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Publication Date

1978-02-01

Submitted to Nuclear Physics B

LBL-7515 C.2
Preprint

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February 15, 1978

Prepared for the U. S. Department of Energy
under Contract W-7405-ENG-48

LBL-7515 C.2

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ON A CONNECTION BETWEEN CHARGE CONJUGATION, QUARK
STRUCTURE AND ORDERED S-MATRIX THEORY*

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ABSTRACT

We show that the representation of particles and antiparticles by the same graphical structure but by opposite orientations yields the quark structure of mesons and baryons and justify zero triality for all hadrons. Such an approach explains the necessity and sufficiency of three colors and justifies the exclusive use of 2- and 3- vertices in the "Ordered Hadronic S-matrix."

* This work was supported by the U. S. Department of Energy.

I. Introduction

In quark models, mesons and baryons are depicted as quark-antiquark objects and three-quark objects, respectively. Furthermore, three colors are introduced to explain certain features of the experimental data. The objective of this paper is to present a connection between the requirement of charge conjugation on the one hand and the quark structure of hadrons on the other. Our topological formalism exhibits features similar to those of quark models but uses less ad-hoc hypothesis. In particular, the number "three" (colors) emerges naturally from topological considerations. The formalism also provides a justification for the basic "building blocks" used in the ordered hadronic S-matrix (i. e., the two- and three-vertices of ordered graphs).¹

As in the "topological expansion",² the starting point is to associate one or more graphs with each transition amplitude. The structure of the graph and the description of particles however differ considerably from the topological expansion. The point of view taken here emphasizes the notion of particle (i. e., pole in an amplitude) rather than the notion of quark constituents. The structure of mesons and baryons as $q\bar{q}$ and qqq respectively will emerge as a natural consequence of the formalism.

In the next section we state the two basic postulates adopted for this study and describe their immediate consequences. In Section III we present the formal argument, based on graph theory. Section IV describes the connection between this formalism and the formalism of the "ordered hadronic S-matrix" and Section V summarizes the results.

II. The basic postulates and their consequences

The basic ingredients of this study are the concepts of graphs and hypergraphs³ which we now define. Consider a set of points $V = \{v_1, \dots, v_n\}$ called a vertex set and a family of subsets of V called an edge set E . $E = \{(v_1 v_2), (v_1 v_3 v_4), \dots\}$. The elements of E are called edges if they are defined by two vertices and hyperedges of order n (or n -edges) if they are defined by n vertices ($n \geq 2$). A hypergraph is defined as the couple $H = (V, E)$ and it is called a graph if all elements of E are edges.

The vertices of any given edge or hyperedge are said to be adjacent. Thus, for all practical purposes in this paper, an n -edge can be associated with - or replaced by - a complete graph of order n : K_n (i. e., a graph with n vertices, all adjacent to one another) so long as the edges of K_n are not considered as elements of E . (This latter point is relevant only if one is interested in the incidence matrix of H which is of no concern in this paper.)

The edges of a graph can be given an orientation if one assumes that $(v_i v_j) \neq (v_j v_i)$ and, correspondingly the hyperedges of a hypergraph can be given an orientation if $(v_1 v_j, \dots, v_k) \neq (v_j v_1, \dots, v_k)$. In this case the corresponding complete graph K_n is no longer unique: some or all of its edges may be oriented (in the sense defined above.) We denote such a partially or totally oriented complete graph by $\bar{K}_n$. We can now state the two postulates adopted for this study.

(a) First postulate: To each state in the physical Hilbert space corresponds a labeled connected hypergraph (or graph). The labels and the connectedness structure completely specify the state.

All possible hyperedges are allowed a priori. We shall see however that a second postulate will considerably reduce the number of allowed states.

(b) Second postulate: A particle and its antiparticle are represented by labeled graphical structures differing only by the "orientation". Conversely, if two labeled graphical structures differ only by their orientation, they are related by charge conjugation.

The orientation of a graph is given by the orientation of each of its edges or hyperedges. Reversing the orientation of a graph means reversing the orientation of all edges and hyperedges in the graph. To satisfy our second postulate, all hyperedges in particle graphs must have two and only two possible orientations. The notion of orientation of a hyperedge will be given a specific mathematical meaning which, in conjunction with a theorem due to Kagno,⁴ will enable us to exhibit a connection with concepts relevant to hadron physics.

Let us immediately consider the possibility of associating certain single-particle states with connected hypergraphs that contain a single orientable hyperedge. The simplest orientable hyperedge is the 2-edge. Let $E = (v_1 v_2)$ be a 2-edge (as shown on

Fig. 1-a). Orienting the edge means, in this case, that the couples (v_1v_2) and (v_2v_1) have different signification. These two oriented edges are shown on Figs. 1-b and 1-c. The "arrow" on the edge defines the orientation we are referring to. The next level of complexity is the 3-edge. As will be shown later, there is only one way to define a 3-edge such that it has two and only two orientations. These are represented in Figs. 2-b and 2-c.

One might suppose that this process can be continued with any n-edge, but we shall see that such is impossible. First, an important remark is in order.

Assuming for the moment that Figs. 1 and 2 represent the only possible way of orienting the 2- and 3-edges, respectively (as will be shown later), one can identify the usual quark structure associated with mesons and baryons. As one can easily guess, the 2-edges are associated with mesons. One of the vertices may be associated with the quark while the other may be associated with the antiquark. The orientation of the connecting edge provides in this case a natural way to distinguish between them. The vertices of the 3-edge are also associated with quarks, but there the particular orientation of the edge prohibits any distinction in the role played by any of the vertices so the associated particle must be of the type qqq as postulated in the quark models. Note that the structures of the oriented edges are not tailored to obtain this result. They are imposed by the requirement that to each particle

corresponds a unique antiparticle (and hence two and only two orientations are required). Finally, the requirement of the two orientations to represent a pole of the scattering amplitude prohibits association of a single vertex with a particle (pole) since a vertex standing alone is not orientable. This latter statement is equivalent to quark confinement. We shall discuss in Section IV in what sense the "quarks" of a 2-edge are equivalent to those of a 3-edge.

To summarize, the requirement that particle and antiparticle are represented by graphical structures differing only by orientation leads to the quark structures of mesons and baryons. Furthermore, as we shall discuss next, no higher order hyperedges possesses two and only two orientations. So all hadrons must correspond to connected hypergraphs built from hyperedges of the type of Figs. 1 and 2 (b,c). Zero triality for all hadrons is thereby assured.

III. Defining "orientation" and formalism.

III.1 Connection between groups and graphs⁵

Consider a set of n elements (vertices) V .

$V = \{v_1, \dots, v_n\}$ and a one to one mapping of V onto itself:

$$P : V \rightarrow V$$

P is called a permutation.

Now we define an automorphism of a graph G as a permutation acting on the set of vertex labels of G preserving adjacency. That is, if E is the set of edges of G and P is the automorphism then:

$$(u, v) \in E \Rightarrow (P(u), P(v)) \in E.$$

The set of all permutations on a graph G , preserving adjacency together with the usual composition law forms a group called the automorphism group of G and is denoted $\Gamma(G)$.

III.2 Equivalent ways of labelling a graph

Given an unlabeled graph G of degree n (n vertices) there are $n!$ ways to label it with n labels. Let G_i and G_j represent two different ways of labeling G . We say that G_i and G_j are equivalent if and only if there is a permutation

P of $\Gamma(G)$ connecting the two graphs:

$$G_1 \approx G_2 \Leftrightarrow \exists P \in \Gamma(G) \text{ such that } G_1 = P(G_2).$$

This is obviously an equivalence relation. The number of inequivalent ways of labeling a graph is thus given by the number of class of equivalences S : namely:

$$(1) \quad S = \frac{n!}{|\Gamma(G)|}$$

where $|\Gamma(G)|$ is the order of $\Gamma(G)$. This result is a corollary of the Lagrange theorem.

III.3 Identifying "orientable" hyperedges

The first postulate introduced in Section II implies that some subset of physical states are represented by a single labeled hyperedge (as a special case of a connected hypergraph). These states are defined in terms of the vertex labels only since the connectedness structure is trivial in that case (Recall that in a hyperedge all vertices are adjacent and one could formally represent a unique hyperedge by a complete graph). However, since in general $(v_1 v_j, \dots, v_k) \neq (v_j v_1, \dots, v_k)$, the set of labels may not completely define the state. Thus, for this particular subset of states, we seek hyperedges - or complete

graphs - such that the association of a label with a particular vertex is irrelevant modulo 2 (for particle-antiparticle representation) - that is to say, based on Eq. (1), we seek hyperedges - or complete graphs $\bar{K}_n$ - such that

$$(2) \quad |\Gamma(\bar{K}_n)| = \frac{n!}{2}.$$

Now, since $\Gamma(\bar{K}_n)$ is a group of permutation of n objects it must be a subgroup of S_n . Furthermore, since the order of that group is fixed by Eq. (2), the subgroup can only be the alternate subgroup. Thus:

$$(3) \quad \Gamma(\bar{K}_n) = A_n.$$

Now we examine the consequences of this result for each value of n separately.

For $n = 1$: Equation (2) would yield $|\Gamma(K_1)| = \frac{1}{2}$ which is impossible and corresponds to the statement that a single vertex cannot have two orientations!

For $n = 2$: $\Gamma(K_2) = A_2 = \{\text{identity}\}$. Only one permutation should be allowed for $\bar{K}_2$: the identity permutation. This graph is represented in Fig. 1-b or 1-c (we see that the other possible permutation P_{21} would not preserve the orientation of the arrow). The graph 1-a would allow both permutations as automorphisms and is therefore not oriented.

For $n = 3$: $\Gamma(K_3) = A_3 = C_3$. Thus, here $\bar{K}_3$ is the graph corresponding to the cyclic group C_3 . This graph is shown on Fig. 2-b (or 2-c). Again, graphs like those in Fig. 3 do not have C_3 as automorphism group and must be rejected.

For $n = 4$: $\Gamma(\bar{K}_4) = A_4$. It turns out, however, that there is no graph of degree 4 (with 4 vertices) whose automorphism group is A_4 (Theorem due to Kagno⁴). Therefore, it is impossible to label a 4-edge in two and only two inequivalent ways. We conclude that there are no particles corresponding to hyper-edges of degree 4. The same result holds for all $n \geq 4$ (Kagno⁴).

We now make two remarks: First, the result above does not preclude the existence of particles (poles) represented by graphical structures containing more than three vertices; it only precludes the use of orientable hyperedges of order higher than three in graphs representing physical processes. Second, it is interesting to note that if any other number of orientations were required the situation would have been radically different. For instance, if antiparticles did not exist, then only one orientation would be required and the relevant group would have been $\Gamma(\bar{K}_n) = S_n$ (since $|S_n| = n!$) and the relevant graph would have been the complete graph K_n which of course exists for all values of n , including $n = 1$. So the limitation of fundamental "building blocks" is directly related to the existence of antiparticles.

To complete the discussion we now define the charge conjugation operation as the operation mapping one orientation onto the other. The operation can be defined on a complete scattering amplitude as well and corresponds to reversing the orientation of every two- or three- edge separately in the graph representing the amplitude.

III.4 Note on baryonium

The results above do not forbid the existence of poles like baryonium ($qq\bar{q}\bar{q}$). It merely specifies that baryonium or, for that matter, any pole, other than mesons and baryons, is to be represented by a superposition of two- and three-edges. For instance, baryonium is represented by two 3-edges with "opposite" orientations as shown on Fig. 4. (The orientation of 3-edges is henceforth denoted by a + or - sign inside the 3-edge to clarify the graph).

IV. Scattering amplitudes and connection with the Theory of Ordered Hadronic amplitudes

IV.1 Color

The question of baryon representation and orientation needs to be clarified. To make contact with physics we assume that the vertices of our edges correspond to the usual quarks. The natural way of labeling the vertices would then be according to the flavors. This mechanism is acceptable for mesons. For instance, $(p\ n)$ would represent a π^+ and $(n\ p)$ would represent its anti-particle (the π^-). Edges like (n,n) are unaffected by inversion of orientation as it should be.

The scheme, however, does not work for baryons where it is impossible a priori to distinguish between the vertices. For instance, a 3-edge labeled (ppp) can represent either Δ^{++} or $\bar{\Delta}^{++}$. To solve this question we must introduce another label which we call "color" and which always distinguishes between the three vertices. Three labels $(1,2,3)$ are necessary and sufficient and we can make the convention that arrows running $(1 \rightarrow 2 \rightarrow 3 \rightarrow 1)$ correspond to the baryon whereas the opposite orientation $(1 \rightarrow 3 \rightarrow 2 \rightarrow 1)$ corresponds to the antibaryon. Of course, the flavor label is in addition to the color label.

The vertices of a 2-edge (meson) are labeled with the same color label. A priori, nothing precludes the existence of 2-edges labeled with two different colors but one can readily show

such a difference to be in contradiction with ordered unitarity requirements (Weissman⁶). The argument is related to that connecting 2-edge quarks to 3-edge quarks. We discuss the rule for the connection in the next paragraph.

IV.2 Scattering Amplitudes

Initial and final states of a scattering amplitude are represented by connected structures of 2- and 3-edges. If vertices are superposed into one vertex, the overall flavor and color are attached to the resulting vertex. For instance, the reaction $\text{baryon} + \text{antibaryon} \rightarrow \text{meson} + \text{meson}$ can be expressed as in Fig. 5.

To make the discussion more formal, a sum is defined on the set of vertex-labels (thus defining an abelian group). Two vertices labeled a and b superposed become a simple vertex $(a + b)$. Now we adopt the following convention:

- a + sign for every vertex belonging to a positively oriented 3-edge (i. e., representing a baryon) and a - sign for every vertex negatively oriented (anti-baryon).
- a + sign for the vertex at the tail of the arrow in a 2-edge (meson) and a - sign for the other vertex.

In order to make contact with the "Ordered Hadronic Amplitudes", we restrict ourselves to graphs such that all superposed vertices correspond to the neutral element (o) . That is, a vertex with

(flavor) label a and (color) label i can only be superposed with the same vertex but with opposite sign (its inverse). The graph of Fig. 5 has been written with this convention. This representation is not crossing symmetric. A crossing symmetric notation can be obtained by superposing all vertices. That is, there should be no pendant vertices. The reaction of Fig. 5 is represented in Fig. 6 in a crossing symmetric notation.

Although this notation possesses the required properties it can be very cumbersome in practice. It turns out to be much simpler to use the edge-vertex reversed graphs. These graphs are obtained simply by replacing every edge (or hyperedge) by a vertex and every vertex by a two-edge in the crossing symmetric graphs.* Figure 7 is the edge-vertex reversed graph of Fig. 6.

Contact is made with the "Ordered Hadronic S-matrix" if one orients the edges of these new graphs from the + 3-vertices toward the - 3-vertices (although this edge-orientation is not necessary if the + and - signs are kept at each 3-vertex).

Thus the analysis of the present paper provides justification for the exclusive use of 2- and 3-vertices as well as for the use of edge coloring in the ordered graphs of the "Ordered Hadronic S-matrix".

* This notation was first proposed by G. F. Chew.

V. Conclusion

We have shown that representation particles and anti-particles by the same graphical structure but by opposite orientations yields uniquely the quark structure of mesons and baryons and zero triality for all hadrons. Such an approach explains the necessity and sufficiency of three colors and justify the exclusive use of 2- and 3-vertices in the "Ordered Hadronic S-matrix".

A more detailed analysis of the topological properties of the graphs representing amplitudes, using the methods of Algebraic Topology may yield further insight on the basic mechanisms suppressing certain transitions relative to others (baryonium decay into mesons, etc.)

Acknowledgments

I would like to thank Professor G. F. Chew for the countless discussions and the constant encouragement he provided during the course of this work.

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FIGURE CAPTIONS

- Fig. 1: Single 2-edge: (a) non oriented and (b) and (c) the two orientations corresponding to a meson and its antiparticle.
- Fig. 2: Single 3 edge: (a) non oriented and (b) and (c) the two orientations corresponding to baryon and antibaryon respectively.
- Fig. 3: Unacceptable "orientation" of a 3-edge.
- Fig. 4: Representation of a baryonium.
- Fig. 5: Representation of the reaction baryon + antibaryon
→ meson + meson.
- Fig. 6: Crossing symmetric notation for the reaction of Fig. 5.
- Fig. 7: Edge-Vertex reversed graph of Fig. 6.

Fig. 1-a

Fig. 1-b

Fig. 1-c XBL782-254

Fig. 2-a

Fig. 2-b

XBL782-255

Fig. 2-c

Fig. 3-a

Fig. 3-b

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Fig. 4

XBL782-258

Fig. 5

XBL782-259

Fig. 6

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Fig. 7

This report was done with support from the Department of Energy. Any conclusions or opinions expressed in this report represent solely those of the author(s) and not necessarily those of The Regents of the University of California, the Lawrence Berkeley Laboratory or the Department of Energy.

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