yes
4	*Soledad's Math Analysis class:* Soledad reviewed with the whole class how to find the domain of the function representing the volume of a box. She led the students through a sequence of IRE-style questions that mimicked the process of finding the domain of the function. Eventually, the correct process was completed and the class had the domain of the function.	No	No	No
5	*Jamie's Alg 2 class:* Four students worked together on a worksheet comparing linear and compound growth. One of the students voiced the answer to one of the questions several times, without response from her groupmates. When a second student voiced the same answer, the other groupmates agreed to it. The second student then incorrectly wrote an exponential function to model linear growth, which his groupmates also agreed to. Jamie came over and helped the students understand their error. In conversation with Jamie, the second student developed the correct linear function.	Yes	No	Yes

As shown in Table 5-4, none of the videos watched in Sessions 2-5 met all three criteria. Perhaps most notably for this chapter's focus on teacher talk about the role of interaction among

students and collaboration in perseverance, only two of the four videos showed students talking with each other about mathematics. However, as shown in Table 5-5 below, the teachers used group representations of perseverance even when discussing videos that did not show students engaging with each other about mathematics.

Table 5-5: Individual and group representations of perseverance during video-watching.

Session	Representations of perseverance in video-watching		**Which video conditions were met?**
	Individual	**Group**	
Session 2 *Jamie's video*	4	3	Student-student Struggle
Session 3 *Leo's video*	2	2	Struggle
Session 4 *Soledad's video*	0	0	None
Session 5 *Jamie's video*	3	4	Student-student Struggle

Notably, Tables 5-4 and 5-5 together highlight Session 3 as a session of interest. The video watched in Session 3, from Leo's class, did not show students interacting with each other about mathematics. It showed Leo and a student struggling with a math problem having a routine interaction. Nonetheless, while discussing this video, the teachers used group representations of perseverance. They then planned a group-oriented intervention in the planning activity that followed. This analysis indicates that the role of interaction among students in perseverance was somehow made transparent enough through collaborative discourse in Session 3 for teachers to use group representations of perseverance as they discussed the video and plan a group-oriented intervention.

Tables 5-4 and 5-5 also show that only the video shown in Session 4, Soledad's video, had none of the features desired by Elizabeth and Anna. Soledad's video consisted of sequences of IRE led by Soledad. Students uttered one sentence or less in response to Soledad's questions. The absence of mathematical conversation involving students and mathematical struggle in this video was significant for the discussion among the teachers that followed. As shown in Table 5-5, the teachers did not explore their problem of practice in the conversation they had after watching video in Session 4. The teachers did not use either representation of perseverance in that conversation. Instead, participants attended to moves that Soledad made, such as giving wait-time after asking a question and using a visual model for the math problem. In the other three sessions, the teachers' conversations about the video included individual and group representations of perseverance. And in Sessions 3 and 5, video-watching led to planning in which the teachers planned a group-oriented intervention to support their students' perseverance.

Therefore, analysis of the videos that the teachers watched and the conversations that followed shows that meeting all three criteria was not essential for teachers' generative collaborative discourse. However, meeting none of the criteria correlated with poor discourse. Some mathematical struggle, student-student interaction, and/or evidence of the teacher using the planned intervention in the video watched correlated with discourse that supported the patterns identified in Chapter 4. This analysis raises the question of how generative discourse was supported in the sessions for which the video shown did not meet all of the criteria. The next part of this chapter takes up that question, focusing on the pivotal Sessions 3 and 5.

How watching video and planning interventions, with the TRU Framework and CI, supported generative discourse in Sessions 3 and 5

Video-watching and planning in Sessions 3 and 5 were pivotal in supporting teachers to take up group representations of perseverance over time. In the cases analyzed in this final part of Chapter 5, I attend to how the participants interacted with each other, activities, and resources—in particular the two frameworks used, the TRU Dimensions and CI— and how those interactions supported their inquiry into their problem of practice. This part provides detailed discourse analysis that supports conclusions about *how* PD conditions supported generative discourse among the teachers. Detailed discourse analysis of a sequence of cases shows how discourse among the teachers unfolded and was supported or constrained by PD conditions.

I find that in Sessions 3 and 5, discussing video clips and planning an intervention *together* broadened the teachers' horizons of observation (Hutchins, 1996) by expanding their discourse around perseverance to include group representations. These activities helped the teachers to see and imagine classroom interactions and perseverance in ways that made group representations of perseverance salient. Discussions of the videos in both sessions highlighted how teacher-student and student-student interactions contributed to the perseverance or lack thereof that the teachers saw in the video. Teachers used individual representations of perseverance alongside of group representations throughout both sessions. However, the teachers were often able to reason in favor of group representations using evidence from the video, PD resources, and support from facilitator questions. Features of the videos themselves, the TRU and CI frameworks, and interactions among participants oriented teachers to ways in which perseverance was co-constructed by students and teachers building on each other's thinking. In the planning that followed, the teachers drew on representations of perseverance that were salient during the video conversations to devise group-oriented interventions and imagine what would happen if they tried them. Taken together, watching and discussing video and then planning interventions to support perseverance expanded how the teachers saw and imagined perseverance in their classrooms.

In what follows, I provide cases of video-watching and planning in Sessions 3 and 5. I start with video-watching in Session 3, the case and analysis. I then move on to planning in

Session 3, the case and analysis, followed by summative analysis of both activities in Session 3. My analysis of video-watching and planning in Session 5 is in the same format.

Session 3

Watching Leo's video

Watching video fell in the middle of Session 3. In the episode analyzed here, the teachers watched a video clip from Leo's classroom. All four teachers were present, as were Elizabeth, the facilitator, and Anna from the research and design team. In addition the teachers had a visitor, Kyle, a resource teacher who worked with special education students at Benton.

Overview of Session 3 discussion before the selected case

In the beginning of the session, the teachers recalled what had happened when they tried Three Things, the teaching strategy they selected to support students' perseverance. As summarized in Table 5-3, reproduced below, teachers predominantly used individual representations of perseverance during the sharing activity.

Table 5-6/5-3 (reproduced from earlier in the chapter): Individual and group representations of perseverance across activities in Sessions 3 and 5.

	Session 3		Session 5	
Activity	**Individual**	**Group**	**Individual**	**Group**
Total	**10**	**5**	**5**	**8**
Share classroom events	5	1	N/A	N/A
Watch video together	2	2	3	4
Plan together	3	2	2	4
Intervention	Group-oriented		Group-oriented	

Jamie shared last before they transitioned to watching video. He recalled that his Algebra 1 students completed very little of a matching activity that they were supposed to be working on in groups. He attributed lack of perseverance to individual student decisions. He shared that when he asked students for three things they knew, the students refused to respond. They pushed back on his questioning, telling him, as he paraphrased, "Mr. Rossi, we're simple people. We don't need this kind of thing." He also shared a poster from the class to illustrate their lack of

perseverance. The poster was largely blank. He commented that groups with "an academically strong student" made more progress, attributing some groups' perseverance to individual students' talents.

After Jamie finished sharing, Elizabeth transitioned the group to watching video.

Elizabeth launches video-watching and introduces the TRU Look-Fors

Elizabeth began the video-watching activity by reminding the teachers of a video-watching tool they had used in the previous session (Session 2). This tool was the TRU Observation Guide Student Look-Fors (called the Look-Fors; see Appendix C for the complete tool).

The Look-Fors is a strengths-based list of behaviors teachers might see if students are engaged in mathematical reasoning with peers, organized by TRU Dimension. In Session 2, the teachers had chosen several behaviors from the list that they thought they might see if students were persevering. Their conversation around the Look-Fors in Session 2 involved teachers nominating Look-Fors. The teachers did not explain why they chose the Look-Fors nor were they pressed to do so. The brief discussion they had as teachers nominated Look-Fors mainly consisted of Elizabeth affirming the teachers' selections.

In Session 3, Elizabeth reminded the teachers of the Look-Fors they had chosen. She asked if they wanted to use the same Look-Fors again when watching this video. They agreed. The Look-Fors they chose are listed in Table 5-7.

Table 5-7: Look-Fors used when watching video in Sessions 2 and 3.

Look-For	TRU Dimension
Explains their thinking, even if somewhat preliminary.	Formative Assessment
Builds on the contributions of others and helps other see or make connections.	Agency, Ownership, Identity
Explains their reasoning processes as well as their answers.	Mathematics

As described in Chapter 4, these Look-Fors had the potential to orient the teachers to perseverance as a group accomplishment. Each had an interactional component, some more obviously than others. The first and third Look-Fors asked teachers to attend to evidence of students explaining their thinking and reasoning. These had the potential to support teachers to notice interactions in which students shared mathematical ideas essential to helping them make progress on tasks together, although the teachers could just notice one side of those interactions. The second Look-For explicitly asked teachers to attend to evidence of students co-constructing mathematics. This Look-For had the greatest affordance for supporting the teachers to notice students persevering together.

Leo's video

The video had limitations, as analyzed in the previous part of this chapter. The video showed Leo talking with Eden, a student in his Math Analysis class. In the video, Eden called Leo over because she was stuck on a math problem. The students had been working mostly individually on using the vertex formula and plugging-in to graph parabolas. In the video, it became clear that Eden did not know how to use the vertex formula. Leo asked Eden questions that uncovered what she did and did not know. Together, Leo and Eden did an example problem that he had presented to the whole class before shifting to individual work. Eden continued to ask questions as he explained until she let him know that she understood. (A full vignette of the video is provided in Appendix B.) In sum, the task was primarily procedural, providing few opportunities for productive struggle, and the video did not show students interacting with one another.

Before they watched the video, Elizabeth explained that she and Anna had chosen this clip because in it, Leo used a close approximation of the Three Things strategy.

Elizabeth: And while this isn't an example of, like, you're not explicitly doing Three Things in this, I think it, it gets at kind of like the essence of Three Things? Where you're giving opportunity for students to share their thinking.

In this explanation, Elizabeth described "the essence of Three Things" as an interaction between a teacher and a student in which the student has "opportunity… to share their thinking." Her description made no mention of how Three Things might support students to persevere together. The teachers had not yet articulated such a use of Three Things, and Elizabeth's summary reflected their shared understanding.

They then watched the two-and-a-half minute video clip and took notes on the three Look-Fors.

Elizabeth gave the teachers a few minutes to discuss the video in small groups. Then she launched the whole-group discussion.

Jamie describes Leo and Eden as building on each other's contributions

Jamie shared first, speaking for himself, Cindy, and Leo. He shared that his group had noticed Eden building on Leo's ideas, an example of the AOI Look-For, "Builds on the contributions of others and helps others see or make connections":

1 Jamie: Uh, so in our conversation, Cindy pointed out that it felt like Eden in the video was really building on Leo as a thought partner? So this, so there's an— there's a moment here for AOI in which he's uh she's building on the contributions of others, right? So like every question he's asking is prompting her to continue to build her explanation?

Cindy's claim, as shared by Jamie, represented Eden and Leo's interaction as collaborative. To make this representation, Cindy drew on a non-normative use of the Look-For, "Builds on the contributions of others and helps others see or make connections." The notion of building on the contributions of others is intended to describe the affordances of student-student interactions. Cindy and Jamie expanded it to include students and teachers building on each other's ideas. In this way, Cindy and Jamie created an opportunity to represent what might otherwise be seen as a student's perseverance with a teacher's help as a group accomplishment between Eden and Leo. Their interaction with the Look-For made the role of interaction in perseverance transparent for their colleagues.

Soledad offers an individualist-oriented alternative interpretation

However, Soledad had another interpretation of Eden's interaction with Leo. Speaking for herself, Kyle, and Anna, she wondered whether Eden actually knew the answer but lacked the confidence to share it. Soledad wondered whether Eden had asked for Leo's help to get him to confirm her answer. She expanded on her claim when prompted by Elizabeth.

2	Soledad:	We— we came up with a question on, does Eden actually know it, but she wanted affirmation?
3	Jamie:	Mmm.
4	Soledad:	And is that why she was asking Leo and then waiting til she was getting bits from Leo to go on with the next step. Was it like a worry about being vulnerable about her, about what she held? You know?
5	Jamie:	Mmhmm.
6	Soledad:	Or was it that, was it that her, like, thinking there was still very partial. And so she, um, she like was really wanting or needing, uh, the bits and pushes from Leo.
7	Elizabeth:	What made you think that that might be a possibility?
8	Soledad:	Because I think that lots of our students, um, lack confidence in their math ability? Um. Or, maybe, and maybe some don't lack confidence but some are for some reason, like, reticent to share? Um. I mean it could be either way, but for some reason there's something that keeps them from, like, sharing completely what's like, what they're what they're thinking and what they have? Um. But some I do think, like, have, have some ideas and are still putting together the puzzle pieces.
9	Jamie:	Mmhmm.

Soledad drew on individual representations of perseverance in her alternative interpretation of the video. She suggested that Eden actually knew how to solve the problem, but

lacked the confidence to share her answer. In Soledad's telling, Eden's lack of confidence made her "reticent to share." This latter part of Soledad's claim was drawn out in response to prompting by Elizabeth for further explanation. In this representation, Soledad attributed what she saw as Eden's lack of perseverance—choosing not to share her thinking—to Eden's lack of confidence, a character trait. Soledad framed her claim a challenge to Jamie and Cindy's, asking (emphasis mine), "does Eden *actually* know it…?"

Leo, Jamie, and Cindy reject Soledad's interpretation

When Elizabeth asked the teachers to respond, Leo, Jamie, and Cindy rejected Soledad's hypothesis. Leo and Jamie drew on their personal knowledge of Eden's character to argue against Soledad's claim.

10	Leo:	Well, I personally think that Eden has no issue with being wrong. She, like, voices her opinions. And I think she was just really confused.
11	Elizabeth:	Mmhmm.
12	Jamie:	Although I will say knowing Eden, I was surprised she wasn't willing to at least formulate something. Because she usually in my class last year was like, even even at the most, like, "I-duh-know [shrugs], this is way off, I'm confused as hell," she was always willing to offer something. And eager. Um. When focused. So, I was surprised that it didn't seem like she was explaining her reasoning process or explaining her thinking? Um. And saying, like, "I can't— I can't say it, I can show it, I can't say it, I can show it."

In this exchange, Jamie and Leo used evidence that only they were privy to in order to build a character-based argument against Soledad's claim that Eden was reticent to share her thinking. Jamie and Leo's claims were meant to push back against Soledad's, but they perpetuated the individual representations of perseverance that Soledad used.

Cindy, however, drew on evidence of Eden and Leo's interaction that she observed when watching the video.

13	Cindy:	Um, from that again, like, like I haven't taught her, I just know her from passing periods and other places. But, like, I e-even I would side with folks who've taught her, like, I don't, I don't think she was try— she knows it and trying to get affirmation because if she really wasn't able to complete any of her sentences in the beginning? And yet, like, after some prompting or some questioning, like, I— I was just saying in this group, some students would be like, "Oh, ok," and then they'd move on to their work. And she actually kept him there a little longer, and like, "So…" And then she either questioned,

give some questions to get confirmation or give some statements to get confirmation on what she just discovered?

In her turn, Cindy drew out a contrast between the interaction she observed in the video, in which Eden engaged with Leo's explanations, and her own experience of students listening to her explanations without interacting with them. In doing so, Cindy highlighted the role of interaction, rather than individual character, in what the teachers' characterized as Eden's perseverance. It is notable that Cindy made this pivot without prompting from or framing by the facilitator.

Soledad offers a group-oriented summary

The question that Soledad asked when Cindy completed her turn indicated that Soledad had picked up on some of what Cindy shared. To confirm that she understood, Soledad suggested a new Look-For from the AOI Dimension that might better describe Eden's behavior.

14 Soledad: So I think she was, "Asking questions and making suggestions to support analyzing, evaluating, applying and synthesizing mathematical ideas"?

Soledad's suggestion continued to highlight the role of Eden and Leo's interaction in the perseverance that the teachers observed. In sharing this, Soledad indicated that she was convinced by Cindy's argument and had come to see the evidence from the video as Cindy represented it. Again, it is notable that Soledad summarized Cindy's argument and drew on a Look-For to do so without prompting from the facilitator.

At this point, Elizabeth transitioned into planning their next intervention strategy. I will describe their engagement with the planning activity after analysis of the video-watching activity just described.

Analysis of Watching Leo's Video

Workgroup conditions supported the teachers to make perseverance as a group accomplishment transparent in a video with limited affordances.

As previously discussed, the video clip that the teachers watched had limited affordances for supporting the teachers' inquiry into perseverance as a group accomplishment. The video clip that the teachers watched showed a routine interaction between a student struggling with a math problem and a teacher helping her by asking questions and engaging her in an example problem. The teachers probably had several similar interactions with students during each lesson, each day. The routine nature of the interaction in the video and that it was between a student and a teacher, rather than multiple students, constrained the video's affordances for making transparent perseverance as a group accomplishment.

Nonetheless, in the discussion that unfolded, the teachers at times represented Leo and Eden as co-constructing perseverance. Cindy, as represented by Jamie, expanded the Look-For, "Builds on the contributions of others and helps others to see or make connections," to incorporate students and teachers building on each other's thinking (turn 1). Cindy and Jamie's adaptation of the Look-For opened up the possibility that the teachers could come to see Leo and Eden's interaction as a perseverance as a group accomplishment. When Soledad disagreed with Cindy and Jamie's interpretation of the video and argued that Eden had chosen not to share what she knew (turns 2-8), the other teachers countered her claims. Although Leo and Jamie drew on individual representations of perseverance to argue that Leo and Eden had persevered together (turns 10-12), the discussion culminated in consensus that the perseverance in the video was not solely the result of Eden's character or choices but also tied to interaction between Eden and Leo. By the end of the discussion, Soledad had come to see the perseverance in the video as Eden and Leo's joint accomplishment. She drew on the Look-Fors to express her understanding, turning to another AOI Look-For that emphasized building understanding through interaction (turn 14).

As was noted in the description of the case, it is notable that this largely unfolded without prompting or framing by the facilitator. The nature of facilitation is thus just one of several PD conditions that can support or constrain collaborative teacher discourse. This video-watching conversation from Session 3 showed the teachers coming to represent perseverance as facilitated by interactions, albeit still drawing on individual representations of perseverance. They were supported to represent perseverance in this way by the TRU Look-Fors and discussion of the video in which they explicitly challenged, responded to, and built on each other's claims. Drawing on direct evidence from the video, as Cindy did in turn 13, proved generative of their consensus around group representations. Thus, through their interactions with each other and PD resources around the video, the teachers together made transparent ways in which perseverance was a group accomplishment, albeit between a teacher and a student. As the following analysis will show, video-watching launched planning that continued to build perseverance as a group accomplishment.

From video-watching to planning to help students build on each other's thinking

Elizabeth transitioned the group from discussing Leo's video to planning their next experimental intervention. She encouraged them to reflect on what they had learned from the video as they discussed their next steps. She also prompted them to revise the Three Things strategy to incorporate what they had learned and further their inquiry. Cindy responded first. She noted how each teacher had innovated on Three Things over the last several weeks.

The discussion that followed exemplified Cindy's observation. Over the next five minutes, the teachers suggested several different, new ways to use Three Things to support and learn more about their students' perseverance. For instance, Jamie suggested that they use Three Things exclusively with concept-building matching tasks, such as the one he had shared before

they watched video. Leo suggested that they use it to prompt students to remember task directions. The teachers devolved into joking and laughter about their ninth graders' failings at following directions. In an attempt to get them back on track, Elizabeth asked a focusing question. She asked them to pinpoint what about perseverance they hoped to support with their strategy.

15 Elizabeth: So what is it about perseverance that we actually want to get at with Three Things. Because I hear Leo talking about this idea of like, content focus versus directions focused. So when you're thinking about, we want to build student perseverance, like what are we hoping this strategy solves. Or attempts to solve.

16 Leo: That's a great question.

Soledad and Leo represent Three Things as a strategy that supports students to "move on," alone

Soledad made the first attempt. She suggested that Three Things was a strategy to help students move on instead of giving up. She hoped that the students could learn to use Three Things on their own.

17 Soledad: I was thinking that it's a bridge so they move on. So that, you know, when they're stuck and they're like [mimes pushing with hands] "Oh, I don't know what to do, so I'm gonna" [grabs phone and pretends to play with it under her desk]. You know?

18 Elizabeth: Mmhmm.

19 Soledad: Um. That instead of doing that, that the that the asking of the three things will become more automatic for them, that they start asking themselves? And that they move forward. Autonomously. Tomorrow. Starting on Monday. [Laughs]

20 Jamie: Like, immediately. [Laughs]

Soledad's suggestion that Three Things was "a bridge so they move on" represented perseverance as an individual student's choice to not give up. In her telling, Three Things could give students somewhere to turn if they felt as though they did not know what to do. It could serve as "a bridge" between feeling stuck and continuing to make progress. Initially teachers would show students that bridge. Eventually—and, as she and Jamie said, hopefully sooner rather than later—students would be able to use the bridge alone. The bridge could become a personal perseverance strategy that each student could turn to when they got stuck.

Jamie agreed with Soledad. After Elizabeth invited others to weigh in, Leo built out her idea, drawing on his experience playing video games.

21 Elizabeth: Is that how you guys are thinking about it?
22 Leo: Yes, that even if they can't move forward, if they move to a place where they know what they should be, like, what should come next, you know? Like, I play video games and I make plans— like, when you're playing, you're like, "Okay, I need to go here, I need to go there, I need to go there." But, I just don't stop and don't do anything. I [holds his fingers wide and moves his hand away from his forehead].
23 Soledad: Yeah.
24 Leo: So, even if you know content-wise what you're supposed to do, you could be like, "Okay, if I ask Lin this, and I find this out, then I'll be able to do this." Or, "If I ask somebody else this, then I'll be able to do this." Um, instead of just getting stuck. So, that's sort of why I brought up the directions because they don't— some, some kids really just don't know the directions
25 Jamie: Yeah.
26 Leo: and if they knew the directions, they would try something.

Leo's analogy to playing video games, which for him was a solitary activity, further drew out the individual representation of perseverance. While this is not the topic of this chapter, Leo's individual-oriented analogy drew on a discourse of gaming that exists primarily outside of mathematics education but is at times leveraged in mathematics education discourses about "growth mindset" (Boaler, 2015; Kiili et al., 2015; O'Rourke et al., 2014). Growth mindset has been embraced by some discourses of ambitious mathematics instruction despite having an individualist bent. The interaction between growth mindset discourses and uses of individual representations of perseverance in the Benton teachers' conversations will be taken up in Chapter 6.

Cindy disagrees and represents Three Things as a mechanism to support students to make sense of math together and see each other and themselves as smart

Cindy did not agree with Soledad, Jamie, and Leo, however. Elizabeth noticed Cindy shaking her head and invited Cindy to share. Cindy described how she had moved away from using Three Things in one-on-one teacher-student interactions. Instead, she wanted to use it in more public settings so that students could hear each other sharing useful ideas. She hoped that if students heard each other's ideas, they could build on each other's ideas and come to see each other as strong math thinkers.

27 Elizabeth: Cindy, I saw you shaking your head earlier.
28 Cindy: Yeah, well cuz it has started to evolve for me. Uh, I keep bringing up, like, I want to engage the unengaged. So, like, I'm starting to see this not as a one-on-one strategy? Like, me and another student? But, like, something public.

		Because it's— I, I want to be able to publicly acknowledge that somebody is successful, like they, they get— "Yeah, that's something that we could see, and point out, you're right." Like, "You know something here."
29	Elizabeth:	Mmm.
30	Cindy:	So, that's why I'm like, I really like that idea [gestures to Jamie], of the doing it in a group, cuz then maybe it's not three things, maybe it's four things, or customize it to how— whatever the size of the team is,
31	Jamie:	Mmhmm.
32	Cindy:	like, so the team [mimes pointing to members of a team] "What's something you know? What's something you know? What's something you know?"
33	Jamie:	Mmm.
34	Anna:	Mmhmm.
35	Cindy:	So then everybody—
36	Jamie:	I like that.
37	Cindy:	--gets a say and everybody's like, "Got it." And so hopefully what that does for the person who's unengaged is that, "Oh, ok, I do know something." And, and that, that, that's my goal. Like, to get the unengaged engaged, now.

Cindy's strategy presented an important pivot from how the teachers had seen Leo use Three Things in the video they watched earlier in the session. In Leo's video, the teachers saw Leo use a variant of Three Things to co-construct with a student the knowledge needed to persist with a mathematical task. As previously discussed, the teachers represented the perseverance in the video as a teacher and student building on each other's knowledge. Cindy aimed to support perseverance in a similar way but also made an important change. Instead of a teacher and student building on each other's ideas, students would build on each other's ideas. Using Three Things in this way would, as Cindy put it, make sure "everybody gets a say" in how the group moved forward and "publicly acknowledge that somebody is successful." The resulting interaction would help everyone contribute to decisions about next steps and build each student's mathematical identity.

Cindy's suggestion of using Three Things to support group work was an implicit nod towards a type of ambitious instruction pedagogy that she and Jamie aimed for called Complex Instruction (CI) (Cohen, 1998; Cohen et al., 1999). CI was discussed briefly in Chapter 4, as an example of a discourse of ambitious instruction that does not directly mention perseverance but implicitly represents perseverance as a group accomplishment. Pedagogies that explicitly acknowledge the contributions of individual students to collective learning, or, in Cindy's words, that "publicly acknowledge that somebody is successful" (turn 28) are central to CI's goals of disrupting problematic status issues that position some students as more mathematically capable than others and supporting student collaboration. As described in Chapter 3, Cindy and Jamie described themselves as trying to use CI in their classrooms.

Elizabeth's facilitation boosts Cindy's group-oriented suggestion

Elizabeth amplified Cindy's suggestion and its connection to perseverance as a group accomplishment by reminding the teachers of their research theme's emphasis on students' "building capacity to struggle productively, *together*."

38 Elizabeth: Which I think also speaks to that piece in your research theme about "together"? Like, you'd, um, "building the capacity to struggle, like, together," so they're using all the, everyone's pieces to come together, to persevere. How do you guys feel about implementing it in the way that Cindy was thinking, in terms of, like, groups, "What do you know, what do you know, what do you know?"

39 Soledad: Yes. That sounds good.

In this interaction, Elizabeth emphasized the connection between Cindy's new strategy suggestion and the representation of perseverance as building on each other's thinking by reminding the teachers of their research theme: "Which I think also speaks to that piece in your research theme about 'together'?"

After Cindy explained, Leo asked questions to learn more about how she would implement her idea. As described in Chapter 3, Leo did not describe himself as using CI pedagogy. In his questioning, Leo detailed how Cindy would ask each student in a group of four students, "What do you know, what do you know, what do you know, what do you know."

Then, Jamie shared a concern he had about using Three Things with groups of students, and a possible remedy:

40 Jamie: I think I could try it, but I'm already anticipating, like,

41 Cindy: Yeah.

42 Jamie: "I don't know anything." Um. I would, if, I'm already feeling like in that situation, um, cuz a lot of kids do that, but I've been really pushing this year, is like, "Ok, I'll come back to you. Cindy, what do you know?" And then when Cindy says something, and I go, "Alright, Elizabeth, please restate what Cindy said in your own words." So that they're like

43 Elizabeth: Mmhmm.

44 Jamie: internalizing whatever, whatever the other folks are saying. And they'd say, "I didn't, I wasn't listening," and I'm like, "Great. Cindy, restate it again." [Points to the imaginary other student] Like [laughs].

Jamie's concern, that students might claim that they "don't know anything," and his potential remedy, encouraging students to share something another student had already contributed instead of their own ideas, had potential to further support students to build on each

other's knowledge. His remedy could facilitate students to interact with each other's ideas beyond simply listening to them as each student took their turn sharing.

Leo summarizes Three Things as a strategy to build group perseverance

After considerable laughter about Jamie's imagined situation, Elizabeth pushed the group to come to consensus on what strategy they would try in the next two weeks. She also pushed them to explain what they hoped would happen if they tried the strategy and how it would build students' perseverance. Leo described the new intervention:

55	Elizabeth:	Summarize for me, what are we all trying between now and 10/25. [Laughter]
56	Leo:	Um. First stab at it.
57	Elizabeth:	Mmhmm.
58	Leo:	Uh, what do you know, in a public setting. And I don't necessarily know what— but whether in groups or in, in whole-class, soliciting knowledge from multiple students.

In this interaction, Leo took the lead on summarizing the new strategy. As Leo summarized it, they planned to use Three Things "in a public setting… soliciting knowledge from multiple students." Leo mentioned group work as a possible public setting but also included whole-class instruction, pivoting away from the implicit use of CI pedagogy around group work.

Elizabeth facilitates a group-oriented summary

Then, Elizabeth pressed the teachers to elaborate on what they hoped would happen when they tried their new intervention. Implicit in her questions was a prompt to connect the new intervention to students' perseverance.

59	Elizabeth:	To do so, and if we do that, then what's gonna happen? What do we hope happens? Or like, why are we doing it that way? I just want to make sure we're all on the same page.
60	Cindy:	Um, hold on, let me see your sentence. [Looks at the whiteboard where Elizabeth's notes are being projected] Um. Oh, ok.
61	Soledad:	So that they push, push forward?
62	Elizabeth:	[Reading aloud what she's typing] Students can push forward. And I also hear a piece about, like, using each other's ideas.
63	Jamie:	Mmhmm.
64	Elizabeth:	[Reading aloud what she's typing] Each other's and their own ideas. Good? Ok.

In response to Elizabeth's question about what the teachers hoped would happen when they tried the strategy, Soledad briefly returned to individual representations of perseverance. But Elizabeth redirected towards perseverance as a group accomplishment by adding on. She in effect answered her own question about how the public use of Three Things would support students' perseverance, "And I also hear a piece about, like, using each other's ideas." Her statement summarized their planning conversation as having developed a strategy that would support students to persevere by building on each other's thinking.

Satisfied with their agreement on the strategy, Elizabeth pivoted to their next activity, brainstorming a math topic for their first research lesson.

Analysis of planning

This episode of planning ended with teachers coming to consensus around a group-oriented intervention. It began with teachers suggesting diverse and unfocused possible next interventions, many with an individual emphasis. Nonetheless, the participants eventually circled back to and, importantly, expanded on, where they had landed at the end of their video discussion: perseverance is co-constructed through interaction, but now *among students*. As Leo summarized it, they planned to use Three Things "in a public setting… soliciting knowledge from multiple students" (turn 58). They hoped that using Three Things in this way would, as Cindy put it, make sure "everybody gets a say" in how the group moved forward and "publicly acknowledge that somebody is successful" (turns 30-37). Cindy implicitly drew on discourse associated with CI when making her suggestions and elaborating on their potential impacts on students' perseverance. The intervention supported groups of students to persevere together by building on each other's ideas. It also presented the idea that how students see each other is important to each student's sense of themselves as capable of doing math.

Two features of the teachers' discourse in this case stand out.

The teachers hybridized a PD-provided pedagogical strategy to better support collaborative perseverance.

The teachers' new strategy represented a significant revision of Three Things as it was initially presented to and taken up by them. In its original form, presented in the list of strategies provided by the PD (TRU-LS Team, n.d.), Three Things asked teachers to push students stuck on a problem to share three things they knew about the problem and three things they wanted to know about the problem. As Elizabeth represented it at the beginning of their video-watching activity, the "essence of Three Things" was teachers supporting students to share their thinking. Three Things in its original form said nothing about supporting students to build on each other's thinking after they shared it. It also did not specify how Three Things could be used in groups of students to surface different students' thinking and thereby support students to see themselves and each other as capable. While in the context of the orientation provided by the TRU Framework, it may have been implied that Three Things should also be used to surface student

thinking so that students could collaboratively build on that thinking together, this subsequent purpose was not explicit in the text of the strategy. Through interaction supported by collaborative video-watching and the TRU Look-Fors, however, this rich collaborative use of the strategy was made explicit. The teachers' revision of Three Things specified new interactional routines that the teachers could use explicitly to support students to persevere together by seeing each other as capable and building on each other's thinking.

Collaborative planning supported the teachers to deepen the representations of perseverance as a group accomplishment surfaced during video-watching.

This intervention represented an expansion of the vision of perseverance, developed during the video discussion, as evolving through interaction between a teacher and a student towards a vision of students persevering together. To make this pivot, participants built on representations of perseverance used during the video-watching activity. Cindy's suggestion of using Three Things in a public setting built on the group representation of perseverance that was salient to the teachers as they interpreted what happened in the video. In their discussion of Leo's video, the teachers came to agree that Eden's perseverance was the result of the interaction between her and Leo, in which the two built on each other's thinking by asking and answering questions. Cindy's strategy imagined a way in which students could persevere together by building on each other's thinking. While students persevering by building on each other's thinking was not transparent in Leo's video, discussions of Leo's video paved the way for Cindy's expansion of perseverance to students building on each other's thinking. These representations helped the teachers imagine how a novel intervention could help support perseverance as a group accomplishment, broadening the teachers' horizons of observation. That Leo, Soledad, and Jamie took up Cindy's new strategy indicated that they were receptive to this representation of perseverance as building on each other's thinking.

Summary of video-watching and planning in Session 3: PD conditions supported generative discourse that led to collaborative learning.

Through discussing Leo's video and then planning an intervention in Session 3, the teachers took up group representations of perseverance alongside of and, sometimes, in place of individual representations of perseverance. As the conversation progressed, moving through the video-watching activity and into the planning activity, the teachers' representations of perseverance became more complex and group-oriented.

Teachers' interactions with each other around Leo's video and then around potential interventions made transparent in their conversations the role of interaction in perseverance. Important to this growing transparency was the teachers explicitly challenging and responding to each other's claims about the video and goals for classroom interventions. Elizabeth's prompts encouraging the teachers to elaborate on their claims and orient to their problem of practice also supported generative discourse. Teachers uses of TRU tools such as the Look-Fors and list of

instruction strategies as well as implicit references to CI pedagogical practices supported representations of perseverance as a group accomplishment in their discourse. Analysis of this session thus shows the teachers learning as defined in this dissertation, engaging in an iterative process of defining, studying, and attempting to resolve problems of practice in and across their workgroup and classrooms, supported by engagement within the PD. Taking up group representations of perseverance, part of iteratively *defining* the problem of practice, through watching video, part of *studying* the problem of practice, supported the teachers to expand what they saw in classroom interactions and what they could collaboratively plan to do to support students, part of *attempting to resolve* the problem of practice.

Overview of Session 4 and transition to analysis of Session 5

Each teacher tried Three Things in a public setting over the two weeks between Sessions 3 and 4. However, they did not recall students building on each other's thinking when they reported in Session 4 on what happened when they tried the strategy. Instead, they returned to individual representations of perseverance. I elaborate on the tendency of the teachers to return to individual representations of perseverance when sharing classroom events in Chapter 6. The teachers ended Session 4 by committing to brainstorm possible interventions that would intentionally support students to persevere individually, without relying on a teacher or other students. Session 4 did not end as Session 3 did, with the teachers collaboratively planning an intervention aimed at their problem of practice. Nonetheless, in the next session, Session 5, group representations of perseverance predominated. From Session 5 onwards, teachers used group representations of perseverance more often than they used individual representations of perseverance. As in Session 3, in Session 5 watching video and planning an intervention supported the uptake of group representations of perseverance, broadening teachers' horizons of observation.

Session 5

Watching Jamie's video

Watching video in Session 5 followed an activity discussing the logistics of the teachers' upcoming research lesson. Leo, Cindy, and Jamie were present, as were Elizabeth, the facilitator, and Anna from the research and design team. Before Elizabeth launched the video-watching discussion, Jamie shared background about the task and what students engaged with the day following the lesson in the video.

Elizabeth launches video-watching with an individualist prompt

When Elizabeth launched the video-watching in Session 5, she reminded the teachers of their Session 4 conclusions, elaborated in Chapter 6. She built on the teachers' thinking about perseverance as individual inclination or ability to participate. She also drew in references to students persevering as a group:

> Elizabeth: And as we're watching this video, we've— we've been working on the Three Things strategy, but we've also recognized that it comes with a lot of — or it doesn't work for everything, right? Like, Three Things we've been working on when the teacher is present, but we're also interested in what happens when kids are individually persevering or indi—like, persevering as a group. So I think that's what we want to be looking at in this video, is like how are— how does each student, each of the four students we're going to see, persevere to get through this problem, or how do they not persevere. And what are the different reasons behind, like what are the root causes of those issues that come up.

Elizabeth represented the situation in the video as "each of the four students… persever(ing) to get through this problem." Her representation portrayed the students as persevering as individuals, despite working together in a group. Her representation also hid the productive struggle essential to perseverance behind language of "get(ting) through the problem." Elizabeth did not guide the teachers to use the Look-Fors when watching this video, which had supported the teachers to see perseverance as afforded by interaction during Session 3. Elizabeth's launch, therefore, lacked supports that could encourage the teachers to inquire into perseverance as a group accomplishment.

Jamie's video

The video was about seven minutes long. The teachers' conversation mostly focused on an approximately two-minute-long span of the video, however, starting at the beginning of the clip. In the video, four students, Zakira, Mohammed, Omar, and Khalid, worked together on a task about comparing simple and compound interest using descriptions of situations, tables, graphs, and equations. A full vignette is provided in Appendix B. Here I provide a description of the part of the video on which the teachers' conversation focused most.

The exchange that the teachers attended to most covered the students' work on a problem in which they matched a simple interest situation to one of three curves.

Zakira's group-mates ignore her bids for collaboration

At the beginning of the clip, Omar and Khalid conversed in Arabic, mostly out of view of the camera. Zakira talked to Mohammed about whether the simple interest situation described in the problem was best represented by Graph B, an increasing linear graph, or Graph C, an

increasing convex curve. Zakira first suggested Graph B, the linear graph. She asked Mohammed if he agreed. He did not respond. Then, she suggested that Graph C might be correct. She noted that the y-intercept of the linear graph was not zero, representing the $200 that Mary initially invested. She asked Mohammed which graph he thought was correct, B or C, and he did not respond. She then asked the person recording the video, Elizabeth, for help. Elizabeth prompted Zakira to talk to her groupmates. Zakira's move to engage Elizabeth indicated that she was craving interaction about the problem, interaction that Mohammed was not providing.

Omar shares the answer

Then, Omar switched to English, directly engaging with Zakira and Elizabeth. He stated that the graph was neither C nor A, the increasing concave curve, without explanation. After a pause, he added that he thought the correct graph was B. Immediately, Mohammed announced his agreement, with Zakira following. Zakira celebrated Omar and Mohammed's agreement with her, saying that their affirmation of her initial thinking showed that she was smart. The four students moved to write explanations as prompted by the worksheet. As they wrote, Zakira, Mohammed, and Omar restated aspects of their reasoning. When Omar restated reasoning that Zakira had shared, Zakira expressed her frustration that he was saying something she had already said. Then the students transitioned to a new question, a segment of the video to which the teachers largely did not attend in their conversations.

Analysis of positioning, status, and perseverance in Jamie's video

This segment of the video showed a series of fraught interactions among the students with implications for their collaborative perseverance. In the exchange that followed Omar's entry to the conversation, Omar, Mohammed, and Zakira all explained key elements of the reasoning behind why Graph B best represents the growth of Mary's money. However, the positioning of the students as mathematical authorities that unfolded as the students interacted hindered their opportunities to persevere together. Omar quickly and coherently provided a correct answer to the question, indicating that he was not struggling with the problem. The students' engagement with Omar's answer cut off opportunities to make other students' thinking and struggle transparent or collaborative. Mohammed agreed with Omar's answer before Omar provided an explanation—even though Omar's answer was the same as Zakira's, which Mohammed had tacitly dismissed moments before. His immediate agreement diminished Zakira's mathematical authority in the group. It signaled that in this group, Omar was assumed correct even without justification while Zakira's mathematical thinking was not worthy of engagement. Zakira asserted authority by naming her own smartness in getting the same answer as Omar, even though none of the students gave her credit, voicing her reasoning after Omar, and calling Omar out when she felt that he had taken her ideas as his own. However, the other students did not attend to Zakira's thinking or her objections. During the clip, Khalid said

nothing in English. The video showed him writing on the worksheet but did not show what he wrote. None of the students moved to engage him in their English-language conversation about the problem.

Return to watching Jamie's video

After watching the clip, the teachers read and organized their notes. Then Elizabeth launched their discussion of the clip:

Elizabeth: What do we notice about each student's level of participation and how they persevered or did not persevere?

Elizabeth's launch again framed their inquiry as focused on individual students. The teachers responded in kind, by describing how individual students participated. They noted that they could not tell how Khalid was participating because he spoke in Arabic throughout the clip. They also noted which students were writing on their worksheets.

Leo, Jamie, and Cindy describe a "cute moment"

Finally, they described the moment when Omar switched from talking to Khalid in Arabic to talking with Zakira and Elizabeth in English about the curve-matching problem. Leo provided a positive assessment of the interaction between Zakira and Omar, without evidence: "He [Omar] provides a good balance to her [Zakira]." Jamie agreed, also without providing evidence.

Elizabeth then jumped in with a replay of Zakira's attempts early in the video clip at engaging her peers with her reasoning. Although she began her replay by saying, "Yeah," as if she was also agreeing with Leo, Elizabeth's replay tacitly problematized the claim that Omar provided "a good balance" to Zakira.

1	Elizabeth:	Yeah, there's a moment when she's eliminating and I was like, 'Come on, ask her some question about that!' Like, 'Ask her why she's eliminating A!'
2	Anna:	Yeah.
3	Elizabeth:	And, um. But they were just both kind of like so in.

Elizabeth tacitly problematized the claim that Omar was a productive collaborator with Zakira by pointing out that none of the students—Omar included—engaged with Zakira as she attempted to work on the graphing question. However, her problematizing was undercut by her explicit vocal agreement with Leo and Jamie (beginning her replay by saying, "Yeah") and unsubstantiated concluding comment that Omar and Zakira "were just both kind of like so in."

Jamie then offered evidence of Omar and Zakira's "balanced" relationship through a replay of the moment when Omar made his first English-language contribution. In describing that moment, Jamie attended to Zakira's frustration that Omar seemingly restated her idea without giving her credit. However, he did not attend to the problematic nature of the interaction for the students' collaborative perseverance.

4	Jamie:	So Zakira is like, 'Well it's adding the same.' No, Zakira said something like it was going up by the same amount, which is 20. And then, Omar, 'Yeah it's adding the same number every time.' There's that interesting moment where it's like not clear if they're actually listening to each other? And Zakira is like, 'Yeah, I said that.' Like [laughs] 'I already said that.' Sort of like, 'You're not listening to me.' Um. Yeah, and Omar I think was just providing a little more specificity in the explanation? But.
5	Elizabeth:	Mmm.
6	Jamie:	That was kind of a cute moment. Where it's not clear whether they're listening to each other. Or whether Zakira is saying, 'You are not listening to me.'

Jamie interpreted the interaction between Zakira and Omar about the answer to the curve-matching problem as relatively benign. He claimed that when Omar explained his answer to the problem, Omar "was just providing a little more specificity in the explanation" (turn 4). Jamie also called the interaction "cute." Jamie's replay of and claims about the video did not attend to the problematic positioning that unfolded as the students affirmed Omar's answer without probing when they had not afforded the same status to Zakira's identical answer. As such, Jamie's interpretation did not make transparent the implications of the interaction for the students' collaborative perseverance.

Cindy added her take immediately afterwards. Cindy saw the interaction as potentially positive for Zakira.

7	Cindy:	Um. Well, with that exchange I put down a note for myself, like, there's validation. And I wonder, like, that's what continued to help drive them to become instead a foursome? And, um, she's like, 'Oh yeah, I'm hella smart!' Right? That's what Zakira said, right?
8	Jamie:	She did say that.
9	Cindy:	She needs to get that validation from Omar, and and, maybe that's that helped them continue on together.

Cindy interpreted Omar's restatement of Zakira's idea as validating Zakira's contribution. She added a claim that what she saw as Omar's validation supported the group's collaboration, saying "that's what continued to help drive them to become instead a foursome"

(turn 7). Cindy's replay and claim represented an attempt to inquire into how the students' interactions supported collaborative perseverance. However, although Omar, Zakira, and Mohammed began talking about the problem after Omar asserted his answer, their work was not collaborative. Furthermore, Zakira's assertion of her own intelligence was not affirmed by the group, undercutting Cindy's claim that Omar had validated Zakira.

Elizabeth problematizes Jamie and Cindy's interpretations

Elizabeth then drew the teachers' attention to the chain of events that led to the encounter which Jamie had classified as "cute" and Cindy had classified as validating Zakira: Zakira asserted the correct answer; no one attended; Omar asserted the correct answer; Mohammed and Zakira immediately agreed.

10 Elizabeth: Yeah. I was really interested in that moment where, you know, Zakira was like, 'It's B, I think it's B.' Omar said, 'It's B.' And everyone was like [pounds table], 'Yeah, it's B.' What do we— what do we think around, um, that moment? How students were acting.

In replaying this series of interactions in this way, highlighting how the students only agreed with Zakira's answer when Omar said it, Elizabeth tacitly problematized them. While she did not explicitly name perseverance, in the context of the conversation Elizabeth's replay implied that the interactions had implications for the students' perseverance. Elizabeth asked the teachers for their thoughts.

Cindy and Jamie wonder how to "interrupt" status dynamics problematic for collaborative perseverance

In his reply, Jamie admitted that he had not seen the interaction in that way. Elizabeth's replay had made the interaction among the students transparent for Jamie in a way that was new to him. Cindy did, however. She responded to Elizabeth's question by characterizing Omar as the "leader" of the group and then problematizing that characterization.

11 Jamie: I'm not sure I caught that as clearly.

12 Cindy: Well, I, uh, saw when you [Jamie] were next to them, and, you know, Zakira was like, 'Oh, I'm writing in pen, so you're not sure of this equation? You don't know it?' Like, so, and then, it's like, and then while he [Omar] was talking to you— you [to Elizabeth] you were panning your camera to other students who were writing stuff down, and so, you know, you could see, Omar's kind of the lead of the whole group.

13 Jamie: Mmhmm.

14 Cindy: And they, they're all, like, waiting to see what he does
15 Jamie: Right.
16 Cindy: or what he writes. And so, um, I see two things— I mean, one is that, yeah, a team needs a leader? I guess. But, also, like, um, what can we do about it? Because right now, we're— we're taking notes and we have two people that we know a lot about, and two who we don't. So is that a spot to interrupt? Or to encourage, like, just I think that's a moment where we could do something.

In this exchange, Cindy explicitly attended to the ways in which the interactions among the students had positioned Omar as having greater mathematical authority than the others. She provided evidence from the video for her claim (turn 12). Cindy was of two minds about the influence of Omar's outsize authority on the students' collaborative work. She claimed both that "a team needs a leader" (turn 16), although she did not specify why, and that Omar's high status had precluded participation and, implicitly, perseverance by other students. She seemed to come down on the side of Omar's status being problematic, as she called for the teachers to "interrupt" (turn 16) interactions such as those captured in the video. Cindy's use of the word "interrupt" was an implicit nod to CI pedagogy's attention to status among students in group work and the teacher's role in disrupting problematic patterns.

In his response to Cindy's replays, claims, and call for action, Jamie laid the burden of "breaking up that status" on himself. He recalled that when he approached the students, he talked mainly to Omar and did not direct his questions to the other students in the group.

17 Jamie: Yeah, I noticed that I did not do a great job of breaking that status up there. [Laughs] There was like a conversation with me and Omar. Um. I missed an opportunity to ask any of the others, like, directly.

Jamie's experience with CI can be seen in his explicit mention of "status."

The teachers look to CI pedagogy for potential interventions

Elizabeth then restated Cindy's observation hinting that Omar's high-status participation may have blocked opportunities for Zakira, Khalid, and Mohammed to meaningfully engage. She explicitly named perseverance, which had been implied in the previous conversation. She asked the teachers what they could do to broaden participation.

18 Elizabeth: Cindy, I think you bring up an interesting question about, like, what can we do about it? Um. Because, like you said, like a group needs a leader? That, I think that helps them persevere. But, like, how do we— what do we do to break that up, or to make sure everyone is participating or persevering.

In restating Cindy's claims, Elizabeth also restated the unelaborated and unjustified claim that "a group needs a leader…that helps them persevere" (turn 18). Elizabeth also did not elaborate on why a group of students might need "a leader," especially when such status-laden participation precluded meaningful engagement from other students. Nonetheless, Elizabeth asked the teachers to brainstorm interventions that would ensure that all students had the opportunity to persevere. Her question made more explicit the previously implicit claim that Omar had persevered, while the other students had not had the opportunity to do so. The claim that Omar had persevered ignored that his engagement showed no evidence of productive struggle.

Jamie responded, reflecting on a CI pedagogical strategy that he often used in other situations to break up status and distribute authority.

19	Jamie:	So, I— I will say for myself, I feel like one of the protocols I've been upholding well this year is around the Resource Manager?
20	Cindy:	[Nods]
21	Jamie:	And I feel like part of the reason in that clip I didn't do a good job of breaking up, like, who's— who's participating, and calling other voices into the conversation, is because they were done ahead of time. And so usually what's happening is when, um, when a group is stuck, the Resource Manager calls me over and I'll pick on anyone, right, I'll just choose someone randomly, and they know that. But this was not a case when they were stuck. They were confident, we're done. And so I didn't, um, I didn't follow in my head my usual protocol, like, I'm gonna— I'll, like, 'Cindy! What's the team question?' And then it's like, addressing it to Cindy. Um. I— I think it was just out of my routine, when I was like, 'Oh, ok, they're, like, a little bit ahead.' And I see that they're making the common mistake that I was expecting on the equation. But I didn't do my usual interruption there.

Assigning team roles such as Resource Manager and asking team questions as Jamie described are two pedagogical strategies important in CI. In his description of his team questions strategy, Jamie noted that he often used the strategy but had not used it in his interaction with this group because the students were not obviously stuck on the math problem.

Leo hybridizes the Team Questions strategy

Next, Leo probed into how Jamie's interaction with this group was different from ones in which Jamie normally used the strategy. Jamie explained that he typically used the team questions strategy when students called him over with the question-less complaint, "We don't know what to do!" But in this lesson, students had called him over because they thought they had finished the sheet. His interactions with these students were less about getting them back on track

and more about troubleshooting mistakes, and this took Jamie by surprise. With this added context, Leo suggested a revision to Jamie's team questions strategy.

22	Leo:	I think you could just extend what you usually do. So, 'What's the team—what's the team answer?'
23	Elizabeth:	Mmhmm.
24	Leo:	'Ok. Cindy, can you tell me how you guys got there. Do you agree, Elizabeth?' And, whatever.
25	Jamie:	Yeah.
26	Leo:	You just got caught off-guard.
27	Jamie:	Yeah, I got caught off-guard. Because, like, in first period, they were genuinely getting stuck. And so, like, teammates were— Resource Managers would call me over, and I'm like, 'Ok, Team Captain. What have you guys done so far?' So it was easy for me to do that with Period 1. But Period 3, they're all, 'We're good! We're done! Can we move on?'
28	Leo:	[Nods]
29	Jamie:	I let my guard down with them. Damn you, Period 3. Let my guard down.
30	Cindy:	It's a pleasant surprise, right?
31	Jamie:	It is a pleasant surprise. 'We got this!' I'm like, 'Great.'

Instead of asking students for the "team question," Leo suggested that they ask students for the "team answer." Leo ended his suggestion by defending Jamie for not using the strategy, saying Jamie was "caught off guard." In response, Jamie elaborated on how and why he "let [his] guard down." Cindy empathized. This interaction between Leo, Jamie, and Cindy highlighted the treacherous waters that the teachers navigated as they problematized their own and each other's practice. Jamie made himself vulnerable in this interaction. His response to Leo's questioning can be read in part as self-defense, but also as providing his colleagues with a window into the complexity he faced when engaging with the students in the video, complexity that was not entirely transparent in the video. It also highlighted how Leo deftly navigated those treacherous waters by asking Jamie for more context and making a suggestion while not blaming Jamie for not having thought to use the pedagogical move himself, in the complexity of the moment.

Leo inquires into implications for Zakira's mathematical identity and brainstorms strategies to support collaborative building of students' positive mathematical identities

Then, Leo turned the teachers' attention back to Zakira's assertions of authority throughout the video. Where Cindy had seen the other students validating Zakira's intelligence (turns 7 and 9), Leo saw Zakira's diminished authority in the group and the implications of her low status for her mathematical identity. Leo asked the teachers how they could support the other students to support Zakira to see herself as mathematically capable.

32	Leo:	In the context of Three Things, um, sort of. So Zakira said the things that people have mentioned, like, 'Oh, shit, I'm hella smart,' 'So you had me writing in pen? But you don't have no equation?' Um. 'I think it's B, do you think it's B?' She never said that, 'I'm not good at math,' but I think it sort of stems from that?
33	Jamie:	Mmm.
34	Leo:	That she wants to be right, and, she just needs to be right. Um. Sort of a different way to rephrase, 'I'm not good at math.' Um. And it's how— I was just wondering how the group can support that? Like, as teachers we know, we know more of what her needs are? But how can, um, we support the kids to support her.
35	Jamie:	Right, so that missed opportunity, somebody ask her a question, why did she eliminate A [one of the graphs].
36	Cindy:	Oh. [Nods]
37	Jamie:	We haven't— I haven't figured out how to get them to do those questioning strategies with each other.

In this exchange, Leo claimed that in asserting authority throughout the video, Zakira was actually announcing to her groupmates that she felt that she was "not good at math" (turns 32 and 34). Leo's claim captured the effect of interactions among the students that positioned Zakira as having less mathematical authority in the group. However, his claimed missed the cause of Zakira's low-status positioning: the interactions themselves that positioned her as having less authority. Implicitly, Leo argued that the feeling, "I'm not good at math," was part of Zakira's character, not the result of a problematic learning environment. In Leo's telling, Zakira's lack of confidence—the result of repeatedly being ignored and shut down by her groupmates—was actually Zakira's fault. His claim thus implicitly reproduced an individual, character-based representation of perseverance.

Nonetheless, Leo, and then Jamie, looked to interactions among the students to build Zakira's mathematical identity. Leo asked, "how can we support the kids to support her" (turn 34). He also drew in the strategy Three Things, which in Session 3 the teachers had altered to encourage the students to build on each other's thinking and see each other as mathematically capable. Jamie elaborated on Leo's mention of Three Things. He noted that none of the students had asked Zakira to explain her thinking (turn 35) and argued that the teachers had yet to find ways to get the students to use questioning strategies like Three Things "with each other" (turn 37).

Leo had a suggestion for how the teachers could "support the kids to support" each other, foreshadowing the planning discussion that would follow. Leo suggested that Jamie could ask students to restate each other's thinking. He hoped that this would in time encourage the students to preemptively check in with each other, without needing a teacher's prompting. Anna added that this strategy might draw students into the conversation.

38	Leo:	I think when you do the Resource Manager thing?
39	Jamie:	Mmhmm.
40	Leo:	'Jamie, what does Elizabeth think about it.' 'Jamie, what does Cindy think about it.' I may— I mean, I'm basically asking the question, but I'm having you ask the questions.
41	Cindy:	[Nods]
42	Leo:	Because you— then you'll figure out that I will ask you, then you'll do it beforehand, hopefully.
43	Anna:	I bet that could be a way to get the kids we didn't hear anything from to as well? Because it's like a really low-stakes thing, for you to just ask somebody else what they know?
44	Jamie:	Mmhmm. Mistakes.
45	Anna:	Right? Instead of having to share your own thinking, you can— you can be like, 'Well, I can just talk about what this person said.'

At this point, Elizabeth transitioned into their next activity, choosing a new intervention to further inquire into their problem of practice. I will describe their engagement with the planning activity after analysis of the video-watching task.

Analysis of watching Jamie's video

In the conversation from Session 5 analyzed here, the teachers continued to deepen their collaborative reasoning about perseverance as a group accomplishment, building on sense-making from Session 3 that represented students' building on each other's thinking as central to perseverance. In this session, the teachers considered how problematic positioning and status among students can impede students from persevering together, with negative consequences for individual students' mathematical identities. Three main findings stand out from this analysis.

PD conditions—including video resources and facilitation—supported generative discourse that deepened the teachers' reasoning about perseverance as a group accomplishment.

Unlike the video clip from Session 3, the video clip that the teachers watched in Session 5 showed interactions that the teachers would not typically see in their classrooms. Most of the video focused on the students' interactions without Jamie present. Teachers do not often have the luxury of eavesdropping on students working together without the students knowing they are there. And students interact differently when they are talking to a teacher. Students may not build on each other's thinking when talking to a teacher. One student may do most of the reporting on the group's questions or answers—even if those questions and answers were built through collaboration and even if the teacher directs their questions to the whole group. As Jamie pointed out in his reflection on this video, if a teacher does not intentionally move, physically or

discursively, when talking to students to disrupt unequal participation patterns, those patterns will persist. Therefore, the video itself had the potential to broaden the teachers' horizons of observation by showing them interactions among students that they would not otherwise see.

However, it was not obvious from the beginning of their discussion of the video that the teachers would attend to the influence of student interactions on perseverance. Elizabeth's launch did not orient the teachers' to group representations of perseverance. Before they watched the video, Elizabeth re-voiced their Session 4 conclusions that individual perseverance is important. Then, right before their discussion, Elizabeth prompted them to talk about how each student participated and persevered. This framing treated perseverance as an individual endeavor by asking teachers to attend to individual behaviors. While it did not exclude attention to interactions among students and how those interactions might support perseverance, it did not explicitly attend to them. The teachers also did not use the TRU Look-Fors, which in Session 3 had supported them to make interaction between Leo and Eden and its role in Eden's perseverance transparent. Leo, Jamie, and Cindy's early contributions did not problematize the ways in which Mohammed and Omar's interactions with Zakira positioned her as less mathematically capable. In fact, their contributions framed Omar and Zakira's interactions as in "balance" and productive. Thus the beginning of the teachers' discussion of this video highlights how what about classroom practice is transparent in a video depends on participants' interactions with each other, not just on the contents of the video itself.

However, seemingly in response to Leo, Jamie, and Cindy's unproblematized interpretations in turns 1-9 of how Zakira was positioned as a mathematical authority, Elizabeth explicitly highlighted in turn 10 how the students' interactions had diminished Zakira's authority. Her carefully constructed replay helped shift the teachers' conversation to how issues of status that unfold through group interactions influence students' opportunities to persevere. Elizabeth's facilitation supported the group to make transparent the role of interaction in students' perseverance.

In the conversation that unfolded after Elizabeth's pivot, the teachers largely represented the unequal opportunities for perseverance in the group as a group accomplishment and looked to group-oriented interventions to increase opportunities for perseverance in the future. Cindy and Jamie in particular drew on discourses from CI to make sense of issues of status among the students and the role of a teacher in intentionally disrupting positioning that confounds collaboration.

Individual representations still retained some explanatory power.

The teachers did not consistently use group representations of perseverance, however. Leo in turns 32 and 34 represented Zakira's lack of mathematical confidence and, implicitly, resulting lack of perseverance, as an individual failing. Leo's use of individual representations of perseverance to explain Zakira's lack of perseverance, while pivoting to group representations when he looked for solutions, was an example of a larger trend in the teachers' use of individual

and group representations. This trend will be further examined in Chapter 6. Nonetheless, at other times in the conversation (namely, Cindy in turn 16 and Jamie in turn 17), the teachers attributed the inequitable participation that unfolded in the video to inequitable status in the group, which was a group accomplishment. By reflecting on how his teaching moves did not break up status among the students (turns 17-21), Jamie put himself in a vulnerable position. But he also made space for the conversation that unfolded about the relationship between teacher's moves, group interactions, status, and students' perseverance as a group.

Ambitious instruction discourse from Complex Instruction afforded robust reasoning about group perseverance, even for a teacher who did not use CI in his teaching.

The pivotal role of CI discourse in this discussion is notable, especially given the Benton teachers' variable experience with it. Discussion of the importance of CI discourse to the conversation that unfolded in this chapter was begun in Chapter 4. Of the four teachers, Jamie alone had formal training in CI. Jamie and Cindy intentionally used aspects of CI pedagogy in their classrooms, including group roles and strategies for teachers interacting with students who are working in group meant to broaden participation and disrupt problematic status dynamics. Leo rarely supported his students to formally work in groups, a key component of CI pedagogy. In the conversation analyzed here, Jamie and Cindy first introduced CI discourse. Nonetheless, in turn 38, Leo introduced a variation to the pedagogical routine he called the "Resource Manager thing" that had potential to support students to learn about and build on each other's thinking when working in groups. Thus, CI discourse was used by all three teachers, despite their variable experience with it, and pivotally so.

In contrast, none of the participants explicitly referenced the TRU Dimensions. Elizabeth did not prompt the teachers to use the TRU Look-Fors or connect their claims about the video to the TRU Dimensions, as she often did in other sessions. Importantly, the TRU Dimensions are not consistent with her initial launch, which prompted the teachers to focus on perseverance as a trait or choice of individual students. That the teachers did not draw on the TRU Dimensions as they made and elaborated claims about the video is not surprising given that they were not prompted to do so. Notably, however, CI discourse supported the teachers to reach conclusions that were consistent with the values in the TRU Framework.

The teachers began planning a group-oriented intervention during their video discussion. The following analysis looks at what happened when they formally transitioned into the planning activity.

From watching Jamie's video to planning to support students to share their thinking

As noted in the previous analysis, the teachers had suggested several possible interventions to support perseverance during the video activity. A brief recap: Jamie discussed how he had forgotten to ask for a team question, a strategy that he often used when approaching

a group of students who called him over because they had a question about a task. He had missed an opportunity to use this strategy because the group in the video did not call him over to ask a question, but, rather, to share an answer. Leo suggested that in a similar situation, Jamie could ask for a "team answer." Later, Leo wondered whether asking students to share an idea from another student in the group could encourage broader participation during group work by compelling students to ask each other questions. He wondered whether such a strategy could also build the confidence of students who were routinely ignored or shut down. After Leo shared this idea, Elizabeth formally transitioned to the planning activity. She summarized Jamie and Leo's suggestions.

The teachers look to the TRU strategies resource for planning ideas

As Elizabeth launched the planning activity, Cindy pulled out a document that the teachers had used in their very first planning activity in Session 1 when they chose Three Things. It was a list of high-leverage instructional strategies compiled by the TRU-LS research team (TRU-LS Team, n.d.). Reminded by Cindy, Elizabeth passed out a copy of the list to each teacher.

On the list, each strategy is given a name. Then the strategy is briefly described. The description of each strategy includes what the teacher should do, what students will hopefully do in response, and a few details about situations in which the strategy may be useful. In Session 1, the teachers had chosen Three Things, "If students are stuck on a problem, ask them to state three things they know about the problem and three things they are wondering about. This can help students realize that they are making progress and have information that they can potentially draw on to make more progress."

As the teachers perused the list of strategies, Anna asked if there was a strategy on the list similar to the strategy Leo had suggested. Before the teachers shifted into reading quietly, Cindy made a tentative suggestion of "Group Questions: The teacher only answers group questions (questions that have been agreed upon by all group members). One student raises their hand when the group has a question. The teacher asks a different student in the group what the question is and only answers the question if all group members agree it's a question."

After quiet reading, Anna returned to Cindy's initial suggestion of Group Questions. But Cindy pivoted away from it. She explained that what they were actually looking for was "more of a variation on Three Things, right? Like, here, 'What do you know?' It's, 'What, what, what does she know?'" Cindy's clarification emphasized the teachers' focus on finding a strategy that would explicitly engage the students with each other's ideas. It also pointed to the teachers' realization that the Three Things strategy's structure of teacher-student-teacher-student interaction did not adequately support students to engage with each other's ideas. She emphasized the importance of a strategy that put students in direct interaction with each other's thinking.

In response, Anna suggested strategy "number 2." "Number 2" was the strategy "Share What Your Partner Said: Before a group discussion, have students turn and talk to a neighbor. When the discussion begins, have students who want to talk share their neighbor's idea, not their own."

Leo joined in with another suggestion, the fifth strategy on the list, "Name It: When a student shares an idea, write their name on the board next to it when you record it. Refer to that idea as 'so-and-so's idea' or 'so-and-so's strategy'. Keep track of whose ideas have been highlighted from class to class." Leo suggested combining "Name It" with "Share What Your Partner Said." He rehearsed the combined strategy. Students would say what another student in the group had said, but name the student in the process, as in, "Cindy said blah-blah-blah-blah." Anna added that this strategy would not be used "in a whole-class discussion context," as was specified in the strategy description.

Then Jamie also suggested "Share What Your Partner Said." He made the suggestion as if it were a new idea, although Anna and Leo had already discussed it. He shared how he already used it in his classes.

46	Jamie:	What about "Share What Your Partner Said," number 2?
47	Anna:	Yeah.
48	Jamie:	Because that seems to be more aligned with what we were talking about. Practicing active listening. And restating. I do— I do that a lot in whole class discussion?
49	Anna:	Mmhmm.
50	Jamie:	Where either I'll, um [clicks tongue], I'll cold-call on a student? Like, well, "Somebody, uh, So-and-so restate what Elana [name of student] just said?"
51	Anna:	Mmhmm.
52	Jamie:	Or, I'll ask for a volunteer, depending on what it is.

In this exchange, Jamie briefly explained how Share What Your Partner Said could support the teachers' goals of encouraging students to engage with each other's ideas, by getting students "practicing active listening" and "restating" (turn 48).

Elizabeth then moved for consensus on the strategy Share What Your Partner Said.

Leo describes how getting students to restate each other's ideas can support collaborative perseverance

After getting their agreement, she asked the teachers to describe what they hoped would change if they used the strategy. Jamie was the first to answer her question.

53	Elizabeth:	And so if we are having students share what their partner or group-member said, what are we hoping to see change?

54 Jamie: I mean I would think in an ideal world, that when the teacher is not present, students are restating each other's ideas.
55 Elizabeth: Mmhmm.
56 Jamie: Either paraphrasing or restating or re-explaining. Cuz this is the scaffold, right. Like, we're asking them to restate another, another teammate's or a partner's thinking. So, ideally they'd then come to internalize that and do it on their own.
57 Elizabeth: Mmhmm.

In his answer, Jamie returned to the theme of students "practicing" a behavior with teachers that they would eventually use on their own, raised in turn 48. He also reiterated the function of the strategy, having students "paraphrasing or restating or re-explaining" (turn 56) other students' ideas. But, he did not elaborate on why it was important for students to engage in these behaviors or practice them with a teacher before doing them on their own. In his telling, students' "paraphrasing or restating or re-explaining" each other's thinking without teacher prompting was an end in itself.

Leo expanded and deepened Jamie's initial explanation. In doing so, Leo articulated the mechanism that connected students' re-stating each other's thinking, eventually without teacher prompting, with students co-constructing mathematical ideas and, implicitly, persevering together. As Leo rehearsed the ideal interaction among students, Jamie joined in.

58 Leo: And I think past that, my goal would be to [clears throat]. So if I'm summarizing what Cindy said, hopefully Cindy would be listening to what I'm saying that she said.
59 Jamie: Mmhmm.
60 Anna: Hmm.
61 Leo: So she'd be like, "No, that's not what I said." [Laughter] "What I meant was this." Or she'd have to process what I'm saying and making meaning out of that.
62 Jamie: Right.
63 Leo: Or as I'm paraphrasing, I'm like, "Wait, that doesn't make any sense." Or, "That brings up another question that I have."
64 Anna: Mmhmm.
65 Leo: "Which is, blank." So, yes, first step is to practice paraphrasing. And I think then the subsequent step is to have everybody involved start analyzing what I'm saying
66 Anna: Mmhmm.
67 Leo: for next steps.
68 Jamie: And ideally, the third person's like, "That's not what Cindy said!"
69 Leo: Yeah.

70 Anna: Mmhmm.
71 Jamie: “You don’t make any sense!” “Try again!”

Leo’s explanation and his and Jamie’s rehearsal of a possible interaction among students co-constructed a robust representation of how an implementation of the strategy Share What Your Partner Said could support students to struggle productively, together. Leo articulated and then enacted with Jamie how close engagement with each other’s ideas, supported by a routine in which students restated each other’s thinking, could lead students to productively question each other’s thinking. Leo and Jamie outlined roles for multiple students in co-constructing shared understanding. Through their rehearsal, Leo and Jamie built a live representation of perseverance as a group accomplishment.

Specifically, Leo and Jamie outlined roles for three students. This prompted Cindy to ask, in a joking manner, “Where’s the fourth person?” Her question reminded Elizabeth of a connection to the video they had just watched. Elizabeth recalled the student Khalid, who had made no contributions in English during the video clip.

72 Cindy: Where’s the fourth person? [Laughs]
73 Anna: Yeah, I know. We’ve got three out of four! [Laughs]
74 Elizabeth: Poor Khalid!
75 Jamie: Poor Khalid.

At this point, Jamie recounted how Khalid had been a vocal participant in the whole-class discussion before the group work episode they watched. After Jamie finished his story, Elizabeth thanked Jamie for giving them the opportunity to watch the video from his class and transitioned to closing the session.

Analysis of planning

Two main findings stand out from this analysis.

As in Session 3, collaborative planning built on group-oriented conclusions from video-watching to galvanize potentially ambitious changes in classroom practice.

In this planning activity, the teachers drew directly on representations of perseverance used in the video-watching activity that preceded it. They had begun brainstorming interventions during the video-watching activity that they hypothesized would help them deal with some of the perseverance issues identified in the video. The teachers had concluded that the group of students in the video did not fully persevere because some students were shut out from meaningful engagement with the task. In planning, the teachers built on the representations and claims made during video-watching to articulate and come to consensus on the strategy Share What Your Partner Said.

The teachers' choice of Share What Your Partner Said represented a deepening of the teachers' collective understanding of the role of interaction in perseverance. In the new intervention, students would be intentionally supported to share their thinking and build on each other's thinking. None of their previous interventions had afforded such intentional support for collaborative perseverance. The influence of CI and its attention to disrupting status and carefully facilitating students' interactions in groups can be seen in the teachers' choice and justifications of this intervention, as can representations of perseverance constructed by the group during the video-watching part of this session and during Session 3. The teachers described their new strategy as a status treatment and way of building collaborative discourse among students.

Individual representations again retained some explanatory power.

As was often the case, individual representations of perseverance resurfaced in this discussion. The extent of this trend, highlighted several times already in this chapter, will be taken up in Chapter 6. Jamie's argument that he hoped students would "come to internalize" the strategy "and do it on their own" (turn 56) evoked individual representations of perseverance. Leo's follow-up put Jamie's answer in the context of a group-oriented representation of perseverance. Leo added that after students "practice[d] paraphrasing" (turn 65), the strategy would support them to think and respond critically to each other's ideas. Once a student attempted to restate another student's idea, Leo surmised, both students could evaluate the idea and its restatement. Maybe then the students could work together to get closer to a correct response. Leo's explanation of how the new strategy could support perseverance represented the strategy as one for facilitating the communication among students necessary for students to build on each other's thinking—and persevere. Such a complex representation of perseverance as a group accomplishment coming from Leo, who throughout Phase 1 drew heavily on individual representations and rarely used group representations, is a marker of how the Benton teachers' collaborative inquiry continued to deepen as they worked together in the context of the TRU-LS PD activities and frameworks.

Discussion of video-watching and planning in Sessions 3 and 5

Video-watching and planning in Sessions 3 and 5 showed the teachers pivoting towards and complexifying group representations of perseverance when explaining what unfolded in the videos and collaboratively planning interventions to support students' perseverance. As was discussed in the analysis of the four cases, the teachers did return to representations of perseverance as individual students' capacity or choice to work throughout. The next chapter examines trends in when and how the teachers returned to individual representations and what explanatory power those representations had. Nonetheless, the teachers' use of group representations to elaborate claims during video-watching and planning in Sessions 3 and 5 was

significant. The teachers ended each video-watching and planning activity in both sessions in consensus about video interpretations that emphasized perseverance as a group accomplishment and interventions that supported students to persevere together, by building on each other's thinking and coming to see each other as mathematically competent.

This discussion focuses on three findings that cut across the four cases analyzed in this chapter.

Collaborative video-watching and planning in a supportive PD environment—including PD resources, frameworks, activity structures, and facilitation— broadened the teachers' horizons of observation about how perseverance is co-constructed.

These interpretations and interventions were evidence of how the teachers' collaborative engagement with videos and planning broadened the teachers' horizons of observation (Hutchins, 1996), supporting them to see and then imagine student perseverance in group-focused ways. The trajectory of the teachers' video interpretations and planned interventions across the two sessions stands out for its movement towards more nuanced understandings of how students persevere together. In Session 3, the teachers came to consensus that Leo and his student, Eden, had persevered together in the video of their interaction by building on each other's thinking. This interpretation of the video represented perseverance as a joint accomplishment of a teacher and student. Notably, this video from Leo's classroom only met one of the three criteria that Elizabeth and Anna set for videos watched during TRU-LS PD sessions, namely that the video show a student struggling with mathematics. It did not show students interacting with one another or a teacher using the planned intervention. Nonetheless, the teachers co-constructed an interpretation of the video that centered the role of interaction in perseverance. They were supported to this by the TRU Look-Fors, which highlighted the importance of participants building on each other's thinking in math class. The teachers' use of collaborative discourse in which they challenged and responded to each other's thinking was also important in their co-construction of this interpretation of the video. Making the role of interaction in perseverance transparent in this video thus took careful collaboration among the teachers and support from a PD artifact.

In Session 5, the teachers expanded this interpretation to include students persevering together. While discussing Jamie's video in Session 5, the teachers elaborated on claims about how problematic positioning that unfolded through student-student interactions of students as having or lacking mathematical authority constrained students' opportunities to persevere. The teachers did not automatically discuss the role of interaction in perseverance in this video, however, despite its focus on four students struggling on a math problem together. Elizabeth's facilitation eventually made transparent for the teachers the status issues among the students in the video. CI discourse was useful for the teachers as they made sense together of the problematic interactions that unfolded in the video and a teacher's role in disrupting them.

Video-based sense-making that expanded teachers' horizons of observation catalyzed collaborative planning around potentially powerful ambitious instruction pedagogies.

The interventions that the teachers planned in Sessions 3 and 5 represented similar movement towards student-focus and complexity in representations of perseverance as a group accomplishment. In Session 3, the teachers revised the Three Things strategy to be intentionally used with students working in groups. The teachers planned to ask each student in a group to share something that they knew about the problem they were working on together. They argued that this strategy would support students to build on each other's thinking and see each other as mathematically capable by letting students hear their groupmates' ideas. By the end of discussing Jamie's video in Session 5, however, the teachers collectively recognized that a teacher prompting students to share thinking was not enough to encourage students to actively build on each other's ideas. As a result, they planned a new strategy, Share What Your Partner Said, that would prompt students to restate each other's thinking. The teachers articulated a clear mechanism through which a teacher prompting students to restate each other's thinking would lead to students co-constructing mathematics: as students attempted to restate each other's ideas, other students would challenge those restatements and, together, build better ideas. Thus, the strategy that the teachers built together in Session 5 provided a clearer vision of how the teachers would support students to persevere together, a vision that built on what the teachers learned in Session 3 and watching video in Session 5. Group representations built during video discussions and as the teachers explained and rehearsed their strategies, supported by CI discourse, helped the teachers envision how their interventions would support perseverance as a group accomplishment.

What is transparent to teachers in a workgroup in a video of classroom practice is co-constructed by those teachers.

Moving from Session 3 to Session 5, then, the teachers took up increasingly student-focused, complex representations of perseverance as a group accomplishment to explain what they saw in videos and mobilized around interventions that took on ambitious mathematics instruction. The increasingly complexity and orientation towards ambitious instruction in their discourse unfolded iteratively, as they moved between making sense together of a representation of practice and organizing for change in their classroom instruction. The teachers' interactions in these two activities of video-watching and planning served to broaden their horizons of observation (Hutchins, 1996). Teachers' collaborative discourse made aspects of classroom interactions that supported them to represent perseverance as a group accomplishment transparent in representations of practice. At the same time, use of group representations of perseverance made new aspects of classroom interactions transparent in those same representations of practice. This analysis shows how transparency of practice is co-constructed by participants in teachers' collaborative workgroups.

Essential to the co-construction of transparency was the teachers' and facilitator's willingness to challenge and respond to each other's interpretations of video and classroom interventions. As analysis showed, the first interpretation of a video or potential classroom intervention offered was not often the one finally taken up by the group. Across Sessions 3 and 5, each teacher and Elizabeth either explicitly challenged, built on, or responded another participant's thinking. Each time, that discourse move served to make transparent something new about classroom practice and deepen the participants' collaborative discourse. Even with imperfect videos, use of PD resources, and, at times, facilitation, participants iteratively broadened, deepened, and complexified their representations of perseverance by explicitly challenging, responding to, and building on each other's thinking.

Conclusion

This chapter explored the question, *How did engagement with PD activities support teachers to take up different representations of the role of interaction in perseverance?* It built on analysis in Chapter 4, which showed that over time, the Benton teachers' collaborative discourse incorporated more complex representations of the role of interaction in perseverance and took on more representations of perseverance as a group accomplishment. This chapter began by showing that increasing complexity in representation of perseverance in the teachers' collaborative discourse correlated with their engagement with the PD activities of watching video together and planning classroom interventions. Importantly, analysis showed that engaging in these activities in combination, back to back, across several PD sessions, correlated with the increasing complexity and movement towards group representations of perseverance in teachers' discourse. By collaboratively watching video of classroom practice to make sense of students' perseverance and then planning classroom interventions aimed at supporting students' perseverance, across several sessions, the teachers iteratively deepened and complexified the representations of perseverance used in collaborative discourse.

This chapter then zoomed in on teachers' discourse in two pivotal sessions, Sessions 3 and 5. Analysis highlighted how the transparency of the role of student-student interaction in perseverance was co-constructed by the participants as they collaboratively made sense of classroom video and planned interventions, supported by TRU and CI discourses. Analysis showed that the contents of the classroom videos themselves—whether they showed student-student interaction, teachers using planned interventions, or mathematical struggle—did not have to align perfectly with the facilitator and researcher's design goals to support generative discussion. Transparency of practice is not a property of video representations, independent of participants' interaction with them. In these cases, the teachers and facilitator engaged together, with the support of TRU and CI, in making sense of complex interactions in the videos, and the meanings and implications of those interactions were emergent in their conversations. Teachers' collaborative discourse was imperfectly but adequately supported by productive discourse practices such as challenging and building on each other's thinking as well as PD resources, facilitation, and ambitious instruction discourses from the TRU Framework and CI.

That this analysis primarily focuses on the teachers' work in only two sessions is a limitation of this analysis. While a detailed look at discourse patterns within several cases provides the in-depth perspective essential to understanding how participants' interactions with and within the PD led to shifts in collaborative discourse, such a limited time frame precludes broad conclusions about the efficacy of the PD activities and materials in other settings.

Quantitative evidence presented in this chapter indicated that the teachers were more likely to use individualist representations of perseverance when engaging in the activity of sharing classroom events, and less likely to use individualist representations when watching video and planning together. It is a limitation of this analysis that it does not and cannot, because of data limitations, explore whether the uptick in individual representations of perseverance when sharing classroom events correlated with the teachers continuing to use individualist teaching practices in their classrooms, even as they planned group-oriented interventions. Such an analytic limitation motivates future research that investigates teachers' collaborative workgroup discourse and classroom practice in tandem.

From the standpoint of PD evaluation, an overly simple conclusion of this chapter might be that the PD "worked." PD activities and frameworks created conditions largely supportive of generative discourse. However, as analysis in this chapter shows, claiming that the PD activities and frameworks "worked" obscures the at times fraught and imperfect but consistently complex interactional work that took place among the participants, PD activities, and frameworks. Analysis shows that drawing arrows from PD conditions, video and planning activities, tools, and frameworks, directly to the outcomes—shifts in collaborative discourse towards ambitious mathematics instruction—is insufficient methodologically to show *how* PD and workgroup conditions enabled generative discourse that leads to learning. The next chapter further complicates an attempt to construct a clear-cut evaluation of the PD, as the teachers continued to use individual representations of perseverance throughout.

Throughout this chapter and the previous one, the teachers' continued use of individual representations of perseverance—even as they leveraged more group representations of perseverance to make potentially powerful shifts in their classroom practice—stood out. The next chapter takes on the questions of when and how the teachers continued to use individual representations of perseverance, further exploring the complexity of their collaborative learning.

Chapter 6 Individual Representations of Perseverance Persist

Chapter 4 showed that the Benton teachers expanded how they represented perseverance over time. In the beginning of their work with their research theme "How do we build student perseverance and capacity to struggle productively, together?" the teachers tended to represent perseverance as individual students' choice or ability to work. As the sessions progressed, the teachers increasingly took up representations of perseverance as a group accomplishment. Chapter 5 showed how this expansion came about. It highlighted the roles of two central PD activities, watching video of the teachers' own practice and planning interventions together, in the teachers adopting more group representations of perseverance.

Throughout Chapters 4 and 5, however, it became apparent that while group representations of perseverance became more prevalent in the teachers' conversations over time, they continued to use individual representations of perseverance. This chapter turns to the persistence of individual representations of perseverance in the teachers' conversations. I find that across the teachers' work with their research theme, they continued to represent perseverance as individual students' choice or ability to work even as they took up group representations. I find patterns in what the teachers emphasized when they returned to individual representations of perseverance and in the explanatory power that individual representations seemed to have for them as they made sense of their students' perseverance. This chapter explores those patterns.

The first section of this chapter motivates the focus on the persistence of individual representations of perseverance even as the teachers made strides in their collaborative discourse towards group representations. It situates the analysis in this chapter broadly in the field of conceptual change (diSessa & Sherin, 1998; Vosniadou, 2009) and more narrowly in prior research on the persistence of traditional discourses of mathematics instruction in teacher learning (Louie, 2016; Philip, 2011). The next section examines how the teachers continued to use individual representations of perseverance in their conversations, amidst the changes described in Chapters 4 and 5. Cases illustrate patterns in how the teachers leveraged individual representations to make sense of their students' perseverance. That the Benton teachers returned to individual representations of perseverance even as they worked to make sense of perseverance as a group accomplishment attests to the pervasiveness of individualism in traditional discourses of mathematics education, which itself is part of individualist-centric ideologies that motivate education beyond mathematics and the structure of society at large (Apple, 2006; Giroux, 2002; Sengupta-Irving & Vossoughi, 2019; Valero, 2017). It also demonstrates the fragmentary nature of ideologies as they are held and used by people and communities and the slow pace of learning at the ideological level (Philip, 2011) and the slow and fragmented nature of conceptual change in general (diSessa & Sherin, 1998; Schoenfeld et al., 1993; Vosniadou, 2009). This analysis implies that professional learning that takes on ideological change requires a slow pace and possibly more direct interventions from facilitators and other non-teacher participants.

Part 1: Perseverance and individualist ideologies, in mathematics education and beyond

The Benton teachers' movement towards using more group representations of perseverance as they made sense of students' behavior and planned classroom interventions together was powerful. Chapters 4 and 5 showed how representing perseverance as a group accomplishment in collaborative professional discourse supported the teachers to see students in new, complex ways and organize together to shift their classroom instruction towards supporting students to make sense of mathematics together. As part of discourses of ambitious mathematics instruction (R. Gutiérrez, 2012; Lampert et al., 2013; Schoenfeld, 2014), group representations of perseverance empower teachers to collaborate around more robust visions of students and mathematically rich, equitable classroom practices. Ambitious mathematics instruction conceptualizes perseverance as a group accomplishment. Whether focusing on the perseverance of individual students within a social context or students working in a group, discourses of ambitious mathematics instruction represent perseverance as co-constructed by students within a classroom environment (Sengupta-Irving & Agarwal, 2017). Analysis in Chapters 4 and 5 showed how the Benton teachers were able to use group representations of perseverance to substantiate situated, growth-oriented claims about students' behavior that positioned the teachers to plan interventions aimed at supporting students to persevere together.

However, analysis in Chapters 4 and 5 also indicated that individual representations of perseverance persisted in the teachers' conversations, even as they took up more group representations. As shown in Figure 6-1 below, the Benton teachers continued to use individual representations of perseverance throughout their collaborative work.

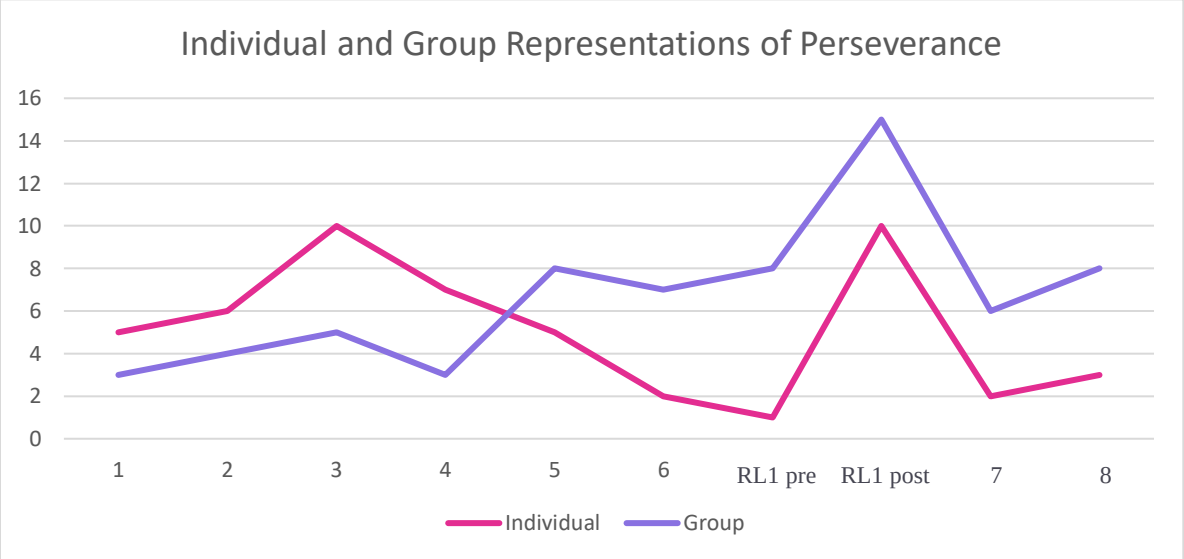


Figure 6-1: Graph of instances of individual and group representations of perseverance across the eight sessions and Research Lesson 1.

As Figure 6-1 shows, even as group representations overtook individual representations in frequency in the teachers' conversations, individual representations persisted. But why does the pattern of individual representations persisting matter to analysis of and conclusions about the Benton teachers' learning? The following sections motivate analysis of these patterns and situate that analysis in prior research.

Why examine patterns in how the teachers continued to use individual representations?

From certain research perspectives, including perspectives predominant in PD evaluation research, the persistence of individual representations may be seen as less significant than the pattern of group representations coming to predominate over time, through engagement with and within the PD. As described in Chapter 2, typical literature evaluating teachers' learning in PD programs seeks to identify whether a PD program "worked"—namely, whether it produced the shifts in workgroup discourse and/or classroom practice desired by the PD designers. Analysis in Chapters 4 and 5 showed shifts in collaborative discourse towards representations consistent with ambitious mathematics instruction and the TRU Framework, the framework that guided the PD program. However, as described in Chapter 2, this dissertation takes a different approach from that of typical PD evaluation literature. The approach taken by this dissertation does not see the persistence of individual representations as less significant than the powerful shifts towards group representations described in the previous chapters. This dissertation conceptualizes teaching and the learning of teaching as complex activities that cannot be fully delineated by PD designers seeking to evaluate for reachable benchmarks. This dissertation views teachers' learning and practice as situated in a variety of diverse discourse communities, traditional and ambitious, that themselves are situated in broader cultural contexts. How teachers' work is situated in larger social and political landscapes complicates the job of shifting teachers' discourse towards a non-traditional form of mathematics instruction.

In particular, the persistence of individual representations of perseverance in the Benton teachers' conversations matters because of the pervasiveness and self-evident nature of individualist approaches to perseverance in mathematics education, education broadly, and society at large. As described in Chapter 4, traditional discourses of mathematics instruction represent perseverance as individual students' capacity or choice to work. However, individualist representations of perseverance are not unique to traditional mathematics education discourses. They are also ingrained in broader discourses of education and society at large. In traditional mathematics instruction discourses, a student's perseverance or lack thereof is explained by looking to the student's individual attributes. Traditional mathematics instruction discourses tend to represent students and their individual attributes in deficit-oriented ways, seeking to explain students' individual failures to achieve mathematically with descriptions of traits and choices represented outside of the broader classroom context (Bannister, 2015; Horn, 2007). As

discussed in Chapter 4, the individualist nature of traditional mathematics instruction discourses impedes teachers from devising ways to shift their instruction to support perseverance.

From these perspectives, then, the persistence of individual representations is an important dimension of the project of characterizing the Benton teachers' learning together in TRU-LS PD. Identifying patterns in when and how the Benton teachers continued to use individual representations of perseverance, even as they used more group representations, can shed light on the teachers' learning processes, in particular on how those learning processes interacted with discourse communities, and inform the design of PD programs. The analysis in this chapter builds on prior research into the persistence of traditional mathematics instruction discourses in teachers' collaborative conversations, but also takes that research in new directions. It suggests that major changes in perspective will not respond to quick interventions. Change in discourse will be gradual and may be best facilitated by long-term, collective, reflective activities.

What prior research about conceptual change tells us about the persistence of prior knowledge, discourses, representations

The analysis in this chapter is situated in a long history of research attempting to make sense of the persistence of knowledge, discourses, and representations even as learners take up new knowledge, discourses, and representations. This section positions this chapter in that rich literature as it exists outside of literature on teachers' learning. The next section zooms in on literature on teachers' learning in particular.

Research in "misconceptions" and conceptual change, fields devoted to understanding how and why learners hold onto knowledge or discourses even as they see that their prior knowledge or discourse is inconsistent with new knowledge systems and newly valued discourses, has shown that learning in complex fields is slow and piecemeal (diSessa & Sherin, 1998; Smith et al., 1994). While this research has largely focused on learning by individuals in science domains (Vosniadou, 2009), it has bearing on teachers' learning in collaborative groups around social domains (with pivotal differences outlined at the end of this section). This research shows that in the learning process, persistence of representations that are problematic and inconsistent with new discourses or knowledge systems should be expected. Even in domains that seem circumscribed, learners can develop intricate, if fragmented, nonstandard understandings through interactions with that domain that they use to make sense of the world in some capacity and that are difficult to shift (diSessa, 2010). These nonstandard understandings may be commonsense to the learners, in the sense that they seem self-evident and do not require justification for individuals or groups to believe them (diSessa, 1993). Thus, disrupting them requires more than a clear explanation from an "expert." Rather, learners must have repeated experiences that prompt them to reorganize and recharacterize how they make sense of the world.

For example, Schoenfeld, Smith, and Arcavi (Schoenfeld et al., 1993) traced one mathematics student's learning about graphing simple algebraic functions. Even within the

seemingly narrow domain of graphing linear functions on the Cartesian plane, Schoenfeld and colleagues found that the student's learning process was complex and misconceptions persisted. Although the student had learned about algebraic functions and graphing in school, she entered their project with what these researchers called "misconceptions," or nonstandard understandings of the mathematical content. Debunking these misconceptions proved more challenging than the researchers anticipated. While they did not align with correct mathematical reasoning, the learner found the misconceptions powerful as she attempted to solve challenging mathematics problems. As she worked with the researchers over seven weeks, misconceptions were repeatedly challenged but then resurfaced until the student had sufficient experience with the mathematics to build robust connections that made the misconceptions obsolete.

This research and others from the field of conceptual change highlights that scholars across domains—including in teacher learning— should expect prior knowledge, representations, and discourses to persist even as learners make shifts. Conceptual change research also highlights that learners' nonstandard understandings of domains may make sense in some capacity, or at least hold significant explanatory power for the leaners. This latter implication is relevant in teacher learning research in that it directs scholars of teachers' learning to understand the explanatory power of nonstandard but persistent representations. In addition to these notable analytic implications, this research has methodological implications for research on teachers' learning. These implications substantiate this dissertation's approach of moving beyond PD evaluation, towards building a robust picture of teachers' learning processes within complex discourses. Conceptual change researchers identified the persistence of knowledge and discourses when they conceptualized learners as sensible and learning as inquiry into complex systems. Treating learners as sensible thinkers and the domain and the entities that learners learn as sufficiently complex are prerequisites for understanding the persistence of nonstandard knowledge and representations (diSessa & Sherin, 1998). For research on teachers' learning, this means considering the work of teaching and discourses around teaching and learning as complex and teachers as inherently oriented towards making sense whatever they encounter. Conceptual change research also highlights the importance of following the learning process over a long period of time. The depth and breadth of a learner's misconceptions or nonstandard knowledge organization only becomes apparent after extensive and repeated interactions (Schoenfeld et al., 1993; Vosniadou, 2009). In research on teachers' learning, this means following teachers for a long time, many months or even years, possibly beyond the point when, from an evaluation perspective, it may seem that they have "learned."

While research on misconceptions and conceptual change outside of teacher learning has bearing on this dissertation's analysis, implications are limited by several essential differences. The domain of sense-making is an important difference. Teachers making sense of problems of teaching practice together are working in a social domain. To distinguish the study of teachers' conceptions of social and ideological domains from the study of learners' conceptions of science, the field traditionally studied in conceptual change research, Philip coined the term "ideology in pieces" (Philip, 2011). The discourses with which teachers work as they learn about teaching

seek to explain human relationships and social structures, not scientific phenomena. The social nature of the domains of teachers' problems of practice has implications for how teachers make sense of concepts, representations, and evidence in those domains. While learners build preliminary understandings of scientific phenomena through contact with the physical world, teachers build representations relevant to their teaching work through interaction with ideologies and discourses used by members of their communities to make sense of the social world (Philip, 2011). As described in Chapter 2 and in the next sections of this chapter, the ideologies and discourses that professional learning initiative seek to move teachers away from are likely to be "traditional," widespread, and commonsense in society at large, with deep social and political implications. While nonstandard understandings of science may be intuitive and even widely held (diSessa, 1993), they are not woven into the fabric of society. Challenging traditional social discourses and ideologies is tantamount to challenging the social order that those ideologies and discourses uphold (Ledgerwood et al., 2011; Philip, 2011). Teachers themselves may be positioned as lacking power by those ideologies and discourses, with further implications for their sense-making about them. With the exceptions of historical examples such as Copernicus's challenges to geocentrism and modern examples of conceptual change around politicized science issues such as vaccinations and climate change (Ranney & Clark, 2016), science learners today are unlikely to face such power-laden barriers to conceptual change. Typical studies of conceptual change do not tend to take up learning around these politicized science topics (Vosniadou, 2009).

The emphasis on individual sense-making taken by most research on conceptual change is another important difference between conceptual change research and research on teachers' collaborative learning around problems of practice. Literature on conceptual change largely examines how individual learners navigate complex conceptual landscapes (Vosniadou, 2009). The example from Schoenfeld and colleagues illustrates this individual-centered approach (Schoenfeld et al., 1993). While Schoenfeld and colleagues considered the student's interactions with her instructor as they traced the evolution of her learning and misconceptions, they positioned the individual student and *her* misconceptions as the unit of analysis. Philip's work on ideology in pieces also took an individualist stance, examining the sense-making of one teacher even as that teacher worked in a collaborative workgroup (Philip, 2011). Research on teachers' learning in collaborative workgroups, such as in this dissertation, seeks to understand the sense-making of the teachers *together,* as well as of the teachers as individuals. This joint collaborative and individual approach is particularly important for studying sense-making about social domains because of how discourses and ideologies are co-constructed and reproduced by communities.

While these important differences between typical conceptual change research and research on teachers' collaborative learning around problems of practice limits what teacher learning can glean from scholarship on conceptual change, they also outline some of the contributions that research on teachers' collaborative learning can make to the broad and robust field of conceptual change. In particular, this dissertation contributes to research on "ideology in

pieces." It also shows how nonstandard or problematic representations may persist or change through collaborative discourse.

The next section zooms in on prior research on teachers' learning and the persistence of traditional discourses of mathematics instruction.

What prior research about teachers' learning around equity tells us about the persistence of traditional discourses in teachers' collaborative work

Research into teachers' collaborative learning about equity has shown that shifting communities of math teachers away from traditional discourses and towards ambitious discourses is slow work. Chapter 2 reviewed some literature in this vein (Bannister, 2015; Horn, 2007; Kazemi & Franke, 2004; Louie, 2016), with the goal of highlighting successes and challenges in teachers' collaborative learning around ambitious mathematics instruction discourses. This literature points to the tenacity of traditional mathematics instruction discourses even as teachers work towards ambitious mathematics instruction under favorable workgroup conditions.

It also shows that those discourses persist in teachers' collaborative talk in part because traditional mathematics instruction discourses have roots that reach beyond mathematics education and its associated communities (R. Gutiérrez, 2018; Martin, 2013; Oakes & Rogers, 2007). Connecting predominant themes in mathematics education discourses with themes relevant outside of mathematics education is not a main focus of PD evaluation literature or most other literature on teachers' collaborative learning (Lefstein et al., 2020). However, observing that traditional representations that hold strong in math teachers' collaborative discourse also retain a grip on discourse in society at large helps explain the slow, fragmented nature of teachers' learning towards ambitious mathematics instruction. Bannister and Horn, among others, have connected pervasive inequitable, deficit-oriented representations and categorizations of students, in particular students of color, used by teachers as part of traditional mathematics education discourses with broader deficit-oriented and racist category systems and ideologies that are commonsense in society at large (Bannister, 2016; Horn, 2007). That inequitable representations central to traditional mathematics instruction discourses are situated in broader discourses and ideologies make those representations more tenacious, likely to persist in teachers' discourse in fragmented but still meaningful ways (Philip, 2011).

More relevant to this dissertation's focus on perseverance, individualist and character-based representations of perseverance that are commonsense in traditional mathematics education discourses are also commonsense in society beyond mathematics education. Prior research has not explored patterns in teachers' uses of traditional discourses as they pertain to perseverance, nor how math teachers' discourse around perseverance is situated in broader discourses and ideologies. However, sociological research from outside of education indicates that individualist discourses are pervasive and powerful in society. The following section reviews this research.

Challenges to teachers' inquiry into perseverance presented by broader cultural discourses

Teachers making sense of perseverance together in a collaborative PD workgroup are doing that sense-making in a broader societal context in which individualist representations are taken for granted. The notion of perseverance as individual hard work is central to the ideology of meritocracy, a set of definitions, narratives, and representations that attributes economic success and US cultural hegemony to individual hard work—or, in other words, individual perseverance (Kunovich & Slomczynski, 2007; Ledgerwood et al., 2011). Each individual's success attains value as it reflects well on economic growth and the increase of the nation's standing on the world stage. Meritocracy is endemic in US culture at large. This is despite decades of evidence that wealth, rather than individual perseverance, begets more wealth and individuals from minoritized and systemically marginalized backgrounds face barriers to economic, social, and political success not encountered by White, English-speaking men (Chetty et al., 2017; Cooper, 2015; M. D. Foster & Tsarfati, 2016).

Individualist ideologies inform the institutions, values, and practices of education and, in particular, mathematics education. Scholars have long noted how ideologies that cement racial inequity in society at large shape educational institutions, values, and practices (Martin, 2013; Oakes & Rogers, 2007); the same is true for individualist ideologies. In education communities and discourses, mathematics has been represented as an individual activity in which innate ability matters more than opportunities for learning (Boaler, 2002b). A main goal of traditional mathematics education is identifying individuals with the most potential for advancement in mathematics, part of the larger goal of cultivating a mathematically-powerful workforce that can create the technological break-throughs a nation needs to remain scientifically and militarily dominant (for example, National Research Council, 1989; see also Giroux, 2002). Simultaneously, mathematics education is represented as an important tool for community advancement by providing individuals from historically marginalized communities with the knowledge they need to attain high-paying jobs (for example, Moses & Cobb, 2001). The inherent contradiction between representing mathematics education as both a community-building tool and an ability-centered, individualist enterprise is not lost on a number of mathematics education researchers, in particular those who study how students of color experience mathematics schooling (R. Gutiérrez, 2017; Martin, 2013; Sengupta-Irving & Vossoughi, 2019; Valero, 2017).

Those contradictions may not be transparent for teachers, however, who work in settings steeped in traditional individualist discourses. Individualist approaches to mathematics and mathematics education have long shaped the work and learning environments in US schools. Starting in the late twentieth century and picking up with No Child Left Behind and the growth of the charter school movement, neoconservative and neoliberal educational reformers have embedded the individualist ideology of meritocracy in the structure of mathematics schooling. Policy-makers behind high-stakes mathematics testing, charter schools that advertise rigorous mathematics instruction for working-class students of color, and national mathematics standards make alternately hegemonic and equity-based arguments for these policies built around

individualistic ideologies (Ravitch, 2011; Valero, 2017). At the same time, literature and programming aimed at teachers of all subjects and mathematics teachers in particular that discusses how teachers can build the "perseverance" character trait have grown in popularity (SRI International, 2018). Videos and books for teachers about building students' "grit" (Duckworth, 2016; Kirchgasler, 2018; Ris, 2015; Tough, 2013) garner thousands of viewers and readers.

Even as individualist approaches to perseverance pervade the cultures in which teachers work and learn, ambitious mathematics instruction discourses lack clarity in how they represent perseverance in individualist or group-oriented ways. Chapter 4 outlined how two predominant frameworks of discourses of ambitious mathematics instruction, the TRU Framework (Schoenfeld & the Teaching for Robust Understanding Project, 2016) and Complex Instruction (CI) (Cohen, 1998; Cohen et al., 1999), differently emphasize perseverance as a group accomplishment. CI explicitly rejects individualist, character- or disposition-based representations of students' perseverance in mathematics (Cohen et al., 1999). The TRU Framework also represents perseverance as a group accomplishment, but how empowering mathematical interactions among students and classroom environments position students to productively struggle with mathematics together is explicit in only one of the five dimensions, Agency, Ownership, and Identity (Schoenfeld, 2014). It is implicit in the other four dimensions. The popular theory of "growth mindset" is similar to the TRU Framework in its implicit, rather than explicit and central, representation of perseverance as a group accomplishment. The theory of growth or fixed mindsets specifies how students' dispositions towards the struggle and failure inevitable in learning influence their capacity to persevere through challenge and learn (Dweck, 2006). This theory has been specifically applied to mathematics, where widely-held beliefs about the importance of innate skill and speed in mathematical ability, traits which traditional discourses have typically attached to White males, make growth mindsets towards mathematics particularly hard to attain for students who do not fit the traditional mold (Boaler, 2015). Mindset theory has grown popular among reform-oriented mathematics educators. Its challenge to traditional gender and racial norms in mathematics and stance that mindsets that dispose students to mathematical success are learned, not innate, make it compatible with many of the values of ambitious mathematics instruction. However, the theory's emphasis on individual students' mindsets reflects an individualist orientation. Applications of mindset theory to educational practice have sought to design classroom environments that cultivate growth mindsets. Nonetheless, the theory looks to individual students' *dispositions* first to explain perseverance or its lack.

The Benton teachers pursued their problem of practice, "How do we build student perseverance and capacity to struggle productively, together?" in a culture of schooling embedded with explicit, implicit, and ambiguous individualistic orientations, through literature on grit and growth mindset, individualistic schooling structures, and, in general, the common-sense discourse of individualism. That they picked up group representations of perseverance at

all is notable. However, as Chapters 4 and 5 highlighted and this chapter expands on, individual representations of perseverance remained salient to the Benton teachers. The literature reviewed in this section motivates analysis of *when* and *how* individual representations held explanatory power for the Benton teachers, even as group representations became salient. When and how do teachers continue to use individualist representations of perseverance? What is the power of those representations in teachers' collaborative discourse, even as group representations come to predominate? This chapter addresses those questions.

Part 2: The persistence of individual representations of perseverance over time

Analysis of the teachers' conversations across their first two lesson study cycles shows that the teachers continued to use individual representations of perseverance throughout both phases of activity identified in Chapter 4. As shown in Figure 6-1, reproduced here, before Session 5 the teachers predominantly used individual representations of perseverance (Phase 1). From Session 5 onwards the teachers used group representations of perseverance more often than individual representations of perseverance (Phase 2). Chapter 5 analyzed how this uptake of group representations was supported by the PD.

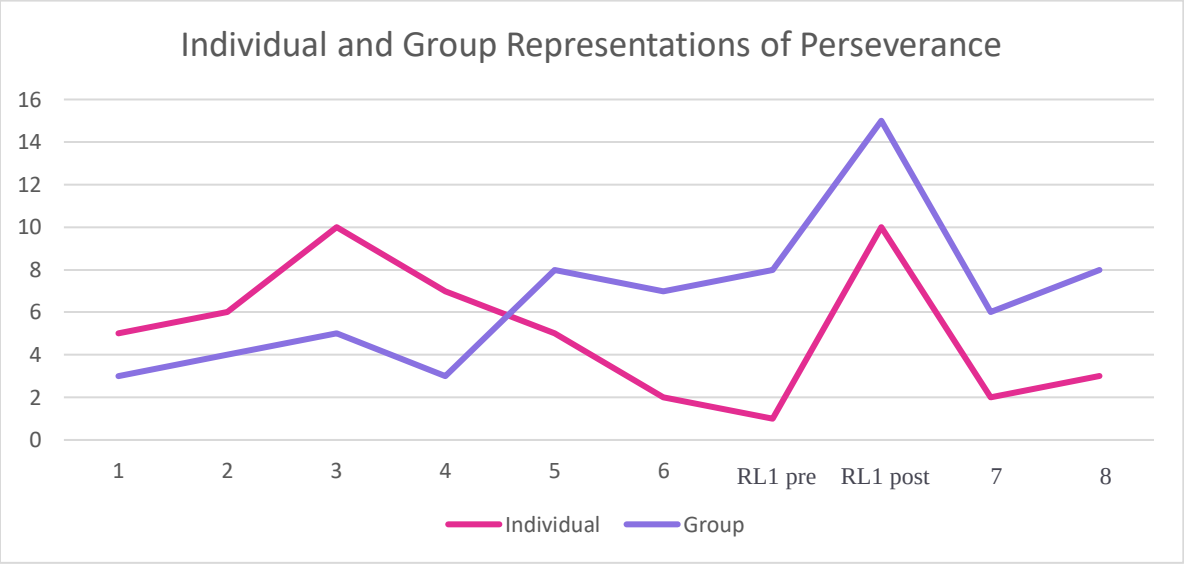


Figure 6-2/6-1: Graph of instances of individual and group representations of perseverance across the eight sessions and Research Lesson 1.

However, as Figure 6-1 shows, the teachers still often turned to individual representations of perseverance. Notably, for example, participants used individual representations of perseverance ten times during the post-lesson discussion following Research Lesson 1. Individual representations of perseverance made up almost half of the representations of perseverance used by participants in that session. The prevalence of individual representations of perseverance even in Phase 2 (Sessions 5-8) indicates that individual representations remained salient to the teachers. Individual representations of perseverance retained significant

explanatory power for the teachers. They retained that power even when the teachers had planned classroom interventions aimed at supporting students to persevere together, as in the case of Research Lesson 1.

In what follows I describe patterns in what the teachers attended to when using individual representations of perseverance. These patterns surfaced the ways in which the individualistic approaches to education framed the teachers' perspectives on their work with their students.

Three findings emerged from analysis of the teachers' continued use of individual representations of perseverance. These are that the teachers:

1. Tended to turn to individual representations first when reasoning about their students' perseverance.
2. Described the goal of supporting their students' perseverance as preparing their students for participation in the economy.
3. Often emphasized perseverance as independence, in particular from teachers and other authority figures.
4. Often leveraged individual representations when describing or making claims about times when they felt that their students had failed to persevere.
5. Assigned non-normative meanings to the TRU Dimensions while building individual representations of perseverance, and were not redirected by PD materials or facilitation.

These findings point to the explanatory power that individual representations of perseverance had for the teachers, even as group representations took on explanatory power in their collaborative discourse. The teachers often looked first to individual representations of perseverance when making sense of student behavior or reasoning about the goals of a planned intervention. This finding is the complement of a key finding from Chapter 4, that as the sessions progressed the teachers tended to reason away from individual representations of perseverance in favor of group representations. Even though collaborative reasoning often eventually moved towards group representations, individual representations stood out for their initial salience. Individual representations of perseverance helped the teachers make sense of why their students failed to persevere. They also helped the teachers characterize their own roles in supporting students to persevere, albeit in a negative sense—in emphasizing independence, the teachers were *not* supposed to help the students. Individual representations provided the foundation for the teachers' collective professional work, preparing their students to participate in the economy. And the PD, here manifested by the TRU Dimensions and facilitation, at times did not push back on or problematize individual representations of perseverance. The following sections provide analysis leading to the identification of these findings and cases of each.

Connecting patterns identified in Chapter 6 to patterns identified in Chapter 4

The discussion of this chapter considers relationships between these four patterns in how the teachers continued to use individual representations and findings from previous chapters in how the teachers came to use group representations to make sense of student behavior and plan interventions. Two of the four findings reported in Chapter 4 dealt directly with how the teachers came to use group representations in their collaborative discourse over time. They were:

1. Over time, group representations of perseverance became salient to explain cases of successful student perseverance.
2. Over time, the teachers came to conclude that the classroom activity of students' building on each other's thinking was important to perseverance, and that teachers should work to support that activity.

While in-depth discussion of this chapter's findings in the context of patterns identified in Chapter 4 is left for the end of this chapter, I remind readers of these two salient findings from Chapter 4 here because they are useful to keep in mind while reading this chapter's analysis. The teachers continued to frame their goals for their students in individual economic terms, represent perseverance as independence, and turn to individual representations to make sense of students' failures to persevere at the same time as they increasingly used group representations to explain students' perseverance and planned to support their students to build on each other's thinking.

Findings 1 and 4 in the Post Lesson Discussion following Research Lesson 1: Turning to individual representations of perseverance for initial interpretations and claims about students' failings

The first case analyzed here comes from the post-lesson discussion following Research Lesson 1. Before presenting the case analysis, I motivate the selection of a case from this particular session.

Quantitative evidence: Tracing an increase of individual representations of perseverance in the Research Lesson 1 Post Lesson Discussion

The post-lesson discussion following Research Lesson 1 stands out in the trajectory of the teachers' collaborative sense-making about perseverance. Quantitative evidence, shown in Figure 6-3 reproduced here, pointed to a distinctive uptick in the use of individual representations of perseverance in the post-lesson discussion following the Benton teachers' first research lesson.

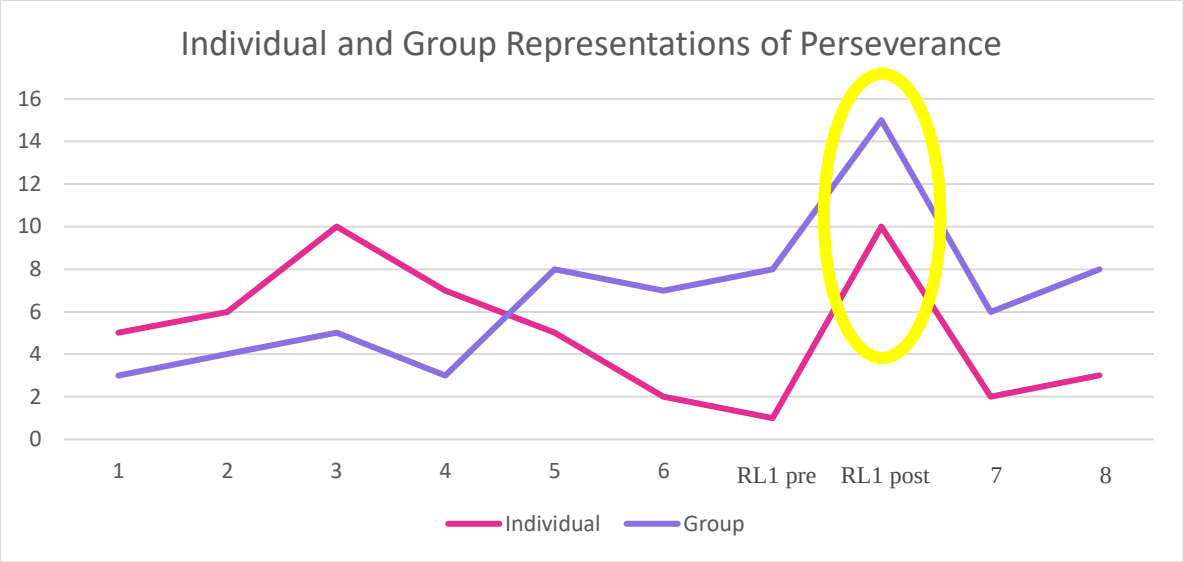


Figure 6-3: Graph of instances of individual and group representations of perseverance across the eight sessions and Research Lesson 1, with RL 1 post circled to highlight the uptick in individual representations.

As shown in Figure 6-3, use of both representations increased in the post-lesson discussion—correlating with a longer conversation that itself included more-than-usual micro-activities. However, the ratio of individual-to-group representations also increased in the RL 1 post-lesson discussion, following a steady decrease in that ratio starting in Session 4. The ratio of individual-to-group representations starting in Session 4 is shown in Table 6-1, with the ratio in the RL 1 post-lesson discussion highlighted.

Table 6-1: The ratio of individual-to-group representations from Sessions 4 to 8, including RL 1.

Session	Ratio of individual to group representations
4	2.33
5	0.625
6	0.29
RL 1 pre	0.125
RL 1 post	**0.67**
7	0.33
8	0.375

As shown in Table 6-1, the ratio of individual-to-group representations steadily decreased from Session 4 through the Research Lesson 1 pre-lesson discussion, from more than two to 0.125. However, it increased to 0.67 in the post-lesson discussion. Thus, quantitative evidence

shown in Figure 6-3 and Table 6-1 indicates that group representations of perseverance were particularly salient to the teachers in the post-lesson discussion following Research Lesson 1, even as group representations of perseverance had been and continued to be more salient. For these reasons, Research Lesson 1 merits further analysis.

The case analyzed here from the Research Lesson 1 post-lesson discussion illustrates how participants used individual representations of perseverance in this conversation. This case also points to two of the five patterns described in this chapter: that the teachers tended to turn to individual representations *first*, before reasoning in favor of group representations, and that individual representations were used to explain students' failures to persevere.

Case analysis: Making sense of why students do and do not draw on resources in the Research Lesson 1 Post-Lesson Discussion

In the post-lesson discussion following their first research lesson, the teachers debriefed the data they collected while watching the research lesson. They also generated questions about what they had observed and discussed one of those questions. This case focuses on their whole-group discussion of the question they chose. All four teachers were present, as were Elizabeth, the facilitator, and Anna from the research team. In addition, there were several visitors: Lucy, a science coach based at the school; Catherine Lewis, a member of the research team and an expert in Lesson Study; and Lara, at the time a doctoral candidate in math education and an expert in teacher learning and collaboration in math classrooms. Lara also contributed to the development of the TRU Dimensions and TRU PD tools. Lara was the final commentator. In lesson study, the final commentator's primary role is to deliver reflections at the end of the post-lesson discussion. The final commentator highlights observations and conclusions essential to the research theme and theory of action. The final commentator can also contribute during the pre- and post-lesson discussions, as Lara did.

Overview of discussion leading up to the analyzed case

The participants began the post-lesson discussion by eliciting initial reactions to the lesson from the teachers who had co-taught it, Soledad and Leo. Soledad and Leo's initial thoughts were largely negative. Soledad had forgotten to transition the students from the group-work part of the activity to a gallery walk, during which the students were to give each other feedback on posters produced in small groups. Soledad shared her disappointment in herself at having failed to set the timer that would have reminded her to make the transition. She also shared that the students had gotten stuck in places in the task where she and Leo had not anticipated that they would get stuck. Leo shared a more upbeat claim that none of the students had given up on the task despite its challenges.

Then Elizabeth prompted the participants to individually organize the notes they had taken during the research lesson. The participants then discussed their observations in small groups of three. Finally, participants shared key claims and questions from their small-group

discussions with the whole group. Jamie, speaking for his group consisting of himself, Anna, and Lara, offered a question about why some students had drawn on many material resources available in the classroom as they struggled with the task, while other students had not. Jamie's question and other questions shared by the participants were recorded on a centrally-viewable white board.

After each small group of participants shared claims and questions, and some of the claims and questions were discussed by the group, Elizabeth asked Soledad and Leo to choose one question from the list for the participants to discuss in more depth. Case analysis picks up here.

Soledad choses Jamie's question, and Lara suggests a more group-oriented phrasing

After conferring with Leo about which question to choose, Soledad named Jamie's question about students' resource use for the whole group to discuss.

1 Soledad: So, why do some students go to resour— um, go to resources and why do some not.

Soledad's phrasing of the question was broad, but it implicitly represented students' use of resources as an individual choice, not situated in any context. Seemingly picking up on this implicit representation, Lara, the final commentator, suggested a different way to phrase the question. She suggested that they think about "what supports students to draw on resources" instead of why some students drew on resources and others did not. Lara's rephrasing explicitly situated students' drawing on resources in classroom interactions.

In response to Lara's suggestion, Elizabeth indicated that her phrasing might be a useful addition to the original question because thinking about how to support students to draw on resources was "where we're gonna go next." But Lara further explained her rephrasing, arguing that simply asking why some students did not go to certain resources might lead to speculative and unfruitful individualist thinking.

2 Elizabeth: We wanna start also thinking about the supports because that's where we're gonna go next.
3 Lara: Or maybe something like, for me it's helpful to be like, when students are
4 Elizabeth: Yeah.
5 Lara: drawing on resources? How and why. Like what are, what's getting them to do that.
6 Elizabeth: Mmhmm.
7 Lara: Rather— you know, I feel like sometimes if I just say the why are some and why are some not, it'll lead me to places that aren't that useful. Like, because some students are having a bad day, or whatever, which we don't, you know.

		Whereas, like, when it's happens well, like why is it happening well, or what is supporting
8	Elizabeth:	Yeah.
9	Lara:	it to happen well. You know what I mean? I feel like that might be, like, generative to a start on that question.
10	Elizabeth:	That's helpful. Let's do some little think-time.

In this exchange, Lara justified asking the question "what's getting them [students] to" draw on resources, instead of "why do some students… go to resources and why do some not" because responses to the latter question could include claims not situated in the classroom environment. Lara gave an example of such a claim, which had an individualist and deficit orientation (turn 7): "Like, because some students are having a bad day, or whatever." She argued that thinking about "what is supporting" students to draw on resources could be more "generative."

Elizabeth agreed to Lara's rephrasing and prompted the participants to think about the question for a few minutes. Think-time was interrupted by a brief discussion about whether asking the teacher a question counted as drawing on resources. The participants decided that it did not.

Finally, Elizabeth invited the participants to share "what supports help students draw on resources".

Cindy claims that students drew on the resources that they had made themselves

Cindy went first. She noted that the students she observed had looked to material resources, such as notes and posters, that they themselves had made. Soledad quickly agreed.

11	Cindy:	Um. They, they may, they wrote their notes. They made the posters. Like, the people who were drawing on the resources are the people who made the resources.
12	Lucy:	Hmm!
13	Soledad:	Yeah. So like if they had the worksheets and they're able to pull them out. But if they'd, they'd not done their work, if they'd thrown it away
14	Cindy:	Mmhmm.
15	Soledad:	then they don't have those to draw on anymore?

Soledad continued by sharing a specific example of a time when she saw a student using a resource that she herself had made. Cindy and Soledad's claim, supported with evidence from the Research Lesson, did not draw on either individual or group representations of perseverance. While it was a claim about individual students, it did not attribute successful use of resources—and, implicitly, perseverance—to either individual character/ acontextual choices or interpersonal interaction.

Leo adds an individualist interpretation

However, Leo then added reasoning to Cindy and Soledad's claim that drew on a representation of perseverance as individual students' character and acontextual choices. He hypothesized that some students failed to draw on resources such as notes because they were too disorganized to keep track of them. Cindy, Soledad, and Lucy verbally or physically indicated their agreement.

16	Leo:	And I think related to that is organization?
17	Cindy:	[Nods]
18	Lucy:	Mmhmm.
19	Soledad:	Yeah.
20	Leo:	Like I think, I think some— I didn't, I didn't see it but I think some kids probably had their stuff, it's just [throws up his hands] in a jumble, so they were like, "I'm not gonna get that."

Leo's argument drew on a trait-based representation to argue why some students failed to persevere by drawing on resources. Notably, he made this argument without direct evidence. As he said in turn 20, he had not witnessed the asserted disorganization first-hand.

Lara pivots to a group-oriented interpretation

Lara quickly offered her own interpretation. Beginning her argument and reasoning with the word "But," to indicate that it countered Leo's argument, Lara emphasized the positive aspect of Cindy and Soledad's observation, that students had used resources that they had made. In her reasoning, Lara implicitly drew out that students finding and using material resources that they had made was an interactive feat.

21	Lara:	But I, I, I was interpreting something related, that maybe you [points to Cindy] meant, that wasn't just about if they had their stuff. Or like, and I, I hadn't noticed this before, so thank you. I think the group, they were, they, that I was watching was looking at the posters on walls, on the walls. And I think the poster that they were most looking at was the one they had made.
22	Jamie:	That they'd made.
23	Cindy:	Really? [Laughs]
24	Catherine:	Whoa!
25	Lara:	Is that what you were saying?
26	Cindy:	Well, I, I, kind of. But. Yeah! That— she [inaudible, because Lara began to talk]

27	Lara:	Because if they have work on the wall, they know that it's there, they know it's—
28	Cindy:	And what she said—
29	Lara:	there, they know it contains something useful.
30	Cindy:	She said, "I think I did that one?" Like she doesn't, she didn't remember if, if she did that one or not? [Laughs]
31	Lara:	That's super interesting to me, like,
32	Elizabeth:	Mmhmm.
33	Lara:	it makes a lot of sense, I hadn't thought of it that way. Right? That they would know it's there, know it's useful, and be ready to orient to it if they'd made it.

In these turns, Lara answered the question she had posed, "what supports students to draw on resources". Her answer outlined a series of interactions between students, material resources, and, implicitly, teachers that supported the students to use posters during the research lesson. Namely, the students looked to posters with which they had a relationship, because the students had made those posters. That a teacher had hung posters made by the students in the room for students to use during future lessons was an implicit but important interactive support.

Lara positions student collaboration as a resource

Then Lara asked the participants a question: What did they mean by resources? She explained how she saw resources as not just material resources, such as notes, but also students explaining their thinking and asking each other questions. She used student conversation drawn from her observations to explain her reasoning.

34	Lara:	I was also wondering a little bit about what we mean by resources? Because I saw some really rich drawing on what I call resources, that had nothing to do with notes?
35	Elizabeth:	Mmhmm.
36	Lara:	Or, like, information? But were, like. So I saw some. "When, when I'm stuck I can ask you to explain it again." That's a resource.
37	Cindy:	Mmhmm. [Nods]
38	Lara:	Right? Um. A res— I know that your explanation, my teammate, your explanation is a resource to me. So I know that I can ask for it. When I— even if you maybe just said it. But if I'm, like, not there yet, I can ask. That was certainly being drawn on. Um. There was the question, "How do you know that?" Was a resource for them, they were drawing on it, they were using it, and it was getting them further. When they said, "How do we know that?" the question, the, the conversations that came after that question often,

> in many cases got them to a new place. That they hadn't been. Even if the person who was answering the question thought they were just restating something they had already said.

While perseverance as a group accomplishment had been subtle in Lara's reasoning in turns 21-33, here it took center stage. In these turns, Lara argued that students asking other students to explain their thinking constituted drawing on a resource and, implicitly, persevering. Lara provided a replay vague on details but rich with reasoning to explain how students asking each other, "How do we know that?" supported the students to build new and important mathematical knowledge together. Lara's turns here constitute a robust representation of perseverance as a group accomplishment.

Transition to and summary of Lara's final commentary

After Lara's final turn, Elizabeth wrapped up the conversation. She transitioned to Lara's final commentary. In her final commentary, Lara shared that she thought the students were "brilliant." She drew attention to the word "together" in the teachers' research theme. She went on to highlight how the students she observed had used questions probing each other for explanation as resources, thereby persevering together. She also discussed an observation she had made that students grew less confident with the task and became more dependent on the teacher after they spoke with a teacher. This observation further highlighted for her how Soledad's students persevered by working together. It also highlighted for her how the presence of a teacher changes group dynamics among students.

Discussion of Case 1

This analysis highlighted how the teachers returned to individual representations of perseverance amidst a general shift towards group representations of perseverance. Three findings stand out in this analysis.

The teachers turned to individual representations of perseverance as a first resort.

In this conversation, the teachers offered questions or claims about students' perseverance that initially drew on individual representations of perseverance. Soledad's phrasing of Jamie's question about students drawing on material resources highlighted use of resources as an acontextual choice. Later in the conversation, Leo claimed that some students were disorganized to reason about why some students did not use resources that they had made. In both of these instances, the teachers navigated towards individual representations of perseverance *first* when attempting to make sense of students' perseverance during the research lesson.

Turning to individual representations as a first resort was a pattern across the sessions. Chapter 4 pointed to other examples of the teachers using individual representations to make initial claims about students' perseverance in several instances, ranging from interpretations of students' behavior captured by a video to reasoning about the goals of a planned intervention. In the case analyzed in this chapter as well as the cases from Chapter 4, collaborative reasoning eventually illuminated the explanatory power of group representations. However, that the teachers tended to look to individual representations first attests to the common-sense nature of individualist discourses.

Individual representations helped the teachers explain students' failures to persevere.

Leo's reference to students' character deficits—that they were disorganized—to explain why some students did not draw on resources and, implicitly, persevere, stands out in this case. In arguing that some students were too disorganized to find and use material resources that they themselves had made (turns 16-20), Leo named a disadvantageous character trait to explain those students' lack of perseverance. Leo admitted that he had no evidence for his claims. His claims also did not answer the question that the teachers were supposed to be discussing, "what supports students to draw on resources," a question that intentionally pivoted away from individualist responses. Lara's response to Leo's claim looked to students' strengths and interactional supports for perseverance.

This finding, that much of the explanatory power of individualist discourses was tied to deficit-oriented claims, is further elaborated in the two cases that follow the one analyzed in this section.

The uptick in individual representations in the RL 1 post-lesson discussion was not an anomaly.

Quantitative evidence presented at the beginning of this section pointed to a decisive uptick in use of individual representations of perseverance in the post-lesson discussion following the first research lesson. As shown in Table 6-1, reproduced here, the ratio of individual-to-group representations increase in the RL 1 post-lesson discussion after having steadily decreased, and then decreased again following it.

Table 6-2/6-1: The ratio of individual-to-group representations from Sessions 4 to 8, including RL 1.

Session	Ratio of individual to group representations
4	2.33
5	0.625
6	0.29
RL 1 pre	0.125
RL 1 post	**0.67**
7	0.33
8	0.375

However, case analysis uncovered patterns in the participants' uses of individual representations of perseverance that appear across the sessions. That patterns in how the participants turned to individual representations of perseverance in the RL 1 post-lesson discussion match patterns across other sessions indicates that discussion in the RL 1 post-lesson discussion was not anomalous. It was, in fact, indicative of the teachers' tendencies to return to individual representations of perseverance as they pursued their inquiry together.

The next two cases pursue those tendencies further.

Findings 2, 3, and 4 in Session 1: Perseverance, independence, collaboration, and preparation for the modern economy

During the first session, the teachers discussed why they chose to focus their first two lesson study cycles on supporting students' perseverance. While they did not return to this discussion in subsequent sessions, their description of their goals set the tone for their subsequent inquiry. In this discussion, the teachers claimed that their students needed to learn to persevere and work together in school so that they would be prepared to participate in the modern economy. At the same time, the teachers claimed that students needed to learn to be independent and asserted that this was something students could not currently do. All four teachers were present, as were Elizabeth, the facilitator, and Anna and Matt from the research team.

Elizabeth asks the teachers why their research theme is important

After they had arrived at the research theme, "How do we build student perseverance and capacity to struggle productively, together?" (described in Chapter 4) and agreed that encouraging students to explain their thinking would support perseverance, Elizabeth prompted the teachers to think about why their research theme was important to them.

Elizabeth: So now maybe we should go through and think about, like, why these things are important to us?...Is that something that we want them to be able to do when they leave Benton?

Elizabeth's prompt was derived from framing typical to certain traditions of Lesson Study (Lewis, 2009), which guide teachers to think about their students' development and lives beyond the classroom. In her prompt, Elizabeth indicated that the teachers should think about their goals for their students after they left school. She explicitly represented perseverance as an out-of-school skill, implicitly drawing out the importance of perseverance for work.

Leo emphasizes the importance of independence outside of school

Leo picked up on her question, answering that students needed to learn to justify their thinking on their own in school because outside of school, they would not have teachers to prompt them to do so.

Leo: Yes. Because they won't have us to tell, um, they won't have us to tell them what to do anymore. So they have to be able to do it themselves.

In his response, Leo built on Elizabeth's initial representation of perseverance as a skill that should be learned in school because it would be required outside of school. At the same time, he represented it as normal in school for teachers to "tell" students "what to do". This statement implicitly represented students as without agency and needing strong scaffolding from teachers, a deficit-centric representation. Again implicitly, Leo represented perseverance as independence, in this case from an authority such as a teacher. Leo's answer implicitly represented the world outside of school as a place for independent work, in which overly dependent people do not thrive.

Soledad represents working with other people as an important out-of-school skill

Next, Elizabeth asked the teachers about the "together" part of their research theme. Soledad drew out the connection between perseverance, the skill of collaboration, and thriving in the world outside of school in her subsequent response.

Elizabeth: Why is it important to us that students work together?
Soledad: That's the reality out there. [Laughter] Right? Not going to be able to, when they're out there working or doing anything, they're not gonna be alone, right, then?
Jamie: Mmhmm.

Soledad: So when, teach them, teach them those skills, um, how, how to work with people effectively and inter-connectedly.

Elizabeth: Um like just to see each other as resources. Not that like you have to do it all on your own.

Elizabeth's question, highlighting part of the teachers' research theme, and Soledad's response drew out a tension between two seemingly conflicting views of the world outside of school. In Leo's representation, the world outside of school required independence, namely from an authority. In Soledad's representation, the world outside of school required people to work together "effectively". Despite this seeming tension, which will be taken up further in the discussion of this part of the chapter, both Soledad and Leo represented perseverance as necessary in the real world and, implicitly, in the workplace.

Elizabeth revoiced Soledad's claims, although in revoicing she altered the representation Soledad had constructed. Elizabeth did not represent working together as required by work in the real world. Instead, she implicitly represented working together as adding quality to life outside of school, saying that students might "see each other as resources." She also implicitly represented working together as something that students might choose to do, saying that students do not "have to do it all on your own."

Jamie returns to independence

Jamie then returned to Leo's initial representation of perseverance as independence.

Jamie: Well and that they don't have to rely on an authority or an expert. So-called authority or expert.

Leo: Yep.

[5 second pause]

In this exchange, Jamie corrected Elizabeth's revoicing of Soledad's claim. He highlighted the importance of students being able to apply the skills of justifying, perseverance, and working together without relying "on an authority or expert," continuing the theme first raised by Leo. Leo immediately said, "Yep," explicitly affirming the content of Jamie's claims and, implicitly, his correction of Elizabeth and Soledad's representations of the working world as requiring collaboration.

Cindy aims to build students' positive mathematical identities

After a pause, Cindy described her goals. She hoped that through seeing other students justify mathematical thinking, students would come to see themselves as mathematically capable.

Cindy: I think it also points to, like, like, back to that dimension, like, AOI? Like, I, I think if they see each other as resources, then they see each other as, like, "Oh, you, you know the math. And I know some of the math, too!" Like, hopefully that bounces back.

Elizabeth: Mmm.

Cindy returned to Elizabeth's representation of working together as students "see(ing) each other as resources." In doing so, she built on Elizabeth's representation of perseverance-as-collaboration as adding value to students' lives. Cindy continued in that vein by elaborating on how students' senses of their own value and mathematical capability could be enhanced by seeing each other as resources. Cindy thus provided an enhanced representation of perseverance as a group accomplishment and how collaboration could improve students' lives, unrelated to work.

At this point Elizabeth transitioned the conversation to the next topic, choosing a pedagogical strategy to try in their classrooms to begin their inquiry.

Discussion of Case 2

This case highlights how many representations and themes were at play when the teachers talked about the importance of building their students' perseverance, even during the first session. Both individual and group representations of perseverance surfaced during this brief conversation. Perseverance as independence *and* as collaboration, students as lacking agency *and* as needing to work without help, perseverance as necessary for work in the real world *and* as adding value to students' senses of self, were all raised in this exchange. The teachers' claims and representations were fragmented and often in tension, even though that tension was not explicitly addressed.

Despite the diversity of claims and representations, two patterns in the teachers' use of individual representations stand out.

The teachers represented perseverance as valuable outside of school, specifically at work.

Many of the teachers' contributions, encouraged by Elizabeth's initial question, took up perseverance as valuable in the real world and at work. Soledad most clearly tied perseverance to work when she claimed that students needed to learn to work together "effectively," implying effectively for their jobs. While Leo did not explicitly name the workplace as a site where perseverance would be valuable, he claimed that perseverance-as-independence was required outside of school but not in school. In Leo's representation, students were dependent on teachers in school. Soledad continued his and Elizabeth's theme of perseverance outside of school by explicitly extending it to work. As described in the beginning of this chapter, a common trend in

traditional education discourses and broader ideologies is to value math education for the workplace preparation it provides to individual students (Apple, 2006; Valero, 2017), rather than for humanistic reasons (R. Gutiérrez, 2018). Soledad explicitly and Elizabeth and Leo implicitly drew out these traditional values in their explanations of why they wanted to build students' perseverance.

Perseverance as independence was dominant.

Leo and Jamie together developed a representation of perseverance as independence. They did so forcefully, as Jamie explicitly rejected the group representation of perseverance that Soledad and Elizabeth had begun to build. Leo drew on a deficit-focused representation of his students as currently unable to work independently, characterizing them as being told what to do by teachers. Notably, while Soledad represented perseverance as students working together, her phrasing and reference to an effective workplace hinted that for her, "working together" meant individuals working side-by-side, rather than truly collaborating. A representation of "working together" as cooperation or concatenation, rather than collaboration, is consistent with an individualist-centered discourse. In the technology-driven modern economy, each individual brings their own expertise and skill to the larger enterprise. Success and failure are not distributed among all; rather, the individual as part of a whole is centered in success and failure. Hints of this individual-focused discourse appear in Soledad's representations of perseverance and working together. Cindy represented working together in a more group-oriented sense, however, when she picked up Elizabeth's revoicing of Soledad's claims. Cindy elaborated on how students working together come to see each and other and themselves as mathematically capable. None of the participants engaged with Cindy's claims, however.

Notably, the notion of independence from an authority that Leo and Jamie began to develop in this first session did not rule out the possibility of students persevering alone *after* helping each other or collaborating, as in co-constructing knowledge in partnership. Soledad, Elizabeth, and Cindy's claims were not inconsistent with Leo and Jamie's, although Leo and Jamie positioned their claims as more important. Furthermore, Leo and Jamie's attention to independence was not inconsistent with the pattern identified in Chapter 4, that the teachers over time came to see the importance of students building on each other's thinking to students' perseverance. This compatibility of representing perseverance as independence with supporting students to build on each other's thinking will be further discussed at the end of this chapter. Nonetheless, Leo and Jamie's attachment to perseverance as independence from an authority at times interfered with their sense-making around group representations of perseverance. The case described in the following section, from Session 4, highlights the teachers' treatment of perseverance as working without help from an authority and how the individual representations of perseverance involved in developing that idea undercut the teachers' efforts to build perseverance as a group accomplishment.

Findings 3, 4, and 5 in Session 4: Perseverance means not needing help, and individual representations and failure

As the previous case began to highlight, perseverance as independence or not needing help was a major theme when the teachers turned to individual representations of perseverance. In the previous case, for example, Leo and Jamie drew out the importance of students learning to work without relying on, as Jamie put it, "an authority or an expert." The notion of perseverance as the ability to work and overcome challenges without help from "an authority" recurred when the teachers drew on individual representations of perseverance. The following case from Session 4 draws out that pattern in how the teachers used individual representations of perseverance. It also highlights how representing perseverance as independence from authority constrained the Benton teachers' roles as mentors in their classrooms.

The following case also exemplifies another trend in how the teachers used individual representations of perseverance: drawing on individual representations to explain lack of perseverance. Quantitative evidence and case analysis illustrate this pattern.

Quantitative evidence: The salience of individual representations to explain failure

The teachers drew heavily on individual representations of perseverance when discussing evidence of students not persevering and analyzing why students might not persevere. Table 6-3 shows the number of times teachers used individual and group representations of perseverance when sharing classroom events or discussing video, categorized by whether teachers used the representations to explain lack of perseverance or perseverance without valence.

Table 6-3: Individual and group representations of perseverance when the teachers discussed evidence of students failing to persevere, while sharing classroom events and discussing video.

Students not persevering		**All references to perseverance**	
Individual	*Group*	*Individual*	*Group*
21	10	38	38

As shown in Table 6-3, across the eight workgroup sessions, the teachers drew on twice as many individual representations of perseverance as group representations of perseverance when sharing or discussing what in the teachers' eyes was evidence of students not persevering. However, individual and group representations of perseverance were evenly balanced. Table 6-3 indicates that the teachers found explanatory power in individual representations of perseverance when describing and making claims about students' *failure to persevere*. Chapter 4 illustrated how the teachers came to use group representations of perseverance when describing and making claims about students' *successful perseverance.* These two patterns in the teachers' uses of different representations of perseverance are not in direct conflict, much as the teachers'

tendency to represent perseverance as independence is not in direct conflict with their coming to value students building on each other's thinking. The compatibility of, or at least lack of glaring tension between, these otherwise disparate ways of representing perseverance will be further discussed at the end of this chapter. The case from Session 4 discussed in this section exemplifies the tendency of the teachers to return to individual representations to make sense of students' failure to persevere.

In the activity in the following case from Session 4, the teachers described and interpreted what happened in their classrooms after trying a group-oriented intervention planned in Session 3 (analyzed in Chapter 5). As they shared replays and claims, the teachers did not address students persevering by building on each other's thinking. Instead, they drew on representations of perseverance as individual students' ability or choice to work. Their discussion highlighted the importance to them of students working independently. It also highlighted the teachers' reliance on individual representations to make sense of why their students did not persevere. The remainder of this section analyzes their conversations in this session.

Case analysis: Discussing the results of a group-oriented intervention in Session 4

This case from Session 4 spanned twenty minutes of workgroup conversation. (A full transcript with line numbers is provided in Appendix A). All four teachers were present, as were Elizabeth, the facilitator, and Anna and Matt from the research team.

Overview of conversation before the analyzed case

The teachers began Session 4 as they began many sessions, by describing and interpreting what happened when they tried the intervention they had planned in the previous session. In Session 3, described in Chapter 5, the teachers had committed to trying an intervention that they hoped would support students to persevere by building on each other's thinking. They agreed to try the strategy 'Three Things' (i.e. ask students to share three things they know and three things they want to know about a challenging math problem) with groups of students or in whole-class discussions. The teachers called this variation on strategy "Three Things in a public setting." Cindy suggested this variation and explained how students building on each other's thinking could support perseverance. The other three teachers enthusiastically agreed to it. They agreed to wait for several students to share something that they knew. In using this strategy, the teachers hoped that students would come to see themselves and others as valuable contributors, much as Cindy had hoped in Session 1. They also hoped that students would use each other's ideas to continue working on challenging mathematical tasks.

The teachers write reflections about what happened when they tried Three Things in a public setting

Elizabeth launched Session 4 by prompting the teachers to type into a shared document their observations from trying Three Things in a public setting. The document, shown in Table 6-4, prompted the teachers to describe "what happened" when they tried the intervention and to elaborate on the "impact" they saw along the TRU Dimensions. The teachers had available brief descriptions of the dimensions, shown in Table 6-5. She then gave the teachers a few minutes to read what was written.

Table 6-4: Teachers' written reflections on what happened when they tried Three Things in a public setting.

Name	**What happened when you tried 3 Things in a public setting (in front of the whole class or in front of a small group)? How did students react?**	**What impact did you see in the different TRU Dimensions? (Example: How did Three Things impact student Access?)**
Leo	Slightly more responses, non-regular participators participated. Providing follow-through on incorrect information helped students further thinking	Students were able to articulate something that they knew and thought was important. [in contrast, when i asked for answers, i would get one, and others wouldn't answer if they had something else.] Students seemed more likely to follow-up with explanation or next steps, if they had made an initial suggestion.
Cindy	In small groups, some students offer neighbors help.	Equitable access
Jamie	In Algebra 2, in a small group setting, students were more willing to surface their background knowledge and problem-solving process. This gave me an opportunity to ask specific questions targeted to the needs of the group	Cognitive Demand: Keeping the rigor sufficiently high without having to provide just-in-case scaffolds.
Soledad	In small groups, students responded positively. Sometimes a groupmate supported and added on the each other's answers. In the smaller setting, though i tried to also remember to face opposite the questioner and this may have encouraged whole group to engage in question.	Students engaged. There were still some students who i could not get to engage (those that seemed that they rarely if had ever engaged.)

Table 6-5: The TRU Dimensions, as the Benton teachers had them during Session 4.

The Five Dimensions of Mathematically Powerful Classrooms				
The Mathematics	**Cognitive Demand**	**Access to Mathematical Content**	**Agency, Ownership, and Identity**	**Formative Assessment**
The extent to which the mathematics discussed is focused and coherent, and to which connections between procedures, concepts and contexts (where appropriate) are addressed and explained. Students should have opportunities to learn important mathematical content and practices, and to develop productive mathematical habits of mind.	*The extent to which classroom interactions create and maintain an environment of productive intellectual challenge conducive to students' mathematical development. Students should be able to engage in sense-making and productive struggle.*	*The extent to which classroom activity structures invite and support the active engagement of all of the students in the classroom with the core mathematics being addressed by the class. No matter how rich the mathematics being discussed, a classroom in which a small number of students get most of the "air time" is not equitable.*	*The extent to which students have opportunities to conjecture, explain, make mathematical arguments, and build on one another's ideas, in ways that contribute to their development of agency (the capacity and willingness to engage mathematically) and ownership (recognition for being mathematically solid), resulting in positive identities as doers of mathematics.*	*The extent to which the classroom activities elicit student thinking and subsequent instruction responds to those ideas, by building on productive beginnings or addressing emerging misunderstandings. Powerful instruction "meets students where they are" and gives them opportunities to deepen understanding.*

In their reflections, shown in Table 6-4, the teachers described increased student participation. Leo and Jamie described being able to engage in formative assessment as students shared more thinking, correct and incorrect. Cindy described students offering each other help, while Soledad highlighted that students sometimes "supported and added on" to each other's ideas. Only Soledad's reflections directly responded to the intent behind the intervention, to support students to persevere by building on each other's thinking.

The second column in table 6-4 prompted the teachers to connect their observations to the TRU Dimensions. Only Cindy and Jamie named a specific dimension, and only Jamie provided a normative explanation of the dimension in connection to his observations. Jamie described how students' sharing their thinking helped him provide support that the students actually needed and keep "the rigor sufficiently high without having to provide just-in-case scaffolds." While Cindy did not elaborate on her reference to the Equitable Access dimension, her reference to it was tacitly individualist in that it framed the benefit of students' offering each other help as landing on individual students' access to content. It is unclear how Leo and Soledad were thinking about the TRU Dimensions as they answered the questions in the table because they did not refer to any specific dimensions.

As the teachers finished reading, Elizabeth invited them to discuss what they noticed and wondered about their students' perseverance.

The teachers make claims about individual students' participation patterns

Jamie began the discussion. He commented that he felt that Three Things in a public setting had encouraged more students to verbally engage. He described his comment as building on what Cindy had written.

1	Jamie:	I know Cindy put Equitable Access. It feels like almost everyone is responding in a way that's showing that more students are open to participating verbally?
2	Elizabeth:	Mmm.
3	Jamie:	And sharing ideas?
4	Elizabeth:	Yeah. Cindy, I was wondering if, like, if that translated then to on paper, did you see, you, like, they were talking about it? Did you also see that translate to their, like, work product?
5	Cindy:	Mmm? A little bit. Yeah. [Nods]

Jamie's comment did not clearly build on Cindy's written reflection (shown in Table 2), "In small groups, some students offer neighbors help." Cindy's written reflection referred to the stated goal of the Three Things intervention, albeit not closely, turning the goal of supporting students to build on each other's thinking into a weaker but still important goal of supporting students to help each other. Instead, Jamie's comment pivoted the discussion towards verbal participation, broadly represented, without attention to students interacting with each other.

Soledad then added on with her own observations. She agreed with Jamie that more students had participated but added that the strategy still failed to engage some students. In her interpretation of why some students still failed to engage, Soledad highlighted individual students' choices to not participate.

6	Elizabeth:	Other noticings?
7	Soledad:	Um. So my noticing was that for, like, the students that were kinda borderline, like, sometimes would, like, show with body-language or, like, kinda, like, s-, like, kinda semi-engaged, that sometimes, like, it got those students? But the students who were the heads-down, the, um, you know, a bit more resolute in their th— it seemed like they're more resolute in the thought that, "I am not going to engage,"
8	Elizabeth:	Mmhmm.
9	Soledad:	"like, no matter what." That I, does, it still did not capture those students. That the ones that were like on the border [nods], it engaged those students.
10	Elizabeth:	[Nods] Okay. That seems like a common, like a, a tension we keep having in these meetings? That this is addressing one type of disengagement
11	Anna:	Mmhmm.
12	Elizabeth:	but not total disengagement. From.

13 Jamie: Hmm.

14 Elizabeth: And I'm thinking back to our first meeting where Cindy was sharing one of her struggles with the students who just don't [moves hands up and down and grimaces] engage at all. Okay.

Soledad's claims about why students did not persevere drew on individual representations of perseverance. When describing evidence that students were not persevering [turn 8], Soledad referred to individual body-language signs. She described students whom the intervention did not reach as "more resolute in the thought, 'I am not going to engage" [turn 7], representing their lack of perseverance as a willful and acontextual choice. Implicit in Soledad's argument was the claim that some students are inherently unreachable. In turn 14, Elizabeth added on to Soledad's claims with a recollection of Cindy's replay from a previous session of students "who just don't engage at all." Intentionally or not, Elizabeth's contribution reinforced the implicit narrative that perseverance and lack thereof are immutable character traits.

Leo pivots to a discussion of perseverance as independence

Next, Leo offered his interpretation of what happened when they tried Three Things in a public setting. Leo began by agreeing with Jamie and Soledad that using Three Things in a public setting had increased students' engagement. However, he argued that this increased engagement had come at the cost of perseverance. His claims framed the theme that perseverance is independent work.

15 Leo: So in my mind persistence is, you get stuck. And then you try something else. Or you do, there's some strategy that you use. But we're providing a way to, to avoid that resistance. To avoid the wall. So you don't necessarily see the persistence.

16 Elizabeth: Hmm.

17 Leo: You know what I mean? So. Once the strategy, once they, like, offer Elianna [name of student] as help, then they get stuck, then we can see the persistence. But. I mean, I think this is great. They're participating more. Um. But for me I just don't see the persistence.

In this exchange, Leo shared his view that by prompting students to share what they knew, the teachers had helped students bypass the moment when individual students would "get stuck" and need to get themselves unstuck. Leo claimed that in using Three Things to encourage students to share what they knew in front of the whole class, teachers were "providing a way to avoid that resistance" (turn 15). Leo's representation of perseverance was two-fold. First, he represented perseverance as students getting unstuck. By offering a way for students to "avoid the wall," Leo claimed that the teachers were removing the struggle to get unstuck essential to

perseverance. Second, he represented interaction and collaboration among students as getting in the way of perseverance, here by making it possible for students to continue working without first getting stuck. In essence, Leo claimed that if students got unstuck in the course of interacting with other students or a teacher, they had not persevered. In Leo's telling, perseverance only happens when students struggle alone.

Leo's representation of perseverance as individual students struggling to get unstuck raised an important potential problem with the Three Things strategy, that it may have removed opportunity for students to struggle productively with challenging math problems. Leo did not provide a replay or other evidence grounded in classroom experience to justify his claim, however, so precisely what impact the Three Things strategy had on productive struggle was not transparent in Leo's argument. However, Leo's individual representation of perseverance overlooked how productive struggle can be uncovered when students share their thinking while interacting with each other. Leo claimed that the Three Things strategy prevented him from "see(ing) the persistence" (turn 15). However, the Three Things strategy had the potential to encourage students to recount productive struggle, including productive struggle that happened even when students did not get noticeably "stuck" while solving a problem. His individual representation of perseverance also overlooked how collaboration expands students' individual and collective zones of proximal development (ZPDs), supporting productive struggle that would not be possible for students working alone (Goos et al., 2002).

Elizabeth next invited the other teachers into the conversation. In the exchange that ensued, Jamie asked Leo to clarify what he meant by persistence, drawing out the individual aspect of Leo's argument. Leo expanded on his claim that perseverance is working through struggle alone, without support.

18	Jamie:	When you— when you're saying persistence in this context, Leo, you mean, like, your own self-management?
19	Leo:	When I think persistence, I think, "I'm stuck. What can I do?"
20	Jamie:	Mmhmm.
21	Leo:	Like, there's an obstacle in front of me. How do get around this, how do I work around this. What we're providing is a way around it without having them stop. You know what I mean?
22	Jamie:	Mmhmm.
23	Leo:	So they're not coming up with the strategy, they're just using what they know. Which is great. I don't know if that. [Sighs] So not bad. It's just, it doesn't address the persistence.

Here Jamie focused on the individualist component of Leo's representation of perseverance by asking Leo whether or not he thought perseverance was "your own self-management" (turn 18). While Leo did not explicitly affirm Jamie's interpretation, his elaboration included a rehearsal of how an individual student "self-managed" getting unstuck. As

part of his response to Jamie's question, Leo argued that participation after getting help and without first getting stuck was not perseverance. He said (turn 21), "What we're providing is a way around it without having them stop…. So not bad. It's just, it doesn't address the persistence." Then added that students were "not coming up with the strategy" (turn 23), implying that perseverance required each student to generate strategies themselves. In this way, Leo and Jamie co-constructed a representation of perseverance as independence.

Soledad, Jamie, and Elizabeth continued to ask Leo questions to better understand his argument. They attended to Leo's claims about the connection between "stuckness" and perseverance. They problematized Leo's claim that perseverance required students to first "get stuck." Soledad asked, "And how about if they [students] don't ever come to a complete stop, could they still have persistence?" In response to her question, Leo, Jamie, and Soledad rehearsed several forms that students' struggle could take that did not involve, as Leo called it, "a debilitating stop." In his comments on Soledad and Jamie's rehearsals, Leo emphasized the importance of struggle. He also implicitly reinforced the representation of perseverance as individual, independent work.

24	Leo:	Like, "I don't know what to do next?" [More laughter] I think about it. "Oh, I can do this."
25	Jamie:	Mmhmm.
26	Leo:	S- so, just at a point where the tools are not, like, immediately come to mind? And they need to struggle a little bit for that. Because I think about like, you know, you can coast through high school. Are you persisting? If you have straight As, are you persisting? Maybe, like, not really? But when you get to college, you're gonna get stuck. Right? And, you, you can freak out and fail. So that's not persistence. Um.

Here, Leo emphasized that to count as perseverance, students "need to struggle a little bit" (turn 26). Implicit in his comment was a rebuke of Three Things, that Three Things made it so that students did not need to struggle by offering them a crutch. Also implicit in his argument was the notion that struggle should be independent. Leo conducted his rehearsal of struggle with singular pronouns. Leo's claims continued to overlook how students can be supported to struggle together and students' prior struggle can be made transparent when they share their thinking, as the Three Things strategy encouraged.

Elizabeth prompts the teachers to reflect on what the TRU Dimensions can add to the conversation

Then Elizabeth shifted the discussion. She asked the teachers if they could use the TRU Dimensions to brainstorm reasons why students might persevere. However, when asked to clarify her prompt, Elizabeth restated it in a way that removed the push to use TRU to explain

why students persevere. Instead, Elizabeth's rephrased prompt asked the teachers to describe "different types of perseverance."

27	Elizabeth:	I wonder if this idea of the different reason, like the different ways that students show persis— persistence? If we like looked at the TRU Dimensions, to figure out, like, if those could explain different reasons? Like I could see, Jamie, in your comment around, like some people don't experience persistence at all because they're just, going through? Like, that seems like a Cognitive Demand issue, where the Cognitive Demand might be too low.
28	Jamie:	Mmhmm.
29	Anna:	Mmhmm.
30	Elizabeth:	For that student. But then, could we think of other ways, like. Like an AOI issue that would cause, that would affect student persistence? Or an Access issue? Or is that not a helpful [laughs] place for us to go right now.
31	Soledad:	Wait, can you ask the question one more time?
32	Jamie:	[Laughs]
33	Elizabeth:	Let me see if I understand my own question. [Laughs] Um. So I'm thinking about how we've been talking about this idea of persistence, and how there's different types of persistence. And so I'm thinking about trying to define the different types of persistence? And if the TRU Dimensions can help us define those different types.

In restating her question, Elizabeth shifted it from a causal analysis question to a descriptive question. In turn, she made the question less powerful for the teachers' inquiry into how to support their students' perseverance. The teachers answered neither question. Instead, they made claims about why students *did not* persevere. Their claims attributed lack of perseverance to individual students' abilities and choices and at times drew on non-normative interpretations of the TRU Dimensions, a discussion to which I turn next.

Jamie uses non-normative interpretations of the TRU Dimensions and mindset theory to make individualist claims about students' failures to persevere

Jamie answered Elizabeth's question first. In his responses, he drew on individual representations of perseverance to explain why his students failed to persevere. Jamie began by describing and interpreting student behavior that he claimed showed that some students did not persevere because they had a "fixed mindset" about mathematics. In so doing, Jamie drew on a term, "mindset," that has wide circulation in ambitious mathematics instruction discourses.

34 Jamie: Well so you're, you're kinda referring to, like, the difference between a student that you are perceiving as not perseverant because they have, they have no feeling of agency around mathematics, right?

35 Elizabeth: Mmhmm.

36 Jamie: Like, they have the sense of, [pounds table with fist] "I can't do math. I'm not a math person." Right? They have that very fixed mindset.

37 Elizabeth: Mmhmm.

In this exchange, Jamie represented "fixed mindset" as a static character trait that impeded perseverance. He drew on the AOI dimension as he attributed students having "no feeling of agency around mathematics" (turn 34) to the "fixed mindset" trait. This was a non-normative use of the AOI dimension and a debatably non-normative use of the concept of growth or fixed mindset. While both the TRU Framework and the literature around mindsets (Boaler, 2015; Dweck, 2006) represent students' individually-held dispositions towards mathematics as influential over their mathematical agency, these literatures also elaborate on how mathematical mindsets and identities develop in response to learning environments. In particular, the TRU Framework emphasizes that students' mathematical identities and dispositions are neither static nor universal across all contexts in which students might encounter mathematics (Schoenfeld, 2014). In characterizing fixed mindset as a trait, Jamie tacitly invoked the related concept of "grit," a character trait that some education scholars and reformers argue inclines students who possess it towards resilience in the face of adversity (Duckworth, 2016; Tough, 2013). Grit and growth mindset are conceptually distinct concepts. Grit is explicitly a character trait, while growth or fixed mindset are sets of inclinations and assumptions held by individuals about their capacity to improve when faced with challenges. However, the fuzzy boundary between these concepts in Jamie's talk draws out their shared attribution of students' perseverance or lack thereof to *individually-held* traits or dispositions. Both sets of literature center their nominal individually-held trait or belief system and its asserted positive effect on students' perseverance, while placing the role of classroom context and interaction among students in cultivating the trait or dispositions in a secondary position (Boaler, 2015; Duckworth, 2016; Dweck, 2006).

Jamie went on to contrast "fixed mindset" students with those who had positive mathematical identities but could be intimidated by challenging, unfamiliar content.

38 Jamie: Versus a student who is looking at it and is like, "This is really challenging. This is way in over my head. I don't understand this content."

39 Anna: Mmm.

40 Jamie: Right? Cuz like, I have those kids that, like. We're finishing up stats in Algebra 2. And I think that they actually think of themselves as strong mathematical thinkers, but because statistics has been a hole in their overall mathematical experience, they're looking at this like, "Why are we doing this, this is so hard. I've not had to think about data and numeracy in this way."

That's more like a Cognitive Demand thing because I'm asking them to do something challenging, that they haven't done before. But they've probably experienced success in prior math classes, so they normally would feel like, you know, "When are we going to get to the real math." That's the most common thing that we hear in stats. "When do we get to the real math?" You know, the one that has naked numbers and.

In these turns Jamie added a claim that even students who did not "have that very fixed mindset" (turn 36) were discouraged by unfamiliar content. While he did not cite specific classroom experiences, he enacted student behavior and conversation that he portrayed as derived from his experience with students. He tied this claim to the Cognitive Demand dimension. Attributing lack of perseverance to inappropriate challenge is a normative interpretation of the Cognitive Demand dimension. However, Jamie cast his students' lack of perseverance as willful rejection of unfamiliar content, not inability to struggle productively with unreasonably challenging content. In Jamie's replay, his students asked, "When are we going to get to the real math?" (turn 40), instead of choosing to engage with new material. Thus Jamie continued to leverage representations of perseverance as individual students' abilities and choices as he made sense of students' lack of perseverance.

Soledad relays an anecdote about math, its relevance to students' lives, and perseverance

Then, Leo and Soledad described attempts to make math more relevant to their students and interpreted their students' reactions. Leo claimed that students complained about the irrelevance of math to avoid doing work. His claim that some students did not persevere because they complained that math they did in school was not relevant to their lives represented lack of perseverance as individual students' choice to avoid math. Then, Soledad described at length a way in which she had used art in her Geometry classes and how students had been excited by the connection between geometry and art.

41 Soledad: I think it's a strategy, too, but today I did something and I accidentally, like, I thought I was— I was trying to bring in all the poster paper from home? And I accidentally brought one where, already had work on it? And it was, like, uh, sketches my sister and I had done? It was, like, really large, like on a [holds hands several feet apart] maybe 5 yards? Of paper? And I unrolled it, and my kids are going through, um, rigid transformations? And I brought it out for them. They were saying something I said, and I showed the artwork. And, like, "Oh my god! You're right! That's…" Like, when we did constructions, like, that piece is constructions, this piece is this. And it's really big and it's tribal art. So they were like. And especially like the Islander kids really, um, connected to it? I try to post, like, uh. I dunno, I

always, I dunno, I like art, so I always post, um, math and art. And I think it's easy in Geometry. I post a lot of, um, geometry— I mean, lots of art that has math. And I, like, sent one to the teachers about a piece that was found in Downtown Oakland just in the last few weeks of Trump? It has a whole bunch of transformations in it. Um. So [sighs] I try art because I think that it's something that kids can connect to more easily? Um. But today was the first time there was like, "Oh my god! Yeah!" Like, I thought was genuine. But most of the time I think it's not, like. I'm like trying and trying, and nothing's connecting. But today I felt like, oh my god, I think that… they liked that. [Laughs] And it was just like whoa! The first time. The first time, I think.

Soledad's anecdote countered Leo's assertion that students complained math was not relevant simply to evade working by demonstrating students' excitement about relevant math. She replayed her students' unusually positive reaction to an activity that combined art, geometry, and politics, saying, "today was the first time there was like, 'Oh my god! 'Yeah!'" Her anecdote conveyed hope that students could be engaged and countered deficit-oriented, individualist narratives about students' lack of perseverance put forth by Leo and Jamie. Her story had the potential to help the teachers orient to affordances for students' perseverance provided by classroom environments. However, her anecdote was not probed or taken up by the other participants.

Leo returns to independence

When Soledad finished, Leo returned the conversation to the discussion of reasons why students fail to persevere. He claimed that some students did not persevere because they did not have the tools to make progress. He described individual students as each having "a toolbox."

42	Leo:	So for persistence for Access, I would say, like, sometimes kids don't really feel like they have a toolbox? Um.
43	Jamie:	Hmm.
44	Leo:	So. If we were doing an activity, and they got stuck and they go, and they said to themselves, "Oh, wait. What do I know about this problem?" I would count that as persistence. Because it's, it's, uh, student-initiated. They recognize, they just got stuck. They go, "Okay, this is a strategy I have, so that's what I'm gonna do." But I'm just saying that when we do it, when we question like that, from, when we initiate it, it's not necessarily persistence, you know.
45	Elizabeth:	Um, you're providing some sort of scaffold, then.

46	Leo:	Yeah. Like, “Go this way!” [Gestures with his arm, pretending to guide] We’re not. [Gestures again] “That way, try it. It might get you something.” And then [nods].

Leo’s abrupt shift returned the conversation to claims about why students fail to persevere and individual representations of perseverance. It also focused the conversation again on the theme of perseverance as independence. In his telling, when students got stuck, they each turned to their toolboxes for the resources to get unstuck. Leo further elaborated on his earlier claim that Three Things in a public setting did not support perseverance because in using the strategy, teachers were helping students (turns 44 and 46). This time, his elaboration left room for students to help each other. Leo explained that if students got stuck and “said to themselves, ‘Oh, wait. What do I know about this problem?’ I would count that as persistence” (turn 44). Leo used plural and singular pronouns in this statement, so it is unclear whether he was rehearsing a student’s internal monologue or a conversation between students. Either way, while Leo’s explanation did not rule out that students might use Three Things together it also did not draw out the intent of the intervention, supporting students to persevere by building on each other’s thinking.

Jamie then reminded the group of his previous claims connecting fixed-mindset to lack of perseverance (from turns 34-36). He reinforced his static, trait-based representation of fixed-mindset by describing his students’ forceful refusals to do math: “It’s just that blank gaze that some of them have, no matter what you put in front of them, it’s like, ‘Nope! Freakin’ out! Freakin’ out!’” Elizabeth added on with a question about whether students who “always feel like they’re good at math” fail to persevere when faced with a challenge because they also have fixed-mindsets.

Then, Matt revisited previous conversation about how students’ perceptions of the relevance of math influences their perseverance. He attributed that discussion to Elizabeth and did not acknowledge Soledad’s contribution in turn 41. Jamie joined in by continuing his story from turn 40 about students rejecting statistics because it did not resemble math they had done before. He represented statistics as “real math” and especially relevant to students because of its profitability.

47	Jamie:	I love doing stats when they’re like, “When are we doing real math?” I’m like. “This is as real as it gets.” [Laughter] This is, like. If you wanna talk about real,
48	Elizabeth:	Yeah.
49	Jamie:	statistics is where math gets very, very real.
50	Elizabeth:	Well, like Leo’s comment earlier, like, “When am I going to see this in real life?” Like. [Laughter]
51	Leo:	It’s currently very profitable. Statistics.
52	Jamie:	Yep. Tell them that, too. That’s where the money is.

In this exchange, Jamie, Elizabeth, and Leo co-constructed students' negative feelings about statistics as misguided. They argued that students should find statistics more relevant that other types of math they encountered in school because, as Jamie said, "That's where the money is" (turn 52). In this argument, the three participants implied that the value of mathematics education lay in the job prospects and potential wealth it afforded students. Their representation contrasted with Soledad's from turn 41. Soledad's detailed replay had demonstrated students finding math relevant for its political and artistic connections, not its potential to earn them money. While Soledad's replay represented math in a humanistic manner, Jamie, Elizabeth, and Leo's represented math through a meritocratic lens and demeaned students for not seeing its potential value.

Elizabeth summarizes the conversation with individualist conclusions

At this point, Elizabeth shifted to an activity aimed at supporting the teachers to summarize their conclusions about Three Things in a public setting and think about next steps. She introduced an inquiry reflection tool she called XYZ. The teachers were to: (X) recall their hypotheses for how Three Things in a public setting would support perseverance; (Y) describe and interpret what happened when they tried Three Things in a public setting; and, finally, (Z) brainstorm what to try next. She re-voiced their hypothesis that Three Things in a public setting would support perseverance by helping students share and build on each other's ideas. Then she invited the participants to provide the Y, a description and interpretation of what happened.

Anna responded by returning to Leo's claim that the strategy took away the opportunity for students to decide how to make progress without a teacher's suggestions. Elizabeth summarized the previous twenty minutes of conversation by suggesting that Three Things in a public setting had given students more access to content, but less access to individual perseverance. Leo, Jamie, and Soledad gave their assent to Elizabeth's summary.

53	Elizabeth:	So maybe more access to content? But less access to perseverance?
54	Soledad:	Yeah. I think that, yes. I think that's the most clear way it's been stated so far. That more access to content, less access to perseverance.
55	Elizabeth:	And can I even add in, individual perseverance?
56	Anna:	Hmm.
57	Jamie:	[Nods]
58	Leo:	[Nods]

After prompting from Elizabeth, Cindy gave a thumbs-up (the only substantive contribution she made to their discussion). This is notable because during Session 3, Cindy suggested the Three Things in a public setting intervention and described how she thought helping students to build on each other's thinking would support perseverance. However, she did

not share her experiences with the intervention during Session 4 beyond her brief notes in the written reflections. Cindy's silence in this session, followed by a non-verbal agreement added ad hoc at the close of the conversation, can be interpreted as resignation to the will of the group. Cindy's participation in Session 5 leads to this conclusion. In Session 5, Cindy pushed her colleagues to return to group-oriented representations of perseverance and led the group to co-construct a new intervention aimed at helping students to build on each other's thinking. In this session, however, Cindy did not contribute as the conversation moved in a direction with which her participation in other sessions indicates she did not agree.

The teachers fail to plan an intervention

After asking the teachers if they had anything else to add, Elizabeth transitioned the teachers into their next activity, planning an intervention together. However, during this conversation the teachers did not build or come to consensus on an intervention, as they had at the end of the previous three sessions. Instead, the teachers left with a vague agreement to continue using Three Things, despite having decided that Three Things worked against their goal of building perseverance, and to brainstorm strategies to support students' individual perseverance. Elizabeth emailed this agreement to the teachers following the session:

> "We also agreed to continue our investigation on Three Things, collecting evidence/reflections on how that strategy is impacting student perseverance. We did notice, however, that the strategy gave students more access to content, but not to individual perseverance (as we provided the scaffold that we hope they would be doing on their own). In addition to continuing with Three Things, we agreed to brainstorm other strategies that would get a building *individual* perseverance, for discussion on 11/8."

Reflecting this lack of an agreement to try a pedagogical strategy, Elizabeth did not begin Session 5 by asking the teachers to share what happened in their classrooms between sessions. Instead, she began the session with video-watching, as described in Chapter 4.

Discussion of Case 3

Discussion of the preceding analysis is divided into two parts: discussion of the teachers' collaborative discourse, and implications for the PD.

Collaborative discourse about perseverance in Session 4

In this twenty-minute discussion, disagreement surfaced about the utility of Three Things in a public setting for supporting perseverance. Nonetheless, by the end of the conversation, the teachers explicitly and tacitly came to consensus that Three Things in a public setting diminished

opportunities for students to draw on *individual* resources to get themselves unstuck. This consensus effort was led by Leo, reinforced by Elizabeth's facilitation, and at times bolstered or simply not disputed by the other teachers. The conversation that led to this consensus reflected three patterns of how the teachers drew on individual representations of perseverance outlined in the introduction to this chapter. One of those patterns, that the teachers leveraged non-normative meanings of the TRU Dimensions as they build individual representations of perseverance, is taken up in the discussion of implications for the PD. The other two are discussed here.

Led by Leo, the teachers represented perseverance as independence.

In this conversation, Leo developed the claim that student engagement with mathematics is perseverance only if the students draw on individual tools and resources to persist through challenge. He represented perseverance as independence. He built an image of students looking in individually-held toolboxes, rather than turning to peers or a teacher, to figure out what to do when faced with a challenge. Leo constructed this claim as a rejection of Three Things in a public setting, which he represented as having fed students information that led them around challenges. Leo's rejection of Three Things tacitly rejected the goal of the strategy, which was to support students to persevere *together* by building on each other's thinking. Rather than explicitly condemning students building mathematical knowledge and struggling productively together, Leo centered his criticism on the role of the teacher in Three Things. In so doing, Leo's claims further undercut the broader goal of Three Things, which was to have the teachers change their classroom environments in a way that had the potential to support students' perseverance. The claim that students were not persevering if they worked through challenges with teacher help provided a negative imperative for teachers—do not help students. The claim did not come with a suggestion for how the teachers could help students build and learn to draw on their hypothetical individual toolboxes.

Notably, none of the teachers explicitly ruled out that students might help each other or build on each other's thinking—the intent of the intervention—and still persevere. Nonetheless, their conversation did not develop the idea that students can persevere by building on each other's thinking. Opportunities arose during the conversation for the teachers to discuss the relationship between collaboration and perseverance. The intervention they had tried in the previous weeks, Three Things in a public setting, was meant to support students to persevere by building on each other's thinking. Soledad shared brief evidence of this happening in her written response at the beginning of the session, shown in Table 2. Soledad and Leo also made claims at the end of the session about the intervention encouraging students to see diverse solutions to problems. However, these claims and observations were not taken up by the group. The facilitator also did not bring them up, a finding that will be discussed in the next section.

All teachers except for Cindy relayed an example of students failing to persevere, substantiated with individual representations of perseverance.

Concurrent with developing the claim that perseverance is an independent activity, all teachers except Cindy leveraged individual representations of perseverance to explain why students did not persevere. Soledad (turns 7-9), Jamie (turns 34-40), and Leo (described before turn 41) used character- and choice-based individual representations of perseverance to develop claims about why their students did not persevere. Jamie bolstered his claims with references to work on mindsets, itself an individual- and trait-centered body of research, and non-normative interpretations of the TRU Dimensions.

Importantly, building individual representations of perseverance as they made sense of students' non-perseverant behavior hampered the teachers' inquiry into their problem of practice, supporting their students' perseverance. While the teachers left Session 3 with a shared agreement to try a thoughtfully-revised version of the Three Things strategy (as described in Chapter 5), they left Session 4 with a vague imperative to "brainstorm" strategies to support individual perseverance. The individualist claims and representations of perseverance that the teachers used in this conversation did not position the teachers to transform their practice to better respond to their students' experiences of mathematics instruction. Instead, having collectively reproduced common sense, traditional, individualist claims, the teachers effectively threw up their hands. They had implicitly characterized themselves as unable to empower students they represented as constitutionally incapable of persevering, present their students with relevant mathematical experiences, or support students who they represented as needing to persevere independently. Thus, this analysis demonstrates how individualistic approaches to student perseverance hamstrings teachers' collaborative inquiry into problems of practice. If success is determined by character only and each individual must go it alone, there is little that teachers can do. Sociologists working outside of education have identified how people leverage the ideology of meritocracy to defend the status quo when confronted by evidence that the structure of society systemically underprivileges individuals from historically marginalized backgrounds (Ledgerwood et al., 2011). Implicitly, at least, the Benton teachers did precisely that in this conversation in Session 4.

Implications for the PD

Throughout, the teachers did not provide transparent representations of practice or students' experiences as they made claims in support of individual representations of perseverance. They also were not prompted to do so by the PD activity of sharing classroom experiences or the facilitator. Instead, they relied on impressions and detail-poor replays laden with interpretation. For example, in turn 15 Leo said (emphasis added), "So *in my mind* persistence is, you get stuck. And then you try something else. Or you do, there's some strategy that you use. But we're providing a way to, to avoid that resistance. To avoid the wall. So you don't necessarily see the persistence." In this turn Leo made two claims: that perseverance is individual work to get unstuck, and that Three Things helped students circumvent that individual work. He justified both with impressions, not transparent representations of students'

experiences of perseverance and Three Things. His representations were not problematized, nor was he prompted to elaborate on his impressions with evidence that the other teachers could problematize. Nonetheless, even without a transparent representation of practice, the other teachers eventually agreed with Leo. Their agreement relied on a shared impression of perseverance as individual work made possible by individual ability and choices, a shared impression consistent with common sense discourses of mathematics education, learning generally, and success in society at large (Apple, 2006; Sengupta-Irving & Agarwal, 2017; Valero, 2017).

Notably, the PD itself did not support the teachers to share transparent evidence or problematize each other's individualist claims and representations. The structure of the activity itself tacitly legitimized shallow, evidence-poor replays. The conversation analyzed here was part of the routine TRU-LS PD activity of sharing classroom experiences with planned interventions (described in Chapter 3). In this activity, the teachers were encouraged to represent their experiences through replays, without non-memory-based representations of practice such as video or student work. They largely took turns sharing, a form of participation that did not encourage questioning or problematizing each other's replays.

In this conversation from Session 4, facilitation and PD materials did not push back on or problematize the poorly transparent replays or individualist representations that were developed. The facilitator did not herself often question or problematize what the teachers shared, nor did she push for more depth from their replays. The teachers were encouraged to relate their replays to the TRU Dimensions. This activity structure had the potential to support push-back on individualist representations and claims, as the TRU Dimensions are not consistent with individual representations of perseverance. However, analysis showed that the teachers associated the TRU Dimensions with non-normative, individualist representations of students' perseverance. Participants did not use the TRU Dimension materials provided in this session, namely the names of the dimensions and brief descriptions of them, in ways that had potential to redirect the teachers' non-normative meaning-making. The facilitator also did not redirect the teachers towards more normative meanings, either directly or tacitly by prompting the teachers to return to the document of the TRU Dimensions as they substantiated their individualist claims. Evidence of the teachers assigning non-normative meanings to the TRU Dimensions consistent with traditional mathematics education discourses in other sessions also surfaced in Chapter 4, indicating that this was a pattern rather than an isolated incident.

Taken together, then, three features of the PD workgroup environment failed to sufficiently support generative discourse: an overly-vague framework that did not orient discussions at a level of detail that supports discussions of central issues and examples; PD activities that did not elicit concrete examples that made core ideas transparent for the participants; and facilitation that did not problematize teachers' discourse, take advantage of framework affordances, or push for examples. Without transparent representations of students' experiences and classroom events or opportunities to problematize individualist claims and reasoning, the teachers' inquiry was stunted. The lack of support provided by these three features

of the PD environment in Session 4 contrasts with the robust supports provided for generative discourse in Sessions 3 and 5. In those conversations, analyzed in Chapter 5, the teachers' inquiry into their problem of practice was facilitated by collaborative viewing and discussion of classroom videos and detailed replays of students' experiences of pedagogical strategies. Teachers' inquiry was also facilitated by explicit use of a different TRU resource than was used in Session 4, the TRU Look-Fors. As described in Chapter 5, the Look-Fors provide detailed descriptions of student behaviors that positively align with the TRU Dimensions. The teachers were intentionally prompted to refer to the Look-Fors as they reasoned about classroom videos. Finally, in Sessions 3 and 5 teachers' inquiry was supported by the facilitator, who probed the teachers for more information and explicitly problematized their individualist takes. Analysis of those conversations from Sessions 3 and 5 showed how the teachers' collaborative work with classroom videos broadened their horizons of observation (Hutchins, 1996), enabling them to see perseverance in new ways. In this conversation from Session 4, however, the teachers did not engage with transparent representations of practice in ways that afforded collaborative meaning-making and broadening of horizons. They used the TRU Framework in nonstandard ways to support individualist reasoning, and facilitation did not problematize teachers' claims, evidence, or reasoning.

Conclusion

This chapter characterized when and how the Benton teachers returned to individual representations of perseverance even as they took up representations of perseverance as a group accomplishment. Analysis showed that the teachers leveraged representations of perseverance as individual students' capacity or choice to work throughout the two lesson study cycles examined in this dissertation. This was even as the PD and teachers' interactions with it encouraged uptake of representations and pedagogical practices that centered collaboration. Thus, the teachers' discourse around perseverance was fragmented. Individualist representations of perseverance continued to have explanatory power for the teachers. These findings are characteristic of the body of research on conceptual change (Vosniadou, 2009), especially from a "knowledge" or "ideology in pieces" perspective (diSessa, 2010; Philip, 2011).

Analysis highlighted five patterns in how the Benton teachers drew on individual representations of perseverance. First, the teachers tended to turn to individual representations for initial claims and reasoning about their students' perseverance. Second, the teachers represented mathematical perseverance as valuable to students' lives outside of school as workers and wage-earners. Third, the teachers tended to represent perseverance as independence, in particular from teachers and other authority figures. Fourth, the teachers returned to individual representations of perseverance in particular when making sense of their students' failures to persevere. And fifth, the teachers assigned non-normative meanings to the TRU Dimensions while building individual representations of perseverance, and were not redirected towards normative, group-oriented meanings by PD materials or facilitation.

I elaborated on these five patterns in three cases, one from the post-lesson discussion following Research Lesson 1, a second from the first workgroup session, and the third from Session 4. Session 4 was particularly notable in how it fell between the two sessions analyzed in Chapter 5, in which the teachers took up group representations of perseverance and collaborated to hybridize a pedagogical practice to better support students to build on each other's thinking. Analysis of teachers' conversation in Session 4 in particular highlighted how the teachers did not introduce transparent representations of practice and student thinking as they made claims about perseverance as an individual accomplishment. Without transparent representations of practice or prompting from PD activity structures, materials, or facilitation, problematizing claims, reasoning, and representations was out of reach. The teachers returned to common-sense individual representations of perseverance and struggled to move away from them without the explicit problematizing that took place in Sessions 3 and 5. Taken together with analysis in Chapter 5, this analysis highlights the importance of PD activities, materials, and facilitation that explicitly problematize common-sense, traditional representations whose perpetuation impedes teachers' collaborative work.

Although how the teachers returned to individual representations of perseverance draws a contrast with how the teachers increasingly turned to group representations, as shown in Chapters 4 and 5, the particular patterns in how the teachers used individual and group representations over time are not entirely contradictory. While the teachers' sense-making about perseverance was piecemeal, it was not illogical. Chapter 4 elaborated on four patterns in how the teachers used representations of perseverance over time, two of which dealt exclusively with how the teachers used group representations. They were that:

1. Over time, group representations of perseverance became salient to explain cases of successful student perseverance.
2. Over time, the teachers came to conclude that the classroom activity of students' building on each other's thinking was important to perseverance, and that teachers should work to support that activity.

In this conclusion, I would like to draw attention to the relationship between these two patterns and two of the five patterns in how the teachers returned to individual representations of perseverance elaborated on in this chapter. They were that the teachers:

3. Often emphasized perseverance as independence, in particular from teachers and other authority figures.
4. Often leveraged individual representations when describing or making claims about times when they felt that their students had failed to persevere.

Taken together, Findings 1 and 4 indicate that over time, the Benton teachers were drawn to group representations to make sense of *successful* perseverance, but returned to individual

representations to make sense of *failed* perseverance. These findings are not inherently contradictory. A teacher can look for group-oriented reasons why a student persevered—such as classroom practices that encouraged students to build on each other's reasoning— while still arguing that individual traits or acontextual choices led a student to not persevere. Furthermore, they are consistent with traditional mathematics instruction discourses' attention to students' *deficits* and ambitious mathematics instruction discourses' attention to students' *potential for growth*. That the teachers might hold onto traditional representations of perseverance while reasoning about a main focus of traditional discourses but take up ambitious representations of perseverance while reasoning about a main focus of ambitious discourses is reasonable. It is also a reason for discourses of ambitious mathematics instruction to be more explicit in how they represent the role of group activity and interactional context in perseverance.

A limitation of this analysis is that the Benton teachers never explicitly discussed equity and race. As described in Chapter 2 and Part 1 of this chapter, traditional mathematics instruction discourses inherit inequitable and racially problematic representations of students' individual capacities to succeed from broader discourses and ideologies (R. Gutiérrez, 2018; Martin, 2013; Oakes & Rogers, 2007). Scholars who have previously investigated patterns in how teachers leverage traditional representations in collaborative workgroup sensemaking and connected those patterns to larger common-sense discourses have found how teachers represent equity, mathematical competency, and race to be key markers of discursive and ideological inclination (Bannister, 2016; Horn, 2007; Philip, 2011). Scholars have also found that students of color are more likely to be seen as lacking the capacity to persevere in mathematics and to have their character or disposition evaluated in lieu of social or environmental factors (Sengupta-Irving & Vossoughi, 2019; Valero, 2017). However, the Benton teachers *never* mentioned equity or race as they inquired into their students' perseverance. Students at Benton High School were overwhelmingly people of color (as described in Chapter 3). In a society in which who is and is not seen as mathematically perseverant is racialized, the Benton teachers' discussions of their students' perseverance were also racialized, even though the teachers never explicitly mentioned race. Nonetheless, the absence of explicit mentions of equity or race and of prompts encouraging the teachers to discuss equity and race as it related to their inquiry into their students' perseverance made discourse analysis of this dimension of their talk methodologically challenging.

A second limitation of this analysis is lack of attention to how the teachers were positioned with respect to each other in the workgroup and within society by individualist or group-oriented representations of perseverance. As discussed in the first part of this chapter, traditional, individualist discourses and ideologies of perseverance help uphold an inequitable social structure (Ledgerwood et al., 2011). Challenging those discourses and ideologies of perseverance is tantamount to challenging that social structure. The differential social power that the teachers were afforded because of how they were positioned with respect to each other in the workgroup and within society may have supported some teachers to use discourse in ways that

implicitly or explicitly challenged the pervasive, inequitable social structure, and constrained other teachers from doing the same. Analysis in this chapter does not shed light on this.

These limitations aside, however, the analysis in this chapter points to the slow, piecemeal nature of teachers' learning around concepts with representations deeply embedded in commonsense, traditional discourses, discourses that cross into mathematics education but are also widely used in society at large. It also points to the importance of clarity and explicit problematization in shifting teachers' discourse away from individualist representations and towards group-oriented, ambitious representations, representations that support the kind of powerful collaborative inquiry and impetus to pedagogical change identified in Chapter 5. Each of these implications will be further discussed in the next chapter, the conclusion to this dissertation.

Chapter 7 Conclusion

In this dissertation, I aimed to investigate the potential for teachers' learning towards ambitious mathematics instruction when they collaboratively inquire into problems of practice in PD. To do this, I examined patterns in how teachers engaged in collaborative learning in a PD program represented core ideas of teaching and learning central to their problem of practice over time. I also looked for workgroup conditions with the potential to support collaborative inquiry and examined the relationship between those conditions and the representational patterns that unfolded. These investigations painted a picture of (1) how teachers' collaborative discourse can move towards representations of students and teaching consistent with ambitious mathematics instruction; (2) the affordances those representations offered to support teachers to plan together to shift classroom practice in powerful directions; and (3) workgroup and PD conditions that support teachers to build ambitious, group-oriented representations. These investigations also showed that despite shifts towards discourses of ambitious mathematics instruction that unfolded, traditional discourses persisted in the teachers' conversations. Representations of students consistent with traditional discourse retained significant explanatory power for the teachers whose learning was examined in this dissertation, even as representations consistent with ambitious discourses gained prominence.

The investigations in this dissertation have implications for research about teachers' learning in PD and the practice of PD. In the first part of this chapter, I summarize the contributions made by each chapter of this dissertation. I follow that summary with a discussion of the key conclusions of this dissertation and implications of this research.

Summaries by chapter

Chapter 1 introduced the dissertation. It summarized the dissertation's main findings and situated the dissertation in prior research into teachers' learning towards ambitious mathematics instruction in PD. Chapter 1 showed that research was needed into how mathematics teachers make sense of a commonly-discussed idea in teaching, perseverance, and what leads to perseverance in mathematics classrooms.

Chapter 2 outlined two theoretical frameworks central to this dissertation: a framework for traditional and ambitious mathematics instruction discourses, and a framework for the PD and workgroup conditions that support generative collaborative discourse for teachers. Together, these frameworks guided this dissertation's investigation of teachers' learning. In outlining these frameworks, Chapter 2 showed that research into teachers' learning towards ambitious mathematics instruction in PD could benefit from approaches that go beyond evaluation of PD effectiveness and situate teachers' learning in broader cultural discourses.

Chapter 3 described the setting of this research and methods used. It outlined the PD program, TRU-Lesson Study, and introduced the teachers and facilitator. It also provided overviews of the teachers' collaborative work before and after the PD sessions analyzed in this dissertation. Finally, Chapter 3 described the analytic methods used in this dissertation.

Chapters 4, 5, and 6 presented the analysis. Chapter 4 showed that the participants leveraged two representations of perseverance as they inquired into their problem of practice: perseverance as individual students' capacity or acontexual choice to work, and perseverance as a group accomplishment. Chapter 4 showed that over time, the teachers used more complex representations of perseverance that increasingly reflected perseverance as a group accomplishment. Using representations of perseverance as a group accomplishment empowered the teachers to notice student behavior in more robust, complex ways and plan interventions aligned with ambitious mathematics instruction. However, individual representations of perseverance persisted in their talk. Chapter 4 also showed how well-timed problematizing by the facilitator supported generative discourse among the teachers.

Chapter 5 dove into the question of *how* the PD workgroup conditions supported the patterns in representation uncovered in Chapter 4. Analysis in Chapter 5 pointed to the combination of collaborative video-watching following by planning as supportive of generative discourse. How iteratively engaging in this pair of activities supported deepening reasoning and richer pedagogical planning was uncovered through analysis of conversation in two pivotal sessions. Analysis in Chapter 5 also showed how the participants collaboratively made perseverance as a group accomplishment transparent in videos that did not offer clear affordances for generative discourse.

Chapter 6 returned to analysis of patterns in the teachers' uses of the two representations of perseverance over time. Analysis was framed in terms of conceptual change literature that seeks to explain learners' lingering mis- or prior-conceptions and fragmented sense-making about observed scientific and social phenomena (Philip, 2011; Vosniadou, 2009). This chapter looked at the persistence of individual representations in the teachers' discourse, even as they complexified their discourse to include more group representations of perseverance. This chapter uncovered five patterns that point to the explanatory power and resilience of individual representations for teachers: (1) the teachers turned to individual representations *first*, (2) individual representations correlated with talk about perseverance as an out-of-school, workplace skill, (3) the teachers tended to represent perseverance as independence, (4) individual representations were useful to the teachers as they made sense of failed perseverance, and (5) the teachers used the TRU Dimensions in non-normative ways as they build individual representations of perseverance.

The findings of the three analytic chapters point to several key conclusions for research on teachers' learning towards ambitious mathematics instruction and PD workgroup conditions that support generative collaborative discourse. The next part of this chapter examines those conclusions.

Key conclusions

In these next two sections, I highlight key conclusions from this dissertation about the two bodies of research on which this work built: research on teachers' learning towards

ambitious mathematics instruction discourses, and research on PD workgroup conditions that support generative discourse for teachers.

Conclusions about teachers' learning towards ambitious mathematics instruction discourses

This dissertation added another robust case to a growing body of literature on how teachers' discourses around traditional and ambitious mathematics instruction are complex, piecemeal, and slow to change—but that they can and do change under supportive conditions. Building on work by Horn, Bannister, Nasir, and others (Bannister, 2015; Horn, 2007; Kazemi & Franke, 2004; Nasir et al., 2014), this dissertation showed that through sustained collaborative inquiry into a rich problem of practice, teachers' collaborative discourse can move away from traditional, individualist mathematics instruction discourses and towards ambitious, group-oriented mathematics instruction discourses. This dissertation also highlighted the power of representations consistent with ambitious mathematics instruction discourses for teachers' collaborative sense-making about students' behavior. As shown in this dissertation, sustained inquiry supported the teachers to iteratively broaden their horizons of observation (Hutchins, 1996), coming to see students' perseverance in more robust, ambitious ways over time. Sustained inquiry followed by collaborative planning positioned the teachers to organize for potentially powerful classroom change. Thus, in addition to providing a proof of concept that teachers' discourse can change towards ambitious instruction discourses, this dissertation provided a robust case of how teachers' collaborative workgroup discourse around key teaching ideas affords potential changes in classroom practice (Horn & Kane, 2015).

This dissertation's finding that teachers leverage group-oriented and individualist representations to make sense of students' mathematical perseverance is new. Prior research has developed the field's understanding of how teachers make sense of other key ideas of mathematics teaching and learning, including students' capacity for rich mathematical sense-making (Beisiegel et al., 2018; Kazemi & Franke, 2004) and equity (Bannister, 2015; Horn, 2007; Jilk, 2016; Nasir et al., 2014). Teachers' collaborative discourse about perseverance has not been studied, despite the pervasiveness of "perseverance" and related ideas (including persistence, mindset, and "grit") in both traditional and ambitious mathematics instruction discourses. This dissertation's findings about the common-sense and deficit-oriented nature of individualist representations of perseverance in the teachers' talk is consistent with prior work on teachers' discourse about equity. This dissertation's findings about the resilience of individualist representations of perseverance in teachers' discourse is also consistent with prior work on equity.

This dissertation also showed that teachers move between traditional and ambitious discourses, highlighting how teachers' uses of those discourses are not uniform. These findings build on work on learning across domains in literature on conceptual change (diSessa & Sherin, 1998; Vosniadou, 2009), and in teacher-learning specifically (Louie, 2016; Philip, 2011). As analysis showed, representations from both discourses can hold explanatory power for teachers

at the same time. Analysis showed that teachers' uses of those discourses are situated in broader cultural discourses about individualism and perseverance, which themselves lack clarity. Furthermore, this dissertation showed that teachers' piecemeal discourses are not inherently contradictory. How teachers make use of both traditional and ambitious discourses at the same time might, in fact, be sensible; this mixing of perspectives is entirely consistent with diSessa's "knowledge in pieces" framework (diSessa, 1988). This latter finding in particular shows that research that takes an evaluative stance—looking for learning *towards* ambitious instruction—misses the internal logic teachers' piecemeal discourse. It also shows that evaluative research that treats traditional and ambitious discourses as dual, rather than multiple, and uniform, rather than diverse, oversimplifies what is actually a very complex learning space for teachers. These findings have implications for future research on teachers' collaborative discourse, which will be discussed in the next part of this chapter.

Conclusions about PD workgroup conditions that support generative collaborative discourse

This dissertation examined three PD workgroup conditions that may support or constrain generative collaborative discourse for teachers: facilitation, PD activities, and frameworks. This section takes up each of these conditions in turn.

Facilitation

Analysis found the facilitation of the Benton teachers' learning to offer mixed affordances. At times, the facilitator mirrored the teachers' individualist discourse, implicitly supporting it even though it contradicted the PD's guiding framework. At other times, however, the facilitator took an explicit problematizing stance. This dissertation showed how the facilitator's explicit problematizing was powerful for setting generative, collaborative discourse among the teachers in motion. It also identified moments in which more problematizing by the facilitator could have been productive. Notably, the teachers themselves often did significant heavy lifting to make conversation generative. That the teachers' could do this heavy lifting points to the strength of their collaboration. However, there were opportunities for the facilitator to shape conversation by problematizing teachers' comments and directing teachers towards normative uses of the TRU Framework. The work that a facilitator does of problematizing and guiding conversation is essential to generative discourse.

Analysis also called into question whether more problematizing by a facilitator is always better. A key conclusion of this research is that a facilitator's work requires a delicate balance of building trust among participants, possibly by mirroring their discourse even if misdirected, and pushing participants' discourse through problematizing. Nonetheless, a key conclusion of this dissertation is that facilitation need not be "perfect," whatever that might mean, to support generative collaborative discourse among teachers. The Benton teachers had rich conversations about perseverance even as facilitation offered mixed affordances.

PD activities

This dissertation showed that engaging in the activities of collaborative video-watching and planning, in succession and iteratively over multiple sessions, supported inquiry-oriented discourse around a problem of practice. Representations of perseverance developed through collaborative sense-making about a video episode were deepened as the teachers used them to plan an intervention to support their students' perseverance. The iterative nature of these activities across multiple sessions was also important. The teachers hybridized an intervention over time, changing it so that it better supported students to persevere by building on each other's mathematical thinking.

This finding that engaging in a hands-on manner with real classroom episodes and planning, rather than simply talking about big issues of teaching and learning, supported deep collaborative engagement is consistent with broader literature on learning for students of all ages and in all content areas. In particular, it is consistent with the literature on conceptual change reviewed in the beginning of Chapter 6. While there are diverse views on precisely how to bring about conceptual change, they all center on learners engaging with real phenomena in ways that surface and problematize implicit representations and connections (Schoenfeld et al., 1993; Vosniadou, 2009). For teachers learning together about problems of practice, the same holds. Interacting with peers around rich representations of practice, such as classroom videos, and in activities grounded in practice, such as planning, has the potential to support generative conversation that in turn supports shifts in discourse.

This dissertation also showed that collaborative sense-making about a problem of practice through watching video together supported discourse that broadened the teachers' horizons of observation about perseverance. While this finding is in itself not new (cf. van Es & Sherin, 2010), this dissertation shed new light on how that broadening came about and the role of the video itself in supporting generative discourse. Through conversation in which the participants offered claims about the video and problematized each other's claims, the participants collaboratively made perseverance as a group accomplishment transparent in the videos they watched. They accomplished this even when discussing videos that were not "optimal," in that the videos did not meet all or even most of the criteria for generative videos built by the PD designers and facilitators. Thus, this analysis shows that the transparency of practice in video of classroom interactions is *co-constructed* by participants as they interact with videos.

Frameworks

The Benton teachers engaged with two frameworks for ambitious mathematics instruction: the TRU Framework, which grounded the PD, and Complex Instruction, which was introduced by the teachers in conversation. One teacher also used the theory of mindsets in a conversation analyzed in Chapter 6, although mindset theory was not a main framework used by

participants in this dissertation. Analysis showed that when frameworks offered an explicit, detailed perspective on perseverance as a group accomplishment, the teachers used those frameworks to make sense of perseverance as a group accomplishment. CI and the TRU Look-Fors tool offered explicit, detailed visions of perseverance as a group accomplishment and supported generative discourse.

However, when frameworks were not explicit about the role of collaboration or individual character in perseverance, the teachers assigned non-normative meanings to key ideas in the frameworks to support common-sense individualist representations. And, importantly, PD tools consistent with those non-explicit frameworks were not sufficient to afford pushing back on teachers' non-normative meanings and uses. The TRU Framework in its overview form and the theory of mindsets did not offer explicit, detailed visions of perseverance as a group accomplishment. As such, teachers tended to assign non-normative meanings to the TRU Dimensions and the theory of mindsets as they built individualist representations of perseverance. Speaking broadly, if a framework provides clear guidance—and users know how to find that guidance—discourse can be supported by the framework to move in normative directions. When the framework does not provide clear guidance (or users do not know how to find it), users will return to what is familiar to them. This is precisely what the Benton teachers did in their conversations.

Implications for research and practice

In these next two sections, I highlight implications of this dissertation for research and practice in teachers' learning towards ambitious mathematics instruction discourses and PD workgroup conditions that support generative discourse for teachers.

Implications for research

On teachers' learning towards ambitious mathematics instruction discourses

Research on mathematics teachers' learning continues to need more cases of successful change, even if partial, to build out robust visions of trajectories for ambitious growth. Teachers' learning around ambitious mathematics instruction can take many forms. Regardless of the form it takes, that learning is slow, piecemeal, and complex. Researchers should continue to add more cases to the map so that teacher learning scholars can better support teachers to learn in such a complex space.

More research is needed into how teachers' sense-making draws on both traditional and ambitious mathematics instruction discourses and interweaves the two discourses to make sense of the complex work of teaching. Few studies have attempted to uncover the explanatory power that traditional discourses of mathematics instruction *continue to hold* for teachers even as they engage in learning together around ambitious mathematics instruction. More research that takes an exploratory, sense-making approach, rather than an evaluative approach, is needed to better

understand when and how traditional and ambitious mathematics instruction discourses make sense to groups of teachers.

Finally, this dissertation points to a need for more research into teachers' learning about *perseverance*. As discussed throughout this dissertation, perseverance is a pervasive but under-studied idea in mathematics education and math teachers' collaborative discourse. At the same time, perseverance is woven into broader discourses and ideologies that people use beyond mathematics education to make sense of struggle, success, and failure in society. More research is needed into the challenges teachers face in moving from a primarily individualist perspective to a collectivist or collaborative perspective, centered in sense-making about perseverance in addition to other important ideas in teaching and learning. More research is also needed into how teachers' discourse about perseverance overlaps with teachers' discourse about race and equity, as situated in broader cultural discourses and ideologies. A limitation of this dissertation was its methodological inability to examine the Benton teachers' reasoning about perseverance, race, and equity at the same time. However, perseverance is racialized in mathematics education and society at large. Understanding teachers' discourse on perseverance is incomplete without examining crossovers between discourse on perseverance and other racialized concepts with implications for equity.

This dissertation also did not analyze the relationship between the teachers' discourse on perseverance and *productive mathematical struggle*. It was noted in this analysis that the teachers tended to represent perseverant behavior as *working*, without direct regard to mathematical challenge and the relationship between challenge, productive struggle, and learning. Analysis into how discourse about struggle and learning was used by the teachers would provide a fuller picture of the teachers' reasoning about perseverance.

On PD workgroup conditions that support generative collaborative discourse

This dissertation's non-evaluative, grounded approach to uncovering PD workgroup conditions and their affordances for generative collaborative discourse pointed to gray areas in how facilitation, PD activities, and frameworks may or may not support generative collaborative discourse. Prior research on PD workgroup conditions has tended to take an evaluative "this or that" approach, essentially asking: Which produces more/better changes in teachers' discourse and practice? Stock video *or* own-video, expert facilitation *or* teacher-led facilitation, video club activities *or* collaborative planning? Research with such an orientation essentially skips over generative collaborative discourse, tying PD workgroup conditions directly to content-focused discourse outcomes. It also produces black-and-white conclusions that do not match with the complexity, connectedness, and "grayness" of PD workgroup conditions and their affordances for generative collaborative discourse. This dissertation implies that more grounded research into PD workgroup conditions and their affordances for generative discourse is needed. In general, analysis in this dissertation indicates that documenting *that* PD "works," and not *how* it works, is not enough.

A limitation of this research was that it did not take a structured approach to coding for features of generative collaborative discourse. While the narrative identified instances in which the teachers engaged in features of generative collaborative discourse, including reasoning with evidence, problematizing, and building on each other's ideas, analysis did not code for these features. Therefore, their presence or absence was not systematically identified. Delineating and coding for specific features of generative collaborative teacher discourse is a burgeoning field of research (Lefstein et al., 2020). Coupling systematic analysis of features of generative discourse with the analysis of PD workgroup conditions undertaken in this dissertation could prove fruitful.

Implications for practice

A central aim of this dissertation was to provide research that could help improve the design and implementation of practice-based professional education programs for teachers. In this section, I outline implications for the practice of PD along three dimensions.

Facilitation

This dissertation's finding that generative discourse among the participants unfolded even amidst facilitation with variable affordances points to the collaborative, co-constructive nature of teachers' learning in PD workgroups. Analysis showed that at various times, contributions made by the teachers problematized individualist representations and promoted ambitious teaching practices. The facilitation implicitly and explicitly empowered the teachers to make these generative discursive moves. Specifically, then, this dissertation implies that the best facilitation is not "done to" participants. Instead, it gives teachers agency, providing them opportunities to making meaningful contributions and move the group towards their collective goals.

PD activities

This dissertation points to the promise of activities that *intertwine* collaborative video-watching and planning. Practice-based PD programs tend to be siloed in the kinds of representations of practice with which they engage teachers. They are often video clubs *or* focused on analysis of student work *or* devoted to planning interventions or focal lessons. However, this dissertation found that engaging teachers in both video-watching and collaborative planning, iteratively over time, supported generative collaborative discourse. These two activities together cut across several central practices of teaching: noticing, planning, and evaluating interventions. This dissertation indicates that engaging teachers in activities that integrate teaching practices that are often siloed in typical PD, iteratively and over time, can support generative collaborative discourse and learning.

Frameworks

Some of the most pointed implications of this dissertation for PD practice relate to the design of frameworks and framework-derived tools used in PD. Specifically, this dissertation found that to shift teachers' discourse around pervasive, common-sense representations derived from traditional discourses, representations that are also embedded in common-sense discourses and ideologies outside of mathematics education, frameworks should be explicit and detailed. Implicit and broad representations of common-sense but also high-stakes ideas such as perseverance and individualism leave the door open for non-normative meaning-making that perpetuates problematic common-sense reasoning. Similar arguments have been made about frameworks and tools around equity and race (Martin, 2013). This dissertation implies that the same is true for perseverance and individualism. Frameworks and framework-derived tools should speak clearly about ideas that pervade teachers' discourses and have significant implications for equity in mathematics teaching.

And yet addressing ideas about teaching and learning as teachers represent them does not mean that frameworks should *prescribe* how teachers should represent them instead. As the analysis in the dissertation shows, discourse shifts happen slowly. They cannot be forced. Instead, they are co-constructed by the learners themselves in *supportive*, but not *prescriptive*, environments that include facilitation, activities, and frameworks.

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Appendix A Selected Transcripts

This appendix contains full transcripts of episodes for which analysis in Chapters 4, 5, and 6 included summaries in place of transcript. Note that line numbers in these transcripts do not match line numbers in Chapters 4, 5, and 6.

Chapter 4 transcripts

Extended transcript of episode Case 1a from Session 1

1	Elizabeth:	Ok, so. We're gonna share out your two successes and challenge and relate them to a TRU Dimension. And I want to chart these, Anna do you want to chart?
2	Anna:	Yeah.
3	Elizabeth:	Ok, great. Do we have a brave volunteer who wants to go first?
4	Soledad:	Can we have the TRU Dimensions posted because I left my folder--
5	Elizabeth:	Yes!
6	Soledad:	I left my folder at home. No, I actually just organized it yesterday because I had so many papers.
7	Elizabeth:	Does that work?
8	Cindy:	I'll go first.
9	Elizabeth:	Okay.
10	Cindy:	One success I had, uh, I feel like I'm teaching with more intention now um because I've seen the Algebra 2 curriculum last year and Algebra 1, same thing, I've seen it many times. So more it's like, when I look at a lesson I kinda can I remember how it's contributing to like a whole unit and then what I need to emphasize or demphasize within the lesson to make sure that they're gonna be ready for something later on or they're gonna be ready for another course later on. Like, I'm starting to think that way now. Um. Success number two?
11	Elizabeth:	Uh, which of those dimensions--
12	Cindy:	Oh!
13	Elizabeth:	-- do you see that filling in?
14	Cindy:	Oh, the Mathematics!
15	Elizabeth:	Sure. Seeing those connections.
16	Cindy:	Yes! Uh. Success two, I actually I created a team task for Algebra 1 just last week and I'm pretty proud of myself for that. And um, the hope is that I give students more opportunity to do team work, to rely on each other. And, um. That would be Equitable Access to Content.
17	Jamie:	Are you talking about the Error Analysis, Cindy?

18	Cindy:	[Nods]
19	Jamie:	That was a really good lesson.
20	Cindy:	Well it didn't completely, like, students didn't do the whole thing but I feel like the set up was there--
21	Jamie:	Yeah.
22	Cindy:	-- for it. Later.
23	Elizabeth:	Yeah I definitely see that as Access but I'm also wondering if that can also be AOI?
24	Cindy:	Mmm.
25	Elizabeth:	And how students rely on each other more and have those opportunities to share.
26	Cindy:	Mmhmm.
27	Jamie:	Yeah.
28	Cindy:	So! One challenge. Um. How can I reengage students who are quickly disengaging? Particularly those who don't like math or those who are repeating the math class? So that's AOI right there. Agency, Ownership and Identity.
29	Elizabeth:	Thank you, Cindy. Anyone else want to go?
30	Soledad:	I'll go. Ok um. This one is for Access to Mathematical Content. And it was because we did um goals for our Smarty Goals and I-and the rest of the math team chose up like cultural teaching? Or teaching um through broad cultural context. So I made a short chant. And some student did it, some students didn't. But the warm up the next day, although like I you know remembering what you can construct with a compass is really simple, it really helped some students. So like, our chant is, compass constructs a circle and arc, compass constructs a circle and arc. So some students did it and some students would go, blaaaaaa.
31	Anna:	[Laughs]
32	Soledad:	And that was the warmup that day. Things you can do with a compass umm. And then lots of students didn't get it which surprised me but I would come up (inaudible) and then next day students got it and now today everyone remembers what a compass is used to construct. And then we also talked about what a compass measures. We added that to it after the second day. And so so I did that for Access to Mathematical Content, so I got pretty much 100 percent on assessments for that. Now um, today, this morning and (inaudible) getting 100 percent on that on that warmup um beginning part of that warmup. And I think another success is for Agency, Ownership, Identity? Um. I'm doing everything really group heavy, and in Math Analysis I have them do everyhing not only in um teams but um like as a like you-- two week project. The first one the umm the maximizing volume

of the rectangular prism was one and half weeks. And I had them do pre-posters before their poster presentation. And they would do things like, What is the problem? What is your parent formula? You know, How can you arrange data? And I'd have them do gallery and after they do gallery everyone was required to write assessments, too, of each other. So then it helped them to see like, Oh how can I-- they're not being clear, how can I be more clear? So they all start to really own the math a lot more and the process. Um and they're really taking serious like assessing each other. And we're really clear like that in the process of assessing each other, the pre-posters, that they're all helping each other earn A's on their final product. So.

33 Elizabeth: So cool.

34 Soledad: I think it's working because it seems to so far. So the first posters for volumizing-- I mean maximizing volume of rectangular prism-- worked. I'm kind of thinking the kids-- I don't see it in the, I don't see it in our-- I mean maybe in the end, um, intro to calculus? But they-- I'm gonna-- I'm thinking that I might check to see if they can do like first derivatives, like now that we done some volumizing and maximizing-- I mean, maximizing volume and maximizing other thigs. I don't know. I'm still thinking because I haven't seen them stuck yet but i'm thinking when they're stuck, I'm going to use some of Leo's work. Because he said I could. So like, if they don't um if they're not getting whatever I'd go back to that or whatever. But so far they seem to be taking ownership um, like, very like they're really um forming agency over their work in the math. Um. A question, a challenge is I think mine's similar to um Cindy's is um I have some repeat students. And some of them are like, Pshhh, I am not gonna get a D and F again. So they're just like self-motivated in that sense. But there are some who are not. And I don't know. I'm doing it a lot different than last year and some of them didn't have me last year for first semester. But I guess, you know, had not passed it the first time with any other geo teacher. Um. But yeah, how to keep repeat students engaged when they feel like, oh I kind of done material like this before.

35 Elizabeth: Mmhmm. And which dimension do you see that or which multiple dimensions could you see that fitting into?

36 Soledad: I mean I guess it could be any of them because if I give them a different way to Access Mathematical Content-- Access to Content, um, it might engage them. Right? Um. Ummmm. Uh but. I think like I could really quickly see it fall under Agency and Ownership? Um. I'm still trying to see which of my students identify, how they how they-- I kind of think it's probably most likely in AOI.

37 Elizabeth: I think we're also seeing that like that like um lots of different dimensions and all these dimensions kind of work together and can't really be isolated.

38 Soledad: Yeah.

39 Elizabeth: What about you guys?

40 Jamie: I'll go next. I feel like I've been really enforcing team roles. And so especially the resource manager is the one I've been focusing on the most so that they're forced to work together. I've already seen like, in my first period this morning there were a couple of those like, (raises hand) Mr. Rossi! I'm like, you're not the resource manager.

41 [Laughter]

42 Jamie: And they're just so angry with me. There was one team--

43 [Laughter]

44 Jamie: there was one team that like, Ugh, Luis! Luis raises his hand. And I'm like, Rigo, what's the question? And they're like, they don't know, they haven't discussed it. And I walk away and I come back. And then there's another team and I'm like, you're not the resource manager, and they never call me back because they figure it out. So.

45 Leo: Score.

46 Jamie: Yeah so score. Uh so I feel like the Access to the Mathematical Content fits in there? Team roles so that it's enforced, like, everyone needs to participate, everyone has a part here, um. And if we're not playing our parts then I guess we're not getting all the support that we need. Or that we could be getting. Um. So I feel like that's been really successful. I also have them uh grading each other on their collaborative work. So as part of their weekly, um, stamp sheets they actually grade everyone in their team how well they played their role. Um. Which I think has forced a little more harmony (laughs) than perhaps in years past. Um. I also feel like I've been doing more formative assessment. So I've been building into my lessons a lot more of like debriefing, ok you're getting up to at least this point, now we're gonna share some student work or we're going to get some ideas out there? And it really-- and I'm seeing a lot more of the mistakes coming out before the lesson ends rather than after when I look at their work. Um so I've built that in a lot more. And I was telling you all the example from first period this morning when uh they were making histograms and one kid showed off his histogram at the doc cam here and then we had a very spirited discussion about how to properly label the x axis. Um. And one student was like, I like this class! I'm gonna be able to argue! I'm like (thumbs up.) Uh questions, challenges that I still have--

47 Anna: Aaaa!

48 Jamie: Sorry, Anna, I'm just gonna keep talking randomly until--

49	Anna:	Hold on I didn't get your, um, TRU Dimensions though. Hold on, hold on.
50	Jamie:	Oh sorry! That would be Formative Assessment.
51	Anna:	Ok this one's Formative Assessment.
52	Jamie:	Yeah.
53	Anna:	And this one's-- you said was?
54	Jamie:	Access to Mathematical Content.
55	Anna:	Access. Ok. And then I captured a third one for you?
56	Jamie:	Uh the grading each other, I also, I put that as Access.
57	Anna:	It's like related to this one.
58	Jamie:	Yeah so like they're forced to--
59	Anna:	Mmhmm.
60	Jamie:	Be pro-social and civil, sort of. To the extent possible.
61	Anna:	[Laughs]
62	Jamie:	And questions. So I've, uh, I have a really chaotic 9th grade class?
63	Cindy:	Mmhmm.
64	Jamie:	And just, uh. The question or challenge is when students-- I'm already sensing that when students are not um familiar with the format in which they are expected to collaborate and expected to do mathematics that is not individualized sit in a seat, like seatwork worksheets, um. I'm just I'm still confused about how to enforce those expectations and how to hold the whole class accountable? Um. I have like one class of 9th graders that are doing it pretty expertly? Like, astoundingly? Like unnervingly? I think there's something wrong with them. Um.
65		[Laughter]
66	Leo:	The sixth, right?
67	Jamie:	Yeah, the sixth. You observed them, right? They're like mostly on task and mostly following instructions and mostly like oh cool we get to talk together and spend time together. And my fifth is like, We can do whatever we want! It's a madhouse in here! Because there's not like an individual paper assignment we're supposed to complete and turn in. Um. So how do I get them acculturated to that format? Actually having collaborative conversations and discussions. Um. I'm not sure if that's Agency, Ownership, Identity or Cognitive Demand? Like I'm still not sure what it is that's tripping them up? Is it just the format of like, we get to sit together and talk together and it's a screw around period, or is it like, this is harder than we're used to? So. Anything is better than doing this. Yeah my sixth period is unnerving. I'm like, why are you all so great?
68		[Laughter]
69	Jamie:	I'm just waiting for the other shoe to drop, like there are too many ninth graders in here for it to be this calm.
70	Elizabeth:	And sixth period is towards the end of the day, right?

71	Jamie:	Mmhmm.
72	Elizabeth:	That's kind of nice to end your day with a class like that?
73	Jamie:	Yeah and fifth period comes in right after lunch and they're like, Yeaaaahhh! We can do whatever we want!
74	Soledad:	Mine's like that too, my fifth period is a zoo and my sixth period is like, pretty awesome.
75	Jamie:	Yeah.
76	Elizabeth:	They burn all their energy out.
77	Jamie:	Yeah. I mean fifth period is right after lunch. It's right after lunch.
78	Soledad:	Yeah so.
79	Jamie:	Yeah, that's part of it. It's that time of day. Like, I feel like some of it's not being used to that kind of structure even though we've been like rehearsing and and reinforcing those expectations but. Who knows.
80	Elizabeth:	Thank you.
81	Leo:	Uh yeah so continuing from last year one success is more intentional open ended questions. And so still the questioning strategies for me, I'm still using those. And being aware that I am, I need to do them and I am doing them. That falls under... I guess the way I'm asking could be AOI.
82	Anna:	How so?
83	Leo:	In that-- in that I'm asking them for their opinions. So it's it's what they think. It's what they're doing as opposed to what do I want. And it's what are you thinking, what are you saying. And hopefully in my classes I'm making connections between multiple representations. That's I guess just math. It's just math. Uh.
84	Anna:	It's not "just math."
85		[Laughter]
86	Leo:	You know.
87	Jamie:	Cindy, “The Mathematics.”
88	Cindy:	It’s the title.
89		[Laughter]
90	Jamie:	You just said it with such authority. "The Mathematics."
91		[Laughter]
92	Leo:	Just the math. Just.
93	Elizabeth:	Do you find that in both geometry and math analysis? Or is more in one--
94	Leo:	More math analysis. Because I, now that I'm saying it, you know, I should do it in that one also.
95	Elizabeth:	Mmhmm.
96	Leo:	And my challenge is um, I I frequently think about this, is the balance between discovery and straight skill building, that kind of thing. Um I don't know I don't know what term is commonly used for that, but discovery

		versus how much rote instruction should there be. Again I'm thinking of math analysis because at this point this may be their last math class. So they need to discover, but they need to know these things.
97	Jamie:	Mmhmm.
98	Cindy:	Mmhmm.
99	Leo:	So. And I think that's, either Math or Cognitive Demand.
100	Elizabeth:	I feel like that's a challenge that we all face when teaching is that like that balance. Like how do I get through everything but also present it in a way that I want students to see it in. Thanks for sharing these. I think these are definitely going to help us when we're thinking about what's our long term goal or research theme. Like I'm seeing some themes kind of come out of those questions and challenges. We will revisit these in a moment.

Extended transcript of episode Case 1b from Session 1

1	Elizabeth:	So I thought we could just start this conversation with like just kind of making a list and then we could tweak, decide on which which research theme we wanna take up. And this could be a draft right now it doesn't have to be total finalized, we can revisit it. Um. But are there ones that we to add to those possible themes?
2	Cindy:	[Whispers] I like them.
3	Elizabeth:	You like them?
4	Anna:	You like them both?
5	Elizabeth:	I'm seeing that in your questioning strategy-- questions and challenges, there's a lot of things coming up around, like, reengaging students.
6	Anna:	Mmhmm.
7	Elizabeth:	Um. Which I think can fit into the building student perseverance. And you could also call that out in a different?
8	Soledad:	And I also like the balance of discovery and rote instruction. That's definitely a daily thought struggle for me.
9	Anna:	Mmhmm.
10	Elizabeth:	Mmhm. And what we're eventually gonna do is we're gonna come up with a theory of action. Like how are we going toget this? And so that feels like that could also be part of the Theory of Action?
11	Soledad:	Yeah.
12	Elizabeth:	Is like, if we’re able to balance discovery and rote instruction, then our students will better with— prepare our students to persevere.
13	Anna:	Yeah.
14	Elizabeth:	A lot of key words.
15	Soledad:	That could connect to that one, yeah.

16 Elizabeth: Other? And, like, thinking to your, like, school-wide goals? Like I know there's, like, Soledad you mentioned, like, the cultural that you guys all come back to cultural…?

17 Soledad: Well, the math team did— uh, uh, there were other people, too. I think there were— were there four or five? I missed that initial part of it. There were four or five that we broke off into different groups and this group, um, was teaching with, through cultural lens?

18 Elizabeth: Mmhmm. [Nods] Is that something that we would wanna add to our research theme? Or do we see that being more as a way to get, to achieve the goal.

19 Soledad: I see. I think it's, it could be part of achieving the goal because using cultural tools?

20 Anna: Mmhmm.

21 Soledad: Um. And cultural knowledge? To build student perseverance.

22 [Pause 5 seconds]

23 Elizabeth: I feel like you guys always just agree about everything so quickly. [Laughs] I was expecting this to be like a long conversation! Are there other topics that are just really on your mind?

24 Jamie: I'm still thinking.

25 Elizabeth: Ok.

26 Jamie: I'm, like, left.

27 Elizabeth: Yeah.

28 Jamie: I don't agree yet. I'm getting there. [Laughs]

29 Elizabeth: What don't you agree with.

30 Jamie: No, I mean I'm still thinking, I'm not sure.

31 Elizabeth: Oh ok.

32 Jamie: I'm just being contrarian.

33 [Pause 5 seconds]

34 Elizabeth: So maybe we should go about this—

35 Jamie: [Waves his hand]

36 Elizabeth: Oh, go.

37 Anna: Yeah.

38 Jamie: Oh, I was just going to say, for the second one, "Building student perseverance," um. I feel like what's missing there that we brought up here is around, is something around, like, promoting that collaborative work.

39 Anna: Mmm.

40 Elizabeth: Mmm.

41 Soledad: [Nods]

42 Jamie: So it's there, it's just not explicit?

43 Anna: Mmhmm.

44	Jamie:	Um. But I feel like each of us in one facet or another mentioned how we can get students to work together better or, um, what it, what it would require or take to promote that kind of collaborative work so that students are hopefully taking ownership as much as possible. So there's, so there's something there that's missing for me that wasn't, that wasn't a big part of our discussion last year but that is already surfacing this year.
45	Elizabeth:	Maybe something like, "Building student perseverance and the capacity to struggle productively, to engage in group work?" Or, "to [shrugs]…"
46	Jamie:	That's a good question.
47	Anna:	Yeah, I'm hearing something like a goal, or like a why.
48	Elizabeth:	Mmhmm.
49	Anna:	Like a, why are they doing that? Or.
50	Jamie:	Why are they doing what?
51	Anna:	Why are they persevering? Or like.
52	Elizabeth:	Persevering to do what?
53	Anna:	Mmhmm!
54	Jamie:	Hmm.
55	Anna:	Or. I don't know.
56	Jamie:	It's not formed yet. [Laughs]
57	Anna:	Yeah, that's fine.
58	Elizabeth:	Maybe we just add something around working together.
59	Soledad:	Maybe it's like the capacity to struggle productively and collaborate. Is that what it is?
60	Cindy:	Struggle collaboratively. [Laughs]
61	Elizabeth:	Collaboratively productively. [Laughter] Productively collaboratively.
62	Cindy:	Struggle together, maybe. Or.
63	Anna:	Ok, together?
64	Jamie:	I mean, we're just talking themes, right?
65	Elizabeth:	Yeah.
66	Anna:	Mmhmm!
67	Elizabeth:	So, like--
68	Anna:	And it's, this is developing through, over the long-term.
69	Jamie:	Yeah.

Extended transcript of episode Case 2 from Session 3

Transcript included under "Chapter 5 transcripts."

Extended transcript of episode Case 3 from Session 5.

Transcript included under "Chapter 5 transcripts."

Chapter 5 transcripts

Extended transcript of episodes from Session 3

1	Elizabeth:	So last time [lots of laughter, because Jamie told a joke] — last time we looked at student video, we had used this document which I know is, um, there's a lot of text on here. But we had identified three different Look-For?
2		[Side chatter of Soledad looking for her materials]
3	Elizabeth:	Um, so if you can find in formative assessment, we had identified, "Explains their thinking, even if somewhat preliminary," as something that we wanted to look for--
4	Jamie:	Mmhmm.
5	Elizabeth:	for student perseverance? We also identified in Agency, Ownership, Identity, um, "Builds on the contributions of others and helps others see or make connections." And then in the Math, "Explains their reasoning processes as well as their answers." Do those still feel like the three that we'd wanna look at?
6	Jamie:	What was the second one again? I'm sorry, Elizabeth.
7	Elizabeth:	In AOI, it was, "Builds on the contributions of others and helps others see of make connections." So that can just— so we're not trying to look for everything when we watch this video, we can kind of try to hone our eyes on, on those three.
8	Soledad:	Yeah, circle them [inaudible].
9	Elizabeth:	Student perseverance. Yeah. And Leo, do you want to give us a little framing on what you were doing the day of this video? Like what the math task was?
10	Leo:	Um, we're basically starting graphing quadratics by finding the vertex and uh direction. Which way it opens, and uh, being smart about the values you pick for the table.
11	Elizabeth:	Do you remember, I know you just kind of skimmed through this video, but do you remember what, where she was on this worksheet?
12	Leo:	Uh, I have, uh, probably somewhere in here. [Gestures to the bottom half of a sheet.]
13	Elizabeth:	Ok.
14	Leo:	This we did as a whole group, whole class, and she was prob— I don't remember.
15	Anna:	She was, yeah, I think that they were working on this first one, over here. And I think as a class, as a whole class you'd done this stuff.
16	Leo:	Yeah.

17	Anna:	Oh so this was, um, oh just a little more framing— The, the segment I picked, I tried to pick a time when Leo was talking to a student. As an example of perseverance, um, where like a student might be confused. Um. And Leo was talking to the student. This is, I think this is towards the end of the period? And they're working, um, on the sheet.
18	Leo:	Ok, so it might not be the one you guys are thinking about. It might be on the back. Which some of you guys don't have. Um.
19	Anna:	Oh yeah. Mmm, maybe. Here, do you want— yeah.
20	Elizabeth:	And while this isn't an example of, like, you're not explicitly doing Three Things in this, I think it, it gets at kind of like the essence of Three Things? Where you're giving opportunity for students to share their thinking.
21		[Participants watch Leo's video]
22		[Participants talk in small groups]
23	Elizabeth:	I'm interested to see which each group, like, focused your conversations on. [Pause 5 seconds] Now you're silent. [Laughter]
24	Soledad:	Well, now we said it all.
25	Jamie:	We're very simple people.
26	Cindy:	Just replay the video. [Laughter]
27	Anna:	[Laughing] Just hit play, what was said.
28	Elizabeth:	Just rewind.
29		[Laughter]
30	Jamie:	Uh, so in our conversation, Cindy pointed out that it felt like Eden in the video was really building on Leo as a thought partner? So this, so there's an— there's a moment here for AOI in which he's uh she's building on the contributions of others, right? So like every question he's asking is prompting her to continue to build her explanation?
31	Soledad:	We— we came up with a question on, does Eden actually know it, but she wanted affirmation?
32	Jamie:	Mmm.
33	Soledad:	And is that why she was asking Leo and then waiting til she was getting bits from Leo to go on with the next step. Was it like a worry about being vulnerable about her, about what she held? You know?
34	Jamie:	Mmhmm.
35	Soledad:	Or was it that, was it that her, like, thinking there was still very partial. And so she, um, she like was really wanting or needing, uh, the bits and pushes from Leo.
36	Elizabeth:	What made you think that that might be a possibility?
37	Soledad:	Because I think that lots of our students, um, lack confidence in their math ability? Um. Or, maybe, and maybe some don't lack confidence but some are for some reason, like, reticent to share? Um. I mean it could be either

		way, but for some reason there's something that keeps them from, like, sharing completely what's like, what they're what they're thinking and what they have? Um. But some I do think, like, have, have some ideas and are still putting together the puzzle pieces.
38	Jamie:	Mmhmm.
39	Elizabeth:	What do you guys think?
40	Leo:	Well--
41	Jamie:	Go ahead.
42	Leo:	Well, I personally think that Eden has no issue with being wrong. She, like, voices her opinions. And I think she was just really confused.
43	Elizabeth:	Mmhmm.
44	Jamie:	Although I will say knowing Eden, I was surprised she wasn't willing to at least formulate something. Because she usually in my class last year was like, even even at the most, like, "Iuhknow [shrugs], this is way off, I'm confused as hell," she was always willing to offer something. And eager. Um. When focused. So, I was surprised that it didn't seem like she was explaining her reasoning process or explaining her thinking? Um. And saying, like, "I can't— I can't say it, I can show it, I can't say it, I can show it."
45	Cindy:	Um, from that again, like, like I haven't taught her, I just know her from passing periods and other places. But, like, I e- even I would side with folks who've taught her, like, I don't, I don't think she was try— she knows it and trying to get affirmation because if she really wasn't able to complete any of her sentences in the beginning? And yet, like, after some prompting or some questioning, like, I— I was just saying in this group, some students would be like, "Oh, ok," and then they'd move on to their work. And she actually kept him there a little longer, and like, "So…" And then she either questioned, give some questions to get confirmation or give some statements to get confirmation on what she just discovered? Um. And. My, my next thing is like, um, I, I wonder why some students do that, right? Like she did. Like she actually pressed further to make sure she got it. And then some students would just go back, like, and not, not make sure to get that sort of double-checking.
46	Soledad:	So I think she was "Asking questions and making suggestions to support analyzing, evaluating, applying and synthesizing mathematical ideas"?
47	Cindy:	Okay?
48	Soledad:	[Laughs] Well, and—
49	Cindy:	That's somewhere?
50	Soledad:	It's in, it's in AOI?
51	Cindy:	Mmhmm?

52 Soledad: Just above the one we're supposed to be looking at?
53 Cindy: Mmhmm?
54 Soledad: Maybe that's what she was doing?
55 Elizabeth: So in thinking about our research theme of, like, trying to build students' perseverance and capacity to struggle, is after watching this, are you guys thinking about continuing with Three Things? Continuing with it and adding something to it? Like, is there something missing that we, that we want to investigate? Or do we feel comfortable with the research that we're doing in our classes right now on Three Things.
56 Cindy: So far what I'm noticing is that, we, we do ask for things that students know, but it's not necessarily like that one straight question, like, "What do you know?" It's, um, some form of it. Like it's our own style of it.
57 Anna: Mmhmm.
58 Elizabeth: Mmhmm.
59 Cindy: Like we, we're all, we're modifying the heck out it. Anyway, and sometimes it's more than three things. Like, um. Just that interaction between Leo and Eden, it's, like, you you don't say, "What do you know?" But you're pointing at the, the points and asking her questions about it anyway. Like, I— I counted that as, one, two, three, like it probably more than three things right there--
60 Anna: Mmhmm.
61 Elizabeth: Mmhmm.
62 Cindy: --in that interaction.
63 Soledad: So should we, um, create a uniform question that we're all gonna do?
64 Jamie: I think if we were to continue with this, I'd be more curious about where we chose to do it. Like at what points in the lesson. Is it when we're opening up a question for the whole class? Like something they're seeing for the first time and exploring for the first time? Is it more like when they're in the weeds of problem-solving? And they're saying, "I don't know anything, I'm stuck, I don't know what to do, you didn't give us anything, you suck, I hate you." Right? [Laughs] Like. Like at what point, at what point. I'd be curious to know at what point this is most useful or resourceful for students.
65 Elizabeth: Mmhmm. What do you guys think?
66 Cindy: Um, I had that moment just the other day. It was the— it was a, like, there was a graph? And there was that paragraph that sorta give the beginning of the story of that graph? And so, and so this is after I know the students can give some descriptions of a graph.
67 Jamie: Mmhmm.
68 Cindy: That I ask them specifically for three things that you know. So, like, and I'm already sort of anticipating some of these answer, like, I'm hoping that they

would say something about the x-y labels, or how it was scaled, or that what's the title, or, um, and. And this is after they read the learning target about increasing decreasing constants. So, like, some students brought that up, like, "It's going up, then it's going down, then it's going up," and then some students said, "Yeah, it's increasing, then decreasing, then increasing," like that. Um. So. I, I guess, I mean to, for me, it was a combination of things? But one of them was just knowing that there will be some successes cuz they've seen it once before, so--

69 Jamie: Mmm.

70 Cindy: when I say three things, like, there will be something they could say.

71 Elizabeth: Hmm.

72 Cindy: Um. Yeah.

73 Jamie: Mmhmm.

74 Elizabeth: Any thoughts for next steps in terms of implementing a strategy to discuss at the next 10/25 meeting? I hear Jamie's really wanting to kinda pin point like when to use that? Do you feel like you guys all wanna be using it at the same points in lessons? Or not.

75 Cindy: I like that you [to Jamie] were using it with the posters, like when they were doing team stuff. So I'm gonna try that next time.

76 Jamie: Or— or even thrown back to what Soledad had said earlier when she was thinking internally, like the KWL strategy, like when— what kinds of problems make the most sense for a strategy like this with students. So not, not necessarily like where in the lesson do you place that, but like, ok a matching activity where it's sort of open-ended and students are struggling. I wonder, yeah. I just wonder if that's more or less helpful than, say, compared to what Leo had going on with Eden around like, they're doing an individual, like, assignment about vertex.

77 Elizabeth: Mmhmm.

78 Leo: So I think that [clears throat] I think the straw— I think our three things that we've been focusing on is content knowledge? I think it could be, um, take a step back for directions? So I know you checked for understanding, for instructions before you started the activity.

79 Jamie: Mmhmm.

80 Leo: But that could be even something they know. That, like, "What are we supposed to be doing? We're supposed to be putting these matches together."

81 Jamie: Mmhmm.

82 Leo: Um. Like three are red and, "These go together. What do you think? Do," like, see I'm not asking a question. "What do you know about these three, should they go together?" Uh, so it's really directions-focused, and not

		content-focused? Because it's something they also know. A-a-and I think ninth graders have a lot of trouble with directions also.
83	Cindy:	Whaaat.
84	Leo:	Surprise! [Laughter] Surprise!
85	Jamie:	All my— all my ninth graders always know exactly what they're supposed to be doing at all times. [Laughs]
86	Leo:	Yeah, "What are you supposed to be doing, making dots on my poster? Is that right, Mr. Rossi? Is that right?" "No."
87	Elizabeth:	I guess like what is it about perseverance--
88	Jamie:	Sorry. [Laughter]
89	Elizabeth:	that we're trying to--
90	Jamie:	Sorry, Elizabeth. [Laughter]
91	Elizabeth:	No! [Laughter] I have been finding you very funny today. [Laughter] But I have to like make sure that I don't get too distracted so that we actually end at 11!
92	Jamie:	Thank you.
93	Cindy:	Oh that's right!
94	Elizabeth:	Um. Ok. So what is it about perseverance that we actually want to get at with Three Things. Because I hear Leo talking about this idea of like, content focus versus directions focused. So when you're thinking about, we want to build student perseverance, like what are we hoping this strategy solves. Or attempts to solve.
95	Leo:	That's a great question.
96	Soledad:	I was thinking that it's a bridge so they move on. So that, you know, when they're stuck and they're like [mimes pushing with hands] "Oh, I don't know what to do, so I'm gonna" [grabs phone and pretends to play with it under her desk]. You know?
97	Elizabeth:	Mmhmm.
98	Soledad:	Um. That instead of doing that, that the that the asking of the three things will become more automatic for them, that they start asking themselves? And that they move forward. Autonomously. Tomorrow. Starting on Monday. [Laughs]
99	Jamie:	Like, immediately. [Laughter]
100	Elizabeth:	Is that how you guys are thinking about it?
101	Leo:	Yes, that even if they can't move forward, if they move to a place where they know what they should be, like, what should come next, you know? Like, I play video games and I make plans— like, when you're playing, you're like, "Okay, I need to go here, I need to go there, I need to go there." But, I just don't stop and don't do anything. I [holds his fingers wide and moves his hand away from his forehead].

102 Soledad: Yeah.

103 Leo: So, even if you know content-wise what you're supposed to do, you could be like, "Okay, if I ask Lin this, and I find this out, then I'll be able to do this." Or, "If I ask somebody else this, then I'll be able to do this." Um, instead of just getting stuck. So, that's sort of why I brought up the directions because they don't— some, some kids really just don't know the directions--

104 Jamie: Yeah.

105 Leo: and if they knew the directions, they would try something.

106 Jamie: Hmm.

107 Elizabeth: Cindy, I saw you shaking your head earlier.

108 Cindy: Yeah, well cuz it has started to evolve for me. Uh, I keep bringing up, like, I want to engage the unengaged. So, like, I'm starting to see this not as a one-on-one strategy? LIke, me and another student? But, like, something public. Because it's— I, I want to be able to publicly acknowledge that somebody is successful, like they, they get— "Yeah, that's something that we could see, and point out, you're right." Like, "You know something here."

109 Elizabeth: Mmm.

110 Cindy: So, that's why I'm like, I really like that idea [gestures to Jamie], of the doing it in a group, cuz then maybe it's not three things, maybe it's four things, or customize it to how— whatever the size of the team is--

111 Jamie: Mmhmm.

112 Cindy: like, so the team [mimes pointing to members of a team] "What's something you know? What's something you know? What's something you know?"

113 Jamie: Mmm.

114 Anna: Mmhmm.

115 Cindy: So then everybody--

116 Jamie: I like that.

117 Cindy: gets a say and everybody's like, "Got it." And so hopefully what that does for the person who's unengaged is that, "Oh, ok, I do know something." And, and that, that, that's my goal. Like, to get the unengaged engaged, now.

118 Elizabeth: Which I think also speaks to that piece in your research theme about "together"? Like, you'd, um, "building the capacity to struggle, like, together," so they're using all the, everyone's pieces to come together, to persevere. How do you guys feel about implementing it in the way that Cindy was thinking, in terms of, like, groups, "What do you know, what do you know, what do you know?"

119 Soledad: Yes. That sounds good.

120 Leo: [To Cindy] Oh, you were asking different groups?

121	Soledad:	But, well, Leo doesn't do group work really, though.
122	Cindy:	No, I--
123	Leo:	I mean I could do it publicly, that's just [inaudible].
124	Cindy:	Yeah, but that's still whole class. Like, I, just, I've done one whole-class, but now I'm very interested in seeing how it would work out in groups.
125	Leo:	So, if you have a group of four, you'd be like [mimes pointing to students], "What do you know, what do you know, what do you know, what do you know."
126	Cindy:	Right. Like, when I do that matching activity next week.
127	Jamie:	Mmhmm.
128	Cindy:	Um. Yeah.
129	Jamie:	I think I could try it, but I'm already anticipating, like--
130	Cindy:	Yeah.
131	Jamie:	"I don't know anything." Um. I would, if, I'm already feeling like in that situation, um, cuz a lot of kids do that, but I've been really pushing this year, is like, "Ok, I'll come back to you. Cindy, what do you know?" And then when Cindy says something, and I go, "Alright, Elizabeth, please restate what Cindy said in your own words." So that they're like--
132	Elizabeth:	Mmhmm.
133	Jamie:	internalizing whatever, whatever the other folks are saying. And they'd say, "I didn't, I wasn't listening," and I'm like, "Great. Cindy, restate it again." [Points to the imaginary other student] Like [laughs]
134		[Video skips]
135	Elizabeth:	summarize for me, what are we all trying between now and 10/25.
136		[Laughter]
137	Leo:	Um. First stab at it.
138	Elizabeth:	Mmhmm.
139	Leo:	Uh, what do you know, in a public setting. And I don't necessarily know what— but whether in groups or in, in whole-class, soliciting knowledge from multiple students.
140	Elizabeth:	To do so, and if we do that, then what's gonna happen? What do we hope happens? Or like, why are we doing it that way? I just want to make sure we're all on the same page.
141	Cindy:	Um, hold on, let me see your sentence. [Looks at the whiteboard where Elizabeth's notes are being projected] Um. Oh, ok.
142	Soledad:	So that they push, push forward?
143	Elizabeth:	[Reading aloud what she's typing] Students can push forward. And I also hear a piece about, like, using each other's ideas.
144	Jamie:	Mmhmm.

145 Elizabeth: [Reading aloud what she's typing] Each other's and their own ideas. Good? Ok.

Extended transcript of episodes from Session 5

1 Elizabeth: Um. So Jamie, do you want to share a little bit of, like, context, and, like, the mathematical goal of the lesson, that we will see?

2 Jamie: Yeah. So. I can show you the assignment. [Passes assignment out to each participant] Uh? They started it yesterday, they're gonna finish it today. Um. It's the Making Money assignment from Algebra 2? And so, looking at comparing simple versus compound interest? Um. Yesterday we really only got to simple interest. And that's because I spent way too long lecturing and answering their questions about what is interest and how it works. So I told them, um, the background context of interest, and simple versus compound, and then I gave an example of what simple interest is like. And then they were supposed to complete the front side of this assignment? For the simple interest example. And then today they're gonna complete the compound interest and then debrief it. Um. And so basically after way too long of me talking, and I warned them it's gonna be way too long me talking, and asking, and answering tons and tons of really awesome questions that students had about, like, what is interest, how does it work, how do I get more of that money. [Laughs]

3 Elizabeth: How do I cheat the system.

4 Jamie: How do I cheat the system. There were some really good ideas, I was like, "Ooo. Ooo. I like that." Um. They. I mean, basically, we'll hopefully see what happens today, afterwards. Unfortunately you're not gonna see that. But you're gonna see in the video them doing group-work on the front side of this assignment? And working together to try and ultimately, like, derive the equation, which is a linear equation. So I anticipated that because we've been doing exponential functions, um, most students are going to accidentally write the standard form for an exponential function instead of linear.

5 Elizabeth: Mmhmm. Um. And as we're watching this video, we've— we've been working on the Three Things strategy, but we've also recognized that it comes with a lot of — or it doesn't work for everything, right? Like, Three Things we've been working on when the teacher is present, but we're also interested in what happens when kids are individually persevering or indi— like, persevering as a group. So I think that's what we want to be looking at in this video, is like how are— how does each student, each of the four students we're going to see, persevere to get through this problem, or how do they not persevere. And what are the different reasons behind, like what are the root causes of those issues that come up. So last time we met we

		talked, we broke it down a little bit by these three dimensions. Um. And we had some predictions that like, within Cognitive Demand, students might be saying, like, why are we doing this, this is hard, when do we get to real math, like that's one reason that students might have difficulty persevering? And Access, kids might not feel like they have the right tools. Or AOI with fixed mindset. So I wanna keep this kinda as a frame for us to see if we want to expand our list here. Um. Or if we can like diagnose these students, in the video.
6	Jamie:	Yeah. And Elizabeth, uh, zoomed in on the land of misfit toys, so it'll be really fun to watch.
7	Elizabeth:	[Laughs] That is a really— I think— I think it's just like a nice example of what happens in group work. Like, we have all different types of students in our classes.
8	Jamie:	It's a team that on their first day together were like, 'I don't wanna work with him! I hate him!' Like, really violently opposed to working together, and still are but in ways that are cute and adorable. Like, 'Why do you say that? You're not listening to me!' 'Ugh, shut—what do I do?' They're— they're more entertaining at this point than they are--
9	Elizabeth:	They're very entertaining.
10	Jamie:	irritating.
11	Elizabeth:	You'll see me in the video, um. They— I tried to cut it out, but my skills are just not that great. Um. They're, at one point, Zakira tries to ask me for help? And I didn't realize that the camera, you can't hear my responses when you're behind the camera? But, um, I was saying that I can't do that, she should ask her group. [Laughs] And I apologize for the videoing, I am not a videographer. [Laughs] It's not my skill set. Ok.
12		[Participants watch Jamie's video]
13	Elizabeth:	What do we notice about each student's level of participation and how they persevered or did not persevere? And remember we're really sticking to low-inference data. So just what we noticed, we're not making any assumptions.
14	Leo:	I couldn't tell if Mohammed was participating at all.
15	Jamie:	I want to add about Khalid. I think it was hard to tell because he was speaking so much in Arabic, and I'm not sure— I don't know what he was saying to Omar. There were multiple points where he is talking to Omar, but it is not clear if it's about the mathematics or not. So he could be, but he might not be.
16	Elizabeth:	Right.
17	Cindy:	Yeah, I'm— I was looking at all of the papers, like, and noticing that there were a lot of notes on Omar's and Zakira--

18 Anna: Yeah.

19 Cindy: but not a lot on Mohammed and Khalid. But I did have, like, you know, Khalid at some point sh— shared notes with Omar, right? And I think that— I, I, I note— I noted Access and AOI there. Just like, you know, one of the comments was whether they have a toolbox, and they do, they they have their notes with them. And so I think that's a point for Khalid, in that case. He's got something to share.

20 Elizabeth: Yeah.

21 Jamie: Yeah, and he's decoding it seemingly well.

22 Cindy: Mmhmm.

23 Leo: Yeah, for the record he's using blue notes as they were intended to be used, as like reference material. Not just old work but, like, actual reference material.

24 Jamie: Yeah. Um. I also noticed that at the beginning, there were two pairs. They were not working cohesively as a group. So it looked like Omar and Mohammed were working together, and Zakira and Khalid were working together, in that first clip?

25 Elizabeth: Mmhmm.

26 Jamie: And then. It seemed like that pattern was breaking as they continued.

27 Elizabeth: Mmhmm.

28 Jamie: But it sort of started off like Zakira and Khalid working across and Omar kind of doing his own thing and unclear whether Mohammed is working with him or not.

29 Elizabeth: Yeah I saw that moment as like, when they were in this [pointed to the graphs part of the sheet] section--

30 Cindy: [Nods]

31 Jamie: Mmhmm.

32 Elizabeth: and Zakira was like, 'It's B, or C,' and Khalid, we couldn't— he wasn't speaking but he looked like he was thinking about it, like he was staring at it, and so the fact that the pair looked like it was kind of stuck.

33 Jamie: Hmm.

34 Elizabeth: And then eventually Omar was like, 'What are you guys on?' and then they started working as a foursome.

35 Jamie: Mmhmm.

36 Leo: I think this is low inference. Um. I think Khalid was helping Zakira with like, um, calculation kind of stuff? Like, calculators, how this— how we got this. That kind of thing? And then when it was more conceptual, like, which one is it, then Omar jumped into it more, was more at, at that point?

37 Elizabeth: Mmhmm.

38 Anna: Mmhmm.

39	Jamie:	Mmhmm.
40	Leo:	Um. But that's what I seemed to notice.
41	Jamie:	But I also noticed that Zakira was engaged with the conceptual questions as well.
42	Leo:	Yeah, yeah.
43	Jamie:	Like she's the first one offering--
44	Leo:	Right.
45	Jamie:	her ideas about why— uh, which graph it would be? And she's eliminating, like, she's like it can't be A. She didn't explain why, from what I could hear, but she's— she's offering that up. Like for some reason she knows in her mind that it can't be A. Um. And that— and that's where it seems like Omar jumps in and starts providing a counterbalance to what she's saying.
46	Cindy:	Mmhmm.
47	Leo:	He provides a good balance to her.
48	Elizabeth:	Mmhmm.
49	Jamie:	They both provide a good balance to each other.
50	Cindy:	[Laughs]
51	Elizabeth:	Yeah, there's a moment when she's eliminating and I was like, 'Come on, ask her some question about that!' Like, 'Ask her why she's eliminating A!'
52	Anna:	Yeah.
53	Elizabeth:	And, um. But they were just both kind of like so in.
54	Jamie:	But then they both— they both had, um, really good observations. Once they decided on B, they both phrased how come it was B. Like why, why is it graph B, why is it that linear function.
55	Elizabeth:	You say both. You're saying Zakira and--
56	Jamie:	Zakira and Omar. So Zakira is like, 'Well it's adding the same'—No, Zakira said something like it was going up by the same amount, which is 20. And then, Omar, 'Yeah it's adding the same number every time.' There's that interesting moment where it's like not clear if they're actually listening to each other? And Zakira is like, 'Yeah, I said that.' Like [Laughs] 'I already said that.' Sort of like, 'You're not listening to me.' Um. Yeah, and Omar I think was just providing a little more specificity in the explanation? But.
57	Elizabeth:	Mmhmm.
58	Jamie:	That was kind of a cute moment. Where it's not clear whether they're listening to each other. Or whether Zakira is saying, 'You are not listening to me.'
59	Cindy:	Umm. Well, with that exchange I put down a note for myself, like, there's validation. And I wonder, like, that's what continued to help drive them to become instead a foursome? And, um, she's like, 'Oh yeah, I'm hella smart!' Right? That's what Zakira said, right?

60 Jamie: She did say that.

61 Cindy: She needs to get that validation from Omar, and and, maybe that's that helped them continue on together.

62 Elizabeth: Yeah. I was really interested in that moment where, you know, Zakira was like, 'It's B, I think it's B.' Omar said, 'It's B.' And everyone was like [pounds table], 'Yeah, it's B.'

63 Jamie: Mmm.

64 Anna: Mmhmm.

65 Elizabeth: What do we— what do we think around, um, that moment? How students were acting.

66 Jamie: I'm not sure I caught that as clearly.

67 Cindy: Well, I, uh, saw when you [Jamie] were next to them, and, you know, Zakira was like, 'Oh, I'm writing in pen, so you're not sure of this equation? You don't know it?' Like, so, and then, it's like, and then while he [Omar] was talking to you— you [to Elizabeth] you were panning your camera to other students who were writing stuff down, and so, you know, you could see, Omar's kind of the lead of the whole group.

68 Jamie: Mmhmm.

69 Cindy: And they, they're all, like, waiting to see what he does--

70 Jamie: Right.

71 Cindy: or what he writes. And so, um, I see two things— I mean, one is that, yeah, a team needs a leader? I guess. But, also, like, um, what can we do about it? Because right now, we're— we're taking notes and we have two people that we know a lot about, and two who we don't. So is that a spot to interrupt? Or to encourage, like, just I think that's a moment where we could do something.

72 Jamie: Yeah, I noticed that I did not do a great job of breaking that status up there. [Laughs] There was like a conversation with me and Omar. Um. I missed an opportunity to ask any of the others, like, directly.

73 Elizabeth: Well, at the end you did tell him to make sure he could explain it to the rest of his group.

74 Cindy: Mmhmm.

75 Jamie: Right. And they— and they did start doing that--

76 Elizabeth: And he did.

77 Anna: He did.

78 Jamie: he was able to explain it. I just don't know because we didn't see it on the video afterwards? Like, what— what happened as a result. Was there any conversation after that, or they just accepted it as truth and move on.

79 Cindy: Well, um, Zakira did ask him to move on to the back, like she was starting to question.

80 Jamie: She was ready to go.

81 Elizabeth: She really wanted to move on to the back.

82 Cindy: [Laughs]

83 Jamie: I was like, we haven't done compound interest yet. You're ok, you're ok. 'But I wrote in pen!' She's hilarious.

84 Elizabeth: Cindy, I think you bring up an interesting question about, like, what can we do about it? Um. Because, like you said, like a group needs a leader? That, I think that helps them persevere. But, like, how do we— what do we do to break that up, or to make sure everyone is participating or persevering.

85 Jamie: So, I— I will say for myself, I feel like one of the protocols I've been upholding well this year is around the Resource Manager?

86 Cindy: [Nods]

87 Jamie: And I feel like part of the reason in that clip I didn't do a good job of breaking up, like, who's— who's participating, and calling other voices into the conversation, is because they were done ahead of time. And so usually what's happening is when, um, when a group is stuck, the Resource Manager calls me over and I'll pick on anyone, right, I'll just choose someone randomly, and they know that. But this was not a case when they were stuck. They were confident, we're done. And so I didn't, um, I didn't follow in my head my usual protocol, like, I'm gonna— I'll, like, 'Cindy! Whats the team question?' And then it's like, addressing it to Cindy. Um. I— I think it was just out of my routine, when I was like, 'Oh, ok, they're, like, a little bit ahead.' And I see that they're making the common mistake that I was expecting on the equation. But I didn't do my usual interruption there.

88 Leo: Did you go over, or did they call you over?

89 Jamie: Well, I— I did say at some point to the whole class, was like, 'If you feel like you're confident you've got your equation, like let me know' because— the thing is, they were making faster progress than my first period had? So, there were a couple of teams that were like, 'We're done! We're done!' You know, they weren't really calling me over? Because they thought that they were done. But almost every team that was done, was making that same mistake. So I was going over to sort of intervene and just check with them, like, 'How did you come up with this equation? Where are you getting that from? What kind of equation is it?' Um. So I don't think they explicitly called me over there. Except, expect to be like, 'We're done, Mr. Rossi! Can we go on to the next part?' Like, no because you haven't done compound interest yet. So. Because usually they were, like, explicitly calling me over, like, 'We don't know what to do!' So I was breaking with my usual protocol, because it's not typically why they call me over.

90	Elizabeth:	[Laughs]
91	Jamie:	‘We’re done, let’s go!’
92	Leo:	I think you could just extend what you usually do. So, ‘What’s the team—what’s the team answer?’
93	Elizabeth:	Mmhmm.
94	Leo:	‘Ok. Cindy, can you tell me how you guys got there. Do you agree, Elizabeth?’ And, whatever.
95	Jamie:	Yeah.
96	Leo:	You just got caught off-guard.
97	Jamie:	Yeah, I got caught off-guard. Because, like, in first period, they were genuinely getting stuck. And so, like, teammates were— Resource Managers would call me over, and I’m like, ‘Ok, Team Captain. What have you guys done so far?’ So it was easy for me to do that with Period 1. But Period 3, they’re all, ‘We’re good! We’re done! Can we move on?’
98	Leo:	[Nods]
99	Jamie:	I let my guard down with them. Damn you, Period 3. Let my guard down.
100	Cindy:	It’s a pleasant surprise, right?
101	Jamie:	It is a pleasant surprise. ‘We got this!’ I’m like, ‘Great.’
102	Elizabeth:	Any other noticings, or, point that you want to discuss from that group?
103	Leo:	In the context of Three Things, um, sort of. So Zakira said the things that people have mentioned, like, ‘Oh, shit, I’m hella smart,’ ‘So you had me writing in pen? But you don’t have no equation?’ Um. ‘I think it’s B, do you think it’s B?’ She never said that, ‘I’m not good at math,’ but I think it sort of stems from that?
104	Elizabeth:	Oh.
105	Jamie:	Mmm.
106	Leo:	That she wants to be right, and, she just needs to be right. Um. Sort of a different way to rephrase, ‘I’m not good at math.’ Um. And it’s how— I was just wondering how the group can support that? Like, as teachers we know, we know more of what her needs are? But how can, um, we support the kids to support her.
107	Jamie:	Right, so that missed opportunity, somebody ask her a question, why did she eliminate A.
108	Cindy:	Oh. [Nods]
109	Jamie:	We haven’t— I haven’t figured out how to get them to do those questioning strategies with each other.
110	Leo:	I think when you do the Resource Manager thing?
111	Jamie:	Mmhmm.

112	Leo:	‘Jamie, what does Elizabeth think about it.’ ‘Jamie, what does Cindy think about it.’ I may— I mean, I’m basically asking the question, but I’m having you ask the questions.
113	Cindy:	[Nods]
114	Leo:	Because you— then you’ll figure out that I will ask you, then you’ll do it beforehand, hopefully.
115	Anna:	I bet that could be a way to get the kids we didn’t hear anything from to as well? Because it’s like a really low-stakes thing, for you to just ask somebody else what they know?
116	Jamie:	Mistakes.
117	Anna:	Right? Instead of having to share your own thinking, you can— you can be like, ‘Well, I can just talk about what this person said.’
118	Elizabeth:	Uh, so in terms of our research theme around, like, perseverance. And we have the strategy, Three Things. I’ve heard you guys talk a little bit about other strategies we might use specific to group-work. Is there one that we all want to take up between now and the sixth? Like I heard Jamie mentioning really upholding the Resource Manager role? And then possibly Leo adding to that is like using that as an opportunity to, to get at students asking each other questions.
119	Jamie:	Mmhmm.
120	Cindy:	Mmhmm.
121	Elizabeth:	Is that direction like we wanna head? Or are there other ideas about what, what sorts of strategies we might want to try?
122	Jamie:	Look at Cindy, so prepared. You’re a [inaudible]
123	Cindy:	No! I just—
124	Anna:	Oh, sweet!
125	Elizabeth:	Oh yeah!
126	Jamie:	Look at your resources.
127		[Elizabeth passes out pedagogical strategy document]
128	Anna:	Is there one on there that’s like the thing Leo just mentioned?
129	Cindy:	Group questions? No.
130	Anna:	Group questions? Well.
131		[Participants read the strategy document quietly]
132	Anna:	Cindy, you said you thought Group Questions?
133	Cindy:	Yeah, but that’s different.
134	Anna:	Hmm.
135	Cindy:	Because it’s, it’s more of a variation on Three Things, right? Like, here, “What do you know?” It’s, “What, what, what does she know?”
136	Anna:	Yeah. So like number 2? But not quite number 2?
137	Cindy:	Hmm.

138	Anna:	So, like, not quite that.
139	Cindy:	Yeah!
140	Anna:	But.
141	Cindy:	Mmhmm.
142	Leo:	I feel like, um, it's sort of like number 5? Not really number 5, that's not what I, I really want students to do.
143	Anna:	Hmm.
144	Leo:	But to be able to say, "Cindy said blah-blah-blah-blah."
145	Anna:	Mmhmm.
146	Leo:	So, that's "Share what your partner said," but it has a name.
147	Anna:	But with a name, yeah. And maybe not in like a whole-class discussion context?
148	Leo:	Yeah, not like that.
149	Anna:	Yeah.
150	Leo:	And not like, "Oh, like, Cindy said that one time." [Laughter]
151	Anna:	Yeah.
152	Leo:	But just like, she has an idea, you, you, you--
153	Anna:	Yeah.
154	Leo:	you recognized it--
155	Anna:	Yeah.
156	Leo:	and, um.
157	Jamie:	What about "Share what your partner said," number 2?
158	Anna:	Yeah.
159	Jamie:	Because that seems to be more aligned with what we were talking about. Practicing active listening. And restating. I do— I do that a lot in whole class discussion?
160	Anna:	Mmhmm.
161	Jamie:	Where either I'll, um [tsk], I'll cold-call on a student? Like, well, "Somebody, uh, So-and-so restate what [name of student] just said?" Or, I'll ask for a volunteer, depending on what it is.
162	Anna:	Mmhmm.
163	Elizabeth:	Does that seem like the strategy that we want to take up in group work?
164	Leo:	I would like that.
165	Cindy:	[Nods]
166	Elizabeth:	And so if we are having students share what their partner or group-member said, what are we hoping to see change?
167	Jamie:	I mean I would think in an ideal world, that when the teacher is not present, students are restating each other's ideas.
168	Elizabeth:	Mmhmm.

169 Jamie: Either paraphrasing or restating or re-explaining. Cuz this is the scaffold, right. Like, we're asking them to restate another, another teammate's or a partner's thinking. So, ideally they'd then come to internalize that and do it on their own.

170 Elizabeth: Mmhmm.

171 Leo: And I think past that, my goal would be to [clears throat]. So if I'm summarizing what Cindy said, hopefully Cindy would be listening to what I'm saying that she said.

172 Jamie: Mmhmm.

173 Anna: Hmm.

174 Leo: So she'd be like, "No, that's not what I said." [Laughter] "What I meant was this." Or she'd have to process what I'm saying and making meaning out of that.

175 Jamie: Right.

176 Leo: Or as I'm paraphrasing, I'm like, "Wait, that doesn't make any sense." Or, "That brings up another question that I have."

177 Anna: Mmhmm.

178 Leo: "Which is, blank." So, yes, first step is to practice paraphrasing. And I think then the subsequent step is to have everybody involved start analyzing what I'm saying for next steps.

179 Jamie: And ideally, the third person's like, "That's not what Cindy said!"

180 Leo: Yeah.

181 Anna: Mmhmm.

182 Jamie: "You don't make any sense!" "Try again!"

183 Cindy: Where's the fourth person? [Laughs]

184 Anna: Yeah, I know. We've got three out of four! [Laughs]

185 Elizabeth: Poor Mohammed!

186 Jamie: Poor Mohammed.

187 Anna: Aww.

188 Jamie: He was super into the discussion, though.

189 Elizabeth: Yeah.

190 Jamie: He was, he was the one who was like, like [in a hushed voice], "So that's how millionaires get money?" He was like, "So then you can put the money in. And it's always gonna grow that same amount, so I can just take the money out right? It just keeps growing." Like, "Yeah. But there's simple interest, sort of." I have to qualify that later. Cuz he was, he was all like saying, "I put like a thousand in, and then it's like, that, like, three percent is just gonna stay there, and I can just take it out." Like, uuhhh [waves hand to indicate not quite].

191 Elizabeth: He's like, "What about a million?!" [Laughs]

192	Leo:	He’s gonna be super excited for compound.
193	Jamie:	He’s gonna be so excited for compound. Today his head’s gonna explode. [Laughter]
194	Elizabeth:	I wish I could stay for third period today!

Chapter 6 transcripts

Extended transcript of episode from Research Lesson 1

1	Soledad:	So, why do some students go to resour— um, go to resources and why do some not.
2	Elizabeth:	Ok. So, with that in mind, we’re gonna think of, like, what have we learned about why they go? So we’re gonna have, kinda brainstorm a little bit together? And then we’re gonna think about, like, what we would want to try next. So let’s have some, like, individual think-time right now? What do we know?
3	Lara:	Can you say that again? What do we know?
4	Elizabeth:	What do we know so far about why students draw on different resources.
5	Lara:	Hmm.
6	Lucy:	Is this based on what we saw? Or just—?
7	Elizabeth:	Could be what we saw. I can also just be from experience.
8	Anna:	And I think some, some of the things we’ve already said up here I think relate to that.
9	Elizabeth:	Yeah.
10	Lara:	Is that the same as asking what supports students to draw on resources? Or is that just a related question.
11	Lucy:	Mmm?
12	Elizabeth:	Hmm.
13	Lara:	No.
14	Lucy:	Can you say your question one more time? Why do. Is it that one specifically [pointing to the board], why were some groups?
15	Soledad:	Why were. Yeah, why were some groups quick to draw on resources and others not.
16	Elizabeth:	We wanna start also thinking about the supports because that’s where we’re gonna go next.
17	Lara:	Or maybe something like, for me it’s helpful to be like, when students are
18	Elizabeth:	Yeah.
19	Lara:	drawing on resources? How and why. Like what are, what’s getting them to do that.
20	Elizabeth:	Mmhmm.

21	Lara:	Rather— you know, I feel like sometimes if I just say the why are some and why are some not, it'll lead me to places that aren't that useful.
22	Catherine:	Mmm.
23	Lara:	Like, because some students are having a bad day, or whatever, which we don't, you know. Whereas, like, when it's happens well, like why is it happening well, or what is supporting--
24	Elizabeth:	Yeah.
25	Lara:	it to happen well. You know what I mean? I feel like that might be, like, generative to a start on that question.
26	Elizabeth:	That's helpful. Let's do some little think-time.
27		[Participants think quietly]
28	Lara:	Are we assuming that drawing on the teacher is not what we're counting as drawing on resources here?
29	Elizabeth:	I was sort of assuming that, but let us know, Soledad.
30	Lara:	Yeah. Non-teacher-based resources.
31	Soledad:	Right.
32	Jamie:	I'm not. Well, I'm not sure I agree with that. Only because the team that Lucy and I observed, I don't even think they drew on the teacher as a resource. So I--
33	Lucy:	They wanted to.
34	Jamie:	They wanted to but they didn't.
35	Lucy:	Two minutes in they were like, "Ms. Castillo!" And then [closes her mouth and stares, then laughs], he was like, "See, she never listens to me anyway, I give up." [Laughter] And so they didn't call you again for a while! And it just took, they ac, they actually just sat there waiting for someone to walk over.
36	Catherine:	Hmm.
37	Lucy:	They didn't, they weren't advocating for themselves as people who deserved a teacher to be there.
38	Jamie:	Yeah and--
39	Lucy:	To help them!
40	Jamie:	Yeah.
41	Lucy:	And maybe, I don't know.
42	Catherine:	Hmm.
43	Lucy:	I don't know. I feel like it has— I don't know.
44	Elizabeth:	I guess I have a question about when it's actually helps productivity for a--
45	Lucy:	Right.
46	Elizabeth:	teacher to be there.
47	Lucy:	And it would have helped them, if. They didn't get a teacher over there for, until twenty-five minutes in.

48 Jamie: Yeah.
49 Lucy: Um. Which is why they were so lost. I don't, I don't really know. Is that part— was that a part of the task? To be able to call the teacher over when they needed them? I don't know. I actually don't know.
50 Elizabeth: Mmhmm.
51 Lucy: I don't know if that's something you were instructionally, or like, "We'll come to you"? "We're gonna check in with you in at least fifteen minutes"? Or was it like. I don't know. I think it depends on the task.
52 Elizabeth: Mmhmm.
53 Soledad: It was open. I mean, I don't think that we had planned, like, we're going to circulate to every table in some number of minutes.
54 Lucy: Mmhmm.
55 Elizabeth: Well let's, let's start a brainstorm to get some stuff up there. I'm curious to see what you guys were thinking about. Of like what supports help students draw on resources? And what do they--
56 Lucy: Are we just making guesses for why at this point?
57 Lara: I believe so.
58 Lucy: Ok.
59 Elizabeth: Mmhmm.
60 Cindy: Um. They, they may, they wrote their notes. They made the posters. Like, the people who were drawing on the resources are the people who made the resources.
61 Lucy: Hmm!
62 Soledad: Yeah. So like if they had the worksheets and they're able to pull them out. But if they'd, they'd not done their work, if they'd thrown it away--
63 Cindy: Mmhmm.
64 Soledad: then they don't have those to draw on anymore? Like, Briana took pictures of warmups.
65 Cindy: Uh huh.
66 Soledad: She takes pictures of them after they're done.
67 Cindy: Oh, yeah.
68 Soledad: So she went to her phone and she, like, pulled out a picture of her warmup.
69 Jamie: Yeah.
70 Lara: Hmm. [Nods]
71 Jamie: She did do that.
72 Lara: So cool. So resourceful.
73 Soledad: [Nods]
74 Lara: It's like, how'd they think of doing that?
75 Catherine: [Looks at Lara and nods, smiling]

76	Soledad:	They take pictures of like the slides a lot. I mean, they could go to Google Classroom, too?
77	Lara:	Right.
78	Soledad:	To get the slides? But then if they take a picture, then the work's done. Um.
79	Leo:	And I think related to that is organization?
80	Cindy:	[Nods]
81	Lucy:	Mmhmm.
82	Soledad:	Yeah.
83	Leo:	Like I think, I think some— I didn't, I didn't see it but I think some kids probably had their stuff, it's just [throws up his hands] in a jumble, so they were like, "I'm not gonna get that."
84	Soledad:	Hmm.
85	Lara:	But I, I, I was interpreting something related, that maybe you [points to Cindy] meant, that wasn't just about if they had their stuff. Or like, and I, I hadn't noticed this before, so thank you. I think the group, they were, they, that I was watching was looking at the posters on walls, on the walls. And I think the poster that they were most looking at was the one they had made.
86	Jamie:	That they'd made.
87	Cindy:	Really? [Laughs]
88	Catherine:	Whoa!
89	Lara:	Is that what you were saying?
90	Cindy:	Well, I, I, kind of. But. Yeah! That— she [inaudible]
91	Lara:	Because if they have work on the wall, they know that it's there, they know it's—
92	Cindy:	And what she said—
93	Lara:	there, they know it contains something useful.
94	Cindy:	She said, "I think I did that one?" Like she doesn't, she didn't remember if, if she did that one or not? [Laughs]
95	Lara:	That's super interesting to me, like--
96	Elizabeth:	Mmhmm.
97	Lara:	it makes a lot of sense, I hadn't thought of it that way. Right? That they would know it's there, know it's useful, and be ready to orient to it if they'd made it.
98	Elizabeth:	Yeah. I was--
99	Soledad:	Yeah. I think that when they know they made it, or they're [nods], they look at it more.
100	Lara:	Mmhmm.
101	Elizabeth:	And I was also wondering if it was, like, students that have more confidence in math are more willing to look back at resources? Because they have more confidence that they can interpret those resources?

102	Lucy:	Hmm. [Touches her nose and points at Elizabeth]
103	Lara:	Yeah, I was thinking something similar. Like, when students were turning to resources like that, it seemed to me there was some sense they had that they could do something with it.
104	Elizabeth:	Mmhmm.
105	Lara:	Or like, that it would be, that they have the capacity to, you know. Given a piece of information, they know how to use it.
106	Lucy:	Mmhmm. [Nods]
107	Anna:	I think something that I noticed that was related to that was that in, in some groups, they were drawing on resources but maybe there were, other people in the group didn't have the confidence that they would find something useful— that the other student would find something useful there? So they weren't necessarily asking, like it, you know, asking, "What did you find?" "Why— how do you know this?" "What do you," like, inquiring into that resource. I don't know if it's just, like, your own personal confidence.
108	Elizabeth:	Right.
109	Lara:	I was also wondering a little bit about what we mean by resources? Because I saw some really rich drawing on what I call resources, that had nothing to do with notes?
110	Elizabeth:	Mmhmm.
111	Lara:	Or, like, information? But were, like. So I saw some. "When, when I'm stuck I can ask you to explain it again." That's a resource.
112	Cindy:	Mmhmm.
113	Lara:	Right? Um. A res— I know that your explanation, my teammate, your explanation is a resource to me. So I know that I can ask for it. When I— even if you maybe just said it. But if I'm, like, not there yet, I can ask. That was certainly being drawn on. Um. There was the question, "How do you know that?" Was a resource for them, they were drawing on it, they were using it, and it was getting them further. When they said, "How do we know that?" the question, the, the conversations that came after that question often, in many cases got them to a new place. That they hadn't been. Even if the person who was answering the question thought they were just restating something they had already said.
114	Elizabeth:	Mmhmm. And I just, I wanna time check us for a second? So. It's 3:05. Originally— Thank you for joining us today, Lucy.
115	Lucy:	Um. Can I just add. Not knowing which resources align to the task. So--
116	Elizabeth:	Oh.
117	Catherine:	Ah ha.

118	Lucy:	[points to Leo] saying it's an organization problem? They don't know which ones they even need to pull out to go with what they're working on. Um. [She leaves.]

Extended transcript of episode from Session 4

1	Jamie:	I know Cindy put Equitable Access. It feels like almost everyone is responding in a way that's showing that more students are open to participating verbally?
2	Elizabeth:	Mmm.
3	Jamie:	And sharing ideas?
4	Elizabeth:	Yeah. Cindy, I was wondering if, like, if that translated then to on paper, did you see, you, like, they were talking about it? Did you also see that translate to their, like, work product?
5	Cindy:	Mmm? A little bit. Yeah. [Nods]
6	Elizabeth:	Other noticings?
7	Soledad:	Um. So my noticing was that for, like, the students that were kinda borderline, like, sometimes would, like, show with body-language or, like, kinda, like, s-, like, kinda semi-engaged, that sometimes, like, it got those students? But the students who were the heads-down, the, um, you know, a bit more resolute in their th— it seemed like they're more resolute in the thought that, "I am not going to engage,"
8	Elizabeth:	Mmhmm.
9	Soledad:	"like, no matter what." That I, does, it still did not capture those students. That the ones that were like on the border [nods], it engaged those students.
10	Elizabeth:	[Nods] Okay. That seems like a common, like a, a tension we keep having in these meetings? That this is addressing one type of disengagement
11	Anna:	Mmhmm.
12	Elizabeth:	but not total disengagement. From.
13	Jamie:	Hmm.
14	Elizabeth:	And I'm thinking back to our first meeting where Cindy was sharing one of her struggles with the students who just don't [moves hands up and down and grimaces] engage at all. Okay.
15		[5 second pause]
16	Leo:	Um, so I think it does increase student engagement. But. And that sort of decreases our opportunity to see persistence? Because they don't run into that wall?
17	Anna:	Hmm.
18	Leo:	You know what I mean? So in my mind persistence is, you get stuck. And then you try something else. Or you do, there's some strategy that you use.

		But we're providing a way to, to avoid that resistance. To avoid the wall. So you don't necessarily see the persistence.
19	Elizabeth:	Hmm.
20	Leo:	You know what I mean? So. Once the strategy, once they, like, offer Elianna [name of student] as help, then they get stuck, then we can see the persistence. But. I mean, I think this is great. They're participating more. Um. But for me I just don't see the persistence.
21	Elizabeth:	What do you guys think about that?
22	Jamie:	When you— when you're saying persistence in this context, Leo, you mean, like, your own self-management?
23	Leo:	When I think persistence, I think, "I'm stuck. What can I do?"
24	Jamie:	Mmhmm.
25	Leo:	Like, there's an obstacle in front of me. How do get around this, how do I work around this. What we're providing is a way around it without having them stop. You know what I mean?
26	Jamie:	Mmhmm.
27	Leo:	So they're not coming up with the strategy, they're just using what they know. Which is great. I don't know if that. [Sighs] So not bad. It's just, it doesn't address the persistence.
28	Soledad:	So then, it's like the, defining persistence, like it's on— it's persistence if they first come to a complete stop before then they come up with their own solution? [Squints and scrunches her nose at Leo]
29	Leo:	That's how I'm seeing that word.
30	Soledad:	Okay, got it. So what does a complete stop look like, then? And how bout if they don't ever come to a complete stop, could they still have persistence?
31	Elizabeth:	Hmm, that's a good question.
32	Leo:	[Looks at the ceiling and rubs his chin and nose, tilts his head from side to side]
33		[5 second pause]
34	Elizabeth:	Can I rephrase your question to make sure I understand you.
35	Soledad:	Yeah! [Giggles]
36	Elizabeth:	Um. So you're saying, like, if we don't actually see students stop and experience a struggle by, like, getting stuck and stopping? Are they still persisting in getting through the problem.
37	Soledad:	Yeah.
38	Jamie:	Hmm.
39	Soledad:	[Nods]
40	Leo:	I mean, I don't think it has to be a debilitating stop. But [laughter] just a stop. [Holds up hand like a stop sign]
41	Jamie:	[Mimes screeching to a stop]

42	Leo:	Where like, “I don’t know what to do next?” [More laughter] I think about it. “Oh, I can do this.”
43	Jamie:	Mmm.
44	Leo:	S- so, just at a point where the tools are not, like, immediately come to mind? And they need to struggle a little bit for that. Because I think about like, you know, you can coast through high school. Are you persisting? If you have straight As, are you persisting? Maybe, like, not really? But when you get to college, you’re gonna get stuck. Right? And, you, you can freak out and fail. So that’s not persistence. Um.
45	Jamie:	What you’re getting at, then, is the subjectivity of it, right? Cuz, like, for some students, persistence means, “I come to a grinding halt. I reassess and I continue.” For others it means, like, tinkering and continuing along?
46	Leo:	Okay.
47	Jamie:	And for, and for others, they haven’t, because of where the cognitive demand is currently, they haven’t even come to that level of needing to persist, right? Because it’s not challenging, it’s not sufficiently challenging that they’re like, actually persisting, they’re just going. They’re able to continue.
48	Soledad:	Or they’re able to, like, continuously make mistakes without stopping.
49	Jamie:	Mmhmm. That’s what I meant by the, like, keep tinkering, like
50	Soledad:	Yep.
51	Jamie:	students who are just chugging along.
52	Soledad:	Yeah.
53	Jamie:	And they’re like
54	Soledad:	But
55	Jamie:	screwing everything up but then, like, getting back up.
56	Soledad:	But I feel like that that’s persisting because if, like, if you have the capacity to continuously make mistakes and keep going?
57	Jamie:	Mmhmm!
58	Soledad:	And like, “I made a mistake! Okay! Learn from that, go on.”
59	Elizabeth:	Mmhmm.
60	Soledad:	“Made another mistake! Made twenty mistakes!” Twenty’s kind of a lot, right? [Laughter] Maybe a big number. But, you know. Right, even after so many mistakes. That. Right, can you call that persistence, too? Because they’re not coming at a stop, but. They have to, like. They, um, have to stop and that, that process? That thought process? And going, “Oh! That didn’t work.” And then, try and start another train of thought, another train of logic.
61	Jamie:	Yeah, I guess what, I, I guess what I was getting at was for us, on the outside looking in, it’s always gonna be subjective.

62 Soledad: Right.

63 Jamie: Or inter-subjective--

64 Soledad: [Talking simultaneously] Like some--

65 Jamie: --the degree to which were are able to de-

66 Soledad: like, spectrum [holds hards] of, like, what it is, like--

67 Jamie: to determine, is this student actually persisting. Because for some kids, persistence might mean. "Well, I, I started?" [Laughs] "But I'm stuck now! And I'm thinking and I'm thinking and I'm thinking and I'm thinking. I'm not writing anything but I'm [moves fingers in circles around his ears like gears] intently focused." And um. And they're assessing and reevaluating. And for others it's like you said, it's just, like, "Oops, I screwed up. Okay let me try that— oh, okay, kay, screwed up again, let me try— oh, let me try something else," like.

68 Elizabeth: I wonder if this idea of the different reason, like the different ways that students show persis— persistence? If we like looked at the TRU Dimensions, to figure out, like, if those could explain different reasons? Like I could see, Jamie, in your comment around, like some people don't experience persistence at all because they're just, going through? Like, that seems like a Cognitive Demand issue, where the Cognitive Demand might be too low.

69 Jamie: Mmhmm.

70 Anna: Mmhmm.

71 Elizabeth: For that student. But then, could we think of other ways, like. Like an AOI issue that would cause, that would affect student persistence? Or an Access issue? Or is that not a helpful [laughs] place for us to go right now.

72 Soledad: Wait, can you ask the question one more time?

73 Jamie: [Laughs]

74 Elizabeth: Let me see if I understand my own question. [Laughs] Um. So I'm thinking about how we've been talking about this idea of persistence, and how there's different types of persistence. And so I'm thinking about trying to define the different types of persistence? And if the TRU Dimensions can help us define those different types.

75 Soledad: So would we say that there's different types or, like, or a spectrum, I don't know.

76 Jamie: Well so you're, you're kinda referring to, like, the difference between a student that you are perceiving as not perseverant because they have, they have no feeling of agency around mathematics, right?

77 Elizabeth: Mmhmm.

78 Jamie: Like, they have the sense of, [pounds table with fist] "I can't do math. I'm not a math person." Right? They have that very fixed mindset.

79 Elizabeth: Mmhmm.

80 Jamie: Versus a student who is looking at it and is like, “This is really challenging. This is way in over my head. I don’t understand this content.”

81 Anna: Mmm.

82 Jamie: Right? Cuz like, I have those kids that, like. We’re finishing up stats in Algebra 2. And I think that they actually think of themselves as strong mathematical thinkers, but because statistics has been a hole in their overall mathematical experience, they’re looking at this like, “Why are we doing this, this is so hard. I’ve not had to think about data and numeracy in this way.” That’s more like a Cognitive Demand thing because I’m asking them to do something challenging, that they haven’t done before. But they’ve probably experienced success in prior math classes, so they normally would feel like, you know, “When are we going to get to the real math.” That’s the most common thing that we hear in stats. “When do we get to the real math?” You know, the one that has naked numbers and.

83 Cindy: [Whispers] Naked! [Smiles and laughs]

84 Anna: [Laughs]

85 Matt: [Laughs]

86 Jamie: That’s what they’re asking, right? Like.

87 Leo: And then the other side to that is, “When are we ever going to use this in life?” So, the not relevance that, that leads to non-persistence.

88 Jamie: That’s, and that’s AOI.

89 Matt: Do they ever get satisfying answers? Do you ever notice that they get satisfying answers

90 Jamie: No.

91 Matt: to those questions? Why are we doing this, and. It, at least in a way that might assist them in the persistence question?

92 Leo: Not in a genuine way. Like, there’s not, I don’t think there is an answer that’s intellectually satisfying? There’s insulting answers. Like,

93 Anna: Mmm.

94 Leo: you know, “If you’re not gonna work at McDonalds, yes, you’re gonna need this.” That kind of thing deters them?

95 Jamie: [Laughs]

96 Matt: Mmhmm.

97 Leo: But. It’s, I think it’s a strategy to get out of it.

98 Matt: Mmhmm.

99 Soledad: I think it’s a strategy, too, but today I did something and I accidentally, like, I thought I was— I was trying to bring in all the poster paper from home? And I accidentally brought one where, already had work on it? And it was, like, uh, sketches my sister and I had done? It was, like, really large, like on

a [holds hands several feet apart] maybe 5 yards? Of paper? And I unrolled it, and my kids are going through, um, rigid transformations? And I brought it out for them. They were saying something I said, and I showed the artwork. And, like, "Oh my god! You're right! That's…" Like, when we did constructions, like, that piece is constructions, this piece is this. And it's really big and it's tribal art. So they were like. And especially like the Islander kids really, um, connected to it? I try to post, like, uh. I dunno, I always, I dunno, I like art, so I always post, um, math and art. And I think it's easy in Geometry. I post a lot of, um, geometry— I mean, lots of art that has math. And I, like, sent one to the teachers about a piece that was found in Downtown Oakland just in the last few weeks of Trump? It has a whole bunch of transformations in it. Um. So [sighs] I try art because I think that it's something that kids can connect to more easily? Um. But today was the first time there was like, "Oh my god! Yeah!" Like, I thought was genuine. But most of the time I think it's not, like. I'm like trying and trying, and nothing's connecting. But today I felt like, oh my god, I think that… they liked that. [Laughs] And it was just like whoa! The first time. The first time, I think.

100 Leo: So for persistence for Access, I would say, like, sometimes kids don't really feel like they have a toolbox? Um.

101 Jamie: Hmm.

102 Leo: So. If we were doing an activity, and they got stuck and they go, and they said to themselves, "Oh, wait. What do I know about this problem?" I would count that as persistence. Because it's, it's, uh, student-initiated. They recognize, they just got stuck. They go, "Okay, this is a strategy I have, so that's what I'm gonna do." But I'm just saying that when we do it, when we question like that, from, when we initiate it, it's not necessarily persistence, you know.

103 Elizabeth: Um, you're providing some sort of scaffold, then.

104 Leo: Yeah. Like, "Go this way!" [Gestures with his arm, pretending to guide] We're not. [Gestures again] "That way, try it. It might get you something." And then [nods].

105 Jamie: Yeah, and I said under AOI, that fixed-mindset. The, "I'm not good at math, I can't do math, I'm not a math person." It's just that blank gaze that some of them have, no matter what you put in front of them, it's like, "Nope! Freakin' out! Freakin' out!"

106 Elizabeth: I also wonder if that affects the students that always feel like they're good at math? And when they get presented with a challenge, if that persistence looks different.

107 Jamie: Right.

108	Elizabeth:	To them. Um.
109	Matt:	What you mentioned a second ago about unfamiliar math?
110	Elizabeth:	Mmhmm.
111	Matt:	I think that's really important, too.
112	Elizabeth:	"Why are we doing this?"
113	Matt:	Mmhmm.
114	Jamie:	I love doing stats when they're like, "When are we doing real math?" I'm like. "This is as real as it gets." [Laughter] This is, like.
115	Soledad:	[inaudible]
116	Jamie:	If you wanna talk about real,
117	Elizabeth:	Yeah.
118	Jamie:	statistics is where math gets very, very real.
119	Elizabeth:	Well, like Leo's comment earlier, like, "When am I going to see this in real life?" Like. [Laughter]
120	Leo:	It's currently very profitable. Statistics.
121	Jamie:	Yep. Tell them that, too. That's where the money is.
122	Elizabeth:	Well, it sounds we still. Talk about persistence and, and perseverance, which is along our draft research theme. Um. I'm wondering now if it makes sense for us to do. So, when I brought this to the project team, there's an idea to this, like, XYZ, um, inquiry reflection tool? Has anyone done that before?
123		[Heads shake no]
124	Elizabeth:	Okay. Apparently, like, the district, it was one of the district's things. A few years ago. And then it cycled through something else. Um. But. X is like the hypothesis. Um. Y is like what happened when you tried it? And Z is like your next steps, like, based on these two things what are we gonna do next.
125	Soledad:	Or we could think of it as a function. X the input, Y the output, and Z. I mean, we're not, I mean, if you think in three dimensions, the Z doesn't make sense. [Laughter] But. It's like a function! I use them a lot, sorry.
126	Jamie:	It's okay, I thought about it in three dimensions.
127	Elizabeth:	So, you want an f of x, is what you're saying?
128	Cindy:	Yeah!
129	Soledad:	Yes! And then a later, then, somehow, it can, right?
130	Leo:	Yeah that makes sense.
131	Elizabeth:	To inform… Z?
132	Cindy:	[to Jamie] Or it's a compositing function.
133	Anna:	[Laughs]
134	Soledad:	Or maybe it could be, like. It could be a translation or, like, a sheer.
135	Jamie:	I think we should maybe [inaudible]
136	Anna:	Yeah, I was thinking exactly that, yeah.

137	Soledad:	[inaudible] It could be a sheer. [Laughter] We can somehow create a sheer and then we're gonna get a Z.
138	Elizabeth:	A function!
139	Anna:	It could be a rotation!
140	Soledad:	It could be!
141	Anna:	It could be anything!
142	Soledad:	Let hope it's not completely a rotation! Not a 180 rotation, let's hope not. Though it might have to be.
143	Elizabeth:	[Laughs]
144	Anna:	That's okay, you just feed it right back through.
145	Elizabeth:	So I think we defined our X beforehand.
146	Soledad:	Mmhmm.
147	Elizabeth:	If I try this in a public setting, students will be able to work through productive struggle by using their own and each other's ideas? So I wonder what we would say about why, given our conversation. When I tried this in the public setting, students… What actually happened?
148	Jamie:	I feel like one thing that was common was more students joined the conversation.
149	Elizabeth:	Okay. Maybe we'll make, like, a list.
150	Anna:	That feels like, I'm thinking back to what Leo said about the Access and the toolbox? That feels like they got more access? But I wonder if they got more access to the perseverance. Right?
151	Elizabeth:	Mmm.
152	Anna:	Like, cuz you were, you were saying that you felt like that sort of took away the opportunity for them to look in the toolbox. But.
153	Jamie:	Hmm.
154	Anna:	But it sounds like access to me. I just wonder about to what?
155	Jamie:	Mmhmm.
156	Elizabeth:	Is that a fair statement that I just typed? So maybe more access to content? But less access to perseverance?
157	Soledad:	Yeah. I think that, yes. I think that's the most clear way it's been stated so far. That more access to content, less access to perseverance.
158	Elizabeth:	And can I even add in, individual perseverance?
159	Anna:	Hmm.
160	Jamie:	[Nods]
161	Leo:	[Nods]
162	Elizabeth:	I see some nods. Cindy, you've been quiet.
163	Cindy:	[Laughs, gives thumbs up]
164	Soledad:	But she was a thumbs up!

165	Elizabeth:	But she [laughs]. Um, any other things we noticed? Or is that kind of the, the gist of it.
166	Leo:	I think sort of. [Waves hand] I don't have any evidence for this, but I think students opened up to the idea that there are multiple ways to proceed? That there are--
167	Elizabeth:	Mmm!
168	Anna:	Hmm.
169	Leo:	--multiple acceptable answers? Um. That all this will help us get where we need to go. Acceptable answers.
170	Soledad:	I was thinking about Number Talks lately. And I realize, like, all the questions I wrote were so like the same questions you ask in a Number Talk. It's like [makes a face looking down her nose], "Oh! These are the questions I ask, I set up for myself in a Number Talk! Oh, these are the questions!" So, like. Attacking it from different ways is like [shrugs] Number Talks, right? Accessing it? Um. Different like, processes to get the same answer?
171	Elizabeth:	Mmhmm.

Appendix B Vignettes of Classroom Videos

Leo's video, watched and discussed in Session 3

Two-minute video from Leo's Math Analysis class. Leo interacts with a student about finding the vertex of a parabola from a quadratic function written in standard form, using the vertex formula.
***Problem**:* Specifics unknown. In general, the task is to find the vertex of a parabola from a quadratic function written in standard form, using the vertex formula, and then plug in several *x*-values to make a table and eventually graph. ***Setting**:* A student, Eden (participants use her real name), sitting at a desk near the back of the classroom. Desks are arranged in rows. Leo approaches her when she calls him over. ***Video episode**:* Eden called to Leo because she needed help. She started to ask a question about the vertex formula, but stopped, saying that she can't explain her question. Leo described her actions back to her and asked what her question was. Eden pointed to her paper and asked, "How we get a negative three?" *(From context clues, -3 is probably the correct x-value of the vertex.)* Leo leaned over and pointed to her paper. "Do you see where we got that negative six?" he asked. *(From context clues, -6 is probably what Joline had for the x-value of the vertex. However, -6 is probably the "-b" from the vertex formula. She did not divide by 2.)* After thinking for a moment, Eden realized that the negative six was incorrect. Leo walked her through how to use the vertex formula correctly, getting -3. Eden seemed to understand. However, she still had a question that she couldn't explain. By asking questions, Leo led Eden through the process of finding the *y*-value of the vertex once she had the *x*-value. Then he explained that when she graphed the points she had calculated, she would find that the vertex point was "right in the middle." This was why when she plugged in more values to find more points, she would pick some on either side of the vertex. As he explained, Eden said, "Ohhhh." After Leo's explanation, Eden asked, "We only use it once? That equation?" (meaning the vertex formula). Leo answered yes. As Leo walked away, Eden sighed and said, "You could have said that before."

Jamie's video, watched and discussed in Session 5

Seven-minute clip from Jamie's Algebra 2 class, of four students working in a group on an activity about simple interest.
Problem**:* Mary invested $200 at a bank, with simple, non-compounding interest of 10% per year. The worksheet asks for how much interest she will earn per year and for students to fill in a table of how much money she will have each year for seven years, which the students have already completed. Students are working on a problem that shows three increasing curves with the same *y*-intercept-- a concave curve (A), a straight diagonal line (B), and a convex curve (C)—and asks which best represents the growth of Mary's money. There is space for students to explain their answer. Students also work on a problem that asks them for an equation that represents the growth of Mary's money. ***Setting**:* Four students sitting around a table: Zakira, Khalid, Omar, and Mohammed. ***Video vignette: Mohammed and Omar spoke to each other in Arabic for several minutes. (I don't know what they said, but when the camera pans to them it looks like Omar was translating the problem into Arabic for Mohammed.) Khalid told Zakira how to fill in the table, saying that it starts with $200 and

goes up by 20. After they filled it in, Zakira read the graph problem aloud. She said to Khalid, "I think it's B, do you think it's B?" referring to the straight diagonal line. When he didn't answer she pointed to the *y*-intercept of the graph in option B, showing that it matched the situation by starting at $200. But then she pointed to C, the convex curve, and said, "I honestly kind of think that it's this, though." Then she tried to get help from the person recording, Elizabeth, who advised her to ask her group.

Omar announced that it could not be C. Gesturing with his finger upwards in a diagonal line, Omar said, "It's 20 dollars, 20 dollars, 20 dollars." Khalid agreed with him. Zakira announced, "I'm hella smart," and said that it's going up by a certain number. Khalid agreed. Omar explained that it could be a concave curve, as in answer option A, if it were "times 20." But 20 added repeatedly makes a line.

Mohammed appeared to be working on a different problem. Omar named what he thought the equation would be, $y = 200(20^x)$. The other students wrote his answer on their papers.

Jamie approached their table when called over by Omar. Jamie leaned down beside Omar. Omar told Jamie that they wrote a linear equation. Jamie asked, "How do you know?" Omar said that in the situation, Mary's money was adding by 20, so the equation should be linear. Jamie affirmed this answer but questioned the equation they wrote. Omar said that he did not understand the equation he himself had built. Zakira challenged him, "So you had me writing in pen and you don't understand the equation?"

Khalid handed a sheet of notes to Omar, which Omar read. He then said that their equation was exponential but should have been linear. Jamie repeated his explanation and asked how they would write a linear equation. Omar said, $200 + 20x$. Zakira asked if that was right, to which Jamie responded that Omar should explain it and walked away. Omar explained his answer. Zakira proposed that they move on to the back of the sheet, but Omar demanded to know if she understood.

The video jumped to the students doing their exit ticket. [Jamie did not provide the participants with the exit ticket, so we do not know what it was.] The three boys spoke in Arabic as they wrote. Zakira asked if she should divide. Khalid responded that she should multiply, and showed her the calculation on his calculator. Zakira wrote what he explained. Omar looked at the other boys' papers, speaking in Arabic.

Appendix C PD Tools

TRU Observation Guide Student-Look Fors

TRU Observation Guide: Student Look-fors				
The Mathematics	Cognitive Demand	Equitable Access to Content	Agency, Ownership, and Identity	Formative Assessment
How do mathematical ideas from this unit/course develop in this lesson/lesson sequence? How can we create more meaningful connections?	*What opportunities do students have to make their own sense of mathematical ideas? To work through authentic challenges? How can we create more opportunities?*	*Who does and does not participate in the mathematical work of the class, and how? How can we create more opportunities for each student to participate meaningfully?*	*What opportunities do students have to see themselves and each other as powerful mathematical thinkers? How can we create more of these opportunities?*	*What do we know about each student's current mathematical thinking? How can we build on it?*
Each Student… ➢ Engages with grade level mathematics in ways that highlight important concepts, procedures, problem solving strategies, and applications ➢ Has opportunities to develop productive mathematical habits of mind ➢ Has opportunities for mathematical reasoning, orally and in writing, using appropriate	Each student… ➢ Engages individually and collaboratively with challenging ideas ➢ Actively seeks to explore the limits of their current understandings ➢ Is comfortable sharing partial or incorrect work as part of a larger conversation ➢ Reasons and tests ideas in ways that connect to and build on what they know ➢ Explains what they have done	Each student… ➢ Contributes to collective sense making in any of a number of different ways (e.g., proposing ideas, asking questions, creating diagrams…) ➢ Actively listens to other students and builds on their ideas ➢ Supports other students' developing understandings ➢ Explains, interprets, applies and reflects on important	Each student… ➢ Takes ownership of the learning process in planning, monitoring, and reflecting on individual and/or collective work ➢ Asks questions and makes suggestions that support analyzing, evaluating, applying and synthesizing mathematical ideas ➢ Builds on the contributions of others and help others see or	Each student… ➢ Explains their thinking, even if somewhat preliminary ➢ Sees errors as opportunities for new learning ➢ Consistently reflects on their work and the work of peers ➢ Sees fellow students as resources for their own learning ➢ Provides specific and accurate feedback to fellow students

mathematical language ➢ Explains their reasoning processes as well as their answers.	so far before asking for help ➢ Continues to wrestle with an idea after the teacher leaves	mathematical ideas ➢ Participates meaningfully in the mathematical work of the class	make connections ➢ Holds classmates and themselves accountable for justifying their positions, through the use of evidence and/or elaborating on their reasoning	➢ Makes use of feedback in revising their work

TRU-LS PD Pedagogical Strategies List

1. **What's the big picture?** Identify an "essential question" or "big idea" that's important for the math your students are working on. Share it with students at the beginning of the lesson and find time, especially at the end of the lesson, for students to connect what they're doing to the big idea. Check in with students about it while they're working in groups, or have a few students share or lead a class discussion about it at the end of a lesson.

2. **Share what your partner said** Before a group discussion, have students turn and talk to a neighbor. When the discussion begins, have students who want to talk share their neighbor's idea, not their own.

3. **Ask for patterns and outliers** To give students something more to think about while they practice similar problems or do problem sets, ask them to describe any mathematical patterns they noticed while working on similar problems. Or, ask them to find problems that seemed like outliers—ones for which the method didn't work as expected, or with a solution that surprised or confused them. This can be given in addition to a regular homework assignment. Have a few students share their patterns and outliers in class, or have students discuss them in pairs.

4. **Deep dive homework** Instead of turning in an entire problem set from homework, students pick some small, reasonable number of problems that they will show work from in detail and then turn in. The goal is to have them make something they want to show the teacher, whether because they're impressed with what they did, want feedback on that specifically, enjoyed solving it and want to spend more time.

5. **Name It** When a student shares an idea, write their name on the board next to it when you record it. Refer to that idea as "so-and-so's idea" or "so-and-so's strategy". Keep track of whose ideas have been highlighted from class to class.

6. **Invent an argument** Come up with some fictional characters who disagree about a math idea you want to teach and ask students how they would resolve the argument. Make sure students also discuss why each side thinks what they do, even if the characters' arguments aren't correct.

7. **Fishbowl** During class discussions, split the group in half. Have one half sit together so they can see each other, and the other half sit in a circle around them. The inner group talks for a certain amount of time (3 minutes, 5 minutes) with minimal input from you. The outer group writes what they heard during the discussion. Collect their notes; switch groups on different days.

8. **Make categories** After a long problem set or homework assignment filled with similar problems, ask students to group the problems into categories. They can choose the categories they like, but encourage them to be mathematical. For example, after a homework assignment of solving systems of equations, students might choose to categorize the problems according to those that have solutions and those that do not; those that have solutions in different quadrants when graphed; those that were best solved with different methods; etc. Let students be creative! This gives students opportunities to see the bigger picture after doing detailed problems.

9. **Open it up** Reframe a problem or problem set so that there's more for students to discover. For example, when doing proofs with students, instead of asking them to "show" something, such as that the diagonals of a rectangle bisect each other (which assumes that it's already known to be true), pose the thing to be shown as something to discover. Ask, "What do you notice about the diagonals of different quadrilaterals?" and give students time to draw, find patterns, and make claims about what they see.

10. **Wait a turn** During whole class discussions, try letting two (three, four, five…) students speak in a row before you say something.

11. **Reflective journals** Can begin simply, by asking students to respond to a single question like: What did I learn today? Every day, teachers can choose 3 journal entries to read to everyone at the beginning of the next lesson, that revisit content, embody habits they are trying to build in the whole class, etc. Over a couple weeks, all students' journals are chosen. Journals can be a place where students write the problem, record their thinking, how it changed, and why, and reflect on what they did not know previously that they know now.

12. **Confidence thermometer** At the end of a lesson or unit, ask students to rate how comfortable they are with key idea(s), by coloring in an empty bar (or thermometer) next to each key idea. Students can who feel comfortable with a given idea can share tips for studying it or making sense of it.

13. **Time alone** Give students some time (maybe 5 minutes) to work alone on a problem before working in a group, so that they have time to think and prepare their ideas. You may want to structure this alone time by setting expectations as to what students will share with their groups after 5 minutes. For example, tell students, "Be prepared to share one idea that you have for getting started on this problem and one question that you have about the problem." or "Be prepared to share one thing that you know about this problem and one thing that you don't know about this problem."

14. **Student-led questioning routine** Establish a routine of student-led questions and challenges after viewing each student presentation or work. For example, students might routinely finish their presentation by asking the class "Do you have comments or questions?" and calling on classmates.

15. **What do you wish you knew?** At the end of a test or quiz, ask students to choose at least one problem they aren't sure they did correctly and write a quick reflection about what they think they're missing or what tool/method/idea would be useful to solve the problem correctly.

16. **Share wrong answers like they're right answers** Share an incorrect solution to a problem, saying "another student solved the problem this way. Identify similarities and differences between this solution and the solution you have. What makes sense to you? What questions do you have?" Encourage students to share their thinking in a whole class discussion or in small groups about their own solution to the problem as it relates to the incorrect solution the teacher provides. If possible, let the students come to the decision that the provided solution is incorrect and encourage them to justify their thinking.

17. **Three things** If students are stuck on a problem, ask them to state three things they know about the problem and three things they are wondering about. This can help students realize that they are making progress and have information that they can potentially draw on to make more progress.

18. **Choose your color** Each student uses a different color pen/pencil when working on a group problem. Students can use poster paper or larger than normal paper, so that everyone has physical access to the paper.

19. **Extended wait time** After posing a question to the whole class, count to five slowly in your head before calling on a student, hopefully resulting in more hands being raised.

20. **Gallery walk** Students post their individual or group work around the room and then walk around and look at their peers' work. Work may be in finished form or in progress. Gallery walks can be "silent" or interactive. Students can post comments/question using sticky notes or talk through questions and comments. With group work, one student can be chosen to stay behind to present their group's work to the other students.

21. **Always, Sometimes, Never** Pose questions that require explanation and are not straight yes/no or one answer solutions. For example, instead of asking, "Is it true or false that when you multiply two numbers, the answer will always be bigger?" ask, "When you

multiply two numbers, the answer will be bigger. Is this statement always, sometimes or never true. Explain your reasoning." This gives students opportunities to explore more possibilities, such as positive / negative numbers and fractions.

22. **Group questions** The teacher only answers group questions (questions that have been agreed upon by all group members). One student raises their hand when the group has a question. The teacher asks a different student in the group what the question is and only answers the question if all group members agree it's a question.

23. **Idea melting-pot** After a do-now, rather than having students raise their hands to share how they solved a problem, have them turn in their work to you without their names on it. Randomly pick one of the papers and choose a student to present the solution written there. The class can work together to understand what was done.

24. **Start with the end** Start a unit with a rich problem that draws out ideas that will be useful throughout the unit and helps students build them. In some textbooks, these problems typically occur at the end of the unit. In contrast, some research-based textbooks use rich problems to *begin* the unit, so that students can actually develop new procedures or ideas of the unit using their prior knowledge. Such experiences help students realize they can develop procedures they have not yet been taught.

25. **Checkpoints** When students are working on a big, challenging problem or set of problems in groups, plan checkpoints at different stages of the activity when you'll check in with the groups. Checkpoints should be far enough apart that students get a lot of quality time to struggle with the problem, but close enough together that you get a sense for how they're progressing and what they know. Tell students when the checkpoints will be at the beginning of class. Let students know that when they reach the checkpoint, they should call you over and expect that you could call on any member of the group to check in about their progress.

26. **I'm coming back…** When talking with students who may be stuck during a challenging task, instead of giving them all the information you know they need to get unstuck, leave them with an open-ended question that you think they can answer themselves and that will help them make progress once they've answered it. Tell them that you will come back later to see what they've done. Then come back! (This works well in combination with Checkpoints and Group Questions.)

27. **Change one thing** Give students the same basic arithmetic or algebraic expression or function (depending on your grade level). Then, ask each student to change just one thing about it. Have students share what happened when they changed their one thing. In small

groups and/or as a whole class, look for patterns among and connections between what happened when students made their individual changes.

28. **Let's just explore!** In your next lesson or unit, identify something simple that students could play around with. Maybe it's changing the lengths of sides or sizes of angles in a particular shape; maybe changing the slopes of lines and seeing how the graphs shift; anything simple works. Then, give students at least ten minutes during class or for homework to play and explore. Ask students to write or verbally share a reflection about what they noticed and/or wondered about after exploring.

29. **Students make the test** Near the end of a unit, have students spend time in class coming up with 3-5 questions that they think should go on the unit test. Encourage students to make a set of questions that they think tests all the most important ideas and techniques they learned during the unit. If students do this in small groups, the whole class can discuss the questions they made and why they thought the questions tested the important big ideas. You can use the questions they make on the test if you like, or just use the ideas students generated as another way of finding out about what they learned.

30. **Which one doesn't belong?** Project or somehow show students four of something (shapes, numbers, equations, graphs, etc.), each with some difference. Any three of things could make a set, which the fourth wouldn't belong to. Then ask students, Which one doesn't belong? Students can explain what three have in common that one doesn't share. It's important that there be more than one right answer.