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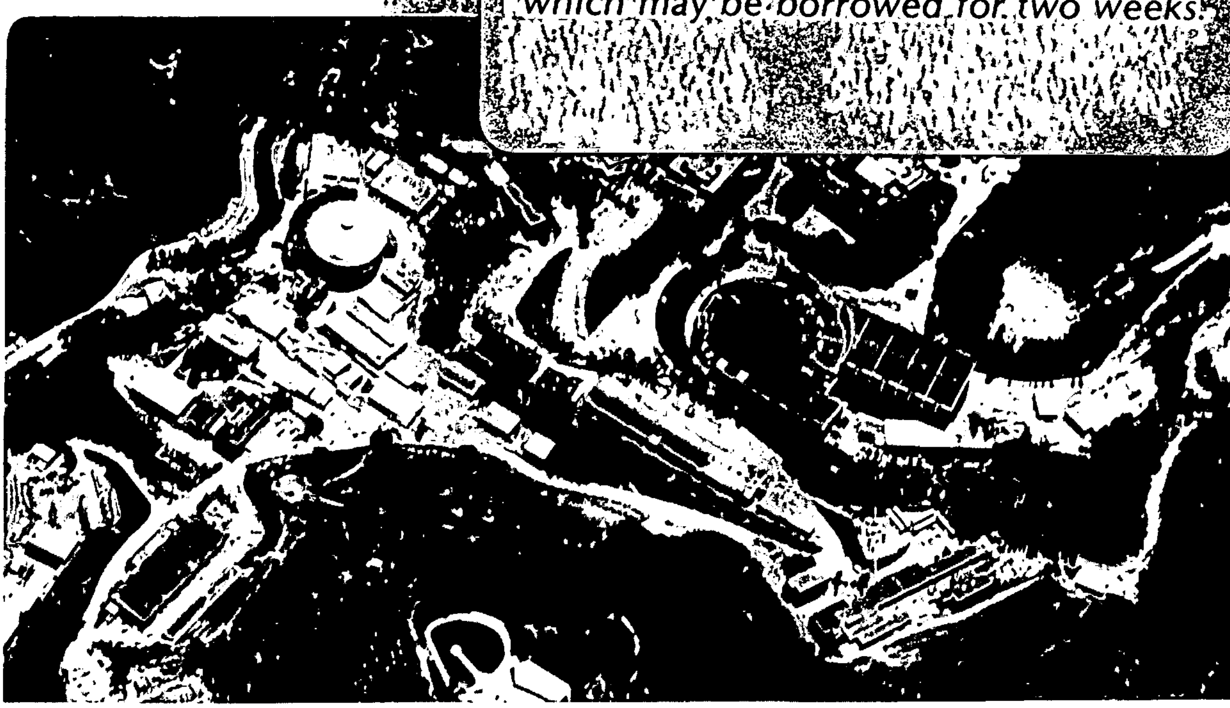

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RADIATIVE CORRECTIONS IN COMPACTIFIED SUPERSTRING MODELS*

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ABSTRACT

We calculate one–loop corrections to the effective potential in models obtained from compactification of ten–dimensional superstring theories. We find that no masses are generated for gauge non-singlet scalars even in the presence of supersymmetry breaking terms induced by gauge and gaugino condensation, but that the gravitino mass is determined at one loop. The scales of grand unification, supersymmetry breaking and condensation are fixed by the gauge singlet scalars and are found to be close to the Planck scale. Requiring $M_{GUT} < M_{\mathrm{Planck}}$ restricts the other parameters of the theory. We also discuss the one-loop effective potential at scales between the condensate and compactification scales with possible implications for the allowed particle content of the effective theory.

In most attempts[1,2] to extract the low energy phenomenology from the effective Lagrangian[3] of N = 1 supergravity, an effective renormalizable theory is specified at a scale near the Planck scale κ^{-1} by taking the limit $\kappa \to 0$ after appropriate shifts have been performed on those fields that acquire vacuum expectation values (vevs) of order or greater than the Planck scale. The usual renormalization group methods are used to study the theory and its vacuum structure at lower energies. This procedure can be misleading if radiative corrections themselves play a role in specifying some large vevs. This is in particular the case for models of the no–scale type[4] that apparently emerge from the reduction [5] of 10 dimensional superstring theories to four dimensions, where one scalar field remains undetermined at tree level.

In this letter we present the one loop corrections to the effective potential in these models[5], with and without the contributions[6] to the effective tree potential that are induced by gauge and gaugino condensates in the hidden sector. The results obtained without the condensate contribution are valid at scales above the condensate scale μ. The stability of the one-loop effective potential in this case provides a constraint on the number of gaugino degrees of freedom allowed at these scales. When this constraint is satisfied, the one loop potential has the interesting property that the quadratically divergent terms can be reabsorbed into wave function and/or coupling constant renormalizations.

The more interesting case is the one including condensate– induced terms, which is valid below the condensate scale. These terms break supersymmetry at tree level in the effective potential but the scale of supersymmetry breaking remains undertermined at that level; it has been argued[7,8] that squark and Higgs masses will be generated, leading to (via renormalization group effects in the gauge non-singlet sector[2]) spontaneous breaking of the electroweak symmetry and the simultaneous determination of the supersymmetry breaking scale.

We find instead that the vev of the scalar that is undetermined at tree level is fixed by the one-loop corrections in the gauge–singlet sector of the theory (the dilaton, another scalar field coming from the 10– dimensional metric and two axions[5]). This in turn fixes the "grand unification" mass M_{GUT} and the gravitino mass at values near the Planck scale that are relatively insensitive to choices of the "grand unification" gauge coupling α_{GUT} of the observable sector and the β–function of the strongly interacting hidden gauge sector, although requiring $M_{GUT} < M_{Planck}$ restricts the allowed range of these parameters. On the other hand, the gauge

non–singlet scalars remain massless at the one-loop level.

The one loop effective potential is given by

$$V_{eff}(Z) = V_{tree}(Z) + \frac{1}{2(4\pi)^2} STr \int dp^2 p^2 \ln[p^2 + M^2], \qquad (1)$$

where M^2=$M^2(Z)$ is the squared mass matrix evaluated for expectation values Z of the scalar fields. If we choose for the gravitino ψ_μ the gauge condition $\gamma^\mu\psi_\mu$=0, the supertrace of a function $F(M^2)$ of $M^2(Z)$ is:[9]

$$STrF(M^2) = 3TrF(M_V^2) + TrF(M_S^2) - 2Tr(M_F^2) + 2F(4m_{\tilde{G}}^2) - 4F(m_{\tilde{G}}^2) \qquad (2)$$

where $M^2_{V,S,F}$ are the squared mass matrices for vectors, scalars and spin-1/2 fermions, respectively, and

$$m_{\tilde{G}}^2 = e^{\mathcal{G}}, \qquad (3)$$

with $\mathcal{G}$ the Kähler potential, is the value of the gravitino mass at tree level.

The integral in (1) can be regularized by introducing a cut–off $\bar{\mu}^2$ or by a (double) subtraction procedure that introduces a scale parameter $\bar{\mu}^2$; in either case $\bar{\mu}$ is interpreted as the scale at which new physics damps the integral. The result is:

$$V_{eff} = V_{tree} + \frac{1}{2(4\pi)^2}\left[\eta\bar{\mu}^2 STrM^2 + \frac{1}{2}STrM^4\ln(M^2/\eta'\bar{\mu}^2)\right], \qquad (4)$$

where η and η' are constants of order unity that depend on the regularization procedure; the cut-off prescription gives $\eta = 1$, $\eta' = e^{1/4}$. In the numerical analysis we take $\eta = \eta' = 1$; our results are very insensitive to the precise values of these parameters.

The general form of the field-dependent mass matrix has been derived previously.[10] The fermion and vector boson masses are extracted from the effective tree Lagrangian after a (scalar field dependent) renormalization that transforms their kinetic energy terms to canonical form. The scalar mass matrix is given by a similar field dependent renormalization applied to the covariant second derivative[10] of the tree potential, where here covariance is meant with respect to general transformations among the scalar fields. This procedure yields an effective one–loop potential that is independent of the choice of field variables and that retains the invariance properties of the tree potential. The latter feature is not assured if the theory is truncated to an effective renormalizable one before calculation of loop corrections.

We will first study the quadratically divergent terms in (4), proportional to STrM^2. As shown previously,[10] the trace of the second covariant derivative of the potential reduces to the trace of the ordinary second derivative if the standard form[3] is used for the potential with complex scalars of well-defined chirality as field variables.

Let us first consider the effective potential[5] at a scale above the condensate scale, so that condensate effects are negligible. The effective Lagrangian is obtained using the prescription of Ref. 3 with the Kähler potential[5]

$$\begin{aligned}\mathcal{G} &= -\ln(s+s^*) + G + \ln|W(\phi)|^2 \\ G &= -3\ln(t+t^* - k|\phi|^2), |\phi|^2 = \sum \phi_i \phi^i\end{aligned} \tag{5}$$

where the complex scalar fields s and t are gauge singlets and the ϕ_i are non–singlets; k is a normalization constant. W(ϕ) is the superpotential for the non-singlet fields, at least cubic in those fields[5] and including all possible terms compatible with the symmetries.[7] The gauge and gaugino fields are remormalized by the field dependent function (in the notation of Cremmer et al.[3]):

$$f_{\alpha\beta}(Z) = \delta_{\alpha\beta}\lambda(s+s^*) \equiv \delta_{\alpha\beta} f \tag{6}$$

where λ is a constant. Equations 5 and 6 completely specify the tree Lagrangian; in particular the tree potential takes the form (dimensionful quantities are expressed throughout in Planck mass M_P units):

$$V_{tree} = \hat{V} + e^{\mathcal{G}} + \mathcal{D} \tag{7}$$

with

$$\begin{aligned}e^{\mathcal{G}} &= e^G |W|^2/(s+s^*) \\ \hat{V} &= \frac{1}{3k} e^{2G/3} \bar{W}_i W^i/(s+s^*), W^i = \frac{\partial W}{\partial \phi_i} = (\bar{W}_i)^* \\ \mathcal{D} &= \frac{g^2}{2f} D^\alpha D^\alpha, D^\alpha = 3k e^{G/3} \bar{\phi}^i T_i^{\alpha j} \phi_j\end{aligned} \tag{8}$$

where $g = \sqrt{4\pi\alpha_{GUT}}$ is the gauge coupling of the observable sector and the matrices T^α represent the generators of the gauge group. The tree potential (7) is positive definite and vanishes for $\phi = 0$. The vevs of the fields s and t are undetermined at tree level.

Using (5) and (6) in the general form[3] for the effective Lagrangian, we obtain the contributions to the supertrace:

$$\begin{aligned}TrM_V^2 &= \frac{4}{3}\mathcal{D} + 2\mathcal{K} \\ TrM_F^2 &= \frac{19}{6}\mathcal{D} + 4\mathcal{K} + \frac{14}{3}\hat{V} + \mathcal{W} + e^{\mathcal{G}}(10 + \frac{N_G}{4}) \\ TrM_S^2 &= 4(2 + \frac{N}{3})\mathcal{D} + 2\mathcal{K} + 2(5 + \frac{2N}{3})\hat{V} + 2\mathcal{W} + 2(N+6)e^{\mathcal{G}} \\ m_{\tilde{G}}^2 &= e^{\mathcal{G}}\end{aligned} \tag{9}$$

where N is the number of non-singlet complex chiral multiplets and N_G is the effective number of gaugino degrees of freedom; above the condensate scale this includes the gauginos of the hidden E_8 sector. In the limit $\kappa \to 0$ the only surviving contributions would be the dimension-two operators

$$\mathcal{K} = 3k\frac{g^2}{f} e^{G/3} \bar{\phi}^i K_i^j \phi_j, \tag{10}$$

where the matrix K = $\sum T^\alpha T^\alpha$ represents the Casimir operator of the observed gauge group, and

$$\mathcal{W} = \frac{1}{(3k)^2} \frac{e^{G/3}}{(s+s^*)} \bar{W}_{ij} W^{ij}, \; W^{ij} = \frac{\partial^2 W}{\partial\phi_i \partial\phi_j}. \tag{11}$$

The dimension-two operators cancel in the supertrace, Eq. (2):

$$STrM^2 = \frac{1}{3}(4N+17)\mathcal{D} + \frac{2}{3}(1+2N)\hat{V} + (2N - 4 - \frac{N_G}{2})e^{\mathcal{G}}, \tag{12}$$

as required by the mass-squared sum rule of rigid supersymmetry.

Here $\bar{\mu}$ represents the compactification or grand unification scale, and for $\bar{\mu} \sim M_P$, the residual contribution (12) cannot be dropped. It has however the same structure as the tree potential (7), and as long as

$$\frac{N_G}{2} + 4 - 2N < 2(4\pi)^2 \frac{M_P^2}{\eta \bar{\mu}^2} \simeq 315\, M_P^2/\bar{\mu}^2 \eta \tag{13}$$

the potential remains positive definite, so that the vacuum structure is unchanged. In this case, the quadratically divergent contribution to the one–loop corrected potential can apparently be absorbed in coupling constant renormalization and/or field rescaling; the validity of this interpretation depends, of course, on the structure of the other quadratically divergent contributions to the full one-loop corrected effective Lagrangian.

Since $e^{\mathcal{G}} \sim |\phi|^6$, while $\mathcal{D}, \hat{V} \sim |\phi|^4$, a negative coefficient for $e^{\mathcal{G}}$ would in general distabilize the potential. Stability could be restored by the logarithmic term in (4), but this would presumably generate some non singlet vevs $<\phi> \sim M_P$. If such a result is to be avoided, (13) restricts the particle content, particularly the gauginos in both the hidden and observed sectors, that may be permitted to survive at the highest scales where a description in terms of a 4–dimensional field theory becomes a good approximation. Consider for example the $E_8 \times E_8'$ superstring model of Candelas et al.[11] If E_8' remains unbroken, N_G receives a contribution equal to dim E_8' =248 from the hidden sector which consists of vector superfields in the adjoint representation of E_8'. The gauge group is a rank five or rank six subgroup of E_6 and contains at least $SU(3) \times SU(2) \times U(1) \times U(1)$.[12] Therefore N_G >261. On the other hand the matter fields belong to N_f families of 27 representations of E_6, plus some self-conjugate part of $(27 + \overline{27})$'s.[7,12] Considering the minimal case–minimal gauge group, $N_f = 3$ and no contribution from the antifamilies-we have $N_G = 261$, N = 81, in which case $\frac{N_G}{2} + 4 - 2N = -27.5 < 0$ and (13) is satisfied trivially. This is general: since $N_G <$ dim $E_8 \times E_6 = 326$ and N > 81, we have $\frac{N_G}{2}$ + 4 - 2N < 5 and (13) does not yield any useful restriction. The limit would become relevant only if gauge groups larger than 0(32) or $E_8 \times E_8$ were found.

We now turn to the effective one loop corrected potential below the scale μ of the gaugino condensate of the hidden sector, under the assumption that (13) is satisfied for $\bar{\mu} \sim M_P$. The superpotential is now modified according to[6]

$$W \to W_c = W(\phi) + h(c + e^{-3s/2b_0}) \tag{14}$$

$$\mathcal{G} \to \mathcal{G}_c = -\ln(s + s^*) + G + \ln|W_c|^2 \tag{15}$$

where b_0 determines the β-function of the strong gauge group of the hidden sector:

$$\mu_o \frac{dg_o(\mu_o)}{d\mu_o} = -b_o g_o^3. \tag{16}$$

The tree potential is modified from (7) to:

$$V^c_{tree} = \hat{V} + U + \mathcal{D} \tag{17}$$

with

$$U = e^{\mathcal{G}_c}(s + s^*) \left| \frac{\partial \mathcal{G}_c}{\partial s} \right|^2 = \frac{e^G}{s + s^*} \left| W(\phi) + hc + h(1 + \alpha) e^{-\frac{\alpha}{2}} e^{-\frac{i\beta}{2}} \right|^2 \tag{18}$$

where we have defined

$$\alpha = \frac{3Res}{b_o}, \beta = \frac{3Ims}{b_o} \tag{19}$$

and c is a constant. Each term in the potential (17) is still positive definite. $\hat{V}$ and $\mathcal{D}$ vanish for $\phi \to 0$ and U vanishes for

$$\begin{aligned} \beta = \beta_o: &\quad Re(ce^{i\beta_o/2}) = -|c|, \\ \alpha = \alpha_o: &\quad |c| = (1 + \alpha_o) e^{-\alpha_o/2} \end{aligned} \tag{20}$$

The scalar field Re t and therefore the value $m_{\tilde{G}} = e^{\mathcal{G}_c}$ of the gravitino mass remain undetermined at tree level.

As before, the squared mass matrix is obtained from the general Lagrangian of Cremmer et al.[3] using (6) and (14), (15). We find the contributions:

$$\begin{aligned} TrM_V^2 &= \frac{4}{3}\mathcal{D} + 2K \\ TrM_F^2 &= \frac{19}{6}\mathcal{D} + 4K + \frac{14}{3}\hat{V} + \mathcal{W} + U(10 + \frac{N_G}{4}) \\ &\quad - 4(U - e^{\mathcal{G}_c}) + \frac{3|h|^2}{2b_o} e^{\mathcal{G}} \alpha^3 e^{-\alpha} \\ TrM_S^2 &= 4(2 + \frac{N}{3})\mathcal{D} + 2K + 2(5 + \frac{2N}{3})\hat{V} + 2\mathcal{W} + 2(N + 6)U \\ &\quad - 2(U - e^{\mathcal{G}_c}) + \frac{3|h|^2}{b_o} e^G \alpha^3 e^{-\alpha} \\ m_{\tilde{G}}^2 &= e^{\mathcal{G}_c} \end{aligned} \tag{21}$$

where $\mathcal{K}$ and $\mathcal{W}$ are the same as above (Eqs. 10, 11). Again, the dimension–two operator cancel in the supertrace.

$$STrM^2 = \frac{1}{3}(4N+17)\mathcal{D} + \frac{1}{3}(4N+2)\hat{V} + (2N-4-\frac{N_G}{2})U + 2(U - e^{\mathcal{G}_c}) \tag{22}$$

where now N_G includes only the gauginos of the low energy observed gauge sector. The case without condensate effects is recovered in the limit $h \to 0$, for which $U \to e^{\mathcal{G}}$, $\mathcal{G}_c \to \mathcal{G}$, and Eqs. (21, 22) reduce to Eqs. (9, 12) above.

The cancellation of dimension–two operators even in the presence of induced supersymmetry breaking effects has as a consequence that the quadratically divergent one–loop corrections do not induce mass terms for the gauge non–singlet scalars. Indeed, the $\mathcal{W}$ term coming from fermion loops (in Tr M_F^2) which was thought to give rise to a mass for the scalars[8] is exactly cancelled by a similar contribution from the scalar sector (in Tr M_S^2). We will see below that the gauge non singlet scalars remain massless when the full one–loop corrections are considered.

The one loop corrections yield, however, a potential that is no longer positive definite. This can easily be seen by writing the potential (4) with STr M^2 as given by (22) in the form

$$V_{eff} = a_1\mathcal{D} + a_2\hat{V} + a_3 U + \frac{1}{2(4\pi)^2}[-\eta\mu^2 2e^{\mathcal{G}_c} + \frac{1}{2}STrM^4\ln M^2/(\mu^2\eta')], \tag{23}$$

with

$$a_i = 1 + \frac{\eta\mu^2}{2(4\pi)^2}\delta a_i, \delta a_1 = \frac{1}{3}(17+4N), \delta a_2 = \frac{1}{3}(4N+2), \delta a_3 = 2N - 2 - \frac{N_G}{2}.$$

The first three terms in (23) are positive definite and vanish for $\phi = 0$ and $S = S_o$, as determined by the tree potential [Eq. (20)]. This leaves Re t undetermined; its value is determined by the last term in (23), which is not positive definite. (Strictly speaking, the contributions to (23) that do not vanish for $h \to 0$ should be cut off at the Planck mass scale; however these terms play no role in the analysis below. Alternatively, the fields and coupling constants in (23) might be interpreted as the renormalized ones with corrections from internal momenta at scales $\mu^2 \le |p|^2 \le M_P^2$ already included.)

In general, we should minimize the corrected potential (23) with respect to all the fields. However, if the loop expansion is meaningful, s and ϕ should be shifted from their ground state values by corrections of order of the loop expansion parameter $\hbar$: $s = s_o + \hbar\varepsilon$, $\phi = \hbar\delta$. Since $\mathcal{D}$ and $\hat{V}_c$ are $O(\phi^4)$ and V is proportional to

$$\left|\frac{\delta g}{\delta s}\right|^2 = |f(s,\phi) - f(s_o, 0)|^2 = 0(\hbar^2)$$

the one loop vev of Re t is determined by the last term in Eq.(23) with ε and δ neglected.

Once we set ϕ and s at their tree level ground state values, it is easy to evaluate Eq. (24) as a function of Re t. The fermion mass-squared matrix is diagonal with non-zero elements:

$$\begin{aligned}(M_F^2)_s^s &= Ae^G\alpha^3 e^{-\alpha}\\ (M_F^2)_t^t &= Ae^G 4\alpha e^{-\alpha}\\ A &\equiv 3|h|^2/(2b_o)\end{aligned} \tag{24}$$

Note that no gaugino masses are generated at tree level. The scalar mass matrix is of the general form:

$$M_S^2 = \begin{pmatrix} \rho_i^{\ j} & \rho_{ij} \\ \rho^{ij} & \rho^i_{\ j}\end{pmatrix} \tag{25}$$

with[10] $(z = s, t, \phi_i)$:

$$\left.\begin{aligned}\rho_i^{\ j} = \rho^j_{\ i} &= (\mathcal{G}^{-1/2})_i^k \frac{\partial^2 V_{tree}}{\partial \bar{z}^k \partial z_\ell}(\mathcal{G}^{-1/2})_\ell^j\\ \rho^{ij} = (\rho_{ij})^* &= (\mathcal{G}^{-1/2})_k^i\left[\frac{\partial^2 V_{tree}}{\partial z_k \partial z_\ell} - \mathcal{G}_m^{k\ell}(\mathcal{G}^{-1})_n^m \frac{\partial V_{tree}}{\partial z_n}\right](\mathcal{G}^{-1/2})_\ell^j\\ \mathcal{G}_i^j &= \frac{\partial^2 \mathcal{G}}{\partial z_j \partial \bar{z}^i}, \mathcal{G}_m^{k\ell} = \frac{\partial^3\mathcal{G}}{\partial\bar{z}^m\partial z_k\partial z_\ell}\end{aligned}\right\} \tag{26}$$

In other words, for the "diagonal" part ρ_i^j of the squared scalar mass matrix, the covariant second derivative is the same as the ordinary second derivative, while for the "off–diagonal" parts ρ_{ij} and ρ^{ij} there is an additional term. However, since we are here evaluating M_S^2 at the tree ground state values of s and ϕ, for which V_{tree} is an absolute minimum, $\partial V_{tree}/\partial z_i = 0$, and the additional term vanishes. The only non-vanishing elements of M_S^2 are:

$$\begin{aligned} m_s^s &= Ae^G\alpha(1+\alpha^2)e^{-\alpha} \\ m_{ss} &= m^{ss} = -Ae^G 2\alpha^2 e^{-\alpha} \end{aligned} \tag{27}$$

The resulting matrix, Eq. (26), is easily diagonalized with non–zero eigenvalues

$$m_\pm^2 = (m_s^s \pm m_{ss}) = Ae^G\alpha(1\pm\alpha)^2 e^{-\alpha}. \tag{28}$$

Finally, the squared gravitino mass is

$$m_{\tilde{G}}^2 = e^{\mathcal{G}_c} = Ae^G\alpha e^{-\alpha}. \tag{29}$$

assembling these results, the one–loop potential (4) takes the form:

$$\left.\begin{aligned} V_{eff} &= \frac{1}{(4\pi)^2}\frac{A^2}{|h|^2}e^{-\alpha}v \\ v &= u^2 S + u^3[T_1\ln u + T_2] \end{aligned}\right\} \tag{30}$$

with:

$$\begin{aligned} &u = e^{2G/3} = 1/(t+t^*), \Lambda^2 = M^2A^{-1}\alpha^{-1}e^{\alpha}e^{-G} \\ &S = -2\eta e^{-\alpha/3} STr\Lambda^2, T_1 = \frac{|h|^2}{4}\alpha^2 e^{-\alpha} STr\Lambda^4 \\ &T_2 = T_1\left[2\ln\alpha + \ln(|h|^2/4) - 2\alpha/3 - \ln\eta'\right] \\ &\quad + \frac{|h|^2}{4}\alpha^2 e^{-\alpha} STr(\Lambda^4\ln\Lambda^2) \end{aligned} \tag{31}$$

and we have used the relation for the condensate scale[6]:

$$\mu^2 = \frac{6}{b_o}\alpha^{-1}e^{-\alpha/3}/\left(2Ret - k\left|\phi^2\right|\right) \tag{32}$$

at $|\phi|^2 = 0$. From the above evaluation of the non–zero elements of M^2 we obtain:

$$STr\Lambda^2 = -2 \tag{33}$$

$$STr\Lambda^4 = 2(6\alpha^2 - 1) \simeq 12\alpha^2 \text{ for } \alpha^2 >> 1 \tag{34}$$

$$\begin{aligned} STr\Lambda^4\ln\Lambda^2 &= -2\alpha^4\ln\alpha^2 + (1+\alpha)^4\ln(1+\alpha)^2 + (1-\alpha)^4\ln(1-\alpha)^2 \\ &\simeq 2\alpha^2(7 + 12\ln\alpha) \text{ for } \alpha^2 >> 1 \end{aligned} \tag{35}$$

The stability of the potential (30) requires $T_1 > 0$ and consequently STr $\Lambda^4 > 0$, or from (34):

$$\alpha^2 > 1/6 \tag{36}$$

This in turn places a restriction on the gauge coupling constant of the observable sector[6]

$$\alpha_{GUT} = \frac{1}{4\pi Res} = \frac{3}{4\pi b_o\alpha}. \tag{37}$$

The restriction (36) corresponds to $\alpha_{GUT} \leq 1$ if the string gauge group is E_8 and is weaker for smaller groups; we assume (36) is satisfied and obtain a relation for the vev of Ret:

$$\frac{\partial v}{\partial u} = 0 = 2S + u\left[3T_1\ln u + 3T_2 + T_1\right]. \tag{38}$$

Equation (38) is solved by:

$$x = u\exp\left(\frac{3T_2 + T_1}{3T_1}\right), x\ln x = \frac{-2S}{3T_1}\exp\left(\frac{3T_2+T_1}{3T_1}\right). \tag{39}$$

Using the large α^2 approximation (which will be justified a posteriori) for the supertraces, Eqs. (34) and (35), the right hand side of Eq. (39) becomes a pure number:

$$\left.\begin{aligned} x\ln x &= \frac{\eta}{\eta'}\frac{2}{9}e^{3/2} = \eta/\eta' 1.00 \\ x &= 1.76 \text{ for } \eta = \eta' = 1 \end{aligned}\right\} \tag{40}$$

This in turn determines the GUT mass scale:

$$M_{GUT} = \left\langle \frac{1}{\sqrt{ResRet}} \right\rangle_{vac} = \left(\frac{6\sqrt{u}}{\alpha b_o} \right)^{1/2}$$
$$= \frac{2}{3}(\eta' x)^{1/4} e^{-3/8} \frac{b_o}{\sqrt{|h|}} (4\pi \alpha_{GUT})^{3/2} \exp\left(\frac{1}{8\pi b_o \alpha_{GUT}} \right) \quad (41)$$

(where we used Eq. (37) to eliminate α), the gravitino mass:

$$m_{\tilde{G}} = e^{\mathcal{G}_c/2} = \frac{2}{9}(\eta' x)^{3/4} e^{-9/8} \frac{b_o^2}{\sqrt{|h|}} (4\pi\alpha_{GUT})^{5/2} \quad (42)$$

and the condensate scale:

$$\mu = M_{GUT} e^{\frac{-1}{8\pi b_o \alpha_{GUT}}} = \frac{2}{3}(\eta' x)^{1/4} e^{-3/8} \frac{b_o}{\sqrt{|h|}} (4\pi\alpha_{GUT})^{3/2}. \quad (43)$$

The value of M_{GUT}, Eq.(40) is smallest for $\alpha_{GUT} = (12\pi b_o)^{-1}$; this means

$$M_{GUT} \geq \frac{2}{9} \frac{e^{9/8}}{\sqrt{3b_o |h|}} (\eta' x)^{1/4} \quad (44)$$

If we impose $M_{GUT} \leq M_P$ we find $b_o \geq 0.21$ for $h \sim \eta' \sim 1$. For example $SU(3)$ with $b_o \simeq$0.06 would be too small a group unless h >3.6. Adopting the E_8 value $b_o = 0.57$, we find that $M_{GUT} \leq M_P$ for values of α_{GUT} in the range $1/44 \leq \alpha_{GUT} \leq 1/8 (18.5 \leq \alpha \leq 3.5)$, with a minimum value $M_{GUT} \geq 0.6 M_P$ for $\alpha \simeq 1/21.5$. This gives ($\eta' \sim h \sim 1$):

$$1.5 \times 10^{-3} \leq m_{\tilde{G}}/M_P \leq 0.1$$
$$0.05 \leq \mu/M_P \leq 0.55.$$

There is a degree of ambiguity in our analysis in that we included the scalar field dependence of the condensate scale, Eq. (32), in the minimization of the effective potential (4). If instead we treat μ^2 as a constant, and for consistency set

$$\mu^2 = < \frac{6}{b_o} \alpha^{-1} e^{-\alpha/3} u^{1/2} >_{vac} . \quad (45)$$

after minimization, the equation (38) that determines $< u >$ is modified to

$$0 = S + u[2T_1 \ln u + 2T_2 + T_1]. \quad (46)$$

This modification has an imperceptible effect on the above numerical analysis. We also remark that as the cut–off parameter $\bar{\mu}$ appearing in Eq. (4) is physically meaningful, evaluating (7) by dimensional regularization would not be a correct procedure. However, had we done so, we would have obtained instead of Eq. (40):

$$x \ln x = 0, x = 1,$$

which again has only a slight effect on the numerical analysis.

Finally, we look for radiatively induced masses for the non–singlet scalars. These are determined by the terms quadratic in ϕ in the expression (4) for the effective potential and we can obtain them by expanding (4) near $\phi = 0$. Expanding the squared mass matrix M^2 about its value M_0^2 at the ground state:

$$M^2(Z) = M_0^2 + \Delta$$

it follows from the identity:

$$TrF(M^2) = TrF(M_0^2) + Tr[\Delta F'(M_0^2)] + 0(\Delta^2),$$

for any matrix–valued function F of M^2, that only those elements M^2_{ij} of M^2 that are non vanishing at the ground state, $(M_0^2)_{ij} \neq 0$, need be retained in the evaluation of the logarithmic term in the effective potential, Eq. (4). Then one can check that, to order $|\phi|^4$, the ϕ– dependence of the effective potential (4) is only through the combinations [using either of the prescriptions (32) or (45)]:

$$c \to c + W(\phi)/h = c + 0(\phi^3)$$
$$\frac{1}{t+t^*} \to \frac{1}{t+t^* - k|\phi|^2} = u^{1/2} + uk|\phi|^2 + 0(\phi^4) \quad (47)$$

so that:

$$V_{eff}(Z) = V_{eff}(Z_0) + k|\phi|^2 y^2 \frac{\partial}{\partial y} V_{eff}(Z_0) + 0(\phi^3) \quad (48)$$

Since by construction $\frac{\partial}{\partial y} V_{eff}(Z_0) = 0$, no terms quadratic in the non–singlet fields ϕ are generated by the one loop corrections.

A possible caveat is the observation that a non vanishing, negative cosmological constant appears at the one loop level (independently of the prescription used).

Depending on the scalar field dependence of whatever term(s) we have missed that serves to cancel the cosmological constant, our results could be modified. It has recently been argued[13] that such a cancellation is assured by a residual unbroken invariance under a non-compact transformation on the scalar fields. However, the symmetries of the tree Lagrangian should also insure that the $|\phi|$ dependence of the resulting effective potential arises only through the substitutions (47), so that no scalar masses should result. We further see no reason why the orders of magnitude obtained above for the scales $M_{GUT}, m_{\tilde{G}}$ and μ should be significantly altered. On the other hand, in order that superstring models lead to a successful phenomenology at low energies, it is important to have a source for scalar masses[7,8,14]. They will therefore be viable only if radiative corrections at higher orders or higher modes of the string provide such a source. In any case, in order to discuss the phenomenology of these models, one should set the common mass scale m of the gauge non-singlet scalars at grand unification as a free parameter.[14] Its order of magnitude is presumably smaller than the one that one- loop radiative corrections would have given, that is (using Eq. (4, 11, 12))

$$m << \mu \left(\frac{e^{G/3}}{s+\bar{s}} \right)^{1/2} \tag{49}$$

or, with (41)

$$m << \mu \frac{M_{GUT}}{M_P}. \tag{50}$$

In conclusion, we have shown that no scalar masses are generated by one loop radiative corrections in the presence of a gaugino condensate that serves to break supersymmetry in the ground state. On the other hand, the vacuum degeneracy of the tree potential is removed, and the scales M_{GUT}, μ and $m_{\tilde{G}}$ that govern the scales of grand unification, condensation and supersymmetry breaking respectively, are found to be close to Planck scale. The condition $M_{GUT} \leq M_P$ implies that the hidden E_8 cannot be broken to arbitrarily small group and restricts the value of α_{GUT}. A more detailed discussion of our results will be given elsewhere.[15]

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