An EZ-circular diffusion model of continuous decision processes

Hassan Qarehdaghi (h.qarehdaghi95@gmail.com)
Institute for Cognitive and Brain Sciences (ICBS), Shahid Beheshti University, Tehran, Iran

Jamal Amani Rad (j.amanirad@gmail.com; j_amanirad@sbu.ac.ir)
Institute for Cognitive and Brain Sciences (ICBS), Shahid Beheshti University, Tehran, Iran

Abstract

Because there are many situations in our daily life in which the option space is not discrete but continuous, recently developed decision models have been able to examine the cognitive processes underlying choice in laboratory tasks with a continuous outcome space. One of the most important of these continuous models is the circular diffusion model (CDM) by Smith ((Smith, 2016)), which has been shown to account for continuous space data from a wide range of paradigms, including color identification, orientation, brightness, pricing. However, in addition to the inherent complexity of this model, it has become more complex in order to predict reliable data patterns, making it a tool only for experts. Here we propose a more easy version of the CDM, the EZ version, to fit the model on continuous scale data. The EZ-CDM for continuous choice space tasks can estimate the parameter values for the cognitive processes underlying without considering the response time distribution but only using traditionally favored summary statistics (i.e. the mean and variance of response time, and angular variance of accuracy.) by simple formulas that can be computed easily and needs neither theoretical knowledge of model fitting nor much programming skills. Here, we formulate the EZ method and show that, despite being easy and fast to calculate, it's performance in recovering true parameters is acceptable.

Keywords: Decision making, Continuous response, Cognitive model, Circular diffusion model, Response time, Complexity.

Introduction

Behavioral data is the main source of information in psychological research. Experimental psychologists often obtain behavioral data by conducting tasks that involve decision making by subjects. At this point, researchers confront two issues to make inferences from these behavioral decision data. First, choice and response time data is often variable (even in very similar trial conditions), and secondly, different factors affect the results and the data comprise mixture effects of these factors. The primitive approach performed by researchers to extract stable information from variable data is to use summary statistics of data, like mean and variance. Although this approach deals with the issue of variability, but still the statistics contain a mixture of information about different factors. The remedy to overcome both issues has been introduced by the cognitive modeling approach. Cognitive models of decision making, introduce a particular way of interaction between variability and individual factors to generate data.

Diffusion Decision Model (DDM) of Ratcliff (Ratcliff, 1978; Ratcliff & Mckoon, 2008; Ratcliff & Rouder, 1998) is one such model of choice and response time data generated from underlying cognitive processes in simple two-choice decisions. More precisely, this model is a quantitative counterpart of the conceptual model, describing decision making as an accumulation of information over time to reach the criterion for choosing each alternative. The main source of variability proposed by DDM, is a stochastic diffusion process of information accumulation, and the main influential factors are captured by: the systematic tendency of the stochastic process to approach each alternative (called drift rate) as the relative speed of information accumulation in favor of alternatives; the criterion as the amount of information needed to choose each alternative (called boundary separation), and non-decision time as time consumed by other processes than decision, like encoding stimulus and executing response (Ratcliff, 1978). DDM has been used numerous times in research involving a vast range of decision tasks about different cognitive processes, and successfully accounted for behavioral and neurophysiological data (Evans & Brown, 2017; Ratcliff, Huang-Pollock, & McKoon, 2018; Evans, Bennett, & Brown, 2018; Fontanesi, Gluth, Spektor, & Rieskamp, 2019; Pedersen, Frank, & Biele, 2017; Krajbich, Lu, Camerer, & Rangel, 2012; Forstmann, Ratcliff, & Wagenmakers, 2016; Gold & Shadlen, 2007).

Two-alternative decision tasks are mainly used in psychological research, but recently there has been an increasing interest in tasks involving continuous decision alternative space (e.g. (Itti & Koch, 2001)). Unlike two-alternative decisions, this allows getting distribution for accuracy, instead of just a single correct response rate. Also, there are many situations in our daily life in which the option space is not discrete but continuous (Yoo, Hayden, & Pearson, 2020). For example, when a driver needs to avoid a dog that suddenly ran in the middle of a road, their options are not discrete: the steering angle of a car is typically 60 degrees wide. Many other activities, such as evaluating the selling or buying price of a product (Kvam & Busemeyer, 2020), involve selecting the best action from a continuous action space. Therefore, these tasks could be more informative and natural than traditional two-alternative decision tasks.

Recently, cognitive models with the concept of noisy information accumulation have been proposed for continuous tasks. The spatially continuous diffusion model (Ratcliff, 2018) is one such model, but it has conceptual and practical

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complexity (Smith, Saber, Corbett, & Lilburn, 2020). Also multiple anchored accumulation theory (Kvam, Marley, & Heathcote, 2021) and geometric similarity representation (Kvam & Turner, 2021) have been proposed, but in addition to being complex, they are somewhat general and need to become specific suitably for the particular task at hand.

The circular Diffusion Model (CDM) (Smith, 2016) is simpler, yet insightful model of circular tasks. Circular decision tasks are commonly used as continuous tasks that the subject has to choose one point on a circle. Examples of such tasks are judgement about orientation, direction, shape, location, or color of stimuli (Unsworth, Fukuda, Awh, & Vogel, 2014). The circular diffusion model is based on the same theory of DDM but extended to the continuous outcome decisions and has almost the same parameters. It represents information accumulation as a stochastic 2D Wiener diffusion process on the interior of a disk whose bounding circle represents the decision criterion.

Despite the obvious advantages of this model for explaining the observed data in continuous scale tasks based on underlying psychological meaningful processes, unfortunately, it has not been used much so far. The use of this model seems to be complicated due to the nature of the model. In fact, in order to become commonly used, various methods such as maximum likelihood estimation (MLE) and Bayesian approaches are needed to fit the model and extract parameter values from observed data. Theoretically, the MLE and Bayesian approaches are suitable methods for parameter estimation but they require some knowledge about fitting routine and programming skills to some degree, which remains the use of this model a suitable approach for experts.

This challenge also existed in the DDM of Ratcliff, which is much simpler than the CDM, but (Wagenmakers, Van Der Maas, & Grasman, 2007) were able to solve this problem by introducing a simple approach named the "EZ diffusion model". In EZ-DDM, the parameters are calculated from summary statistics of data by quite simple formulas. Here, we introduce a new EZ method for parameter estimation of CDM where it enables the computation of CDM parameter values, from mean and variance of response time and circular variance of accuracy. We test the performance of the EZ method by comparing its parameter recovery with the so-called theoretically favored MLE method.

Circular Diffusion Model

CDM is a model of decision making in circular decision tasks, proposing a procedure for generation of choice and response time, from underlying cognitive components. It assumes that decision is the result of a noisy accumulation of information represented in 2D evidence space. State of evidence at any time is represented by a point in a plane. Also, the direction of this point could be thought of as representative of the alternative with most evidence and its norm, as the magnitude of evidence for that alternative (Kvam, 2019). As the magnitude of evidence favoring one alternative, reaches a criterion, a choice is made. So the decision criteria is a circle of which each point corresponds to a decision alternative. Furthermore, the radius of this circle determines the amount of evidence needed to respond. Mathematically, dynamic of evidence state in time is modeled by the following stochastic differential equation:

$$dX_t = vdt + \sigma dW_t, \tag{1}$$

where X_t is state in time t, v is drift rate vector, σ is diffusion coefficient, and W_t is a two-dimensional Wiener process. The drift rate vector shows the systematic tendency of state change over time. In fact, its direction and length, represent identity and quality of inner representation of stimulus, respectively. On the other hand, the diffusion coefficient determines the range of noisy change in state. As it is a scaling variable (multiplying v, a, and σ by constant term, wouldn't change the predictions of the model) it is taken to be constant $\sigma = 1$. The process starts from origin of zero coordinate in plane, and changes according to the above dynamic equation until it reaches the boundary of the circle. The point of intersection determines the choice alternative and the time it takes from start to boundary hit, determines the decision process time. Response time here is the sum of decision process time and all other time-consuming processes included in decision making, called non-decision time (T_{er}). Figure (1) illustrates a schematic of the model in a circular decision task sample.

EZ-circular diffusion model

The EZ-CDM proposed here is based on reparameterizing the CDM with cumulants of predicted distributions and then easily estimating these cumulants by summary statistics. To do so, we need to get from these cumulants to original parameters, but this is not straightforward. However, the inverse operation (calculating cumulants from model parameters) could be done more easily. So we first formulate equations to get the cumulants of predicted response time and accuracy distributions of CDM. As the model has three parameters, three cumulants will be needed for reparameterization. We use the first two cumulants (expected value and variance) of response time and second circular cumulant (circular variance) of accuracy.

By accuracy, we mean the angular distance of the chosen point from the right answer point of the trial on the circle. For the sake of simplicity in equations, from now on, we will use the symbol v as the drift length, not the drift vector itself. We will estimate only the length of the drift vector. Drift vector direction (that represents bias) could be estimated by angular mean of accuracy.

As stated in (Smith, 2016), the predicted distribution of accuracy is a von Mises distribution with concentration parameter av. So the angular variance of accuracy (VACC) will be:

$$VACC = 1 - \frac{I_1(av)}{I_0(av)}, \tag{2}$$

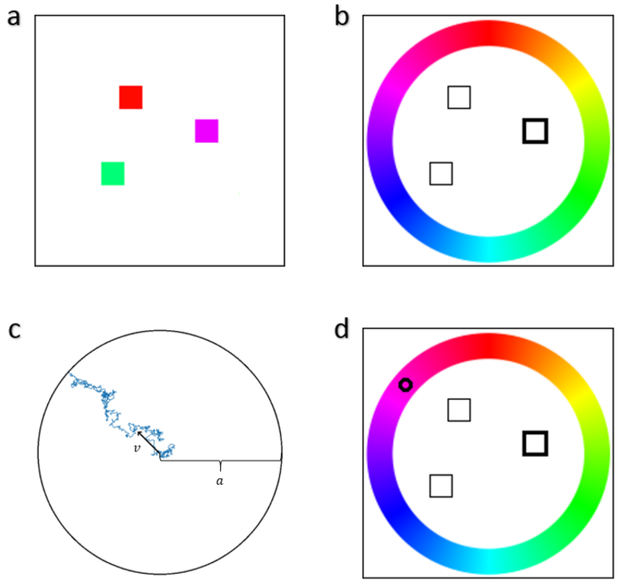

Figure 1: CDM model in one trial of a typical circular decision task. (a) Subject is presented with three different colored squares. (b) The subject is asked to choose the color of the probed square by moving the mouse pointer to a position on the circle. (c) CDM model assumes that the process of decision making consists of a 2D Wiener diffusion process with a drift vector v, representing the encoded stimulus. The process runs until it hits the boundary circle of radius a. (d) The hitting point determines the decided color and the response time is determined by the time it takes for the process to hit the boundary plus non-decision time T_{er}.

where I_0 and I_1 are the first kind modified Bessel functions of order zero and one, respectively. To calculate cumulants of the predicted distribution of response time, we can use its moment generating function (Laplace transform) given by (see appendix in (Smith, 2016)):

$$E[e^{-\lambda T}] = \frac{e^{-\lambda T_{er}} I_0(av)}{I_0(a\sqrt{2\lambda + v^2})}.$$

The first moment which equals mean reaction time (MRT), can be calculated from the moment generating function as:

$$MRT = E[T] = -\frac{d}{d\lambda} E[e^{-\lambda T}]\Big|_{\lambda=0} = T_{er} + \frac{a}{v}\frac{I_1(av)}{I_0(av)}, \quad (3)$$

where we have used the simple formula $(d/dx)I_0(x) = I_1(x)$. Also the second moment will be:

$$\begin{aligned} E[T^2] &= \frac{d^2}{d\lambda^2} E[e^{-\lambda t}]\Big|_{\lambda=0} \\ &= \frac{1}{I_0^4}\Big(I_0^4 t_0^2 + 2\frac{a}{v} I_0^3 I_1 t_0 + 2\frac{a}{v^3} I_0^3 I_1 - \frac{a^2}{v^2} I_0^4 \\ &\quad + 2\frac{a^2}{v^2} I_0^2 I_1^2\Big), \end{aligned}$$

where all Bessel functions are calculated at av and superscripts are power. We used the fact that $(d/dx)I_1(x) = I_0(x) - I_1(x)/x$ in the above calculations. Now we can calculate the variance of response time (VRT):

$$VRT = E[T^2] - E[T]^2 = \frac{a^2}{v^2}\frac{I_1^2(av)}{I_0^2(av)} + \frac{2a}{v^3}\frac{I_1(av)}{I_0(av)} - \frac{a^2}{v^2}. \quad (4)$$

Now we need to solve the algebraic system of three equations (2), (3), and (4):

$$\begin{cases} VACC = 1 - \dfrac{I_1(av)}{I_0(av)}, \\ MRT = T_{er} + \dfrac{a}{v}\dfrac{I_1(av)}{I_0(av)}, \\ VRT = \dfrac{a^2}{v^2}\dfrac{I_1^2(av)}{I_0^2(av)} + \dfrac{2a}{v^3}\dfrac{I_1(av)}{I_0(av)} - \dfrac{a^2}{v^2}. \end{cases} \quad (5)$$

Now, we first solve the first equation for av and by means of the third equation, we get the a and v, then calculate the T_{er} from the second equation. It's easier to take $R = 1 - VACC$ and $\kappa = av$. So the first equation will be:

$$R = \frac{I_1(\kappa)}{I_0(\kappa)}.$$

Here, we use an approximate solution for this equation given in (Banerjee, Dhillon, Ghosh, Sra, & Ridgeway, 2005):

$$\kappa = \frac{R(2 - R^2)}{1 - R^2}.$$

But this approximation is not accurate enough for our purpose, so we use Newton-Raphson iteration as suggested in the reference article:

$$\kappa_1 = \kappa - \frac{\frac{I_1(\kappa)}{I_0(\kappa)} - R}{1 - \frac{I_1^2(\kappa)}{I_0^2(\kappa)} - \frac{I_1(\kappa)}{\kappa I_0(\kappa)}}.$$

It seems that one iteration is satisfactory for our use here as the ratio of the estimate to the exact solution of the equation is between 0.995 and $1 + 10^{-15}$ and we have checked that this magnitude of error has no considerable effect on the parameter recovery of the proposed EZ method.

Now we rewrite the third equation with respect to v, by replacing the a with κ_1/v, and $I_1(av)/I_0(av)$ with R:

$$v^4 = \frac{1}{VRT}\Big(\kappa_1^2 R^2 + 2\kappa_1 R - \kappa_1^2\Big). \quad (6)$$

So a could be computed as:

$$a = \kappa_1/v. \quad (7)$$

Then the T_{er} can be calculated from the second equation:

$$T_{er} = MRT - \frac{a}{v}R. \quad (8)$$

Now that we completed the reparametrization of CDM to cumulants, summary statistics can be used to estimate these cumulants:

$$\begin{cases} VACC = 1-\bar{R} = 1-\frac{1}{N}\sqrt{\left(\sum_{n=1}^{N}\cos(\theta_n)\right)^2+\left(\sum_{n=1}^{N}\sin(\theta_n)\right)^2}, \\ MRT = \frac{1}{N}\sum_{n=1}^{N} t_n, \\ VRT = \frac{1}{N}\sum_{n=1}^{N}\left(t_n-\frac{1}{N}\sum_{n=1}^{N} t_n\right)^2, \end{cases} \tag{9}$$

where N is the number of data points of response time t and accuracy θ (in radian). It should be noted that the first equation is the maximum likelihood estimation of circular variance for von Mises distribution, and the $\bar{R}$ above is equivalent of R in previous calculations.

In summary, the procedure for EZ fitting consists of calculating summary statistics ($\bar{R}$, MRT and VRT) from (9) and computing the parameter values in the following order:

$$\begin{cases} \kappa = \frac{\bar{R}(2-\bar{R}^2)}{1-\bar{R}^2}, \\ \kappa_1 = \kappa - \frac{\frac{I_1(\kappa)}{I_0(\kappa)}-\bar{R}}{1-\frac{I_1^2(\kappa)}{I_0^2(\kappa)}-\frac{I_1(\kappa)}{\kappa I_0(\kappa)}}, \\ v = \sqrt[4]{\frac{1}{VRT}\left(\kappa_1^2\bar{R}^2+2\kappa_1\bar{R}-\kappa_1^2\right)}, \\ a = \kappa_1/v, \\ T_{er} = MRT - \frac{a}{v}\bar{R}. \end{cases} \tag{10}$$

The "scipy.special.iv" function in Python and "besseli" in Matlab and "BesselI" in Wolfram Mathematica could be used to calculate the modified Bessel function of the first kind (I_0 and I_1) in the second equation above.

Results and comparison

Here, we analyze the ability of the EZ method to correctly recover the true parameters of CDM. To do this, we first simulate artificial data from CDM model and use EZ method to recover the parameters and calculate the difference between true, and estimated parameter values. To have a reference for examining EZ performance, we also recover parameters with MLE and compare the EZ results with recovery results from MLE.

We use a span of parameter values containing previously estimated values resulted from fitting CDM on empirical data (Kvam, 2019; Smith et al., 2020; Zhou, Osth, Lilburn, & Smith, 2021). Three values of $1,2,3$ are used for criteria a, and three values of $1.5,3,4.5$ are used for drift length. Also one additional value of zero is used for drift length to check any possible deficiency in EZ performance in limiting cases where drift length approaches zero. The non-decision time is fixed to zero because it only shifts all response time data points and this will just shift the recovered non-decision time by both EZ and MLE methods.

For simulating a trial, we run a discrete random walk version of dynamic equation (1) started from the origin and changed according to:

$$\begin{cases} \Delta X_t^{(1)} = v\Delta t + \xi^{(1)}\sqrt{\Delta t}, \\ \Delta X_t^{(2)} = \xi^{(2)}\sqrt{\Delta t}, \end{cases}$$

where superscripts indicate coordinated components and ξ is a sample of standard normal distribution. We used $\Delta t = 0.001$ seconds and checked that the simulated data with this value of Δt is indistinguishable from finely grained data with number of trials we use. The absence of drift component in the second equation is because we take the drift vector to lie on the horizontal axis, having zero component on vertical coordinate. According to symmetry in the model, this assumption does not reduce the generality of results.

The number of trials has three levels of 50, 150, and 800 which represents the number of trials taken from subjects in psychological research and modeling analysis, respectively. For every three levels of trial number and 12 parameter sets (3 criteria $\times$ 4 drift), 100 data sets are simulated. Note that the EZ method is implemented as procedure discussed above, and MLE is performed using the Likelihood function given in (Smith, 2016):

$$\frac{1}{2\pi a^2}\exp\left(va\cos(\theta)-\frac{v^2 t}{2}\right)\sum_{i=1}^{\infty}\frac{j_{0,i}}{J_{1(j_{0,i})}}\exp\left(-\frac{j_{0,i}^2}{2a^2}t\right),$$

where J_1 is the first-order Bessel function of the first kind and $j_{0,i}$ is the i-th zero of the zero-order Bessel function of the first kind. We simulated data with a drift vector of zero angle, so we only entered the horizontal coordinate of the hitting point in the above formula, restricting the estimated vector to have angle zero. The infinite sum is calculated till 50 which gives a good approximation of likelihood function, except for values of very small t where actual likelihood will be so small and we chose an infinitesimal constant value for these data points. Also, the Nelder–Mead optimization method with true parameter values is used to find the maximum likelihood parameters.

Parameter recovery results are presented in Figures (2), (3), and (4).

The middle points in the figures are representing the bias and the error bars show the deviation in the recovery of parameters. By increasing the number of trials, bias and deviation of EZ method recovery are decreasing. The decline in bias is very similar to the MLE method, except for v in the case that the actual drift length was zero. As mentioned this situation is not expected in real data. Also the deviations are reasonably low and a little more than MLE deviations, especially for v and a.

These results show that the EZ method is capable of extracting true parameter values and detecting differences in values of parameters.

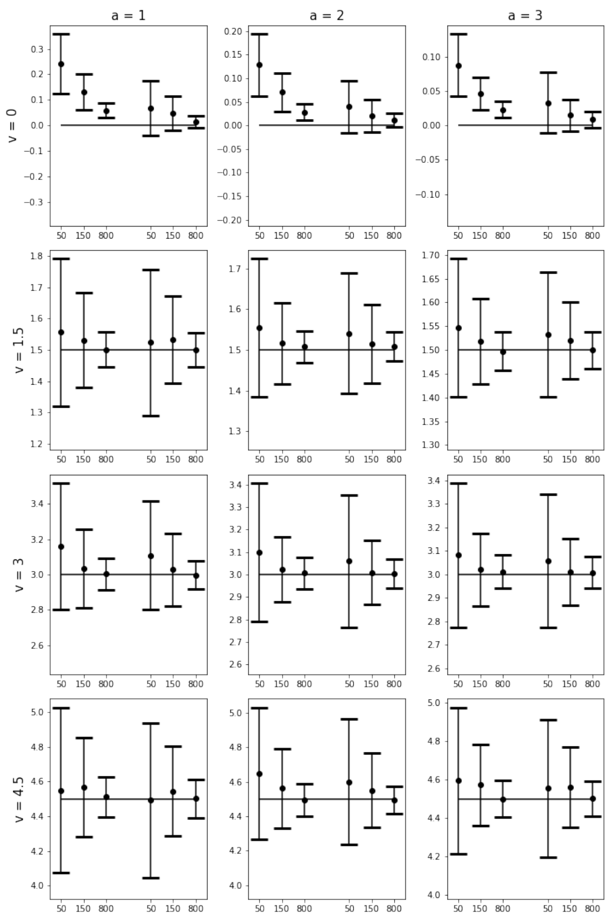

Figure 2: Parameter recovery results for drift length *v*. True drift length parameter for each row is 0, 1.5, 3, and 4.5 from top to bottom. True criteria value parameter for each column is 1, 2, and 3 from left to right. In each figure, the three bars on left, show the results for the EZ method in 50, 150, 800 trial number levels, and the bars in right are for the MLE method. The middle point is the mean of 100 recovered parameter values for *v* and the error bars show the one standard deviation above and below the mean value. Horizontal lines show the true value for parameter *v*.

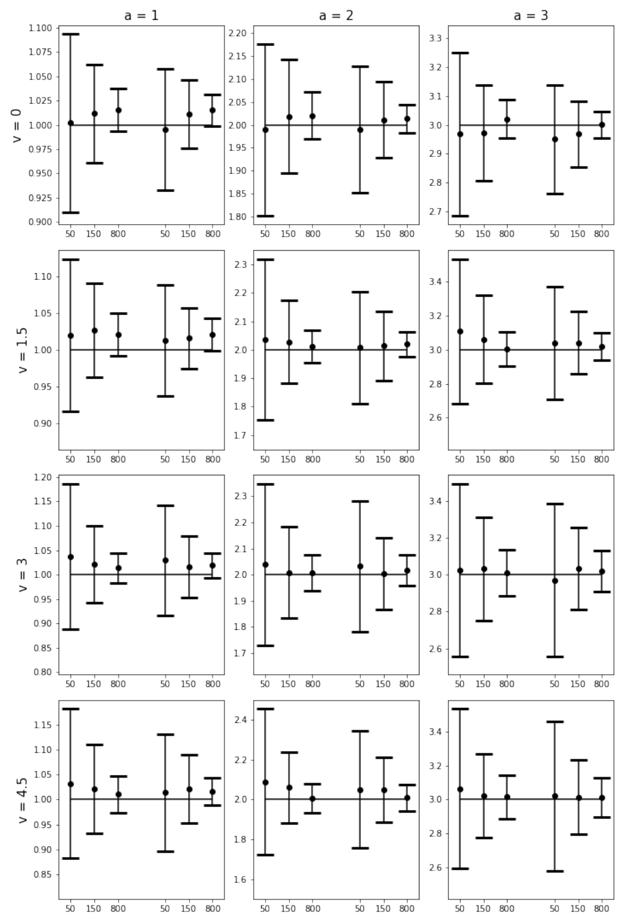

Figure 3: Parameter recovery results for criteria value *a*. True drift length parameter for each row is 0, 1.5, 3, and 4.5 from top to bottom. True criteria value parameter for each column is 1, 2, and 3 from left to right. In each figure, the three bars on left, show the results for the EZ method in 50, 150, 800 trial number levels, and the bars in right are for the MLE method. The middle point is the mean of 100 recovered parameter values for *a* and the error bars show the one standard deviation above and below the mean value. Horizontal lines show the true value for parameter *a*.

Discussion

The EZ-DDM facilitated the use of the cognitive decision model in the psychology community for two-alternative decision tasks. Here we proposed the basis for the same facilitation for continuous decision tasks.

Previous experience from investigations on EZ-DDM, shows some pros and cons of using this method. It could face some problems when the data is contaminated (Ratcliff, 2008), or when there is some between trial variability in parameters. But still, investigations show that it successfully captures the differences between experimental conditions and individual and group differences (van Ravenzwaaij & Oberauer, 2009; Ratcliff & Childers, 2015; van Ravenzwaaij, Donkin, & Vandekerckhove, 2017). We think these situations exist in EZ-CDM as well, but need to be carefully considered in the future.

Another existing subject is guessing data. In currently used circular tasks, there is an assumption that some proportion of data is the result of guess. To use the EZ method, one needs to separate this guess data from the rest, because they have different nature. One way to do so, is to take confidence in each trial and only enter the high confidence data into the EZ fit procedure. The other method could be fitting the mixed model of uniform plus von Mises distribution on accuracy data and calculating the circular variance of von Mises part of accuracy distribution (Zhang & Luck, 2008). The other two statistics for response time could be calculated with all data points as the successful fit of encoding failure model

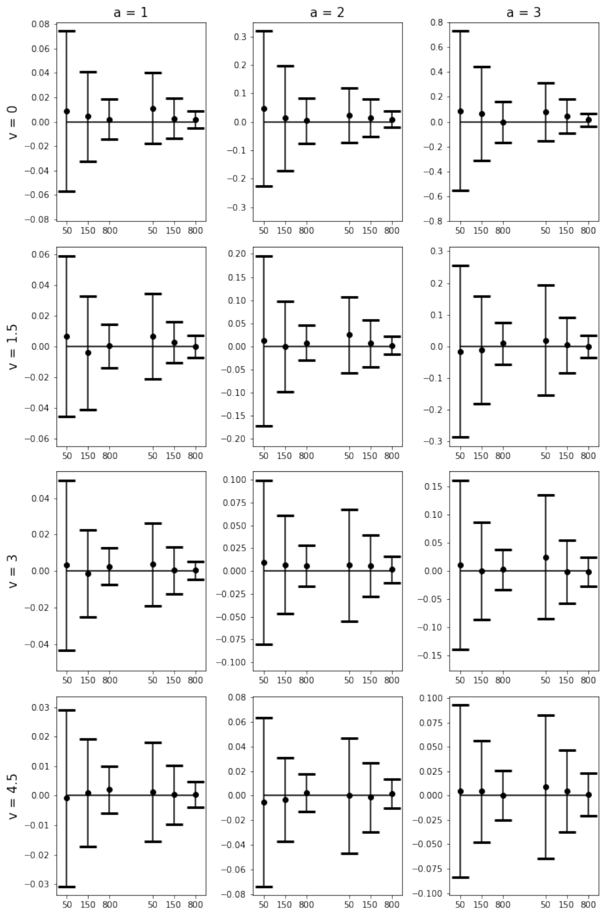

Figure 4: Parameter recovery results for non-decision time T_{er}. True drift length parameter for each row is 0, 1.5, 3, and 4.5 from top to bottom. True criteria value parameter for each column is 1, 2, and 3 from left to right. In each figure, the three bars on left, show the results for EZ method in 50, 150, 800 trial number levels, and the bars in right are for the MLE method. The middle point is the mean of 100 recovered parameter values for T_{er} and the error bars show the one standard deviation above and below the mean value. Horizontal lines show the true value for parameter T_{er}.

on empirical data, suggests that the response time for guess and non-guess choices are nearly equal. But this should be investigated further for different tasks.

Conclusion

Researchers traditionally base their inference on summary statistics which summarize denoised information of data. Because they are easy to compute, a stable summary of variable data, but they lack meaningful information about distinct underlying components, since the effect of these components is mixed in them. For example important results in visual working memory research come from investigating variance of accuracy (e.g. (Zhang & Luck, 2008)) but as discussed in (Smith et al., 2020), this quantity depends on the multiplication of criteria value and drift length, so any change in variance of accuracy should be traced to detect the change in each parameter. The method of EZ enables this separation of parameter values from traditionally favored summary statistics by simple formulas that can be computed easily and needs neither theoretical knowledge of model fitting nor much programming skills. Here, we formulated the EZ method and showed that, despite being easy and fast to calculate, its performance in recovering true parameters is acceptable.

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