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# MEAN PION MULTIPLICITY OF THE HIGH MISSING MASS*

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## ABSTRACT

The mean pion multiplicity of the high value missing mass is investigated from a realistic version of the ABFST multiperipheral model. It is shown that, for small invariant momentum transfer, the multiplicity is proportional to the logarithm of the missing mass. This can be tested experimentally at a single energy of the big accelerators.



## I. INTRODUCTION

Methods of searching for hadron states are conventionally categorized into resonance formation experiments and resonance production experiments. In the latter category, for instance in the $\pi p \to Xp$, one detects the recoil proton in order to search for bumps in the mass distribution of the object X. In this example, we have $X = \pi$, a single pion, and next $X = \rho$ which decays into two pions, and next $X = A_1$ which decays (finally) into three pions, and so on. When we move to higher and higher values of the mass of X (the "missing mass" $m^{*2}$), there are more and more resonance states produced which might overlap each other. Thus the states are so overlapping that we might not bother to distinguish which is which of the individuals. Then we may ask the question: For a given value of $m^{*2}$, what is $\bar{N}$, the mean number of pions "decaying" from it?

In this paper we shall show, with the ABFST multiperipheral model,[1] that in the other extreme end of the spectrum, we have the simple law

$$\bar{N} \underset{(m^{*2})^{-\frac{1}{2}} \to 0}{\sim} C \, \ell n \, m^{*2} + \text{constant} . \qquad (1.1)$$

In this equation C is just the coefficient in the corresponding law for the better-known case of resonance formation experiments.

Notice that Eq. (1.1) can be tested at a single energy, provided that the energy is high enough to give a large range of the value of $m^{*2}$.

Let us next study the kinematic aspect of the problem before we go to the model derivation of the law.

## II. KINEMATICS

Consider Fig. 1 for the process $ab \to Xd$. (The line e in this figure is just a symbol for the momentum transfer t. It need not be associated with an exchanged virtual particle at this point.) We now investigate, for a fixed total energy s, the maximum value of the missing mass $m^{*2}$ as a function of t.

Let the rapidity difference between the rest frame of particle a and the rest frame of particle b be $\eta$, and the rapidity difference between the rest frame of particle a and the "rest frame" of "particle e" be $q^a$. [By the "rest frame" of "particle e" we mean the Lorentz frame in which the momentum of e takes the form $(p^0,p^1,p^2,p^3) = \left(0,0,0,(-t)^{\frac{1}{2}}\right)$.] And similarly we have the definition for $q^b$. The two "rest frames" of "particle e" have their rapidity difference $\xi$ [an O(1,2) rotation angle]. These quantities follow the BCP trigonometry:[2]

$$\cosh \eta = \cosh q^a \cosh q^b \cosh \xi + \sinh q^a \sinh q^b , \qquad (2.1)$$

where, in terms of invariant variables,

$$\cosh \eta = \frac{s - m_a^2 - m_b^2}{2m_a m_b} , \qquad \sinh \eta = \frac{\Delta^{\frac{1}{2}}(s,m_a^2,m_b^2)}{2m_a m_b} , \qquad (2.2)$$

$$\cosh q^a = \frac{\Delta^{\frac{1}{2}}(m^{*2},m_a^2,t)}{2m_a(-t)^{\frac{1}{2}}} , \qquad \sinh q^a = \frac{m^{*2} - m_a^2 - t}{2m_a(-t)^{\frac{1}{2}}} , \qquad (2.3)$$

$$\cosh q^b = \frac{\Delta^{\frac{1}{2}}(m_d^2,m_b^2,t)}{2m_b(-t)^{\frac{1}{2}}} , \qquad \sinh q^b = \frac{m_d^2 - m_b^2 - t}{2m_b(-t)^{\frac{1}{2}}} , \qquad (2.4)$$



where $\Delta(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2zx$. At the boundary of the phase space, $\xi = 0$. Thus from Eq. (2.1) we get

$$m_{max}^{*2} = m_a^2 + t + 2m_a(-t)^{\frac{1}{2}}(\sinh \eta \cosh q^b - \cosh \eta \sinh q^b) . \qquad (2.5)$$

When $a = \pi$, $b = p$, $d = p$, and s is much greater than the masses and t, we have the good approximate expression

$$m_{max}^{*2} \approx \frac{(-t)^{\frac{1}{2}}s}{2M^2} [(4M^2 - t)^{\frac{1}{2}} - (-t)^{\frac{1}{2}}] . \qquad (2.6)$$

This boundary at $s = 400$, 1000, and 2500 GeV$^2$ is plotted in Fig. 2. Thus, when $s = 1000$ GeV$^2$ and $t = -0.3$ GeV$^2$ for example, we can vary $m^{*2}$ from $\mu^2 = 0.02$ GeV$^2$ to 440.0 GeV$^2$.

## III. THE MODEL

To be specific we shall consider the process $\pi p \to Xp$. It will become clear from the model presented below that essentially the same conclusion applies for all processes ($\pi p \to \pi X$, $pp \to Xp$, etc.).

Assume the production processes to be given by Fig. 3a and b. Then the missing mass differential cross sections for the production of $n$ pions, when expressed in terms of invariant variables,[3] are, for

$n$ = even number :

$$\frac{d^2\sigma_n}{dm^{*2}\,d(-t)} = \sum_i \frac{1}{16\pi^3 s^2}\,\pi(-t)g_i^{\,2}\,F(t)\,\frac{A_n(m^{*2},0;\,t)}{(t-\mu^2)^2}\,, \qquad (3.1)$$

where $\sum_i$ = the sum performed over the charge state of the left-most exchanged pion,

$g_i^{\,2}$ = $14.5 \times 4\pi$ for $\pi^0 pp$ coupling,

$F(t)$ = the off-shell form factor for the $\pi pp$ vertex, and we shall use the Dürr-Pilkuhn form factor[4]

$$F(t) = \left(\frac{r^{-2}-\mu^2}{r^{-2}-t}\right) \text{ with}$$

$r^{-2} = 10\mu^2$, based on the experience[5] in low-energy singly peripheral process,

$A_n$ = the imaginary part of the (off-shell) amplitude for producing $n\pi$'s in $\pi$-$\pi$ scattering, the normalization is $A_n = \Delta^{\frac{1}{2}}\sigma_n$.

And, for $n$ = odd number:



$$\frac{d^2\sigma_n}{dm^{*2}\,d(-t)} = \sum_i \frac{1}{16\pi^3 s^2}\int_{x^-}^{x^+} dx \int_{\tau^-}^{\tau^+} d\tau \int_{s_B^-}^{s_B^+} ds_B$$

$$\times\ \frac{|T_i(s_B,t;\,\tau)|^2 A_{n-1}(s'',0;\,\tau)}{16\pi^2(\tau-\mu^2)^2[(s_B^+-s_B)(s_B-s_B^-)]^{\frac{1}{2}}}\,, \qquad (3.2)$$

where $T_i$ = the $\pi p$ elastic amplitude,

$$x \equiv \frac{s''}{m^{*2}}\,, \qquad (3.3)$$

$$x^- = \frac{4\mu^2}{m^{*2}}, \qquad x^+ = \left[1-\left(\frac{\mu^2}{m^{*2}}\right)^{\frac{1}{2}}\right]^2, \qquad (3.4)$$

$$\tau^{\mp} = t+\mu^2 - \frac{(m^{*2}+t-\mu^2)(m^{*2}+\mu^2-s'')}{2s}$$

$$\mp\,\frac{\Delta^{\frac{1}{2}}(m^{*2},t,\mu^2)\,\Delta^{\frac{1}{2}}(m^{*2},\mu^2,s'')}{2s}\,, \qquad (3.5)$$

$$s_B^{\mp} = M^2 + \tau - \frac{(s-\mu^2-M^2)(s''-\mu^2-\tau)}{2\mu^2}$$

$$+\,\frac{2\mu^2}{\Delta^{\frac{1}{2}}(s',\mu^2,t)}\left[(M^2-t-M^2)+\frac{(s-\mu^2-M^2)(m^{*2}-\mu^2-t)}{2\mu^2}\right]$$

$$\times\left[(\mu^2-t-\tau)+\frac{(m^{*2}-\mu^2-t)(s''-\mu^2-\tau)}{2\mu^2}\right]$$

[Equation (3.6) continued, next page]

[Equation (3.6) continued]

$$\mp 2\,\frac{\{\Delta(s,\mu^2,M^2)\Delta(m^{*2},\mu^2,t)-[2\mu^2(M^2-t-M^2)+(s-\mu^2-M^2)(m^{*2}-\mu^2-t)]\}^{\frac{1}{2}}}{2\mu\Delta^{\frac{1}{2}}(s',\mu^2,t)}$$

$$\times\,\frac{\{\Delta(m^{*2},\mu^2,t)\Delta(s'',\mu^2,\tau)-[2\mu^2(\mu^2-t-\tau)+(m^{*2}-\mu^2-t)(s''-\mu^2-\tau)]\}^{\frac{1}{2}}}{2\mu\Delta^{\frac{1}{2}}(m^{*2},\mu^2,t)}\,. \tag{3.6}$$

Note that we do not assume $s_B$ to be large, its range is determined by this limit of integration which is in turn determined by other variables. Up to this point we have only used essentially the Deck model for $d^2\sigma_n/[dm^{*2}d(-t)]$, because we have not yet specified what the mathematical expression is for $A_m$ in Eq. (3.1) and Eq. (3.2). Now the ABFST multiperipheral model prescribes $A_m$ to be the structure shown in Fig. 3. We may then calculate $\bar{n}(m^{*2},t)$, the mean $\pi$-multiplicity detected around $m^{*2}$ and $t$, from $d^2\sigma/[dm^{*2}d(-t)]$ according to the method of the model. The expression for $d^2\sigma/[dm^{*2}d(-t)]$ is obtained by summing Eq. (3.1) and Eq. (3.2) with $A_m$ replaced by $A$, the forward imaginary part of the (off-shell) elastic amplitude.

Now since we are interested in the asymptotic form for $\bar{n}(m^{*2},t)$ we need only the high energy part of $A$, which contains the Pomeranchuk-pole term alone. Thus we use

$$A(s,0;\,u)\;\sim\;\tilde{b}_\alpha(0;\,u)\left(\frac{s}{s_0}\right)^{\alpha}, \tag{3.7}$$



where $\alpha = \alpha(0)$, the intercept of the Pomeranchuk-pole,

$\tilde{b}_\alpha$ = the imaginary part of the corresponding residue,

$s_0$ = the scale factor,

$u$ = the mass of the virtual pion.

We assume that the expression thus obtained for $d^2\sigma/[dm^{*2}d(-t)]$ is good for the region of phase space with, say, 20 $\text{GeV}^2 \lesssim m^{*2} \lesssim s$. We may emphasize at this point that such a model is known to be able to account for many experimental results. (See further discussion of this below.)

But here we concern ourselves only with the multiplicity of the high missing mass and we readily deduce the result:

$$\bar{n}(s,m^{*2},t)\;\underset{\substack{\text{fixed } s\\ (m^{*2})^{-\frac{1}{2}}\to 0}}{\sim}\;C\left\{\ell n\left(\frac{m^{*2}}{s_0}\right)+R(s,m^{*2},t)\right\}+1\,. \tag{3.8}$$

The coefficient $C$ will be discussed later. The remainder term $R$ involves a quotient of integrals. The numerator integral has factor $\partial\tilde{b}_\alpha/\partial\alpha$ in the integrand. All the $t$ dependence is contained in $R$. In the following, with additional specifications of the model, we shall demonstrate that $R$ is independent of $m^{*2}$ (and $s$). This will be the most elaborate part of our work.

Let us first examine two simple cases: From a $\phi^3$ type model with factorizable approximation to the kernel, we can see immediately that the corresponding $R$ is only a function of $t$. Or, if we assume a Regge form for the elastic $\pi p$ amplitude $T \sim (s_B/s_0)^{\alpha(t)}$ {thus $d^2\sigma/[dm^{*2}d(-t)]$ is represented by a triple-Regge formula}, then again we can see immediately that $R$ is only a function of $t$.

Thus the conclusion we hope to establish for our model is also true in these simple cases, we, however, would not satisfy ourselves with these arguments alone. For, in the first case we cannot supply any logical inference which enables us to apply the conclusion from an unphysical $\phi^3$ model to our much more realistic model or to the physical situation. In the second case the assumption $s_B/s_0 \gg 1$ [thus $s/m^{*2} \gg 1$ from Eq. (3.6)] holds only for a rather small region of the phase space. And since duality (in the sense of Finite Energy Sum Rule) does not apply for the full elastic amplitude, we are not confident to push the triple-Regge formula to other regions of the phase space where $s/m^{*2} \gtrsim 1$.

To see the functional behavior of $R$ in Eq. (3.8), there seems no substitute for a direct integration. To this end we make these additional specifications:

1) The high-energy off-shell amplitude $A$ is taken to be Eq. (3.7) with

$$\tilde{b}_\alpha(0;\, u) = \bar{\beta}_\alpha(0)\left(\frac{s_0}{s_0 - u}\right)^{\alpha+1}, \qquad (3.9)$$

$\alpha = \alpha(0) = 1$, and we shall use $s_0 = 1\ \mathrm{GeV}^2$ (this scale factor is related to the masses of the prominent $\pi$-$\pi$ resonances). We don't have to specify explicitly the functional form for $\bar{\beta}_\alpha(0)$ at this stage. This form of Eq. (3.9) is suggested by the approximated solution to the ABFST integral equation.[6]

2) The off-shell $\pi p$ elastic amplitude is taken, for simplicity and reality, to be



$$T_i(s_B, t;\, \tau) = T_i(s_B, 0)\, e^{\gamma t}\left(\frac{s_0}{s_0 - \frac{1}{2}(\tau + \mu^2 - \frac{t}{2})}\right)^{\bar{\alpha}+1}. \qquad (3.10)$$

In this expression $T_i(s_B, 0)$ is the on-shell forward $\pi p$ elastic amplitude, for which we shall use the experimental distribution.[7] The factor $e^{\gamma t}$ is suggested by experimental parametrization of the t-dependence, and we shall use $\gamma = 4\ \mathrm{GeV}^{-2}$. The off shell factor is again suggested by the approximate solution of Ref. 6. This factor, with $\alpha$ taken to be 0.7, represents the gross feature of the off-shell dependence of both the Pomeranchuk and non-Pomeranchuk part of the amplitude.

With Eqs. (3.9) and (3.10) in $d^2\sigma/[dm^{*2}\, d(-t)]$, it has been shown to be very successful to account for the magnitude and variable dependences of the missing mass distribution.[8]

So the remainder term in Eq. (3.8) becomes

$$R(s, m^{*2}, t) = \tilde{R}(s, m^{*2}, t) + \frac{1}{\bar{\beta}_\alpha}\frac{\partial \bar{\beta}_\alpha}{\partial \alpha}, \qquad (3.11)$$

where

$$\tilde{R}(s, m^{*2}, t) = \frac{U(s, m^{*2}, t)}{L(s, m^{*2}, t)}, \qquad (3.12)$$

$$U(s,m^{*2},t) = \sum_i \pi(-t)\, g_i^2\, F(t)\, \frac{1}{(t-\mu^2)} \left(\frac{s_0}{s_0 - t}\right)^{\alpha+1} \ln\left(\frac{s_0}{s_0 - t}\right)$$

$$+ \sum_i \int dx \int d\tau \int ds_B \frac{|T_i(s_B,t;\,\tau)|^2\, x^{\alpha}\left(\frac{s_0}{s_0-\tau}\right)^{\alpha+1}}{16\pi^2(\tau-\mu^2)^2[(s_B^+ - s_B)(s_B - s_B^-)]^{\frac{1}{2}}} \ln x\left(\frac{s_0}{s_0-\tau}\right); \tag{3.13}$$

$L(s,m^{*2},t)$ has the same expression but without the $\ln$ factors.

With all these inputs, $\tilde{R}(s,m^{*2},t)$ is computed numerically for different values of $m^{*2}$ with $s = 2500$ GeV$^2$ and $t = -0.1, -0.2, -0.3$, and $-0.4$ GeV$^2$. The results are shown in Fig. 4. We have also computed $\tilde{R}$ for different values of $m^{*2}$ with $s = 400$, and 1000 GeV$^2$, and the same four values of $t$. Those results are practically the same as that shown in Fig. 4, except of course that for the same value of $t$ and smaller $s$ we have a smaller upper limit for $m^{*2}$.

If we are willing to go to a larger value of -t, we certainly have a longer range of the missing mass $m^{*2}$ for a given $s$. (On the other hand, extension of the multiperipheral model prediction to -t values beyond 1 or 2 GeV$^2$ is doubtful.) Experimentally, the study of $\bar{n}$ for larger -t values is just as simple as for the smaller -t values, except there are less events for the former case.

We close this section with the following conclusions:

1) $\tilde{R}$ is (practically) independent of $m^{*2}$ and $s$.

2) The $t$ variation of $\bar{n}$ is quite slow, at least for those $t$ values we have studied. This variation seems independent of $m^{*2}$ and $s$.



3) If we add up all the $t$ intervals, we have the similar law: $\bar{N} \sim C \ln m^{*2}$ + const.

## IV. DISCUSSION AND CONCLUSION

It may be illuminating to separate the predictions from Eq. (3.8) into two classes. The first class comprises those qualitative predictions such as the $\ln m^{*2}$ dependence, the functional behavior of $R$, etc. These properties may be commonly true for a general scheme known as the multiperipheral model. The second class comprises those quantitative predictions such as the magnitude of the coefficient $C$, the numerical value of $R$, etc. These properties depend on the detail features of the particular version of the multiperipheral model under consideration. The ABFST model we used here, if we make a factorizable approximation to the meson-meson resonance kernel of the integral equation,[6] then we can readily give a crude estimate of the magnitudes:

$$C = 2\left[-\frac{\partial\kappa}{\partial\lambda}\right]^{-1}_{\lambda=\alpha(0)=1} \approx \frac{3}{4}, \tag{4.1}$$

and

$$\frac{1}{\bar{\beta}_\alpha}\frac{\partial\bar{\beta}_\alpha}{\partial\alpha} \approx \frac{43}{12}, \tag{4.2}$$

where $\kappa$ appears in the Fredholm determinant $D(\lambda) = 1 - \kappa(\lambda)$. These magnitudes may not be very far away from experimental values. [From Eq. (4.1) we get $C_{charge} \approx \frac{1}{2}$ which seems about two times too small.]

Next, let us emphasize that what we have done in this study is to transform the experimental observation of the interaction from the system of the two particle a and b to the system of the particle a and the "particle" e. Theoretically we can make successive transformations along the multiperipheral chain (visually along the gaps in between the produced particles in the rapidity plot). Whenever we fix the distances from the two ends and observe the interaction in between, we shall see Regge behavior of the subcross section and $\ln$ growth of the corresponding multiplicity as that subenergy variable increases. In practice we can make only one or two transformations because of the difficulty of detecting the neutral particles as well as achieving high energy in those subsystems.

In conclusion, we have established theoretically that the mean pion multiplicity of the missing mass is asymptotically a $\ln$ function of the value of the missing mass (we have neglected the 10% production of other particles). This can be tested at a single energy of the accelerators now beginning operation or of the future ones. If the observation turns out to be substantially different from this law, that will by itself be a great discovery. On the other hand, an accurate verification of this law will be another big triumph for the multiperipheral model.

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## FOOTNOTES AND REFERENCES

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+ Member of the Mathematics and Computing Group; participating guest in the Theoretical Physics Group.

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## FIGURE CAPTIONS

Fig. 1. Kinematics.

Fig. 2. Physical boundary of Eq. (2.6) of the missing mass experiment.

Fig. 3. Production amplitude of $\pi p$ scattering of

(a) even number of final $\pi$'s, and

(b) odd numbers of final $\pi$'s.

Fig. 4. $\tilde{R}$ [in Eq. (3.12)] as a function of $m^{*2}$ and $t$.

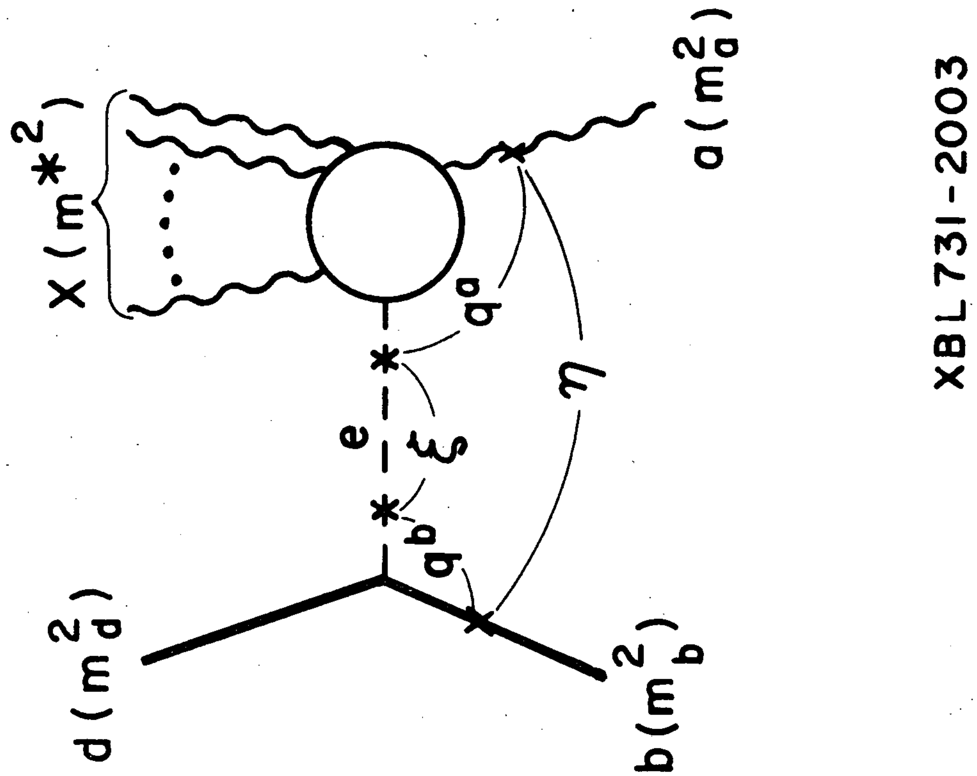
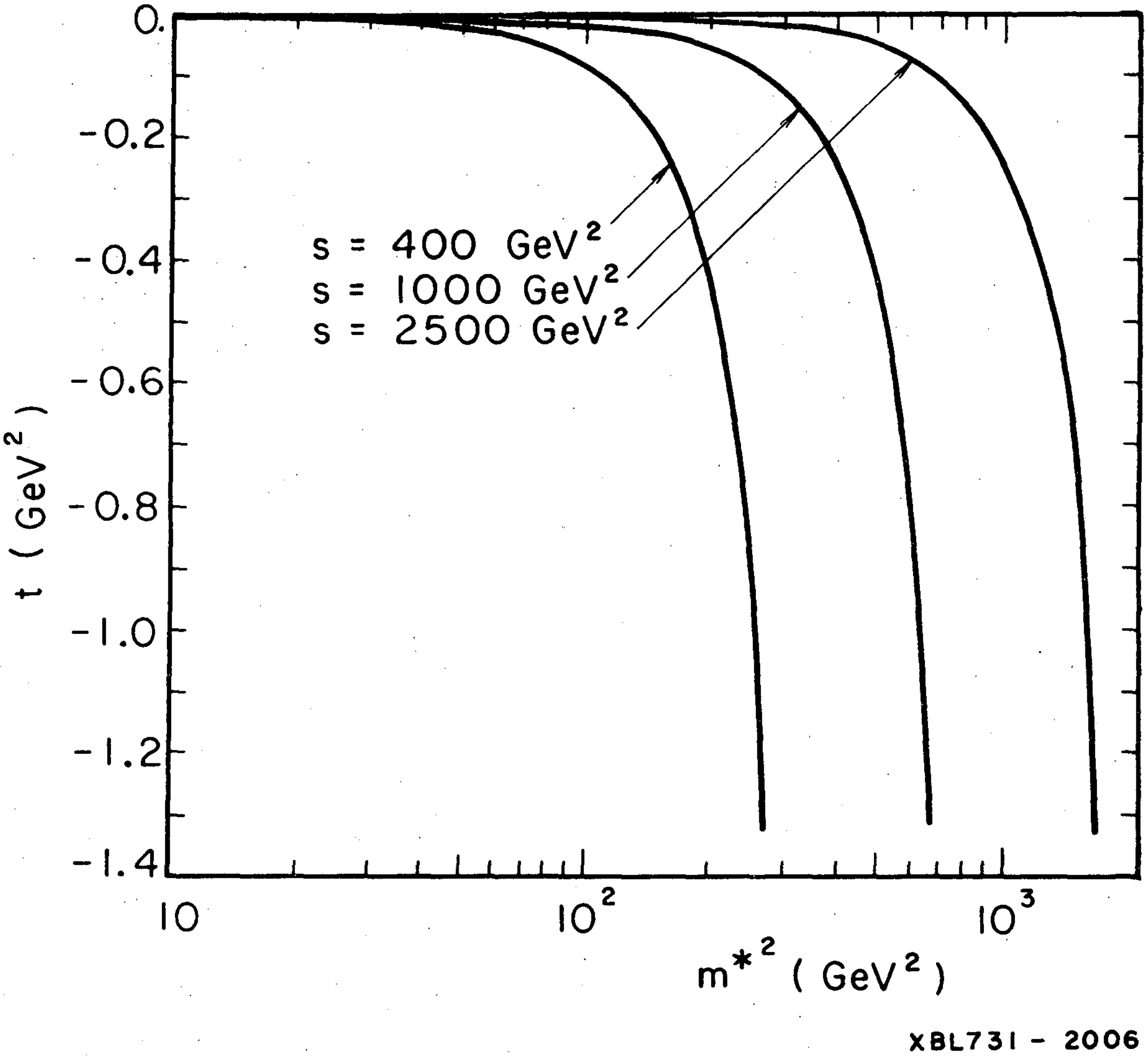



Fig. 1



0.
-0.2
-0.4
-0.6
-0.8
-1.0
-1.2
-1.4
t ( GeV²)
10
10²
10³
m*² ( GeV² )
s = 400 GeV²
s = 1000 GeV²
s = 2500 GeV²
XBL731 - 2006


Fig. 2

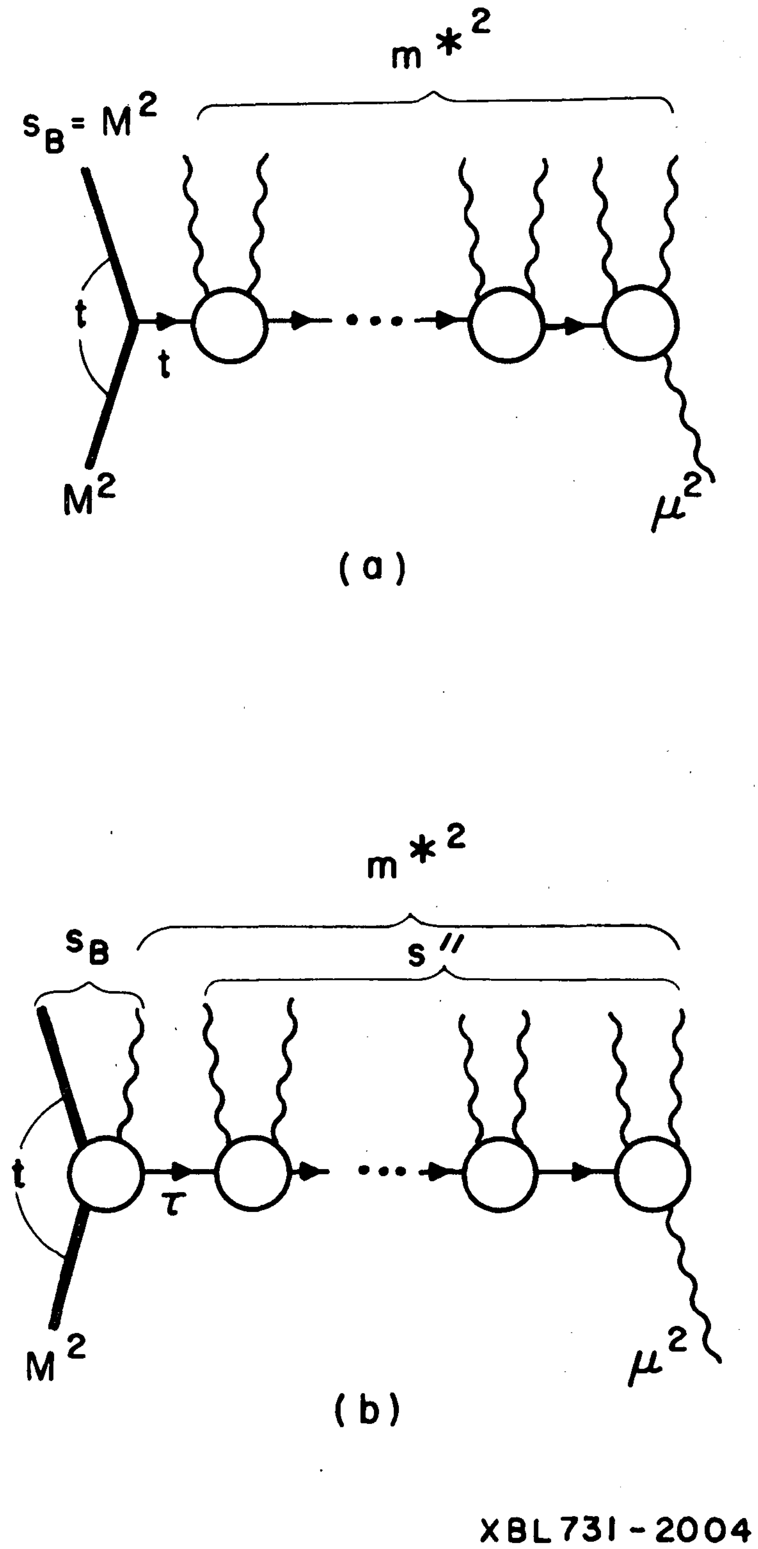



Fig. 3

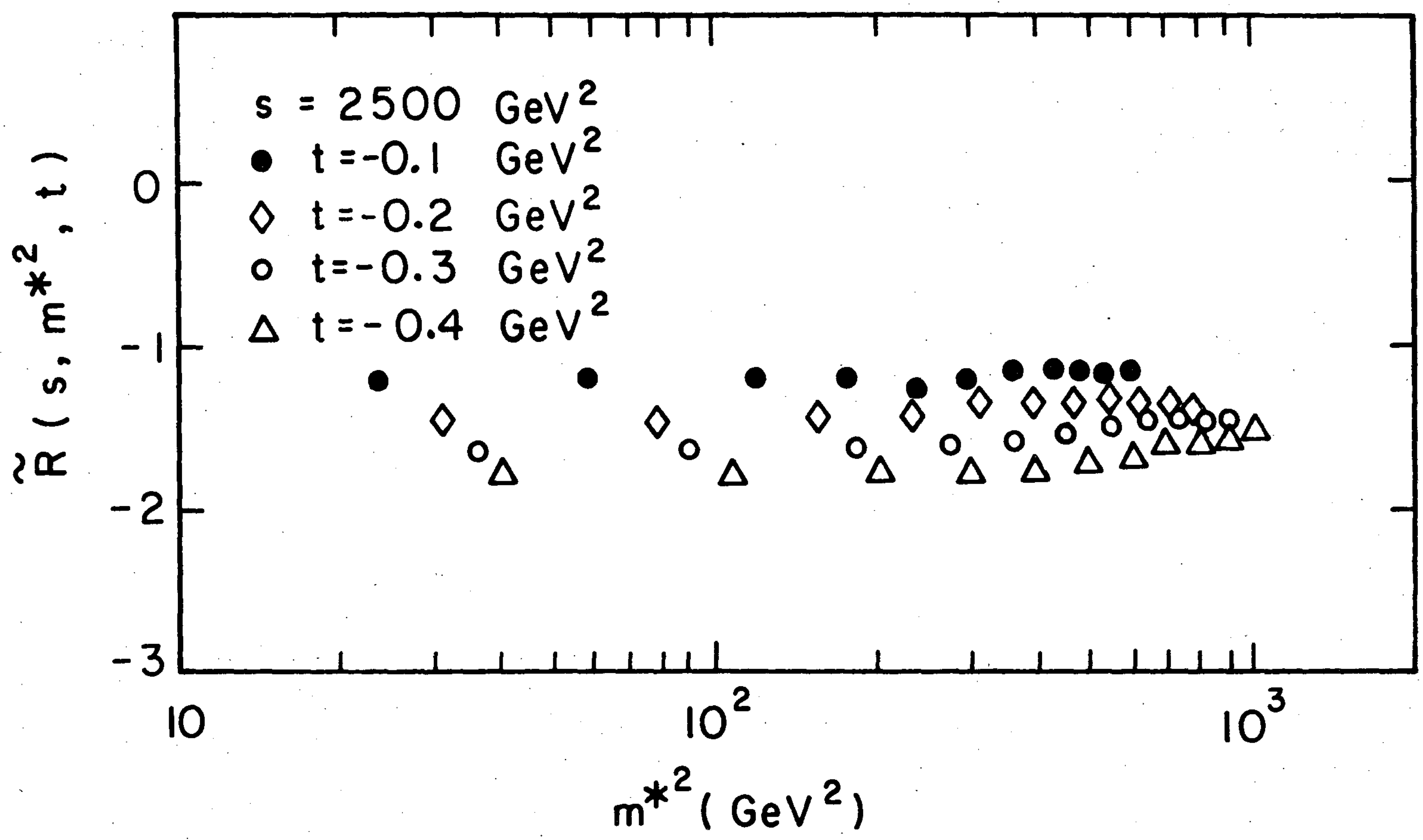

s = 2500 GeV$^2$
t=-0.1 GeV$^2$
t=-0.2 GeV$^2$
t=-0.3 GeV$^2$
t=-0.4 GeV$^2$
$\tilde{R}(s, m^{*2}, t)$
0
-1
-2
-3
10
$10^2$
$10^3$
$m^{*2}$ (GeV$^2$)
XBL731-2005

Fig. 4