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SHEAR STRENGTH
OF REINFORCED CONCRETE BEAMS

A Report of an Investigation

by

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to the

REINFORCED CONCRETE RESEARCH COUNCIL
BUREAU OF YARDS AND DOCKS, DEPARTMENT OF THE NAVY
OFFICE OF CHIEF OF ENGINEERS, DEPARTMENT OF THE ARMY
ENGINEERING DIVISION, DEPARTMENT OF THE AIR FORCE

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I. INTRODUCTION

1. General Remarks

The problem of determining the shearing strength of reinforced concrete beams has received a great deal of attention in the technical literature. A large number of laboratory investigations have been reported both in the United States and abroad, and empirical methods have been proposed for predicting the shearing strength of beams without and with web reinforcement[(1-13)*]. However, the complexity of the problem is so great that as yet no adequate analytical solution of the problem has been developed.

While the basic variables governing the shearing strength of reinforced concrete beams were correctly appraised by Talbot in 1909[(1)], the general nature of the mechanism of failure in all its various aspects has emerged only recently. This mechanism may be described as follows.

In beams wherein shear effects are significant, diagonal cracks are formed due to "diagonal tension" resulting from a combination of shearing and flexural tension stresses. Following formation of these "diagonal tension" cracks a redistribution of stresses takes place leading to ultimate failure. This redistribution of stresses results in the following:

(a) Increase in shearing and compressive stress in the compression zone of the beam above the crack.

(b) Increase in tension stress in the longitudinal reinforcement at the crack.

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* Numbers in parentheses indicate references listed in Section V of this report.

(c) Development of transverse shear and local bending in the longitudinal reinforcement at the crack due to the resistance of this reinforcement to transverse displacement.

(d) Development of tension, together with some shear and bending, in the web reinforcement at the crack due to the resistance of this reinforcement to relative displacement.

The degree of importance of the various stresses noted above depends on the geometry of the beam, the nature of loading, amount and distribution of the reinforcement, and on the mechanical properties of concrete and reinforcement. In some beams without web reinforcement, the primary cause of failure is the splitting along the longitudinal reinforcement in the tension zone caused partly by transverse shear in the reinforcement; in other beams without web reinforcement the primary cause of failure is crushing in the compression zone resulting from the combined state of shear-compression in the concrete. In some beams with web reinforcement the failure is due to initial yielding of the web and/or longitudinal reinforcement, which leads to relative rotation of beam segments adjacent to a diagonal crack about some point in the compression zone. This rotation may be characterized as "shear hinge" action. In some cases failure is due to crushing in the compression zone resulting from a critical state of combined stress, without significant relative rotation of the segments.

While these principal characteristics of the failure mechanism are generally recognized, no general analytical method for the determination of the various forces causing failure has been formulated, and most of the special methods rely on numerous simplifying assumptions.

In the absence of analytical solutions, design criteria must be formulated from empirical data with "adequately conservative premises" as bases for such criteria. Views as to what constitutes "adequately conservative premises" vary widely. For example, in the past most European specifications required that the web reinforcement be designed to carry the total shear thus disallowing any shear capacity of the concrete compression zone. On the other hand U.S. codes traditionally have allowed a portion of the total shear to be carried by the concrete, as empirical data seemed to warrant such an allowance.

Similarly two points of view have been expressed in the technical literature with regard to the shear strength criterion for beams without web reinforcement. One states that the load corresponding to the formation of a "critical diagonal tension crack" should be considered as the limit of useful capacity of the beam, even though in some cases the beam may be capable of carrying additional load prior to failure. The other point of view contends that the state of stress in the uncracked compression zone is the proper criterion for determining the shear capacity. The ASCE-ACI Joint Committee on Shear and Diagonal Tension recommended adoption of the former criterion.

Another difference in opinion found in the technical literature deals with the definition of the shear capacity of a reinforced concrete beam with web reinforcement. As the mechanism of failure of a beam with web reinforcement differs significantly from that of a beam without web reinforcement, the usual assumption of superposition of the concrete shear capacity (determined for a beam without web reinforcement) and the web reinforcement capacity calculated on the basis of a horizontal projection of an idealized diagonal crack is not considered rigorously valid. Yet, a desire for a simple criterion recommended this procedure in the past as it could be justified empirically. A large amount of data on beams with heavy web reinforcement indicated that the simple superposition would result in an "adequately conservative" design criterion. However, only scant data on the behavior and strength of beams with normal and light web reinforcement was available prior to 1958.

From a designer's point of view the following questions were raised:

1. For a beam with a given type of loading, geometry, and properties of materials, what is the minimum amount of web reinforcement necessary to increase the shearing strength of the beam to a particular value V greater than its cracking strength V_{cr}?

2. For a beam with a given type of loading, geometry, and properties of materials, what is a minimum amount of web reinforcement necessary to develop the full flexural strength of this beam?

2. Objectives and Scope

The investigation described in this report was carried out to answer partly the questions stated above. The immediate objectives were to observe the general behavior and to determine the cracking load and ultimate strength of a specially designed series of twelve beams. All of these beams were to have shear spans a/d in the range between four and seven, and except for the three control beams without web reinforcement, the beams were to be reinforced by means of vertical stirrups with rf_y values ranging from 50 to 100. To minimize the possibility of flexural failure high-strength longitudinal steel reinforcement was used in all beams.

3. Acknowledgments

The investigation reported here was carried out during the year 1960 at the Engineering Materials Laboratory of the University of California at Berkeley under sponsorship of Reinforced Concrete Research Council, Bureau of Yards and Docks - Department of the Navy, Office of Chief of Engineers - Department of the Army, and Engineering Division - Department of the Air Force.

The task committee for this project appointed by the Reinforced Concrete Research Council was constituted as follows: W. E. Schaem (Chairman), C. A. Willson, D. E. Parsons, E. Hognestad, and E. Cohen. The sponsors' generous support of the investigation and the helpful suggestions of the Task Committee are gratefully acknowledged.

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4. Notation

The letter symbols used in this report are usually defined when they are introduced. They are listed below alphabetically for convenient reference:

a = Shear span = $L/2$ for beam under center point load

A_s = Area of longitudinal tension reinforcement

A_s' = Area of longitudinal compression reinforcement

A_v = Area of web reinforcement

b = Width of beam

d = Effective depth of beam

E_c = Secant modulus of elasticity of concrete

E_s = Modulus of elasticity of steel

f'_c = Compressive strength of 6 x 12 in. concrete cylinder

f'_t = Modulus of rupture of concrete

f_s = Stress in longitudinal tension reinforcement

f_v = Stress in web reinforcement

f_y = Yield point of steel reinforcement

f_u = Ultimate strength of steel reinforcement

h = Over-all depth of beam

K = Constant depending on angle of inclination of web reinforcement; $K = 1$ for vertical stirrups

L = Span length

M = Bending moment at a section

n = Number of stirrups crossing a diagonal crack

p = Tension - steel reinforcement ratio = A_s/bd

p' = Compression-steel reinforcement ratio = A'_s/bd

P_{cr} = Load producing initial diagonal tension crack

P_f = Calculated ultimate load as governed by flexure

P_v = Calculated ultimate load as governed by shear

P_u = Ultimate test load

q = Longitudinal reinforcement index = $(p - p') f_y/f'_c$

r = Web reinforcement ratio = A_v/bs

s = Longitudinal spacing of web reinforcement

v_c = Ultimate shearing stress for beams without web reinforcement

v_u = Ultimate shearing stress for beams with web reinforcement

V = Total shear at a section

V_s = Total shear taken by web reinforcement

$\alpha = v_c/\sqrt{f_c'}$

$\beta = f_v/f_y$

$\triangle$ = Midspan deflection

λ = Ratio of the length of the horizontal projection of a diagonal crack to the effective depth

II. EXPERIMENTAL PROGRAM

1. Description of Test Beams

In designing the test beams the following criteria were considered:

a. Nominal rf_y values for web reinforcement were to be 0, 50, 75, and 100.

b. Nominal a/d ratios were to be 4, 5, and 7.

c. Calculated ultimate loads were to be governed by shear rather than flexure.

d. Bond or anchorage failures were to be prevented.

e. The effective depth of all beams was to be the same.

f. The required rf_y value was to be obtained mainly by varying the width of the specimen.

g. The spacing of the stirrups was to be no greater than half the effective depth.

h. Main londitudinal reinforcement in all cases was to be made up of the same size high strength steel bars. The number of bars was to be varied to achieve the desired steel percentage.

A number of different types of cross-sections and reinforcement arrangements were considered in an attempt to satisfy the above criteria.

Cross-sectional properties for each of the 12 beams finally selected and tested to failure are given in Fig. 1 and beam elevations are shown in Fig. 2. All beams were of rectangular cross-section and had the same nominal over-all depth of 21 3/4 in. Main longitudinal reinforcement consisted of from two to six No. 9 high strength steel deformed bars placed in the bottom of the beams at two or three levels. The nominal effective depth to the centroid of this reinforcement was 18 in. in all cases. Actual beam dimensions obtained by measurements prior to each test are given in Table 9. All stirrups were made from No. 2 intermediate grade steel deformed bars bent, lapped, and welded to form box-type stirrups. For beams with stirrups two No. 4 longitudinal reinforcing bars of intermediate grade steel were placed at the top of the beam to facilitate the spacing of stirrups and acted as compressive steel. Percentages of steel reinforcement and stirrup spacing are given in Fig. 2 and in Table 9.

Three beam widths - 6, 9, and 12 in. and three simple span lengths - 12, 15, and 21 ft. were used to obtain the desired variations in a/d ratios and rf_y values. All beams were subjected to a single center-point load at midspan. The test beams were grouped into four series OA, A, B, and C with each series containing three specimens. The beam designations are summarized below:

Beam Width	Span Length 12 ft.	15 ft.	21 ft.	Remarks
12 in.	OA-1	OA-2	OA-3	without stirrups
12 in.	A-1	A-2	A-3	with stirrups
9 in.	B-1	B-2	B-3	with stirrups
6 in.	C-1	C-2	C-3	with stirrups

Nominal strengths of the concrete used in the 12, 15 and 21 ft. span beams were 3500, 3500, and 5000 psi respectively.

To prevent bond failures due to possible insufficient anchorage after the formation of diagonal tension cracks, "Howlett" grip anchor nuts were attached to the No. 9 longitudinal bars which protruded from the ends of the specimens about 6 inches. 1 3/8 in. thick steel plates were used at the ends of the beams to provide bearing for these nuts. Details of the bar anchorage and the "Howlett" grip anchor nuts are shown in Fig. 3.

2. Fabrication

All reinforcing steel was thoroughly cleaned before assembly into a reinforcing cage. The reinforcing cages were assembled prior to placement into the forms. The steel assembly was securely held in the proper location in the forms by means of specially fabricated chairs which were spaced 2 ft. apart throughout the length of the specimen. Lifting lugs were also provided for transporting the finished specimen.

The beams were cast in wooden forms made of plywood with a plastic coating to give a smooth and impervious surface. The forms were designed so that they could be adjusted to the desired width and length of each test specimen.

The concrete was mixed in 6 cu. ft. capacity horizontal, non-tilting drum-type mixer. Each batch averaged about 5 1/4 cu. ft., while the total number of batches required for a single beam together with control specimens varied between 3 and 9. Aggregates were blended and moisture contents were determined the day prior to casting. The dry materials were first blended in the mixer for one minute, then the water was added and the entire contents mixed for three additional minutes. The concrete was transported to the forms in buggies and placed into the forms in two layers. Each layer was vibrated internally with a high frequency vibrator (8000 to 10,000 cycles per second).

Forms were stripped 4 days after casting. All specimens were cured moist for 7 days using wet burlap and then left air dry until testing at the age of 13 days.

3. Materials and Control Specimens

Concrete mixes were designed by the trial batch method to achieve a 3500 psi mix and a 5000 psi mix. Type I Portland cement and locally available Elliot sand and Fair Oaks gravel were used in all of the mixes.

The cement was purchased in a one lot from a single mill run. A chemical analysis of the cement is given in Table 1. As needed the cement was blended in 20 sack batches and stored in steel drums.

Petrographic analyses of the aggregates are given in Table 2 and the results of sieve analyses on the aggregates are given in Table 3. The maximum size of the coarse aggregate was 3/4 in.

The 3500 psi concrete mix, which was used in the 12 and 15 ft. span test beams, had a cement factor of 5.3 sacks per cu. yd. The water-cement ratio was 0.56 by weight or 6.32 gallons per sack. Mix proportions were 1.00:2.96:3.77 by weight. These aggregate weights are based on a saturated surface dry condition. Consistency measured by means of a Kelly-ball average about 3 in. slump-equivalent.

The 5000 psi mix, which was used in the 21 ft. span test beams, had a cement factor of 7.9 sacks per cu. yd. The water-cement ratio was 0.39 by weight or 4.40 gals per sack. Mix proportions were 1.00 : 1.64: 2.57 by weight and consistency averaged about 3 in. slump-equivalent.

Concrete control specimens consisted of from ten to twenty-four 6 x 12 in. cylinders and four 6 x 6 x 20 in. beams for each test specimen. The control specimens were cured in the same manner as the test beams. Values of compressive strength f'_c and secant modulus of elasticity E_c at 1000 psi obtained from the 6 x 12 in. cylinders are given in Tables 4A, 4B, 5A, and 5B. Values of modulus of rupture f'_t obtained by loading the 6 x 6 x 20 in. beams at the third points of an 18 in. span are shown in Table 4C and 5C. Fig. 4A and 4B depict the stress-strain relationships for the concrete of the 3500 and the 5000 psi mixes respectively.

Three reinforcing bar sizes were used in the beams. The bottom tension steel was made up of No. 9 high strength deformed bars having a minimum yield point of 80 ksi. Two No. 4 intermediate grade bars were used as compression steel for each of the beams with stirrups. No. 2 intermediate grade deformed bars were used for the stirrups.

Control specimens for each bar size were tested in tension to determine the yield strength f_y, ultimate strength f_u, modulus of elasticity E_s, and per cent elongation in an 8 in. gage length. These results together with values obtained for deformation spacing and heights, weight per ft., and nominal areas are tabulated in Tables 6, 7, and 8. Typical stress-strain diagrams for each bar size are shown in Fig. 5A and 5B.

4. Method of Loading and Instrumentation

The loading arrangement and instrumentation are shown in Fig. 6. The centerpoint load was applied by means of a 4,000,000 lb. universal testing machine. An 8 in. spherical loading block was utilized at the load point. One end of the beam was supported on a 6 in. spherical bearing block while the other end was supported on a 3 in. diameter roller.

Midspan deflections were obtained by two methods. In the first method a simple dial gage with a least count of 0.001 in., supported by a floor stand and bearing on the bottom of the beam at midspan was used. In the second method a scale graduated in 0.01 in. and a mirror were glued to the beam on each face at midspan. A piano wire was then stretched between the support points on each face to obtain deflection readings.

Changes in the over-all depth of the beam due to diagonal cracking were measured by means of specially designed yoke extensometers. These measurements were taken at six separate stations on each beam. The yoke extensometers consisted of two 1/4 x 1 1/2 x 16 in. steel bars clamped to the beam, one across the top and one across the bottom.

These two bars were connected vertically on each side of the beam by means of a light steel chain and a dial gage. Relative movements between the top and bottom surfaces of the beam were registered on the dial gages which read to the nearest 0.0001 in. Details of the extensometers are shown in Fig. 7.

To facilitate the recording of cracks and the visual observation of the beam behavior during testing, the entire beam was first white-washed and a ruled grid was then marked on the two sides of the beam. For beams with stirrups vertical grid lines were placed at stirrup locations so that during testing the number of stirrups being crossed by a particular crack could immediately be discerned.

5. Test Procedure

Twelve days after casting, the beam to be tested was placed in position under the testing machine after which it was white-washed and the yoke-extensometers and deflection gages were installed. All beams were tested under centerpoint load at an age of 13 days.

The beams were first loaded to about 30% of ultimate in two or three increments and then the load was removed. The load was re-applied in 10 kip increments to a point near failure and then in 5 kip increments until failure occurred.

Deflection and yoke-extensometer readings were taken at the beginning and end of each load increment. Cracks were plotted at the end of each load increment directly on the beam and also on specially prepared data sheets. After failure a careful visual inspection of the beam was made and several photographs were taken. Total testing time for a single beam varied between 1 1/2 and 3 hours.

III. EXPERIMENTAL RESULTS AND ANALYSIS OF DATA

1. General Behavior

Beam behavior in general agreed with that described by numerous other investigators[6, 12]. Typical initial flexural cracks appeared first, followed by the appearance of diagonal tension cracks, usually in the middle third of the over-all beam depth and at various sections along the span. These diagonal cracks extended both upwards and downwards with further increase in load.

Three general modes of failure were observed in this series of tests. These may be differentiated as diagonal tension (D-T) failures, shear-compression (V-C) failures, and flexure-compression (F-C) failures, as defined below. Diagonal tension failures were observed in all the beams without web reinforcement; shear-compression failures were observed in intermediate span beams with web reinforcement; flexure-compression failures were observed in long span beams with adequate web reinforcement.

The general behavior of the various test specimens may be interpreted through a study of the crack patterns Fig. 8A to 8L, the load deflection curves Fig. 9, and the yoke data Fig. 10A to 10L.

Diagonal tension failures. This type of failure occurred in beams OA-1, OA-2, OA-3 which had no web reinforcement. These beams failed shortly after the formation of the "critical diagonal tension crack." The failures occurred as a result of longitudinal splitting in the compression zone near the load point, and also by horizontal splitting along the tensile reinforcement near the end of the beam,

Fig. 8-A, B. C. Failures were sudden; the critical cracks formed at a load of approximately 80 per cent of the ultimate load. Although the beams carried some additional load after the formation of the critical crack, the deterioration was rapid as evidenced principally by opening of the crack, Fig. 10-A, B, C.

Shear compression failures. This type of failure occurred in beams A-1, A-2, B-1, B-2, C-1, and C-2 which had web reinforcement and intermediate span lengths. The shear span-to-depth ratios for these beams had nominal values of either 4 or 5. Failure took place at loads substantially greater than the load at which the initial diagonal tension crack occurred. The diagonal tension cracks formed at approximately 60 per cent of the ultimate load. Additional load caused further diagonal cracking but caused no visible signs of distress. Failures developed without extensive propagation of flexural cracks in the center portion of the span indicating that the mechanism of failure was that of shear-compression, Figs. 8-D, E, G, H, I, K. Final failures occurred by splitting in the compression zone but without splitting along the tension reinforcement which was characteristic of beams without web reinforcement. One observation during the tests of the beams differs somewhat from other investigations. It was noted that the diagonal tension cracks often stopped at the level of the tension reinforcement and did not extend to the bottom surface of the beam prior to failure, Figs. 10-D, E, G, H, I, K. It is believed that this phenomenon can be explained by the high values of pf_y/f'_c, the multi-layered arrangement of the reinforcement, and the effectiveness of the longitudinal reinforcement (if stressed below yield point) to arrest the propagation of diagonal tension cracks.

Flexure-compression failures. This type of failure occurred in beams A-3, B-3, C-3 which had web reinforcement and the greatest span lengths. The shear span ratio for these beams had a nominal value of 7. The beams failed by crushing of the compression zone near midspan at the section of maximum moment. Initial flexural cracks appeared at loads approximately 15 percent of the ultimate load and diagonal tension cracks at about 50 per cent of the ultimate load. However, the diagonal tension cracks never developed into major critical cracks while flexural cracks continued to extend upward until a sudden compression failure occurred such as is typical in over-reinforced concrete beams, Figs. 8-F, J, L and Figs. 10-F, J. L.

2. Load-deflection Relationships

Load-deflection relationships for the beams tested are shown in Figure 9. Each group of curves shows the load deflection relationship for a series of beams of the same span: the upper group (Series-1) includes beams having a 12 ft. span, the middle group (Series-2) includes those with a 15 ft span, and the lower group (Series-3) includes those with a 21 ft span.

Deflection values plotted in this figure are the average values of those recorded at the beginning and the end of the time interval of a particular load application. These values represent the average of readings on the two faces. Only the deflections recorded during the final cycle of loading from zero to ultimate are shown. Earlier cycles of loading resulted in deflections similar to those shown in Fig. 9.

Comparison of the deflections of each beam in the OA series with those of beams in the A series indicates the effect of web reinforcement on the deflections. Comparison of the slopes of the load-deflection curves for the beams without web reinforcement, OA-1, OA-2, and OA-3, indicates that the stiffnesses of companion beams with web reinforcement, A-1, A-2, and A-3, are approximately the same, and thus are not influenced appreciably by the addition of web reinforcement. However, beams with web reinforcement fail at higher loads and are capable of developing substantially higher deflections, thus exhibiting greater "ductility".

3. <u>Yoke Extensometer Data</u>

Vertical displacements of the bottom of the beam with respect to the top surface at selected sections for each of the specimens are shown in Figs. 10-A through 10-L. These sections were selected to correspond to stirrup locations for the beams with web reinforcement. Average values of the displacements observed on the north and south faces are plotted in the figures. The values observed on opposite faces did not vary significantly from the average. The maximum displacement shown on the figures represents the largest value recorded in the test but does not always correspond to the displacement at the ultimate load. Because of danger of impending failure at loads approaching ultimate, it was not always possible for the observers to read the dial gages at the ultimate load.

As seen from the figures, the changes in depth are hardly measurable, with the sensitivity of the yoke extensometers used in this study, until diagonal cracking begins to develop. The diagonal cracking load, Table 9, in general corresponds to the point when the curve of the vertical displacement versus load, Fig. 10-A to 10-L, just deviates from the vertical. With the development of diagonal cracks, these displacements increase rapidly in beams with web reinforcement failing in shear. In beams without web reinforcement or in beams with web reinforcement which fail in flexure, failure occurs before the diagonal cracks fully develop and the relative vertical displacements across a diagonal crack do not reach significant magnitudes.

In beams without web reinforcement, values of vertical displacements are not related to stirrup elongations as no stirrups exist in the beam. For beams with web reinforcement, these displacements are related to the total stirrup elongation. As the strain distribution along the length of the stirrup is not known, a quantiative measure of stirrup strain cannot be obtained. It appears from Figs. 10-J, K, L, that for beams failing in flexure the stirrups have not reached the yield point stress anywhere. For beams with web reinforcement failing in shear, Figs. 10-D, E, F, G, H, I, it appears that local initial yielding may have developed. This is particularly apparent in beams with low values of rf_y, Figs. 10-D, E, F.

4. Shearing Strength Criteria

Before proceeding with the evaluation of test results, some of the implications and limitations of shearing strength criteria are examined below. The ultimate shearing strength of a reinforced concrete beam may be determined using the following equation proposed by the Joint ASCE-ACI Committee on Shear and Diagonal Tension[13]:

$$v_u = \frac{V_u}{bd} = v_c + K\,rf_y = 1.9\sqrt{f'_c} + 2500\,\frac{pVd}{M} + K\,rf_y \qquad (1)$$

where M is the moment at the section. In the above equation M should be taken not greater than $(M_{max} - Vd)$.

As indicated in the Shear and Diagonal Tension Committee report, the contribution of the term $2500\,\frac{pVd}{M}$ may be expressed in terms of steel stress f_s in the main reinforcement. The value of v_c then becomes:

$$v_c = 1.9\sqrt{f'_c}\left[\frac{f_s}{f_s - 2850}\right] = \alpha\sqrt{f'_c} \leq 3.5\,f'_c \qquad (2)$$

For ultimate strength design conditions f_s may reach a value $f_s = f_y$ at the section of maximum moment, and would decrease to zero at the point of inflection in the beam (assuming no axial load). The magnitude of the coefficient α is shown below for various values of f_s.

Values of f_s, ksi	60	50	40	30	20	10	5.2 and less
Values of $\alpha = \frac{v_c}{\sqrt{f'_c}}$	1.98	2.02	2.04	2.09	2.22	2.66	3.5

It is seen that for balanced design when f_s at ultimate approaches a value of f_y, say between 60 and 30 ksi, the value of α deviates only slightly from 2.

The magnitude of v_c is increased substantially at sections where f_s is small, such as at sections near the inflection point, and when f_s is equal to or less than 5.2 ksi the recommended value reaches $v_c = 3.5\sqrt{f'_c}$. For example, at a point of inflection in a beam with $f'_c = 3600$ psi, $v_c = 3.5 \sqrt{3600} = 210$ psi, and therefore no web reinforcement would be required for ultimate shearing stresses up to 210 psi.

The possibility of developing such high shearing stresses in the region of a point of inflection without web reinforcement has yet to be demonstrated experimentally for beams with relatively large values of M/Vd.

Data used in the derivation of the expression $v_c = 1.9 \sqrt{f'_c} + 2500\, p \frac{Vd}{M}$ are based on numerous test results and are shown in Fig. 11. Beams with proportions normally encountered in structural elements have values of p Vd/M less than 0.01, and for all values of f'_c greater than 2500 psi, the values of $1000\, pVd/M \sqrt{f'_c}$ (abscissa in Fig. 11) would fall between 0 and 0.2. For this group of test data, taken alone, it is difficult to justify the proposed straight line equation. Indeed a value of $2 \sqrt{f'_c}$ appears to be just as valid, slightly more conservative for the range, and simpler to use as a design criterion. Furthermore, it does not lead to the apparently excessive values of v_c which would be obtained by Eq. (1) for sections without web reinforcement in the region of the inflection point. Therefore, for the design of reinforced concrete beams of conventional proportions, when the ratio pVd/M is less than 0.01, the use of an ultimate shearing stress $v_c = 2 \sqrt{f'_c}$ seems to be a satisfactory approximation. Thus, a modified criterion for the ultimate shearing strength of reinforced concrete beams without web reinforcements, having pVd/M values less than 0.01, may be stated as follows:

$$v_c = \frac{V_c}{bd} = 2 \sqrt{f'_c} \qquad (3\text{-a})$$

With this modified value of v_c, the value of ultimate shearing strength, v_u, becomes:

$$v_u = \frac{V_u}{bd} = 2\sqrt{f'_c} + Krf_y \qquad (3\text{-b})$$

In Eq. (1) the ultimate shearing strength of a reinforced concrete beam with stirrup web reinforcement is obtained by directly adding the ultimate shearing strength v_c for a beam without web reinforcement to the contribution of the web reinforcement indicated by the term Krf_y. The validity of such a superposition cannot be justified analytically but may be acceptable as an expedient measure until such a time when a more rational solution become available. A critical examination of the assumptions implicit in the superposition of v_c and Krf_y is useful in defining the limitations of Eq. (1).

For a general case the contribution V_s of the web reinforcement to the shearing capacity is a function of the number of stirrups n crossing the diagonal crack and the force being carried by each of these stirrups. The total contribution may be expressed as:

$$V_s = \sum_n f_{vi} A_{vi} \qquad (4)$$

where f_{vi} and A_{vi} are the actual tensile stress and cross-sectional area of the i-th stirrup. If all of the stirrups have the same area A_v and if the ratio of f_{vi}/f_y for the stirrup is denoted by β_i then:

$$V_s = \sum_n f_{vi}A_{vi} = f_y A_v \sum_n \beta_i \qquad (5)$$

If λd represents the horizontal projection of the diagonal

crack, then the number of stirrups crossing the crack is $n = \frac{\lambda d}{s}$.
Thus it is seen that in order to define the contribution of the web reinforcement rationally it is necessary to know the values of the horizontal projection λd of the diagonal crack and the variable tension stress factors β_i for the individual stirrups.

The contribution V_s of the vertical stirrup reinforcement as proposed in Eq. (1) may be written as follows:

$$V_s = r\, f_y\, bd = A_v f_y\, (d/s) \tag{6}$$

It is seen from Eq. (6) that both λ and β_i have been taken to be constants equal to unity. In other words it is assumed that each stirrup is stressed to the yield point, and the number of stirrups so stressed is the number crossing a diagonal crack having a horizontal projection equal to the effective depth d.

In Eq. (1) the value of v_c is taken as the cracking strength of a beam without web reinforcement. It is important to note that for beams with web reinforcement the physical significance of v_c is quite different from the cracking strength, as it represents the shearing strength contribution of both the concrete in the compression zone and the longitudinal steel reinforcing bars. Usually the shear contribution of longitudinal steel, so-called dowel action, has been neglected. Actually, however, it is believed that in beams with stirrups the contribution of longitudinal steel reinforcement may be an important one, particularly in beams where tension reinforcement is arranged in more than one layer.

5. Evaluation of Test Results

Table 9 presents a summary of the test program and Table 10 presents a summary of test results, including values of the diagonal tension cracking load P_{cr}, ultimate load P_u, maximum deflection Δ_{max}, and failure mode for each of the beams tested. Calculated flexural capacity P_f, cracking load P_{cr} and shear capacity P_v are also included in Table 10.

The value of P_f for each beam was determined by trial and error using the Hognestad-McHenry-Hanson stress block with an assumed ultimate compressive unit strain in the concrete of 0.003, and using experimentally determined stress-strain characteristics of the top and bottom longitudinal steel reinforcement.

Two values of P_{cr} and of P_v were calculated and are shown in Table 10. For each beam, values of P_v were first determined using the ultimate strength v_u defined by Eq. (1), as proposed by the Joint ASCE-ACI Committee on Shear and Diagonal Tension, and secondly using the ultimate strength v_u defined by Eq. (3-b) as proposed in this report. Two values of P_{cr} were calculated also: one, based on v_c defined by Eq. (1) and the other based on $v_c = 2\sqrt{f'_c}$, as in Eq. (3-a).

Comparison of test data with calculated values indicates the following.

1. The observed diagonal tension cracking load was in all cases in excess of the calculated values.

2. All beams developed ultimate strengths greater than the calculated values. Three beams, A-3, B-3 and C-3, failing in flexure developed strengths in excess of both the calculated flexural and shearing capacities. The remaining beams failing in shear developed strengths from approximately 30 to 50% greater than the calculated

values of shearing strength. In all but two cases, beams with web reinforcement failing in shear developed ultimate loads in excess of calculated flexural capacities, although substantial distress due to diagonal tension cracking was evident at loads below the calculated flexural capacities.

3. The apparent high reserve strength is believed to be partly due to the shear carried by "dowel action" of the longitudinal reinforcement which is neglected in calculations, and partly due to a greater effectiveness of web reinforcement than that assumed in calculations.

4. It is believed that the shear rigidity of the multilayered tensile reinforcement contributes a significant portion of the calculated reserve shear strength due to the so-called "dowel action". Experimental data are not sufficient to permit a meaningful evaluation and further experimental studies of such variables as reduction of shear rigidity of the tension reinforcement zone effected by placing steel bars in one layer and by cutting off excess reinforcement in the region of low moment is highly desirable.

5. The effectiveness of web reinforcement may be estimated by comparing the shearing strengths of beams OA-1 and A-1, and of beams OA-2 and A-2. Assuming that the additional shearing strength of beams A-1 and A-2 is due entirely to web reinforcement, it is seen that its contribution is about 1/3 greater than indicated by the term $(Krf_y)bd$. Data on two pairs of specimens available in this series are inadequate for a meaningful evaluation and therefore, further experimental study of the effective contribution of web reinforcement to shearing strength of reinforced concrete beams is desirable.

6. In comparing test results with the proposed design equations it is interesting to note the relative contributions of the various terms in Eq. (1) to the value of P_v for each beam, and to consider the reserve capacity in terms of calculated shearing strength. The magnitudes of these relative contributions are shown in Fig. 12.

It is seen that for beams failing in shear the basic term $1.9\sqrt{f'_c}$ contributes from 48.5 to 92.4 per cent of the calculated strength, the factor 2500 pVd/M contributes from 5.8 to 12.3 per cent of the calculated strength, and the rf_y term contributes from 0 (no web reinforcement) to 41.4 per cent ($rf_y = 100$) of the calculated strength. For all beams failing in shear, the reserve strength based on Eq. (1) is found to be from 27 to 49 per cent of calculated strength, with an average of 40 per cent and similar values based on Eq. (3-b) are in the range from 31 per cent to 60 per cent with an average of 47 per cent.

IV. CONCLUSIONS

The limited scope of the investigation reported here substantially restricts the conclusions which can be rigorously supported by the data. Nevertheless, several important points have been demonstrated in the report and are summarized below:

1. Small amounts of stirrup reinforcement, with rf_y values as low as 50, effectively increase the shearing strength of reinforced concrete beams, provided the stirrups are spaced approximately d/2 apart or closer. Investigation of larger stirrup spacing was not included in this study.

2. The ultimate shearing strength of reinforced concrete beams with vertical stirrups may be predicted by either of the following equations:

$$\frac{V_u}{bd} = 1.9 \sqrt{f'_c} + 2500 \, (pVd/M) + rf_y$$

or

$$\frac{V_u}{bd} = 2 \sqrt{f'_c} + rf_y$$

The first of the above equations has been proposed by the ACI-ASCE Joint Committee on Shear and Diagonal Tension. The second equation is a modification of the first proposed by the authors for normal proportions of concrete members, subjected to flexure and shear without axial forces.

3. Multilayered arrangements of tensile steel reinforcement appear to increase shear resistance of reinforced concrete beams.

4. Web reinforcement effectively prevents sudden failures due to shear, and permits development of substantial deflections and almost full flexural capacity prior to ultimate collapse.

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TABLE 1 CHEMICAL ANALYSIS OF CEMENT[1]

Chemical	Percent
SiO_2	21.3
Fe_2O_3	2.9
Al_2O_3	5.6
CaO	63.2
MgO	2.4
SO_3	2.7
Ignition loss	0.9
Insoluble	0.1
Alkalis plus undetermined	0.9

1 Type I, Portland cement, mill analysis supplied by Pacific Cement and Aggregate Company, Davenport, California.

TABLE 2 PETROGRAPHIC ANALYSES OF AGGREGATES

Elliot Sand[1]		Fair Oaks Gravel[2]	
Mineral	Percent	Mineral	Percent
Graywacke	64	Basic igneous rocks	24
Metaigneous rocks	16	Basic metaigneous rocks	42
Gabbro	2	Andesite	14
Jasper	16	Sandstone	11
Vein quartz	2	Quartzite	5
		Slate	2
		Vein quartz, chert, schist	1

1. Data supplied by Pacific Cement and Aggregate Company, Pleasanton, California.
2. Ref. "Test Data Concrete Aggregates in Continental United States", Corps of Engineers, U.S.A., TM No. 6-370.

TABLE 3 SIEVE ANALYSES OF AGGREGATES

Sieve Size	Percentage Retained on Sieve	
	Elliot Sand	Fair Oaks Gravel
3/4 in.		3.1
1/2 in.		(43.5)
3/8 in.		68.1
No. 4	0.5	99.5
No. 8	14.7	100.0
No. 16	43.0	
No. 30	68.7	
No. 50	89.4	
No. 100	97.8	
Fineness Modulus	3.14	6.71

Average of 4 samples of sand and 4 samples of gravel.

TABLE 4A COMPRESSIVE STRENGTH f_c' OF CONCRETE

3500 psi mix; 6 x 12 in. cylinders

1. SSD parts by weight, C : S : G = 1.00 : 2.96 : 3.77
2. W/C ratio = 0.56 by weight = 6.32 gals/sack
3. Unmarked values indicate test at 13 days; *values indicate test at 14 days
4. All values given are in ksi

Spec No. \ Beam	OA-1	OA-2	A-1	A-2	B-1	B-2	C-1	C-2
1A	3.26	3.15	3.44	3.71*	3.85*	3.33	4.11	3.36
1B	3.22	3.51	3.50	3.43	3.72	3.17	4.18	3.40
1C	3.46*	3.06	--	--	--	3.27	--	3.42
1D	3.27	3.43	--	--	--	3.11	--	3.51
2A	3.26	3.29	3.72*	3.31	3.41	3.51	4.47	3.49
2B	3.18	3.12	3.52	3.72	3.56	3.33	4.31	3.44
2C	3.24	3.14	--	--	--	3.42	--	3.40
2D	3.19	3.36	--	--	--	3.41*	--	--
3A	3.37	3.39	3.67	3.75*	3.98*	3.50	4.46	3.41
3B	3.27	3.60	3.49	3.63	3.66	3.47	4.18	3.42
3C	3.32*	3.35*	--	--	--	3.15	--	--
3D	3.47	3.64	--	--	--	3.48	--	3.53
4A	3.43	3.40	3.44*	3.29	3.52	3.15		--
4B	3.38	3.25	3.61	3.19	3.48	3.32*		3.41
4C	3.30	3.11*	--	--	--	3.33		3.51
4D	3.39	3.15	--	--	--	3.26		3.61
5A	3.37*	3.37	3.22	3.67*	3.47	3.31		
5B	3.24	3.31	3.46	3.60	3.32*	3.40		
5C	3.36	3.32	--	--		3.60		
5D	3.14	3.73	--	--		3.53		
6A	3.21	3.57	3.30	3.46		3.37		
6B	3.12	3.56	3.46*	3.54		3.48		
6C	3.17	3.38	--	--		3.51		
6D	3.13	3.37	--	--		3.30*		
7A	3.26	3.32*		3.52				
7B	3.31	3.70		3.45				
7C	3.15	3.93		--				
7D	2.99	3.82		--				
8A		3.61						
8B		3.62						
8C		3.89						
8D		3.54						
Average	3.27	3.44	3.49	3.52	3.59	3.36	4.29	3.45

TABLE 4B SECANT MODULUS E_c OF CONCRETE

3500 psi mix; 6 x 12 in. cylinders;
E_c at 1000 psi

1. Unmarked values indicate test at 13 days; *values indicate test at 14 days
2. All values given are in ksi x 10^3

Spec No. \ Beam	OA-1	OA-2	A-1	A-2	B-1	B-2	C-1	C-2
1	3.55*	3.15*	4.13*	3.69*	3.84*	3.85*	3.73	4.00*
2	3.00*	3.15*	3.88*	3.73*	3.57*	3.88*	3.80	3.89*
3	3.85*		3.76*	3.60		3.66*	3.57	3.80*
Average	3.47	3.15	3.92	3.67	3.71	3.80	3.70	3.90

TABLE 4C MODULUS OF RUPTURE f_t' OF CONCRETE

3500 psi mix; 6 x 6 x 20 in. beams

1. All beams tested on 18 inch span under third point loading.
2. Unmarked values indicate test at 14 days; *values indicate test at 15 days.
3. All values given are in ksi.

Spec No. \ Beam	OA-1	OA-2	A-1	A-2	B-1	B-2	C-1	C-2
1	.514*	.589	.613	.536	.636	.523	.640	.594
2	.570*	.578	.540	.586	.597	.573	.568	.576
3	.629*	.675	.556	.546	.526	.563	.628	.562
4	.585*	.672	.525	.491	.552	.520	-	.547
Average	.575	.629	.559	.540	.578	.545	.612	.570

TABLE 5A COMPRESSIVE STRENGTH f'_c OF CONCRETE

5000 psi mix; 6 x 12 in. cylinders;

1. SSD parts by weight, C : S : G = 1.00 : 1.64 : 2.57
2. W/C ratio = 0.39 by weight = 4.40 gals/sack
3. For compressive strengths unmarked values indicate test at 13 days; *values indicate test at 14 days.
4. Values of f'_c are in ksi.

Beam / Spec No.	OA-3	A-3	B-3	C-3
1A	5.46	5.30	6.00*	5.24
1B	5.91	5.48	5.85	5.44
2A	6.05	5.01	5.80	5.33
2B	5.41	5.28	5.76	5.35
3A	5.64	4.83	6.25	4.95
3B	5.61	4.87	5.53	4.98
4A	5.11	5.18*	5.31	4.86
4B	5.29	4.82	5.59*	4.82
5A	5.27	4.87	5.38	4.96
5B	5.50	5.09	5.47	4.86
6A	5.19	5.16*	5.31	
6B	5.68	5.22	5.67	
7A	5.33	5.10	5.31	
7B	5.33	4.97	5.40*	
8A	5.24	4.90		
8B	5.05	5.37*		
9A	5.39	4.82		
9B	5.63	5.09		
Average	5.45	5.08	5.62	5.08

TABLE 5B SECANT MODULUS OF ELASTICITY E_c

5000 psi mix; 6 x 12 in. cylinders

E_c at 1000 psi

1. Unmarked values indicate test at 13 days; *values indicate test at 14 days.

2. Values of E_c are in ksi x 10^3.

Spec No. \ Beam	OA-3	A-3	B-3	C-3
1	5.00	5.10*	4.92*	4.53
2	4.61	4.64*	4.65*	4.36
3	4.66	4.50*		4.20
4				
Average	4.76	4.75	4.64	4.36

TABLE 5C MODULUS OF RUPTURE f_t' OF CONCRETE

5000 psi mix; 6 x 6 x 20 in. beams

1. All beams tested at 14 days.
2. All beams tested on 18 in. span under third point loading.
3. Values of f_t' are in ksi.

Spec No. \ Beam	OA-3	A-3	B-3	C-3
1	.606	.627	.552	.591
2	.581	.611	.640	.512
3	.607	.634	.648	.594
4	.604	.644	.603	.538
Average	.600	.629	.611	.559

TABLE 6 PROPERTIES OF NO. 9 HIGH STRENGTH STEEL REINFORCING BARS

Sample	1[a]	2[a]	3[b]	3[b]
Yield strength f_y, ksi	82.0	79.0	80.0	80.2
Ultimate strength f_u, ksi	141.0	136.8	135.6	135.0
Mod. of elasticity E_s, ksi	33.5×10^3	29.7×10^3	28.6×10^3	30.9×10^3
% elongation in 8 in.	14.1	10.0*	13.1	14.4
Weight per lineal ft, lb	3.467	3.471	3.461	3.461
Nominal area, in.^{2}	1.020	1.020	1.017	1.017
Average deformation height, in.	.057	.070	.070	.070
Average deformation spacing, in.	.581	.580	.580	.580

a. Heat No. 6153; Chemical analysis supplied by Inland Steel Co., % by weight: 0.61C; 0.92 Mn; 0.015 P; 0.024 S; 0.32 Si; 0.03 Cu; 0.84 Cr.

b. Heat No. 9314: 0.59C; 0.86 Mn; 0.013P; 0.030 S; 0.29 Si; 0.10 Cu; 0.85 Cr.

* Fracture below test section.

TABLE 7 PROPERTIES OF NO. 4 INTERMEDIATE GRADE STEEL REINFORCING BARS

Sample	1	2	3
Yield strength f_y, ksi	50.0	49.9	50.2
Ultimate strength f_u, ksi	79.2	75.7	80.9
Mod. of elasticity E_s, ksi	31.5×10^3	28.3×10^3	27.8×10^3
% elongation in 8 in.	20.6	21.3	18.2
Weight per lineal ft, lb	.657	.673	.664
Nominal area, in^2	.193	.198	.195
Average deformation height, in.	.035	.036	.035
Average deformation spacing, in.	.307	.304	.307

TABLE 8 PROPERTIES OF NO. 2 DEFORMED INTERMEDIATE GRADE STEEL REINFORCING BARS

Sample	1	2	3
Yield strength f_y, ksi	48.7	45.0	47.8
Ultimate strength f_u, ksi	63.6	61.6	61.8
Mod. of elasticity E_s, ksi	28.0×10^3	25.5×10^3	29.0×10^3
% Elongation in 8 in.	16.2*	15.0*	19.0
Weight per lineal ft, lb	.167	.170	.174
Nominal Area, in.^{2}	.049	.050	.051
Average deformation height, in.	.015	.010	.015
Average deformation spacing, in.	.179	.179	.179

*Fracture above test section.

TABLE 9 SUMMARY OF TEST PROGRAM

	Concrete		Beam Dimensions				Ratio	Reinforcement					
Spec. No.	f'_c ksi	f'_t ksi	b in.	h in.	d in.	L ft.	a/d	No. 9 bars [1]	p %	No. 4 bars [2]	p' %	Spacing No. 2 Stirrups	rf_y [3]
OA-1	3.27	.575	12.2	21.9	18.15	12	3.97	4	1.81	0	0	---	0
OA-2	3.44	.629	12.0	22.1	18.35	15	4.90	5	2.27	0	0	---	0
OA-3	5.45	.600	12.1	21.9	18.17	21	6.94	6	2.74	0	0	---	0
A-1	3.49	.559	12.1	22.1	18.35	12	3.92	4	1.80	2	0.180	8 1/4	47.2
A-2	3.52	.540	12.0	22.0	18.27	15	4.93	5	2.28	2	0.182	8 1/4	47.6
A-3	5.08	.629	12.1	22.1	18.35	21	6.91	6	2.73	2	0.182	8 1/4	47.2
B-1	3.59	.578	9.1	21.9	18.15	12	3.95	4	2.43	2	0.243	7 1/2	69.2
B-2	3.36	.545	9.0	22.1	18.33	15	4.91	4	2.43	2	0.243	7 1/2	70.0
B-3	5.62	.611	9.0	21.9	18.13	21	6.95	5	3.06	2	0.245	7 1/2	70.0
C-1	4.29	.612	6.1	22.0	18.25	12	3.95	2	1.80	2	0.361	8 1/4	93.9
C-2	3.45	.570	6.0	22.0	18.28	15	4.93	4	3.66	2	0.366	8 1/4	95.2
C-3	5.08	.559	6.1	21.8	18.06	21	6.98	4	3.63	2	0.363	8 1/4	93.9

Notes: (1) Yield point value f_y = 80.5 ksi for bars in Series -1, and -2, and f_y = 80.1 ksi for bars in Series -3, tension steel.

(2) Yield point value f_y = 50.1 ksi for No. 4 bars, compression steel.

(3) Yield point value f_y = 47.2 ksi for No. 2 bars, stirrups. Nominal area of No. 2, based on weight, equals 0.050 sq. in.

TABLE 10 ANALYSIS OF TEST RESULTS

	Test Values					Calculated Values				Test Value/Calculated Value			
Spec. No.	P_{cr} 1)	P_u 1)	Δ max in.	Fail. Mode	P_f 2)	P_{cr} Eq. 1	P_{cr} Eq. 3	P_v 3) Eq. 1	P_v 3) Eq. 3	P_{cr} Eq. 1	P_{cr} Eq. 3	P_v Eq. 1	P_v Eq. 3
OA-1	60	75	0.26	D-T	114.8	55.0	50.9	55.0	50.9	1.09	1.18	1.36	1.47
OA-2	65	80	0.46	D-T	99.6	55.2	51.5	55.2	51.5	1.18	1.26	1.45	1.55
OA-3	70	85	1.10	D-T	98.1	66.6	64.7	66.6	64.7	1.05	1.08	1.27	1.31
A-1	60	105	0.56	V-C	126.5	56.5	52.3	77.5	73.4	1.06	1.15	1.35	1.43
A-2	55	110	0.90	V-C	108.5	56.1	52.4	77.1	73.3	0.98	1.05	1.43	1.50
A-3	85	105	1.41	F-C	96.0	65.0	63.0	86.0	84.1	1.31	1.35	1.22*	1.25*
B-1	55	100	0.54	V-C	106.5	44.4	39.6	67.1	62.4	1.24	1.39	1.49	1.60
B-2	55	90	0.82	V-C	81.3	41.4	38.3	64.5	61.4	1.33	1.44	1.40	1.46
B-3	60	80	1.39	F-C	78.9	50.5	48.6	73.4	71.5	1.19	1.23	1.09*	1.12*
C-1	45	70	0.70	V-C	67.3	31.2	29.2	52.2	50.4	1.44	1.54	1.34	1.39
C-2	35	73	0.79	V-C	61.0	29.6	25.8	50.5	46.8	1.18	1.36	1.44	1.56
C-3	40	61	1.45	F-C	53.6	33.2	31.4	53.9	52.3	1.20	1.27	1.13*	1.17*

(1) Applied loads - exclusive of weight of specimen.

(2) Critical section at midspan, adjustment made for weight of specimen.

(3) Critical section at midspan, requires no adjustment for weight of specimen.

* Flexural failure.

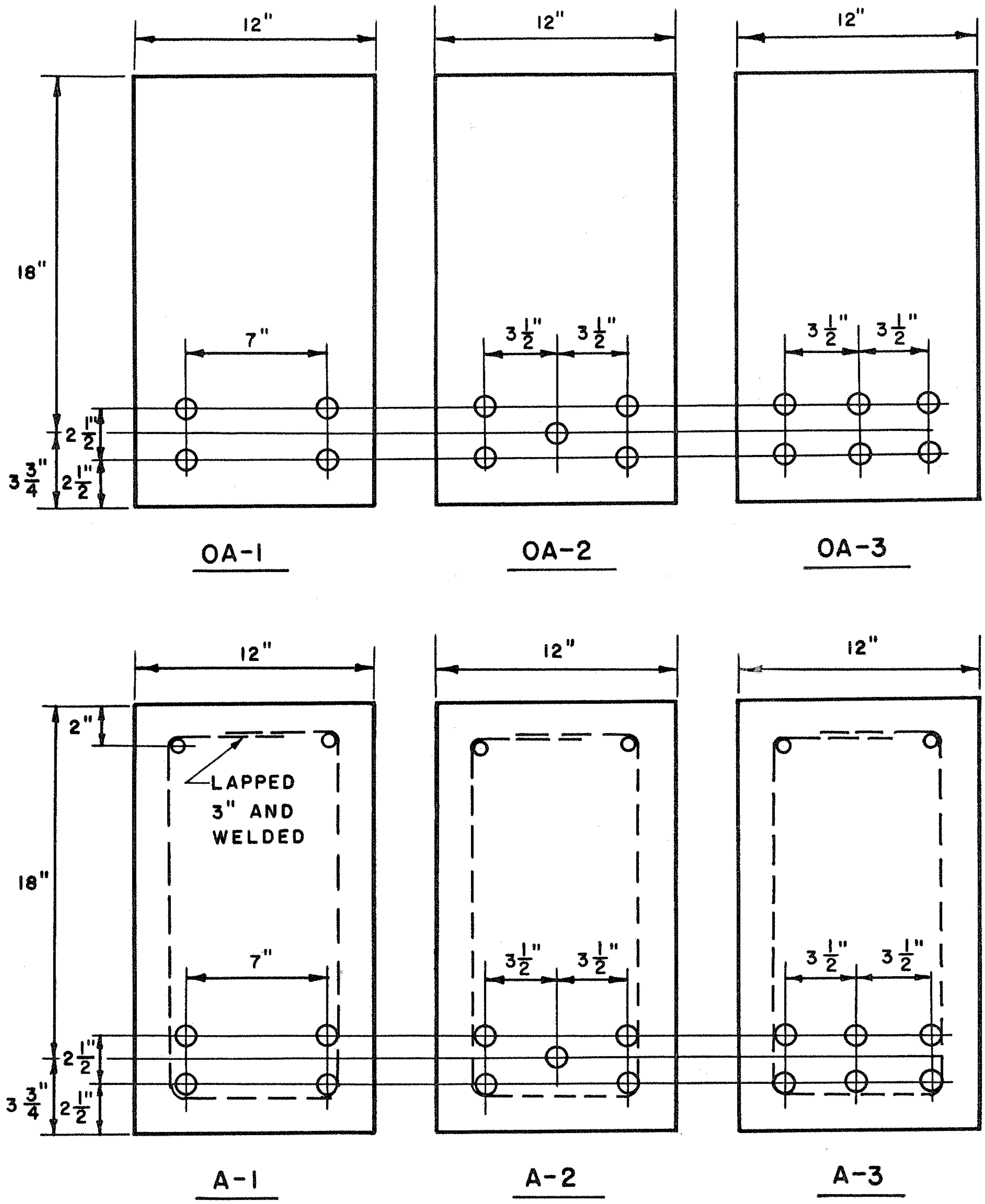


FIG. 1-A SERIES OA AND SERIES A-BEAM CROSS-SECTIONS

1. ALL DIMENSIONS SHOWN ARE NOMINAL; SEE TABLE 9 FOR MEASURED DIMENSIONS.
2. BOTTOM BARS ARE NO. 9, TOP BARS ARE NO. 4, AND STIRRUPS ARE NO. 2.

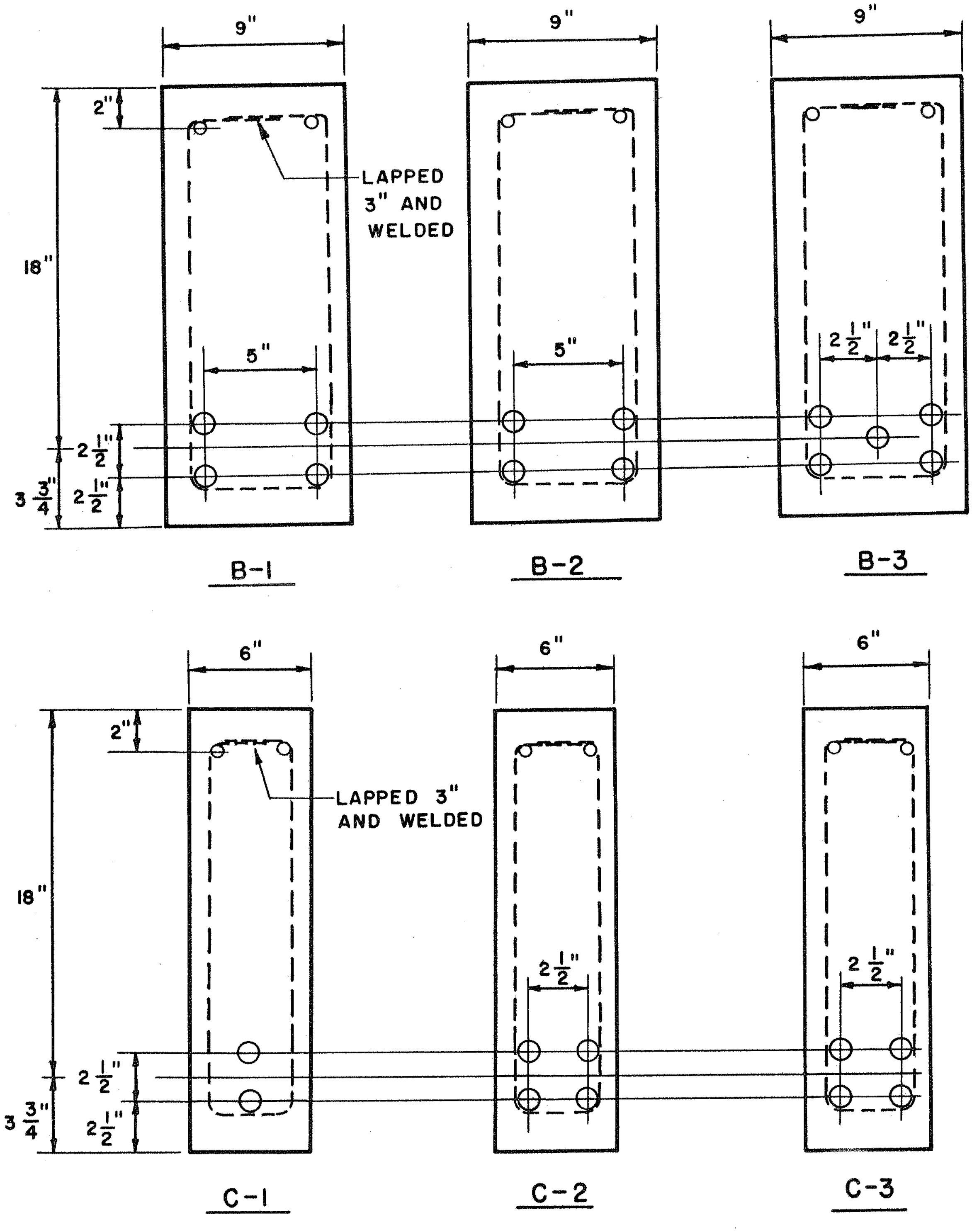


FIG. 1-B SERIES B AND SERIES C-BEAM CROSS-SECTIONS

1. ALL DIMENSIONS SHOWN ARE NOMINAL; SEE TABLE 9 FOR MEASURED DIMENSIONS.

2. BOTTOM BARS ARE NO. 9, TOP BARS ARE NO. 4, AND STIRRUPS ARE NO. 2.

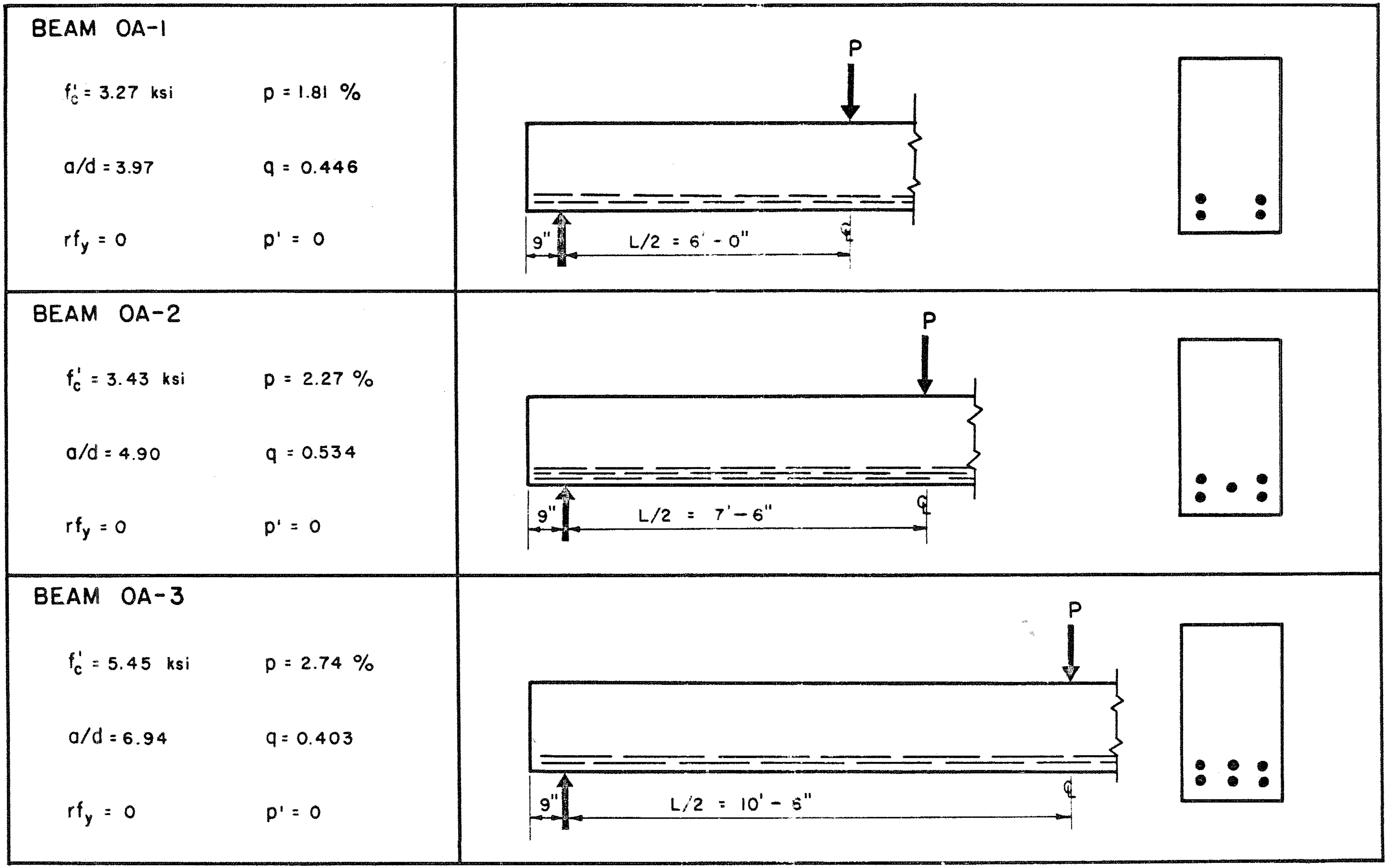


FIG. 2-A SERIES OA - BEAM ELEVATIONS

BEAM A-1		
	f'_c = 3.49 ksi	p = 1.80 %
	a/d = 3.92	q = 0.390
	rf_y = 47.2 psi	p' = 0.180 %
BEAM A-2		
	f'_c = 3.52 ksi	p = 2.28 %
	a/d = 4.93	q = 0.494
	rf_y = 47.6 psi	p' = 0.182 %
BEAM A-3		
	f'_c = 5.07	p = 2.73 %
	a/d = 6.91	q = 0.414
	rf_y = 47.2 psi	p' = 0.182 %

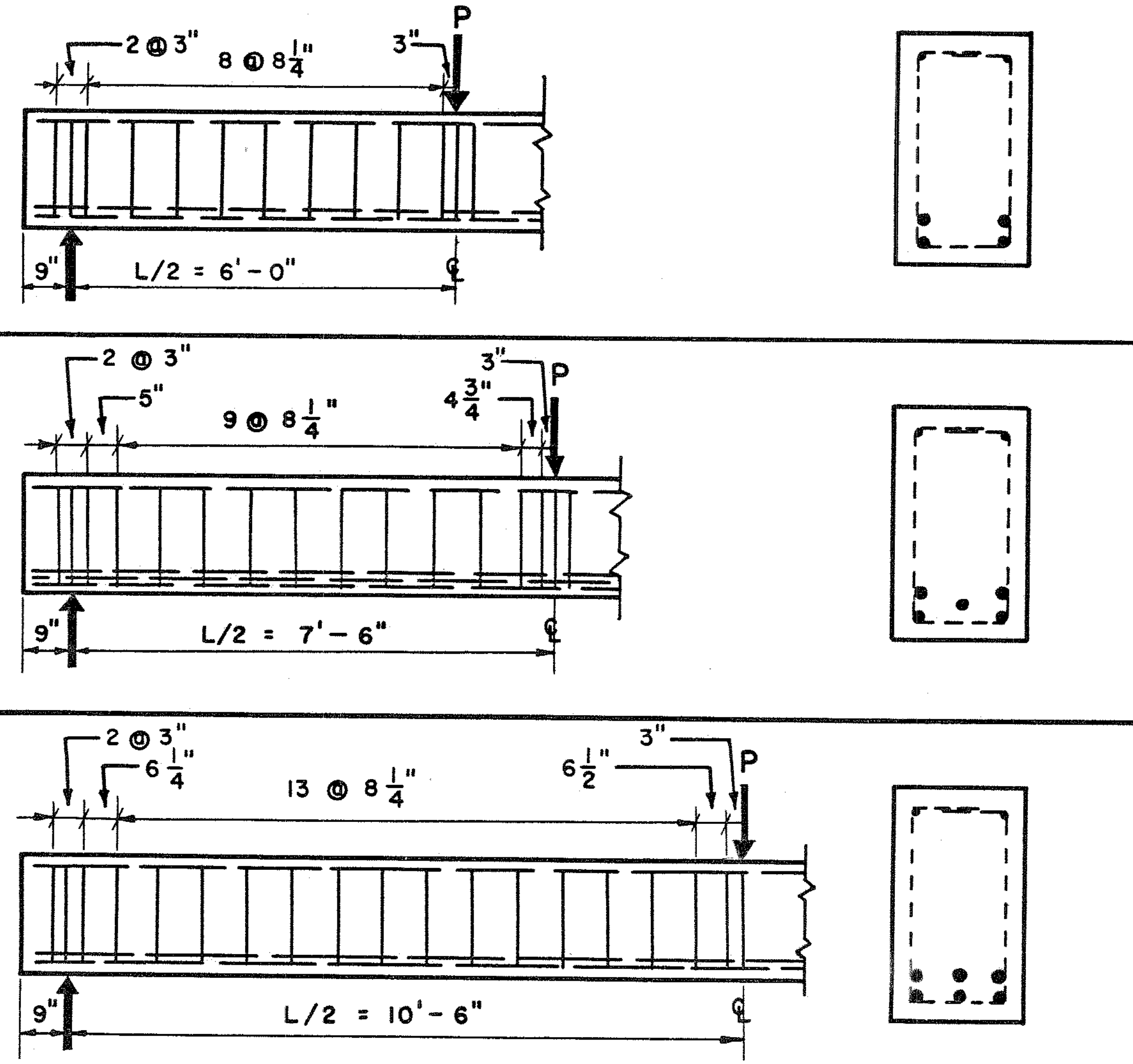


FIG. 2-B SERIES A - BEAM ELEVATIONS

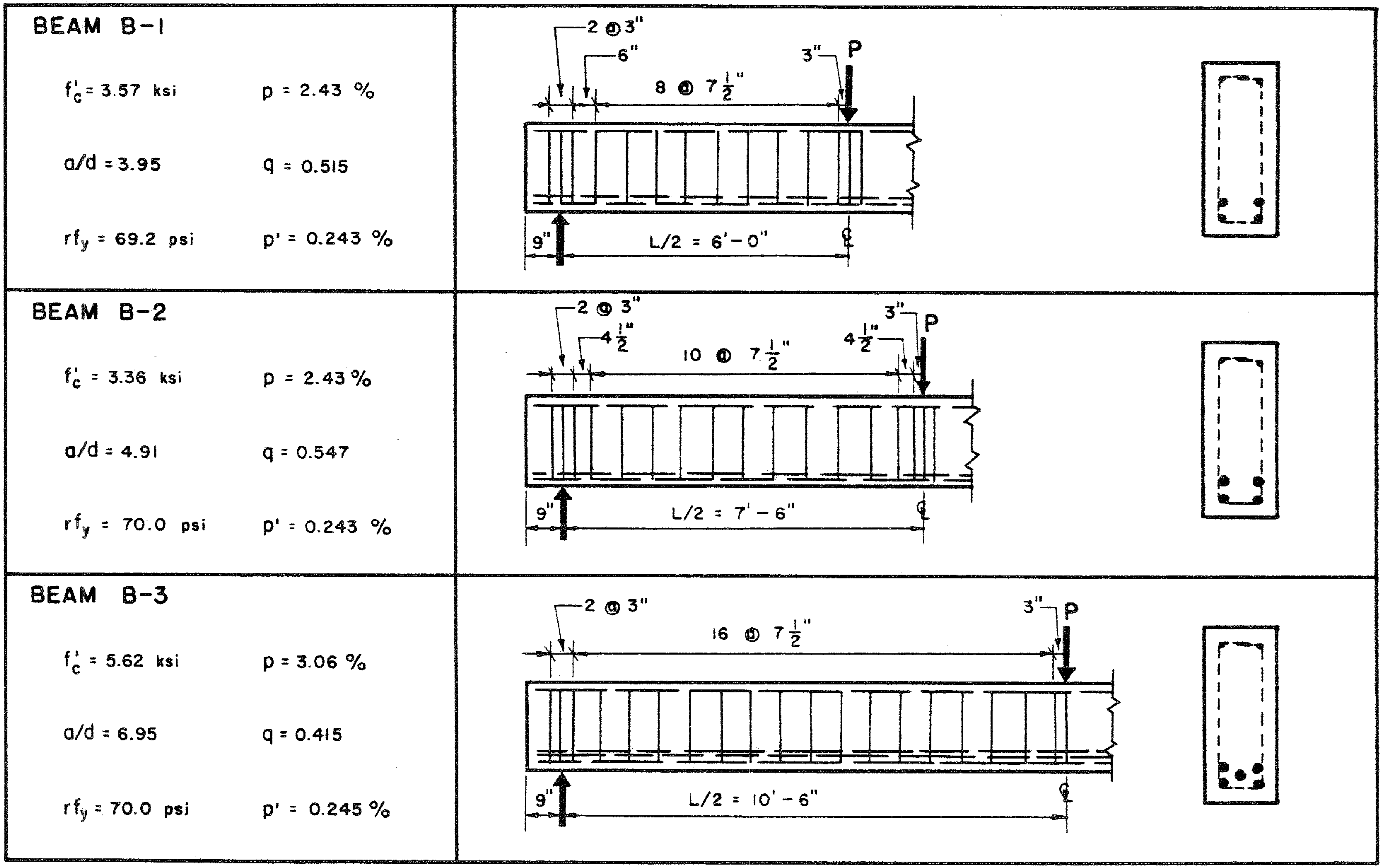


FIG. 2-C SERIES B-BEAM ELEVATION

Beam		
BEAM C-1	f'_c = 4.28 ksi	p = 1.80 %
	a/d = 3.95	q = 0.297
	rf_y = 93.9 psi	p' = 0.361 %
BEAM C-2	f'_c = 3.46 ksi	p = 3.66 %
	a/d = 4.93	q = 0.800
	rf_y = 95.2 psi	p' = 0.366 %
BEAM C-3	f'_c = 5.08 ksi	p = 3.63 %
	a/d = 6.98	q = 0.536
	rf_y = 93.9 psi	p' = 0.363 %

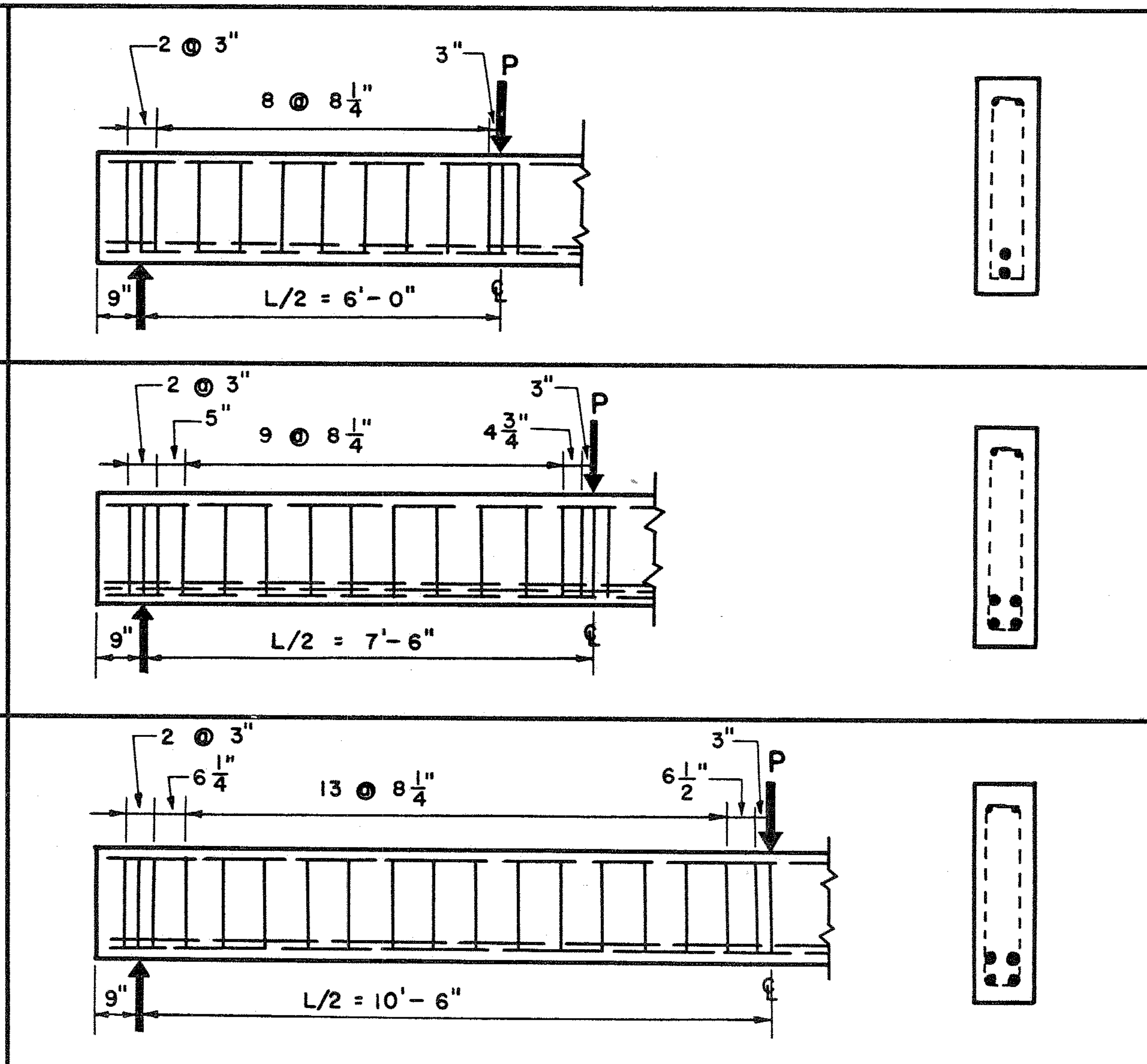


FIG. 2-D SERIES C - BEAM ELEVATIONS

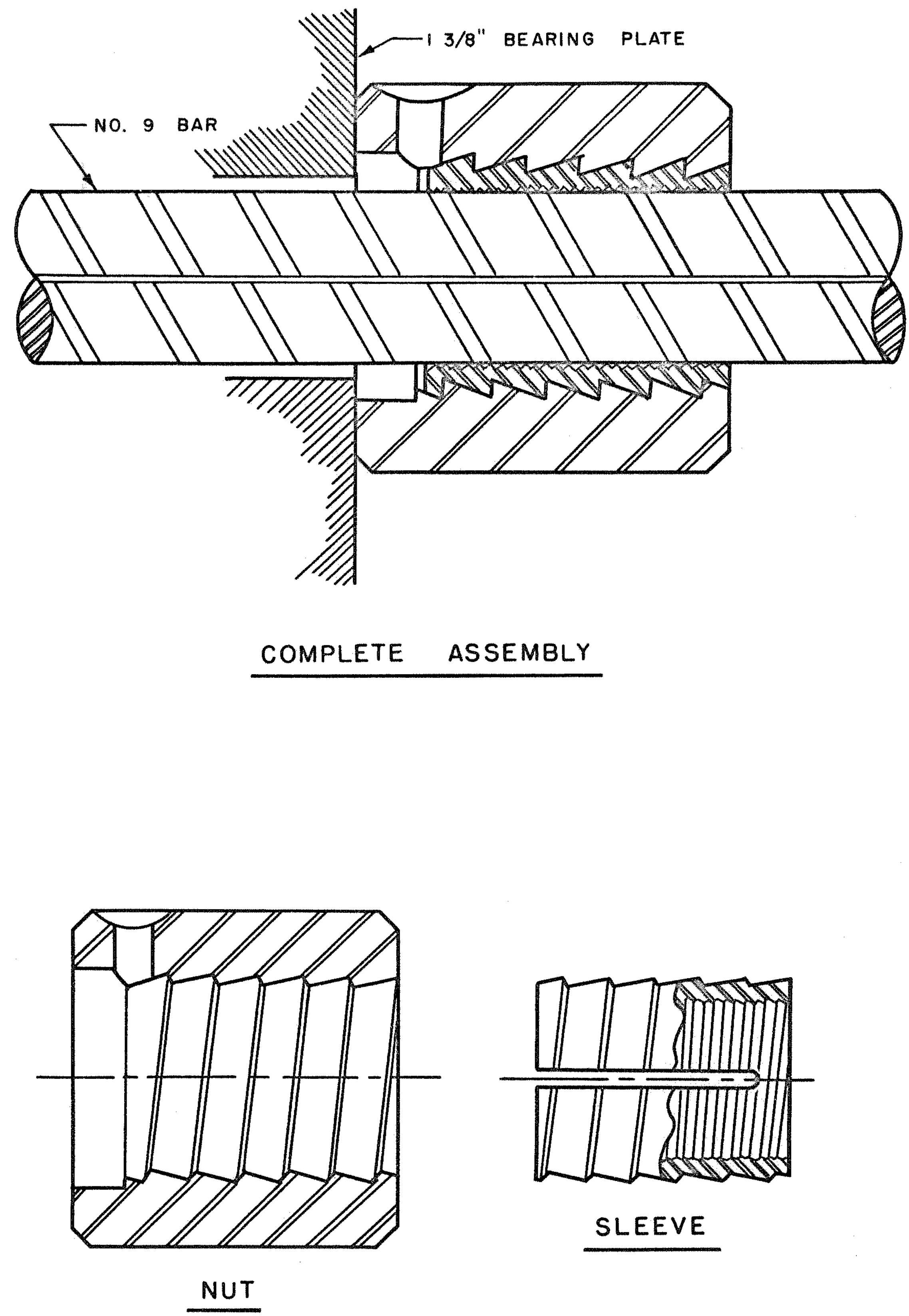


FIG. 3 DETAILS OF HOWLETT ANCHOR NUT

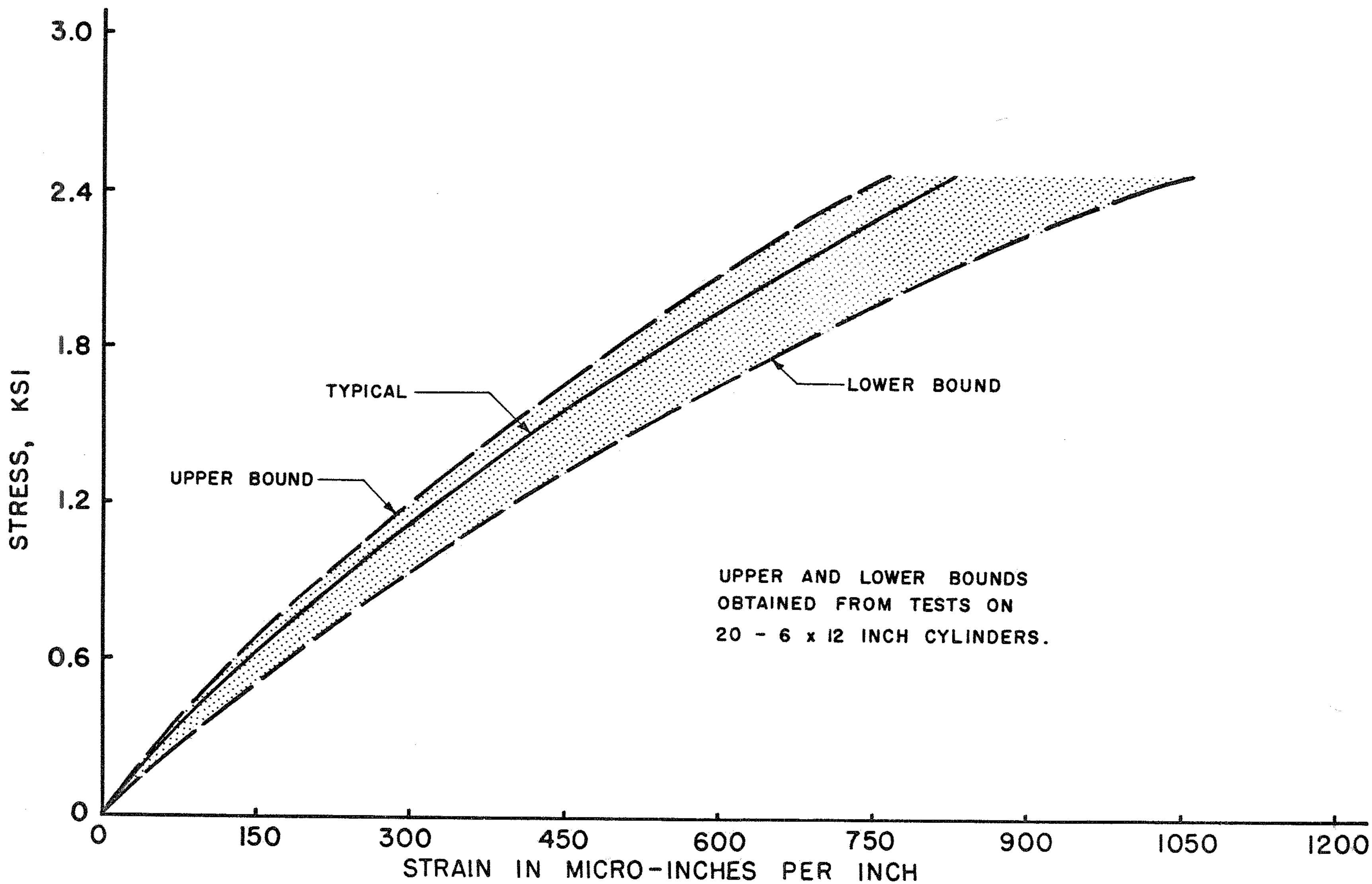


FIG. 4-A STRESS STRAIN RELATIONSHIPS FOR CONCRETE - 3500 PSI MIX

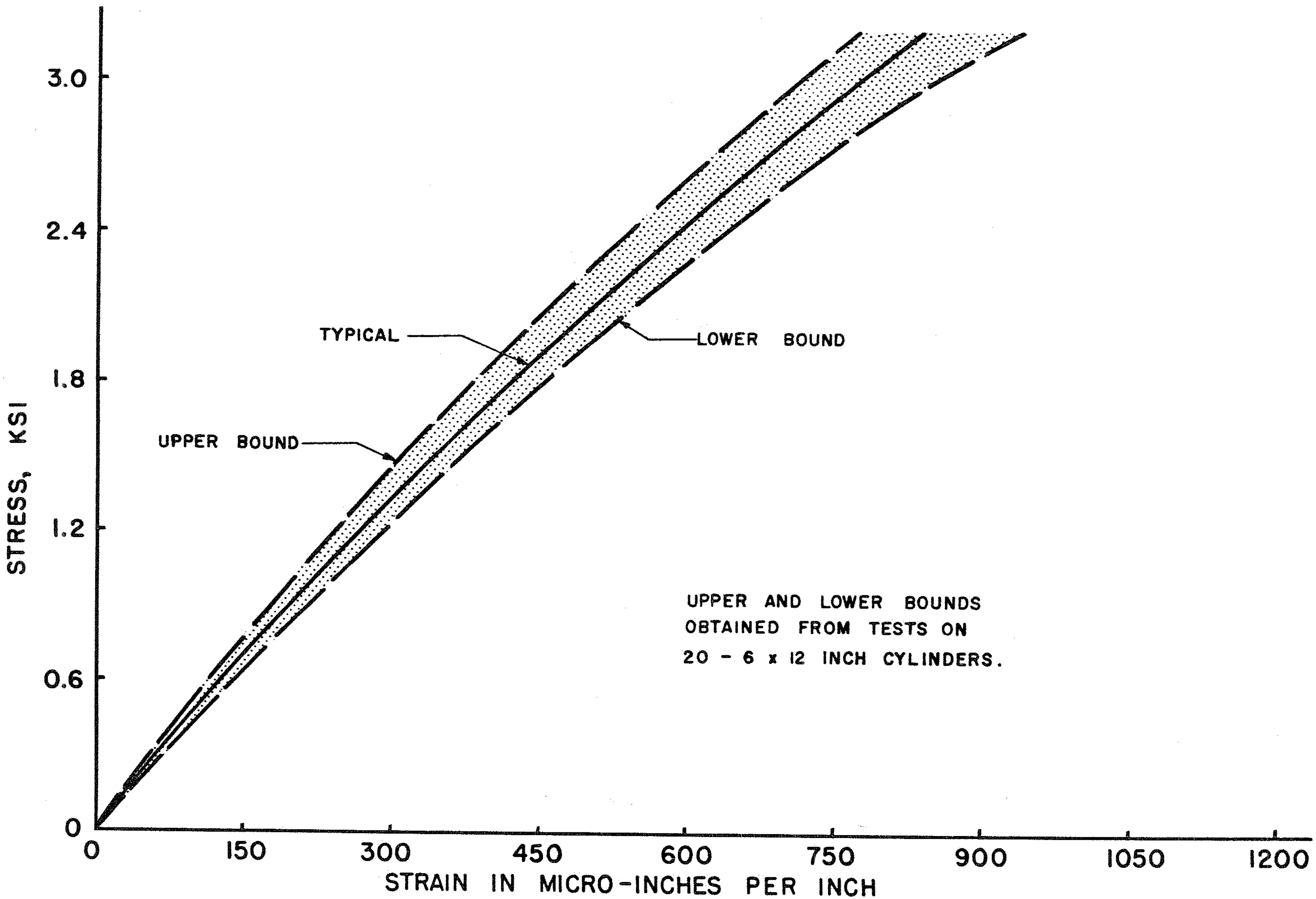


FIG. 4-B STRESS STRAIN RELATIONSHIPS FOR CONCRETE - 5000 PSI MIX

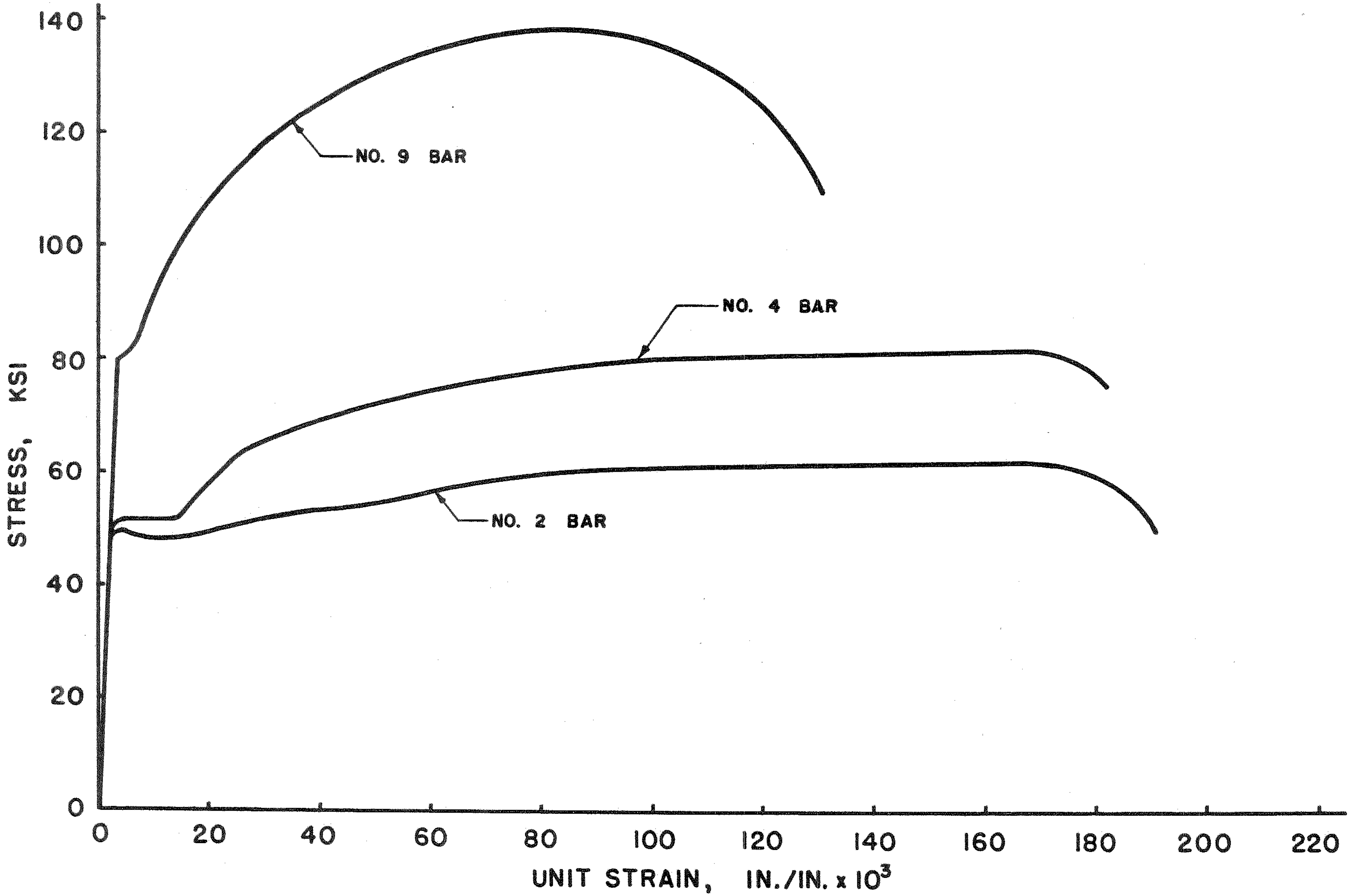


FIG. 5-A TYPICAL STRESS-STRAIN DIAGRAMS FOR STEEL REINFORCEMENT

(FULL RANGE TO FAILURE)

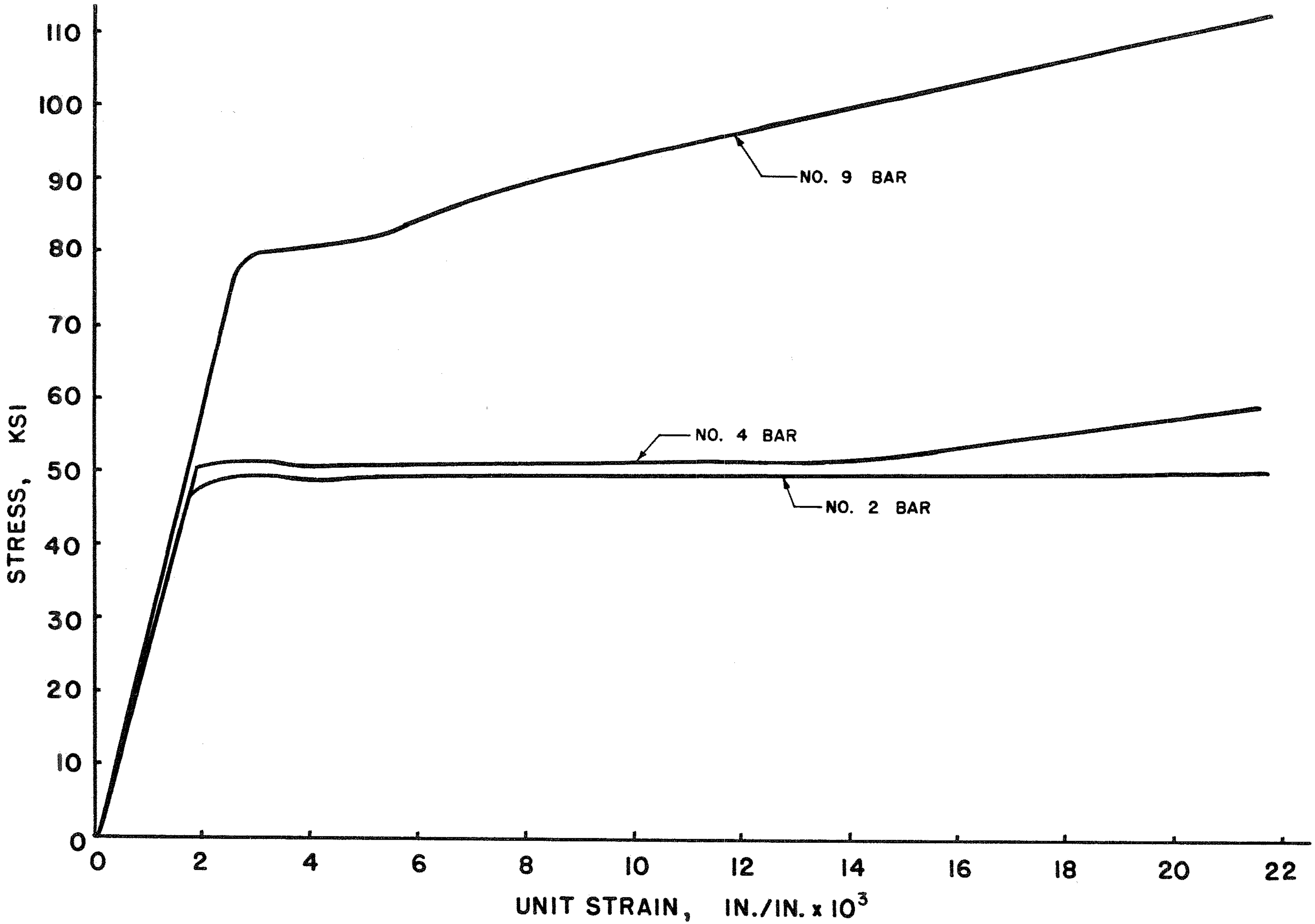


FIG. 5-B TYPICAL STRESS-STRAIN DIAGRAMS FOR STEEL REINFORCEMENT
(THROUGH YIELD RANGE)

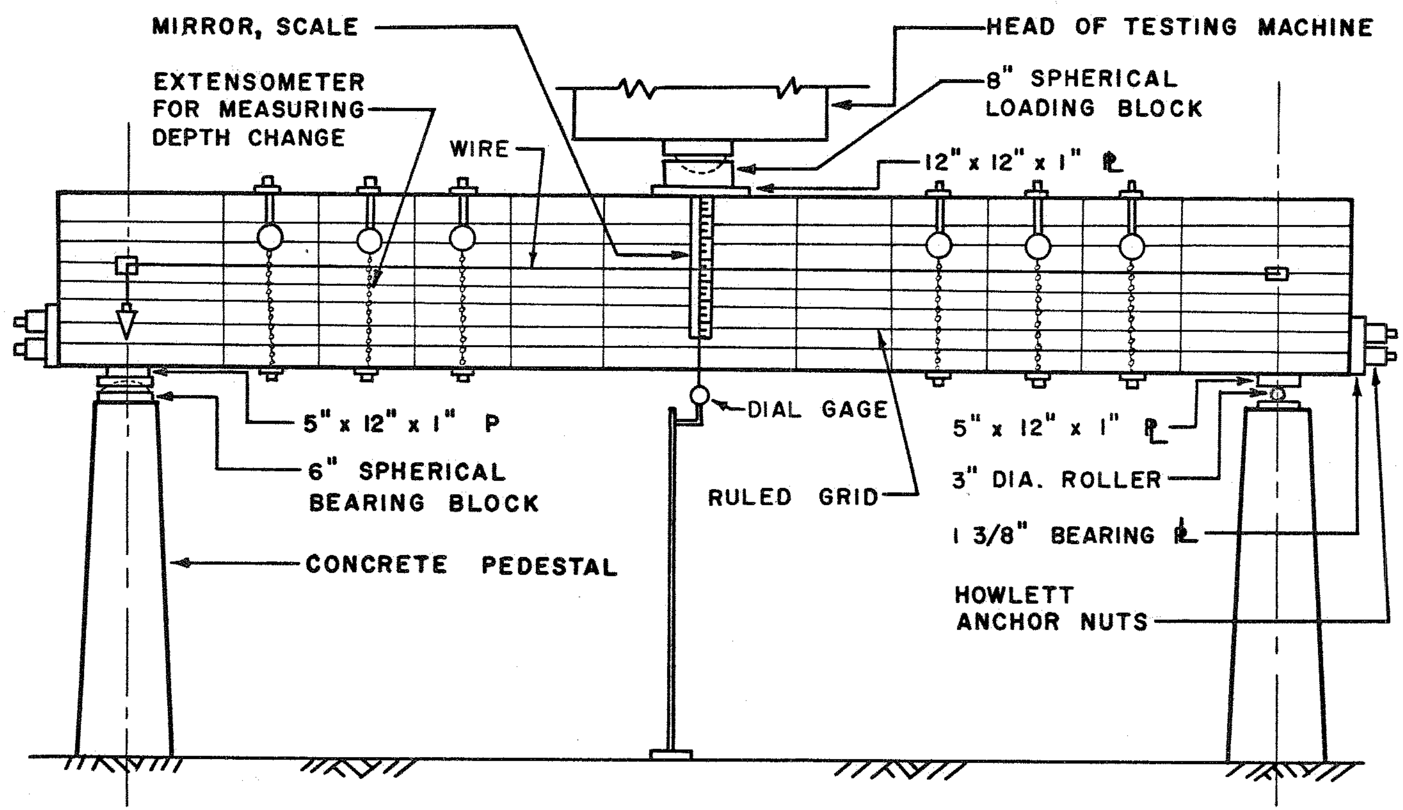


FIG. 6 LOADING ARRANGEMENT & INSTRUMENTATION

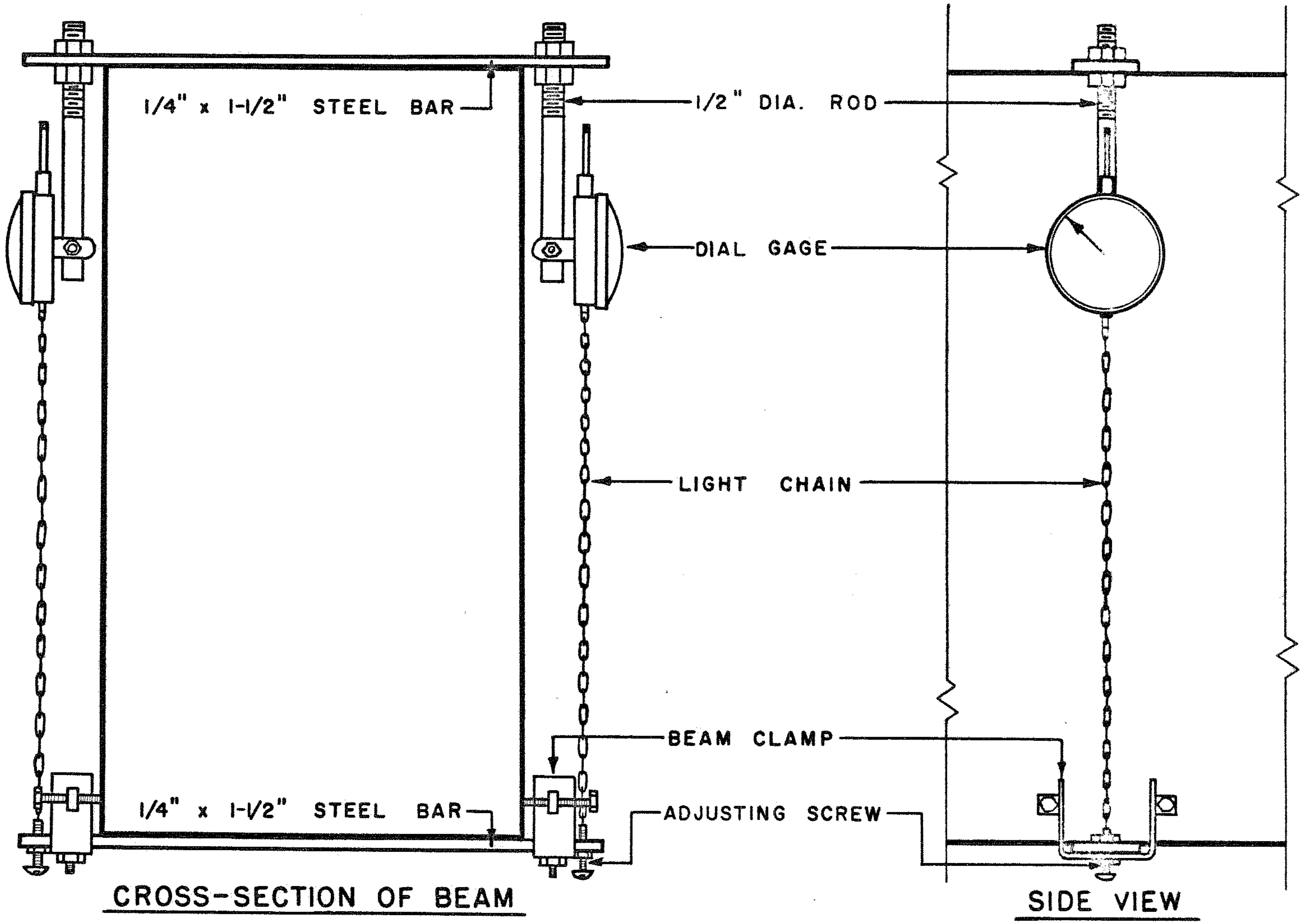


FIG. 7 DETAILS OF EXTENSOMETER FOR MEASURING DEPTH CHANGE

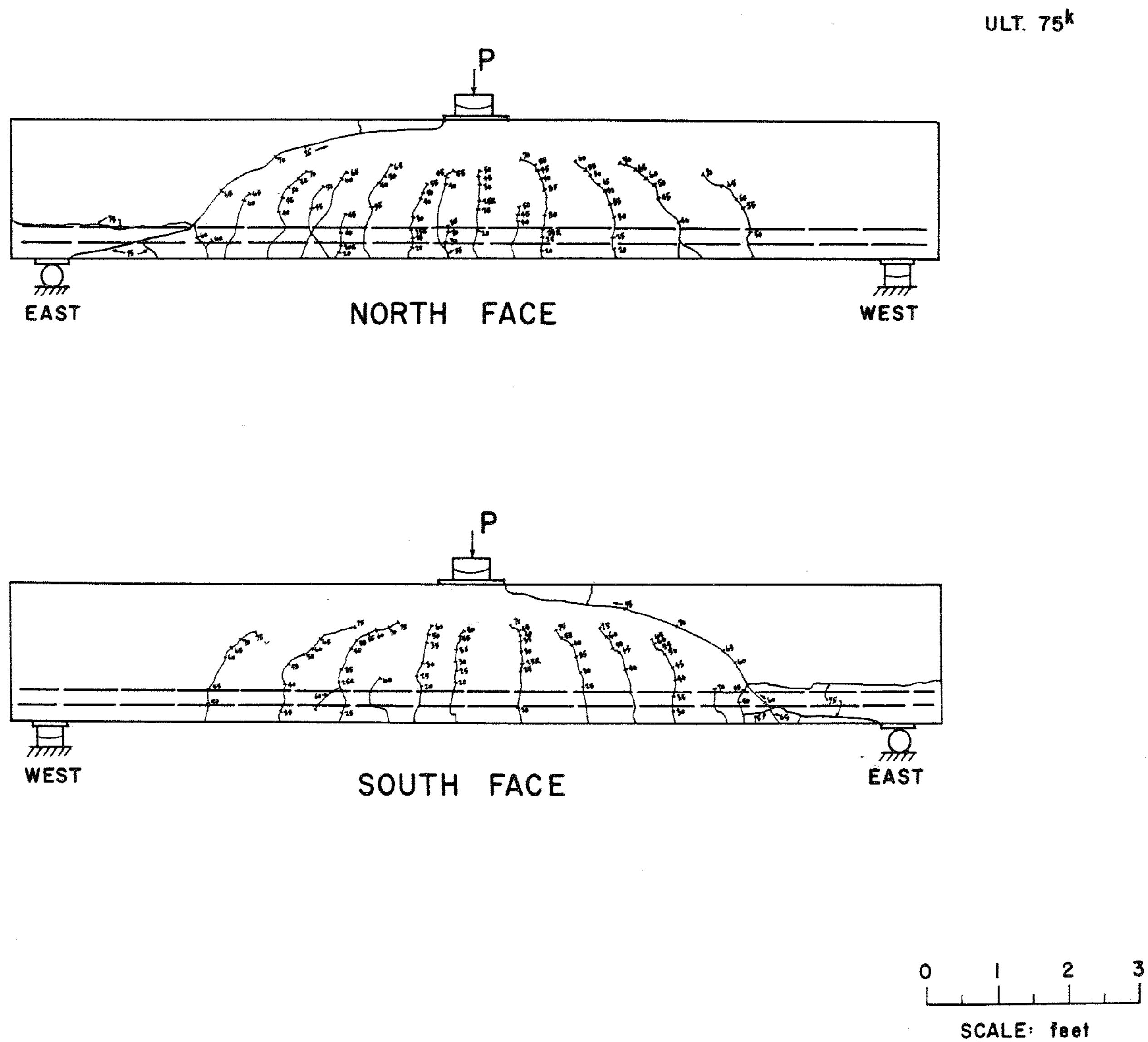


FIG. 8-A BEAM OA-1

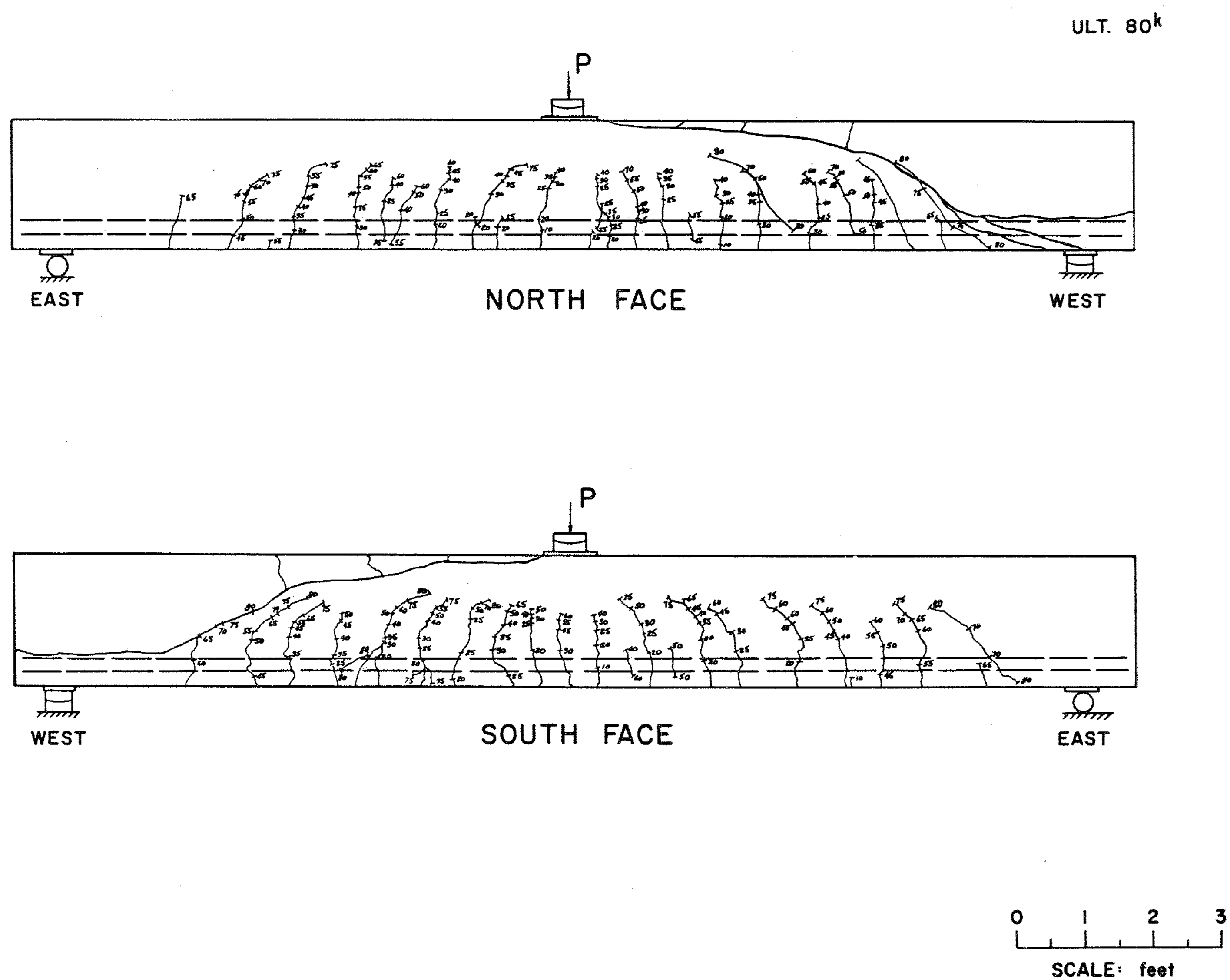


FIG. 8-B BEAM OA-2

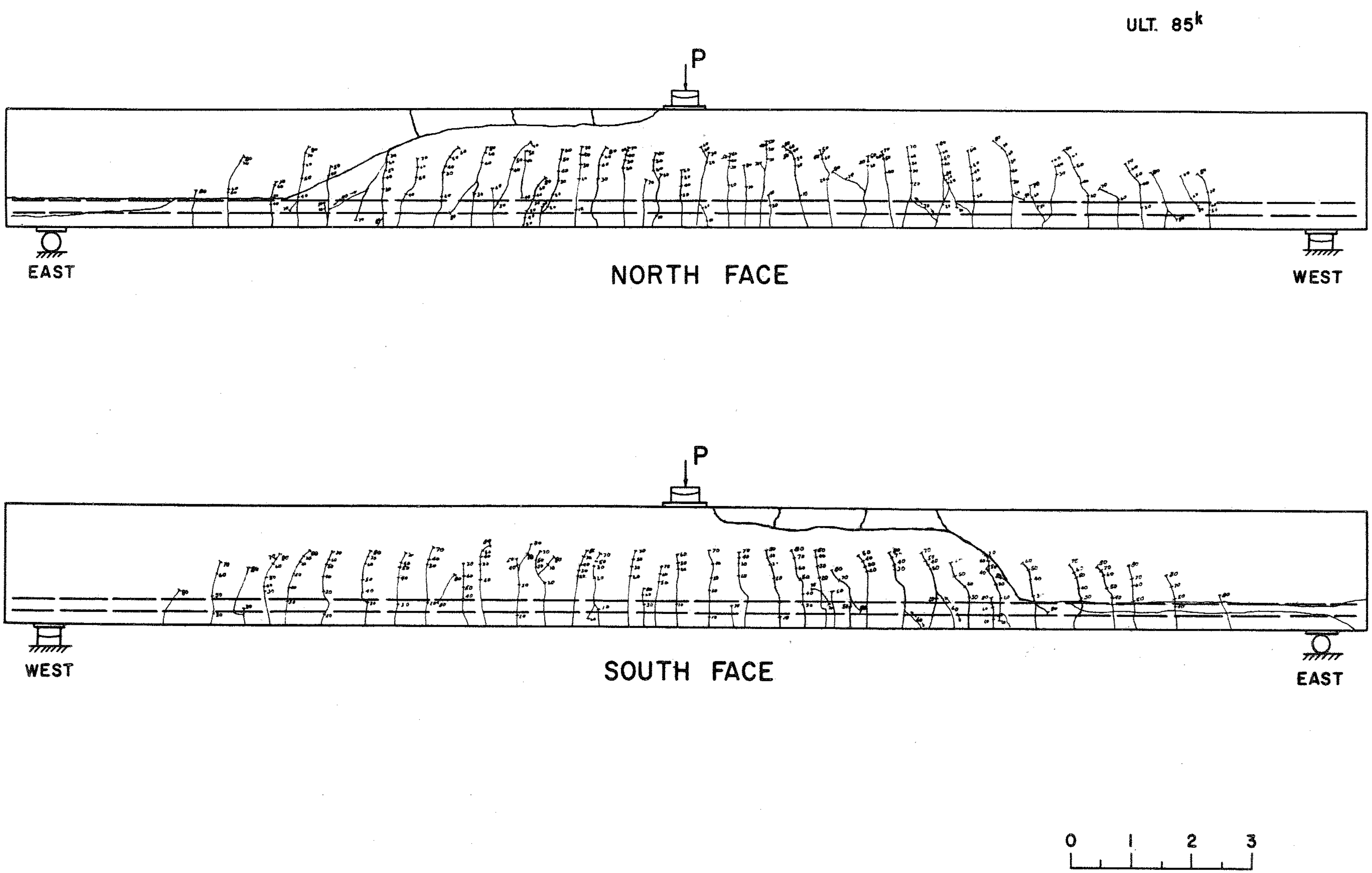


FIG. 8-C BEAM OA-3

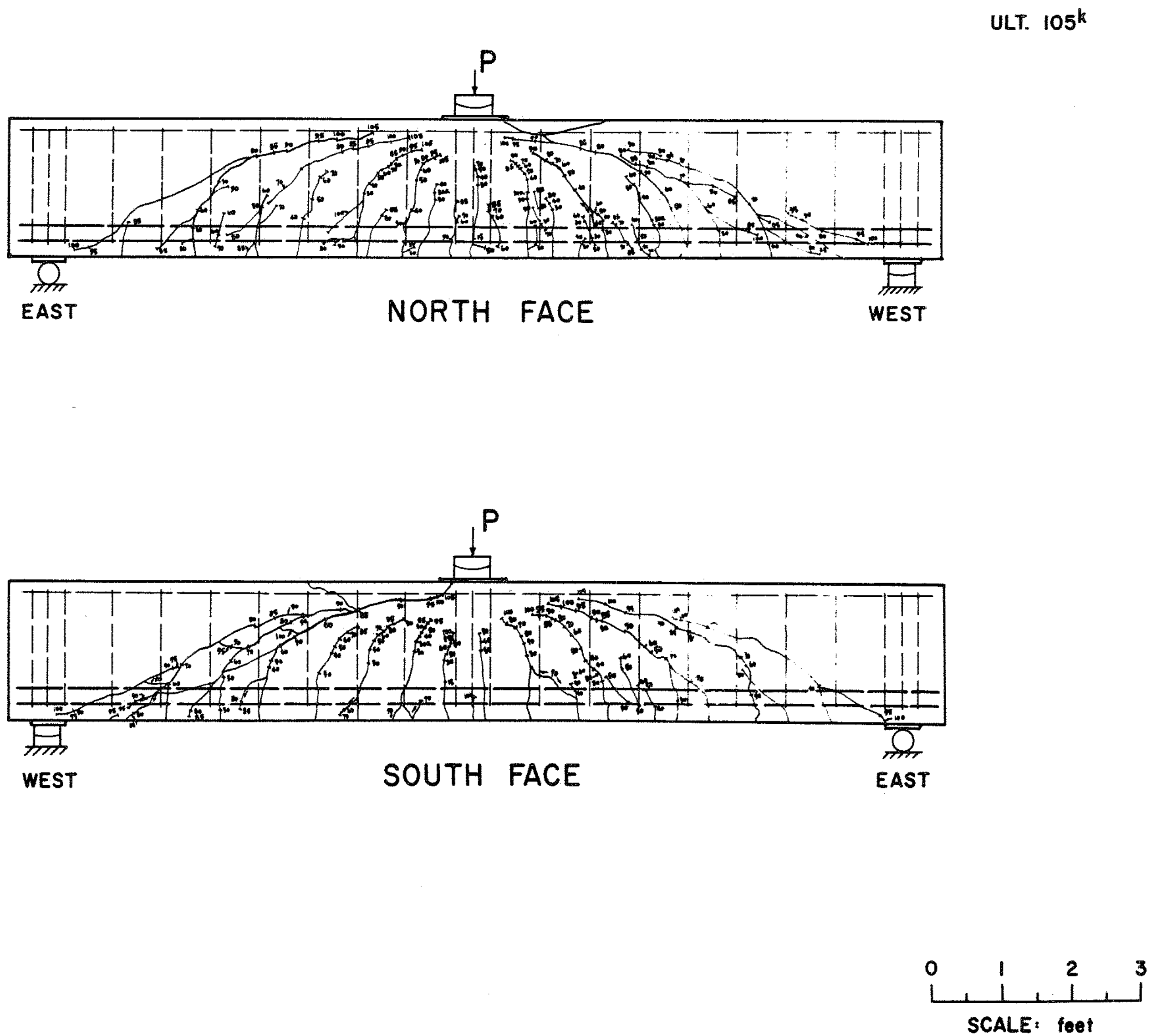


FIG. 8-D BEAM A-1

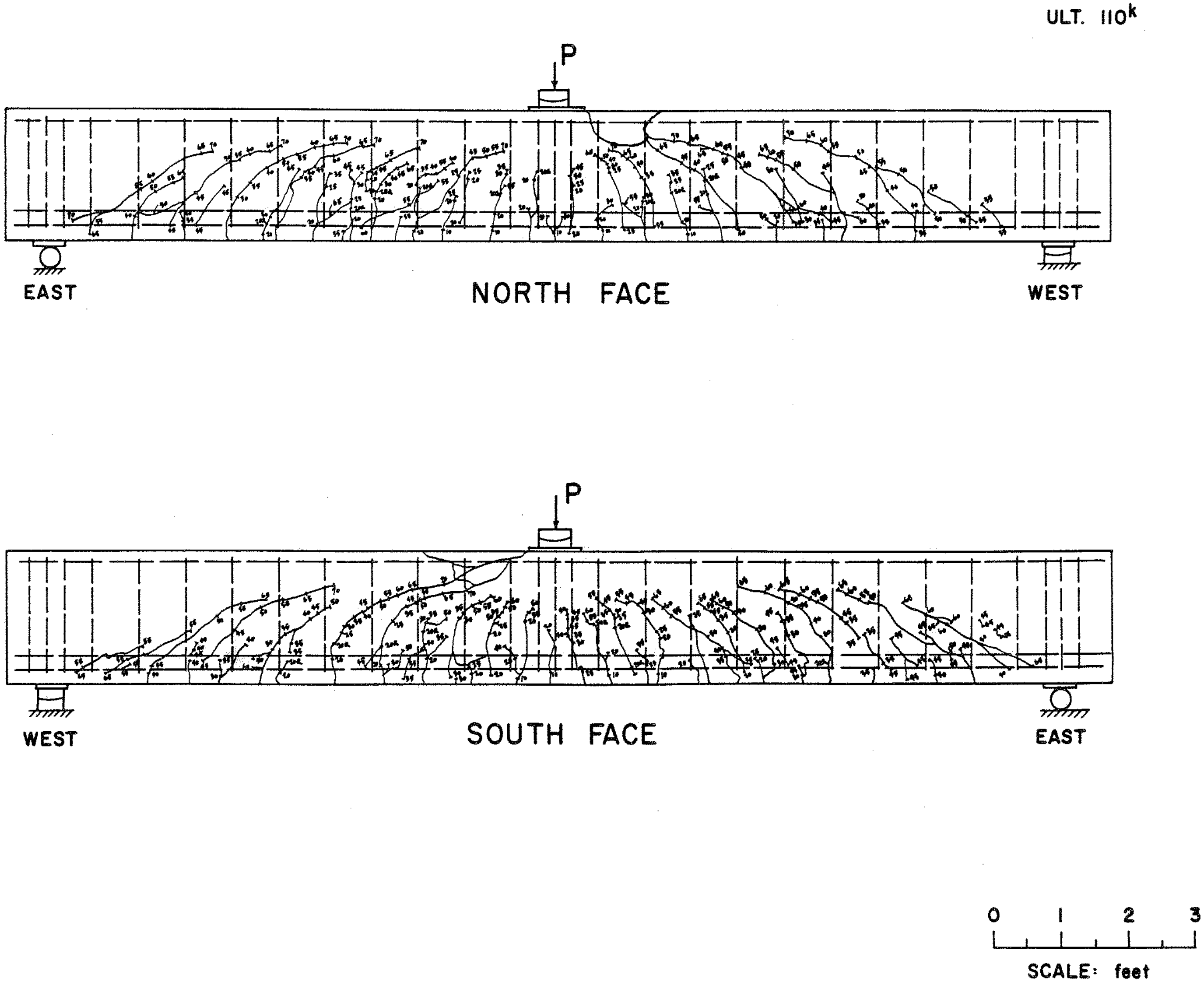


FIG. 8-E BEAM A-2

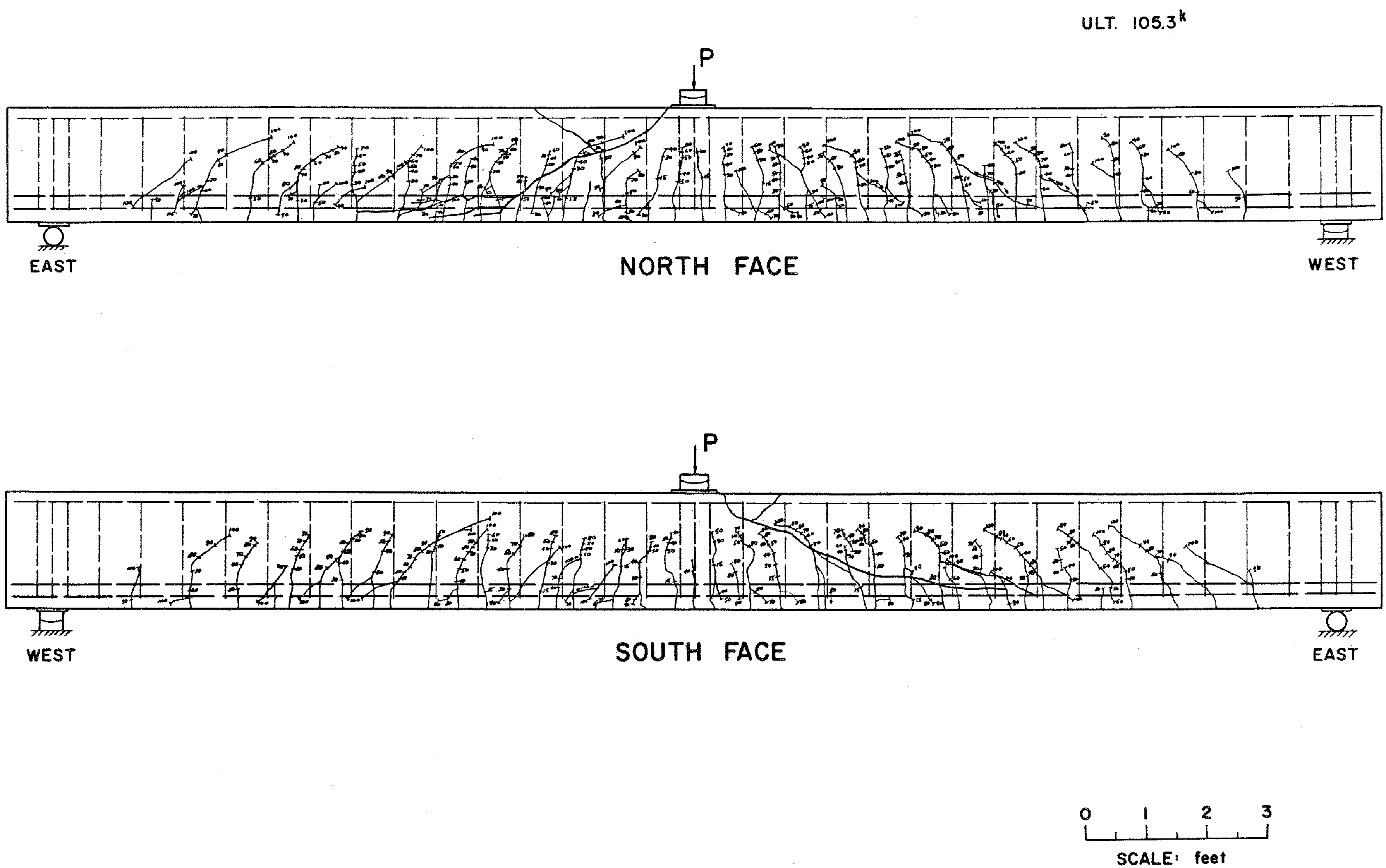


FIG. 8-F BEAM A-3

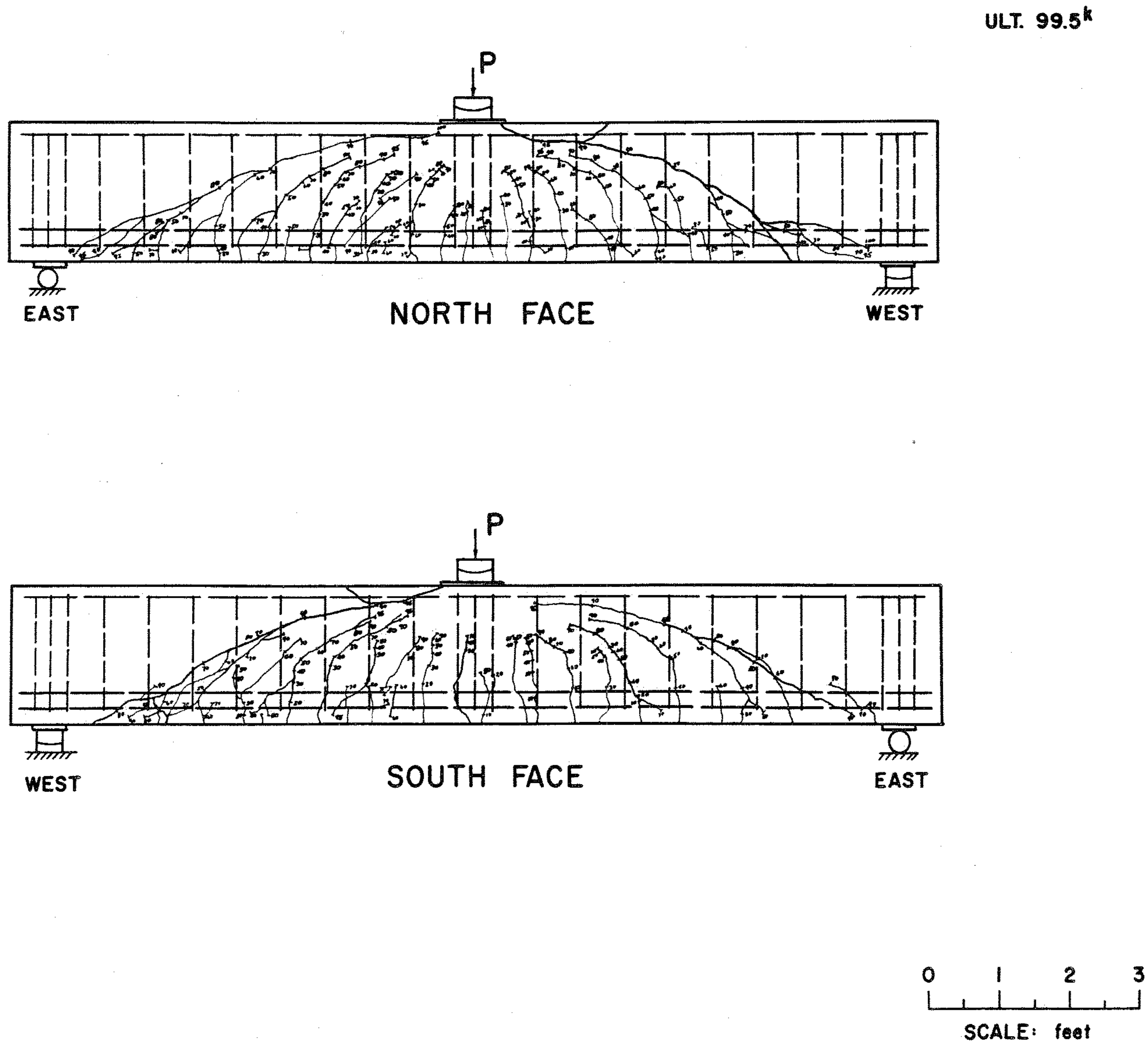


FIG. 8-G BEAM B-1

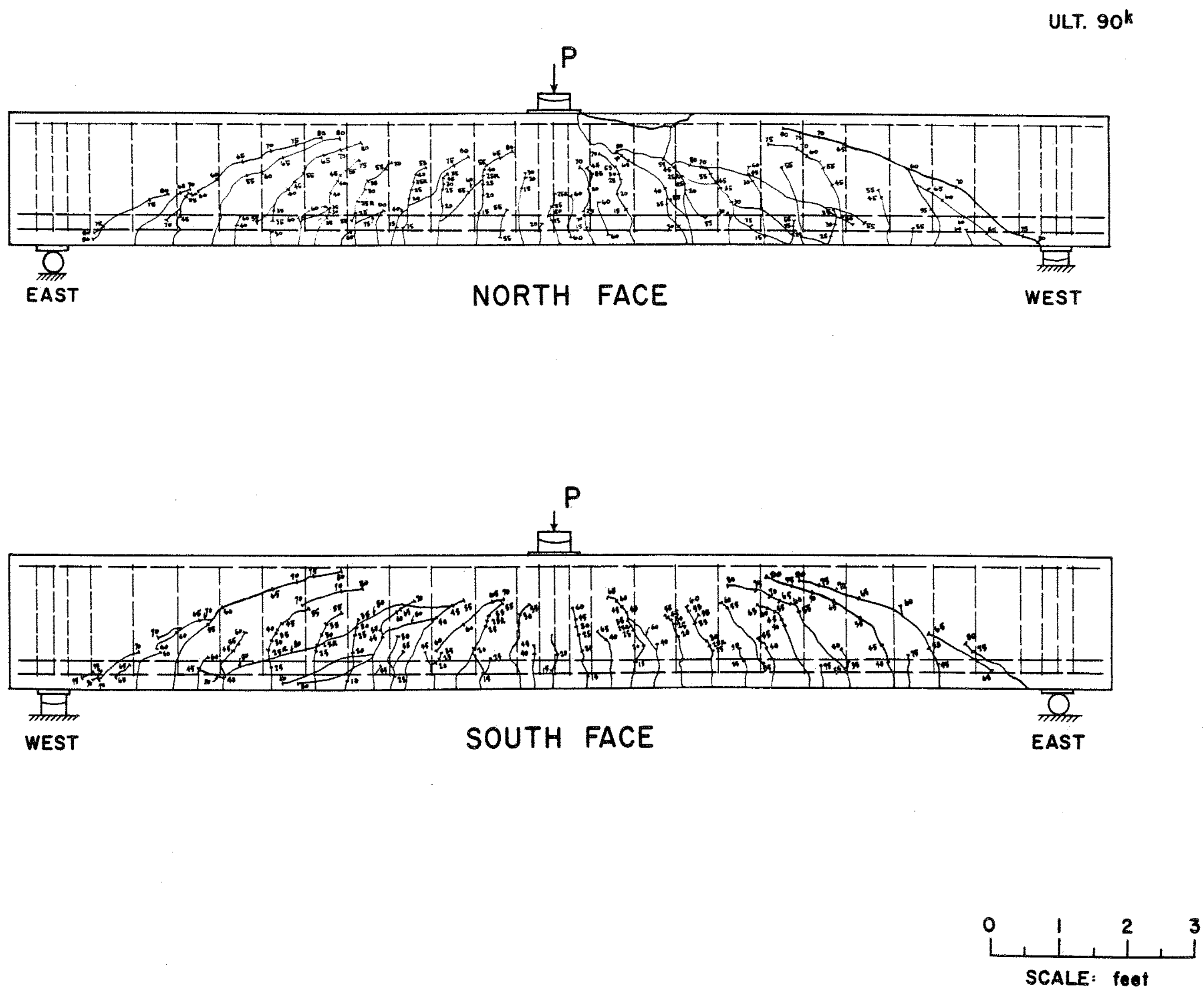


FIG. 8-H BEAM B-2

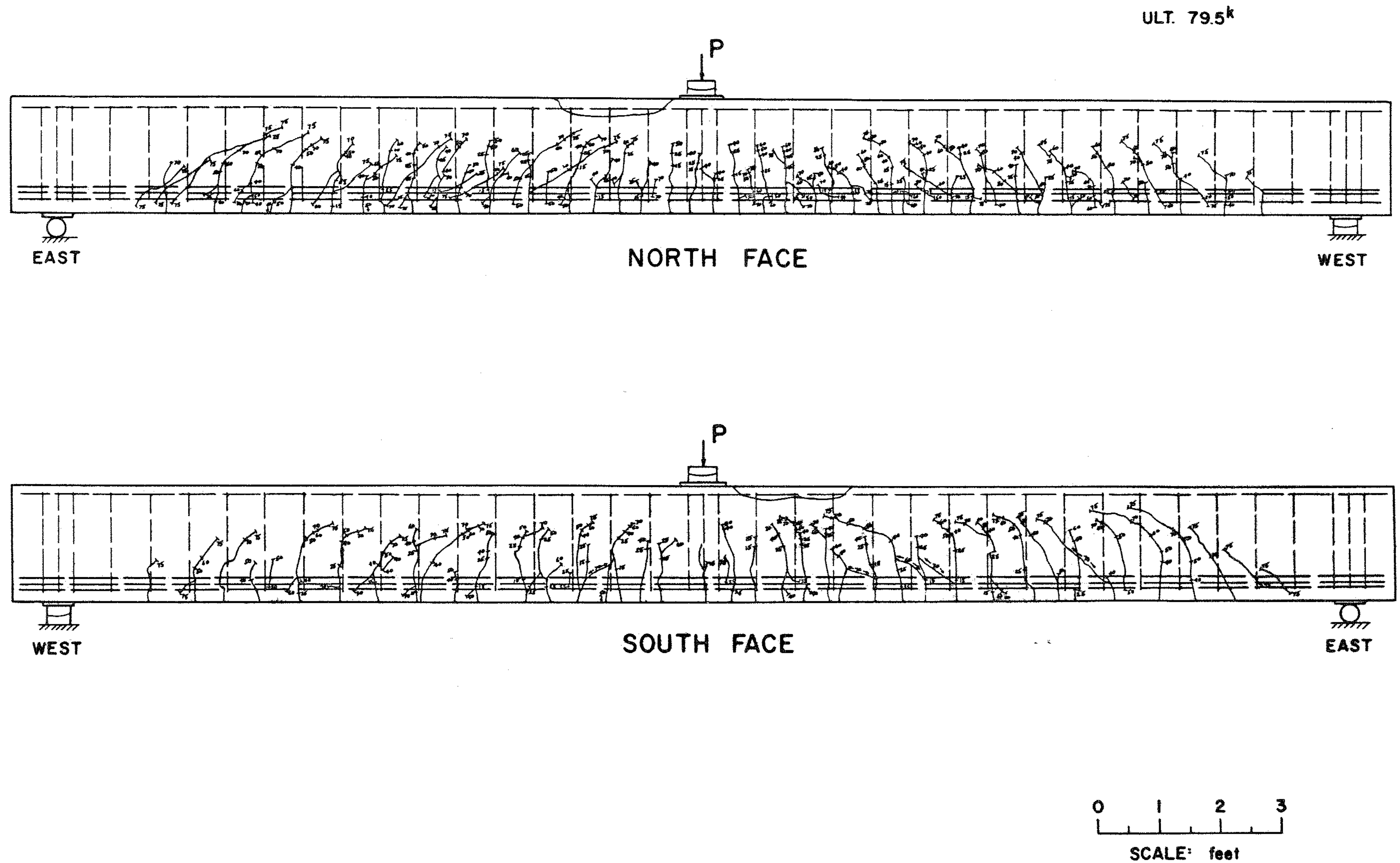


FIG. 8-I BEAM B-3

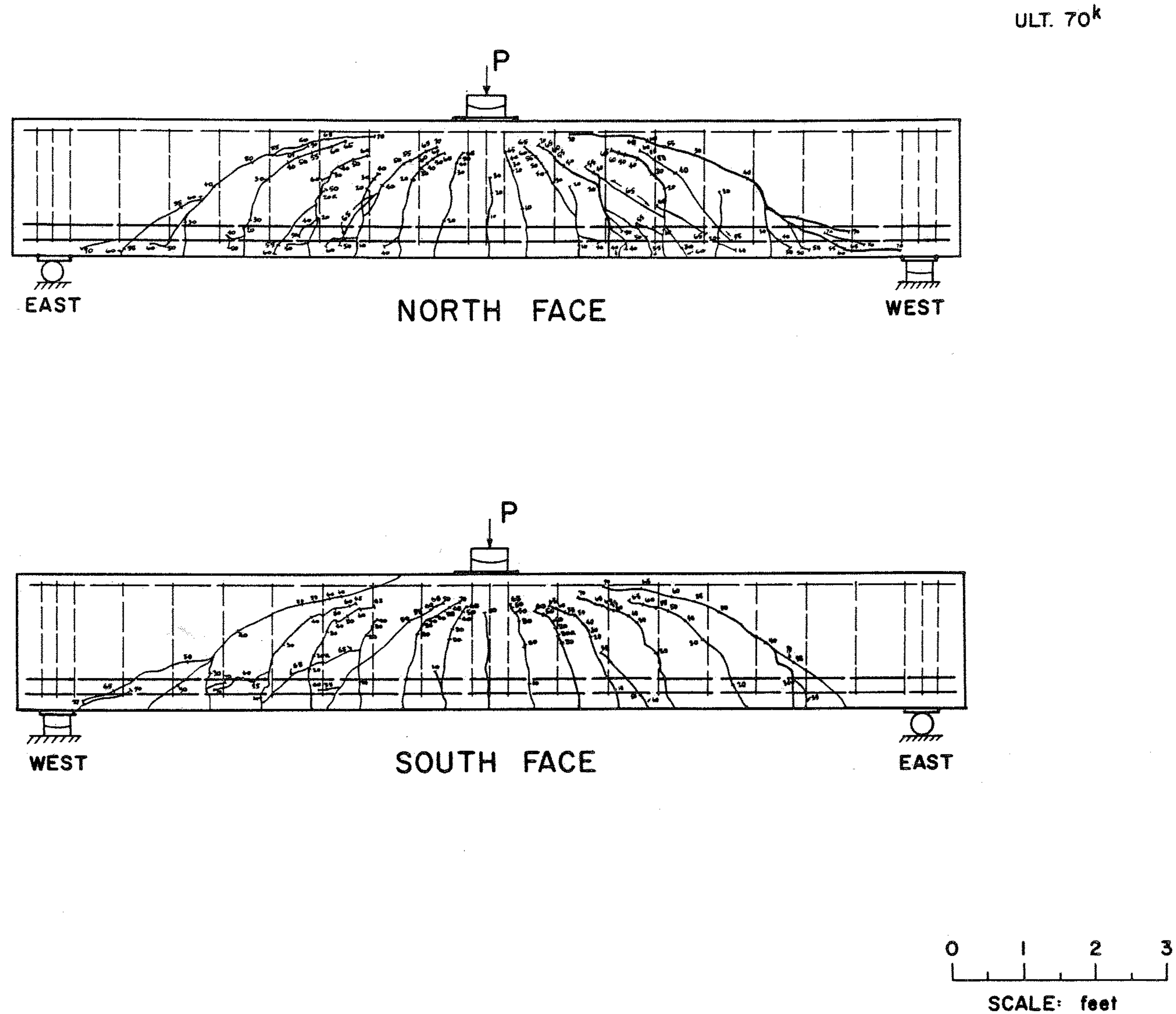


FIG. 8-J BEAM C-I

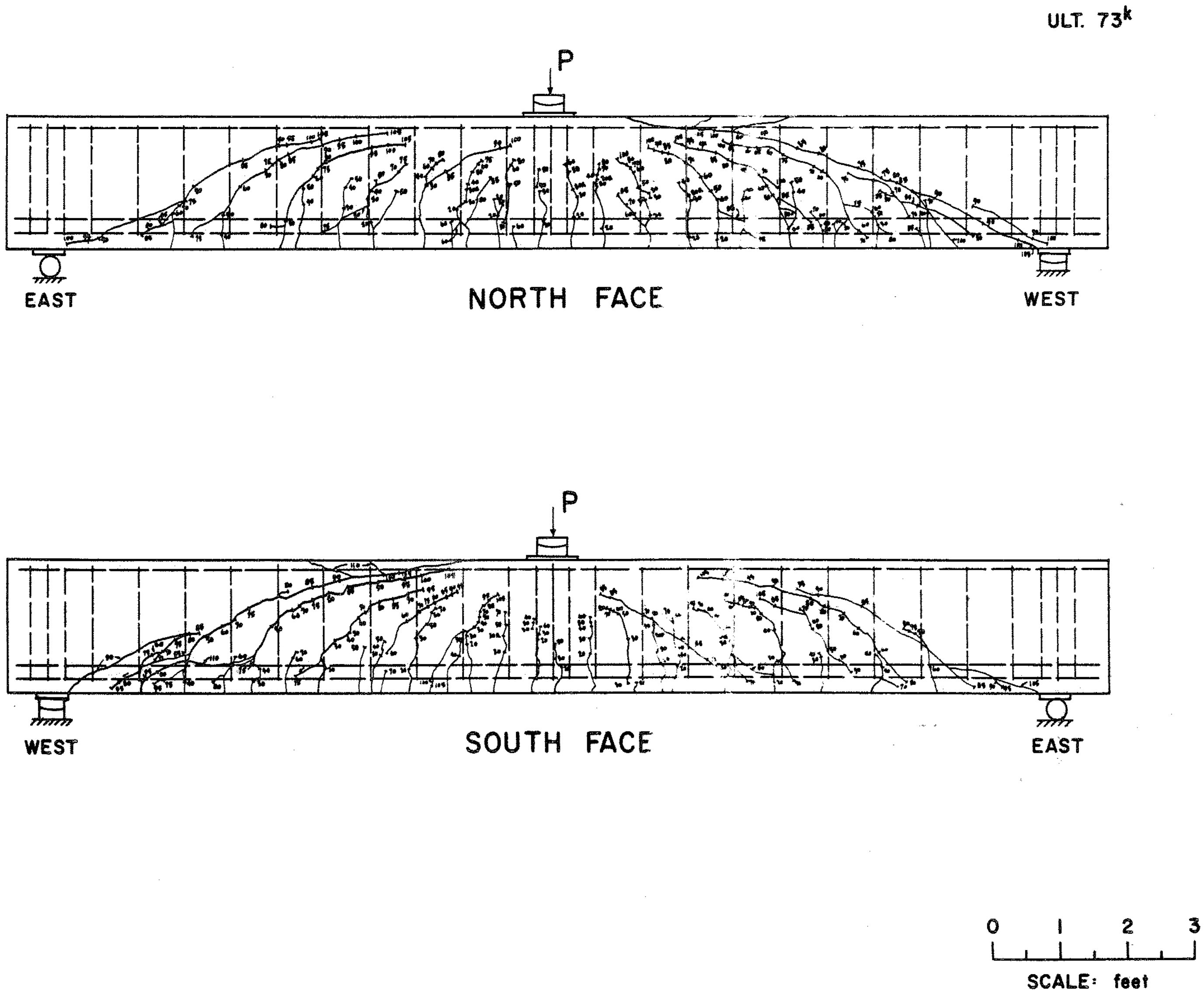


FIG. 8-K BEAM C-2

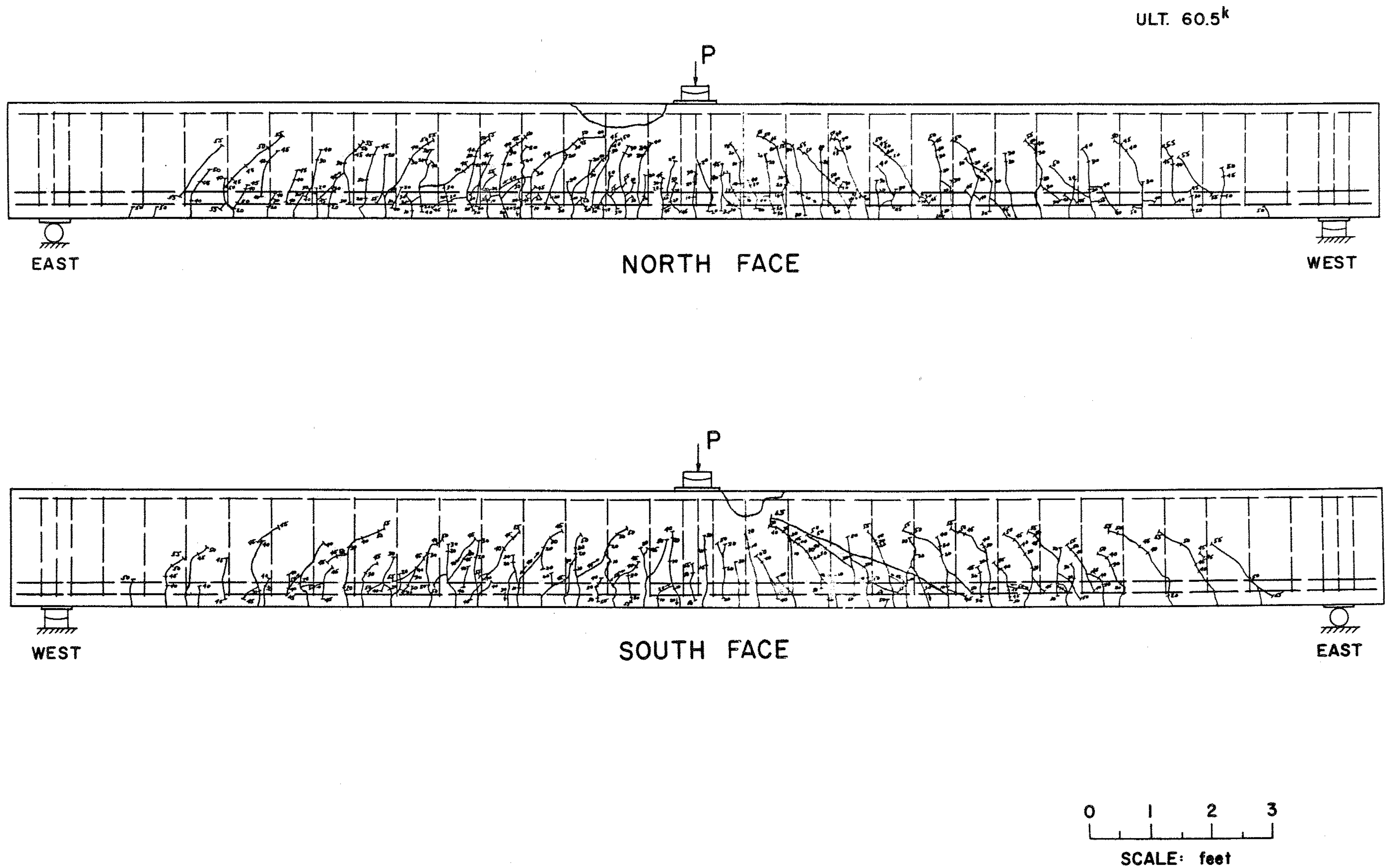


FIG. 8-L BEAM C-3

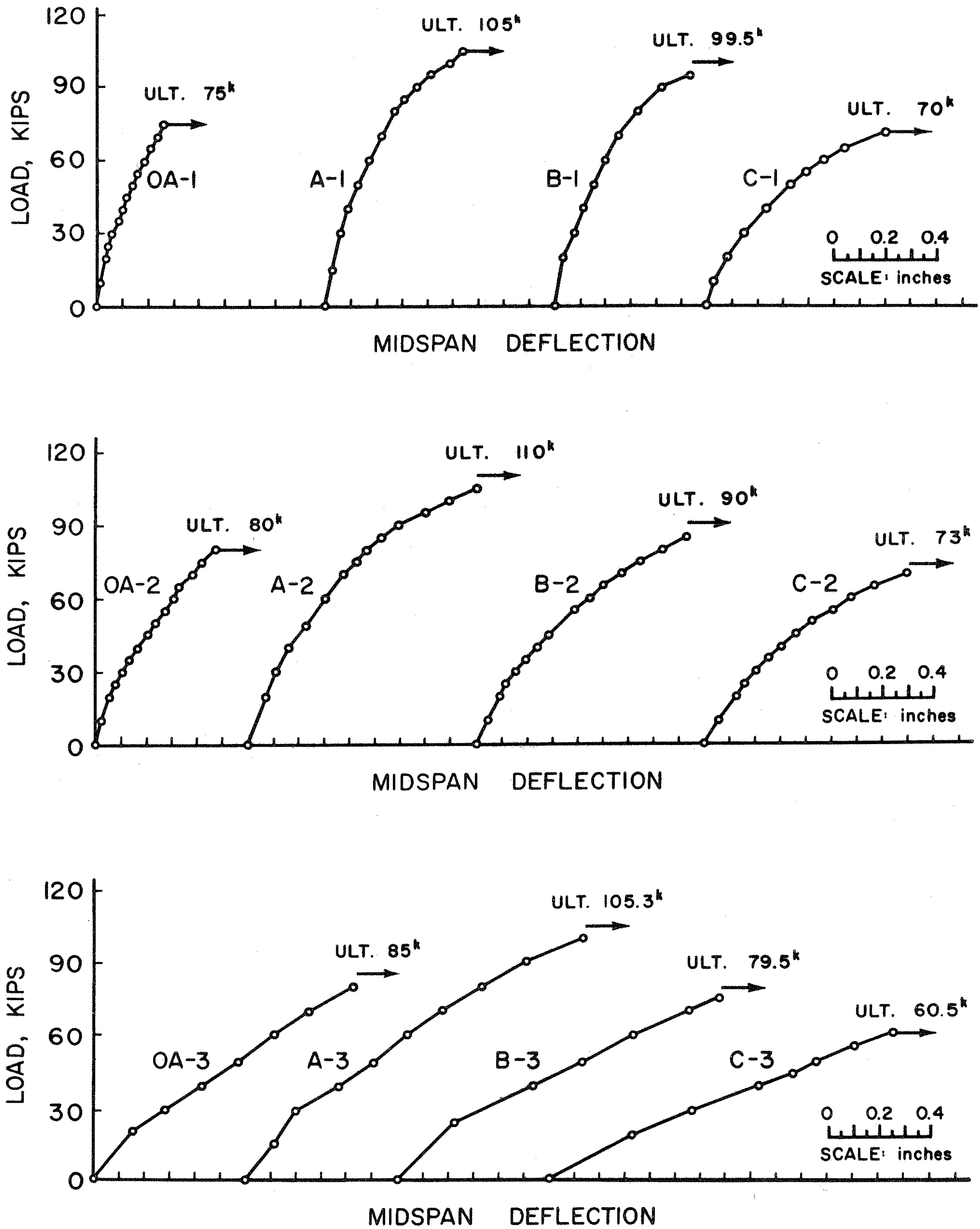


FIG. 9 LOAD DEFLECTION CURVES

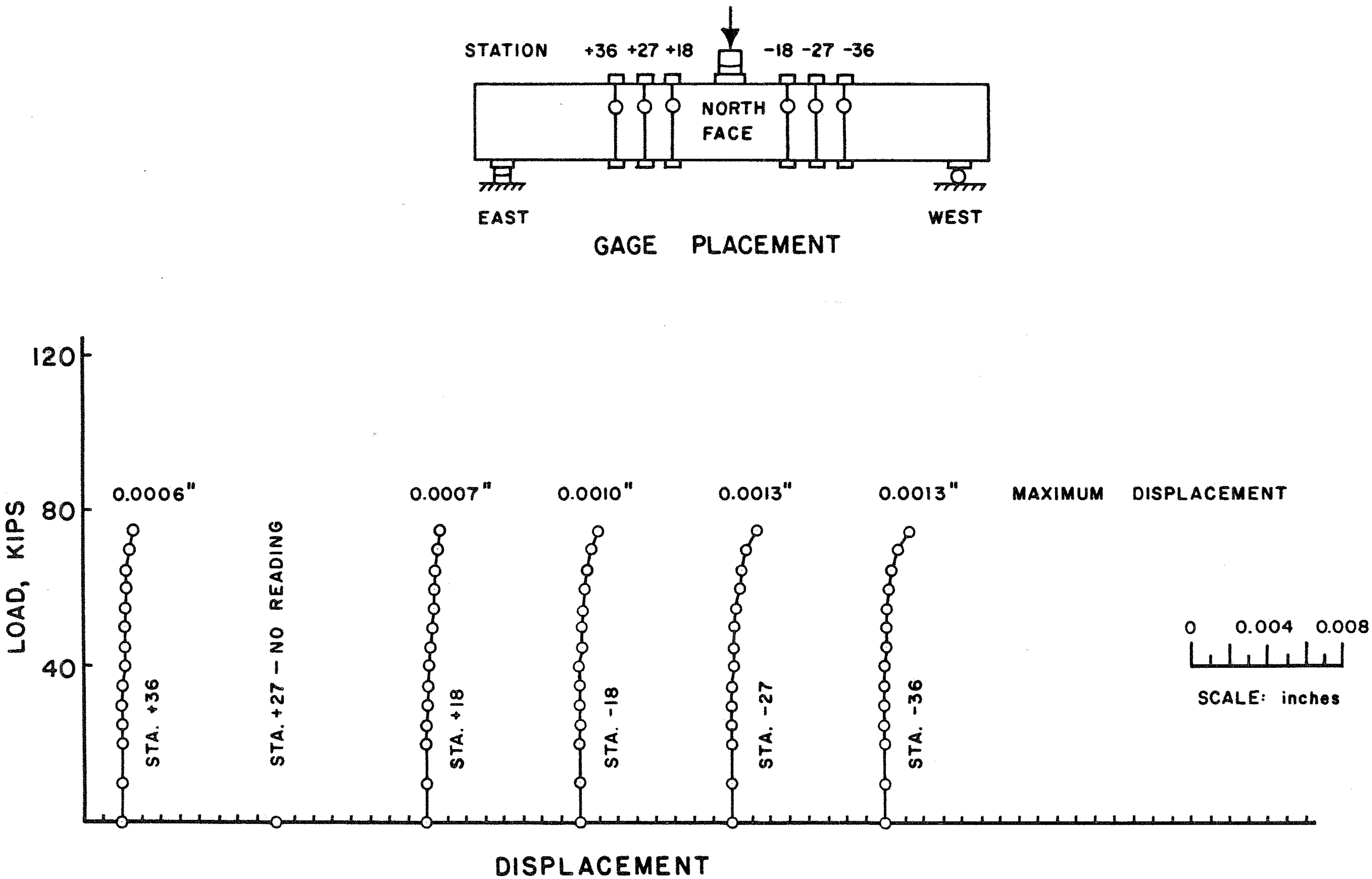


FIG. 10-A YOKE DATA — BEAM OA-1

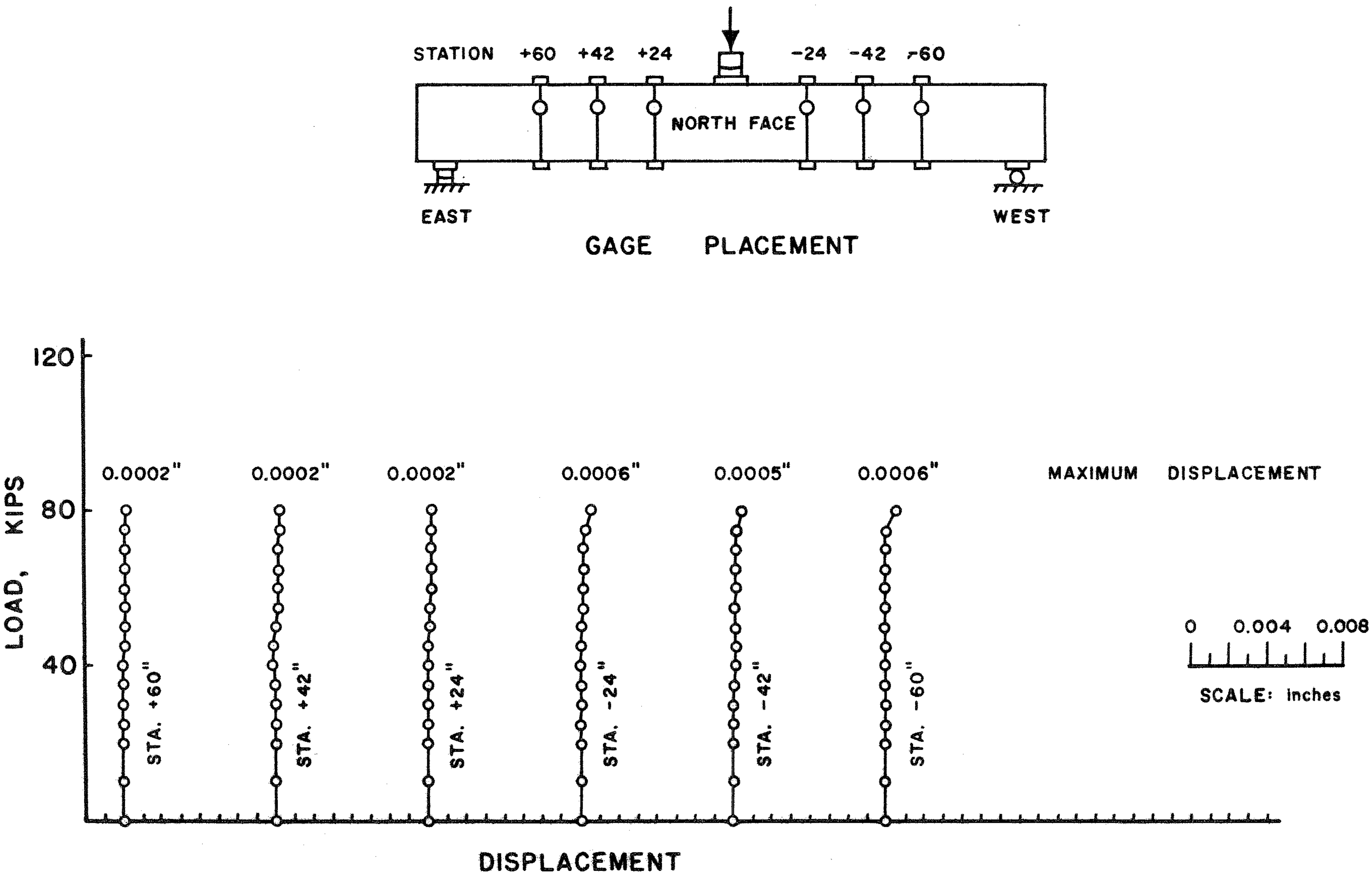
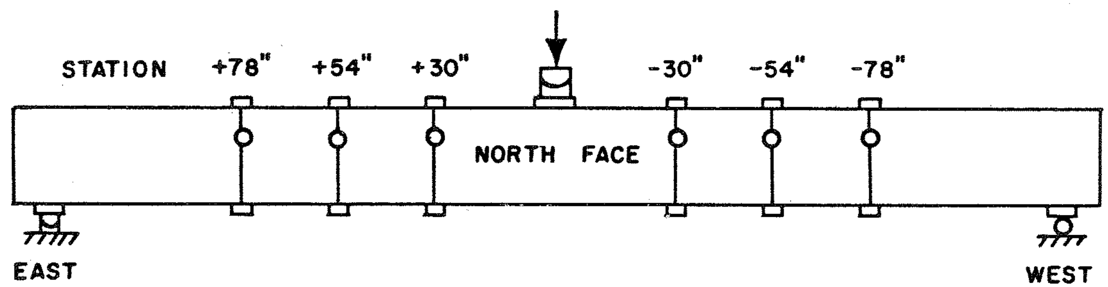


FIG. 10-B YOKE DATA — BEAM OA-2



GAGE PLACEMENT

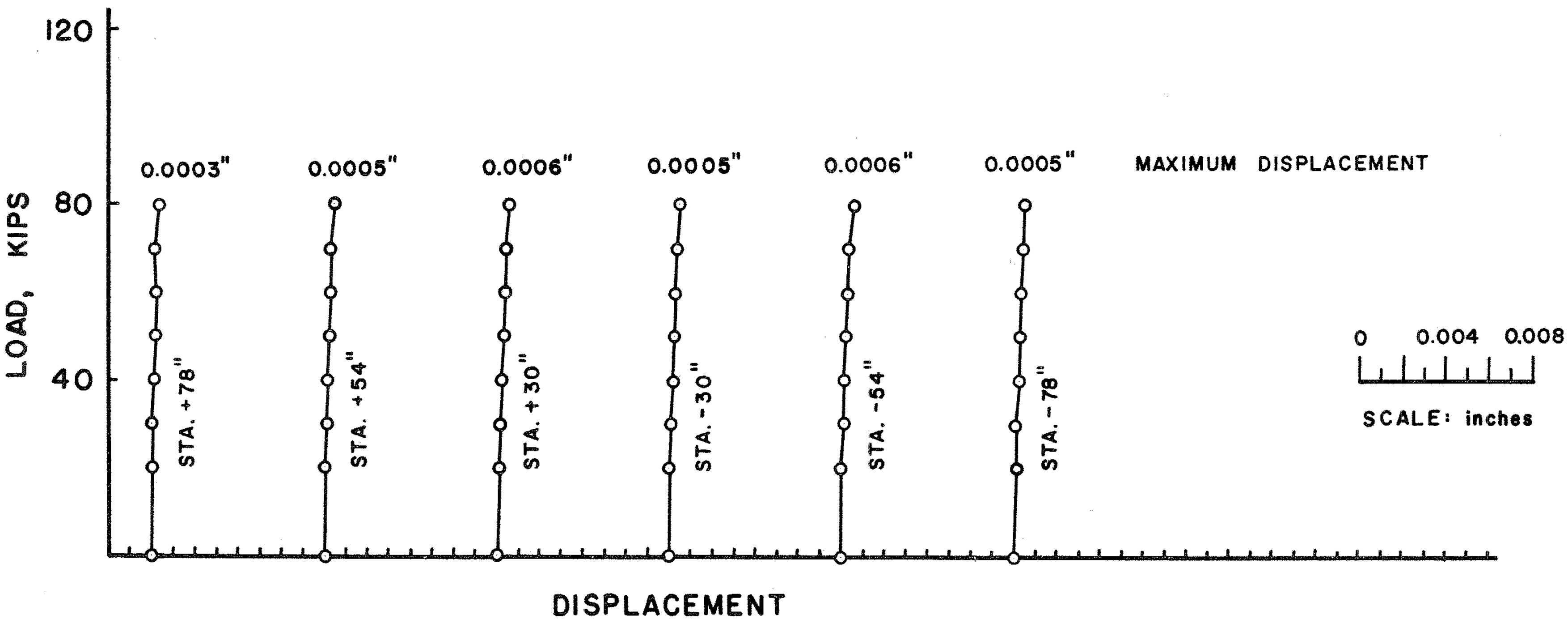


FIG. 10-C YOKE DATA - BEAM OA-3

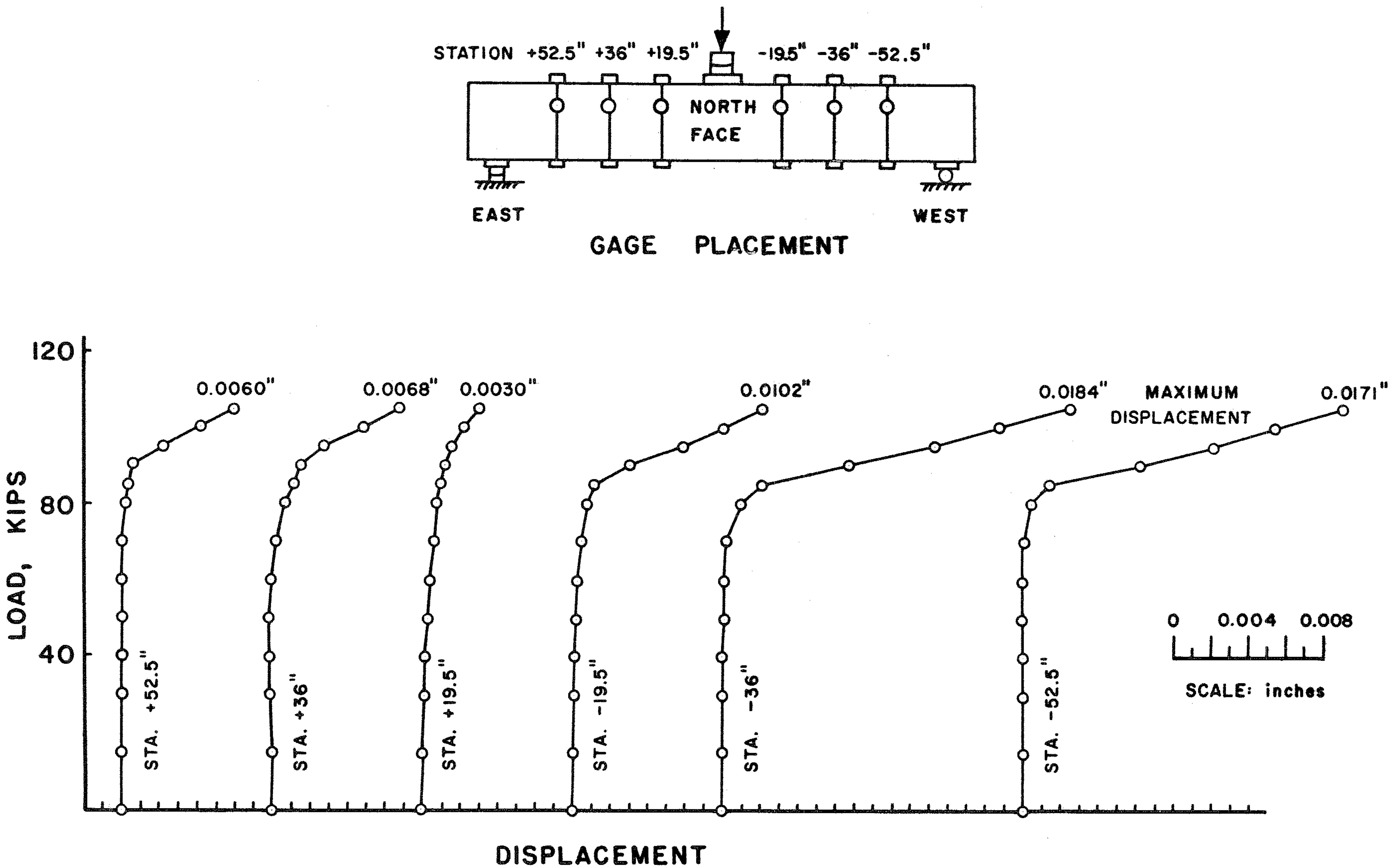


FIG. 10-D YOKE DATA — BEAM A-1

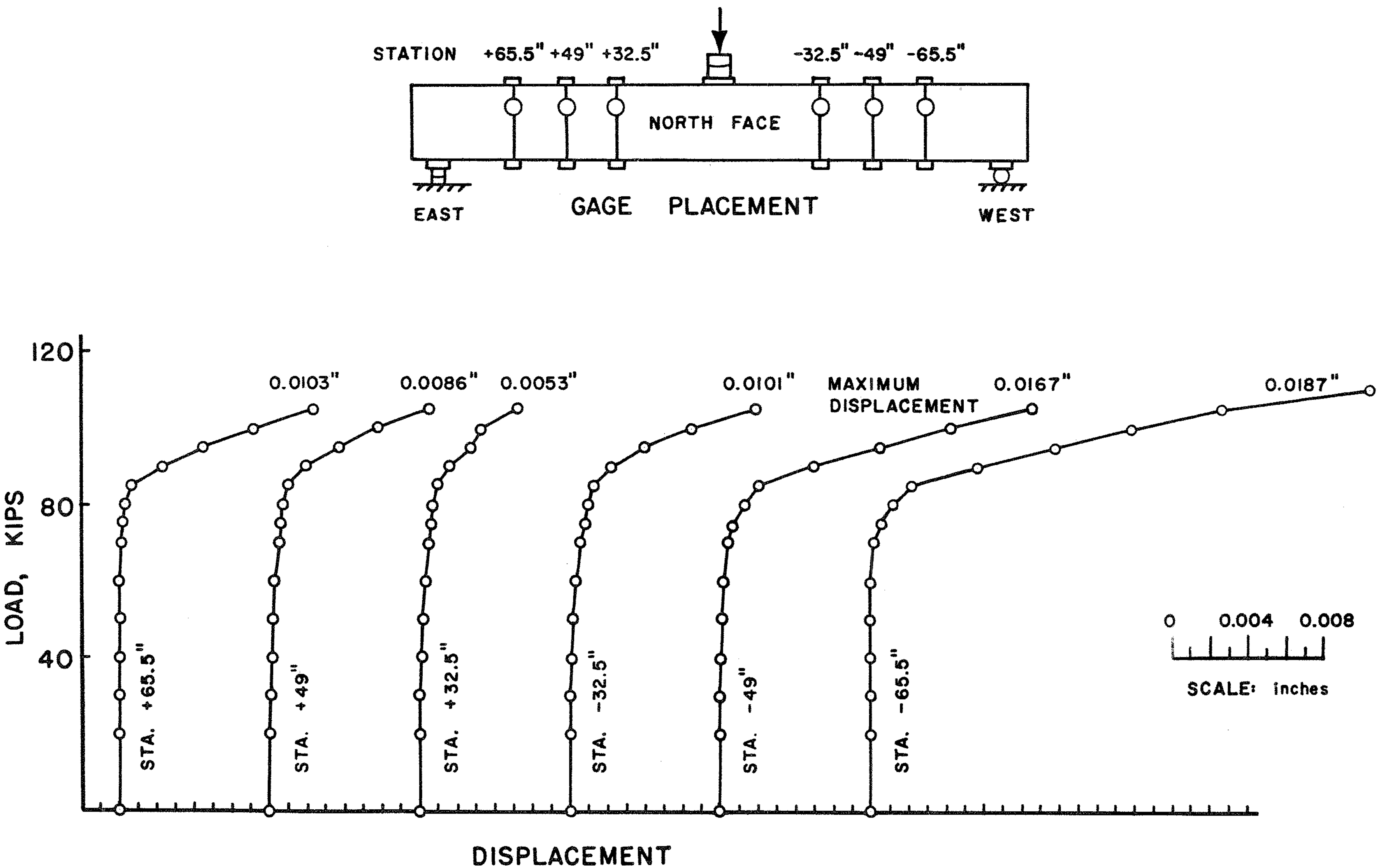


FIG. 10-E YOKE DATA — BEAM A-2

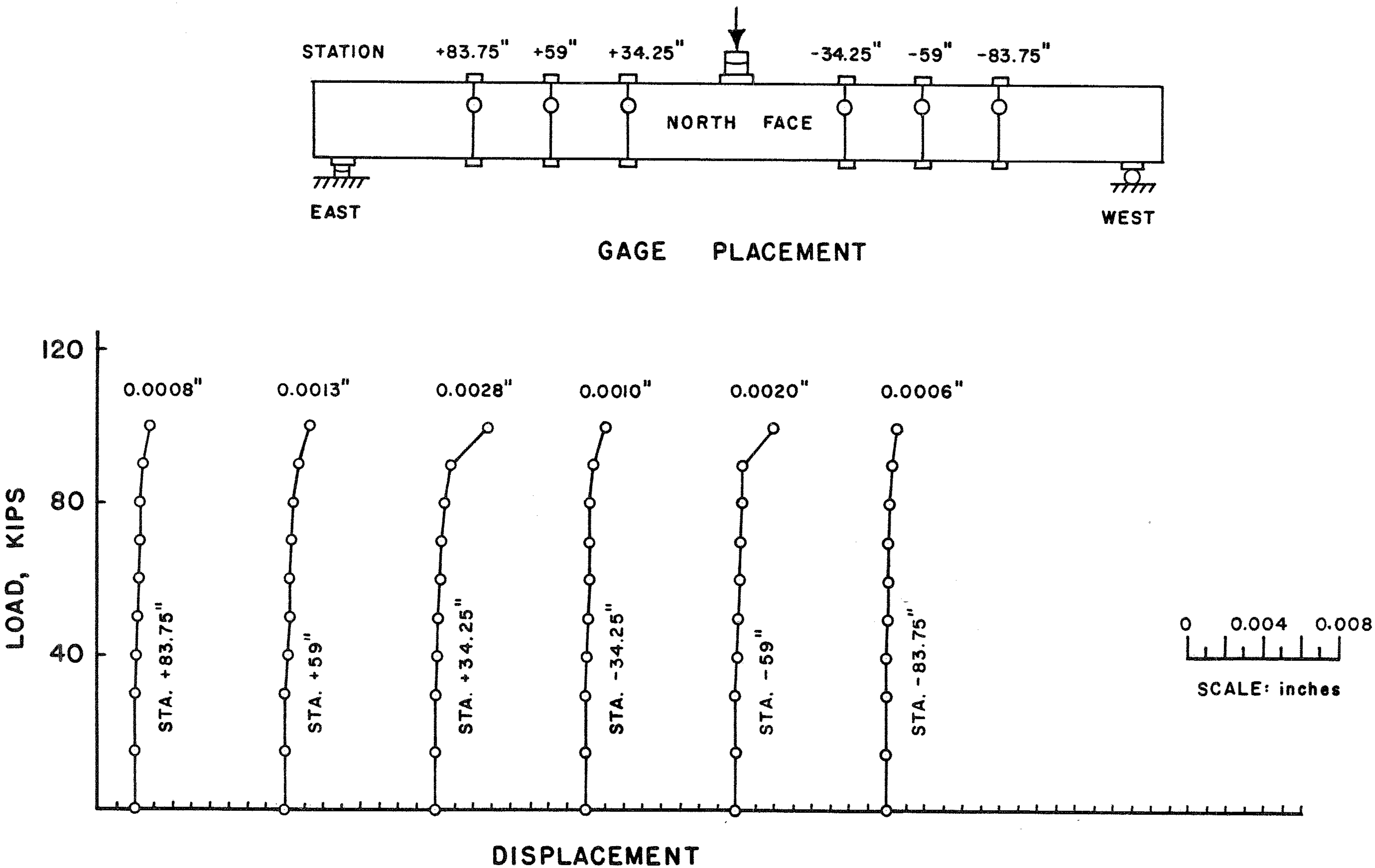


FIG. 10-F YOKE DATA - BEAM A-3

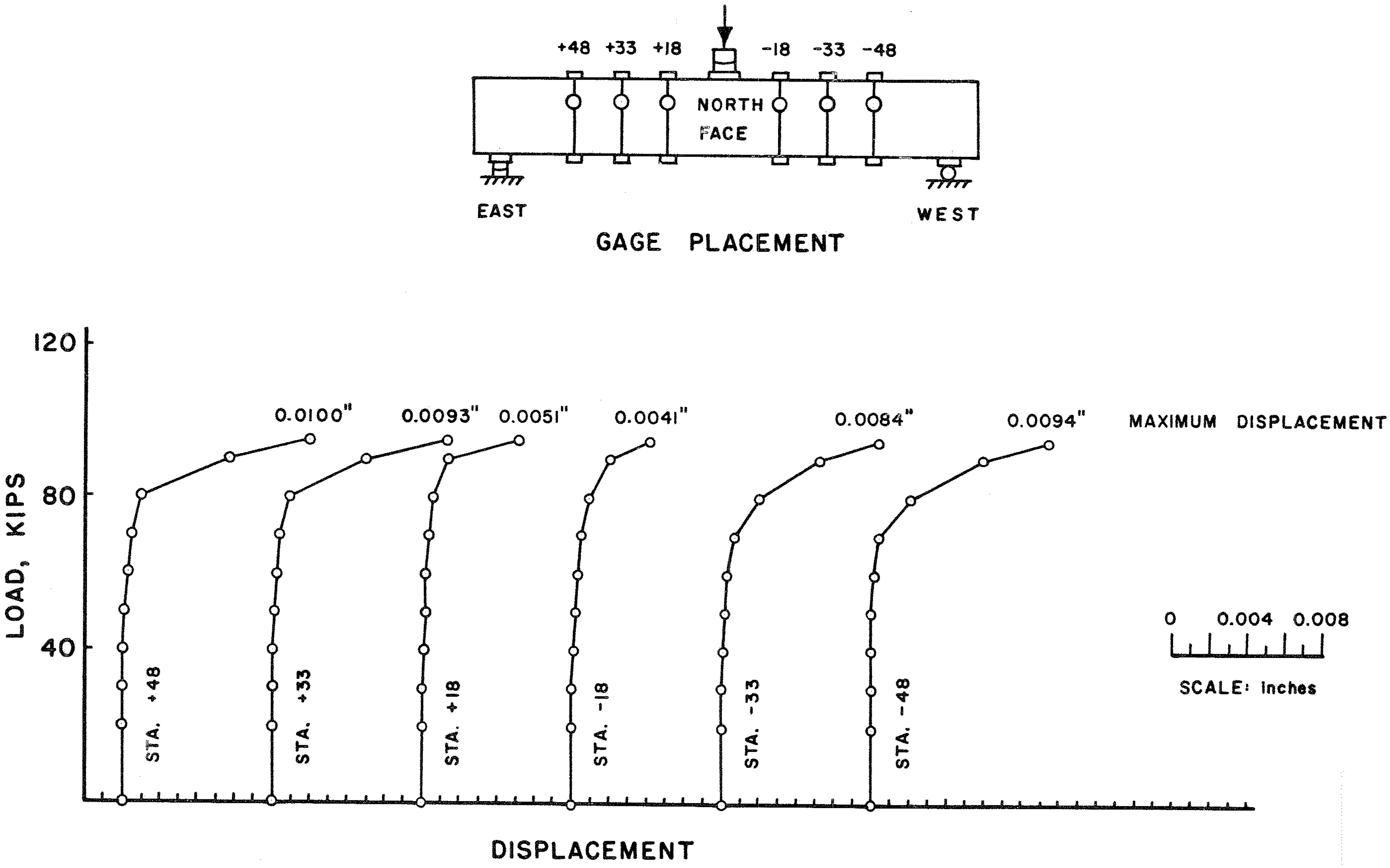


FIG. 10-G YOKE DATA—BEAM B-1

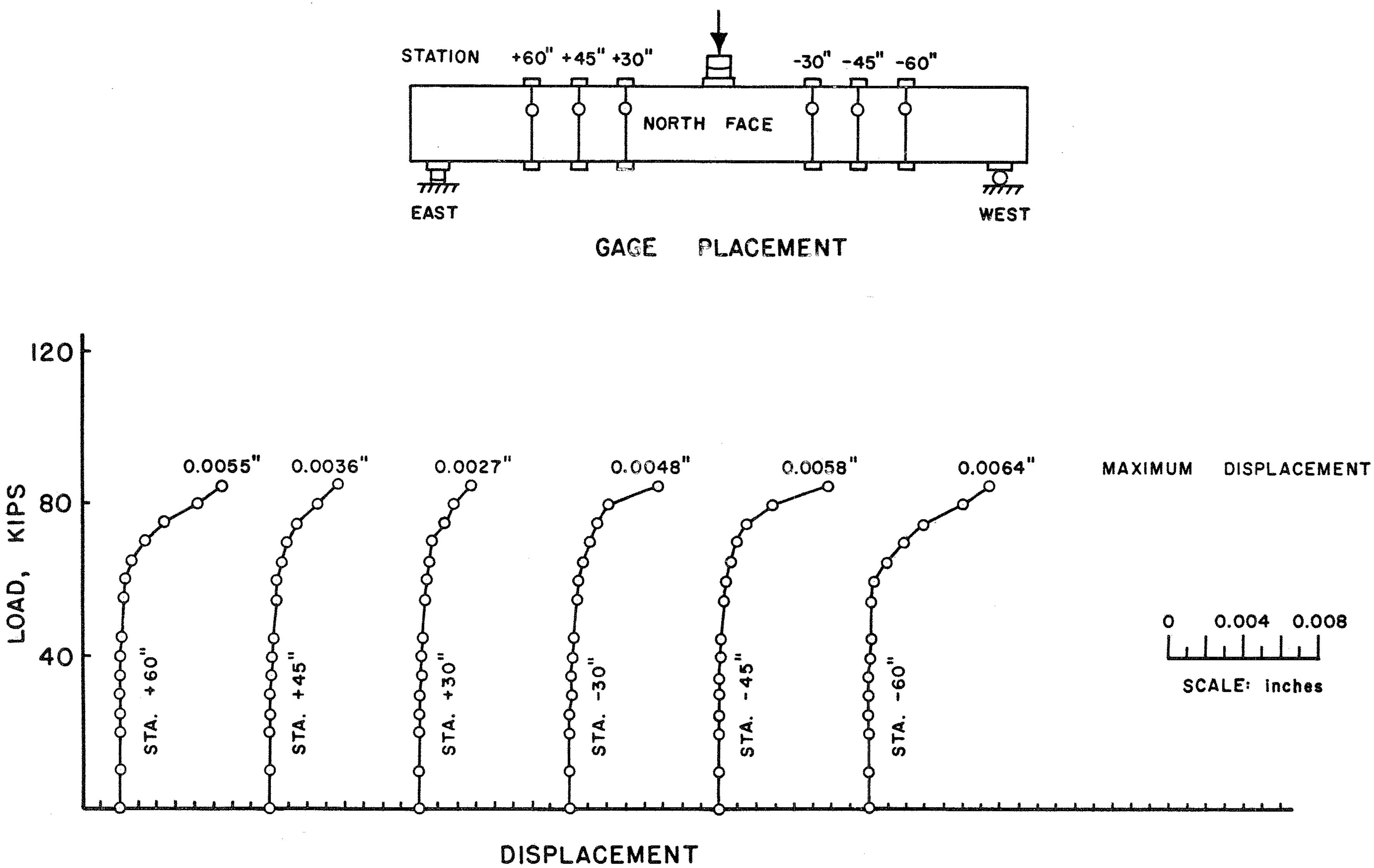


FIG. 10-H YOKE DATA - BEAM B-2

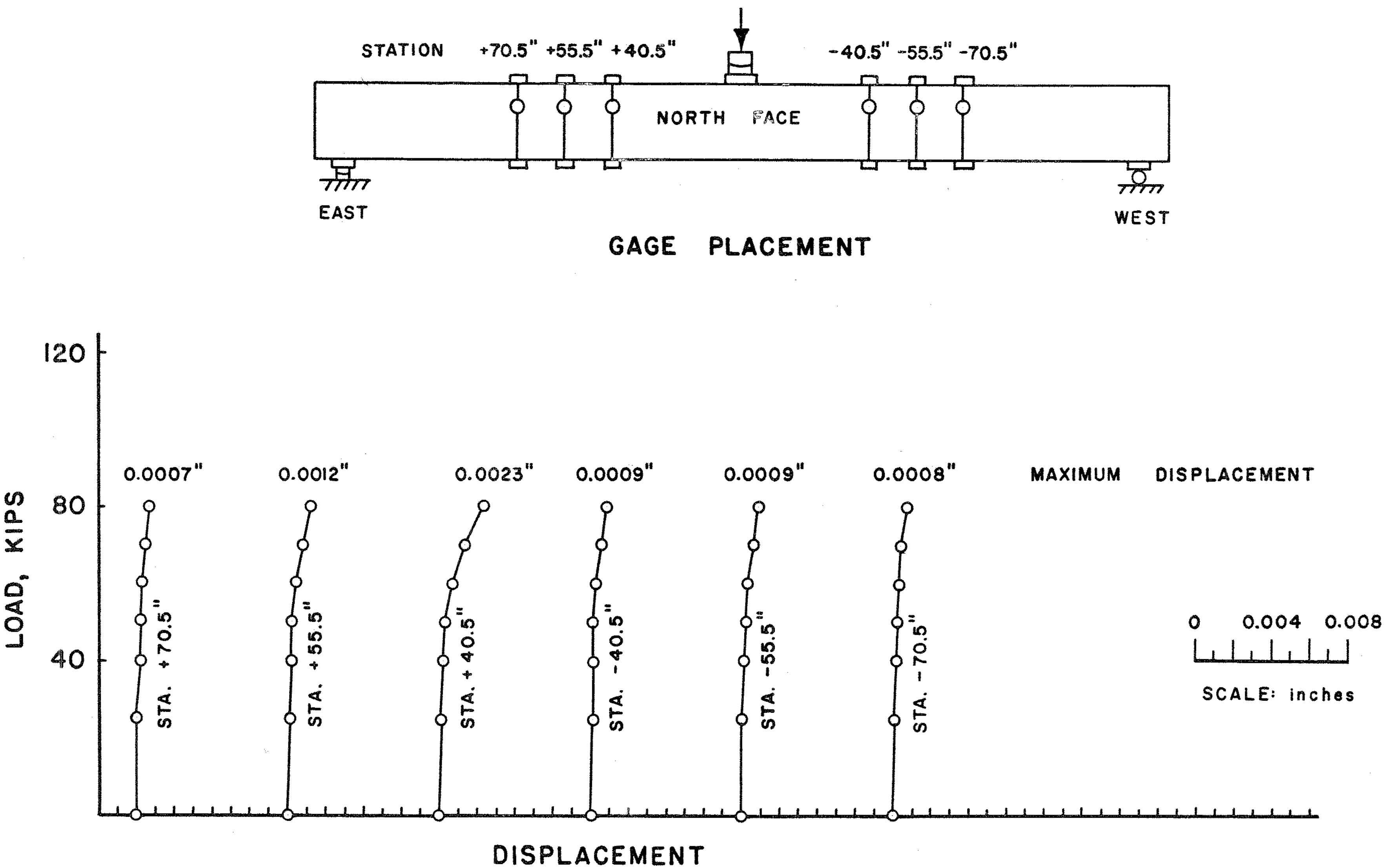


FIG. 10-I YOKE DATA - BEAM B-3

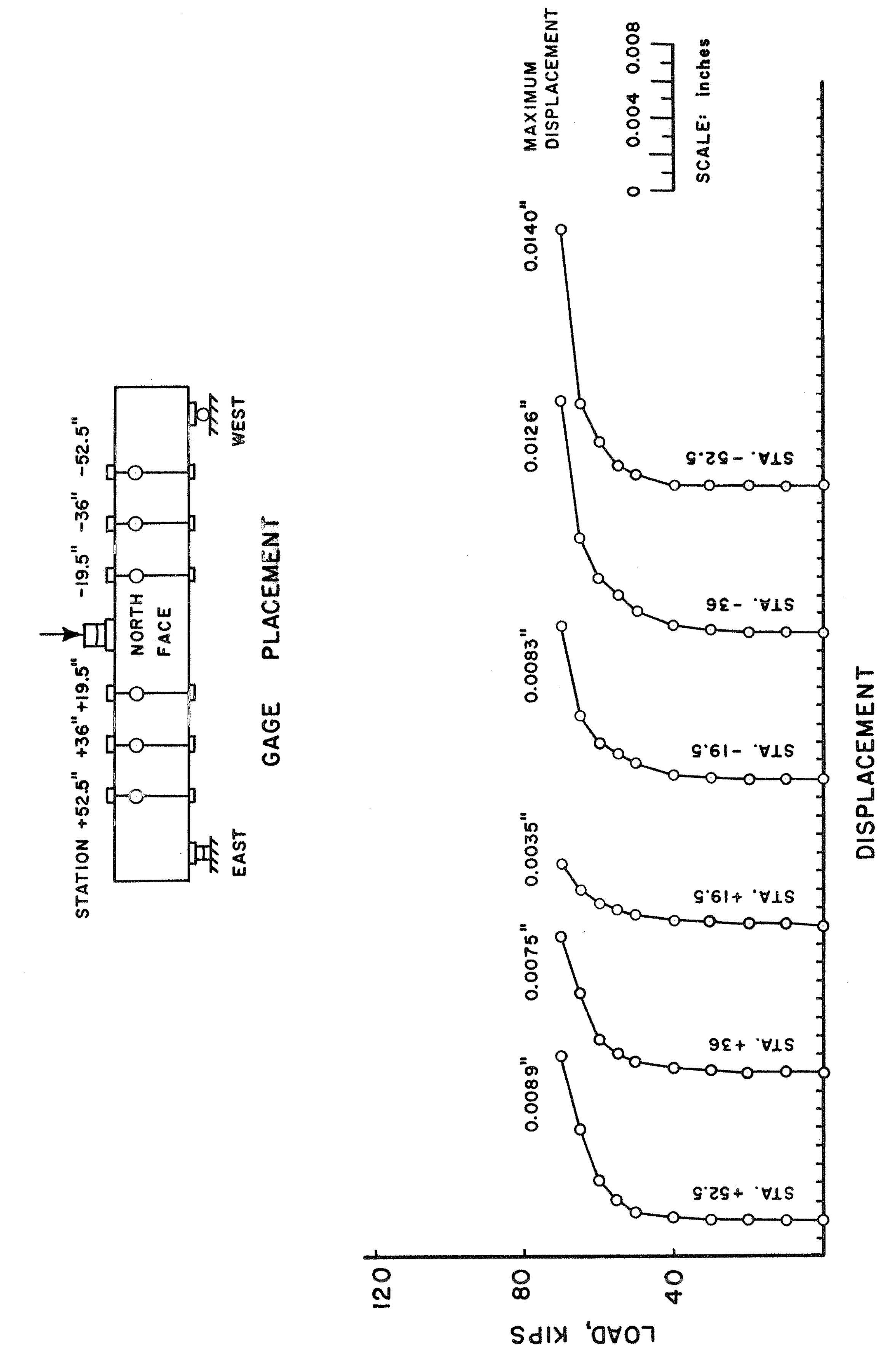


FIG. 10-J YOKE DATA - BEAM C-I

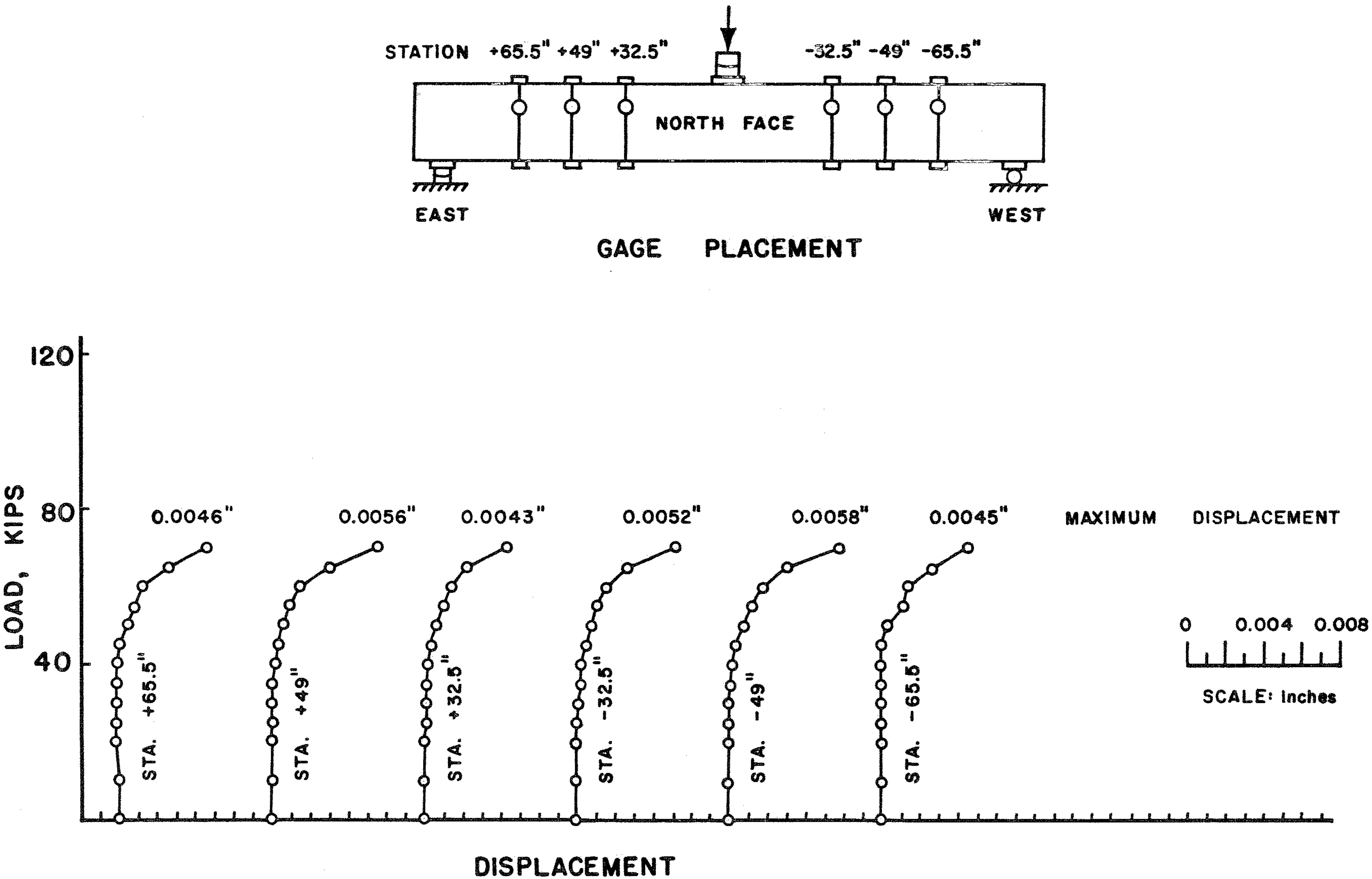
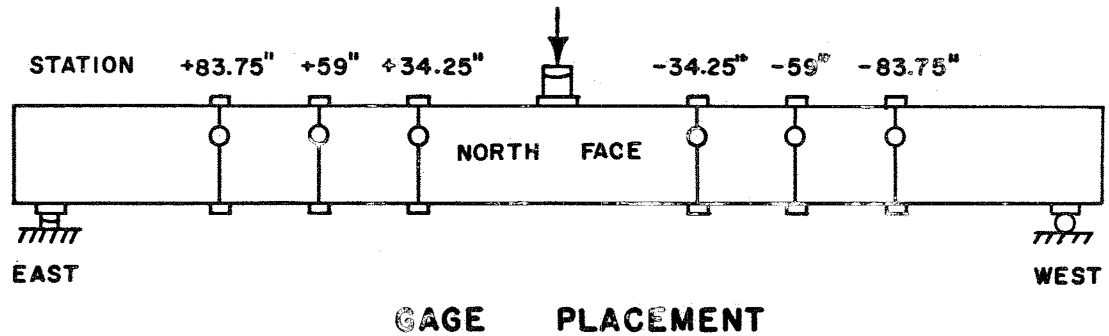


FIG. 10-K YOKE DATA — BEAM C-2



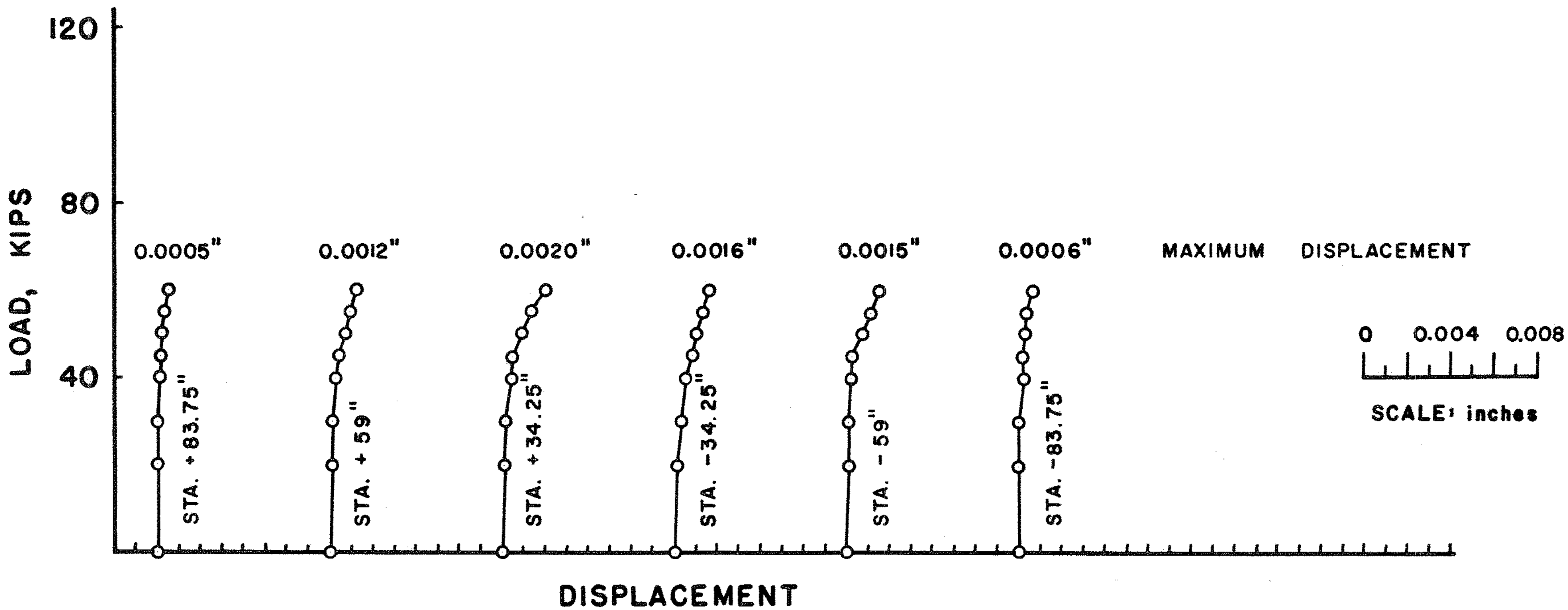


FIG. 10-L YOKE DATA — BEAM C-3

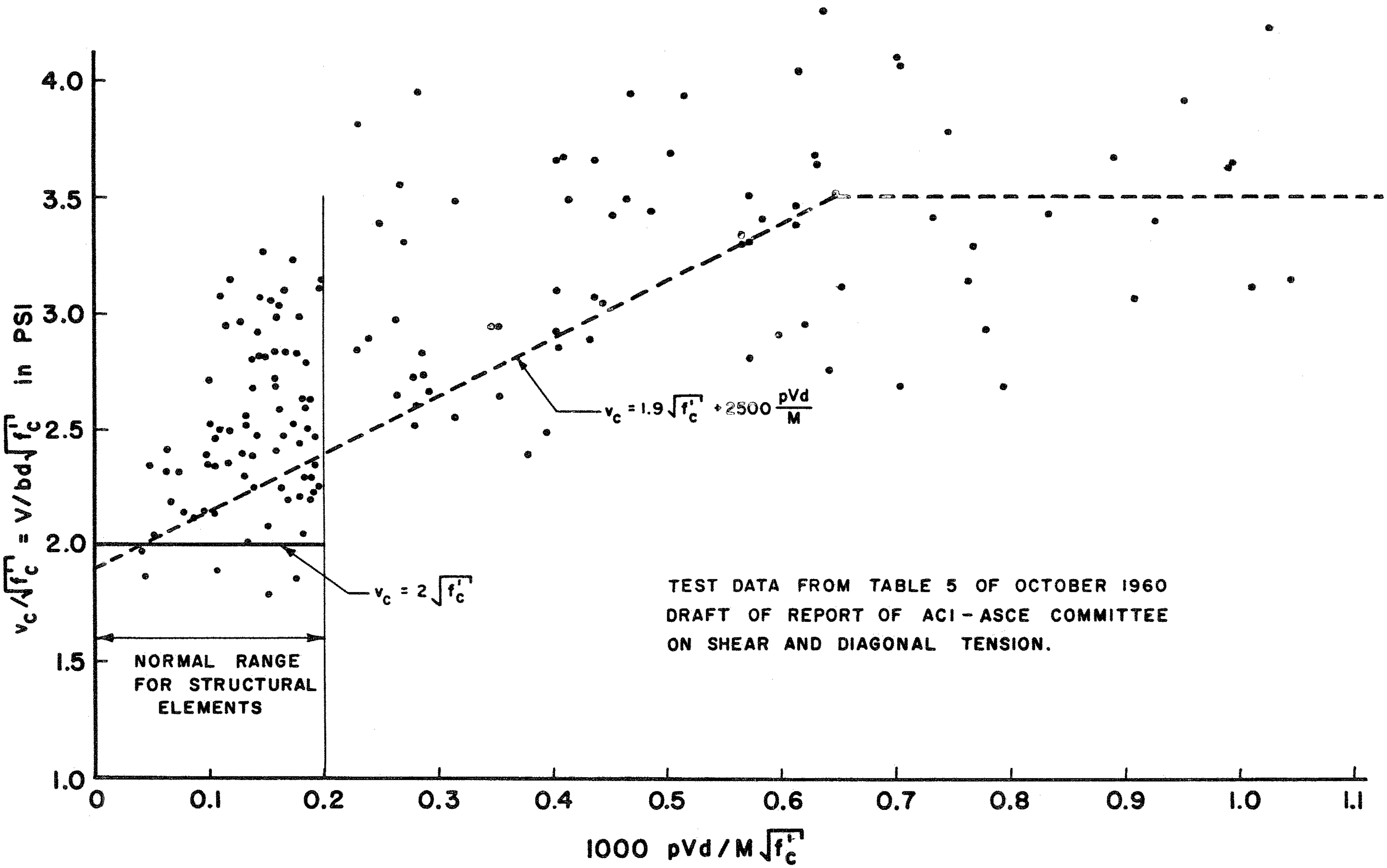
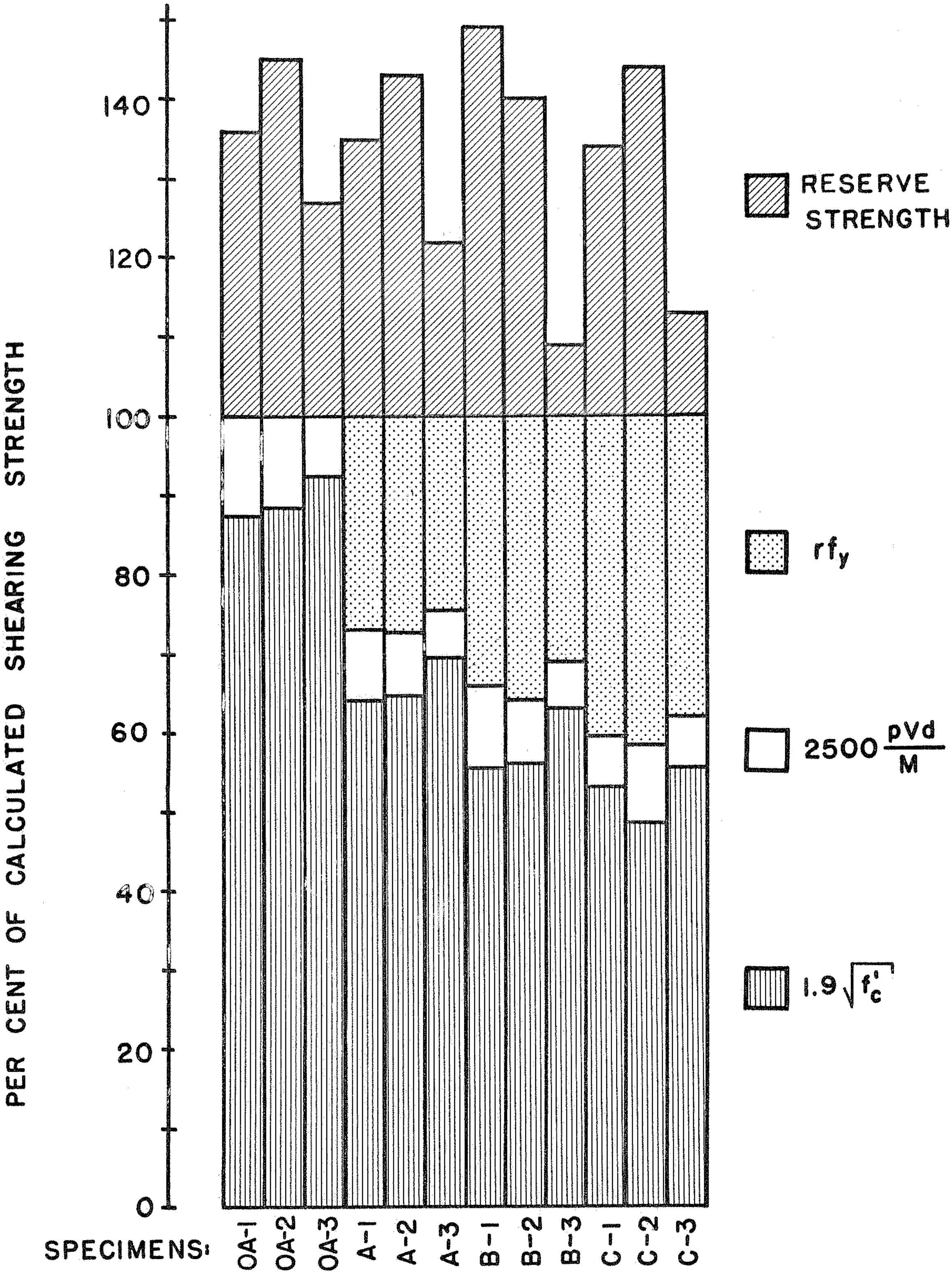


FIG. 11 COMPARISON OF TEST DATA WITH PROPOSED DESIGN EQUATIONS



$$V_{calc.} = bd\left(1.9\sqrt{f'_c} + 2500\frac{pVd}{M} + rf_y\right)$$

FIG. 12 COMPARISON OF CALCULATED AND TEST VALUES OF ULTIMATE SHEARING STRENGTH