Title
Scissor congruence

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On Scissor Congruence

The article "WHAT IS... a Scissors Congruence" in the October Notices reviewed the decomposition of two polyhedra into pairwise congruent subpolyhedra. The term "scissor congruent," introduced in [1], refers to a pair of planar convex bodies $A = A_1 \cup \cdots \cup A_n,\ \ B = B_1 \cup \cdots \cup B_n$, where A_i and B_i are congruent compact 2-cells, and for $i \neq j$ the interiors of A_i and A_j (respectively, B_i and B_j) are disjoint.

The main result is that A and B are scissor congruent iff they have the same area and their boundaries have partitions $\partial A = I_1 \cup \cdots \cup I_m,\ \ \partial B = J_1 \cup \cdots \cup J_m$, where I_k and J_k are congruent 1-cells. It follows that a square and a circle are not scissor congruent, and scissor congruent ellipses are congruent. No analog in higher dimensions is known.

Gold [2] investigated scissor congruence of unobunded convex sets. Gardner [3] looked at more refined equidecomposability questions. Richter [4, 5] considered scissor congruence based on affine motions.

[1] L. Dubins, M. Hirsch, J. Karush: *Scissor conruence.* Israel J. Math. **1** (1963), 239–247

[2] S. Gold: *The sets that are scissor congruent to an unbounded convex subset of the plane.* Trans. Amer. Math. Soc. **215** (1976), 99–117.

[3] R. Gardner: *A problem of Sallee on equidecomposable convex bodies.* Proc. Amer. Math. Soc. **94** (1985), 329–332.

[4] C. Richter: *Affine congruence by dissection of discs— appropriate groups and optimal dissections.* J. Geom. **84** (2005), 117-132.

[5] C. Richter: *Squaring the circle via affine congruence by dissection with smooth pieces.* Beitrge Algebra Geom. **48** (2007), no. 2, 423434.

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