



**Polymeters, Body, and Mind: One Musician's Creative Experiments with  
(Dis)embodied Rhythm**

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## **Polymeters, Body, and Mind: One Musician's Creative Experiments with (Dis)embodied Rhythm**

As a young music student in Vienna, Austria, I attended several lectures by the ethnomusicologist Gerhard Kubik at the University of Vienna. One form of music he presented, the ancient court music of the Kingdom of Buganda, now in Uganda, completely changed my way of thinking about music. I was an aspiring drummer and composer, learning the craft of my trade, but hoping to become an innovative artist. The most inspiring thing I could have asked for was to find out about a conceptual approach to music that was entirely new to me, and the music of Buganda launched me on a road of experimentation and discovery that continues until today. I am not interested in imitating the music of the Baganda or in appropriating it by taking recognizable elements out of context to use as exotic ornaments for my own music otherwise located in my everyday cultural environment. Rather, my music builds upon the way this tradition makes use of a kind of meta-hearing in order to open doors to new ways for musicians to relate to one another in performance. Among the results of my research and experimentation has been the development of techniques of polymetric and polytempo interplay in ensembles, hopefully leading to new ways of listening to and understanding music, for both performing musicians and audiences; this can be done, to very different effect, both with and without computer technology. In this paper, however, I will focus on music I have conceived for myself to perform solo. The Baganda music as de- and transcribed by Gerhard Kubik and other ethnomusicologists (Klaus Wachsmann, Peter Cooke, etc.) is a music performed by ensembles of three or more musicians. But the idea of adapting these approaches to solo performance does not seem far-fetched, as the court music of the Baganda is assumed by researchers to have originally been a solo music that was only later reconceived to become an ensemble music tradition.

In Western traditions, musicians normally play together at the same tempo and share a common beat. A beat is, essentially, a division of time the musicians use to maintain their feeling of, and bearings within, the music. Often considered the basic unit of time, the beat is the moment at which you might tap your foot along with the music. Today's Western concept of the beat, which developed out of the 'tactus' (a term that emerged in the late 15th century through writers like Adam von Fulda, though there doesn't seem to be a total consensus as to a clear definition [Panov 2014]), allows it to be subdivided into any number of sections, creating a sort of internal microstructure within its time period. While Europe's music traditions are extremely diverse, with some styles exhibiting an extremely flexible use of the beat and much variation of tempo generally left up to the interpreting musicians, the musics of Africa are, if anything, even more varied, and it would be a mistake to generalize the concepts of its rhythmic-metric organization. Some African music does not have any constant tempo or beat - for example, the talking drum music of Nigeria. But it would still be accurate to say that a certain approach to thinking about rhythm underlies the majority of musical styles and traditions in Africa south of the Sahara. These musics also use beats, but while European music subdivides a slower beat, African music often employs a fast basic pulse, of which any unit can contain a sound or be a rest. Often, 2, 3, or 4 pulse units will be grouped together, and the beat occurs at the

beginning of every such grouping. One of the most typical and interesting features of African music is that this grouping can be ambiguous - for example, it might be unclear whether 3 or 4 pulses form a beat, and different people might hear, and choose to dance to, different groupings.

I greatly enjoy working with electronics in an African music environment. To many people, that seems counterintuitive due to the inaccurate but widely accepted image of Africa as a continent lacking in development or sophistication. But my feeling is that with its pulse-based architecture, African music is a great fit for digital technology. Computers run based on a fast pulse, for each unit of which electrical current can be sent (1), or not (0), just like the elementary pulses of African music that contain either sound or silence. It could thus be said that pulse-based African music is, in a certain way, inherently digital.

In addition to the ambiguity of pulse groupings, African music also often makes use of illusions as to when the beat occurs relative to significant sound events. Consider, for example, the following Senegalese rhythm:

Fig. 1: ----x-----x-----x-xx-x-xx-x-x-----

(where x is a sounding pulse and - a silent one). Hearing this, even if repeated many times, one will probably ask oneself: Where is the beat? It's difficult to tell: the relatively few sounding pulses, especially during the first half of the pattern, as well as the circumstance that the pattern is quite long at 48 pulses, make it hard to detect any regularity unless one is already "tapping ones foot" at the correct moments. If we see local people dancing to this rhythm, observing when their feet hit the ground will give us important clues. It turns out that the elementary pulses are in groups of 3, and the beat is located on the underlined pulses:

Fig. 2: ---x-----x-----x-xx-x-xx-x-x-----

In witnessing many kinds of African music played in situ across the continent, I have come to regard the 'danceability' of music as a phenomenon of enculturation: Essentially any music can be danced to if you know the code - what to listen for in order to be able to combine the aural and the kinetic. This became clear to me during a tour of Zimbabwe in 1997 when I witnessed villagers of the Batonga tribe in the remote village of Siachilaba, near Lake Kariba, dance to their traditional *Ngoma buntibe* funeral music. They had no difficulty dancing, while I struggled to identify any regular patterns whatsoever. I would even go so far as to say that the music of, say, Pierre Boulez, could be experienced as dance music, even if he probably wouldn't have liked the idea: in order to play the music, the performers must execute certain movements on their instruments. Music and movement are thus inextricably linked, even if the music does not seem to invite physical motion in listeners. Music and dance are two sides of the same coin - related, but with a reversal of cause and effect: in music performance, a dance - a rhythmically controlled succession of movements on the instrument - gives rise to sound. In dance, the sound gives rise to movement.

In many African languages, the words for these two activities are the same; no difference is made.

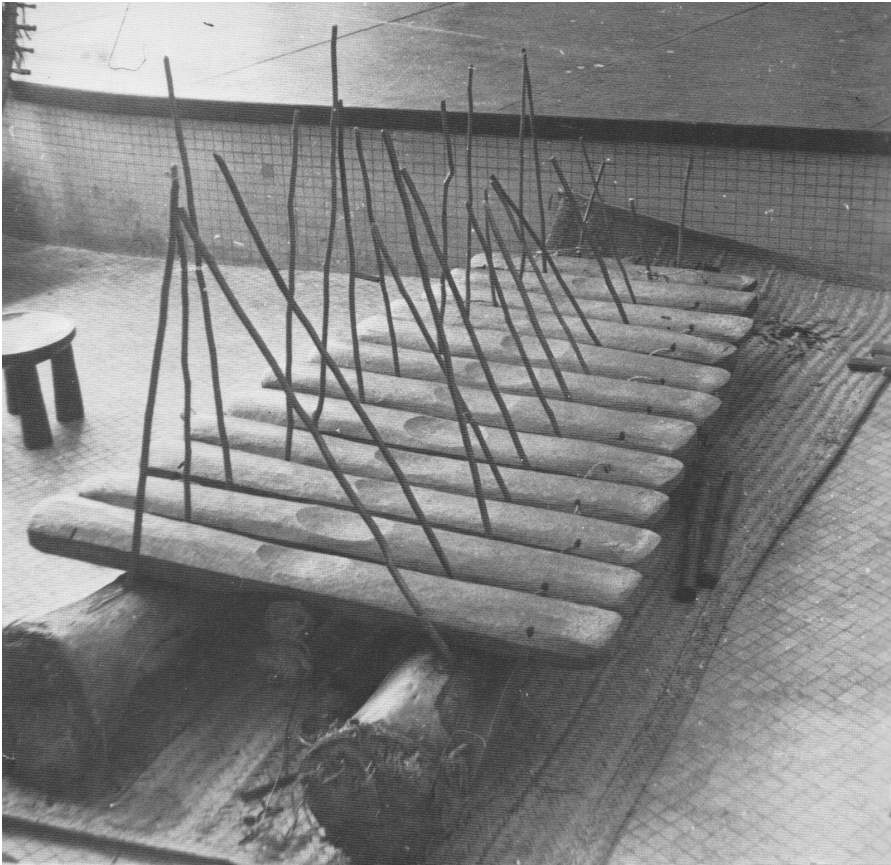


Fig.3: *Amadinda*, from Kubik (1983)

In the music of Buganda, the African interest in beat manipulation is exploited to surprising effect. The music is played on various instruments and in various ensemble configurations, but for the purpose of this brief explanation, we will focus on one typical instrument, the *amadinda* (Fig. 3), a xylophone played simultaneously by three musicians. The music is pentatonic in equidistant tuning; in other words, the octave is divided into five equal intervals (while much Western music is based on so-called equal temperament, where the octave is divided into 12 equal intervals). The *amadinda* has 12 keys: two full octaves plus two extra keys starting a 3rd octave at the high end of the instrument. Two musicians, known as *omunazi* and *omwarwuzi* and sitting facing each other on the two sides of the instrument, play the 10 bottom keys, with a total range of two octaves. The third player, the *omukoonezi*, sits next to the *omunazi* and plays exclusively the highest two keys, which are untouched by the other two players. (Fig. 4)

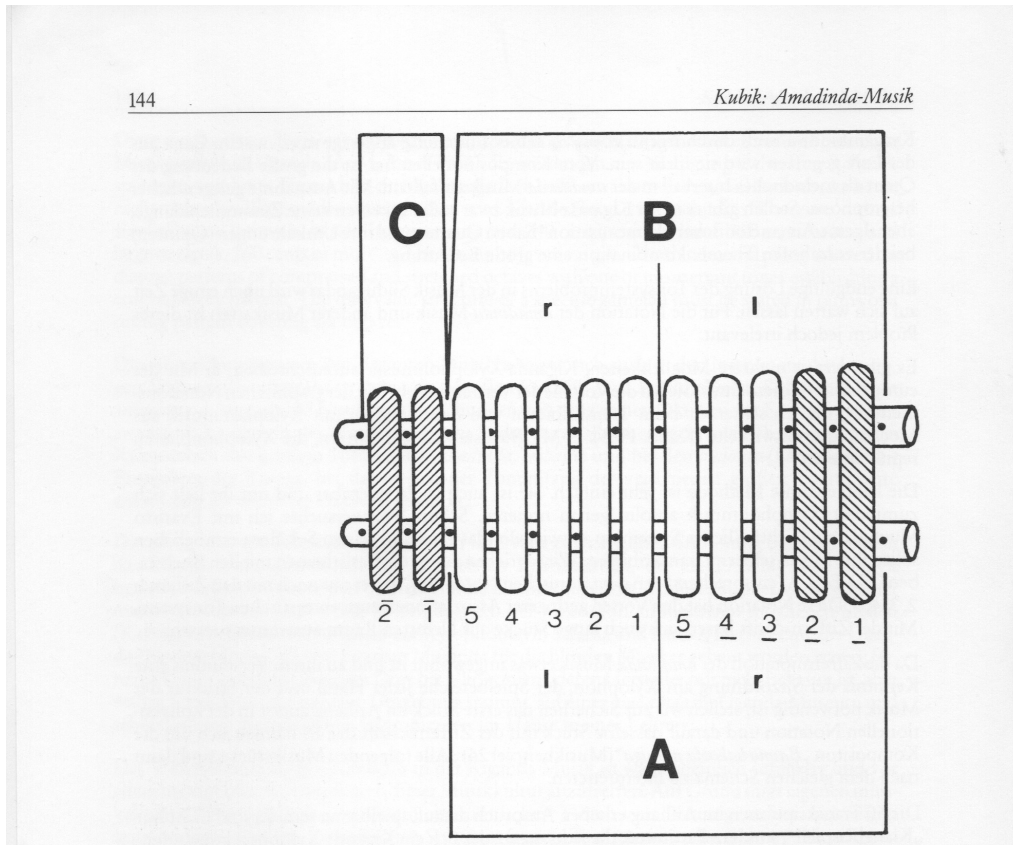


Fig. 4: Schematic drawing of *amadinda* and players' sitting positions. From Kubik (1983)

The omunazi begins to play a melody, called the *okunaga*, always with both hands simultaneously in parallel octaves, at a fast tempo in a steady pulse. The melody, which can be up to 48 pulse units long or in some cases even longer, is repeated over and over again without variation. At a predetermined moment, the omwawuzi starts playing the *okwawula*, a melody with similar characteristics. It can be of the same length, or if not, it is usually shorter so that the length of the *okunaga* is a simple multiple of the *okwawula*. However, the omwawuzi doesn't play at the same time as the omunazi, but rather places his notes exactly in between those of the *okunaga*, so that the two musicians together produce an interlocking structure. At a slow tempo, that wouldn't be too difficult, though the omwawuzi would have to syncopate incessantly. But at the fast tempo of this music - with each musician playing their notes at tempos of up to 250 a minute - syncopation becomes impossible. Any attempt to do so leads to slowing down, and the result would be that the two players gradually go out of phase with each other, a desirable effect perhaps when playing certain compositions by Steve Reich, but not in the music of the Baganda.

So how does the omwawuzi manage to interlock his part? The answer is that while the musicians share an elementary pulse (of which each musician plays every other onset), they both perceive of themselves as playing on the beat and regard their respective opposite as syncopating. In other words, while they are always in coordination with one another, the musicians have different ideas as to

when within the music the beat occurs. This idea of a relative beat over an absolute pulse exists in various forms of Central African music, but not in Western music, and it leads to different functional relationship between collaborating musicians than we know in the West.

Since the two musicians are sitting opposite each other sharing the same keys and playing somewhat similar melodies with obviously the same timbre at a very high tempo, it quickly becomes impossible for the listener to distinguish between the two interlocking melodies. Rather, the listener's brain starts reorganizing the fast stream of pitches according to other criteria, most notably frequency. A melody emerges in the high register, another in the low register, and perhaps even one in the middle. Obviously, none of the melodies are played as such by any one person; they are the result of the combination of the two players' parts. Gerhard Kubik calls these resultant melodies 'inherent patterns'; they are integral to the composition, but not actually played. This is just one of many examples of how African cultural traditions address cognitive phenomena, an interest that can be detected in many forms of music and visual art throughout the sub-Saharan part of the continent.

I have not yet explained the role of the omukoonezi. While the other two musicians play only the lowest two octaves of the amadinda, the omukoonezi plays his part, the *okukoonera*, on the two remaining keys, accurately doubling the inherent pattern that emerges on the two lowest keys, two octaves below. This being an inherent pattern, it isn't actually played as such by either of the other musicians, and as one can imagine, it might be a fairly complicated, jagged rhythm with few obvious regularities. How can the omukoonezi remember this pattern accurately and execute it with precision? It turns out that a mnemonic is available: while this is instrumental music, the compositions were originally songs, with lyrics, sung in Luganda, a tonal language. And the rhythm and pitch inflections of these lyrics underlie this particular inherent pattern. If the musician knows the lyrics, he can replicate this inherent pattern, especially once the other two players provide the elementary pulse.

This short summary barely scratches the surface; more extensive explanations are available in a variety of articles and books, such as Kubik 1994 (pp. 47-86 and 249-328). But I think even this brief summary clearly shows that this is an exceedingly complex and multifaceted form of music, and one that must be completely through-composed rather than improvised in order to maintain its structural integrity.

Armed with this new inspiration, I set out to try to find ways of developing my own music. Experimenting on the drums, I soon came up with what I call a choreographic approach to the performance of polymeters. *Meter* refers to larger groupings of beats; I have always been interested in developing structures based on the simultaneity of different meters and tempos and in how these structures can help us stake out new ways of listening to and understanding music. The simultaneity of different pitches, leading to harmonies, began appearing in Western music some 1000 years ago; just as multiple pitches form consonances and dissonances, I'm interested in consonances and dissonances of tempo.

Polymeters are present and have been explored in many forms of Western music, starting in the Renaissance, but especially in the last 100 years or so. Several composers, such as Conlon Nancarrow, specialized in such structures; nevertheless, they are not strongly represented in the West compared to African music, and overall, I feel that it is an area of music worthy of much more attention, with great potential to lead us towards new discoveries of how musical structure can be perceived and understood.

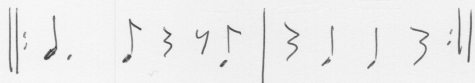
In my choreographic drumming technique, my two hands fulfill roles akin to those of the omunazi and omwawuzi on the amadinda. Each hand plays a 'melody' around the drum set. But the drum set is a very different instrument from the amadinda. On the drums, I work exclusively with indefinite pitches, and each component of the drum set has a different timbre; my melodies, therefore, are timbral melodies rather than pitch melodies. I soon arrived at forms of motion that could be described as drawing circles around the drums - this made it easier for me to perceive the cyclic, repetitive nature of my melodies, which, due to the configuration of my fairly standard drum set, tended to be a lot shorter than the amadinda melodies. For example, I could repeat a pattern of just three timbres with my right hand. Let's say I start with the ride cymbal, continue from there to the right rack tom, and from there to the floor tom, and then start over again. Then, I could add a melody of four timbres in my left hand - let's say I start on the left rack tom, move from there to the snare drum, on to the hi-hat, and back to the snare before starting over again on the rack tom. Unlike the amadinda repertoire, I prefer to keep the melodies short, so that their lengths are not necessarily multiples of each other. By combining my left- and right-hand patterns, I arrive at a pattern  $3 \times 4 = 12$  notes long. But I'm interlocking my hands; therefore, the pattern is actually  $12 \times 2 = 24$  pulse units long.

A special feature of drum set playing is that my feet are also active via pedals; my right foot plays the bass drum while my left plays the hi-hat. To put my feet to use, I can integrate them into my hand patterns; however, this turns out to be quite difficult because it necessitates interrupting the cyclic motion of my hand for a moment while my foot plays, which is not pleasant for my kinesthetic sense. (I later came up with a solution for this by constructing "non-sounding instruments": plastic temple blocks onto which I stuck lots of foam, making them inaudible. By placing these "rest-playing instruments" in strategic positions around my drum set, I was now able to also incorporate rests into my hand patterns while maintaining steady cyclic motion). But I could also use my feet to add further layers to the polymeter I set up between my hand melodies. For example, I could play the hi-hat on every 5th pulse, regardless of which hand it would coincide with. That now gives me a cycle  $24 \times 5 = 120$  pulses long before the first repetition. And then I could add the bass drum on every 7th pulse. Now I already have a pattern  $120 \times 7 = 840$  pulses long! In other words, using a very simple concept, I was able to construct cycles of exploding complexity, not unlike fractal geometry where the iteration of a simple formula leads to structures that become vastly more complex with every pass.

While playing these patterns, I am not feeling a definitive beat; in other words, I can change my perception of the beat as I go along. I can feel it in my right hand

or in my left, in one of my feet, as focused on a particular timbre that recurs every so often, or as any kind of grouping of pulses. Rather than one perspective of the music, multiple, ever-shifting viewpoints are possible, akin to walking around a sculpture and examining it from its different sides. (During a visit to Brasilia in 2013, I saw a sculpture that in many ways exemplifies this effect: *Coluna* by the little-known Austro-Brazilian artist Franz Weissmann in the Itamaraty Palace, the foreign ministry of Brazil, looks entirely different yet nevertheless similar depending on where you stand.) In attempting to find my bearings in this music, I track my limbs and take note of their relative positions and symmetries. It is a strongly visual and motor-memory-driven approach to feeling music, rather than a quantitative one (as in counting 1-2-3-4, etc.). As I play, my mind seems to be sitting somewhere amid my flailing limbs, trying to keep it all under control. Cecilia Sjöholm (2003) wrote: "In order to see the world, we have to be situated in it, and engage in it from a certain viewpoint which we cannot fully detach from our bodies. This is what I would like to call a situation of embodiment. Perception cannot be detached from a particular situation of embodiment."

Of course, in my drum set playing the listener doesn't perceive inherent melodies in different frequency bands. But, by contrast, similar timbres tend to be grouped together, and inherent patterns of tom-toms or cymbals emerge; the image becomes more complex if I incorporate one or several components of the drum set into both hand patterns. The use of cowbells, which have a penetrating timbre very different from other components of the drum set, causes particularly conspicuous patterns that might remind of the *clavé* (or 'asymmetric timeline') patterns of certain styles of African and Afro-Caribbean music. Not a feature of Baganda music, but present in many forms of music in Central and Western Africa, especially near the Atlantic coast, these patterns, repeating over and over again, are usually quite short and serve to give the musicians a metric reference. While such patterns can easily be notated conventionally - for example, *Fig. 5* illustrates a *rumba* pattern - this notation seems to impose a given placement of the phrase relative to the beat and doesn't really convey the cyclic nature of the pattern. More successful in these regards is a circular notation proposed by the mathematician Godfried Toussaint (*Fig. 6*), all the more because it allows for an interesting observation: Listening to lots of *clavé*-driven music, the patterns seem to resemble each other, and it turns out that some of them are actually identical, but with shifted starting points. This once again harkens to the way African music traditions capitalize on ambiguity and how the listener's mind processes this 'relative', rather than 'absolute', information.



*Fig. 5: A rumba clavé*

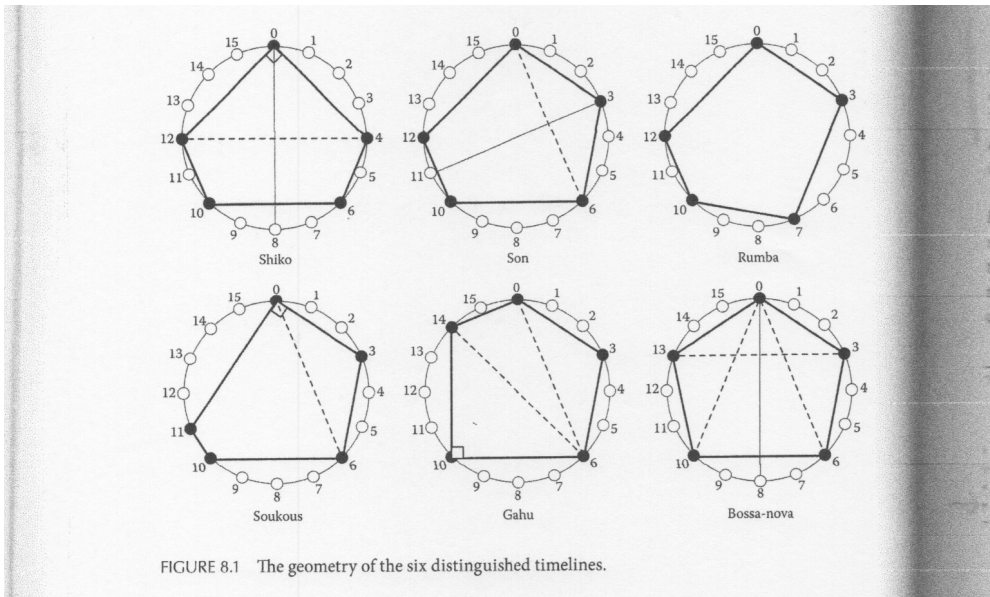


FIGURE 8.1 The geometry of the six distinguished timelines.

Fig. 6: Godfried Toussaint's 'cyclic' notation. From Toussaint (2013)

Since 1994, I have worked frequently with traditional musicians in Africa - initially in Côte d'Ivoire, but later on also in other countries in Western and Southern Africa, as well as, in 2007, in Uganda. In 2000, I worked on a project with musicians from various Caribbean islands living in Miami. I gave them a simple clavé pattern - an indispensable feature of their music tradition - and asked them to play its 'negative': to produce a sound for all pulse units that were normally silent and remain silent at the pulses that usually contain a sound (Fig. 7). To my great surprise, this proved impossible within the short rehearsal time available. While I am not certain of the reason for this, and would have to investigate this problem with a much larger sample of musicians, my hypothesis is as follows: Unlike my West African collaborators, who were generally open to dissecting the structures of their music traditions in any number of ways, my Caribbean collaborators seemed to experience the clavé pattern as one indivisible *gestalt*. Disassembling it seemed like sacrilege. I believe that this was due more to social than to cognitive constraints. As opposed to the African musicians, who may feel a lesser obligation to be agents of preservation, the Caribbean musicians were in a double-diaspora situation of sorts: the traditional patterns had migrated from Africa to Cuba and Puerto Rico, where they were perhaps already somewhat decontextualized. Now, having migrated further to the U.S., it fell upon these musicians to serve their communities by protecting these rhythms from dilution and degeneration.

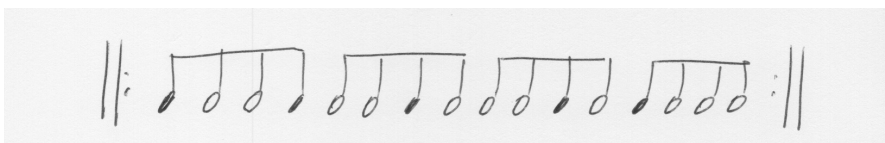


Fig. 7: A son clavé and its 'negative'

Returning to notation, absent lyrics in my patterns, I needed some way to help me remember them, especially because I am an improviser and actually rarely perform a pattern in its original, unaltered way. I tried notating the music on five lines, but the result was a nearly illegible jumble of notes. (I should add, however, that I do notate such patterns on five lines when writing for other musicians, such as Fig. 8 - as an improviser, I am not a particularly skilled sight-reader of music, but musicians for whom I compose are mostly specialized in the interpretation of written music and are thus much more adept at deciphering complex notation.) It then occurred to me that, in playing these patterns, I am not concerned with rhythm, as it is constant in its execution, its complexity emerging only as an auditory 'illusion' through inherent patterns of timbre. Nor am I concerned with pitch, nor even timbre, as that is, again, an automatic consequence of whatever component part of my drum set I hit. I am uniquely concerned with position, and position-based music notation has existed for a long time: it is known as *tabulature*. I therefore came up with a tabulature notation for drum set based on a schematic aerial view of the instrument, with the paths of my hands written in (Fig. 9).

The image shows three systems of musical notation for a solo drum set. Each system consists of three staves: RH (Right Hand), Fr (Front), and LH (Left Hand). The notation is highly complex, featuring numerous rhythmic patterns, rests, and dynamic markings. The first system includes a circled '2' in the top right corner. The second system is labeled 'PART 3' and the third system is labeled 'PART 4'. The notation is dense and intricate, reflecting the complexity of the music described in the text.

Fig. 8: A page from Lukas Ligeti's *Conceivably Infinite* for solo drum set (2015)

With many of these patterns, the dance described by my hands is rather easy to play. To the best of my knowledge, no other drummers play in this way; however, that's not because it is technically difficult, but because drum set playing is normally 'thought' in a different way and my polymetric style is not obviously compatible with the role the drum set fulfills in conventional ensemble playing (although I have occasionally anchored more conventional music with

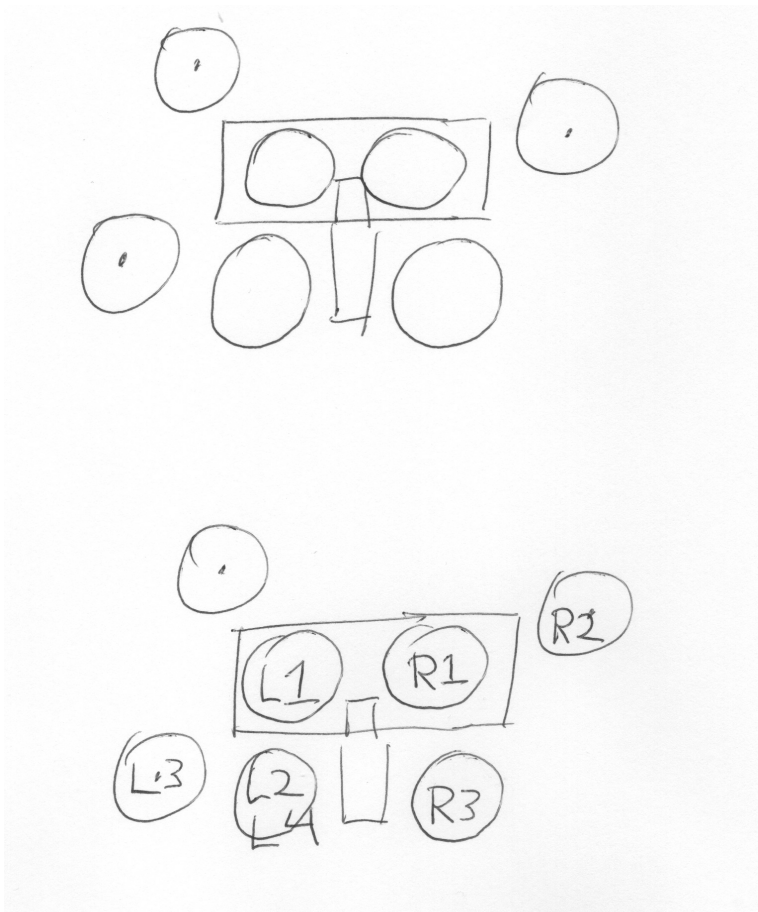


Fig. 9: Lukas Ligeti, Drum Set Tabulature

such rhythmic patterns). But, analogous to the complexity and length of the patterns that explodes with the addition of further limbs, adding the third limb poses a minor challenge while bringing in the fourth makes things very difficult and practice-intensive. At that point, I struggle to achieve the equilibrium of attention between my limbs that allows me to listen to myself in a disembodied way, walking around the 'sculpture' while my limbs move. I am currently beginning to look in a more detailed way at which kinds of patterns pose which kinds of difficulty of coordination and hope to arrive at more systematic conclusions about this in the future. I hope this will give me a more concrete knowledge of how certain forms of physical coordination relate to acoustic and position-based cognition.

But the struggle between embodiment and disembodiment on the drums can also play out in other, rhythmically much simpler, ways. When I began playing drums (some time before I began working on my polymeric patterns), I was strongly dependent on the sound the drums emitted in response to my sticks. One could say that I was 'looking' for certain sounds, and if my sonic expectations were not met, I would become confused and distracted and would struggle to maintain coordination between my limbs. But then, I began spending

time in Africa, where instruments were often in bad condition due to the unavailability of spare parts. In preparation for my first trip to Côte d'Ivoire, I was told I would be provided with a drum set, but that it was "old". I asked whether it would help if I brought new drumheads, as I was told these were not readily available in the country. This met with an enthusiastic reaction, so I asked for the sizes of the drums and arrived in Abidjan, Côte d'Ivoire, with new drumheads, ready to "meet" the drums and change the heads. But the drums were in truly bad shape. Their circular shape was no longer quite intact, and the tuning mechanisms were extremely rusty. Changing the heads proved impossible, so I had to make do with the extant heads, which gave the instrument a "dead", toneless sound. In order to keep the band together, I now had to execute certain motional sequences without having an expectation of hearing the desired sounds, or let's say, I had to project what I wanted to hear onto the actual sound of the drums. This proved enormously educational to me as a drummer: I began to be able to dissociate my body movements from the aural feedback I was receiving. And consequently, this divorce between sound and motion has become of great interest to me, and has helped me not only in my playing of conventional drum set patterns, but also in my polymetric ones. My polymetric patterns are based on motion and structure; strange to say in music, sound is less crucial to their execution. It is extremely important in their conception; when designing such patterns, I need to take into account the timbral 'inherent patterns' that emerge. For the listener, sound is crucial. For the player, however, sound becomes merely a by-product of the movement patterns executed by the limbs.

The use of electronic and computer technology can greatly expand musical and research possibilities in these areas, and as mentioned earlier, I find this particularly rewarding in an African cultural context. I began playing electronic percussion around 1993 and have used various types of electronic percussion instruments in my collaborations with African musicians as well as in solo concerts. While the sound of each component instrument of the traditional drum set can be manipulated in highly nuanced ways depending on the type of stick or mallet used, how and where it is struck, etc., using electronic drum pads gives me an essentially limitless selection of pitches, timbres, etc. More crucially, this limitlessness is available every time I play a pad - in other words, it can have an entirely different sound each time I hit it. Thinking back to the earlier example of the 840-pulse pattern, one can easily imagine the added layers of complexity that emerge when using this possibility of electronics. If I replace my ride cymbal by a pad that cycles through a stack of, say, 3 sounds, playing a different one on every hit, and proceed similarly with the right rack tom but with a stack of 4 sounds, and with the floor tom but with 5 sounds, my right-hand pattern has already expanded from a cycle of 3 notes to one of  $3 \times 4 \times 5 = 60$ . Using this technique, I was soon able to play patterns that last hundreds of thousands or even millions of pulse units until they repeat. The played patterns, meanwhile, remained the same as on the drums - but sound was, once again, separate from the mechanics of playing. Similarly, I have experimented with more conventional drum patterns on electronic percussion, but with the sounds emitted by the pads changing in complex or unpredictable ways that surprise me and force me not to rely on expectations. I am only in the beginning stages of more systematic

research on how this - and the consequent divorce of my internal referencing of my movements from the aural result - might change the way I think about and perceive both the motor-memory and the musical aspects of my drumming. Furthermore, these examples employ the possibilities of electronic technology in very simple ways. The complexity of possible uses seems unlimited.

In the last 10 years, I have begun writing polymetric drum patterns for other musicians. Initially, I regarded this solely as a way for myself to play, and mostly in a solo context. Interpreting drum set players were rare, and the concept of this playing style was so far removed from conventional playing that I felt insurmountable hurdles in communicating it to other musicians. But my feelings about this have changed, particularly with the ascent of a new generation of 'classical' musicians, my age and younger, who are much more open to unusual concepts and technically challenging work. Thus far, I have employed polymetric patterns in pieces for percussionists such as Ben Reimer, David Cossin, Colin Currie, and Ian Ding. I have also adapted ideas from Baganda music to ensemble music, using various techniques both with and without electronics to create environments that facilitate pulse-based rhythmic thinking and multi-tempo structures. This has led to hopefully new ways musicians in an ensemble can interact based on various possibilities of perceiving and thinking about rhythm and meter. While these experiments are beyond the scope of this paper, I hope the preceding descriptions and observations give some indication of the enormous and as yet little explored potential of rhythm and beat cognition for innovative artistic expression.

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