Title
MEASUREMENTS OF VELOCITY FLUCTUATION CORRELATIONS USING A SINGLE-COMPONENT LASER DOPPLER VELOCIMETRY SYSTEM

Author
Ng, T.T.

Publication Date
1981-12-01

LBL-13702
Preprint

Lawrence Berkeley Laboratory
UNIVERSITY OF CALIFORNIA

ENERGY & ENVIRONMENT DIVISION

Submitted to the American Institute of Aeronautics and Astronautics Journal

MEASUREMENTS OF VELOCITY FLUCTUATION CORRELATIONS USING A SINGLE-COMPONENT LASER DOPPLER VELOCIMETRY SYSTEM

T.T. Ng

December 1981

Prepared for the U.S. Department of Energy under Contract W-7405-ENG-48

DISCLAIMER

Measurements of Velocity Fluctuation Correlations Using a Single-Component Laser Doppler Velocimetry System

T. T. Ng

Lawrence Berkeley Laboratory

University of California - Berkeley

Berkeley, California 94720.

Abstract

Methods of measuring several velocity fluctuation correlations: $\overline{u'v'^2}$, $\overline{v'u'^2}$, $\overline{(u'v')^2}$, and $\overline{u'v'(u'^2 + v'^2)}$ using a single-component laser Doppler velocimetry (LDV) system were discussed. The methods offer an alternative to the more complex and costly two-component LDV system for many fluid mechanical measurements. Feasibility of the techniques is demonstrated by studying the turbulent boundary layers over a flat plate with no wall heating, strong stepwise wall heating, and exothermic chemical reaction.

Contents

Laser Doppler velocimetry has become one of the most widely used techniques in fluid mechanical and combustion experiments in recent years. However, its application has been limited mainly to measurements of the mean and the root-mean-square (rms) velocity distributions. In most situations, velocity fluctuations that are relevant to turbulent kinetic energy and momentum transports are valuable in characterizing the flows.

If both velocity components, u in the x-direction and v in the y-direction, are recorded simultaneously, all the correlations involving the velocity fluctuations, u' and v', can be evaluated. This would require a two-component optical system and two frequency measurement units to record two frequency readings simultaneously. The procedures and apparatus involved are a lot more complicated than single-component LDV measurements. By relatively simple methods, however, some important velocity fluctuation correlations can be derived from single-component measurements.

By measuring the velocity component u and two other components, u_1 and u_2, at angles of $\pm\theta$ relative to the x-axis (Fig. 1), it can be shown that

$$u_1 = u \cos\theta + v \sin\theta \qquad (1)$$

$$u_2 = u \cos\theta - v \sin\theta \qquad (2)$$

Using the definition $u_i = \overline{u_i} + u_i'$, where superscript ' denotes fluctuation and $\overline{}$ denotes the mean, the following equations can be derived:

$$\overline{u_1} = \overline{u} \cos\theta + \overline{v} \sin\theta \qquad (3)$$

$$\overline{u_2} = \overline{u} \cos\theta - \overline{v} \sin\theta \qquad (4)$$

$$u_1' = u' \cos\theta + v' \sin\theta \qquad (5)$$

$$u_2' = u' \cos\theta - v' \sin\theta \qquad (6)$$

The derivations of $\overline{v}$, $\overline{v'^2}$, and the Reynolds stress from the three velocity measurements, as described by Durrani and Greated[1], are summarized below:

$$\overline{v} = (\overline{u_1} - \overline{u_2})/2 \sin\theta \qquad (7)$$

$$\overline{v'^2} = [(\overline{u_1'^2} + \overline{u_2'^2})/2 - \overline{u'^2} \cos^2\theta]/\sin^2\theta \tag{8}$$

$$\overline{u'v'} = (\overline{u_1'^2} - \overline{u_2'^2})/4 \cos\theta \sin\theta \tag{9}$$

These procedures are quite standard and have been used in many studies. Several additional higher order correlations, however, can also be derived form the same measurements.

Measurements of $\overline{u'v'^2}$ and $\overline{v'u'^2}$

For convenience of further discussion, define

$$k_1 = u'^2 + v'^2 \tag{10}$$

By evaluating $\overline{(1)^3}$ and $\overline{(2)^3}$, the following equations are obtained:

$$\overline{u_1'^3} = \overline{u'^3} \cos\theta + 3\,\overline{u'v'^2} \cos\theta\sin^2\theta + 3\,\overline{u'^2v'}\cos^2\theta\sin\theta + \overline{v'^3} \sin^3\theta \tag{11}$$

$$\overline{u_2'^3} = \overline{u'^3} \cos^3\theta + 3\,\overline{u'v'^2} \cos\theta\sin^2\theta - 3\,\overline{u'^2v'} \cos^2\theta\sin\theta - \overline{v'^3}\sin^3\theta \tag{12}$$

Adding eq. (11) and (12),

$$\overline{u_1'^3} + \overline{u_2'^3} = 2\,\overline{u'^3} \cos^3\theta + 6\,\overline{u'v'^2} \cos\theta \sin^2\theta$$

or
$$\overline{u'v'^2} = (\overline{u_1'^3} + \overline{u_2'^3} - 2\,\overline{u'^3} \cos^3\theta)/6 \cos\theta \sin^2\theta \tag{13}$$

Subtracting eq. (12) from (11),

$$\overline{u_1'^3} - \overline{u_2'^3} = 2\,\overline{v'^3} \sin^3\theta + 6\,\overline{v'u'^2} \cos^2\theta \sin\theta$$

or
$$\overline{v'u'^2} = (\overline{u_1'^3} - \overline{u_2'^3} - 2\,\overline{v'^3} \sin^3\theta)/(6 \cos^2\theta \sin\theta) \tag{14}$$

In some situations, it may be inconvenient to measure v directly. If θ is chosen to be 60°, the following can still be obtained:

$$\overline{v'k_1} = (\overline{u_1'^3} - \overline{u_2'^3})/(3\sqrt{3}/4) \quad (\text{with } \theta = 60°) \qquad (15)$$

Measurements of $\overline{u'v'k_1}$ and $\overline{(u'v')^2}$

By evaluating $\overline{(1)^4}$ and $\overline{(2)^4}$, the followings are obtained:

$$\overline{u_1'^4} = \overline{u'^4} \cos^4\theta + 4\,\overline{u'v'^3} \cos\theta \sin^3\theta + 6\,\overline{u'^2v'^2} \cos^2\theta \sin^2\theta + 4\,\overline{u'^3v'} \cos^3\theta \sin\theta + \overline{v'^4} \sin^4\theta \qquad (16)$$

$$\overline{u_2'^4} = \overline{u'^4} \cos^4\theta - 4\,\overline{u'v'^3} \cos\theta \sin^3\theta + 6\,\overline{u'^2v'^2} \cos^2\theta \sin^2\theta - 4\,\overline{u'^3v'} \cos^3\theta \sin\theta + \overline{v'^4} \sin^4\theta \qquad (17)$$

Subtracting eq. 17 from eq. 16, one gets

$$\overline{u_1'^4} - \overline{u_2'^4} = 8(\overline{u'v'^3} \cos\theta \sin^3\theta + \overline{u'^3v'} \cos^3\theta \sin\theta)$$

If θ is chosen to be 45°, i.e., $\cos\theta = \sin\theta = \sqrt{2}/2$, it can be shown that

$$\overline{u'v'k_1} = (\overline{u_1'^4} - \overline{u_2'^4})/2 \quad (\text{with } \theta = 45°) \qquad (18)$$

Adding eqs. 16 and 17, one obtains

$$\overline{(v'u')^2} = (\overline{u_1'^4} + \overline{u_2'^4} - 2\,\overline{u'^4}\cos^4\theta - 2\,\overline{v'^4} \sin^4\theta) / 12 \cos^2\theta \sin^2\theta \qquad (19)$$

Example of application

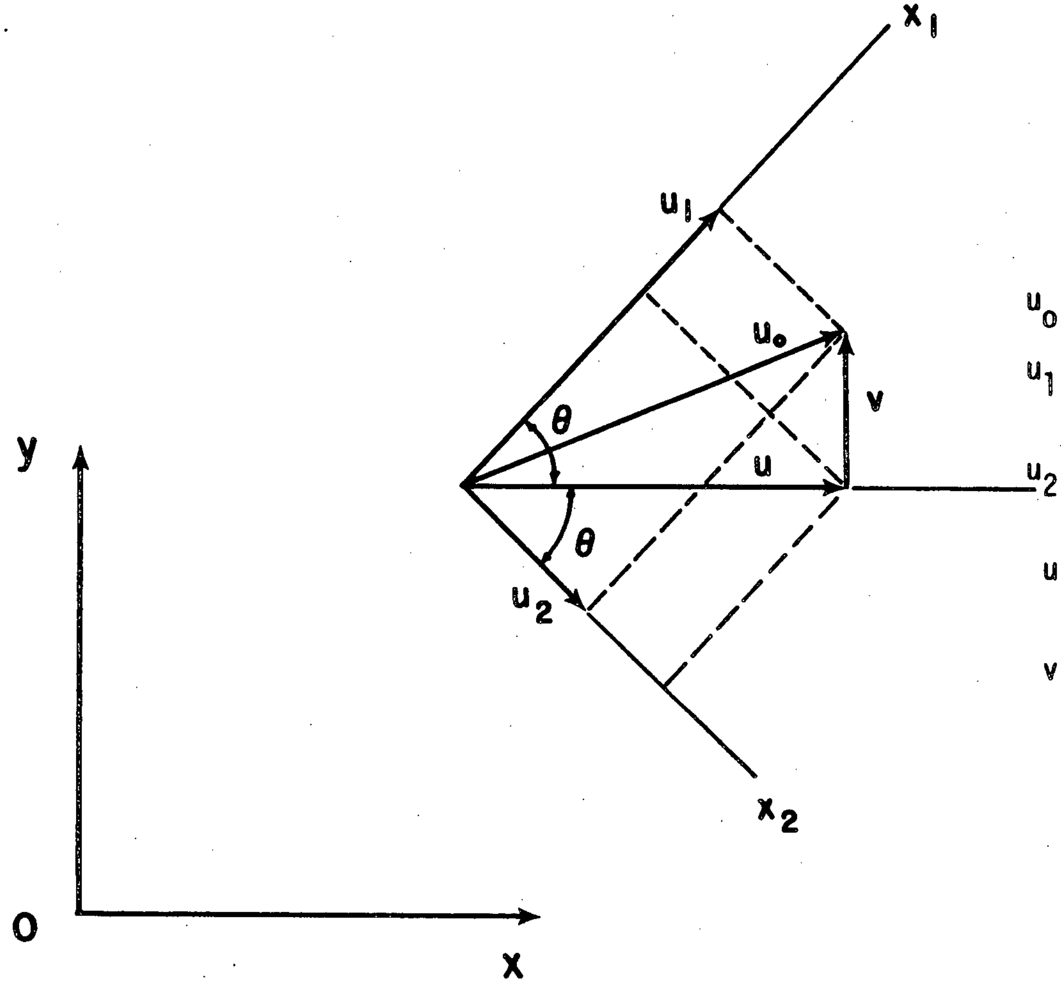
The techniques had been used in studying the boundary layers over a flat plate with no wall heating, strong stepwise heating, and exothermic chemical reaction[2]. Reasonably satisfactory results were obtained. Some examples of the $\overline{u'v'^2}$ and $\overline{v'k_1}$ profiles of the isothermal boundary layer are shown in Fig. 2. More detailed results of the study will be presented in subsequent publications still under preparation.

Acknowledgement

This work was supported by the Assistant Secretary for Energy Research, Office of Basic Energy Sciences, Chemical Sciences Division of the U.S. Department of Energy under Contract W-7405-ENG-48.

References

1. Durrani, T. S. and Greated, C. A. (1977), Laser System in Flow Measurement, Plenum Press, New York.
2. Ng, T. T. (1981), Experimental Study of a Chemically Reacting Turbulent Boundary Layer, Ph.D. thesis, University of California, Berkeley, California.



u_o = Instantaneous Velocity

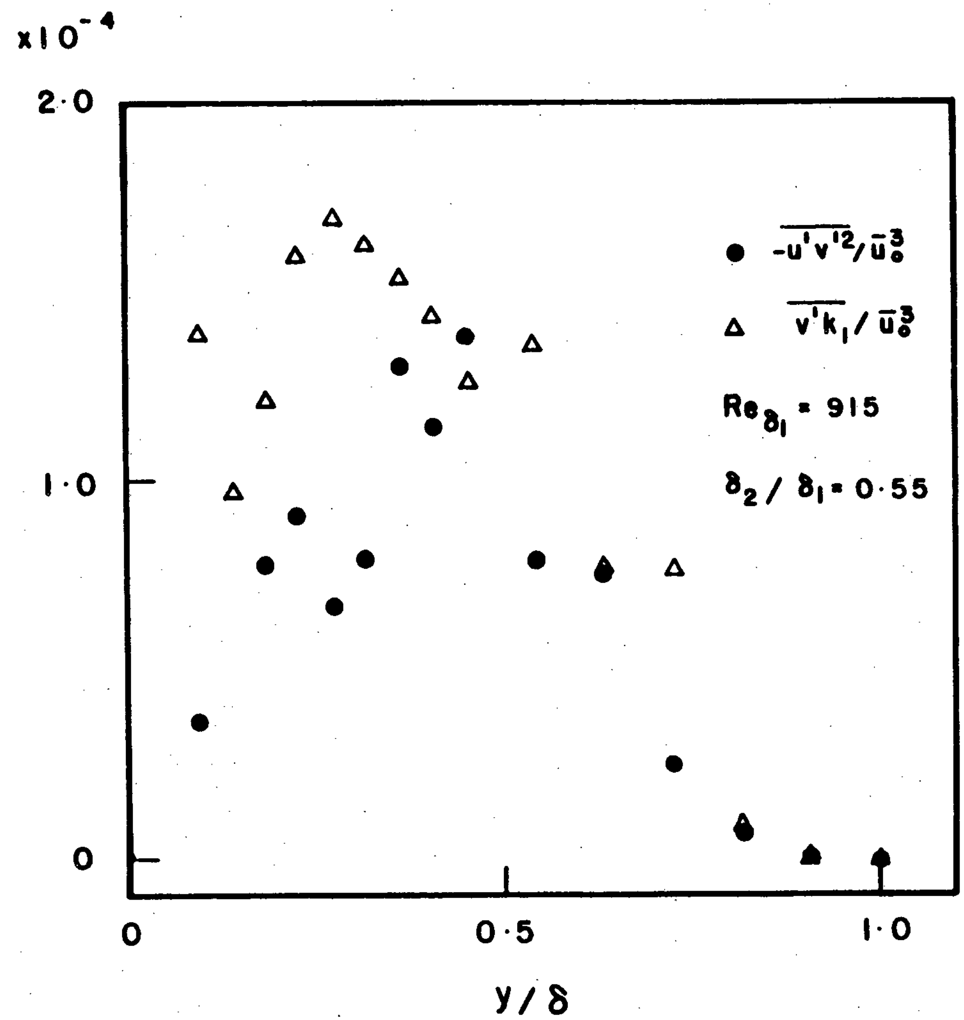
u_1 = Velocity component in x_1-direction

u_2 = Velocity component in x_2-direction

u = Velocity component in x-direction

v = Velocity component in y-direction

Fig. 1   The velocity components



δ_1 = displacement thickness

δ_2 = momentum thickness

Fig. 2 Examples of $\overline{u'v'^2}$ and $\overline{v'k_1}$ profiles of an isothermal turbulent boundary layer over a flat plate.

TECHNICAL INFORMATION DEPARTMENT
LAWRENCE BERKELEY LABORATORY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIFORNIA 94720