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**Publication Date** 2011-10-24



Homophily and Social Distance in the Choice of Multiple Friends: An Analysis Based on Conditionally Symmetric Log-Bilinear Association Model Author(s): Kazuo Yamaguchi Source: Journal of the American Statistical Association, Vol. 85, No. 410 (Jun., 1990), pp. 356-366 Published by: [American Statistical Association](http://www.jstor.org/action/showPublisher?publisherCode=astata) Stable URL: [http://www.jstor.org/stable/2289771](http://www.jstor.org/stable/2289771?origin=JSTOR-pdf) Accessed: 16/05/2011 19:22

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# Homophily and Social Distance in the **Choice of Multiple Friends**

An Analysis Based on Conditionally Symmetric Log-Bilinear Association Models

## **KAZUO YAMAGUCHI\***

This article presents an analysis of friendship choice data by focusing on homophily (or inbreeding bias) and social distance revealed in the patterns of both association between subjects' and friends' statuses and association among friends' statuses. These two aspects of inbreeding bias and social distance are simultaneously taken into account in modeling the data of friendship choice from subjects with different numbers of friends. The statuses of friends are expressed in terms of their combinations rather than their full cross-classifications. Conditionally symmetric log-bilinear partial association models are usefully employed for the analysis. The structural characteristics of inbreeding bias and social distance are identified by comparing nested models and through the interpretation of parameters estimated from models that adequately fit the data.

## 1. INTRODUCTION

#### $1.1$ An Overview of Literature and a Definition of **Key Concepts**

This article is concerned with the analysis of friendship choice data from a sample of subjects having different numbers of friends. The analysis focuses on the modeling of two structural factors, *homophily* (or inbreeding bias) and social distance, which are revealed in the association between subjects' and friends' statuses and in the association among friends' statuses when two or more friends are chosen.

Researchers have long recognized the tendency for people to choose similar others as friends. This tendency occurs with respect to demographic statuses, such as education, occupation, race and ethnicity, religion, marital status, and age (e.g., Fischer 1982; Laumann 1973, 1976; McPherson and Smith-Lovin 1987; Tuma and Hallinan 1979; Verbrugge 1977), attitudes and beliefs (e.g., Ajzen and Fishbein 1980; Berscheid 1985; Cohen 1977, 1983; Hallinan 1974), and social behavior, such as sexual behavior and drug use (Billy, Rogers, and Udry 1984; Kandel 1978). The tendency to choose similar others as friends involves two structural elements, homophily and social distance (Laumann 1973; Marsden 1981). Homophily, or the inbreeding bias of self-selection (Laumann 1976), is the tendency for subjects to choose friends from among those who fall into the same category as themselves, that is, ingroup members, without discriminating among out*group members.* Social distance is the tendency to differentially associate with outgroup members such that subjects more often choose as friends outgroup members who are similar rather than dissimilar to themselves.

The distinction between ingroup members and outgroup members, however, depends on the particular set of categories employed by researchers to characterize statuses. Hence the distinction between homophily and social distance is operationally defined for a given distinction of ingroup and outgroup members.

Researchers usually analyze homophily and social distance in friendship choice by making an artificial one-toone correspondence between subjects and friends. One commonly used method, which selects one friend from each subject, leads to a serious underutilization of information. A second method relies on the unrealistic assumption that each subject–friend dyad is an independent observation. Furthermore, by analyzing friendship choices as one-to-one correspondences between subjects and friends, researchers fail to consider inbreeding bias and social distance among friends' statuses revealed by each subject when two or more friends are chosen.

This article is the first attempt in the literature to model simultaneously (a) the association between the statuses of subjects and friends and (b) the association among the statuses of friends for each subject.

In this article, I define inbreeding bias in the choice of multiple friends as the tendency for subjects to choose as friends individuals who fall into the same status category rather than individuals who fall into different categories, without discriminating among combinations of different categories. On the other hand, I refer to social distance in the choice of multiple friends as the tendency to discriminate among the combinations of friends' different status categories, whereby friends are more likely to be chosen among mutually similar categories rather than mutually dissimilar categories. These two tendencies are defined independent of the statuses of subjects.

### 1.2 Organization of Data and **Analytical Strategy**

The frequency data to be analyzed in this article are constructed by classifying subjects by a discrete ordinal variable, namely their status, and then cross-classifying by the number of friends. For each number of friends, the

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<sup>© 1990</sup> American Statistical Association Journal of the American Statistical Association June 1990, Vol. 85, No. 410, Applications and Case Studies

statuses of subjects are then cross-classified by the statuses of friends. Although each friend is also classified by a discrete ordinal variable that indicates his or her status, the set of friends for each subject is characterized only by the combination of friends' statuses. Since the number of combinations differs depending on the number of friends, the three-way table is an unconventional contingency table. Without loss of generality, the use of combinations of friends' statuses instead of the full cross-classification of friends' statuses considerably reduces the size of the table and the number of cells with small frequencies.

In this article, the following two tables are analyzed. Both tables follow the format described previously.

Table 1 presents a cross-classification of subjects' educational attainments by the combination of friends' discussion statuses. By distinguishing the number of friends, three subtables are generated: a  $5 \times 3$  subtable for subjects with only one friend, a  $5 \times 6$  subtable for subjects with two friends, and a 5  $\times$  10 subtable for subjects with three or more friends.

Table 2 presents a cross-classification of subjects' educational attainments by the combination of friends' educational attainments. Two subtables are distinguished according to the number of friends: a  $5 \times 5$  subtable for subjects with one friend and a  $5 \times 15$  subtable for subjects with two or more friends.

The discussion statuses used to classify friends have three categories: (1) the subject discusses social/political issues with the friend most of the time,  $(2)$  the subject occasionally discusses social/political issues with the friend, and (3) the subject almost never discusses social/political issues with the friend. The combination of friends' discussion statuses 112 in Table 1, for example, represents cases in which the subject discusses social/political issues with two of his or her friends most of the time and discusses those issues with the third friend only occasionally. Here, whereas subjects are classified according to their educational status, friends are classified by a distinct three-category variable for discussion statuses. In Table 2, both subjects and friends are classified by the same five-category variable for educational attainment.

Both data sets were obtained from the 1986 Southern California Social Survey. The survey collected various information for at most three friends per subject. For subjects with four or more friends, the survey randomly chose three friends. In Table 2, I randomly selected two friends for each subject when information about three friends was available.

In analyzing inbreeding bias and social distance in friendship choice, I employ log-bilinear association models originally introduced by Goodman (1979) for the analysis of two-way tables having ordered categories. Clogg (1982) generalized log-bilinear association models for the analysis of higher-order tables, and Goodman (1986) and Becker and Clogg (1989) generalized the models for multidimensional associations. The models they described, however, apply to fully cross-classified data and cannot be directly applied to the data of Tables 1 and 2. In this article, I introduce a group of log-bilinear association models that satisfy conditional symmetry and show that these models can be applied to the data of Tables 1 and 2.

Using conditionally symmetric log-bilinear association models, I test various hypotheses regarding inbreeding bias and social distance revealed in (a) the association between the statuses of subjects and friends and (b) the association among friends' statuses when two or more friends are chosen.

In Section 2 I describe general conditional symmetry

Education Combination of the discussion statuses of friends' 1. Subjects with one friend  $\overline{c}$ 3  $\mathbf{1}$  $0 - 11$ 4 16  $12$ 22  $21$  $10$  $12$  $\frac{27}{7}$  $13 - 15$ 33  $11$  $\overline{2}$ 16 11  $17 +$  $\overline{4}$  $\overline{4}$  $\mathbf{1}$ 2. Subjects with two friends  $11$ 22 33  $12$ 13 23 5 8 10  $0 - 11$ 16 10  $\mathbf{1}$  $12$  $\boldsymbol{6}$ 19 5 13 4 10  $13 - 15$  $10$ 18 5 10  $11$ 24 16 3 7 1 3  $\overline{\mathbf{c}}$  $\overline{\mathbf{c}}$  $\mathbf 0$ 3 0 9  $\overline{c}$  $17 +$  $\Omega$ 3. Subjects with three or more friends 222 333 112 122 113 133 223 233 123 111  $0 - 11$ 4 6 8  $\overline{7}$  $\overline{7}$  $\pmb{0}$  $\overline{A}$  $\overline{A}$ 4  $\overline{4}$ 3 9  $6$  $\overline{7}$ 23 15 5  $12$ 18  $\overline{7}$  $\mathbf{1}$  $\overline{\mathbf{4}}$ 19 23  $13 - 15$  $21$ 41 9 35 25 4 40 24  $\overline{\mathbf{c}}$  $17$ 1 16 3  $\overline{7}$ 16 13 7 1 5  $\mathbf{1}$  $10$  $12$  $\overline{2}$  $\overline{2}$  $\overline{c}$  $17 +$  $\overline{7}$ 

Table 1. Friends' Discussion Status on Social/Political Issues by Subjects' Educations

\*1, discuss social/political issues most of the time: 2, discuss social/political issues occasionally; 3, almost never discuss social/ political issues

Subject's education	Combination of the levels of friends' educations														
						1. Subjects with one friend									
		2	з	4	5										
$1:0 - 11$	10	12	2												
2:12	6	21	21	4											
$3:13 - 15$	11	13	26	8	3										
4:16	0	4	6	9											
$5:17+$	0	0	3	$\overline{c}$	4										
						2. Subjects with two or more friends									
	11	22	33	44	55	12	13	14	15	23	24	25	34	35	45
$1:0 - 11$	13	19	7	0		11	6			10	4	4	4		
2:12	5	27	11	3	3	10	4			27	9	7	8	9	5
$3:13 - 15$	5	33	49	10	10	12	8	6		51	17	9	30	20	11
4:16	$\overline{c}$	9	7	9	5		3		2	10	13	8	14	8	14
$5:17+$		$\overline{c}$	3	4	12	0			0	4	5		10		10

Table 2. Friends' Educations by Subject's Education

models, followed by a derivation of specific conditionally symmetric log-bilinear partial association models. The latter models will be applied to the data of Tables 1 and 2. In Section 3, I describe hypotheses regarding the form of friendship choice. In Section 4, I present an analysis and discussion of the data in Tables 1 and 2.

#### $2.$ **GENERAL CONDITIONAL SYMMETRY MODELS** AND CONDITIONALLY SYMMETRIC **LOG-BILINEAR PARTIAL ASSOCIATION MODELS**

#### $2.1$ **General Conditional Symmetry Models**

In this section I use the term objects instead of friends to refer to the object of choice, thereby making the description of models more general than the particular applications presented in this article. For data sets that characterize the correspondence of each subject with three objects. I derive models for the association between subjects' and objects' categories and between the three sets of objects' categories. Modifications of the models for the correspondences of each subject to two, four, or a larger number of objects are straightforward and are omitted.

When there is a correspondence between subjects and three objects and the order among objects is ignored, then we have an  $I \times J(J + 1)(J + 2)/6$  table to analyze, where I is the number of categories for the variable that classifies subjects and  $J$  is the number of categories for the variable that classifies objects. The first J columns of the table represent combinations of objects' categories when all categories are identical, such as  $(1, 1, 1), (2, 2, 2), (3, 3, 3),$ and so forth. The next  $J(J - 1)$  columns represent combinations of objects' categories when two categories are identical, such as  $(1, 1, 2), (1, 2, 2), (1, 1, 3)$ , and so forth. The last  $J(J - 1)(J - 2)/6$  columns represent combinations of objects' categories when all categories are different, such as  $(1, 2, 3)$ ,  $(1, 2, 4)$ ,  $(2, 3, 4)$ , and so forth.

Before introducing models for this table, let us first consider models for a fully cross-classified four-way table with dimensions  $I \times J \times J \times J$ . For the latter models, we assume *conditional symmetry* between variables  $B, C$ , and  $D$  for each given category of variable  $A$  such that

$$
F_{ijkm}^{ABCD} = F_{ijmk}^{ABCD} = F_{ikjm}^{ABCD} = F_{ikmj}^{ABCD} = F_{imjk}^{ABCD} = F_{imkj}^{ABCD},
$$
\n(1)

where  $F_{ijkm}^{ABCD}$  is the frequency of the  $(i, j, k, m)$  cell expected from the model. Let the saturated log-linear model of this four-way table be such that

$$
\log(F_{ijkm}^{ABCD}) = \lambda + \lambda_i^A + \lambda_j^B + \lambda_k^C + \lambda_m^D
$$
  
+  $\lambda_{ij}^{AB} + \lambda_{ik}^{AC} + \lambda_{im}^{AD} + \lambda_{jk}^{BC}$   
+  $\lambda_{jm}^{BD} + \lambda_{km}^{CD} + \lambda_{ijk}^{ABC} + \lambda_{ijm}^{ABD}$   
+  $\lambda_{ikm}^{AC} + \lambda_{ikm}^{BC} + \lambda_{ijkm}^{ABD}$ . (2)

At each level of cross-classification, a set of lambda parameters satisfies a standard set of linear constraints, whose descriptions are omitted here. The following additional constraints establish conditional symmetry between variables  $B$ ,  $C$ , and  $D$ , given a category of variable  $A$ : (a) ables  $B$ , C, and D, given a category of variable  $A$ : (a)<br>  $\lambda_j^B = \lambda_j^C = \lambda_j^D$  for each j; (b)  $\lambda_{ij}^{AB} = \lambda_{ij}^{AC} = \lambda_{ij}^{AD}$  for each<br> *i* and *j*; (c)  $\lambda_{jk}^{BC} = \lambda_{kj}^{BC} = \lambda_{jk}^{BD} = \lambda_{jk}^{BD} = \lambda_{jk}^{CD} = \lambda_{kj}^{CD}$  for<br>
each *j* and m. With these additional constraints on parameters, the log-linear model has a maximum of  $I/(J + 1)(J + 2)$ 6 parameters. Furthermore, these constraints allow us to use superscript  $B$  in place of superscripts  $C$  and  $D$  without loss of generality: Factors  $\lambda_j^c$  and  $\lambda_j^b$  can be expressed as  $\lambda_j^B$ ;  $\lambda_{ij}^{AC}$  and  $\lambda_{ij}^{AD}$  as  $\lambda_{ij}^{AB}$ ;  $\lambda_{jk}^{BC}$ ,  $\lambda_{jk}^{BD}$ , and  $\lambda_{jk}^{CD}$  as  $\lambda_{ijk}^{BB}$ ;  $\lambda_{ijk}^{ABC}$ ,  $\lambda_{ijk}^{ABC}$ ,  $\lambda_{ijk}^{BC}$ , and  $\lambda_{ijk}^{ACD}$  as  $\lambda_{ijk}^{AB}$ ;  $\lambda_{jkm}^{BC}$  as  $\lambda_{ijkm}^{ABB}$ ; and  $\lambda_{ijkm}^{ABB}$ . In addition, any permutation of subscripts for variable  $B$ (under the new expression of lambdas) does not change the parameters.

The expected frequencies of the original  $I \times J(J + 1)$  $(J + 2)/6$  table can now be defined using the expected frequencies of the conditionally symmetric four-way table described previously:

$$
F_{i[jj]}^{AB} = F_{ijj}^{ABCD}
$$
\n
$$
F_{i[jk]}^{AB} = F_{ijk}^{ABCD} + F_{ikj}^{ABCD} + F_{ikj}^{ABCD}
$$
\n
$$
= 3 F_{ijk}^{ABCD} \quad \text{for } j < k
$$
\n
$$
F_{i[jkk]}^{AB} = F_{ijk}^{ABCD} + F_{ikjk}^{ABCD} + F_{ikkj}^{ABCD}
$$
\n
$$
= 3 F_{ijk}^{ABCD} \quad \text{for } j < k
$$
\n
$$
F_{i[jkm]}^{AB} = F_{ijkm}^{ABCD} + F_{ijmk}^{ABCD} + F_{ikjm}^{ABCD}
$$
\n
$$
+ F_{ikmj}^{ABCD} + F_{imjk}^{ABCD} + F_{imkj}^{ABCD}
$$
\n
$$
= 6 F_{ijkm}^{ABCD} \quad \text{for } j < k < m,
$$
\n(3)

where  $F_{i[jjj]}^{AB}$ ,  $F_{i[jkk]}^{AB}$ ,  $F_{i[jkk]}^{AB}$ , and  $F_{i[jkm]}^{AB}$  are expected frequencies of the  $I \times J(J + 1)(J + 2)/6$  two-way table. It follows that the expected frequencies for the  $I \times J(J + 1)(J +$ 2)/6 table can be expressed log-linearly by the parameters of the conditionally symmetric log-linear model for the I  $\times$  J  $\times$  J  $\times$  J table. Each distinct conditional symmetry model for the four-way table corresponds to a unique model for the  $I \times J(J + 1)(J + 2)/6$  two-way table. The saturated model of the two-way table corresponds simply to the conditional symmetry model for the four-way table without any further constraints on parameters. Similarly, various unsaturated log-linear models for the  $I \times J(J + 1)$  $(J + 2)/6$  table correspond to various conditionally symmetric log-linear models for the  $I \times J \times J \times J$  table.

#### $2.2$ **Conditionally Symmetric Log-Bilinear Partial Association Models**

Since we have ordinal variables for the classification of both subjects and objects, we can apply log-bilinear partial association models introduced by Clogg (1982). Unlike the log-linear version, the log-bilinear association models do not require a correct prior ordering of categories. The partial association models applied in this article have additional constraints on parameters because they satisfy conditional symmetry. In applying the partial association models, we assume the absence of three-factor and higherorder interactions; that is,  $\lambda_{ijk}^{ABB} = \lambda_{ikm}^{BBB} = \lambda_{ikm}^{ABBB} = 0$ .

In addition to log-bilinear association, the models also include a set of parameters for inbreeding bias in the choice of multiple objects. We assume that the effect of the inbreeding bias is specific to each category of objects. The other aspect of inbreeding bias, namely homophily in subjects' choices of each object, is relevant only when there is a one-to-one correspondence between subjects' and objects' categories. We assume that this homophily effect is specific to each category of subjects.

It follows that the two-factor interaction parameters,  $\lambda_{ii}^{AB}$  and  $\lambda_{ik}^{BB}$ , are modeled as follows:

$$
\lambda_{ij}^{AB} = \phi^{AB} u_i v_j + [\delta_{ij} \alpha_i]
$$
  

$$
\lambda_{jk}^{BB} = \phi^{BB} w_j w_k + \delta_{jk} \beta_j,
$$
 (4)

where  $\Sigma_i u_i = \Sigma_i v_i = \Sigma_i w_i = 0$ ,  $\Sigma_i u_i^2 = \Sigma_i v_i^2 = \Sigma_i w_i^2 =$ 1, and  $\delta_{ii}$  is Kronecker's delta that takes 1 only if  $i = j$  and takes 0 otherwise. The factor in the brackets that pertains to the  $\alpha$  parameters is only included in models with a one-to-one correspondence between subjects' and objects' categories.

In Formula  $(4)$ , the  $u$  parameters characterize the pattern of subjects' association with objects; the  $\nu$  parameters characterize the pattern of objects' association with subjects; parameter  $\phi^{AB}$  characterizes the strength of subjectby-object association; the  $\alpha$  parameters characterize homophily (or inbreeding bias) in subjects' choices of each object; the *w* parameters characterize the pattern of symmetric object-by-object association; parameter  $\phi^{BB}$  characterizes the strength of object-by-object association; and the  $\beta$  parameters characterize inbreeding bias in the choice of multiple objects.

Social distance in friendship choice is operationally defined as the log-bilinear association effects. For cells to which parameters  $\alpha$  and  $\beta$  contribute, however, the association parameters also contribute. It follows that parameters  $\alpha$  and  $\beta$ , in fact, represent inbreeding bias over and beyond that characterized by the association parameters. Because of this characteristic, the interpretations of parameters  $\alpha$  and  $\beta$  require qualification.

By applying the specification of  $\lambda_{ij}^{AB}$  and  $\lambda_{jk}^{BB}$  in Equation (4) and allowing all parameters to depend on the number of objects *n*, where  $n = 1, 2, 3$ , we obtain

$$
\log\left(F_{ii}^{AB}\right)
$$

$$
= \lambda_1 + \lambda_{1i}^A + \lambda_{1j}^B + \phi_1^{AB} u_{1i} v_{1j} + [\delta_{ij} \alpha_{1i}] \text{ for } n = 1,
$$
  
\n
$$
\log[F_{i}[jk] \text{perm}(j, k)]
$$
  
\n
$$
= \lambda_2 + \lambda_{2i}^A + \lambda_{2j}^B + \lambda_{2k}^B + \phi_2^{AB} u_{2i} (v_{2j} + v_{2k})
$$
  
\n
$$
+ [(\delta_{ij} + \delta_{ik}) \alpha_{2i}] + \phi_2^{BB} w_{2i} w_{2k} + \delta_{ik} \beta_{2i}
$$

and

$$
\log[F_{i[jkm]}/perm(j, k, m)]
$$
\n
$$
= \lambda_3 + \lambda_{3i}^A + \lambda_{3j}^B + \lambda_{3k}^B + \lambda_{3m}^B
$$
\n
$$
+ \phi_3^A B u_{3i} (v_{3j} + v_{3k} + v_{3m})
$$
\n
$$
+ [(\delta_{ij} + \delta_{ik} + \delta_{im}) \alpha_{3i}]
$$
\n
$$
+ \phi_3^B W_{3j} w_{3k} + w_{3k} w_{3m} + w_{3j} w_{3m})
$$
\n
$$
+ (\delta_{jk} + \delta_{jm}) \beta_{3j} + \delta_{km} \beta_{3k}
$$
\nfor  $n = 3, j \le k \le m$ , (5)

for  $n = 2$ ,  $j \le k$ ,

where the  $\alpha$  parameters in the brackets are included only for relevant models and perm $(j, k)$  and perm $(j, k, m)$  are the number of distinct permutations for  $(j, k)$  and  $(j, k,$ m), respectively. For example, perm $(j, k, m) = 1$  if  $j =$  $k = m$ , perm $(j, k, m) = 3$  if  $j = k < m$  or  $j < k = m$ , and perm $(j, k, m) = 6$  if  $j < k < m$ .

#### 2.3 Method of Parameter Estimation

Newton's unidimensional method, which was employed by Goodman (1979) and Clogg (1982) for log-bilinear association models, is used here for parameter estimation. The iterative estimation of  $\phi^{AB}$  and  $\phi^{BB}$  is constrained to attain convergence; since these two parameters should always be positive, the new estimate at each iteration was allowed to take a value between one-half and two times the size of the previous estimate.

### 3. STRUCTURAL HYPOTHESES

Four sets of structural hypotheses regarding friendship choice are tested. The first set refers to the absence (versus presence) of the four factors that characterize inbreeding bias and social distance. The other three sets pertain to alternative specifications regarding the form of the four factors. Here I switch terminology from *object* to friend to refer to the object of choice. Hypotheses are described for cases with one, two, or three friends.

#### 3.1 Basic Hypotheses

The basic hypotheses pertain to the presence or absence of log-bilinear association and inbreeding bias factors. They are as follows: (a) absence (versus presence) of association between the statuses of subjects and friends, that is,  $\phi_n^{AB} = 0$  for  $n = 1, 2, 3$ ; (b) absence (versus presence) of homophily in subjects' choices of each friend, given a one-to-one correspondence between the statuses of subjects and friends, that is,  $\alpha_{ni} = 0$  for  $n = 1, 2, 3$  and  $i =$  $1, \ldots, I$ ; (c) absence (versus presence) of association among the statuses of friends for each subject, that is,  $\phi_n^{BB} = 0$  for  $n = 2, 3$ ; and (d) absence (versus presence) of inbreeding bias in the choice of multiple friends, that is,  $\beta_{ni} = 0$  for  $n = 2, 3$  and  $j = 1, ..., J$ .

#### 3.2 Homogeneity Hypotheses Regarding the **Effects of the Number of Friends**

Another group of hypotheses pertains to the dependence of parameters on the number of friends. I refer to the independence of factors from the number of friends as *homogeneity* and the dependence of factors on the number of friends as heterogeneity. The following hypotheses on homogeneity (versus heterogeneity) can be tested: (a) homogeneous effects of subjects' statuses in the subject-by-friend association, that is,  $u_{1i} = u_{2i} = u_{3i}$  for each  $i = 1, \ldots, I$ ; (b) homogeneous effects of friends' statuses in the subject-by-friend association, that is,  $v_{1i}$  =  $v_{2j} = v_{3j}$  for each  $j = 1, \ldots, J$ ; (c) the combination of<br>
(a) and (b),  $u_{1i} = u_{2i} = u_{3i}$  and  $v_{1j} = v_{2j} = v_{3j}$ , but<br>  $\phi_1^{AB} \neq \phi_2^{AB}$ ,  $\phi_1^{AB} \neq \phi_3^{AB}$ , and  $\phi_2^{AB} \neq \phi_3^{AB}$ ; (d) homogeneous strength of the subject-by-object association, given that  $u$  and  $v$  parameters are homogeneous, that is,  $\phi_1^{AB} = \phi_2^{AB} = \phi_3^{AB}$ ; (e) homogenous homophily effects in subjects' choices of each friend, given the presence of  $\alpha$ parameters in the model, that is,  $\alpha_{1i} = \alpha_{2i} = \alpha_{3i}$  for each  $i = 1, \ldots, I$ ; (f) homogeneous effects of friends' statuses in the friend-by-friend association, that is,  $w_{2j} = w_{3j}$  for each  $j = 1, \ldots, J$ , but  $\phi_2^{BB} \neq \phi_3^{BB}$ ; (g) homogeneous strength of the friend-by-friend association, given that  $w$ parameters are homogenous, that is,  $\phi_2^{BB} = \phi_3^{BB}$ ; (h) homogeneous effects of inbreeding bias in the choice of multiple friends, that is,  $\beta_{2j} = \beta_{3j}$  for each  $j = 1, \ldots, J$ .

#### $3.3$ A Hypothesis of Common Effects of **Friends' Statuses**

The parameters for the association of friends' statuses with subjects' statuses may be identical to the corresponding parameters for the friend-by-friend association of statuses, that is,  $w_i = v_i$   $(j = 1, \ldots, J)$ .

#### 3.4 Hypotheses of Symmetric Effects

Given that there is a one-to-one correspondence between subjects' and friends' categories, the following two hypotheses of symmetry can be tested: (a) symmetric subject-by-friend association of statuses, that is,  $u_i = v_i$ , for each  $i = 1, \ldots, I$ ; (b) symmetric effects of inbreeding bias in the choice of friends, that is,  $\alpha_i = \beta_i$ , for each  $i =$  $1, \ldots, I$ .

### 4. ANALYSES

In the following analyses of Tables 1 and 2, the likelihood ratio and Pearson's chi squared are used to test the goodness of fit of each model, and the likelihood ratio test is used to compare nested models. There are, however, two limitations in the use of chi-squared tests here. One limitation pertains to how the data were sampled. The data were derived from a stratified one-stage random sampling (random-digit-dialed telephone sampling), not simple random sampling. The bias in chi-squared statistics due to cluster sampling does not apply here, and frequencies can be adjusted for sampling variability across strata. The design effect, however, is not available and is set at 1. Hence, even though parameter estimates may not be biased, chi-squared statistics will be biased (Clogg and Eliason 1987). It follows that comparisons of nested models based on likelihood ratio tests are tentative. Because of this limitation, I identify a group of models that attain a relatively good fit with the data, rather than single out the most parsimoniously fitting model. Significance levels that are marginal (e.g., .10 >  $p$  > .01) will be considered insufficient to make a judgment on the relative goodness of fit.

Second, Haberman (1981) showed that the likelihood ratio chi-squared test of  $\phi^{AB} = 0$  or  $\phi^{BB} = 0$  is not accurate when association parameters are indeterminate. Based on his work, I use the table of statistics available from Pearson and Hartley (1972, table 51) for the test of  $\phi^{AB}$  $= 0$  in such a case. This alternative, however, cannot be directly used to test  $\phi^{BB} = 0$  because of the symmetry of  $w$  parameters. The deviation of the likelihood ratio test statistic from chi squared for the test of  $\phi^{BB} = 0$  arises from the indeterminacy of w parameters when  $\phi^{BB} = 0$ . Therefore, an alternative 1 df test for  $\phi^{BB} = 0$  becomes a valid chi-squared test when the model already assumes either  $w = v$  or fixed scores for w.

#### 4.1 The Analysis of Table 1

The analysis of data in Table 1 is presented in Table 3. Three sets of models are tested, corresponding to (1) basic hypotheses, (2) homogeneity or heterogeneity regarding the effects of number of friends, and (3) modifications of





a HM (homogeneous) and HT (heterogeneous) sets of parameters.

<sup>b</sup> Significant at the 1% level based on the upper percentage points of F(4, 2) presented in table 51 of Pearson and Hartley (1972).

Here  $F(4, 2)$  is the maximum eigenvalue of W(4, 2), where W(4, 2) is the  $4 \times 4$  central Wishart matrix with 2 df (Haberman 1981)

<sup>c</sup> This cannot be accurately tested as a chi-squared test.

various structural hypotheses. Panel 1 of Table 3 presents the chi-squared statistics for three basic models and their degrees of freedom. The first model (Model 1.1) in Table 3 contains, for each of the three subtables of Table 1, only the main effects for subjects' and friends' categories. The second model (Model 1.2) hypothesizes homogeneous sets of  $\phi^{AB}$ , u, v,  $\phi^{BB}$ , w, and  $\beta$  parameters. A single  $\beta$  parameter rather than a set of category-specific  $\beta$  parameters is assumed here, since there are only three categories of friends' statuses, whereas the full set of w and  $\beta$  parameters requires five or more categories. The third model (Model 1.3) hypothesizes heterogeneous sets of  $\phi^{AB}$ , u, v,  $\phi^{BB}$ ,  $w$ , and  $\beta$  parameters, where the parameters vary with the number of friends  $n$ . Model 1.1 does not fit the data, whereas both Models 1.2 and 1.3 attain adequate fits. The comparison of Models 1.2 and 1.3 indicates that Model 1.2 is more parsimonious than Model 1.3. Model 1.2 will be used as the baseline against which other models are compared.

In panel 2 of Table 3, certain homogeneous sets of parameters are replaced by the following heterogenous sets:  $\phi^{AB}$  for Model 2.1;  $\phi^{AB}$  and **u** for Model 2.2;  $\phi^{AB}$  and **v**  for Model 2.3;  $\phi^{AB}$ , u, and v for Model 2.4;  $\phi^{Bb}$  for Model 2.5;  $\phi^{BB}$  and w for Model 2.6; and  $\beta$  for Model 2.7. Comparisons for each of these models with Model 1.2 indicate that Model 1.2 is more parsimonious than all of these models except Model 2.6, and the difference between Models 1.2 and 2.6 is marginally significant (.10 >  $p$  > .05). Hence we may conclude that the following effects are largely independent of the number of friends: the association between subjects' education and friends' discussion statuses; the association among multiple friends' discussion statuses; and the inbreeding bias in the choice of multiple friends with regard to their discussion statuses.

Each of the models in panel 3 of Table 3 omits one of the three basic structural factors, that is, subject-by-friend association, friend-by-friend association, and inbreeding bias. Model 1.2 provides a significant improvement over Models 3.1 and 3.3. Although the likelihood ratio chisquared test for comparing Models 1.2 and 3.2 is not very accurate for the test of  $\phi^{BB} = 0$ , the difference in chi squared seems sufficiently large to reach significance. Hence the main hypotheses regarding the presence of association and inbreeding bias are supported. On the other

hand, the test for the commonality of friends' association parameters in subject-by-friend association and friend-byfriend association, that is, the hypothesis  $\mathbf{v} = \mathbf{w}$ , is inconclusive because of marginal significance (.025 >  $p > .01$ ) (Model 3.4 versus Model 1.2). But the hypothesis  $\phi^{BB}$  = 0 can be clearly rejected by the likelihood ratio chi-squared test when the model already assumes  $v = w$  (Model 3.4) versus Model 3.3).

Table 4 presents the parameter estimates for Model 1.2. The estimates for Models 2.6 and 3.4 are also presented, since these models fit the data nearly as well as Model 1.2. Parameter estimates for Model 1.2 in Table 4 indicate a clear hierarchy among both subjects' educational categories and friends' discussion statuses. These findings agree with expectations based on the substantive contents of these categories.

For subject-by-friend association, the parameters indicate that subjects with four or more years of college education ( $u_4$  and  $u_5$ ) tend to frequently or occasionally discuss social/political issues with friends  $(v_1$  and  $v_2)$ , whereas subjects with less than a full high-school education  $(u_1)$  tend to rarely or never discuss social/political issues with friends  $(v_3)$ . Subjects in the two middle educational categories show middle positions in this respect. The  $w$ parameters for the friend-by-friend association of statuses in Model 1.2 exhibit an almost equidistant structure in the latent position of friends' discussion statuses.

The results from Model 2.6 suggest a possible heterogeneity in the friend-by-friend association that is ignored in Model 1.2. First, the strength of friend-by-friend association, which indicates the tendency to choose friends with mutually similar discussion statuses, may be slightly larger for subjects who chose two friends than for subjects who chose three or more friends. Second, the relative latent position of discussion statuses in the friend-by-friend association may depend on the number of friends. Although the second category (occasional discussion) lies almost exactly in the middle of the three categories for subjects who chose three or more friends, its position moves nearer to category 1 (discuss most of the time) than to category 3 (almost never discuss) for subjects who chose two friends.

If we rely on Model 3.4, which imposes  $v = w$  on Model 1.2, we may lose some information about the characteristics of friends' discussion statuses. As we have seen in the results of Model 1.2, the discussion statuses of friends, when associated with subjects' educational categories, are not equidistant, since the second category is much closer to the first category. Under the imposition of  $v = w$ , however, the estimates for  $v$  parameters become almost equidistant.

#### 4.2 The Analysis of Table 2

Table 5 presents the results of the analysis of data in Table 2. Since there is a one-to-one correspondence between categories of subjects and friends, we can test homophily in subjects' choices of each friend, that is,  $\alpha$  parameters. On the other hand, the data in Table 2 have only two levels for the number of friends, one and two or more. Consequently, we do not test heterogeneity in the set of  $\phi^{BB}$ , w, and  $\beta$  parameters.

The results of four models are presented in panel 1 of Table 5. These models hypothesize, respectively, (1) main effects only (Model 1.1), (2) homogeneous association and homophily for subject-by-friend association and association and inbreeding bias for friend-by-friend association (Model 1.2), (3) heterogeneous association and homophily for subject-by-friend association and association and inbreeding bias for friend-by-friend association, and (4) a modification of Model 1.2 by replacing homogeneous  $\phi^{AB}$ by heterogeneous  $\phi^{AB}$ . Models 1.2–1.4, but not Model 1.1, attain adequate fits with the data. Although the rel-

Models				Subject-by-friend association parameters		Friend-by-friend association parameters					
Model 1.2	$\phi^{BB}$			$u_1$			$\phi^{BB}$		w,		$\beta$
	.586	$-.723$	2 $-.115$	3 $-.089$	4 .349	5 .578	.778	1 .715	2 $-.017$	3 $-.699$	.289
				$\boldsymbol{V}_j$							
			1	$\overline{\mathbf{c}}$	3						
			.477	.335	$-.812$						
Model 2.6	$\phi^{AB}$			$U_i$			$\phi^{BB}$		$W_j$		$\beta$
		1	$\overline{c}$	3	4	5		1	$\overline{c}$	3	
									$n = 2$		
	.584	$-.719$	$-.120$	$-.093$	.352	.580	1.230	.609	.167	$-.766$	.283
									$n = 3$		
				$V_j$			.744	.728	$-.044$	$-.684$	
			1	$\overline{c}$	3						
			.478	.334	$-.812$						
Model 3.4	$\phi^{AB}$			$u_{j}$			$\phi^{BB}$		$w_i = v_i$		$\beta$
		1	$\overline{c}$	3	4	5		1	$\overline{c}$	3	
	.536	$-.720$	$-.123$	$-.028$	.229	.643	.765	.693	.027	$-.720$	.303

Table 4. Parameter Estimates of Selected Models for the Analysis of Table 1





a HM (homogeneous) and HT (heterogeneous) sets of parameters.

<sup>b</sup> Significant at the 1% level based on the upper percentage points of F(4, 4) presented in table 51 of Pearson and Hartley (1972).

Here  $F(4, 4)$  is the maximum eigenvalue of W(4, 4), where W(4, 4) is the  $4 \times 4$  central Wishart matrix with 4 df (Haberman 1981). c This cannot be accurately tested as a chi-squared test.

ative goodness of fit between Model 1.2 and Model 1.3 is inconclusive (.05 >  $p > .025$ ), Model 1.4 significantly improves the fit of Model 1.2 and is not improved significantly by Model 1.3. Hence Model 1.4 serves as the baseline against which other models are compared.

Four modifications of Model 1.4 are tested in panel 2 of Table 5. Models 2.1, 2.2, and 2.3 hypothesize not only that the strength of subject-by-friend association is different, but also that the latent positions of subjects' and/or friends' categories in the subject-by-friend association are different. Model 2.4 hypothesizes heterogeneous homophily effects in subjects' choices of each friend. Comparisons of these models with Model 1.4 show that Model 1.4 is the most parsimonious.

In panel 3 of Table 5, the results from various other models that modify Model 1.4 are presented. Models 3.1-3.4 hypothesize the presence or absence of the four basic structural factors. Models 3.5-3.12 test hypotheses re-

garding symmetry or commonality among certain sets of parameters.

The results unequivocally confirm the presence of three of the four basic structural parameters. Comparisons between Model 1.2 and Model 3.1 and between Model 1.4 and Models 3.2 and 3.4 indicate the presence of subjectby-friend association, homophily in subjects' choices of each friend, and inbreeding bias in the choice of multiple friends. The test for the absence of friend-by-friend association ( $\phi^{BB} = 0$ ) cannot be made directly. When w = **v** is assumed, however, hypothesis  $\phi^{BB} = 0$ , which can then be tested by chi squared, can be rejected (Model 3.6) versus Model 3.3). Note that the hypothesis  $w = v$  itself cannot be rejected here (Model 1.4 versus Model 3.6).

The results from the tests of two hypotheses on symmetry, one for  $\mathbf{u} = \mathbf{v}$  and the other for  $\alpha = \beta$ , are rather ambiguous. The hypothesis  $\alpha = \beta$  cannot be rejected when  $\mathbf{v} = \mathbf{w}$  is not assumed (Model 1.4 versus Model 3.7), but the test attains a marginal level of significance when  $v = w$  is imposed (Model 3.8 versus Model 3.6). The test for the hypothesis of symmetric association, that is,  $\mathbf{u} =$ v, attains a marginal level of significance regardless of whether  $\mathbf{v} = \mathbf{w}$  is imposed or not (Model 1.4 versus Model 3.5 and Model 3.6 versus Model 3.9). When  $\mathbf{u} = \mathbf{v} = \mathbf{w}$ is assumed, however, the hypothesis that the strength of subject-by-friend association is equal to the strength of friend-by-friend association among subjects with two or more friends, that is,  $\phi_2^{AB} = \phi^{BB}$ , cannot be rejected (Model 3.9 versus Model 3.10). When models are compared with Model 3.12, which hypothesizes both  $\alpha = \beta$  and  $\mathbf{u} = \mathbf{v}$ (with  $\phi^{BB} = \phi_2^{AB}$ ) in addition to  $\mathbf{v} = \mathbf{w}$ , neither hypothesis  $\mathbf{v} \neq \mathbf{w}$  nor hypothesis  $\mathbf{u} \neq \mathbf{v}$  provides an improvement in fit (Model 3.11 versus Model 3.12 and Model 3.8 versus Model 3.12), and hypothesis  $\alpha \neq \beta$  attains a marginal level of significance (Model 3.10 versus Model 3.12). When  $\alpha = \beta$  is assumed, however, the hypothesis  $u \neq v$  gains a marginal level of improvement (Model 3.6 versus Model 3.10), whereas hypothesis  $v \neq w$  does not (Model 3.5) versus Model 3.10).

These results lead to the fact that Models 3.6, 3.10, and 3.12 cannot be improved clearly by other models and the relative goodness of fit among them cannot be determined because of marginal levels of significance in comparison tests. Table 6 thus presents parameter estimates from these three models.

Model 3.12, which hypothesizes  $\mathbf{u} = \mathbf{v}$  and  $\alpha = \beta$ , employs the smallest number of parameters among the three "best fitting" models. The structure of distances among the educational categories in the subject-by-friend and friend-by-friend associations revealed by Model 3.12 agrees with the substantive content of the categories, that is, the order of parameters agrees with the order of years of education. The spacing between categories, however, is not equidistant. The distance between neighboring categories is relatively small between categories 1 and 2 and between categories 4 and 5. Furthermore, the number of friends influences the strength of subject-by-object association, and the estimate for parameter  $\phi^{AB}$  is more than twice as large for subjects with only one friend compared with subjects with two or more friends.

Parameter estimates for inbreeding bias indicate that the lowest level of education (fewer than 12 years of ed-

Models				Subject-by-friend	association parameters	Friend-by-friend association parameters							
Model 3.12		$\phi^{AB}$			$u_i = v_i = w_i$			$\phi^{BB} = \phi_2^{AB}$					
				2	$\mathbf 3$	4	5						
	$n =$ $n = 2$	3.625 1.500	$-.564$	$-.390$	$-.044$	.379	.620	1.500					
					$\alpha_i = \beta_i$								
				$\mathbf 2$	3	4	5						
			.931	.119	.378	.207	.313						
Model 3.10		$\phi^{AB}$			$u_i = v_i = w_i$			$\phi^{BB} = \phi_2^{AB}$					
			1	2	3	4	5						
	$n =$ $\overline{c}$ $n =$	3.796 1.499	$-.576$	$-.376$	$-.041$	.371	.623	1.499					
			.635	2 .017	$\frac{\alpha}{3}$ .402	4 .345	5 .257		1 1.568	$\overline{2}$ .380	$\frac{\beta_i}{3}$ .313	4 $-.048$	5 .399
Model 3.6		$\phi^{AB}$			$u_{i}$			$\phi^{BB}$			$w_i = v_i$		
			1	2	3	4	5		1	2	3	4	5
	$\mathbf{1}$ $n =$ $\overline{c}$ $n =$	4.628 1.725	$-.592$	$-.247$	$-.085$	.183	.740	1.210	$-.502$	$-.491$	.003	.401	.588
					$\alpha$						$\beta_{\text{\tiny I}}$		
			1	2	$\mathbf 3$	4	5	$\frac{\epsilon}{\epsilon}$	1	2	3	4	5
			.652	.097	.380	.465	$-0.006$		1.816	.196	.351	$-.099$	.620

Table 6. Parameter Estimates of Selected Models for the Analysis of Table 2

ucation) has the strongest tendency for the bias. Other educational levels have much lower levels of inbreeding bias and, in particular, persons with a four-year high-school education have the lowest level.

Model 3.10 indicates what information we might lose by imposing  $\alpha = \beta$  in Model 3.12. A major difference between the estimates of  $\alpha$  and  $\beta$  parameters in Model 3.10 is the size of effects for the category of fewer than 12 years of education. Although this category has the largest estimated values for both  $\alpha$  and  $\beta$  parameters, the tendency for a subject to choose another friend from among those with fewer than 12 years of education, given that one of his or her friends has this educational level, is much larger than the tendency for a subject with fewer than 12 years of education to choose a friend with the same educational level. Another difference between the estimates of  $\alpha$  and  $\beta$  parameters is the absence of inbreeding bias for distinct educational category, namely four-year highschool education in subjects' choices of each friend  $(\alpha)$ and four-year college education in the choice of multiple friends  $(\beta)$ . As I mentioned before, the absence of inbreeding bias implies a lack of such bias over and beyond that captured by the association effects.

Model 3.6 indicates that information about differences in the structure of distances between educational categories of subjects and those of friends might be lost by relying on Model 3.10 or Model 3.12, which imposes  $\mathbf{u} =$ v. In Models 3.10 and 3.12, we have found proximate positions for categories 1 and 2 and categories 4 and 5. The results from Model 3.6, however, indicate that the latent positions of educational categories 2 and 4 for subjects may in fact be closer to category 3 than to either category 1 or category 5. On the other hand, the latent positions among friends' educational categories show almost no distance between categories 1 and 2 and between categories 4 and 5.

#### 5. CONCLUSION

The use of conditionally symmetric partial log-bilinear association models to analyze friendship choice reveals several noteworthy findings. Regarding the formal structural aspects of friendship choice, we confirmed the following.

1. Not only do homophily (or inbreeding bias) and social distance exist when subjects choose a friend, but inbreeding bias and social distance exist among the statuses of friends when subjects choose two or more friends.

Although the other results may not be generalized over and beyond the present analyses, we also observed the following.

2. The latent distances among categories in both subjectby-friend and friend-by-friend associations and the patterns of inbreeding bias in the choice of friends usually do not depend on the number of friends.

3. The strength of association, however, may depend on the number of friends. We found in the analysis of data in Table 2 that subjects with one friend have a stronger

tendency to choose similar outgroup members as friends than subjects with two or more friends. In the analysis of data in Table 1, we also observed that, compared with subjects with three or more friends, subjects with two friends have a stronger tendency to choose as friends persons who are similar to each other. Based on these two results, we may conjecture that when the strength of subject-by-friend or friend-by-friend association depends on the number of friends, the association will be stronger when the number of friends is smaller.

4. When both subjects and friends are classified by the same status variable, the latent-distance structure among subjects' statuses is not always identical to the latent-distance structure among friends' statuses. Similarly, the latent-distance structure among friends' categories in the subject-by-friend association may be different from that in the friend-by-friend association. The latter may be especially true when different status variables are used to classify subjects and friends.

Finally, although the present analysis is restricted to friendship choice data, conditionally symmetric log-linear and log-bilinear models have a general applicability in the analysis of one-to-many correspondences when the status set of objects is expressed as a combination rather than as a full cross-classification of statuses.

[Received January 1988. Revised August 1989.]

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