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Schatten Class Composition Operators on Weighted Bergman Spaces of Bounded Symmetric Domains (*).

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Summary. - *We obtain trace ideal criteria for* $0 < p < \infty$ *for holomorphic composition operators acting on the weighted Bergman spaces* $A_\alpha^2(\Omega)$ *of a bounded symmetric domain* Ω *in* $\mathbb{C}^n$.

1. - Introduction.

In this paper we obtain trace ideal criteria for all possible values of p for composition operators acting on the weighted Bergman spaces $A_\alpha^2(\Omega)$ of a bounded symmetric domain Ω in $\mathbb{C}^n$. For the unweighted Bergman space of a bounded strongly pseudoconvex domains in $\mathbb{C}^n$ with smooth boundary, this has been done recently by S.-Y. LI [11].

For the unit disc in $\mathbb{C}$, D. LUECKING [13] initiated a systematic study of trace ideal criteria ($0 < p < \infty$) for Toeplitz operators with measures as symbols on some standard Hilbert spaces of holomorphic functions. His condition is expressed in terms of a dyadic hyperbolic decomposition of the unit disc. By an appropriate choice of measure and weight, his result applies to composition operators on the Hardy space and the weighted Bergman spaces.

For values of $p \geq 1$, ZHU [21] extended Luecking's result to the weighted Bergman spaces of a bounded symmetric domain. Although this special case of our main result can be derived from Zhu's work, our methods are different, being based on ideas from [11] and [13], and out result covers all possible values of p.

In another direction, for the Hardy space H^2 and the weighted Bergman (Hilbert) spaces of the unit disc, and for $0 < p < \infty$, LUECKING and ZHU [14] characterized com-

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position operators belonging to the Schatten class in terms of the Nevanlinna counting function, Shapiro's criteria for compactness [18] appearing as a limiting case.

Earlier work on holomorphic composition operators in one and several variables was concerned primarily with compactness. Compactness on the Hardy and Bergman spaces of the unit disc have been studied extensively in the past two decades ([19], [16], [18], [7]), and boundedness ([5], [20]) and compactness ([15], [16], [20]) have been studied in the context of the unit ball B_n in $\mathbb{C}^n$, as well as for bounded symmetric domains [21], [9], and strongly pseudoconvex domains [12].

We now introduce some notation and state our main result.

Let Ω be a bounded symmetric domain in $\mathbb{C}^n$. Let $L^2(\Omega)$ be the usual Lebesgue space over Ω with respect to the Lebesgue volume measure dv of $\mathbb{R}^{2n}$. Let $A^2(\Omega)$ be the holomorphic subspace of $L^2(\Omega)$ and let P: $L^2(\Omega) \to A^2(\Omega)$ be the Bergman projection with Bergman kernel $K(z, w)$. It is well-know that K can be written as $K(z, w) = h(z, \bar{w})^{-N}$ for some positive integer $N = N_\Omega$ and polynomial $h(z, w) = h_\Omega(z, w)$ in both z and w.

By [8], if we let $1/2 \geqslant \alpha_\Omega = N_\Omega^{-1} > 0$ then $C_\alpha = \int_\Omega K(z, z)^\alpha dv(z) < \infty$ for all real numbers $\alpha < \alpha_\Omega$. Then we may define the weighted normalized measures dv^α on Ω as follows: $dv^\alpha(z) = C_\alpha^{-1} K(z, z)^\alpha dv(z)$. We consider the Lebesgue space $L^2(\Omega, dv^\alpha)$ over Ω with respect to the normalized weighted measure dv^α, and let $A_\alpha^2(\Omega)$ be its holomorphic subspace. Let P_α: $L^2(\Omega, dv^\alpha) \to A_\alpha^2(\Omega)$ be the orthogonal projection with reproducing kernel denoted $K^\alpha(z, w)$. It is known that $K^\alpha(z, w) = K(z, w)^{1-\alpha}$. For any holomorphic mapping φ: $\Omega \to \Omega$, we define the *Berezin transform* of φ to be the function B_φ^α defined by

$$B_\varphi^\alpha(z)^2 = \int_\Omega K^\alpha(z, z)^{-1} |K^\alpha(z, w)|^2 dv_\varphi^\alpha(w) = K^\alpha(z, z)^{-1} \int_\Omega |K^\alpha(z, \varphi(w))|^2 dv^\alpha(w).$$

Here v_φ^α, or dv_φ^α is the pull-back measure defined as follows: for each Borel set $E \subset \Omega$, we let $v_\varphi^\alpha(E) = v^\alpha(\varphi^{-1}(E))$.

Let $\beta(z, w)$ be the Bergman metric on Ω. For any $z \in \Omega$ and $r > 0$ we let $E(z, r) = \{w \in \Omega\colon \beta(z, w) < r\}$. Then we let $b_\varphi^\alpha(z, r) = v_\varphi^\alpha(E(z, r))|E(z, r)|^{\alpha-1}$, $|E| = \int_E dv$.

Let φ: $\Omega \to \Omega$ be a holomorphic mapping. The *composition operator* of φ is the operator $C_\varphi u(z) = u(\varphi(z))$ for any function u on Ω. Let $d\lambda(z) = C_\alpha K^\alpha(z, z)\, dv^\alpha(z) = K(z, z)\, dv(z)$. We denote the Schatten p-class of compact operators on the Hilbert space H by $S_p(H)$, $0 < p < \infty$. We propose to prove

THEOREM 1.1. – *Let Ω be a bounded symmetric domain in $\mathbb{C}^n$. Let φ: $\Omega \to \Omega$ be a holomorphic mapping. Then for each $\alpha < \alpha_\Omega$,*

(i) *if $0 < p < \infty$, then $C_\varphi \in S_{2p}(A_\alpha^2(\Omega))$ if and only if $b_\varphi^\alpha(z, r) \in L^p(\Omega, d\lambda)$ for all (or some) $0 < r < \infty$;*

(ii) *if $2(1 - \alpha_\Omega)/(1 - \alpha) < p < \infty$, then $C_\varphi \in S_p(A_\alpha^2(\Omega))$ if and only if $B_\varphi^\alpha \in L^p(\Omega, d\lambda)$.*

Before going any further, let us make a remark on the number $2(1-\alpha_\Omega)/(1-\alpha)$ for the case $\Omega = B_n$, the unit ball in $\mathbb{C}^n$. Since $\alpha_{B_n} = 1/(n+1)$, $2(1-\alpha_\Omega)/(1-\alpha) = 2n/(n+1)(1-\alpha)$. In particular, when $\alpha = 0$, we have $2(1-\alpha_\Omega)/(1-\alpha) = 2n/(n+1)$. Note also that if Ω is the polydisk, $\alpha_\Omega = 1/2$.

We refer to [2], [4], and [21] for the following (asymptotic) properties of the Bergman kernel and metric in bounded symmetric domains. These properties will be used repeatedly in the estimates below in Sections 3 and 4.

- ([4, Proposition 2]). If φ_a denotes the unique automorphism of Ω satisfying $\varphi_a(a) = 0$ and $\varphi_a \circ \varphi_a = \mathrm{Id}$, then for the complex Jacobian

$$|(J_c\varphi_a)(z)| = |k_a(z)|,$$

where $k_a(z) = K(z,a)/K(a,a)^{1/2}$, and $|J_c\varphi_a(0)| = K(a,a)^{-1/2}$.

- ([4, p. 927]). $K(0,w) = K(z,0) = 1$, and $K(z,w) \neq 0$ for all $z, w \in \Omega$.

- ([4, Lemma 6]). For $a, b \in \Omega$ with $\beta(a,b) \leq R$, and $r, s > 0$, there is a constant C depending on R, r, and s such that

$$0 < C^{-1} \leq |E(a,r)|\,|E(b,s)|^{-1} \leq C < \infty.$$

- ([4, Lemma 8]). For $r > 0$ there is a constant C depending on r such that $\forall z \in E(a,r)$,

$$0 < C^{-1} \leq |K(z,a)|^2\,|E(a,r)|\,K(a,a)^{-1} \leq C < \infty. \tag{1}$$

- ([2, Lemmas 5 and 6]). For fixed $r > 0$, there is a sequence $\{w_j\}$ in Ω such that

$$\bigcup_{j=1}^{\infty} E(w_j, r) = \Omega \tag{2}$$

and

(i) There is a positive integer C_0 such that, for any $z \in \Omega$, z belongs to at most C_0 of the sets $E(w_j, 2r)$, where C_0 is independent of r.

(ii) If m is any positive Borel measure on Ω and $F \geq 0$,

$$\sum_{j=1}^{\infty} \int_{E(w_j,r)} F\,dm \leq C_0 \int_\Omega F\,dm.$$

- ([21, Lemma 5]). For $r > 0$ there is a constant C depending on r such that for $p \in [1,\infty)$, $a \in \Omega$ and f holomorphic,

$$|f(a)|^p \leq C|E(a,r)|^{-1} \int_{E(a,r)} |f(z)|^p\,dv(z).$$

- ([21, Lemma 6]). For $r > 0$ there is a constant C depending on r such that for any positive Borel measure μ on Ω and $a \in \Omega$,

$$\mu(E(a, r))^q \leq C|E(a, r)|^{-1} \int_{E(a, r)} \mu(E(z, r))^q dv(z) \quad (0 < q \leq 1).$$

We note the following consequences of eq. (1):

$$K(z, z) \simeq |E(z, r)|^{-1}, \tag{3}$$

$$K(z, w) \simeq K(z, z), \quad (w \in E(z, r)), \tag{4}$$

$$K(z, z) \simeq |E(a, r)|^{-1}, \quad (z \in E(a, r)). \tag{5}$$

2. - Preliminaries.

In this section, we shall prove some preliminary results we shall use later. First let us recall a lemma which can be found in many places, we refer to [1] and references therein.

Let $K_z^\alpha(w) = K^\alpha(w, z)$, $k_z^\alpha(w) = K^\alpha(z, z)^{-1/2} K_z^\alpha(w)$. It is clear that k_z^α is a unit vector in $A_\alpha^2(\Omega)$. We denote the inner product in $L^2(\Omega, dv^\alpha)$ by $\langle \cdot, \cdot \rangle_\alpha$.

LEMMA 2.1. - *Let T be a positive, compact operator on $L^2(\Omega, dv^\alpha)$ with range containe in $A_\alpha^2(\Omega)$. Then*

$$\operatorname{trace} T = \int \langle TK_z^\alpha, K_z^\alpha \rangle_\alpha dv^\alpha(z) = \int \langle Tk_z^\alpha, k_z^\alpha \rangle_\alpha K^\alpha(z, z)\, dv(z).$$

Moreover, for any $1 \leq p < \infty$,

$$\int \langle Tk_z^\alpha, k_z^\alpha \rangle_\alpha^p K^\alpha(z, z)\, dv^\alpha(z) \leq \int \langle T^p k_z^\alpha, k_z^\alpha \rangle_\alpha K^\alpha(z, z)\, dv^\alpha(z) = \operatorname{trace} T^p\,;$$

and, for any $0 < p < 1$,

$$\int \langle Tk_z^\alpha, k_z^\alpha \rangle_\alpha^p K^\alpha(z, z)\, dv^\alpha(z) \geq \int \langle T^p k_z^\alpha, k_z^\alpha \rangle K^\alpha(z, z)\, dv^\alpha(z) = \operatorname{trace} T^p\,.$$

We next start proving some identities.

LEMMA 2.2. - *Let Ω, φ, C_φ, B_φ^α be as defined above. Then*

$$B_\varphi^\alpha(z)^2 = \langle C_\varphi^* C_\varphi k_z^\alpha, k_z^\alpha \rangle_\alpha\,,$$

and for $0 < p < \infty$,

$$\int |B_\varphi^\alpha(z)|^p d\lambda(z) = \int \langle C_\varphi^* C_\varphi k_z^\alpha, k_z^\alpha \rangle_\alpha^{p/2} d\lambda(z).$$

PROOF.

$$\langle C_\varphi^* C_\varphi k_z^\alpha, k_z^\alpha\rangle_\alpha = \langle C_\varphi(K_z^\alpha), C_\varphi(K_z^\alpha)\rangle_\alpha K^\alpha(z,z)^{-1} =$$

$$= \int K_z^\alpha \circ \varphi(w)\, \overline{K_z^\alpha \circ \varphi(w)}\, dv^\alpha(w) K^\alpha(z,z)^{-1} =$$

$$= \int K_z^\alpha(w)\, \overline{K_z^\alpha(w)}\, dv_\varphi^\alpha(w) K^\alpha(z,z)^{-1} = \int |K^\alpha(z,w)|^2 dv_\varphi^\alpha(w) K^\alpha(z,z)^{-1} = B_\varphi^\alpha(z)^2 . \quad \blacksquare$$

As a consequence of Lemmas 2.1 and 2.2, we have

COROLLARY 2.3. – *Let Ω be a bounded symmetric domain in $\mathbb{C}^n$ and let $\varphi\colon \Omega \to \Omega$ be a holomorphic map. Then*

(*a*) *if $2 \leqslant p < \infty$ and $C_\varphi \in S_p(A_\alpha^2(\Omega))$, then $B_\varphi^\alpha \in L^p(d\lambda)$;*

(*b*) *if $0 < p \leqslant 2$ and $B_\varphi^\alpha \in L^p(A, d\lambda)$, then $C_\varphi \in S_p(A_\alpha^2(\Omega))$.*

PROOF. – If $p/2 \geqslant 1$, then by Lemmas 2.1 and 2.2,

$$\|C_\varphi\|_{S_p(A_\alpha^2)}^p = \operatorname{trace}((C_\varphi^* C_\varphi)^{p/2}) \geqslant \int \langle C_\varphi^* C_\varphi k_z^\alpha, k_z^\alpha\rangle_\alpha d\lambda(\lambda) = \int |B_\varphi^\alpha(z)|^p d\lambda(z),$$

so (*a*) follows.

If $p/2 \leqslant 1$, applying Lemmas 2.1 and 2.2 again, we have

$$\int |B_\varphi^\alpha(z)|^p d\lambda(z) = \int \langle C_\varphi^* C_\varphi k_z^\alpha, k_z^\alpha\rangle_\alpha^{p/2} d\lambda(z) \geqslant \operatorname{trace}((C_\varphi^* C_\varphi)^{p/2}) = \|C_\varphi\|_{S_p(A_\alpha^2)}^p$$

and (*b*) follows. $\blacksquare$

Next we shall connect the operator $C_\varphi^* C_\varphi$ to a Toeplitz operator associated to a *symbol* which is our pull-back measure v_φ^α. For any measure μ on Ω define the operator T_μ by the formula

$$T_\mu(f)(x) = \int_\Omega f(w) K^\alpha(z,w)\, d\mu(w)$$

for $f \in A_\alpha^2(\Omega)$ and $z \in \Omega$.

LEMMA 2.4. – *Let Ω be a bounded symmetric domain in $\mathbb{C}^n$. Let $\varphi\colon \Omega \to \Omega$ be a holomorphic map such that C_φ is bounded on $A_\alpha^2(\Omega)$. With the notation above, $C_\varphi^* C_\varphi = T_{v_\varphi^\alpha}$.*

PROOF. – Let $f \in A_\alpha^2(\Omega)$. Then

$$C_\varphi^* C_\varphi(f)(z) = \langle C_\varphi^* C_\varphi(f), K_z^\alpha\rangle_\alpha = \langle C_\varphi(f), C_\varphi(K_z^\alpha)\rangle_\alpha =$$

$$= \int_\Omega f(\varphi(w))\, \overline{K_z^\alpha(\varphi(w))}\, dv^\alpha(w) = \int_\Omega f(w) K^\alpha(z,w)\, dv_\varphi^\alpha(w) = T_{v_\varphi^\alpha}(f)(z).$$

Thus $C_\varphi^* C_\varphi(f) = T_{v^\alpha(\varphi)}(f)$ and the proof is complete. $\blacksquare$

COROLLARY 2.5. – *Let $0 < p < \infty$, let Ω be a bounded symmetric domain in $\mathbb{C}^n$, and let $\varphi: \Omega \to \Omega$ be a holomorphic map. Then $C_\varphi \in S_p(A_\alpha^2(\Omega))$ if and only if $T_{v_\varphi^\alpha} \in S_{p/2}(A_\alpha^2(\Omega))$.*

Combining Lemma 2.2, Corollaries 2.3 and 2.5, and [21, Theorem C], we obtain

COROLLARY 2.6. – *Let Ω be a bounded symmetric domain in $\mathbb{C}^n$, let $\varphi: \Omega \to \Omega$ be a holomorphic map, and let $2 \leq p \leq \infty$. The following are equivalent:*

- $C_\varphi \in S_p(A_\alpha^2(\Omega))$,
- $B_\varphi^\alpha \in L^p(\Omega, d\lambda)$,
- $b_\varphi^\alpha \in L^{p/2}(\Omega, d\lambda)$.

3. – Equivalence of two conditions.

In this section we shall prove a part of our main theorem. For $p \geq 2$, the following theorem was proved by ZHU ([21]; see Corollary 2.6 above).

THEOREM 3.1. – *Let Ω be a bounded symmetric domain in $\mathbb{C}^n$ and $\alpha < \alpha_\Omega$. Let $2(1-\alpha_\Omega)/(1-\alpha) < p < \infty$. Then $B_\varphi^\alpha \in L^p(\Omega, d\lambda)$ if and only if $b_\varphi^\alpha \in L^{p/2}(\Omega, d\lambda)$.*

PROOF. – Suppose first that $B_\varphi^\alpha(z) \in L^p(\Omega, d\lambda)$. We shall show that $b_\varphi^\alpha(z) \in L^{p/2}(\Omega)$. By Lemma 2.2, we have

$$B_\varphi^\alpha(z)^2 = \int_\Omega K^\alpha(z,z)^{-1} |K^\alpha(z,w)|^2 dv_\varphi^\alpha(w) \geq$$

$$\geq \int_{E(z,r)} K^\alpha(z,z)^{-1} |K^\alpha(z,w)|^2 dv_\varphi^\alpha(w) \geq C_r^{-1} K^\alpha(z,z) \int_{E(z,r)} dv_\varphi^\alpha(w) = \quad \text{by eq. (4)}$$

$$= C_r^{-1} K^\alpha(z,z) v_\varphi^\alpha(E(z,r)) \geq C_r^{-1} |E(z,r)|^{-1+\alpha} v_\varphi^\alpha(E(z,r)) = \quad \text{by eq. (3)}$$

$$= C_r^{-1} b_\varphi^\alpha(z).$$

Therefore $\|b_\varphi^\alpha\|_{L^{p/2}(\Omega, d\lambda)} \leq C_r \|B_\varphi^\alpha\|^2_{L^p(\Omega, d\lambda)}$.

Next we shall prove the converse. To achieve this goal, we need the following Forelli-Rudin type inequality from [8]. The notation we use is not exactly the same as it is in [8]. For $\alpha < \alpha_\Omega$, we let

$$I_{a,c}(z) = \int_\Omega K(w,w)^a |K(z,w)|^{1-\alpha+c} dv(w).$$

Let r_Ω be the rank of Ω, and $K(z, w) = h(z, \bar{w})^{-N}$. It is known that if Ω is an irreducible bounded symmetric domain, then $N = a(r_\Omega - 1) + b + 2$ where a, b are nonnegative integers defined in [8]. Then [8, Theorem 4.1] implies the following proposition.

PROPOSITION 3.2. – *Let $\alpha < \alpha_\Omega$. Then*

(i) *If $c < -a(r-1)/2N$, then $I_{\alpha, c}(z)$ is bounded.*

(ii) *If $c > a(r-a)/2N$ then $I_{\alpha, c}(z) \approx K(z, z)^c$, $z \in \Omega$.*

We now come back to the proof of Theorem 3.1. We may assume, by Corollary 2.6 that $p \leq 2$. Since $(1-\alpha_\Omega)/(1-\alpha) < p/2 \leq 1$, we have $1 - p(1-\alpha)/2 < \alpha_\Omega$ and therefore $\int_\Omega K(z, z)^{-p/2(1-\alpha)+1} dv(z) < \infty$. Moreover, by Proposition 3.2,

$$\int_\Omega K^\alpha(z, z)^{-p/2} |K^\alpha(z, w)|^p K(z, z)\, dv(z) = \int_\Omega K(\bar{z}, z)^{1-p(1-\alpha)/2} |K(z, w)|^{p(1-\alpha)} dv(z) =$$

$$= \int_\Omega K(z, z)^{1-p(1-\alpha)/2} |K(z, w)|^{1-(1-p(1-\alpha)/2)+p(1-\alpha)/2} dv(z) =$$

$$= I_{(1-p(1-\alpha)/2),\, p(1-\alpha)/2}(w) \approx K(w, w)^{p(1-\alpha)/2},$$

since

$$c = p(1-\alpha)/2 > [2(1-\alpha_\Omega)/(1-\alpha)](1-\alpha)/2 =$$

$$= 1 - \alpha_\Omega = 1 - 1/N > 1/2 - (b+2)/2N = a(r_\Omega - 1)/2N.$$

Thus, choosing $\{z_j\}$ so that $\Omega = \bigcup_{i=1}^{\infty} E(z_i, r)$,

$$\int_\Omega B_\varphi^\alpha(z)^p d\lambda(z) \leq \int_\Omega \left(\int_\Omega K^\alpha(z, z)^{-1} |K^\alpha(z, w)|^2 dv_\varphi^\alpha(w) \right)^{p/2} d\lambda(z) \leq$$

$$\leq C \int_\Omega \left(\sum_{i=1}^{\infty} \int_{E(z_i, r)} K^\alpha(z, z)^{-1} |K^\alpha(z, w)|^2 dv_\varphi^\alpha(w) \right)^{p/2} d\lambda(z) \leq$$

$$\leq C \int_\Omega \sum_{i=1}^{\infty} \left(\int_{E(z_i, r)} K^\alpha(z, z)^{-1} |K^\alpha(z, w)|^2 dv_\varphi^\alpha(w) \right)^{p/2} d\lambda(z) =$$

$$= C \sum_{i=1}^{\infty} \int_\Omega \left(\int_{E(z_i, r)} K^\alpha(z, z)^{-1} |K^\alpha(z, w)|^2 dv_\varphi^\alpha(w) \right)^{p/2} d\lambda(z) \leq$$

$$\leq C \sum_{i=1}^{\infty} \int_\Omega K^\alpha(z,z)^{-p/2} |K^\alpha(z,w_i)|^p v_\varphi^\alpha(E(z_i,r))^{p/2} K(z,z)\, dv(z) \quad (w_i \in E(z_i,r) \text{ depends on } z) \leq$$

$$\leqslant C\sum_{i=1}^{\infty} K^{\alpha}(w_i, w_i)^{-p/2} K^{\alpha}(w_i, w_i)^p v_{\varphi}^{\alpha}(E(z_i, r))^{p/2} = C\sum_{i=1}^{\infty} K^{\alpha}(w_i, w_i)^{p/2} v_{\varphi}^{\alpha}(E(z_i r))^{p/2} \leqslant$$

$$\leqslant C\sum_{i=1}^{\infty} \int_{E(z_i, r)} [v_{\varphi}^{\alpha}(E(z, r)) K(z, z)^{1-\alpha}]^{p/2} K(z, z)\, dv(z) \quad \text{(by subharmonicity)} \leqslant$$

$$\leqslant C\sum_{i=1}^{\infty} \int_{E(z_i, r)} (b_{\varphi}^{\alpha}(z))^{p/2} d\lambda(z) \leqslant C_0 C \int_{\Omega} (b_{\varphi}^{\alpha}(z))^{p/2} d\lambda(z) = C_{p,r} \|b_{\varphi}^{\alpha}\|_{L^{p/2}(\Omega, d\lambda)}^{p/2}.$$

Therefore $\|B_{\varphi}^{\alpha}\|_{L^p(\Omega, d\lambda)} \leqslant C_{p,r} \|b_{\varphi}^{\alpha}\|_{L^{p/2}(\Omega, d\lambda)}^{1/2}$, and combining the above two steps, the proof of Theorem 3.1 is complete. ∎

Note that the first implication in the preceding proof may be obtained from Corollary 2.3 and Theorem 4.1 in the next section. Here we have given a direct proof.

4. - Proof of main theorem.

In this section, we shall complete the proof of our main theorem, that is, the case $0 < p < 2$. By Corollary 2.5 and Theorem 3.1, it suffices to prove the following theorem.

THEOREM 4.1. - *Let Ω be a bounded symmetric domain in $\mathbb{C}^n$. Let $\varphi\colon \Omega \to \Omega$ be a holomorphic mapping. Then for each $\alpha < \alpha_{\Omega}$, if $0 < p \leqslant 2$, then $C_{\varphi} \in S_{2p}(A_{\alpha}^2(\Omega))$ if and only if $b_{\varphi}^{\alpha}(z, r) \in L^p(\Omega, d\lambda)$ for all (or some) $0 < r < \infty$.*

We shall break the proof of Theorem 4.1 into several lemmas.

LEMMA 4.2. - *Let Ω be a bounded symmetric domain in $\mathbb{C}^n$ and $\varphi\colon \Omega \to \Omega$ be a holomorphic mapping. If $0 < p < 1$, $\alpha < \alpha_{\Omega}$ and $b_{\varphi}^{\alpha} \in L^p(\Omega, d\lambda^{\alpha})$, then $T = T_{v_{\varphi}^{\alpha}} \in S_p(A_{\alpha}^2(\Omega))$.*

PROOF. - Since $0 < p \leqslant 2$, it suffices to prove (cf. [13, Lemma 5]) there is an orthonormal basis $\{\xi_n\}_{n=1}^{\infty}$ of $A_{\alpha}^2(\Omega)$ such that $\sum_{n,k} |\langle T\xi_n, \xi_k\rangle|^p < \infty$. Actually, we shall prove this for an operator $L^* TL$ on an abstract Hilbert space, and appropriate L; then it will follow that $T \in S_p(A_{\alpha}^2(\Omega))$.

Let $\{z_k\}_{k=1}^{\infty}$ be a sequence of points in Ω, and let

$$b_k(z) = K^{\alpha}(z_k, z_k)^{-M-1/2} K^{\alpha}(z, z_k)^{1+M}$$

where M is a positive number to be determined later.

We interrupt the proof to state a proposition which is a comsequence of the proof of Theorems 1 and 2 in [6], and explains the significance of $b_k(z)$.

PROPOSITION 4.3. – *There is a sequence of points* $\{z_k\}_{k=1}^{\infty} \subset \Omega$ *so that* $f \in A_\alpha^2(\Omega)$ *if and only if there is a sequence of numbers* $\{\lambda_k\}_1^{\infty} \in l^2$ *such that*

$$f(z) = \sum_k \lambda_k b_k(z), \qquad z \in \Omega$$

and

$$\|f\|_{A_\alpha^2} \approx \|\{\lambda_k\}\|_{l^2}.$$

The authors wish to thank R. ROCHBERG for showing them how Proposition 4.3 can be proved using Proposition 3.2 and the following two hypotheses.

(H1) The operator $T_{\alpha, M}$ defined as:

$$T_{\alpha, M} f(z) = \int_\Omega |K^\alpha(z, w)|^{1+M} K^\alpha(w, w)^{-M} f(w)\, dv^\alpha(w),$$

is bounded on $L_\alpha^2(\Omega)$.

(H2) $b_k(z) \simeq b_k(z_i)$ on (E_i, r).

(H1) follows from Proposition 3.2 for sufficiently large M and Schur's lemma; and (H2) is true on any bounded symmetric domain by (4) (see [21]). We omit the details here.

In connection with Proposition 3.2, we should point out that whether the Forelli-Rudin type inequality holds or not when $-\alpha(r_\Omega - 1)/2N < c < \alpha(r_\Omega - 1)/2N$ is not completely known. Our definition of b_k with large M avoids this uncertainty. In a general bounded symmetric domain, the Bergman projection may not be bounded on L^p for all $1 < p < \infty$ (for example, see [3]).

We now continue with the proof of Lemma 4.2 by calculating the following quantity:

$$\sum_{n,k} |(Tb_n, b_k)|^p = \sum_{n,k} |(T_{v_\varphi^\alpha} b_n, b_k)|^p \leq \sum_{n,k} \left(\int_\Omega |b_n(z)|\, |b_k(z)|\, dv_\varphi^\alpha(z) \right)^p \leq$$

$$\leq C \sum_{n,k} \left(\sum_i \int_{E(z_i, r)} |b_n(z) b_n(z)|\, dv_\varphi^\alpha(z) \right)^p \leq C \sum_{n,k} \left(\sum_i v_\varphi^\alpha(E(z_i, r))\, |b_n(z_i) b_k(z_i)| \right)^p \leq$$

$$\leq C \sum_{n,k} \sum_i (v_\varphi^\alpha(E(z_i, r)) K(z_i, z_i))^p (K(z_i, z_i)^{-1} |b_n(z_i) b_k(z_i)|)^p =$$

$$= C\left[\sum_i (v_\varphi^\alpha(E(z_i, r)) K^\alpha(z_i, z_i))^p \right] \left[\sum_{n,k} (K^\alpha(z_i, z_i)^{-1} |b_n(z_i) b_k(z_i)|)^p \right].$$

Since $\sum_i (v_\varphi^\alpha(E(z_i, r)) K^\alpha(z_i, z_i))^p \leq C \|b_\varphi^\alpha\|_{L^p(\Omega, d\lambda)}^p$, we would like to prove

$\sum\limits_{n,k} (K^\alpha(z_i, z_i)^{-1} |b_n(z_i) b_k(z_i)|)^p \leq C$, for all $i = 1, 2, 3, \ldots$. However

$$\sum_{n,k} (K^\alpha(z_i, z_i)^{-1} |b_n(z_i) b_k(z_i)|)^p = K^\alpha(z_i, z_i)^{-p} \left(\sum_k |b_k(z_i)|^p \right)^2 =$$

$$= K^\alpha(z_i, z_i)^{-p} \left(\sum_k K^\alpha(z_k, z_k)^{-p/2 - Mp} |K^\alpha(z_i, z_k)|^{p(1+M)} \right)^2 \leq$$

$$\leq CK^\alpha(z_i, z_i)^{-p} \left(\int_\Omega K^\alpha(z, z)^{-p/2 - Mp + 1} |K^\alpha(z_i, z)|^{p(1+M)} dv^\alpha(z) \right)^2 \leq$$

$$\leq CK(z_i, z_i)^{-p} (K^\alpha(z_i, z_i)^{-p/2 - Mp} |K^\alpha(z_i, z_i)|^{p(1+M)})^2 =$$

$$= CK(z_i, z_i)^{-p} (|K^\alpha(z_i, z_i)|^{p/2}))^2 = C_M$$

if we choose M such that $Mp > 1$. Thus we have shown that

$$\sum_{n,k} |(Tb_n, b_k)|^p \leq C \|b_\varphi^\alpha\|_p^p$$

Now we let H be any Hilbert space with $\{e_n\}$ as its orthonormal basis. Let L: $H \to A_\alpha^2(\Omega)$ be defined as follows: $L\left(\sum\limits_{k=1}^\infty c_k e_k \right) = \sum\limits_{k=1}^\infty c_k b_k(z)$. It is clear from Proposition 4.3 that L: $H \to A_\alpha^2(\Omega)$ is a bounded and onto linear map. Since L is onto, it has a bounded right inverse, that is, there is a bounded linear operator R: $A_\alpha^2(\Omega) \to H$ such that $LR = I$: $A_\alpha^2(\Omega) \to A_\alpha^2(\Omega)$. For our T: $A_\alpha^2(\Omega) \to A_\alpha^2(\Omega)$, we have $T = (LR)^* TLR = R^* L^* TLR$: $A_\alpha^2(\Omega) \to A_\alpha^2(\Omega)$. Since $L^* TL$: $H \to H$ is a bounded linear operator and

$$\sum_{k,n} |\langle L^* TL(e_k), e_n \rangle|^p = \sum_{k,n} |\langle TLe_k, L(e_n) \rangle|^p = \sum_{k,n} |\langle Tb_k, b_n \rangle|^p \leq C \|b_\varphi^\alpha\|_{L^p(\Omega, d\lambda)}^p,$$

it follows that $L^* TL \in S_p(H)$ and $\|L^* TL\|_{S_p(A_\alpha^2)} \leq C \|b_\varphi^\alpha\|_{L^p(\Omega, d\lambda)}^p$. Since R^*: $H \to A_\alpha^2(\Omega)$ and L: $H \to A_\alpha^2(\Omega)$ are bounded linear operators, we have $T = R^* L^* TLR \in S_p(A_\alpha^2)(\Omega)$ and $\|T\|_{S_p(A_\alpha^2)} \leq \|R^*\| \|R\| C \|b_\varphi^\alpha\|_{L^p(\Omega, d\lambda)}^p \leq C \|b_\varphi^\alpha\|_{L^p(\Omega, d\lambda)}^p$. Therefore, the proof of Lemma 4.2 is complete. ■

The proof of Theorem 4.1 is now reduced to proving the following lemma. The idea of the proof is similar to one in [13].

LEMMA 4.4. – *Let Ω be a bounded symmetric domain in $\mathbb{C}^n$. Let $\alpha < \alpha_\Omega$ and let $0 < p \leq 1$. If φ: $\Omega \to \Omega$ is a holomorphic map such that $T_{v_\varphi^\alpha} \in S_p(A_\alpha^2(\Omega))$, then $b_\varphi^\alpha \in L^p(\Omega, d\lambda)$.*

PROOF. – Again let $\{z_k\}_{k=1}^\infty$ be a sequence satisfying the density and separation properties of Coifman and Rochberg, that is, $\beta(z_j, z_l) > r$ and $\Omega = \bigcup\limits_{j=1}^\infty E(z_j, r)$.

For $R \gg r$, partition this sequence $\{z_j\}_{j=1}^{\infty} = \bigcup_{k=1}^{C_R} \{z_j^{(k)}\}_{j=1}^{\infty}$ so that

$$\beta(z_j^{(k)}, z_l^{(k)}) > R, \qquad j \neq l, \qquad 1 \leq k \leq C_R.$$

As before, define operators L and L_k from a Hilbert space H into A_φ^α by

$$L\left(\sum_1^\infty c_l e_l\right) = \sum_1^\infty c_l b_l(z), \qquad \text{and} \qquad L_k\left(\sum_1^\infty c_l e_l\right) = \sum_1^\infty c_l b_l^{(k)}(z),$$

where

$b_l^{(k)}(z) = K^\alpha(z_l^{(k)}, z_l^{(k)})^{-M-1/2} K^\alpha(z, z_l^{(k)})^{1+M}$ and $b_l(z) = K^\alpha(z_l, z_l)^{-M-1/2} K^\alpha(z, z_l)^{1+M}$

for some positive number M to be chosen later.

Note that $\|L_k\| \leq \|L\|$, write $\Omega_k = \bigcup_{l=1}^{\infty} E(z_l^{(k)}, r)$, and let χ_k be the characteristic function of Ω_k.

Since we are assuming that $T_{dv_\varphi^\alpha} \in S_p(A_\alpha^2)$, we have $T_{\chi_k dv_\varphi^\alpha} \in S_p(A_\alpha^2)$ and $\|T_{\chi_k dv_\varphi^\alpha}\|_{S_p} \leq \|T_{dv_\varphi^\alpha}\|_{S_p}$. Thus

$$L_k^* T_{\chi_k dv_\varphi^\alpha} L_k \in S_p(A_\alpha^2) \qquad \text{and} \qquad \|L_k^* T_{\chi_k dv_\varphi^\alpha} L_k\|_{S_p} \leq \|L_k\|^2 \|T_{dv_\varphi^\alpha}\|_{S_p}.$$

Fix k and for notation's sake, let $w_l = z_l^{(k)}$, $a_l(z) = b_l^{(k)}(z)$, and $T_k = L_k^* T_{\chi_k dv_\varphi^\alpha \chi_k} L_k$. Write $T_k = D + E$ where $D = \sum_l \langle T_k a_l, a_l \rangle \langle \cdot, e_l \rangle e_l$ and $E = \sum_{n \neq l} \langle T_k a_n, a_l \rangle \langle \cdot, e_n \rangle e_l$. Then

$$\|D\|_{S_p(H)}^p \leq \|T_k\|_{S_p(H)}^p + \|E\|_{S_p(H)}^p \leq \|L\|^{2p} \|T_{v_\varphi^\alpha}\|_{S_p(H)}^p + \|E\|_{S_p(H)}^p.$$

To complete the proof of Lemma 4.4 requires three claims:

CLAIM 1. – $\|D\|_{S_p}^p \geq C_r^{-1} \int_{\Omega_k} (b_\varphi^\alpha(z))^p K(z, z)\, dv(z)$.

CLAIM 2. – For each $1 \leq k \leq C_R$, we have

$$\sup_i \sum_{n \neq l} |a_n(w_i)|^p\, |a_l(w_i)|^p K(w_i, w_i)^{-p} < \varepsilon_R$$

where $\varepsilon_R \to 0$ as $R \to \infty$. (Thus is stated as Lemma 4.5 below.)

CLAIM 3. – $\|E\|_{S_p}^p \leq C\varepsilon_R \int_{\Omega_k} (b_\varphi^\alpha(z))^p K(z, z)\, dv(z)$.

Let us assume these three claims and proceed to finish the proof of Lemma 4.4. We have

$$\int_{\Omega_k} (b_\varphi^\alpha)^p\, d\lambda \leq C\|D\|_{S_p}^p \leq C(\|T_{\chi_k dv_\varphi^\alpha}\|_{S_p(H)}^p + \|E\|_{S_p(H)}^p) \leq C\|T_{dv_\varphi^\alpha}\|_{S_p(A_\alpha^2)}^p + \varepsilon_R C \int_{\Omega_k} (b_\varphi^\alpha)^p\, d\lambda.$$

Therefore, for large enough R,

$$\left(\int_{\Omega_k} (b_\varphi^\alpha)^p \, d\lambda\right)(1 - \varepsilon_R C') \leq C \| T_{dv_\varphi^\alpha} \|_{S_p(A_\alpha^2)}^p ,$$

and hence

$$\int_{\Omega} (b_\varphi^\alpha)^p \, d\lambda \leq \sum_{k=1}^{C_R} \int_{\Omega_R} (b_\varphi^\alpha)^p \, d\lambda \leq C_R C \| T_{dv_\varphi^\alpha} \|_{S_p(A_\alpha^2)}^p ,$$

which completes the proof of Lemma 4.4.

4.1. *Proof of Claim 1.*

$$\|D^k\|_{S_p(H)}^p = \sum_l \langle T_k a_l, a_l \rangle^p = \sum_l \left(\int_{\Omega_k} |a_l(z)|^2 \, dv_\varphi^\alpha(z) \right)^p =$$

$$= \sum_l \left(\int_{\Omega_k} K^\alpha(w_l, w_l)^{-2M-1} |K^\alpha(z, w_l)|^{2+2M} \, dv_\varphi^\alpha(z) \right)^p \geq$$

$$\leq C_r^{-1} \sum_l \left(\int_{E(w_l, r)} b_\varphi^\alpha(z) |E(w_l, r)|^{-1} \, dv(z) \right)^p \geq$$

$$\geq C_r^{-1} \sum_l \int_{E(w_l, r)} (b_\varphi^\alpha(z))^p \, |E(w_l, r)|^{-1} \, dv(z) \geq C_r^{-1} \sum_l \int_{E(w_l, r)} (b_\varphi^\alpha(z))^p K(z, z) \, dv(z) \geq$$

$$\geq C_r^{-1} \int_{\bigcup_l E(w_l, r)} (b_\varphi^\alpha(z))^p K(z, z) \, dv(z) \geq C_r^{-1} \int_{\Omega_k} (b_\varphi^\alpha(z))^p K(z, z) \, dv(z)$$

where $\Omega_k = \bigcup_l E(w_l, r)$.

4.2. *Proof of Claim 2.*

LEMMA 4.5. – *For each* $1 \leq k \leq C_R$, *we have*

$$\sup_i \sum_{n \neq l} |a_n(w_i)|^p \, |a_l(w_i)|^p K(w_i, w_i)^{-p} < \varepsilon_R$$

where $\varepsilon_R \to 0$ *as* $R \to \infty$.

PROOF. – Recall that $K^\alpha(w_n, w_n) \asymp K^\alpha(w, w)$ for $w \in E(w_n, r)$ and that since

$|K^\alpha(w_i, w)|^{(1+M)p}$ is subharmonic, we have

$$|K^\alpha(w_i, w_n)|^{(1+M)p} \leq |E(w_n, r)|^{-1} \int_{E(w_n, r)} |K^\alpha(w_i, w)|^{(1+M)p} dv(w).$$

Thus

$$|a_n(w_i)|^p \leq K^\alpha(w_n w_n)^{-(M+1/2)p} |E(w_n, r)|^{-1} \int_{E(w_n r)} |K^\alpha(w_i, w)|^{(1+M)p} dv(w) \leq$$

$$\leq C \int_{E(w_n, r)} K(w, w)^{-(M+1/2)p} |K^\alpha(w_i, w)|^{(M+1)p} K(w, w)\, dv(w).$$

For $\gamma > 0$, let $\Omega(\gamma) = \{(z, w) \in \Omega \times \Omega: \beta(z, w) > \gamma\}$. Then

$$\sum_{n \neq l} |a_n(w_i)|^p |a_l(w_i)|^p \leq C \sum_{n \neq l} \left[\int_{E(w_l, r)} K^\alpha(z, z)^{-Mp - p/2} K(z, z) |K^\alpha(w_i, z)|^{p+Mp} dv(z) \times \right.$$

$$\left. \times \int_{E(w_n, r)} K^\alpha(w, w)^{-Mp - p/2} K(w, w) |K^\alpha(w_i, w)|^{p+Mp} dv(w) \right] \leq$$

$$\leq C \sum_{n \neq l} \int_{E(w_l, r) \times E(w_n, r)} K^\alpha(z, z)^{-Mp - p/2} K(z, z) |K^\alpha(w_i, z)|^{p+Mp} \times$$

$$\times K^\alpha(w, w)^{-Mp - p/2} K(w, w) |K^\alpha(w_i w)|^{p+Mp} dv(z)\, dv(w) \leq$$

$$\leq C \int_{\Omega(R/c)} K^\alpha(z, z)^{-Mp - p/2} K(z, z) |K^\alpha(w_i, z)|^{p+Mp} \times$$

$$\times K^\alpha(w, w)^{-Mp - p/2} K(w, w) |K^\alpha(w_i, w)|^{p+Mp} dv \times dv(z, w).$$

Now we shall make a change of variables as follows: let φ_i be the automorphism of Ω interchanging 0 and w_i. Then, since $|J_c \varphi_i(z)|^2 = |k_{w_i}(z)|^2 = |K(z, w_i)|^2 / K(w_i, w_i)$,

$$(6) \quad \int_\Omega K^\alpha(z, z)^{-Mp - p/2} K(z, z) |K^\alpha(w_i, z)|^{p+pM} dv(z) =$$

$$= \int_\Omega K^\alpha(\varphi_i(z), \varphi_i(z))^{-Mp - p/2} K(\varphi_i(z), \varphi_i(z)) \times$$

$$\times |K^\alpha(w_i, \varphi_i(z))|^{p+pM} |K(z, w_i)|^2 K(w_i w_i)^{-1} dv(z).$$

Let us calculate each of the factors in the last integral, using the formula

$$K(\varphi(z), \varphi(w)) J_c \varphi(z) \overline{J_c \varphi(w)} = K(z, w).$$

- $K^\alpha(\varphi_i(z), \varphi_i(z)) = (K(z, z)|J_c \varphi_i(z)|^{-2})^{1-\alpha}$;
- $K(\varphi_i(z), \varphi_i(z)) = |J_c \varphi_i(z)|^{-2} K(z, z) = |K(z, w_i)|^{-2} K(w_i w_i) K(z, z)$;
- $K^\alpha(w_i, \varphi_i(z)) = K^\alpha(\varphi_i(0), \varphi_i(z)) = K^\alpha(0, z)[J_c \varphi_i(0) \overline{J_c \varphi_i(z)}]^{-1}$;
- $|K^\alpha(w_i, \varphi_i(z))| = [(|J_c \varphi_i(0)| \, |J_c \varphi_i(z)|)^{-1}]^{1-\alpha} = K^\alpha(w_i, w_i)|K^\alpha(z, w_i)|^{-1}$.

The integrand in question is thus equal to

$$\{K^\alpha(z, z)(|K^\alpha(z, w_i)|K^\alpha(w_i w_i)^{-1/2})^{-2}\}^{-Mp-p/2} \times$$
$$\times \{(|K(z, w_i)|K(w_i, w_i)^{-1/2})^{-2} K(z, z)\} \times$$
$$\times \{|K^\alpha(z, w_i)|^{-p-Mp} K^\alpha(w_i, w_i)^{p+Mp}\}\{|K(z, w_i)|^2 K(w_i w_i)^{-1}\}$$

and therefore eq. (6) becomes

$$\int_\Omega K^\alpha(z, z)^{-Mp-p/2} K(z, z)|K^\alpha(w_i, z)|^{p+pM} dv(z) =$$

$$= K^\alpha(w_i, w_i)^{-Mp-p/2+p+Mp} \int_\Omega K^\alpha(z, z)^{-Mp-p/2} K(z, z)|K^\alpha(z, w_i)|^{2(Mp+p/2)-p-Mp} dv(z).$$

Using the identical calculation in the variable w and noting that for any automorphism φ, $\beta(z, w) = \beta(\varphi(z), \varphi(w))$, we now have

$$K(w_i, w_i)^{-p} \sum_{n \neq l} |a_n(w_i)^p |a_l(w_i)|^p \leq$$

$$\leq \int\int_{\Omega(R/c)} K^\alpha(z, z)^{-Mp-p/2} K(z, z)|K^\alpha(w_i z)|^{Mp} \times$$

$$\times K^\alpha(w, w)^{-Mp-p/2} K(w, w)|K^\alpha(w_i, w)|^{Mp} dv \times dv(z, w) =: I(R, w_i) \text{ say}.$$

Since $-Mp - p/2 + Mp = -p/2 < 0$ and Mp is big enough, for any fixed w', the function

$$K^\alpha(z, z)^{-Mp-p/2} K(z, z)|K^\alpha(w', z)|^{Mp} \times K^\alpha(w, w)^{-Mp-p/2} K(w, w)|K^\alpha(w', w)|^{Mp}$$

is integrable on $\Omega \times \Omega$. In fact, by [8, Theorem 4.1], for big β, we have

$$\int_\Omega K^\alpha(z, z)^{-\beta} K(z, z)|K^\alpha(w', z)|^{\beta+\varepsilon} dv(z) \simeq K(w', w')^\varepsilon.$$

The function $|K^\alpha(\cdot, w)|^{Mp}$ is subharmonic and thus by the maximum principle there is a point w_0 on the boundary of Ω such that $I(R, w_i) \leq I(R, w_0)$. Since $\Omega(R/c) \to$

$\to \emptyset$ as $R \to \infty$, it follows that $I(R, w_i) \to 0$ uniformly in i as $R \to \infty$, completing the proof of Lemma 4.5.

4.3. *Proof of Claim 3.*

$$\|E\|_{S_p}^p \leqslant \sum_{n \neq l} |\langle T_k a_n, a_l \rangle|^p \leqslant \sum_{n \neq l} \left(\int_{\Omega_k} |a_n(z) a_l(z)| \, dv_\varphi^\alpha(z) \right)^p \leqslant$$

$$\leqslant \sum_i v_\varphi^\alpha (E(w_i, r))^p K^\alpha(w_i, w_i)^p \sum_{n \neq l} |a_n(w_i)|^p \, |a_l(w_i)|^p K(w_i, w_i)^{-p} \leqslant$$

$$\leqslant \sum_i (b_\varphi^\alpha(w_i))^p \sum_{n \neq l} |a_n(w_i)|^p \, |a_l(w_i)|^p K(w_i, w_i)^{-p} \leqslant$$

$$\leqslant \varepsilon_R \sum_i (b_\varphi^\alpha(w_i))^p \leqslant \varepsilon_R C \int_{\Omega_k} (b_\varphi^\alpha(z))^p K(z, z) \, dv(z).$$

Combining the all estimates, the proof of Theorem 4.1 is complete. ■

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